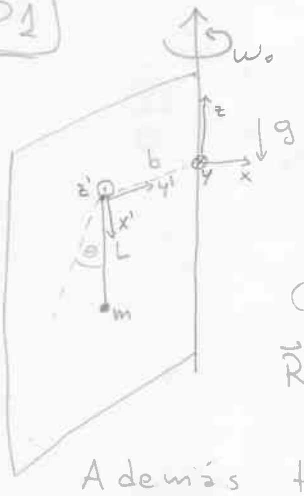
Mecanica

Pauta C3

P1)



Se ubica el sistema inercial a la altura del pivote. Se coloca un sistema no inercial en el pivote, con el eje x' vertical y el eje y' apuntando horizontalmente al origen del sist inercial.

Con esto vemos que:

$\vec{R} = b\,\hat{\rho}$ $\qquad \vec{\Omega} = \omega_0 \hat{k}$

Además tenemos que: $\hat{k} = -\hat{\imath}'$ $\qquad \hat{\rho} = -\hat{\jmath}'$ $\qquad \hat{\phi} = \hat{k}'$

La ecuación de mov $\qquad \Rightarrow r' = L\hat{\rho}'$ $\qquad v = L\dot{\theta}\hat{\theta}'$

$$m\vec{a}' = F - m\ddot{\vec{R}} - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') - 2m\vec{\Omega}\times\vec{v}' - \underbrace{\cancel{m\dot{\vec{\Omega}}\times\vec{r}'}}_{0,\ \Omega = cte}$$

$$m\ddot{\vec{R}} = -mb\omega_0^2\hat{\rho}$$

$$m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = m\omega_0^2 L\,\hat{k}\times(\hat{k}\times\hat{\rho}')$$

$$= m\omega_0^2 L\,\hat{k}\times(\hat{k}\times(\hat{\imath}'\cos\theta + \hat{\jmath}'\sin\theta))$$

$$= m\omega_0^2 L\,\hat{k}\times(\hat{k}\times(-\cancel{\hat{k}\cos\theta} - \hat{\rho}\sin\theta))$$

$$= -m\omega_0^2 L\,\hat{k}\times\sin\theta\,\hat{\phi} = m\omega_0^2 L\sin\theta\,\hat{\rho}$$

$$2m\Omega\times v' = 2m\omega_0\dot{\theta}L\,\hat{k}\times\hat{\theta}'$$

$$= 2m\omega_0\dot{\theta}L\,\hat{k}\times(-\hat{\imath}'\sin\theta + \hat{\jmath}'\cos\theta)$$

$$= 2m\omega_0\dot{\theta}L\,\hat{k}\times(\underbrace{\cancel{\hat{k}\sin\theta}}_{0} - \hat{\rho}\cos\theta) = -2m\omega_0\dot{\theta}L\cos\theta\,\hat{\phi}$$

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$$\vec{F} = -mg\hat{k} - T\hat{\rho}' + N\hat{\phi}$$

$$m\vec{a}' = (-L\dot{\theta}^2\hat{\rho}' + L\ddot{\theta}\hat{\theta} + \underbrace{\cancel{\ddot{z}\hat{k}}}_{0})m$$

$$-mL\dot{\theta}^2\hat{\rho}' + mL\ddot{\theta}\hat{\theta}' = -mg\hat{k} + T\hat{\rho}' + N\hat{\phi} - mb\omega_0^2\hat{\rho} - m\omega_0^2 L\sin\theta\,\hat{\rho} + 2m\omega_0\dot{\theta}L\cos\theta\,\hat{\phi}$$

Escribimos en sist no-inercial

$$-mL\dot{\theta}^2\hat{\rho}' + mL\ddot{\theta}\hat{\theta}' = mg(\cos\theta\,\hat{\rho}' - \sin\theta\,\hat{\theta}') - T\hat{\rho}' + N\hat{k}' + 2m\omega_0\dot{\theta}L\cos\theta\,\hat{k}' + m\omega_0^2(b + L\sin\theta)(\sin\theta\,\hat{\rho}' + \cos\theta\,\hat{\theta}')$$

$\hat{\theta}' \Rightarrow \boxed{\cancel{m}L\ddot{\theta} = -\cancel{m}g\sin\theta + \cancel{m}\omega_0^2(b + L\sin\theta)\cos\theta}$ $\qquad$ a) $\qquad$ 1,5

$\hat{\rho}' \Rightarrow -\cancel{m}L\dot{\theta}^2 + \cancel{m}g\cos\theta - T + \cancel{m}\omega_0^2(b + L\sin\theta)\sin\theta$

$\hat{k}' \Rightarrow -N + 2m\omega_0\dot{\theta}L\cos\theta = 0$

Para buscar θ_{max} hacemos

$$\frac{\partial N}{\partial \theta} = \cancel{+2\, m L \omega_0}\, \frac{\partial}{\partial \theta}\left(\cos\theta \cdot \dot\theta\right) = 0$$

$$= -\sin\theta\,\dot\theta + \cos\theta\,\frac{\partial\dot\theta}{\partial\theta} \qquad \text{Pero} \qquad \ddot\theta = \frac{d\dot\theta}{d\theta}\,\dot\theta \Rightarrow \frac{d\dot\theta}{d\theta} = \frac{\ddot\theta}{\dot\theta}$$

$$= -\sin\theta\,\dot\theta + \cos\theta\,\frac{\ddot\theta}{\dot\theta} = 0$$

$$\Rightarrow \frac{-\sin\theta\,\dot\theta^2 + \cos\theta\,\ddot\theta}{\dot\theta} = 0$$

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Con

$$\frac{\dot\theta^2}{2} = \frac{g}{L}(\cos\theta - 1) + \sin\theta\,\omega_0^2\left(\frac{b}{L} + \frac{\sin\theta}{2}\right)$$

Lo que define implicitamente θ^*

Buscamos $N = 0 \Rightarrow \dot\theta\cos\theta = 0 \Rightarrow$

$$\cos\theta = 0 \Rightarrow \bar\theta = \pm\pi/2$$

$$\dot\theta = 0 \Rightarrow \bar\theta = \theta_{max}$$

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