Mecanica
Pauta C2

P3) a) $-M\rho_0\dot{\phi}_0^2 = -mg$ con $\dot{\phi}_0 = \frac{v_0}{\rho_0}$

$$\Rightarrow \frac{M v_0^2}{\rho_0} = mg \Rightarrow \boxed{v_0 = \sqrt{\frac{m}{M} g \rho_0}} \quad 2\text{pt}$$

b) La fuerza total es $\vec{F}_{T.t} = F_{T.t}\,\hat{\rho}$

$$\Rightarrow \tau = 0 \Rightarrow L = cte = (\rho_0\,\hat{\rho}) \times (M v_0\,\hat{\phi})$$

$$L = \rho_0 \sqrt{M m g \rho}$$

Por otro lado F_0 es tal que

$$\dot{\rho}(t) = -v_1 \Rightarrow \rho(t) = \rho_0 - v_1 t$$

$$\Rightarrow v(t) = \dot{\rho}(t)\,\hat{\rho} + \rho(t)\,\dot{\phi}(t)\,\hat{\phi}$$

$$\therefore L = \rho\hat{\rho} \times \left(M\dot{\rho}\hat{\rho} + M\rho\dot{\phi}\hat{\phi}\right) = M\rho^2\dot{\phi} = L$$

$$\Rightarrow \dot{\phi} = \underbrace{\sqrt{\frac{m}{M} g \rho_0}\,\rho_0^2}_{\alpha} \frac{1}{\rho(t)^2}$$

$$\Rightarrow \phi(t) = \int_0^t \frac{\alpha}{\rho(t)^2}\,dt \qquad \text{Tenemos que } \rho(t^*) = \frac{\rho_0}{2} \Rightarrow t^* = \frac{\rho_0}{2v_1}$$

$$\phi\left(\frac{\rho_0}{2v_1}\right) = \int_0^{\rho_0/2v_1} \frac{\alpha}{(\rho_0 - v_1 t)^2}\,dt$$

CV = $\xi = \rho_0 - v_1 t \Rightarrow dt = -\frac{d\xi}{v_1}$

$$\Rightarrow \phi(t^*) = \frac{\alpha}{v_1}\int_{\rho_0}^{\rho_0/2} -\frac{1}{\xi^2}\,d\xi \Rightarrow \frac{\alpha}{v_1}\int_{\rho/2}^{\rho_0}\frac{d\xi}{\xi^2} = \frac{\alpha}{v_1}\left[\frac{2}{\rho_0} - \frac{1}{\rho}\right]$$

$$\phi(t^*) = \frac{\alpha}{\rho_0 v_1} = \sqrt{\frac{m}{M}\rho_0 g}\cdot\frac{1}{v_1}$$

N° De vueltas $= \frac{\phi(t^*)}{2\pi} = \boxed{\frac{1}{2\pi v_1}\sqrt{\frac{m}{M}\rho_0 g} = N\text{vueltas}}$ 2 pt

c) $M\left[\left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\right] = -T\hat{\rho}$

Pero $\rho = -v_1 t + \rho_0 \Rightarrow \ddot{\rho} = 0$ y

$\dot{\phi} = \frac{\alpha}{\rho^2}$

$\Rightarrow T = \frac{M\alpha^2}{\rho^3}$

Para m $\quad m\ddot{\rho} = T - mg - F_0 \Rightarrow F_0 = T - mg$

$$\boxed{F_0(\rho/2) = \frac{M v_0^2 \rho_0^4}{(\rho_0^3/2)} - gm} \quad 2\text{ pt}$$

$$= \frac{M\frac{m}{M}\frac{g}{\rho_0}\rho_0^4}{\rho_0^3/8} = 8mg - mg = 7mg$$