MA2601 - Ecuaciones Diferenciales Ordinarias. Semestre 2009-03
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Clase auxiliar 01
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1. Variables Separables

$$\begin{aligned}
\frac{dy}{dx} &= \frac{g(x)}{h(y)} \\
\int_{y(x_0)}^{y(x)} h(y)dy &= \int_{x_0}^{x} g(x)dx \\
H(y) - H(y(x_0)) &= G(x) - G(x_0) \\
H(y) &= G(x) + \underbrace{H(y(x_0)) - G(x_0)}_{K}
\end{aligned}$$

Mediante variables separables, resuelva las siguientes EDO's :

1. $\dfrac{dy}{dx} = 3y^{2/3}$
2. $\dfrac{dx}{dy} = 4(x^2+1),\ x(\pi/4) = 1$
3. $\dfrac{dy}{dt} = 6e^{2t-y},\ y(0) = 0$
4. $2t^{1/2}\dfrac{dy}{dt} = cos^2(y),\ y(4) = \pi/4$
5. $\dfrac{dx}{dt} = kx(n+1-x),\ x(0) = n$
6. $x\dfrac{dy}{dx} - y = \sqrt{x^2+y^2}$
7. $\dfrac{dy}{dx} = \dfrac{y}{x(ln(y) - ln(x) + 1)},\ y(1) = e$

Sol. 4:

$$\begin{aligned} 2t^{1/2}\frac{dy}{dt} &= cos^2(y) \\ \int_{y(4)}^{y(t)} \frac{dy}{cos^2(y)}dy &= \int_4^t \frac{1}{2}t^{-1/2}dt \\ tg(y)|_{\pi/4}^{y(t)} &= \sqrt{t}|_4^t \\ tg(y(t)) - 1 &= \sqrt{t} - 2 \\ y(t) &= arctan(\sqrt{t} - 1) \end{aligned}$$

Sol. 6: A priori, esta EDO no es de variables separables. Sin embargo, notemos que

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{y'x - y}{x^2}$$

Luego, dividiendo por x^2, se tiene

$$\begin{aligned} \frac{y'x - y}{x^2} &= \frac{1}{x^2}\sqrt{x^2 + y^2} \\ \frac{d}{dx}\left(\frac{y}{x}\right) &= \frac{1}{x}\sqrt{1 + \left(\frac{y}{x}\right)^2} \end{aligned}$$

Sea $u(x) = \dfrac{y(x)}{x}$:

$$\begin{aligned} \frac{du}{dx} &= \frac{1}{x}\sqrt{1 + u^2} \\ \int \frac{du}{\sqrt{1 + u^2}} &= \int \frac{dx}{x} \\ arcsenh(u) &= ln|x| + K \\ u(x) &= senh(ln|x| + K) \\ y(x) &= xsenh(ln|x| + K) \end{aligned}$$

2. Ecuación lineal de primer orden: factor integrante

$$\begin{aligned} \frac{dy}{dt} + p(t)y &= f(t) & /e^{\int p(t)dt} \\ \frac{d}{dt}\left(ye^{\int p(t)dt}\right) &= f(t)e^{\int p(t)dt} & /\int dt \\ y(t) &= Ke^{-\int p(t)dt} + e^{-\int p(t)dt}\left(\int f(t)e^{\int p(t)dt}dt\right) \end{aligned}$$

1. $\frac{dy}{dt} + 4y = 4t$

2. Encuentre $y(x)$ continua en $\mathbb{R}$, que satisface:

$$\begin{aligned} y' + 4y &= 2 & x \in (-\infty, 0) \\ Ly' + 2xy &= 3x & x \in (0, L) \\ y' - 4y &= 12 & x \in (L, \infty) \\ y(L) &= L \end{aligned}$$