Mecanica

Pauta Ejercicio 1

Datos

$$\dot{\rho} = v_0 \Rightarrow \ddot{\rho} = 0 \quad \wedge \quad \rho(t) = v_0 t + C$$

$$\dot{\phi} = \omega_0 \Rightarrow \ddot{\phi} = 0 \quad \wedge \quad \phi(t) = \omega_0 t + D$$

Imponiendo CI

$$\rho(0) = R \Rightarrow C = R \qquad \phi(0) = 0 \Rightarrow D = 0$$

Usando coordenadas cilindricas

$$\vec{r} = \rho\hat{\rho} = (v_0 t + R)\hat{\rho}$$

$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} = v_0\hat{\rho} + (v_0 t + R)\omega_0\hat{\phi}$$

$$\vec{a} = (\cancel{\ddot{\rho}}^{0} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\cancel{\ddot{\phi}}^{0})\hat{\phi} = -(v_0 t + R)\omega_0^2\hat{\rho} + 2v_0\omega_0\hat{\phi}$$

$\boxed{1\ Pt}$

a) $$\rho_c = \frac{\|\vec{v}\|^3}{\|\vec{v}\times\vec{a}\|} , \qquad \|\vec{v}\|^3 = \left(v_0^2 + (v_0 t + R)^2\omega_0^2\right)^{3/2}$$

$$\vec{v}\times\vec{a} = \left(v_0\hat{\rho} + (v_0 t + R)\omega_0\hat{\phi}\right)\times\left(-(v_0 t + R)\omega_0^2\hat{\rho} + 2v_0\omega_0\hat{\phi}\right)$$ $\boxed{1\ Pt}$

Como $\hat{\rho}\times\hat{\rho} = \hat{\phi}\times\hat{\phi} = 0$ y $\hat{\rho}\times\hat{\phi} = \hat{k}$ $\quad \hat{\phi}\times\hat{\rho} = -\hat{k}$

$$2v_0^2\omega_0\hat{k} - (v_0 t + R)^2\omega_0^3(-\hat{k}) = 2v_0^2\omega_0 + (v_0 t + R)^2\omega_0^3$$

$$\Rightarrow \rho_c = \frac{\left(v_0^2 + (v_0 t + R)^2\omega_0^2\right)^{3/2}}{2v_0^2\omega_0 + (v_0 t + R)^2\omega_0^3}$$ $\boxed{1{,}5\ Pt}$

Ojo si $v_0 \to 0$ La trayectoria es circunferencial

y $\rho_c \to R$

$$\rho_c(v_0 \to 0) = \frac{\left(\cancel{v_0^2} + (\cancel{v_0 t} + R)^2\omega_0^2\right)^{3/2}}{2\cancel{v_0^2}\omega_0 + (\cancel{v_0 t} + R)^2\omega^3} = \frac{(R^2\omega_0^2)^{3/2}}{R^2\omega_0^3} = \frac{R^{\cancel{3}}\cancel{\omega_0^3}}{\cancel{R^2}\cancel{\omega_0^3}} = R$$

$\boxed{0{,}5\ Pt\ *}$

b) $L = \int_{t_0}^{t_f} \frac{ds}{dt}\, dt = \int_{t_0}^{t_f} v\, dt = \int_{t_0}^{t_f} \sqrt{v_0^2 + (v_0 t + R)^2 \omega_0^2}\, dt$ $\boxed{0{,}5\ pt}$

Donde $\rho(t_0) = R \Rightarrow t_0 = 0$

$\rho(t_f) = 2R \Rightarrow t_f = \frac{R}{v_0}$

Ahora para trabajar esta integral usaremos que

$$\rho = v_0 t + R \Rightarrow d\rho = v_0\, dt \Rightarrow dt = \frac{d\rho}{v_0}$$

$$\rho(t_0) = R\ ,\ \rho(t_f) = 2R$$

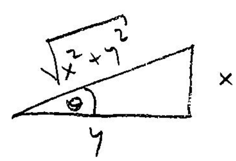
$$\Rightarrow L = \int_R^{2R} \sqrt{v_0^2 + \rho^2 \omega_0^2}\, \frac{d\rho}{v_0} = \int_R^{2R} \sqrt{1 + \rho^2 \frac{\omega_0^2}{v_0^2}}\, d\rho$$

Luego usamos un cambio de variables

$$\frac{\omega_0}{v_0}\rho = \operatorname{tg} \xi \Rightarrow d\rho = \frac{v_0}{\omega_0} \sec^2 \xi\, d\xi$$

$$\int_{\operatorname{arctg} \frac{\omega_0}{v_0} R}^{\operatorname{arctg} \frac{2\omega_0 R}{v_0}} \sqrt{\underbrace{1 + \operatorname{tg}^2 \xi}_{\sec^2 \xi}}\ \frac{v_0}{\omega_0} \sec^2 \xi = \frac{v_0}{\omega_0} \int \frac{d\xi}{\sec \xi} = \frac{v_0}{\omega_0} \int \cos \xi\, d\xi$$

$$= \frac{v_0}{\omega_0} \sin \xi \Big|_{\operatorname{arctg} \frac{\omega_0 R}{v_0}}^{\operatorname{arctg} \frac{2\omega_0 R}{v_0}}$$



$\Rightarrow \operatorname{tg} \theta = \frac{x}{y}$

$\Rightarrow \operatorname{arctg} \frac{x}{y} = \theta$

$\sin \theta = \frac{x}{\sqrt{x^2+y^2}}$

$$L = \frac{v_0}{\cancel{\omega_0}} \left(\frac{2 \cancel{\omega_0} R}{\sqrt{4\omega_0^2 R^2 + v_0^2}} - \frac{\cancel{\omega_0} R}{\sqrt{\omega_0^2 R^2 + v_0^2}} \right) \quad //$$ $\boxed{2\ Pt}$

Nota si $\omega_0 \to 0$

$\Rightarrow L = 2R - R = R$. La partícula viaja en línea recta de $x = R$ a $x = 2R$ $\boxed{0{,}5\ Pt^*}$

$* =$ Bonus