# Rapid Gravity Computations for Two-Dimensional Bodies with Application to the Mendocino Submarine Fracture Zone* 

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#### Abstract

Expressions are derived for the vertical and horizontal components of the gravitational attraction due to a two-dimensional body of arbitrary shape by approximating it to an $n$-sided polygon. These expressions are put in forms suitable for solution by a high-speed digital computer. As an example of the application of this method, the crustal section across the Mendocino fracture zone is deduced from the gravity anomalies. Assuming the crust to consist of a single homogeneous layer, overlain by water and sediment, it is found to be about three km thicker to the north of the fracture zone than to the south of it.


Many geological structures are approximately linear, and the problems connected with them can be solved with two-dimensional forms of analysis. For gravity computations, Nettleton [1940] has determined the criteria for making an adequate two-dimensional computation. Various methods exist for the computation of the gravitational attraction caused by irregularly shaped two-dimensional bodies. These methods can be divided into two categories. In the first category lie those that involve the use of graticules, dot charts, or other such graphical computing aids. While in theory these methods can be made as precise as one pleases, merely by increasing the scale to which the graticule is constructed, in actual practice this may be difficult, if not impossible.

In the second category lie those methods that involve breaking up the irregularly shaped bodies into several smaller bodies of different sizes but of shapes that are regular and for which the gravitational attraction can be easily computed. A convenient form of regular body to use is the rectangular block, as proposed by Vening Meinesz and others [1934]. Here again the method can be made as precise as one pleases by using a sufficiently large number of small blocks [see, for example, Shurbet and others, 1956]. However, the computations become increasingly more tedious as the number of blocks is increased. Further, the blocks may be so small that their

[^0]individual contributions at distant points are neglected even though the total sum of their small contributions is appreciable. This may cause considerable error in computations.

The periphery of any two-dimensional body can be approximated closely by a polygon, by making the number of sides of this polygon sufficiently large. Analytical expressions can be obtained for both the vertical and horizontal components of the gravitational attraction due to this polygon at any given point. These expressions, then, can be used without any limitations to the size or position of the body. The present method involves the use of these expressions. The accuracy depends only on how closely the polygon fits the given body, and can be increased by increasing the number of sides of the polygon. It will be recognized that an irregularly shaped two-dimensional body can be more easily approximated by a polygon than by rectangular blocks. The computations involved in solving the expressions to obtain the components of gravitational attraction are lengthy and tedious, but being iterative are readily programmed for solution by a digital computer. A program for use with the IBM 650 has been made, and the machine time required for obtaining both the vertical and horizontal components of gravitational attraction for an $n$-sided polygon at a single point is approximately equal to $2.5 n$ seconds.
${ }^{\circ}$
Let $A B C D E F$ (Fig. 1) be a given polygon with $n$ sides and let $P$ be the point at which the


Fig. 1-Geometrical elements involved in the gravitational attraction of an $n$-sided polygon
attraction due to this polygon has to be determined. Imagine $P$ to be the origin of an $x z$ system of coordinates, where the polygon also lies in the $x z$ plane. Let $z$ be defined positive downwards (vertical) and let $\theta$ be measured from the positive $x$ axis towards the positive $z$ axis.

It has been shown by Hubbert [1948] that the vertical component of gravitational attraction due to such a two-dimensional body is, at the origin, equal to

$$
2 G \rho \oint z d \theta
$$

the line integral being taken along its periphery, where $G$ is the universal constant of gravitation and $\rho$ is the volume density of the body. It can be shown by a method similar to Hubbert's that the corresponding expression for the horizontal component of gravitational attraction is given by $2 G \rho \oint x d \theta$.

Let us evaluate the two integrals $\mathscr{\mathscr { L }} \mathrm{z} d \theta$ and $\oint x d \theta$ for the above polygon. The contribution to $\mathscr{\infty} z d \theta$ from, say, the side $B C$ of the polygon can be first computed. Produce $C B$ to meet the $x$ axis at $Q$ at an angle $\phi_{i}$. Let $P Q=a_{i}$. Now

$$
\begin{equation*}
z=x \tan \theta \tag{1}
\end{equation*}
$$

for any arbitrary point $R$ on $B C$. Also

$$
\begin{equation*}
z=\left(x-a_{i}\right) \tan \phi_{i} \tag{2}
\end{equation*}
$$

From (1) and (2)

$$
z=\frac{a_{i} \tan \theta \tan \phi_{i}}{\tan \phi_{i}-\tan \theta},
$$

or

$$
\int_{B C} z d \theta=\int_{B}^{C} \frac{a_{i} \tan \theta \tan \phi_{i}}{\tan \phi_{i}-\tan \theta} d \theta \equiv Z_{i} .
$$

Similarly it can be shown that

$$
\int_{B C} x d \theta=\int_{B}^{C} \frac{a_{i} \tan \phi_{i}}{\tan \phi_{i}-\tan \theta} d \theta \equiv X_{i}
$$

The vertical component of gravitational attraction $V$ and the horizontal component $H$, due to the whole polygon, are then given respectively by

$$
V=2 G \rho \sum_{i=1}^{n} Z_{i}
$$

and

$$
H=2 G \rho \sum_{i=1}^{n} X_{i}
$$

the summations being made over the $n$ sides of the polygon.

It now remains to solve the integrals involved in the expressions for $Z_{i}$ and $X_{i}$.

In the most general case it can be shown that

$$
\begin{aligned}
& Z_{i}=a_{i} \sin \phi_{i} \cos \phi_{i}\left[\theta_{i}-\theta_{i+1}\right. \\
& \left.\quad+\tan \phi_{i} \log _{\bullet} \frac{\cos \theta_{i}\left(\tan \theta_{i}-\tan \phi_{i}\right)}{\cos \theta_{i+1}\left(\tan \theta_{i+1}-\tan \phi_{i}\right)}\right] \\
& \\
& \quad \begin{aligned}
X_{i} & =a_{i} \sin \phi_{i} \cos \phi_{i}\left[\tan \phi_{i}\left(\theta_{i+1}-\theta_{i}\right)\right. \\
& \left.\quad+\log _{e} \frac{\cos \theta_{i}\left(\tan \theta_{i}-\tan \phi_{i}\right)}{\cos \theta_{i+1}\left(\tan \theta_{i+1}-\tan \phi_{i}\right)}\right]
\end{aligned}
\end{aligned}
$$

where

$$
\begin{aligned}
\theta_{i} & =\tan ^{-1} \frac{z_{i}}{x_{i}} \\
\phi_{i} & =\tan ^{-1} \frac{z_{i+1}-z_{i}}{x_{i+1}-x_{i}}, \\
\theta_{i+1} & =\tan ^{-1} \frac{z_{i+1}}{x_{i+1}},
\end{aligned}
$$

and

$$
a_{i}=x_{i+1}+z_{i+1} \frac{x_{i+1}-x_{i}}{z_{i}-z_{i+1}} .
$$

Also see Figure 1.
The expressions for $Z_{i}$ and $X_{i}$ reduce to simpler expressions in the following cases:

$$
\begin{aligned}
& \text { Case A-If } x_{i}=0 \\
& Z_{i}=-a_{i} \sin \phi_{i} \cos \phi_{i}\left[\theta_{i+1}-\frac{\pi}{2}\right. \\
& \left.+\tan \phi_{i} \log _{e}\left\{\cos \theta_{i+1}\left(\tan \theta_{i+1}-\tan \phi_{i}\right)\right\}\right] \\
& X_{i}=a_{i} \sin \phi_{i} \cos \phi_{i}\left[\tan \phi_{i}\left(\theta_{i+1}-\frac{\pi}{2}\right)\right. \\
& \left.-\log _{e}\left\{\cos \theta_{i+1}\left(\tan \theta_{\imath+1}-\tan \phi_{i}\right)\right\}\right] . \\
& \text { Case B-If } x_{1+1}=\mathbf{0} \\
& Z_{i}=a_{i} \sin \phi_{i} \cos \phi_{i}\left[\theta_{i}-\frac{\pi}{2}\right. \\
& \left.+\tan \phi_{i} \log _{e}\left\{\cos \theta_{i}\left(\tan \theta_{i}-\tan \phi_{i}\right)\right\}\right] \\
& X_{i}=-a_{i} \sin \phi_{2} \cos \phi_{2}\left[\tan \phi_{2}\left(\theta_{2}-\frac{\pi}{2}\right)\right. \\
& \left.-\log _{\sigma}\left\{\cos \theta_{i}\left(\tan \theta_{i}-\tan \phi_{i}\right)\right\}\right] .
\end{aligned}
$$

Case C-If $\boldsymbol{z}_{\mathbf{i}}=\boldsymbol{z}_{\mathbf{t}+1}$

$$
\begin{aligned}
Z_{i} & =z_{i}\left(\theta_{i+1}-\theta_{i}\right) \\
X_{i} & =z_{i} \log _{a} \frac{\sin \theta_{i+1}}{\sin \theta_{i}}
\end{aligned}
$$

Case D-If $x_{i}=x_{i+1}$

$$
\begin{aligned}
Z_{i} & =x_{i} \log _{e} \frac{\cos \theta_{i}}{\cos \theta_{i+1}} \\
X_{i} & =x_{i}\left(\theta_{\imath+1}-\theta_{\imath}\right)
\end{aligned}
$$

Case $E$-If $\boldsymbol{\theta}_{\boldsymbol{i}}=\boldsymbol{\theta}_{\boldsymbol{i}+1}$

$$
\begin{aligned}
Z_{i} & =0 \\
X_{i} & =0 .
\end{aligned}
$$

Case $F$-If $x_{i}=z_{i}=0$

$$
\begin{array}{r}
Z_{i}=0 \\
X_{i}=0 . \\
\text { Case G—If } x_{i+1}=z_{i+1}=0 \\
Z_{\imath}=0 \\
X_{i}=0 .
\end{array}
$$

Noting that $\theta_{i}, \theta_{i+1}, \phi_{i}$, and $a_{i}$ can all be explicitly expressed in terms of the $x_{i}$ 's and the
$z_{i}$ 's, we are able to obtain expressions for both $V$ and $H$ solely in terms of the $x_{i}$ 's and the $z_{i}$ 's. This is especially advantageous, since one of the simplest ways of defining the periphery of a body is to specify the coordinates of adjacent points at the vertices of the body. These are the coordinates $x_{i}$ 's and $z_{i}$ 's used in the computation. In addition, of course, it is necessary to specify the density of the body and the position of the points at which the attraction is to be calculated.

This method is illustrated by the following example in which the crustal structure along a profile in the Pacific crossing the Mendocino fracture zone has been deduced.

The Mendocino fracture zone has been described by Menard and Dietz [1952] and by Menard [1955]. The map in Figure 2 shows the position of the Mendocino escarpment and the submarine gravity stations in the area made on USS Redfish in 1952 [Worzel and others, 1955] and on the USS Rasher and USS Raton in 1954 [Harrison and others, 1957]. The gravity values at the Redfish stations and at those of the Harrison stations that were used in this study are listed in Table 1.

Table 1-Gravity stations ${ }^{\text {a }}$

| Cruise | Sta. | $\begin{aligned} & \text { Latitude } \\ & \mathbf{N} \end{aligned}$ |  | $\underset{W}{\text { Longitude }}$ |  | Corrected sounding | Free-air anomaly |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | - | , | - | , | m | mgal |
| Redfish | 12 | 38 | 15.0 | 126 | 31.0 | 2503 | -29 |
| (first | 13 | 38 | 38.8 | 127 | 11.3 | 4592 | -44 |
| cruise) | 14 | 39 | 02.4 | 127 | 53.4 | 4396 | -17 |
|  | 15 | 39 | 53.9 | 129 | 28.4 | 4611 | -46 |
|  | 16 | 40 | 18.4 | 130 | 16.1 | 3635 | -38 |
|  | 17 | 40 | 42.3 | 131 | 02.1 | 3767 | -12 |
|  | 18 | 41 | 04.7 | 131 | 44.5 | 3754 | -15 |
|  | 19 | 41 | 30.8 | 132 | 35.3 | 3862 | -19 |
|  | 20 | 42 | 14.6 | 134 | 02.0 | 3887 | -22 |
|  | 21 | 42 | 36.9 | 134 | 47.7 | 3862 | -19 |
|  | 146 | 41 | 40 | 128 | 58 | $3352{ }^{\text {b }}$ | +1 |
|  | 147 | 40 | 44 | 128 | 14 | $3541{ }^{\text {c }}$ | +11 |
|  | 148 | 39 | 46 | 127 | 42 | $4551{ }^{\text {c }}$ | -44 |
|  | 149 | 38 | 58 | 127 | 16 | $4398{ }^{\text {c }}$ | $-19$ |
| Harrison | 61 | 40 | 45.7 | 128 | 57.2 | 3330 | +13 |
| and | 62 | 40 | 34.7 | 128 | 57.2 | 3319 | +19 |
| others | 63 | 40 | 26.0 | 129 | 02.7 | 3319 | + 4 |
| [1957] | 64 | 40 | 12.7 | 129 | 01.3 | 4638 | -79 |
|  | 65 | 40 | 01.8 | 129 | 04.5 | 4738 | $-63$ |
|  | 66 | 39 | 49.2 | 128 | 58.8 | 4738 | -45 |
|  | 67 | 39 | 31.6 | 128 | 58.8 | 4484 | -17 |

[^1]

Fig. 2-Position of Mendocino and Gorda escarpments (modified from Menard and Dietz [1952];) gravity stations within the box are used in this study

While there are three $N-S$ Harrison profiles in this area, only the westernmost one (Stations 61 to 67 and 85 to 90 ) can be considered a typical gravity profile across the Mendocino escarpment. Of the remaining two profiles, one crosses the Mendocino escarpment at its extreme eastern edge, and the other crosses the Gorda escarpment rather than the Mendocino escarpment (see Fig. 2). Even the first profile, north of the escarpment, soon passes into the "Ridge and Trough Province" [Menard and Dietz, 1951], a province of very rough topography. The situation is complicated by the fact that while the topography in this area seems to trend in an approximately SW-NE direction [Menard and Dietz, 1952], the tectonic trends as described by Heezen (personal communication) run in a NW-SE direction. A deduction of structure based on a two-dimensional gravity analysis, using gravity values obtained in such an area, would be inadequate.

However, some distance to the west of the Harrison profile, the main Redfish profile (Sta-
tions 12 to 21) lies north of the escarpment in comparatively smooth topography. But it runs in a NW-SE direction, crossing the $E-W$ trending escarpment at an acute angle. This, again, is not very suitable for a two-dimensional gravity analysis. A compromise was effected by 'projecting' the Redfish gravity profile on to a $N-S$ line $\left(130^{\circ} \mathrm{W}\right)$. The topographic profile along this line was also made up from Redfish soundings. The minimum depth at the top of the escarpment is $3100 \mathrm{~m}(1700 \mathrm{fm})$, and the maximum depth at the foot of the escarpment is $4900 \mathrm{~m}(2700 \mathrm{fm})$. The average slope of the escarpment is about $6^{\circ}$. These figures are in reasonable agreement with those given by Menard and Dietz [1952] for their profiles along the latitudes ( $128^{\circ} 12^{\prime} \mathrm{W}$ and $\left.131^{\circ} \mathrm{W}\right)$. The Harrison stations and the Redfish stations from the profile that contains Stations 143 to 149 were also projected on to the same profile. However, the Harrison Stations 61 and 62 and the Redfish Stations 146 and 147 are at considerably different depths than those corresponding to their respective positions along the
profile. Therefore it was decided not to use these stations in making up the composite profile. They are discussed separately.

The composite profile was terminated on the north at $42^{\circ} \mathrm{N}$. The main Redfish profile north of here diverges considerably from the line of projection, and the Harrison profile, as mentioned earlier, enters the very rough Ridge and Trough Province. Stations east of $127^{\circ} \mathrm{W}$ were also not used in the composite profile. It was felt that the proximity of the continental edge significantly affected the gravity values at these stations.

In the following calculations, the crust is assumed to consist of the water layer underlain by a single homogeneous layer of density 2.84 $\mathrm{gm} / \mathrm{cc}$, but of variable thickness. Beneath the crust, the mantle is assumed to have a density of $3.27 \mathrm{gm} / \mathrm{cc}$. The crust is taken as the layer bounded at the bottom by the Mohorovicic discontinuity. The problem, then, is to compute the attraction of the water layer and subtract this from the observed anomalies. The differences can be treated as 'residual anomalies' and can be ascribed to variations in the thickness of the crust.

Figure 3a shows the observed free-air anomalies; Figure 3b shows the topography. The first step is to compute the attraction due to the water layer $a b c d$. To do this, one has to specify the coordinates of points which, when joined together consecutively, will define the boundary of this

Table 2-List of coordinates used in the deduction of the gravitational effect of the water layer

| Distance |  | Depth |  |
| :--- | :--- | :--- | :--- |
|  | km |  | km |
| $x_{1}$ | $-\infty$ | $z_{1}$ | 0.00 |
| $x_{2}$ | $+\infty$ | $z_{2}$ | 0.00 |
| $x_{3}$ | $+\infty$ | $z_{3}$ | 4.40 |
| $x_{4}$ | 330 | $z_{4}$ | 4.40 |
| $x_{8}$ | 274 | $z_{5}$ | 4.48 |
| $x_{8}$ | 219 | $z_{6}$ | 4.74 |
| $x_{7}$ | 202 | $z_{7}$ | 4.90 |
| $x_{8}$ | 198 | $z_{8}$ | 4.64 |
| $x_{9}$ | 189 | $z_{8}$ | 3.64 |
| $x_{10}$ | 184 | $z_{10}$ | 3.10 |
| $x_{11}$ | 174 | $z_{11}$ | 3.32 |
| $x_{12}$ | 145 | $z_{12}$ | 3.77 |
| $x_{12}$ | 102 | $z_{13}$ | 3.75 |
| $x_{14}$ | 54 | $z_{14}$ | 3.86 |
| $x_{15}$ | 0 | $z_{15}$ | 3.80 |
| $x_{16}$ | $-\infty$ | $z_{16}$ | 3.80 |
| $x_{17}$ | $-\infty$ | $z_{17}$ | 0.00 |
|  |  |  |  |

layer. These are listed in Table 2. It should be noticed that in this particular instance the coordinates of most of the points are determined by the actual corrected soundings and the positions at which these soundings were made.

These data are punched on cards and fed into the IBM 650. It remains to assign a density to this layer. It is found convenient to subtract the densities of all the layers from a constant density of $2.84 \mathrm{gm} / \mathrm{cc}$. (This reduces the density of the crustal layer to zero, and one does not, then, have to make any calculations for it.) Assuming an actual density of $1.03 \mathrm{gm} / \mathrm{cc}$ for sea water, the reduced density assigned to the layer $a b c d$ is $2.84-1.03=1.81 \mathrm{gm} / \mathrm{cc}$.

This value and the coordinates of the points at which the attraction is to be computed are also punched on cards. These latter are points taken at small intervals along the length of the profile. The computed curve is plotted in Figure 3c. This is the correction curve for the water layer. The residual anomalies plotted in Figure 3d are obtained by adding the correction curve to the observed free-air anomalies. Figure 3d may now be used to obtain the variations in crustal thickness.

To the north of the section it is assumed that the water depth is constant at 3.8 km and that the free-air anomalies are constant at -20 mgal . This would require a depth to the Mohorovicic discontinuity of 17.0 km , to be in accord with the standard section of Worzel and Shurbet [1955]. Using this as a fixed level, an approximate estimate of thickness is made along the entire section by the $\sin x / x$ method developed by Tsuboi and Fuchida [1938] and Tomoda and $A k i$ [1955]. (In the $\sin x / x$ method the gravity anomalies chosen along a profile at constant intervals are directly attributed to a mass distribution at a fixed depth. The mass distribution is interpreted as being due to the undulations of a surface, which in this case is the Mohorovicic discontinuity). The IBM 650 is used for this calculation also. The estimated depths to the Mohorovicic discontinuity as determined by Tsuboi's method are shown as the dashed curve in Figure 3e.

The gravity effect of this estimated crustal thickness was evaluated by the polygon method and compared with the residual anomalies. It was found that there was some disagreement. The depths along the discontinuity were then modified


Fig. 3-Various steps involved in the deduction of the depth to the Mohorovicic discontinuity from the free-air anomalies


Fig. 4-Free-air anomalies and the deduced crustal atructure (a) under the Mendocino fracture zone (with no sedimentary layer)
in such a way as to reduce the disagreement. Alterations of crustal thickness were continued until the computed curve fitted the residual anomalies as well as was desired. The final modified crustal thickness is shown as the solid curve in Figure 3e. The computed residual anomaly curve corresponding to this is shown in Figure 3d and is seen to fit most of the points fairly well. (No attempt was made to fit the gravity value at Redfish Station 13, because the unusually large negative anomaly here seems to be of local origin.) It is emphasized here that while Tsuboi's method is very useful in making an approximate estimate of the crustal thickness, it is by no means essential to the solution of the problem. It is quite feasible to guess at an approximate crustal thickness and proceed from there.
Figure 4 shows the final deduced structure (a) and the computed free-air anomaly curve. The
latter is obtained by subtracting the water layer correction curve from the final curve computed to fit the residual anomaly.

The Harrison Stations 61 and 62 and the Redfish Stations 146 and 147, which were not used in the profile, can be considered next. The sounding for the Harrison Station 61 is 3330 m , and the free-air anomaly is +13 mgal . The position of this station along the projected profile would be at 137 km south of $42^{\circ} \mathrm{N}$. Here the depth read off from the topographic profile is 3750 m , which gives a difference of 420 m . In order to make a rough allowance for this depth difference, we can imagine a uniform plate of water 420 m thick and of infinite areal extent. The gravity effect of this layer (computed in the same way as the simple Bouguer correction), using a density defect of $1.81 \mathrm{gm} / \mathrm{cc}$, is 32 mgal . This has to be subtracted from the value of the
correction due to the water layer at 137 km . The latter can be read in Figure 3c as 286 mgal. Then the approximate residual anomaly at the Harrison Station 61 is $13+286-32=267$ mgal. (The same number can be obtained from the value given for the Bouguer anomaly by Harrison and others [1957] at Station 61 after allowing for the fact that they have used a different crustal density in making their computations.) This number differs by only -5 mgal from the value of the computed curve at 137 km . Similarly, the differences for the Harrison Station 62 and the Redfish Stations 146 and 147 are $+1,-13$, and -8 mgal respectively. The comparatively large difference for Station 146 can be attributed to an actual difference in structure, which is expected since the station is at such a great distance from the main Redfish line. This also points to the infeasibility of using both the Redfish and the Harrison stations further north in a single composite profile.

If additional data were available about further layers (as for example a sedimentary layer), the gravity effect of such layers could be 'removed' in the same way as the water layer was removed in the above example. It should be noted that the presence of such layers could alter the deduced section considerably. For instance, a $1-\mathrm{km}$ thickness of sediment of density $2.1 \mathrm{gm} / \mathrm{cc}$ would require the crust to be thinner by about 1.7 km in order to reconcile the computed gravity curve with the observed points.

While, in the absence of any seismic determinations of depth and velocity in this region, it was not possible to introduce any complex modifications to the deduced structure, it was felt that an assumption of a sedimentary layer would make the section geologically more probable. Accordingly, the computations were re-done after a sedimentary layer of density 2.1 $\mathrm{gm} / \mathrm{cc}$ was added to the section at the base of the water layer. This was an average value of density, chosen on the basis of curves given by Nafe and Drake [1957] which relate the compressional velocity and density of ocean sediments. (A value of $2.15 \mathrm{~km} / \mathrm{sec}$ for the compressional velocity was assumed after Raitt [1956].) The thickness of this sedimentary layer was assumed to be $\frac{1}{2} \mathrm{~km}$ over areas of small relief, somewhat greater than $\frac{1}{2} \mathrm{~km}$ at the foot of the scarp, and negligible on the scarp itself. The depths to the Mohorovicic discontinuity were obtained in the same way as
in the previous case, the only difference being that the sedimentary layer as well as the water layer had to be removed. Figure 5 shows the deduced crustal section (b) with the assumed sedimentary layer taken into account.

It can be seen that in this case, as in the case without the sedimentary layer, the crust is thicker to the north of the scarp than to the south of it by about three km . The absolute thickness of the crust may be subject to some uncertainty because the values chosen for the densities and the layer thicknesses are based on a standard oceanic section rather than on any seismic determinations in the area. It is of some interest, therefore, to compare the crustal thickness obtained here with those obtained at the closest seismic refraction stations, even though these are at a considerable distance from the gravity profile. The only published determinations available to the authors were Raitt's Stations $M_{1}$ and $M_{2}$ [Raitt, 1956]. The free-air anomalies in the vicinity of both these stations are of the order of -20 mgal (unpublished Lamont Observatory data). Thus a direct comparison with either end of the gravity profile is in order. At Station $M_{1}\left(27^{\circ} 24^{\prime} \mathrm{N}, 121^{\circ} 35^{\prime} \mathrm{W}\right)$, which lies in Menard and Dietz's Baja California sea mount province, Raitt finds the following structure beneath the water layer: A first layer of thickness 0.26 km and of assumed seismic velocity $2.15 \mathrm{~km} / \mathrm{sec}$, a second layer of thickness 0.93 km and velocity $5.88 \mathrm{~km} / \mathrm{sec}$, and a third layer of thickness 6.24 km and velocity 6.96 $\mathrm{km} / \mathrm{sec}$. Below the Mohorovicic discontinuity the velocity is $8.41 \mathrm{~km} / \mathrm{sec}$. Assuming densities of $2.1,2.6,2.84 \mathrm{gm} / \mathrm{cc}$ for the three layers respectively, and of $3.27 \mathrm{gm} / \mathrm{cc}$ for the material below the Mohorovicic discontinuity, and noting that the water depth here is 4.18 km , one can consider the isostatic balance of this section against the section at the south end of the gravity profile. There the water depth is 4.40 km , and a simple calculation shows that a balance with the seismically determined section at $M_{1}$ requires a depth to the Mohorovicic discontinuity of $\mathbf{1 1 . 6 5}$ $\mathrm{km}_{\text {, considering a single crustal layer of density }}$ $2.84 \mathrm{gm} / \mathrm{cc}$. If allowance is made for an additional sedimentary layer of density $2.1 \mathrm{gm} / \mathrm{cc}$ and thickness $\frac{1}{2} \mathrm{~km}$, the depth to the discontinuity is reduced to 10.8 km . A discrepancy is apparent between these crustal thicknesses and those deduced in the gravity sections (a) and (b), the


Fig. 5-Free-air anomalies and the deduced crustal structure (b) under the Mendocino fracture zone (with assumed sedimentary layer)
latter giving depths of 15.5 km and 14.65 km respectively. This discrepancy has also been pointed out by Menard [1955]. (His choice of densities and the different water depths in the areas for which he has made the comparison leads him to an even greater discrepancy.) However, if the comparison is made with Raitt's Station $M_{2}$ rather than $M_{1}$, the situation is somewhat improved. Proceeding as before, it can be shown that the section at $M_{2}$ is 'equivalent' to a section at the south end of the gravity profile having a total crustal thickness of 13.6 km in the absence of a sedimentary layer, and of 12.8 km in the presence of one.

The south end of the gravity profile lies in the abyssal plain. (Menard terms this the "Deep Plains Province.") Since the crustal thickness as deduced here would be in even better agreement with Raitt's seismic determinations at the deep
ocean stations, this might suggest that it is not the deep structure under the abyssal plain that is unusual, as has been proposed by Menard, but that it is the deep structure under the Baja California sea mount province that might be considered so, especially as one approaches the continental edge. More seismic work in both areas is needed before this point can be definitely settled.

The crustal thickness determination under the Mendocino scarp itself is also of interest. We note that, as determined from the gravity computations, the Mohorovicic discontinuity here seems to have a very steep slope just south of the scarp. Some of this may possibly be a spurious effect arising out of the attempt to explain a gravity anomaly of shallow origin by deeper structure. If a volcanic layer of density $2.60 \mathrm{gm} / \mathrm{cc}$ and a prismatic shape should be
interposed in the section (with the narrow edge of the prism outcropping at the scarp and the base about 100 km wide extending down to the Mohorovicic discontinuity), this would obviate the necessity of thickening the crust under the escarpment, and would reduce the southern slope of the discontinuity. In any event, it is clear that a mass deficiency exists under the Mendocino escarpment, and it seems probable that the depth at which this deficiency exists is not greater than the depth to the Mohorovicic discontinuity. This should be taken into consideration in any theories explaining the origin of the Mendocino escarpment. Also, it should be noted that the mass deficiency or 'root' is somewhat displaced from the theoretical 'root' which can be computed from the topography on the Airy hypothesis. Undoubtedly, this is the cause of the large isostatic anomalies computed for Stations 64 and 65 by Harrison and others [1957].

So far we have assumed that the anomalies across the Mendocino escarpment have their origin in the crust. It would be of some interest to see if variations of density in the mantle could account for these anomalies equally well.

At the two ends of the profile there is a difference of 44 mgal in the residual anomaly (see

Fig. 3). Let us assume that a constant change of density in the mantle occurs across a vertical interface directly below the Mendocino scarp and extends from a depth of 20 km to 200 km . A simple calculation can then be made to show that a density difference of $0.00583 \mathrm{gm} / \mathrm{cc}$ is required to account for the difference of residual anomaly at large distances on either side. Using this density contrast, the gravity curve was computed for the mantle configuration outlined above (Fig. 6). We notice that it fails to fit the anomalies by large amounts in the vicinity of the Mendocino scarp. This further shows that these anomalies must have a shallow origin. While complex modification of the crust, in addition to the density change in the mantle, can probably be made to fit the residual anomaly curve, it is clear that a change of density in the mantle alone cannot account for the observed anomalies.
The actual running time for the computations made in this problem on the IBM 650 was three hours. On a faster computer, the IBM 704, for instance, the running time would be reduced by a factor of at least 30 . Of course, a more complicated problem, one involving more layers, for instance, would require correspondingly more time.


Fra. 6-Attempt to fit the residual anomalies across the Mendocino scarp by a change of density in the mantle (extending to a depth of 200 km )

In conclusion, one must point out that a limitation of this method is the assumed twodimensionality. Though this is not a serious problem for a large number of structures such as continental margins, ocean trenches, or mountain ranges, it might be a handicap for small scale work. A similar method for rapid calculation of the gravitational attraction of three-dimensional bodies is being developed.

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[^0]:    * Lamont Geological Observatory Contribution No. 318

[^1]:    a For other stations in this area see Harrison and others [1957].
    ${ }^{6}$ Based on soundings from U.S.C.G.S. Chart 9000 .
    c Based on soundings from U.S.C.G.S. Chart 5002.

