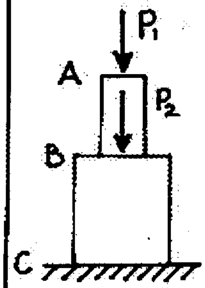
1.2-1

Circular Post in Compression



$P_1 = 1600$ lb

$d_{AB} = 1.2$ in.

$d_{BC} = 2.4$ in.

(a) Normal Stress in Part AB

$$\sigma_{AB} = \frac{P_1}{A_{AB}} = \frac{1600 \text{ lb}}{\frac{\pi}{4}(1.2 \text{ in})^2} = 1415 \text{ psi} \longleftarrow$$

(b) Load P_2 for Equal Stresses

$$\sigma_{BC} = \frac{P_1 + P_2}{A_{BC}} = \frac{1600 \text{ lb} + P_2}{\frac{\pi}{4}(2.4 \text{ in})^2}$$

$$= \sigma_{AB} = 1415 \text{ psi}$$

Solve for P_2: $P_2 = 4800$ lb $\longleftarrow$

Alternate Solution

$$\sigma_{BC} = \frac{P_1 + P_2}{A_{BC}} = \frac{P_1 + P_2}{\frac{\pi}{4} d_{BC}^2}$$

$$\sigma_{AB} = \frac{P_1}{A_{AB}} = \frac{P_1}{\frac{\pi}{4} d_{AB}^2}$$

$$\sigma_{BC} = \sigma_{AB}$$

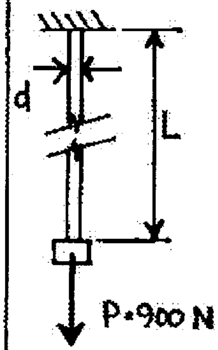
$$\frac{P_1 + P_2}{d_{BC}^2} = \frac{P_1}{d_{AB}^2} \quad \text{OR} \quad P_2 = P_1\left[\left(\frac{d_{BC}}{d_{AB}}\right)^2 - 1\right]$$

$$\frac{d_{BC}}{d_{AB}} = 2$$

$$\therefore P_2 = 3P_1 = 4800 \text{ lb} \longleftarrow$$

1.2-2

Long Steel Rod in Tension



$P = 900$ N

$L = 30$ m

$d = 6$ mm

Weight Density: $\gamma = 77.0 \text{ kN/m}^3$

W = Weight of Rod

$= \gamma(\text{Volume})$

$= \gamma A L$

$$\sigma_{MAX} = \frac{W + P}{A} = \gamma L + \frac{P}{A}$$

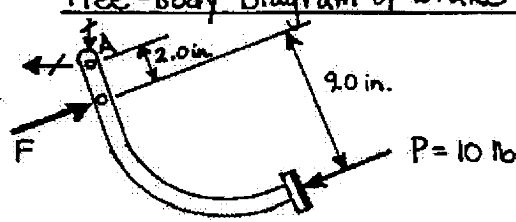
$$= (77.0 \text{ kN/m}^3)(30 \text{ m}) + \frac{900 \text{ N}}{\frac{\pi}{4}(6 \text{ mm})^2}$$

$$= 2.3 \text{ MPa} + 31.8 \text{ MPa}$$

$$= 34.1 \text{ MPa} \longleftarrow$$

1.2-3

Free-body Diagram of Brake Pedal



F = Compressive force in Piston Rod

d = Diameter of Piston Rod

= 0.22 in.

Equilibrium of Brake Pedal

$$\Sigma M_A = 0$$

$$F(2.0 \text{ in.}) - P(11.0 \text{ in.}) = 0$$

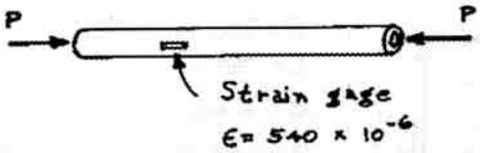
$$F = P\left(\frac{11.0 \text{ in.}}{2.0 \text{ in.}}\right) = (10 \text{ lb})\left(\frac{11.0 \text{ in.}}{2.0 \text{ in.}}\right) = 55 \text{ lb}$$

Compressive Stress in Piston Rod

$$\sigma_c = \frac{F}{A} = \frac{55 \text{ lb}}{\frac{\pi}{4}(0.22 \text{ in.})^2} = 1450 \text{ psi} \longleftarrow$$

1.2-4

Aluminum Tube in Compression



$L = 500\text{ mm}$

$d_2 = 60\text{ mm}$

$d_1 = 50\text{ mm}$

(a) Shortening δ of the bar

$\delta = \epsilon L = (540 \times 10^{-6})(500\text{ mm}) = 0.270\text{ mm}$ ←

(b) Compressive Load P

$\sigma = 40\text{ MPa}$

$A = \frac{\pi}{4}\left[d_2^2 - d_1^2\right] = \frac{\pi}{4}\left[(60\text{ mm})^2 - (50\text{ mm})^2\right]$

$= 863.9\text{ mm}^2$

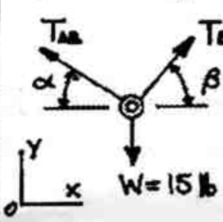
$P = \sigma A = (40\text{ MPa})(863.9\text{ mm}^2)$

$= 34.6\text{ kN}$ ←

1.2-5

Two Steel Wires Supporting a Lamp

Free-body Diagram of Point B



$\alpha = 35°$ $\quad \beta = 50°$

$d = 25\text{ mils} = 0.025\text{ in.}$

$A = \frac{\pi d^2}{4} = 490.9 \times 10^{-6}\text{ in.}^2$

Equations of Equilibrium

$\Sigma F_x = 0 \qquad -T_{AB}\cos\alpha + T_{BC}\cos\beta = 0$

$\Sigma F_Y = 0 \qquad T_{AB}\sin\alpha + T_{BC}\sin\beta - W = 0$

Substitute numerical values:

$-T_{AB}(0.81915) + T_{BC}(0.64279) = 0$

$T_{AB}(0.57358) + T_{BC}(0.76604) = 15\text{ lb}$

CONT.

Solve the Equations:

$T_{AB} = 9.679\text{ lb} \qquad T_{BC} = 12.334\text{ lb}$

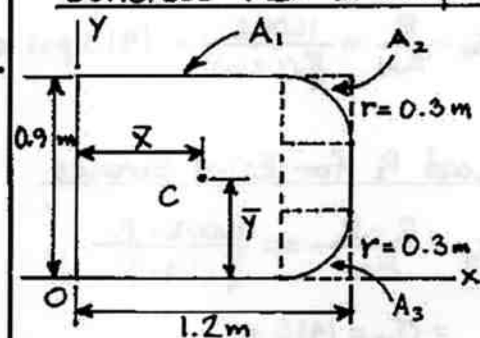
Tensile stresses in the wires

$\sigma_{AB} = \frac{T_{AB}}{A} = 19{,}700\text{ psi}$ ←

$\sigma_{BC} = \frac{T_{BC}}{A} = 25{,}100\text{ psi}$ ←

1.2-6

Concrete Pier in Compression



Load:
$P = 7.5\text{ MN}$

Use the following areas:

A_1 = outer rectangle
$= (1.2\text{ m})(0.9\text{ m}) = 1.08\text{ m}^2$

$A_2 = A_3$ = quarter-circular spandrel
(Appendix D, Case 12)

$A_2 = A_3 = \left(1 - \frac{\pi}{4}\right) r^2 = 0.019314\text{ m}^2$

Area of Pier

$A = A_1 - A_2 - A_3 = 1.0414\text{ m}^2$

(a) Average compressive stress

$\sigma_c = \frac{P}{A} = \frac{7.5\text{ MN}}{1.0414\text{ m}^2} = 7.20\text{ MPa}$ ←

(b) Coordinates of Centroid C

From symmetry, $\bar{y} = \frac{1}{2}(0.9\text{ m})$
$= 0.45\text{ m}$ ←

$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{A}$ (See Chapter 12, Section 12.3)

For Area 1: $\bar{x}_1 = \frac{1}{2}(1.2\text{ m}) = 0.6\text{ m}$

CONT.

1.2-6 CONT.

For Area 2 or 3:

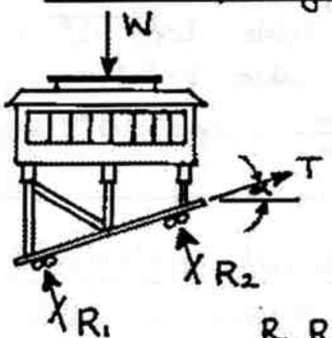
$$\bar{x}_2 = \bar{x}_3 = 0.9\,\text{m} + \frac{2r}{3(4-\pi)}$$
$$= 0.9\,\text{m} + 0.23299\,\text{m}$$
$$= 1.13299\,\text{m}$$

$$\bar{x} = \frac{1}{A}[\bar{x}_1 A_1 - 2\bar{x}_2 A_2]$$
$$= \frac{1}{1.0414\,\text{m}^2}[(0.6\,\text{m})(1.08\,\text{m}^2) - 2(1.13299\,\text{m})(0.019314\,\text{m}^2)]$$
$$= 0.580\,\text{m} \longleftarrow$$

1.2-7

Car on inclined track

Free-body diagram of car



W = Weight of car

T = Tensile force in cable

α = Angle of incline

A = Effective area of cable

R_1, R_2 = Wheel reactions (no friction force between wheels and rails)

Equilibrium in the inclined direction

$\Sigma F_T = 0$ ↗+ ↙− $T - W\sin\alpha = 0$

$T = W\sin\alpha$

Tensile stress in the cable

$$\sigma_T = \frac{T}{A} = \frac{W\sin\alpha}{A}$$

Substitute numerical values:

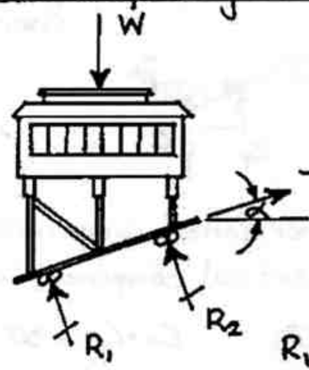
$W = 18\,\text{k}$ $\alpha = 30°$ $A = 0.471\,\text{in.}^2$

$$\sigma_T = \frac{(18\,\text{k})(\sin 30°)}{0.471\,\text{in.}^2} = 19.1\,\text{ksi} \longleftarrow$$

1.2-8

Car on inclined track

Free-body diagram of car



W = Weight of car

T = Tensile force in cable

α = Angle of incline

A = Effective area of cable

R_1, R_2 = Wheel reactions (no friction force between wheels and rails)

Equilibrium in the inclined direction

$\Sigma F_T = 0$ ↗+ ↙− $T - W\sin\alpha = 0$

$T = W\sin\alpha$

Tensile stress in the cable

$$\sigma_T = \frac{T}{A} = \frac{W\sin\alpha}{A}$$

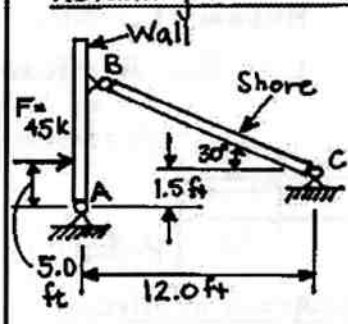
Substitute numerical values:

$W = 110\,\text{kN}$ $\alpha = 32°$ $A = 480\,\text{mm}^2$

$$\sigma_T = \frac{(110\,\text{kN})(\sin 32°)}{480\,\text{mm}^2} = 121\,\text{MPa} \longleftarrow$$

1.2-9

Retaining wall braced by wood shores



A = area of one shore

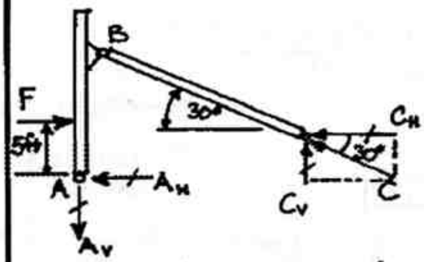
$A = (6.0\,\text{in.})(6.0\,\text{in.})$

$= 36\,\text{in.}^2$

CONT.

1.2-9 CONT.

Free-body diagram of wall and shore



C = Compressive force in wood shore

C_H = horizontal component of C

C_V = vertical component of C

$C_H = C\cos 30°$ $\qquad C_V = C\sin 30°$

Equilibrium

$\Sigma M_A = 0$ ↶+ ↷−

$-F(5\text{ ft}) + C_V(12\text{ ft}) + C_H(1.5\text{ ft}) = 0$

$-(45\text{ k})(5\text{ ft}) + C(\sin 30°)(12\text{ ft}) + C(\cos 30°)(1.5\text{ ft}) = 0$

$\therefore\ C = 30.83\text{ k}$

Compressive stress in shore

$$\sigma_c = \frac{C}{A} = \frac{30.83\text{ k}}{36\text{ in.}^2} = 0.856\text{ ksi}$$

$\sigma_c = 856$ psi ⟵

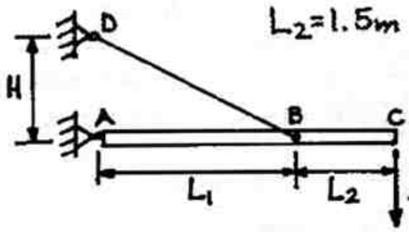
1.2-10

Loading crane with girder and cable

$H = 1.6$ m $\quad L_1 = 3.0$ m

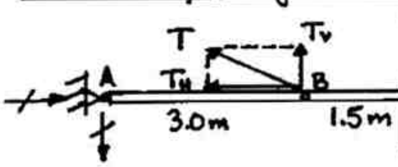
$L_2 = 1.5$ m $\quad$ A = effective area of cable

$A = 481\text{ mm}^2$



Free-body diagram of girder

T = tensile force in cable



P = 32 kN

CONT.

1.2-10 CONT.

Equilibrium

$\Sigma M_A = 0$ ↶+ ↷−

$T_V(3.0\text{ m}) - (32\text{ kN})(4.5\text{ m}) = 0$

$T_V = 48$ kN

$$\frac{T_H}{T_V} = \frac{L_1}{H} = \frac{3.0\text{ m}}{1.6\text{ m}} \qquad \therefore\ T_H = T_V\left(\frac{3.0}{1.6}\right)$$

$$= (48\text{ kN})\left(\frac{3.0}{1.6}\right)$$

$$= 90\text{ kN}$$

Tensile force in cable

$$T = \sqrt{T_H^2 + T_V^2} = \sqrt{(90\text{ kN})^2 + (48\text{ kN})^2} = 102\text{ kN}$$

(a) Average tensile stress in cable

$$\sigma = \frac{T}{A} = \frac{102\text{ kN}}{481\text{ mm}^2} = 212\text{ MPa}$$ ⟵

(b) Average strain in cable

L = length of cable $\quad L = \sqrt{H^2 + L_1^2} = 3.4$ m

δ = stretch of cable $\quad \delta = 5.1$ mm

$$\epsilon = \frac{\delta}{L} = \frac{5.1\text{ mm}}{3.4\text{ m}} = 1500 \times 10^{-6}$$ ⟵

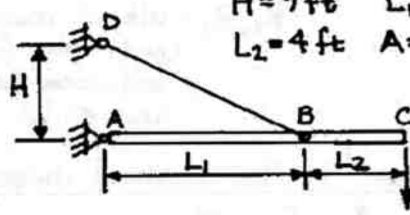
1.2-11

Loading crane with girder and cable

$H = 9$ ft $\quad L_1 = 12$ ft

$L_2 = 4$ ft $\quad$ A = effective area of cable

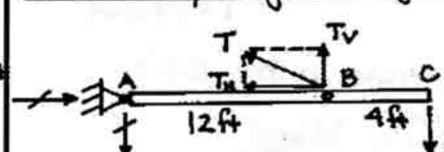
$A = 0.471\text{ in.}^2$



P = 9000 lb

Free-body diagram of girder

T = tensile force in cable



P = 9000 lb

Equilibrium

$\Sigma M_A = 0$ ↶+ ↷−

$T_V(12\text{ ft}) - (9000\text{ lb})(16\text{ ft}) = 0$

$T_V = 12{,}000$ lb

$$\frac{T_H}{T_V} = \frac{L_1}{H} = \frac{12\text{ ft}}{9\text{ ft}} \qquad \therefore\ T_H = T_V\left(\frac{12}{9}\right)$$

$$= (12{,}000\text{ lb})\left(\frac{12}{9}\right)$$

$$= 16{,}000\text{ lb}$$

CONT.

1.2-11 CONT.

Tensile force in cable

$T = \sqrt{T_H^2 + T_V^2} = \sqrt{(16{,}000\text{ lb})^2 + (12{,}000\text{ lb})^2}$
$= 20{,}000\text{ lb}$

(a) Average Tensile stress in cable

$\sigma = \dfrac{T}{A} = \dfrac{20{,}000\text{ lb}}{0.471\text{ in.}^2} = 42{,}500\text{ psi}$ ←

(b) Average strain in cable

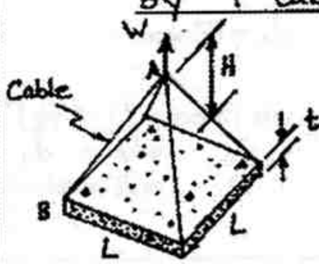
L = length of cable $L = \sqrt{H^2 + L_1^2} = 15\text{ ft}$

δ = stretch of cable $\delta = 0.382\text{ in.}$

$\epsilon = \dfrac{\delta}{L} = \dfrac{0.382\text{ in.}}{(15\text{ ft})(12\text{ in./ft})} = 2120 \times 10^{-6}$ ←

1.2-12

Reinforced concrete slab supported by 4 cables



H = height of hook above slab

L = Length of side of square slab

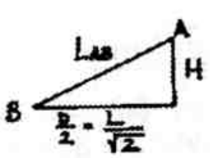
t = thickness of slab

γ = weight density of reinforced concrete

W = weight of slab
$= \gamma L^2 t$

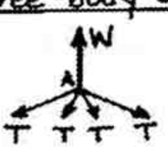
D = length of diagonal of slab
$= L\sqrt{2}$

Dimensions of cable AB



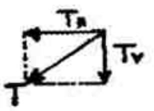
L_{AB} = length of cable
$= \sqrt{H^2 + \dfrac{L^2}{2}}$

Free-body diagram of hook at point A



T = tensile force in a cable

Cable AB: $\dfrac{T_V}{T} = \dfrac{H}{L_{AB}}$



$T_V = T\left(\dfrac{H}{\sqrt{H^2 + L^2/2}}\right)$ (Eq. 1)

CONT.

1.2-12 CONT.

Equilibrium

$\Sigma F_{vert} = 0$ ↑+ ↓−

$W - 4T_V = 0$

$T_V = \dfrac{W}{4}$ (Eq. 2)

Combine Eqs. (1) & (2):

$T\left(\dfrac{H}{\sqrt{H^2 + L^2/2}}\right) = \dfrac{W}{4}$

$T = \dfrac{W}{4}\dfrac{\sqrt{H^2 + L^2/2}}{H} = \dfrac{W}{4}\sqrt{1 + L^2/2H^2}$

Tensile stress in a cable

A = effective cross-sectional area of a cable

$\sigma_T = \dfrac{T}{A} = \dfrac{W}{4A}\sqrt{1 + L^2/2H^2}$ ←

Substitute numerical values and obtain σ_T:

$H = 1.6\text{ m}$ $L = 2.5\text{ m}$ $t = 225\text{ mm}$

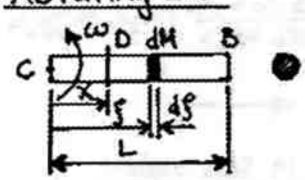
$\gamma = 24\text{ kN/m}^3$ $A = 190\text{ mm}^2$

$W = \gamma L^2 t = (24\text{ kN/m}^3)(2.5\text{ m})^2(225\text{ mm})$
$= 33.75\text{ kN}$

$\sigma_T = \dfrac{W}{4A}\sqrt{1 + L^2/2H^2} = 66.2\text{ MPa}$ ←

1.2-13

Rotating Bar



ω = Angular speed (rad/s)

A = cross-sectional area

γ = weight density

$\dfrac{\gamma}{g}$ = mass density

We wish to find the axial force F_x in the bar at section D, distance x from the midpoint C.

The force F_x equals the inertia force of the part of the rotating bar from D to B.

Consider an element of mass dM at distance ξ from the midpoint C. The variable ξ ranges from x to L.

$dM = \dfrac{\gamma}{g} A\, d\xi$

CONT.

1.2-13 CONT.

dF = Inertia force (centrifugal force) of element of mass dM

$$dF = (dM)(\xi\omega^2) = \frac{\gamma}{g} A\omega^2 \xi \, d\xi$$

$$F_x = \int_B^B dF = \int_x^L \frac{\gamma}{g} A \omega^2 \xi \, d\xi = \frac{\gamma A \omega^2}{2g}(L^2 - x^2)$$

(a) Tensile stress in bar at distance x

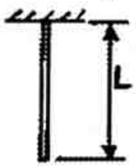
$$\sigma_x = \frac{F_x}{A} = \frac{\gamma\omega^2}{2g}(L^2 - x^2) \longleftarrow$$

(b) Maximum tensile stress

$$x = 0 \qquad \sigma_{max} = \frac{\gamma \omega^2 L^2}{2g} \longleftarrow$$

1.3-1

Hanging wire of length L



W = total weight of copper wire

γ_c = weight density of copper
$= 556 \text{ lb/ft}^3$

γ_W = weight density of sea water
$= 63.8 \text{ lb/ft}^3$

A = cross-sectional area of wire

(a) Wire hanging in air

$$W = \gamma_c A L$$

$$\sigma_{MAX} = \frac{W}{A} = \gamma_c L$$

$$L_{MAX} = \frac{\sigma_{MAX}}{\gamma_c} = \frac{25{,}000 \text{ psi}}{556 \text{ lb/ft}^3}(144 \text{ in.}^2/\text{ft}^2)$$

$$= 6470 \text{ ft} \longleftarrow$$

(b) Wire hanging in sea water

F = tensile force at top of wire

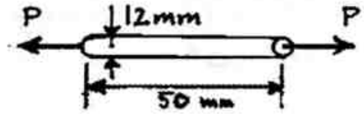
$$F = (\gamma_c - \gamma_W)AL \qquad \sigma_{MAX} = \frac{F}{A} = (\gamma_c - \gamma_W)L$$

$$L_{MAX} = \frac{\sigma_{MAX}}{\gamma_c - \gamma_W} = \frac{25{,}000 \text{ psi}}{(556 - 63.8) \text{ lb/ft}^3}(144 \text{ in.}^2/\text{ft}^2)$$

$$= 7310 \text{ ft} \longleftarrow$$

1.3-2

Tensile tests of three materials



$$\text{Percent elongation} = \frac{L_1 - L_0}{L_0}(100) = \left(\frac{L_1}{L_0} - 1\right)100$$

$$L_0 = 50 \text{ mm}$$

$$\text{Percent elongation} = \left(\frac{L_1}{50} - 1\right)(100) \qquad \text{(EQ. 1)}$$

where L_1 is in millimeters.

$$\text{Percent reduction in area} = \frac{A_0 - A_1}{A_0}(100)$$

$$= \left(1 - \frac{A_1}{A_0}\right)(100)$$

d_1 = final diameter

d_0 = initial diameter

$$\frac{A_1}{A_0} = \left(\frac{d_1}{d_0}\right)^2 \qquad d_0 = 12 \text{ mm}$$

$$\text{Percent reduction in area} = \left[1 - \left(\frac{d_1}{12}\right)^2\right](100)$$

where d_1 is in millimeters

(EQ. 2)

Material	L_1 (mm)	d_1 (mm)	% Elongation (EQ. 1)	% Reduction (EQ. 2)	Brittle or Ductile?
A	54.5	11.46	9.0 %	8.8 %	Brittle
B	63.2	9.48	26.4 %	37.6 %	Ductile
C	69.4	6.06	38.8 %	74.5 %	Ductile

1.3-3

Strength-to-weight ratio

The ultimate stress σ_U for each material is obtained from Table H-3, Appendix H, and the weight density γ is obtained from Table H-1.

The strength-to-weight ratio (feet) is

$$R_{S/W} = \frac{\sigma_U \text{ (ksi)}}{\gamma \text{ (lb/ft}^3)}(10^3 \text{ lb/k})(144 \text{ in.}^2/\text{ft}^2)$$

$$= \frac{\sigma_U}{\gamma}(144 \times 10^3)$$

CONT.

1.3-3 CONT.

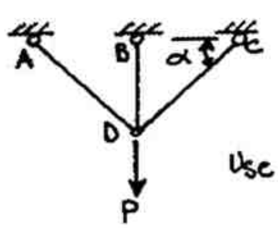
Values of σ_U, γ, and $R_{S/W}$ are listed in the table.

	σ_U (ksi)	γ (lb/ft³)	$R_{S/W}$ (ft)
Aluminum alloy 2014-T6	70	175	58×10^3
Southern pine (bending)	11	37.5	42×10^3
Nylon	9	62.5	21×10^3
Structural steel ASTM-A36	60	490	18×10^3
Titanium alloy	150	280	77×10^3

Titanium has a high strength-to-weight ratio, which is why it is used in space vehicles and high-performance airplanes. Aluminum is much higher than ordinary steel, which makes it desirable for commercial aircraft. Nylon and some woods have a higher ratio than ordinary steel.

1.3-4

Symmetrical framework

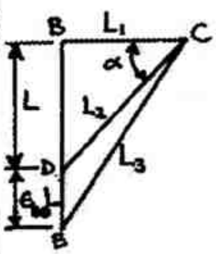


Structural steel

$\alpha = 50°$

$\epsilon_{BD} = 0.0839$

Use stress-strain diagram of FIGURE 1-12.



L = length of bar BD

L_1 = distance BC

$= L\cot\alpha = L(\cot 50°) = 0.8391L$

L_2 = length of bar CD

$= L\csc\alpha = L(\csc 50°) = 1.3054L$

Elongation of bar BD $= \epsilon_{BD}L$

$\epsilon_{BD}L = 0.0839L$

L_3 = distance CE

$L_3 = \sqrt{L_1^2 + (L+\epsilon_{BD}L)^2} = \sqrt{(0.8391L)^2 + L^2(1+0.0839)^2}$

$= 1.3707L$

CONT.

1.3-4 CONT.

δ = elongation of bar CD

$\delta = L_3 - L_2 = 0.0653L$

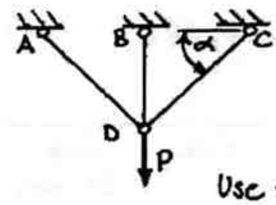
Strain in bar CD $= \dfrac{\delta}{L_2} = \dfrac{0.0653L}{1.3054L} = 0.0500$

From the stress-strain diagram of Fig. 1-12:

$\sigma \approx 49$ ksi or 340 MPa ←

1.3-5

Symmetrical framework

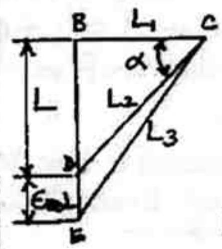


Aluminum alloy

$\alpha = 48°$

$\epsilon_{BD} = 0.0713$

Use stress-strain diagram of Figure 1-13



L = length of bar BD

L_1 = distance BC

$= L\cot\alpha = L(\cot 48°) = 0.9004L$

L_2 = length of bar CD

$= L\csc\alpha = L(\csc 48°) = 1.3456L$

Elongation of bar BD $= \epsilon_{BD}L$

$\epsilon_{BD}L = 0.0713L$

L_3 = distance CE

$L_3 = \sqrt{L_1^2 + (L+\epsilon_{BD}L)^2} = \sqrt{(0.9004L)^2 + L^2(1+0.0713)^2}$

$= 1.3994L$

δ = elongation of bar CD

$\delta = L_3 - L_2 = 0.0538L$

Strain in bar CD $= \dfrac{\delta}{L_2} = \dfrac{0.0538L}{1.3456L} = 0.0400$

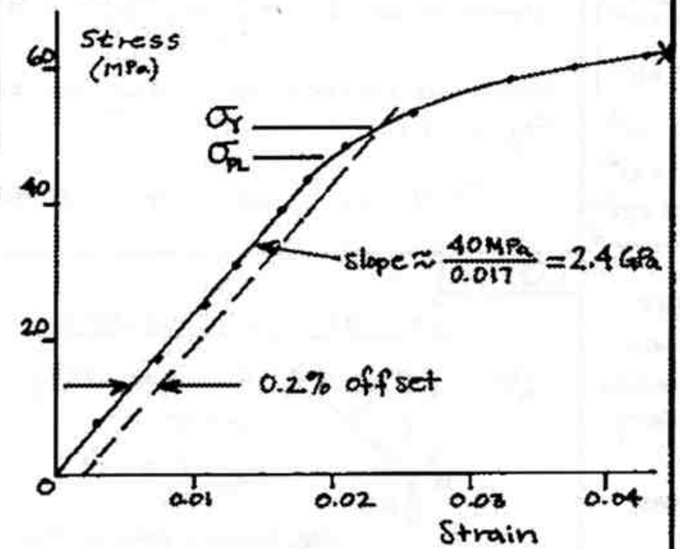
From the stress-strain diagram of Figure 1-13:

$\sigma \approx 32$ ksi ←

1.3-6

Tensile test of a plastic

Using the stress-strain data given in the problem statement, plot the stress-strain curve:



σ_{PL} = proportional limit $\quad \sigma_{PL} \approx 47$ MPa

Modulus of elasticity (slope) ≈ 2.4 GPa

σ_Y = yield stress at 0.2% offset

$\sigma_Y \approx 53$ MPa

Material is brittle, because the strain after the proportional limit is exceeded is relatively small.

1.3-7

Tensile test of high-strength steel

$d_o = 0.505$ in. $\quad L_o = 2.00$ in.

$$A_o = \frac{\pi d_o^2}{4} = 0.200 \text{ in.}^2$$

Conventional stress and strain

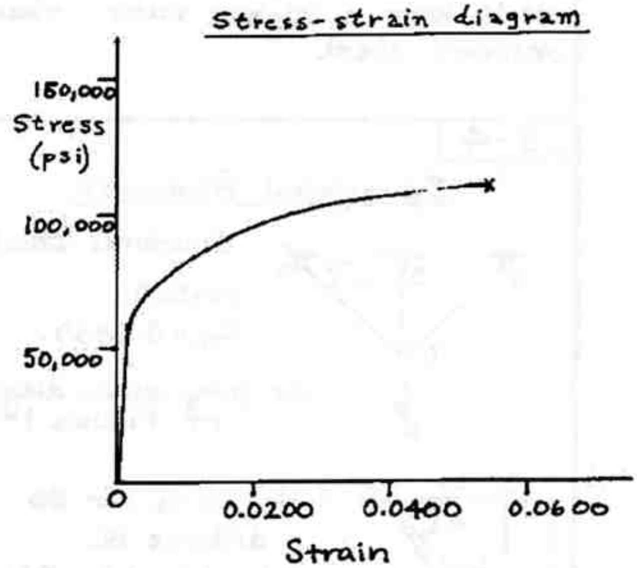
$$\sigma = \frac{P}{A_o} \qquad \epsilon = \frac{\delta}{L_o}$$

CONT.

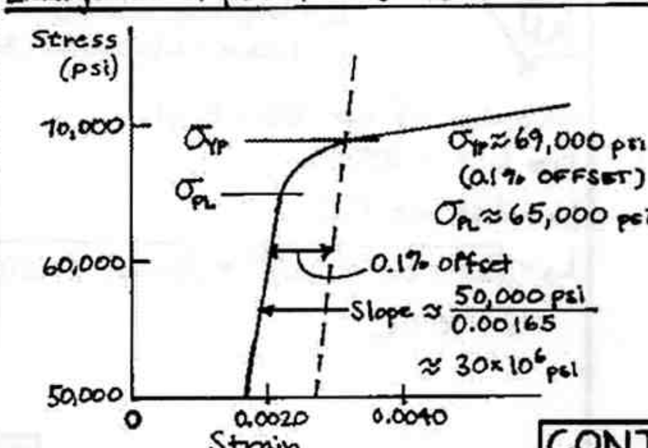
1.3-7 CONT.

Load P (lb)	Elongation δ (in.)	Stress σ (psi)	Strain ϵ
1,000	0.0002	5,000	0.00010
2,000	0.0006	10,000	0.00030
6,000	0.0019	30,000	0.00100
10,000	0.0033	50,000	0.00165
12,000	0.0039	60,000	0.00196
12,900	0.0043	64,500	0.00215
13,400	0.0047	67,000	0.00235
13,600	0.0054	68,000	0.00270
13,800	0.0063	69,000	0.00315
14,000	0.0090	70,000	0.00450
14,400	0.0102	72,000	0.00510
15,200	0.0130	76,000	0.00650
16,800	0.0230	84,000	0.01150
18,400	0.0336	92,000	0.01680
20,000	0.0507	100,000	0.02535
22,400	0.1108	112,000	0.05540
22,600	Fracture	113,000	—

Stress-strain diagram



Enlargement of part of the stress-strain curve



CONT.

1.3-7 CONT.

Results

Proportional limit $\approx 65{,}000$ psi ←

Modulus of elasticity (slope) $\approx 30 \times 10^6$ psi ←

Yield stress at 0.1% offset $\approx 69{,}000$ psi ←

Ultimate stress (maximum stress)

$\approx 113{,}000$ psi ←

Percent elongation in 2.00 in.

$$= \frac{L_1 - L_0}{L_0}(100)$$

$$= \frac{0.12\text{ in.}}{2.00\text{ in.}}(100) = 6\% \leftarrow$$

Percent reduction in area

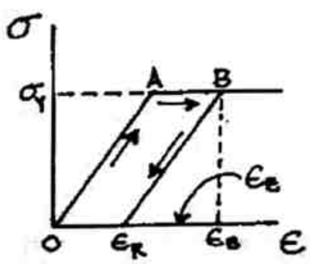
$$= \frac{A_0 - A_1}{A_0}(100)$$

$$= \frac{0.200\text{ in.}^2 - \frac{\pi}{4}(0.42\text{ in.})^2}{0.200\text{ in.}^2}(100)$$

$$= 31\% \leftarrow$$

1.4-1

Steel bar in tension



$L = 5\text{ ft} = 60\text{ in.}$

Yield stress $\sigma_Y = 42$ ksi

Slope $= 30 \times 10^3$ ksi

$\delta = 0.21$ in.

Stress and strain at point B

$\sigma_B = \sigma_Y = 42$ ksi

$$\epsilon_B = \frac{\delta}{L} = \frac{0.21\text{ in.}}{60\text{ in.}} = 0.0035$$

Elastic recovery ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{42\text{ ksi}}{30 \times 10^3\text{ ksi}} = 0.0014$$

Residual strain ϵ_R

$$\epsilon_R = \epsilon_B - \epsilon_E = 0.0035 - 0.0014$$

$$= 0.0021$$

CONT.

1.4-1 CONT.

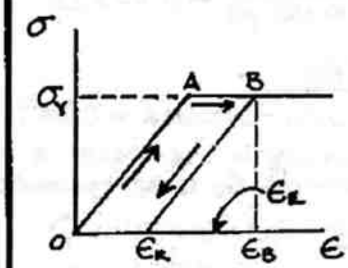
Permanent set $= \epsilon_R L = (0.0021)(60\text{ in.})$

$= 0.13$ in.

Final length is 0.13 in. longer than the original length. ←

1.4-2

Steel bar in tension



$L = 0.8\text{ m} = 800\text{ mm}$

Yield stress $\sigma_Y = 250$ MPa

Slope $= 200$ GPa

$\delta = 2.5$ mm

Stress and strain at point B

$\sigma_B = \sigma_Y = 250$ MPa

$$\epsilon_B = \frac{\delta}{L} = \frac{2.5\text{ mm}}{800\text{ mm}} = 0.003125$$

Elastic recovery ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{250\text{ MPa}}{200\text{ GPa}} = 0.00125$$

Residual strain ϵ_R

$$\epsilon_R = \epsilon_B - \epsilon_E = 0.003125 - 0.00125$$

$$= 0.001875$$

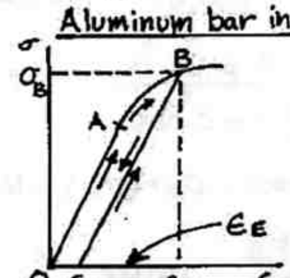
Permanent set $= \epsilon_R L = (0.001875)(800\text{ mm})$

$= 1.5$ mm

Final length is 1.5 mm longer than the original length. ←

1.4-3

Aluminum bar in tension



$L = 16$ in.

$d = 0.70$ in.

$P = 12.5$ k

See Fig. 1-13 for stress-strain diagram

Slope from 0 to A is 10×10^6 psi.

CONT.

1.4-3 CONT.

Stress and strain at point B

$$\sigma_B = \frac{P}{A} = \frac{12.5\ \text{k}}{\frac{\pi}{4}(0.70\ \text{in.})^2} = 32.5\ \text{ksi}$$

From FIG. 1-13: $\epsilon_B \approx 0.05$

Elastic recovery ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{32.5\ \text{ksi}}{10\times10^6\ \text{psi}} = 0.00325$$

Residual strain ϵ_R

$$\epsilon_R = \epsilon_B - \epsilon_E = 0.05 - 0.00325 = 0.047$$

(Note: the accuracy in this result is very poor because ϵ_B is approximate.)

(a) Permanent set $= \epsilon_R L = (0.047)(16\ \text{in})$

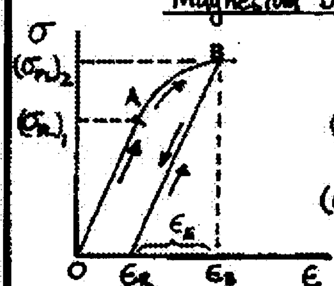
≈ 0.75 in. ←

(b) Proportional limit when reloaded $= \sigma_B$

$\sigma_B = 32.5$ ksi ←

1.4-4

Magnesium bar in tension



$L = 750$ mm

$\delta = 4.5$ mm

$(\sigma_{PL})_1$ = initial proportional limit = 88 MPa (from diagram)

$(\sigma_{PL})_2$ = proportional limit when the bar is reloaded

Initial slope of stress-strain curve

From σ-ϵ diagram:

At point A: $(\sigma_{PL})_1 = 88$ MPa

$\epsilon_A = 0.002$

$$\text{Slope} = \frac{(\sigma_{PL})_1}{\epsilon_A} = \frac{88\ \text{MPa}}{0.002} = 44\ \text{GPa}$$

Stress and strain at point B

$$\epsilon_B = \frac{\delta}{L} = \frac{4.5\ \text{mm}}{750\ \text{mm}} = 0.006$$

From σ-ϵ diagram: $\sigma_B = (\sigma_{PL})_2 = 160$ MPa

Elastic recovery ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{(\sigma_{PL})_2}{\text{Slope}} = \frac{160\ \text{MPa}}{44\ \text{GPa}} = 0.00364$$

CONT.

1.4-4 CONT.

Residual strain ϵ_R

$$\epsilon_R = \epsilon_B - \epsilon_E = 0.006 - 0.00364 = 0.00236$$

(a) Permanent set $= \epsilon_R L = (0.00236)(750\ \text{mm})$

$= 1.77$ mm ←

(b) Proportional limit when reloaded $= (\sigma_{PL})_2$

$= 160$ MPa ←

1.4-5

Wire stretched by forces P

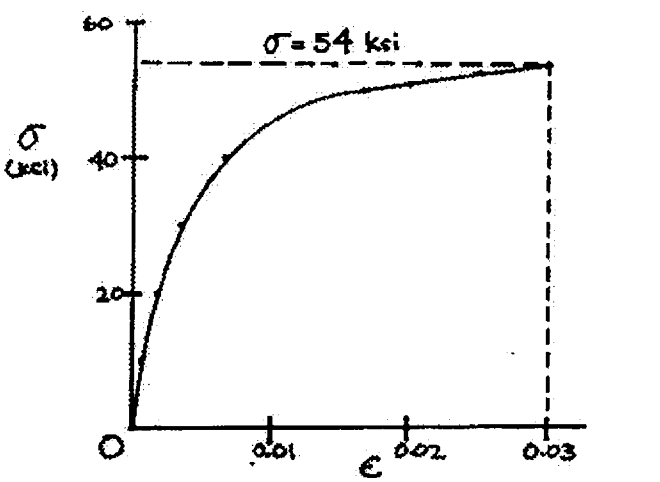
$L = 8$ ft $= 96$ in. $\quad d = 0.0625$ in.

$P = 150$ lb

Copper alloy

$$\sigma = \frac{18{,}000\epsilon}{1 + 300\epsilon} \qquad 0 \le \epsilon \le 0.03 \quad (\sigma = \text{ksi}) \quad \text{(Eq. 1)}$$

(a) Stress-strain diagram



Initial slope of stress-strain curve

Take the derivative of σ with respect to ϵ:

$$\frac{d\sigma}{d\epsilon} = \frac{(1+300\epsilon)(18{,}000) - (18{,}000\epsilon)(300)}{(1+300\epsilon)^2}$$

$$= \frac{18{,}000}{(1+300\epsilon)^2}$$

At $\epsilon = 0$, $\frac{d\sigma}{d\epsilon} = 18{,}000$ ksi

$\therefore$ Initial slope $= 18{,}000$ ksi

CONT.

1.4-5 CONT.

Alternative form of the stress-strain relationship

Solve Eq. (1) for ϵ in terms of σ:

$$\epsilon = \frac{\sigma}{18{,}000 - 3000\sigma} \qquad 0 \le \sigma \le 54\text{ ksi} \qquad (\sigma = \text{ksi}) \qquad \text{(EQ. 2)}$$

This equation may also be used when plotting the stress-strain diagram.

(b) Elongation δ of the wire

$$\sigma = \frac{P}{A} = \frac{150\text{ lb}}{\frac{\pi}{4}(0.0625\text{ in.})^2} = 48{,}900\text{ psi} = 48.9\text{ ksi}$$

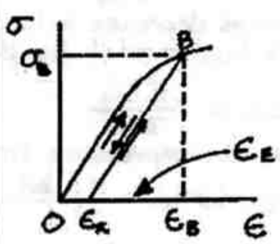
From EQ.(2) or from the stress-strain diagram:

$\epsilon = 0.0147$

$\delta = \epsilon L = (0.0147)(96\text{ in.}) = 1.41\text{ in.}$

Stress and strain at point B

$\sigma_B = 48.9\text{ ksi} \qquad \epsilon_B = 0.0147$



(Diagram not to scale)

Elastic recovery ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{48.9\text{ ksi}}{18{,}000\text{ ksi}} = 0.00272$$

Residual strain ϵ_R

$\epsilon_R = \epsilon_B - \epsilon_E = 0.0147 - 0.0027 = 0.0120$

(c) Permanent set $= \epsilon_R L = (0.0120)(96\text{ in.})$
$= 1.15\text{ in.}$

(d) Proportional limit when reloaded $= \sigma_B$

$\sigma_B = 49\text{ ksi}$

1.4-6 Bar stretched by forces P

$L = 2.5\text{ m} \quad d = 10\text{ mm} \quad P = 20\text{ kN}$

Aluminum alloy

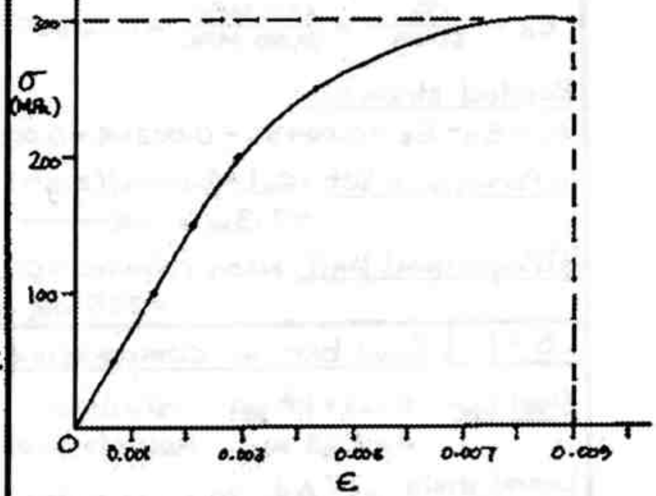
$$\epsilon = \frac{\sigma}{70{,}000}\left[1 + \frac{3}{7}\left(\frac{\sigma}{270}\right)^9\right] \qquad \text{EQ(1)}$$

Where σ = MPa

CONT.

1.4-6 CONT.

(a) Stress-strain diagram



Initial slope of stress-strain curve

Begin by taking the derivative of ϵ with respect to σ:

$$\frac{d\epsilon}{d\sigma} = \frac{\sigma}{70{,}000}\left[\frac{3}{7}\left(\frac{1}{270}\right)^9(9\sigma^8)\right] + \left[1 + \frac{3}{7}\left(\frac{\sigma}{270}\right)^9\right]\left(\frac{1}{70{,}000}\right)$$

$$= \frac{1}{70{,}000}\left[1 + \frac{30}{7}\left(\frac{\sigma}{270}\right)^9\right]$$

$$\text{Slope} = \frac{d\sigma}{d\epsilon} = \frac{1}{d\epsilon/d\sigma} = \frac{70{,}000}{1 + \frac{30}{7}\left(\frac{\sigma}{270}\right)^9}$$

At $\sigma = 0$, $\frac{d\sigma}{d\epsilon} = 70{,}000\text{ MPa}$

$\therefore$ Initial Slope $= 70{,}000\text{ MPa}$

(b) Elongation δ of the bar

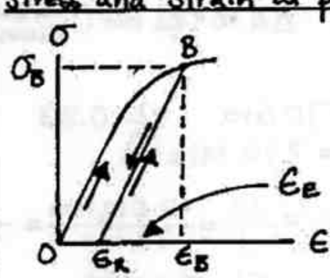
$$\sigma = \frac{P}{A} = \frac{20\text{ kN}}{\frac{\pi}{4}(10\text{ mm})^2} = 255\text{ MPa}$$

From the stress-strain diagram or from EQ.(1):

$\epsilon = 0.00456$

$\delta = \epsilon L = (0.00456)(2.5\text{ m}) = 11.4\text{ mm}$

Stress and Strain at point B



(Diagram not to scale)

$\sigma_B = 255\text{ MPa}$

$\epsilon_B = 0.00456$

CONT.

1.4-6 CONT.

Elastic recovery ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{SLOPE}} = \frac{255\text{ MPa}}{70{,}000\text{ MPa}} = 0.00364$$

Residual strain ϵ_R

$$\epsilon_R = \epsilon_B - \epsilon_E = 0.00456 - 0.00364 = 0.00092$$

(c) Permanent Set $= \epsilon_R L = (0.00092)(2.5\text{ m})$
$= 2.3\text{ mm}$ ←

(d) Proportional limit when reloaded $= \sigma_B$
$= 255\text{ MPa}$ ←

1.5-1 Steel bar in compression

Steel bar $E = 29 \times 10^6$ psi $\quad \nu = 0.30$
$d = 2.25$ in. $\quad$ Max. $\Delta d = 0.001$ in.

Lateral strain $\epsilon' = \dfrac{\Delta d}{d}$ (increase in diameter)

Axial strain $\epsilon = -\dfrac{\epsilon'}{\nu} = -\dfrac{\Delta d}{\nu d}$

(Minus means decrease in length)
Assume Hooke's law is valid for the material.

Axial stress

$$\sigma = E\epsilon = -\frac{E\Delta d}{\nu d}$$

(Minus means compressive stress)

Maximum permissible compressive load

$$P_{max} = |\sigma| A = \frac{EA\Delta d}{\nu d}$$

Substitute numerical values:

$$P_{max} = \frac{(29\times10^6\text{ psi})(\frac{\pi}{4})(2.25\text{ in.})^2(0.001\text{ in.})}{(0.30)(2.25\text{ in.})}$$

$= 171$ k ←

Note: $\sigma = \dfrac{P}{A} = 43$ ksi, which is less than the yield stress for high-strength steel.

1.5-2 Aluminum bar in tension

$d = 12$ mm $\quad \Delta d = 0.012$ mm (decrease in diameter)

6061-T6

Table H-2: $E = 70$ GPa $\quad \nu = 0.33$
Table H-3: $\sigma_Y = 270$ MPa

Lateral Strain $\epsilon' = \dfrac{\Delta d}{d} = \dfrac{-0.012\text{ mm}}{12\text{ mm}} = -0.00100$

Axial Strain $\epsilon = \dfrac{-\epsilon'}{\nu} = \dfrac{0.00100}{0.33}$
$= 0.00303$ (Elongation)

CONT.

1.5-2 CONT.

Axial stress $\sigma = E\epsilon = (70\text{ GPa})(0.00303)$
$= 212.1$ MPa (Tension)

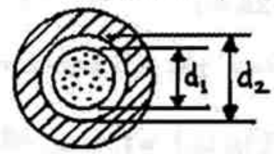
Because $\sigma < \sigma_Y$, Hooke's law is valid.

Tensile force

$$P = \sigma A = (212.1\text{ MPa})(\tfrac{\pi}{4})(12\text{ mm})^2$$

$= 24.0$ kN (Tensile force) ←

1.5-3 Nylon cylinder inside steel tube



Compression
$d_1 = 2.75$ in. $\quad \Delta d_1 = 0.01$ in.
$d_2 = 2.67$ in.
Nylon: $E = 450$ ksi $\quad \nu = 0.4$

Lateral Strain $\epsilon' = \dfrac{\Delta d_1}{d_1}$ (Increase in diameter)

Axial Strain $\epsilon = -\dfrac{\epsilon'}{\nu} = -\dfrac{\Delta d_1}{\nu d_1}$

(Minus means decrease in length)
Assume Hooke's law is valid for the nylon.

Axial stress $\sigma = E\epsilon = -\dfrac{E\Delta d_1}{\nu d_1}$

(Minus means compressive stress.)

Compressive force $P = |\sigma| A = \dfrac{EA\,\Delta d_1}{\nu d_1}$

$A = \dfrac{\pi}{4} d_1^2 \qquad P = \dfrac{\pi E d_1 \Delta d_1}{4\nu}$

Substitute numerical values:

$$P = \frac{\pi(450\text{ ksi})(2.75\text{ in.})(0.01\text{ in.})}{4\,(0.4)} = 24.3\text{ k}$$ ←

Note: $\sigma = \dfrac{P}{A} = 4.1$ ksi, which is less than the yield stress for nylon.

1.5-4 Aluminum bar in tension

$L = 3.0$ m $\quad d = 30$ mm
$E = 73$ GPa $\quad \nu = 1/3$
$\delta = 7.0$ mm

Axial strain $\epsilon = \dfrac{\delta}{L} = \dfrac{7.0\text{ mm}}{3.0\text{ m}} = 0.002333$

Lateral strain $\epsilon' = -\nu\epsilon = -(\tfrac{1}{3})(0.002333)$
$= -0.0007778$
(Minus means shortening)

Decrease in diameter

$\Delta d = |\epsilon'| d = (0.0007778)(30\text{ mm})$
$= 0.0233$ mm ←

CONT.

1.5-4 CONT.

Tensile loads

Axial stress $\sigma = E\epsilon = (73\text{ GPa})(0.002333)$
$= 170.3\text{ MPa}$

(This stress is less than the yield stress, so Hooke's law is applicable.)

$P = \sigma A = (170.3\text{ MPa})\left(\frac{\pi}{4}\right)(30\text{ mm})^2$
$= 120\text{ kN}$ ←

1.5-5 Bar of monel metal in tension

$L = 15\text{ in.}$ $d = 0.30\text{ in.}$ $P = 2500\text{ lb.}$
$E = 25{,}000\text{ ksi}$ $\nu = 0.32$

Axial Stress $\sigma = \frac{P}{A} = \frac{2500\text{ lb}}{\frac{\pi}{4}(0.30\text{ in.})^2} = 35{,}370\text{ psi}$

(This stress is less than the yield stress, so Hooke's law is applicable.)

Axial strain $\epsilon = \frac{\sigma}{E} = \frac{35{,}370\text{ psi}}{25{,}000\text{ psi}} = 0.001415$

Increase in length

$\delta = \epsilon L = (0.001415)(15\text{ in.}) = 0.0212\text{ in.}$ ←

Lateral strain $\epsilon' = -\nu\epsilon$
$= -(0.32)(0.001415)$
$= -0.0004528$

Decrease in diameter

$\Delta d = |\epsilon'|d = (0.0004528)(0.30\text{ in})$
$= 0.000136\text{ in.}$

Decrease in cross-sectional area

Original area $A_1 = \frac{\pi}{4}d^2$

Final area $A_2 = \frac{\pi}{4}(d - \Delta d)^2 = \frac{\pi}{4}[d^2 - 2d\Delta d + (\Delta d)^2]$

Decrease in area $= \Delta A = A_1 - A_2$
$= \frac{\pi}{4}(\Delta d)(2d - \Delta d)$

Percent decrease in area $= \frac{\Delta A}{A_1}(100)$
$= \frac{(\Delta d)(2d - \Delta d)}{d^2}(100)$
$= \frac{(0.000136\text{ in.})(0.5999\text{ in.})}{(0.30\text{ in.})^2}(100)$

$= 0.091\%$ ←

1.5-6 High-strength steel wire in tension

$d = 3\text{ mm}$ $\delta = 37.1\text{ mm}$ $L = 15\text{ m}$
$P = 3.5\text{ kN}$ $\Delta d = 0.0022\text{ mm}$

(a) Modulus of elasticity

$\sigma = \frac{P}{A} = \frac{3.5\text{ kN}}{\frac{\pi}{4}(3\text{ mm})^2} = 495.2\text{ MPa}$

Assume this stress is below the proportional limit so that Hooke's law is valid

$\epsilon = \frac{\delta}{L} = \frac{37.1\text{ mm}}{15\text{ m}} = 0.002473$

$E = \frac{\sigma}{\epsilon} = \frac{495.2\text{ MPa}}{0.002473} = 200\text{ GPa}$ ←

(b) Poisson's ratio

Lateral strain: $\epsilon' = -\nu\epsilon$ (Minus means decrease in diameter.)

$\Delta d = |\epsilon'|d = \nu\epsilon d$

$\nu = \frac{\Delta d}{\epsilon d} = \frac{0.0022\text{ mm}}{(0.002473)(3\text{ mm})} = 0.30$ ←

1.5-7 Hollow bronze cylinder in compression

$d_1 = 1.85\text{ in.}$ $d_2 = 2.15\text{ in.}$
$E = 16{,}000\text{ ksi.}$ $P = 35\text{ k}$
$\Delta d_2 = 0.0017\text{ in.}$ (increase in diameter)

Lateral strain $\epsilon' = \frac{\Delta d_2}{d_2} = \frac{0.0017\text{ in.}}{2.15\text{ in.}}$
$= 790.7 \times 10^{-6}$ (Increase)

(a) Increase in inner diameter

$\Delta d_1 = \epsilon' d_1 = (790.7 \times 10^{-6})(1.85\text{ in.})$
$= 0.0015\text{ in.}$ ←

(b) Increase in wall thickness

$\Delta t = \epsilon' t = (790.7 \times 10^{-6})(0.15\text{ in})$
$= 0.00012\text{ in.}$ ←

Axial stress $\sigma = \frac{P}{A}$ $A = \frac{\pi}{4}(d_2^2 - d_1^2)$
$= \frac{\pi}{4}((2.15\text{ in.})^2 - (1.85\text{ in.})^2)$
$= 0.9425\text{ in.}^2$

$\sigma = \frac{35\text{ k}}{0.9425\text{ in.}^2} = 37.14\text{ ksi}$ (compression)

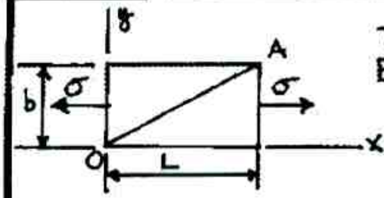
(Assume that this stress is less than the yield stress for the bronze so that Hooke's law is valid.)

Axial strain $\epsilon = \frac{\sigma}{E} = \frac{37.14\text{ ksi}}{16{,}000\text{ ksi}} = 2321 \times 10^{-6}$ (Decrease)

(c) Poisson's Ratio

$\nu = \left|\frac{\epsilon'}{\epsilon}\right| = \frac{790.7 \times 10^{-6}}{2321 \times 10^{-6}} = 0.34$ ←

1.5-8 Plate in tension



ν = Poisson's ratio
E = modulus of elasticity
σ = tensile stress in x direction
ϵ_x = strain in x direction $= \frac{\sigma}{E}$

(a) Slope of diagonal line OA

Elongation in x direction = ΔL

Shortening in y direction = Δb

$\Delta L = \epsilon_x L = \frac{\sigma L}{E} \quad \Delta b = \nu \epsilon_x b = \frac{\nu \sigma b}{E}$

$\text{Slope} = \frac{b - \Delta b}{L + \Delta L} = \frac{b(E - \nu\sigma)}{L(E + \sigma)}$

(Note that the slope decreases)

(b) Increase in area of the front face

Initial area = bL

Final area $= (b - \Delta b)(L + \Delta L)$
$= b\left(1 - \frac{\nu\sigma}{E}\right)(L)\left(1 + \frac{\sigma}{E}\right)$
$= bL\left(1 + \frac{\sigma}{E} - \frac{\nu\sigma}{E} - \frac{\nu\sigma^2}{E^2}\right)$

Increase in area = final area − initial area
$= bL\left(\frac{\sigma}{E}\right)\left(1 - \nu - \frac{\nu\sigma}{E}\right)$

Because $\frac{\sigma}{E}$ is small compared to the other terms, we may disregard the last term.

$\therefore$ Increase in area $= bL\left(\frac{\sigma}{E}\right)(1 - \nu)$ ←

(c) Decrease in cross-sectional area

t = thickness of plate

Δt = decrease in thickness

$\Delta t = \nu \epsilon_x t = \frac{\nu\sigma}{E} t$

Initial area = bt

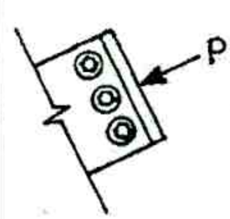
Final area $= (b - \Delta b)(t - \Delta t)$
$= \left(b - \frac{\nu\sigma b}{E}\right)\left(t - \frac{\nu\sigma}{E} t\right)$
$= bt\left(1 - \frac{\nu\sigma}{E}\right)^2 = bt\left[1 - \frac{2\nu\sigma}{E} + \left(\frac{\nu\sigma}{E}\right)^2\right]$

Decrease in area = Initial area − Final area
$= bt\left(\frac{\nu\sigma}{E}\right)\left(2 - \frac{\nu\sigma}{E}\right)$

Because $\frac{\sigma}{E}$ is small compared to the other terms, we may disregard the last term

Decrease in cross-sectional area =
$= bt\left(\frac{2\nu\sigma}{E}\right)$ ←

1.6-1 Brace connected to a column



3 bolts in single shear
P = compressive load in brace = 5.5 k
d_B = diameter of bolts = 5/8 in.
t = thickness of plates = 1/4 in.

(a) Average shear stress in bolts

$\tau_{AVER} = \frac{P}{A_B} \quad P = 5.5^k$

A_B = area of three bolts
$= 3\left(\frac{\pi}{4}\right)d_B^2 = 0.9204 \text{ in.}^2$

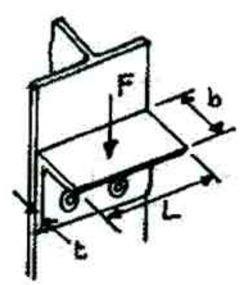
$\tau_{AVER} = \frac{5.5^k}{0.9204 \text{ in.}^2} = 5980 \text{ psi}$ ←

(b) Bearing stress between plates and bolts

$\sigma_b = \frac{P}{A_b} \quad A_b = 3 d_B t = 0.4688 \text{ in.}^2$

$\sigma_b = \frac{5.5 \text{ k}}{0.4688 \text{ in.}^2} = 11{,}730 \text{ psi}$ ←

1.6-2 Angle bracket bolted to a column



Two bolts
d = 15 mm
t = thickness of angle = 12 mm
b = 60 mm
L = 150 mm
p = pressure acting on top of the bracket = 2.0 MPa

F = resultant force acting on the bracket
$= pbL = (2.0 \text{ MPa})(60 \text{ mm})(150 \text{ mm}) = 18.0 \text{ kN}$

Bearing pressure between bracket and bolts

A_b = bearing area of one bolt
$= dt = (15 \text{ mm})(12 \text{ mm}) = 180 \text{ mm}^2$

$\sigma_b = \frac{F}{2A_b} = \frac{18.0 \text{ kN}}{2(180 \text{ mm}^2)} = 50.0 \text{ MPa}$ ←

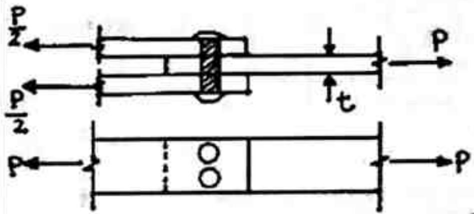
Average shear stress in the bolts

A_s = shear area of one bolt
$= \frac{\pi}{4} d^2 = \frac{\pi}{4}(15 \text{ mm})^2 = 176.7 \text{ mm}^2$

$\tau_{AVER} = \frac{F}{2A_s} = \frac{18.0 \text{ kN}}{2(176.7 \text{ mm}^2)} = 50.9 \text{ MPa}$

$= 50.9 \text{ MPa}$ ←

1.6-3 Three plates joined by two rivets



t = thickness of plates = 5/8 in.

d = diameter of rivets = 1/2 in.

$P = 10.0$ k

$\tau_{ULT} = 32$ ksi (for shear in the rivets)

(a) Maximum bearing stress on the rivets

Maximum stress occurs at the middle plate.

A_b = bearing area for one rivet $= dt$

$$\sigma_b = \frac{P}{2A_b} = \frac{P}{2dt} = \frac{10.0\text{ k}}{2(\frac{1}{2}\text{ in.})(\frac{5}{8}\text{ in.})} = 16.0\text{ ksi} \longleftarrow$$

(b) Ultimate load in shear

Shear force on two rivets $= \frac{P}{2}$

Shear force on one rivet $= \frac{P}{4}$

Let A = cross-sectional area of one rivet.

$$\text{Shear stress } \tau = \frac{P/4}{A} = \frac{P}{4\left(\frac{\pi d^2}{4}\right)} = \frac{P}{\pi d^2}$$

OR, $P = \pi d^2 \tau$

at the ultimate load:

$$P_{ULT} = \pi d^2 \tau_{ULT} = \pi\left(\tfrac{1}{2}\text{ in.}\right)^2(32\text{ ksi})$$

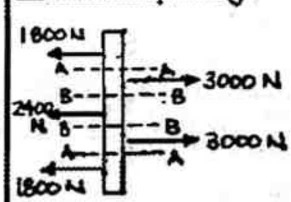
$$= 25.1\text{ k} \longleftarrow$$

1.6-4 Plates joined by a bolt

d = diameter of bolt = 6 mm

t = thickness of plates = 5 mm

Free-body diagram of bolt



Section A-A: V = 1800 N

Section B-B: V = 1200 N

V_{MAX} = max. shear force in bolt = 1800 N

(a) Maximum shear stress in bolt

$$\tau_{MAX} = \frac{V_{MAX}}{\pi\frac{d^2}{4}} = \frac{4V_{MAX}}{\pi d^2} = 63.7\text{ MPa} \longleftarrow$$

CONT.

1.6-4 CONT.

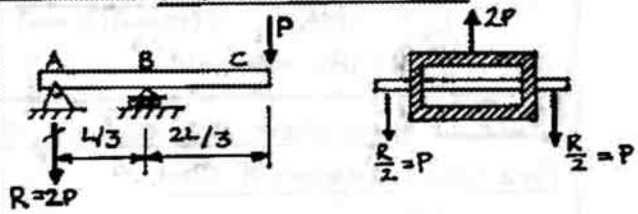
(b) Maximum bearing stress

F_{MAX} = maximum force applied by a plate against the bolt

$F_{MAX} = 3000$ N

$$\sigma_b = \frac{F_{MAX}}{dt} = 100\text{ MPa} \longleftarrow$$

1.6-5 Hollow box beam



$P = 3000$ lb.

d = diameter of pin = 7/8 in.

t = wall thickness of box beam = 1/2 in.

(a) Average shear stress in pin

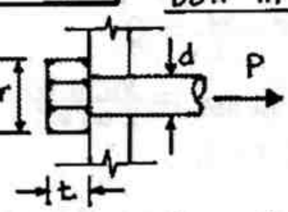
Double shear

$$\tau_{AVE} = \frac{2P}{2\left(\frac{\pi}{4}d^2\right)} = \frac{4P}{\pi d^2} = 4990\text{ psi} \longleftarrow$$

(b) Average bearing stress on pin

$$\sigma_b = \frac{2P}{2(dt)} = \frac{P}{dt} = 6860\text{ psi} \longleftarrow$$

1.6-6 Bolt in tension

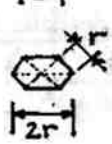


d = 12 mm

r = 10 mm

t = 6 mm

P = 9.0 kN



Area of one equilateral triangle

$$\text{Area} = \frac{r^2\sqrt{3}}{4}$$

$$\text{Area of hexagon} = \frac{3r^2\sqrt{3}}{2}$$

(a) Bearing stress between bolt head and plate

A_b = bearing area

= area of hexagon minus area of bolt

$$A_b = \frac{3r^2\sqrt{3}}{2} - \frac{\pi d^2}{4}$$

$$= \frac{3}{2}(10\text{ mm})^2\sqrt{3} - \frac{\pi}{4}(12\text{ mm})^2$$

CONT.

1.6-6 CONT.

$A_b = 259.8\ \text{mm}^2 - 113.1\ \text{mm}^2$
$= 146.7\ \text{mm}^2$

$\sigma_b = \frac{P}{A_b} = \frac{9.0\ \text{kN}}{146.7\ \text{mm}^2} = 61.3\ \text{MPa}$ ←

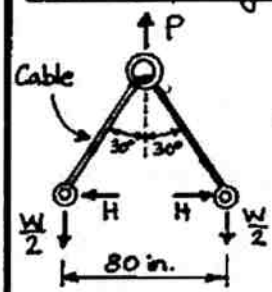
(b) Shear stress in head of bolt

A_s = shear area
$= \pi d t$

$\tau_{AVE} = \frac{P}{A_s} = \frac{P}{\pi d t} = \frac{9.0\ \text{kN}}{\pi(12\ \text{mm})(6\ \text{mm})}$
$= 39.8\ \text{MPa}$ ←

1.6-7 Steel plate hoisted by a sling

Free-body diagram of sling



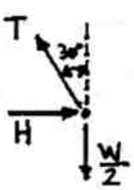
d = diameter of clevis pins
= 0.70 in.
W = weight of steel plate

Weight of steel plate

W = (weight density)(volume)
$= \gamma V$; $\gamma = 490\ \text{lb/ft}^3$
$V = (100\ \text{in.})(50\ \text{in.})(4\ \text{in.})$
$= 20{,}000\ \text{in.}^3$

$W = (490\ \text{lb/ft}^3)\left(\frac{1}{1728}\ \frac{\text{ft}^3}{\text{in.}^3}\right)(20{,}000\ \text{in.}^3)$
$= 5671\ \text{lb}$

Tensile force T in cable



$\Sigma F_{VERT} = 0$ ↑+ ↓−

$T\cos 30° - \frac{W}{2} = 0$

$T = \frac{W}{2\cos 30°} = \frac{5671\ \text{lb.}}{2\cos 30°}$
$= 3274\ \text{lb}$

Shear stress in the pins (double shear)

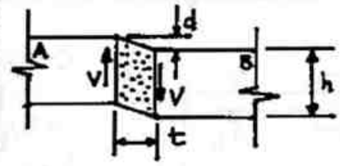
$\tau_{AVE} = \frac{T}{2A_{PIN}} = \frac{3274\ \text{lb}}{(2)(\frac{\pi}{4})(0.70\ \text{in.})^2}$
$= 4250\ \text{psi}$ ←

Bearing stress between steel plate & pins

A_b = (thickness of plate)(diameter of pin)
$= (4.0\ \text{in.})(0.70\ \text{in.}) = 2.80\ \text{in.}^2$

$\sigma_b = \frac{T}{A_b} = \frac{3274\ \text{lb}}{2.80\ \text{in.}^2} = 1170\ \text{psi}$ ←

1.6-8 Epoxy Joint between concrete slabs



h = 100 mm
t = 12 mm
L = 1.0 m
d = 0.048 mm
G = 960 MPa

(a) Average shear strain

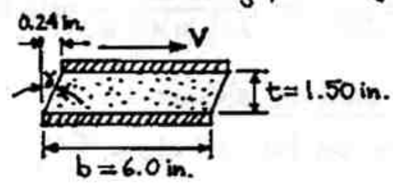
$\gamma_{AVE} = \frac{d}{t} = 0.004$ ←

(b) Shear forces V

Average shear stress : $\tau_{AVE} = G\gamma_{AVE} = 3.84\ \text{MPa}$

$V = \tau_{AVE}(hL) = 384\ \text{kN}$ ←

1.6-9 Bearing pad subjected to shear



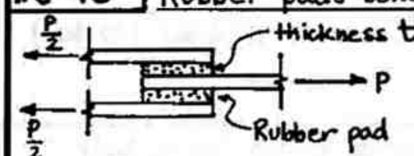
V = 1200 lb
Width of pad:
a = 5.0 in.
Length of pad:
b = 6.0 in.

$\tau_{AVE} = \frac{V}{ab} = \frac{1200\ \text{lb}}{(5.0\ \text{in.})(6.0\ \text{in.})} = 40\ \text{psi}$

$\gamma_{AVE} = \frac{0.24\ \text{in.}}{1.50\ \text{in.}} = 0.160$

$G = \frac{\tau}{\gamma} = \frac{40\ \text{psi}}{0.16} = 250\ \text{psi}$ ←

1.6-10 Rubber pads bonded to steel plates



Rubber pads: t = 12 mm
Length L = 200 mm
Width b = 150 mm
G = 830 kPa
P = 15 kN

(a) Shear stress and strain in the rubber pads

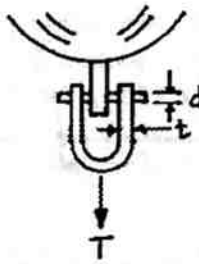
$\tau_{AVE} = \frac{P/2}{bL} = \frac{7.5\ \text{kN}}{(150\ \text{mm})(200\ \text{mm})} = 250\ \text{kPa}$

$\gamma_{AVE} = \frac{\tau_{AVER}}{G} = \frac{250\ \text{kPa}}{830\ \text{kPa}} = 0.301$ ←

(b) Horizontal displacement

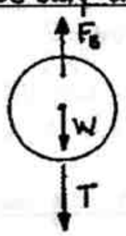
$\delta = \gamma_{AVER}\, t = (0.301)(12\ \text{mm}) = 3.61\ \text{mm}$ ←

1.6-11 Submerged buoy



d = diameter of buoy
$= 60$ in.
T = tensile force in chain
d_p = diameter of pin
$= 0.5$ in.
t = thickness of shackle
$= 0.25$ in.
W = weight of buoy
$= 1800$ lb
γ_W = weight density of sea water
$= 63.8$ lb/ft³

Free-body diagram of buoy



F_B = buoyant force of water pressure (equals the weight of the displaced sea water)

V = volume of buoy

$= \frac{\pi d^3}{6} = 65.45$ ft³

$F_B = \gamma_W V = 4{,}180$ lb

Equilibrium

$T = F_B - W = 2380$ lb

(a) Average shear stress in pin

A_p = area of pin $\quad A_p = \frac{\pi}{4} d_p^2 = 0.1963$ in.²

$\tau_{aver} = \frac{T}{2A_p} = 6060$ psi ⟵

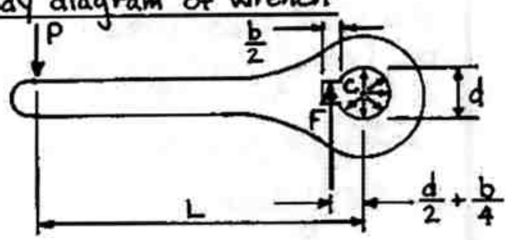
(b) Bearing stress between pin and shackle

$A = 2d_p t = 0.2500$ in.²

$\sigma_b = \frac{T}{A_b} = 9520$ psi ⟵

1.6-12 Wrench with keyway

Free-body diagram of wrench



With friction disregarded, the bearing pressures between the wrench and the shaft are radial. Because the bearing

CONT.

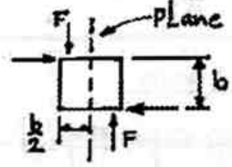
1.6-12 CONT.

pressure between the wrench and the key is uniformly distributed, the force F acts at the midpoint of the keyway.
(Width of keyway $= b/2$)

$\Sigma M_C = 0 \quad PL - F\left(\frac{d}{2} + \frac{b}{4}\right) = 0$

$F = \frac{4PL}{2d + b}$

Free-body diagram of key

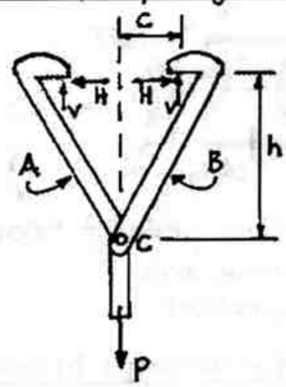


$\tau_{aver} = \frac{F}{bC}$

$= \frac{4PL}{bC(2d+b)}$ ⟵

1.6-13 Clamp supporting a load P

Free-body diagram of clamp



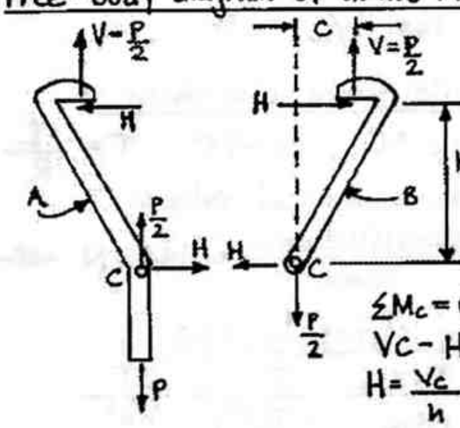
$h = 10.0$ in.
$c = 4.0$ in.
$P = 4{,}000$ lb

From vertical equilibrium:

$V = \frac{P}{2} = 2{,}000$ lb

d = diameter of pin at C
$= 0.5$ in.

Free-body diagram of arms A and B

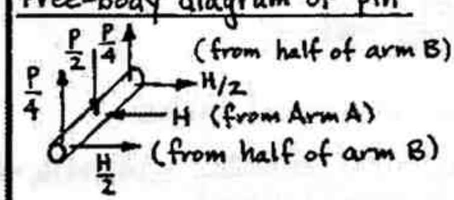


$\Sigma M_C = 0$

$Vc - Hh = 0$

$H = \frac{Vc}{h} = \frac{Pc}{2h} = 800$ lb

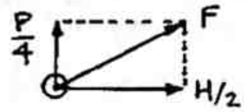
Free-body diagram of pin



CONT.

1.6-13 CONT.

Shear force F in pin

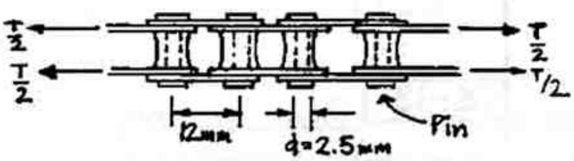


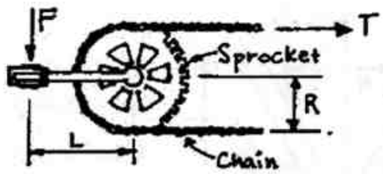
$$F = \sqrt{\left(\tfrac{P}{4}\right)^2 + \left(\tfrac{H}{2}\right)^2} = 1077 \text{ lb}$$

Average shear stress

$$\tau_{AVER} = \frac{F}{A_{PIN}} = \frac{F}{\frac{\pi d^2}{4}} = 5490 \text{ psi} \longleftarrow$$

1.6-14 Bicycle chain





F = Force applied to pedal = 800N
L = Length of crank arm
R = Radius of sprocket

Measurements (for author's bicycle)

(1) L = 162 mm (2) R = 90 mm

Note that $L/R = 1.8$

(a) Tensile force T in chain

$$\Sigma M_{axle} = 0 \quad FL = TR \quad T = \frac{FL}{R} \longleftarrow$$

Substite numerical values:

$$T = \frac{(800\text{N})(162\text{mm})}{90\text{ mm}} = 1440\text{ N} \longleftarrow$$

(b) Shear stress in pins

$$\tau_{AVE} = \frac{T/2}{A_{PIN}} = \frac{T}{2\left(\frac{\pi d^2}{4}\right)} = \frac{2T}{\pi d^2}$$

$$= \frac{2FL}{\pi d^2 R} \longleftarrow$$

Substitute numerical values:

$$\tau_{AVE} = \frac{2(800\text{N})(162\text{ mm})}{\pi(2.5\text{ mm})^2(90\text{ mm})} = 147\text{ MPa} \longleftarrow$$

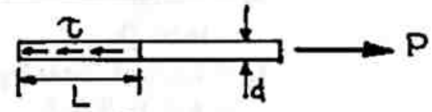
CONT.

1.6-14 CONT.

Note: Both T and τ_{AVE} are proportional to the ratio L/R, which in this case equals 162/90, OR 1.80.

1.6-15 "Pull-out test" of reinforcing bar

(a) Assume shear stress τ is constant



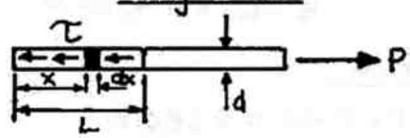
$$\tau_{AVER} = \frac{P}{A_{SHEAR}} = \frac{P}{\pi d L} \longleftarrow$$

Substitute numerical values:

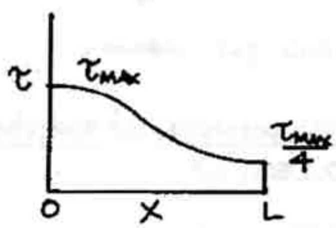
P = 4000 lb d = 0.5 in. L = 12 in.

$\tau_{AVER} = 212$ psi $\longleftarrow$

(b) Assume shear stress varies along the length L



$$\tau = \frac{\tau_{MAX}}{4L^3}(4L^3 - 9Lx^2 + 6x^3)$$



Shear force acting on element dx:

$$dF = \tau(\pi d\, dx)$$
$$= (\pi d)\left(\frac{\tau_{MAX}}{4L^3}\right)(4L^3 - 9Lx^2 + 6x^3)\,dx$$

P = total shear force

$$P = \int dF = \int_0^L (\pi d)\left(\frac{\tau_{MAX}}{4L^3}\right)(4L^3 - 9Lx^2 + 6x^3)\,dx$$

$$= \frac{\pi d\, \tau_{MAX}}{4L^3}\left[4L^3x - 9L\left(\frac{x^3}{3}\right) + 6\left(\frac{x^4}{4}\right)\right]_0^L$$

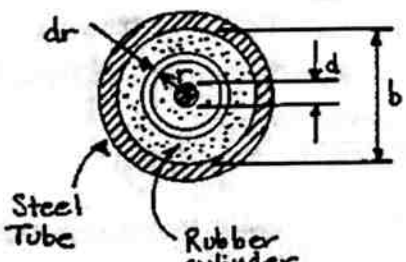
$$P = \frac{5\pi d L \tau_{MAX}}{8}$$

$$\tau_{MAX} = \frac{8P}{5\pi d L} \longleftarrow$$

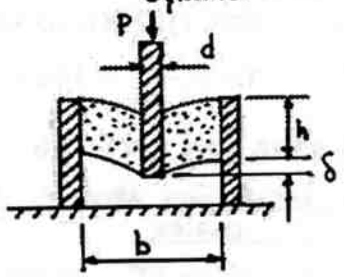
Substitute numerical values:

$\tau_{MAX} = 340$ psi $\longleftarrow$

1.6-16 Shock mount



r = radial distance from center of shock mount to element of thickness dr



(a) Shear stress τ at radial distance r

A_S = shear area at distance r

$= 2\pi r h$

$$\tau = \frac{P}{A_S} = \frac{P}{2\pi r h} \longleftarrow$$

(b) Downward displacement δ

γ = shear strain at distance r

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi r h G}$$

$d\delta$ = downward displacement for element dr

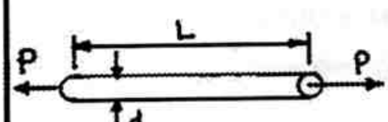
$$d\delta = \gamma\, dr = \frac{P\,dr}{2\pi r h G}$$

$$\delta = \int d\delta = \int_{\frac{d}{2}}^{\frac{b}{2}} \frac{P\,dr}{2\pi r h G}$$

$$\delta = \frac{P}{2\pi h G}\int_{\frac{d}{2}}^{\frac{b}{2}} \frac{dr}{r} = \frac{P}{2\pi h G}\left[\ln r\right]_{d/2}^{b/2}$$

$$\delta = \frac{P}{2\pi h G} \ln \frac{b}{d} \longleftarrow$$

1.7-1 Magnesium bar in tension



$L = 15.0$ in.
$d = 0.25$ in.
$E = 6.2 \times 10^6$ psi

$\sigma_{allow} = 13{,}000$ psi $\quad \delta_{max} = 0.03$ in.

CONT.

1.7-1 CONT.

$$\epsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.03 \text{ in.}}{15.0 \text{ in.}} = 0.0020$$

Maximum stress based upon elongation

$$\sigma_{max} = E\,\epsilon_{max} = (6.2 \times 10^6 \text{ psi})(0.00200)$$
$$= 12{,}400 \text{ psi}$$

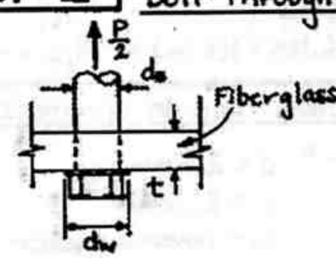
Maximum stress based upon tension

$$\sigma_{allow} = 13{,}000 \text{ psi}$$

Elongation governs

$$P_{allow} = \sigma_{max} A = (12{,}400 \text{ psi})\left(\frac{\pi}{4}\right)(0.25 \text{ in.})^2$$
$$= 609 \text{ lb} \longleftarrow$$

1.7-2 Bolt through fiberglass



$d_B = 6$ mm
$d_w = 22$ mm
$t = 10$ mm

Allowable load based upon shear stress in fiberglass

$\tau_{allow} = 2.1$ MPa

Shear area $A_S = \pi d_w t$

$$\frac{P_1}{2} = \tau_{allow} A_S = \tau_{allow}(\pi d_w t)$$
$$= (2.1 \text{ MPa})(\pi)(22 \text{ mm})(10 \text{ mm})$$
$$= 1451 \text{ N}$$

$P_1 = 2(1451 \text{ N}) = 2900$ N

Allowable load based upon bearing pressure

$\sigma_b = 3.8$ MPa

Bearing area $A_b = \frac{\pi}{4}(d_w^2 - d_B^2)$

$$\frac{P_2}{2} = \sigma_b A_b = (3.8 \text{ MPa})\left(\frac{\pi}{4}\right)\left[(22 \text{ mm})^2 - (6 \text{ mm})^2\right]$$
$$= 1337 \text{ N}$$

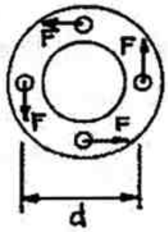
$P_2 = 2(1337 \text{ N}) = 2670$ N

Allowable load

Bearing pressure governs.

$P_{allow} = 2670 \text{ N} \longleftarrow$

1.7-3 Shafts with flanges



T_0 = torque transmitted by bolts

d_B = bolt diameter = 0.75 in.

d = diameter of bolt circle = 6 in.

$\tau_{allow} = 14$ ksi

F = shear force in one bolt

$T_0 = 4F\left(\frac{d}{2}\right) = 2Fd$

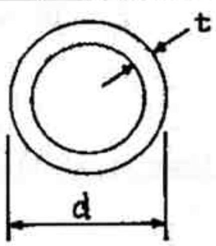
Allowable shear force in one bolt

$F = \tau_{allow} A_{bolt} = (14 \text{ ksi})\left(\frac{\pi}{4}\right)(0.75 \text{ in.})^2$
$= 6.185$ k

Maximum torque

$T_0 = 2Fd = 2(6.185 \text{ k})(6 \text{ in.}) = 74.2$ k-in ⟵

1.7-4 Aluminum tube in compression



$d = 25$ mm

$t = 2.5$ mm

d_0 = inner diameter = 20 mm

$A_{tube} = \frac{\pi}{4}(d^2 - d_0^2) = 176.7 \text{ mm}^2$

$\sigma_Y = 270$ MPa
F.S. = 4
$\sigma_{allow} = \frac{270 \text{ MPa}}{4} = 67.5$ MPa

$\sigma_U = 310$ MPa
F.S. = 5
$\sigma_{allow} = \frac{310 \text{ MPa}}{5} = 62$ MPa

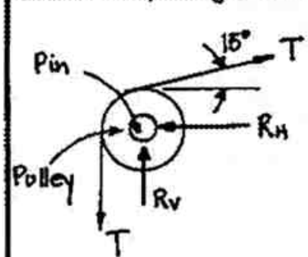
The ultimate stress governs.

Allowable compressive force

$P_{allow} = \sigma_{allow} A_{tube} = (62 \text{ MPa})(176.7 \text{ mm}^2)$
$= 11.0$ kN ⟵

1.7-5 Lifeboat supported by cables

Free-body diagram of pulley



Pin diameter $d = 0.80$ in.

T = tensile force in one cable

$T_{allow} = 1800$ lb

$\tau_{allow} = 4000$ psi

W = weight of lifeboat = 1500 lb

CONT.

1.7-5 CONT.

$\Sigma F_{horiz} = 0 \quad R_H = T\cos 15° = 0.9659T$

$\Sigma F_{vert} = 0 \quad R_V = T - T\sin 15° = 0.7412T$

V = shear force in pin

$V = \sqrt{(R_H)^2 + (R_V)^2} = 1.2175\,T$

Allowable force in cable based upon shear in the pins

$V = \tau_{allow} A_{pin} = (4000 \text{ psi})\left(\frac{\pi}{4}\right)(0.80 \text{ in.})^2 = 2011$ lb

$V = 1.2175T \qquad T = \frac{V}{1.2175} = 1652$ lb

Total allowable load $= 4T = 6608$ lb

Allowable load based upon allowable tension in cables

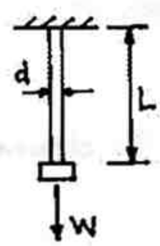
Total allowable load $= 4T_{allow} = 4(1800 \text{ lb}) = 7200$ lb

Shear in the pins governs

Total allowable load = 6608 lb

Maximum load that should be carried =
$= 6608 \text{ lb} - 1500 \text{ lb}$
$= 5100$ lb ⟵

1.7-6 Wire hanging from a balloon



$d = 2.0$ mm

$L = 75$ m

$\sigma_Y = 350$ MPa

Margin of safety = 1.5

Factor of safety = $n = 2.5$

$\sigma_{allow} = \frac{\sigma_Y}{n} = 140$ MPa

Weight density of steel: $\gamma = 77.0 \text{ kN/m}^3$

Weight of wire: $W_0 = \gamma A L = \gamma\left(\frac{\pi d^2}{4}\right)(L)$
$= (77.0 \text{ kN/m}^3)\left(\frac{\pi}{4}\right)(2.0 \text{ mm})^2(75 \text{ m})$
$= 18.1$ N

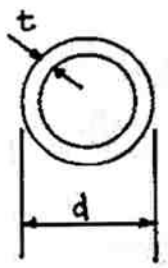
Total load $= W_{max} + W_0$

$W_{max} + W_0 = \sigma_{allow} A = \sigma_{allow}\left(\frac{\pi d^2}{4}\right)$

$W_{max} = \sigma_{allow}\left(\frac{\pi d^2}{4}\right) - W_0$
$= (140 \text{ MPa})\left(\frac{\pi}{4}\right)(2.0 \text{ mm})^2 - 18.1$ N
$= 439.8 \text{ N} - 18.1$ N

$W_{max} = 422$ N ⟵

1.7-7 Cast iron piers in compression



Four piers
$\sigma_U = 50$ ksi
$n = 4$
$\sigma_{ALLOW} = 12.5$ ksi
$d = 4.5$ in.
$t = 0.4$ in.
$d_o = 3.7$ in.

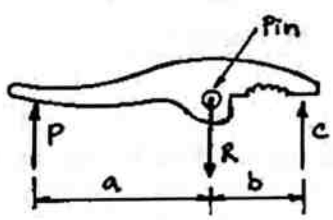
$A = \frac{\pi}{4}(d^2 - d_o^2) = \frac{\pi}{4}[(4.5 \text{ in.})^2 - (3.7 \text{ in.})^2]$
$= 5.152 \text{ in.}^2$

P_1 = Allowable load on one pier
$= \sigma_{ALLOW} A = (12.5 \text{ ksi})(5.152 \text{ in.}^2)$
$= 64.40$ k

Total load $= 4P_1 = 4(64.40 \text{ k}) = 258$ k ←

1.7-8 Forces in pliers

Free-body diagram of one arm



C = clamping force
R = reaction at pin
$a = 90$ mm
$b = 40$ mm
d = diameter of pin = 6 mm

$\Sigma M_{PIN} = 0$

$cb - Pa = 0$

$c = \frac{Pa}{b} \qquad P = \frac{cb}{a}$

$\Sigma F_{VERT} = 0$ ↑+↓−

$P + c - R = 0$
$R = P + C = P(1 + \frac{a}{b}) = c(1 + \frac{b}{a})$

V = R = shear force in pin

Maximum possible clamping force C_{ULT}

$\tau_{ULT} = 320$ MPa
$V_{ULT} = \tau_{ULT} A_{PIN} = (320 \text{ MPa})(\frac{\pi}{4})(6 \text{ mm})^2$
$= 9.048$ kN

$C_{ULT} = \frac{V_{ULT}}{1 + \frac{b}{a}} = \frac{9.048 \text{ kN}}{1 + \frac{40 \text{ mm}}{90 \text{ mm}}} = 6.26$ kN ←

Maximum allowable load P_{ALLOW}

$P_{MAX} = \frac{V_{ULT}}{1 + \frac{a}{b}} = \frac{9.048 \text{ kN}}{1 + \frac{90 \text{ mm}}{40 \text{ mm}}} = 2.784$ kN

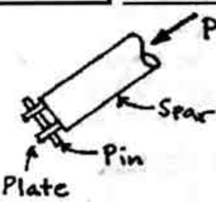
CONT.

1.7-8 CONT.

$P_{ALLOW} = \frac{P_{MAX}}{n} = \frac{2.784 \text{ kN}}{3.5} = 795$ N ←

Note: An applied force P = 20 N produces a clamping force C = 45 N.

1.7-9 Pin connection for a ship's spar



Spar: $d_2 = 3.125$ in.
$d_1 = 2.750$ in.
Pin: $d = 1.0$ in.
Plates: $t = 0.5$ in.

Available load P based upon compression in the spar

$\sigma_c = 11.0$ ksi
$A_c = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[(3.125 \text{ in.})^2 - (2.750 \text{ in.})^2]$
$= 1.7303 \text{ in.}^2$
$P_1 = \sigma_c A_c = (11.0 \text{ ksi})(1.7303 \text{ in.}^2) = 19.03$ k

Allowable load P based upon shear in the pin (double shear)

$\tau_{ALLOW} = 7.0$ ksi
$A_s = 2\left(\frac{\pi d^2}{4}\right) = \frac{\pi}{2}(1.0 \text{ in.})^2 = 1.5708 \text{ in.}^2$
$P_2 = \tau_{ALLOW} A_s = (7.0 \text{ ksi})(1.5708 \text{ in.}^2) = 11.00$ k

Allowable load P based upon bearing

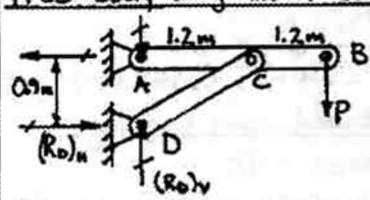
$\sigma_b = 17.0$ ksi
$A_b = 2dt = 2(1.0 \text{ in.})(0.5 \text{ in.}) = 1.0 \text{ in.}^2$
$P_3 = \sigma_b A_b = (17.0 \text{ ksi})(1.0 \text{ in.}^2) = 17.00$ k

Allowable compressive load in the spar

Shear in the pin governs.
$P_{allow} = 11.0$ k ←

1.7-10 Beam ACB supported by a strut CD

Free-body diagram of structure

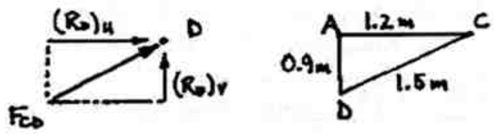


$\Sigma M_A = 0 \qquad -P(2.4 \text{ m}) + (R_D)_H(0.9 \text{ m}) = 0$
$(R_D)_H = \frac{8}{3}P$

CONT.

1.7-10 CONT.

Reaction at joint D



F_{CD} = compressive force in strut

$F_{CD} = (R_D)_H\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)\left(\frac{8P}{3}\right) = \frac{10P}{3}$

Shear in bolt (double shear)

$A_{bolt} = \frac{\pi d^2}{4}$ $\quad V_{allow} = \tau_{allow} A_{bolt}$

$= \tau_{allow}\left(\frac{\pi d^2}{4}\right)$

Allowable compressive force in strut

$(F_{CD})_{allow} = 2V_{allow} = 2\tau_{allow}\left(\frac{\pi d^2}{4}\right)$

Allowable load P

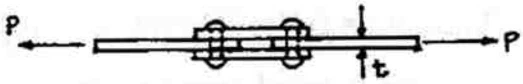
$P_{allow} = \frac{3}{10}(F_{CD})_{allow} = \frac{3\pi}{20}\tau_{allow} d^2$ ⟵

Substitute numerical values:

$\tau_{allow} = 90$ MPa $\quad d = 16$ mm

$P_{allow} = 10.9$ kN ⟵

1.7-11 Splice between two flat bars



Ultimate load based upon tension in the bars

Cross-sectional area of bars: $A = bt$

$b = 0.875$ in.

$t = 0.375$ in.

$P_1 = \sigma_{ult} A = \sigma_{ult} bt = (60\text{ ksi})(0.875\text{ in.})(0.375\text{ in.})$

$= 19.69$ k

Ultimate load based upon shear in the rivets

Double shear $\quad d$ = diameter of rivets

$= 0.5$ in.

A_r = area of one rivet

$= \frac{\pi d^2}{4} = 0.1963\text{ in}^2$

$P_2 = 2\tau_{ult} A_r = 2(30\text{ ksi})(0.1963\text{ in}^2) = 11.78$ k

Ultimate load based upon bearing

A_b = bearing area = dt

$P_3 = \sigma_b A_b = (80\text{ ksi})(0.5\text{ in.})(0.375\text{ in.}) = 15.00$ k

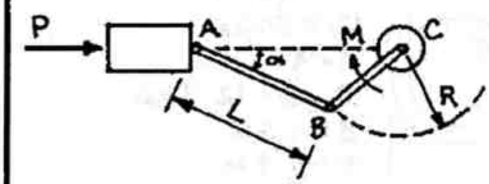
Ultimate load

Shear governs ; $P_{ult} = 11.78$ k

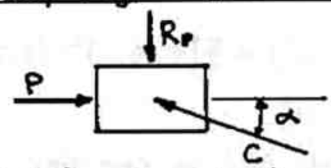
Allowable load

$P_{allow} = \frac{P_{ult}}{n} = \frac{11.78}{2.5} = 4.71$ k ⟵

1.7-12 Piston and connecting rod



Free-body diagram of piston



P = applied force (constant)

C = compressive force in connecting rod

R_P = resultant of reaction forces between cylinder and piston (no friction)

$\Sigma F_{horiz} = 0$ $\xrightarrow{+}$ $\xleftarrow{-}$ $\quad P - C\cos\alpha = 0$

$P = C\cos\alpha$

Maximum compressive force C in connecting rod

$C_{max} = \sigma_c A$

Maximum allowable force P

$P = C_{max}\cos\alpha$

$= \sigma_c A\cos\alpha$

The maximum allowable force P occurs when $\cos\alpha$ has its smallest value, which means that α has its largest value.

Largest value of α

The largest value of α occurs when point B is the farthest distance from line AC. The farthest distance is the radius R of the crank arm.

Therefore,

$\overline{BC} = R$

Also, $\overline{AC} = \sqrt{L^2 - R^2}$

$\cos\alpha = \frac{\sqrt{L^2-R^2}}{L} = \sqrt{1-\left(\frac{R}{L}\right)^2}$

(a) Maximum allowable force P

$P_{allow} = \sigma_c A\sqrt{1-\left(\frac{R}{L}\right)^2}$ ⟵

(b) Substitute numerical values:

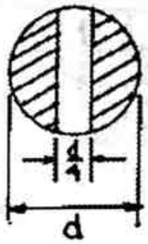
$\sigma_c = 150$ MPa

$A = 63.62\text{ mm}^2$ $\quad R = 0.28L$

$P_{allow} = 9.16$ kN ⟵

1.7-13 Bar with a hole

Cross section of bar



From Case 15, Appendix D:

$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$

$r = \frac{d}{2} \qquad a = \frac{d}{8}$

$b = \sqrt{r^2 - \left(\frac{d}{8}\right)^2} = d\sqrt{\frac{15}{64}} \quad \frac{d}{8}\sqrt{15}$

$\alpha = \arccos \frac{d/8}{r}$

$= \arccos\left(1/4\right)$

$$A = 2\left(\frac{d}{2}\right)^2\left[\arccos\frac{1}{4} - \frac{\left(\frac{d}{8}\right)\left(\frac{d}{8}\sqrt{15}\right)}{(d/2)^2}\right]$$

$$= \frac{d^2}{2}\left(\arccos\frac{1}{4} - \frac{\sqrt{15}}{16}\right) = 0.5380\,d^2$$

(a) Allowable load in tension

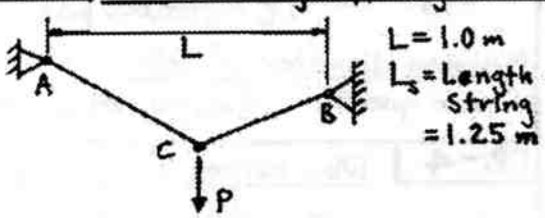
$P_{allow} = \sigma_{allow} A = 0.5380\,d^2\,\sigma_{allow} \longleftarrow$

(b) Substitute numerical values

$\sigma_{allow} = 10\text{ ksi} \quad d = 1.5\text{ in.}$

$P_{allow} = 12.1\text{ k} \longleftarrow$

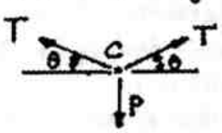
1.7-14 Flexible string supporting a load P



$L = 1.0$ m

L_s = Length of String $= 1.25$ m

Free-body diagram of point C

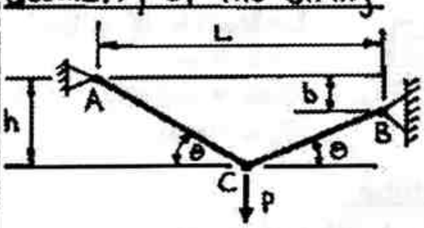


T = tensile force in string

$\sum F_{vert} = 0$

$P = 2T\sin\theta \qquad$ (Eq. 1)

Geometry of the string



Horizontal distance between supports A and B

$L = h\cot\theta + (h-b)\cot\theta = (2h-b)\cot\theta$

$2h - b = L\tan\theta \qquad$ (Eq. 2)

CONT.

1.7-14 CONT.

Length of String ACB

$L_s = \frac{h}{\sin\theta} + \frac{h-b}{\sin\theta} = \frac{2h-b}{\sin\theta}$

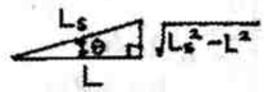
$2h - b = L_s\sin\theta \qquad$ (Eq. 3)

From Equations (2) and (3):

$L\tan\theta = L_s\sin\theta \quad$ or $\quad L = L_s\cos\theta$

$\cos\theta = \frac{L}{L_s} \qquad$ (Eq. 4)

Angle θ (Draw a right triangle)



$\cos\theta = \frac{L}{L_s}$

$\sin\theta = \frac{\sqrt{L_s^2 - L^2}}{L_s}$

$= \sqrt{1 - \left(\frac{L}{L_s}\right)^2} \qquad$ (Eq. 5)

Forces

From Eq. (1): $P = 2T\sin\theta$

$P = 2T\sqrt{1 - \left(\frac{L}{L_s}\right)^2} \qquad$ (Eq. 6)

Allowable tensile force T in the string

S = breaking strength of string

n = factor of safety

$T_{allow} = \frac{S}{n} \qquad$ (Eq. 7)

Allowable load P

From Eq. (6): $P = 2T\sqrt{1 - \left(\frac{L}{L_s}\right)^2}$

$\therefore P_{allow} = 2T_{allow}\sqrt{1 - \left(\frac{L}{L_s}\right)^2}$

$= \frac{2S}{n}\sqrt{1 - \left(\frac{L}{L_s}\right)^2} \longleftarrow$

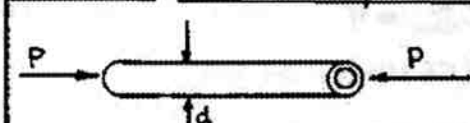
Substitute numerical values:

$S = 200\text{ N} \qquad n = 3$

$L = 1.0\text{ m} \qquad L_s = 1.25\text{ m}$

$P_{allow} = 80\text{ N} \longleftarrow$

1.8-1 Tubular Compression member



$P = 49^k$

$t = 0.4$ in.

$\sigma_{allow} = 9500$ psi

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = \frac{\pi}{4}(4t)(d-t)$

$= \pi t(d-t)$

$P = \sigma_{allow}A = \pi t(d-t)\,\sigma_{allow}$

Solve for d: $\quad d = \frac{P}{\pi t\,\sigma_{allow}} + t$

CONT.

1.8-1 CONT.

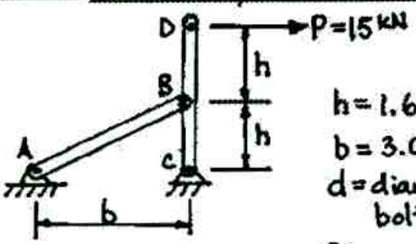
Substitute numerical values:

$$d_{min} = \frac{49\text{ k}}{\pi(0.4\text{ in.})(9500\text{ psi})} + 0.4\text{ in.}$$

$$= 4.105\text{ in.} + 0.4\text{ in.}$$

$$d_{min} = 4.50\text{ in.} \longleftarrow$$

1.8-2 Vertical pipe with a brace



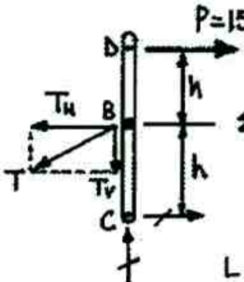
$h = 1.6\text{ m}$

$b = 3.0\text{ m}$

d = diameter of bolt at A

$\tau_{allow} = 55\text{ MPa}$

Free-body diagram of column CD



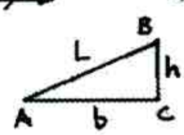
T = tensile force in AB

L = length of AB

$\Sigma M_C = 0$

$T_H(h) - P(2h) = 0$

$T_H = 2P = 30\text{ kN}$



$L = \sqrt{b^2 + h^2} = 3.4\text{ m}$

$$\frac{T}{T_H} = \frac{L}{b} \qquad T = T_H\left(\frac{L}{b}\right) = (30\text{ kN})\left(\frac{3.4\text{ m}}{3.0\text{ m}}\right) = 34\text{ kN}$$

Shear in the bolt at support A

Shear force = T = 34 kN

Double shear

$A_{bolt} = \frac{\pi}{4}d^2$

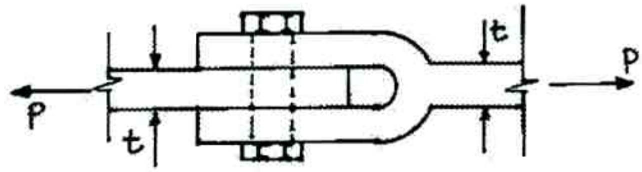
$2A_{bolt}\,\tau_{allow} = T$

$2\left(\frac{\pi d^2}{4}\right)(55\text{ MPa}) = 34\text{ kN}$

$d^2 = 393.55\text{ mm}^2$

$d_{min} = 19.8\text{ mm} \longleftarrow$

1.8-3 Bolted connection



Single bolt in double shear

$P = 1800\text{ lb}$

$\tau_{allow} = 12{,}000\text{ psi}$

$\sigma_b = 20{,}000\text{ psi}$

$t = 5/16\text{ in.}$

Find minimum diameter of bolt.

Based upon shear in the bolt

$$A_{bolt} = \frac{P}{2\tau_{allow}} \qquad \frac{\pi d^2}{4} = \frac{P}{2\tau_{allow}}$$

$$d^2 = \frac{2P}{\pi\tau_{allow}}$$

$$d_1 = \sqrt{\frac{2P}{\pi\tau_{allow}}} = \sqrt{\frac{2(1800\text{ lb})}{\pi(12{,}000\text{ psi})}} = 0.309\text{ in.}$$

Based upon bearing between plate and bolt

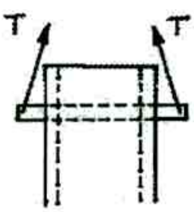
$$A_{bearing} = \frac{P}{\sigma_b} \qquad dt = \frac{P}{\sigma_b}$$

$$d = \frac{P}{t\sigma_b} \qquad d_2 = \frac{1800\text{ lb}}{(\frac{5}{16}\text{ in.})(20{,}000\text{ psi})} = 0.288\text{ in.}$$

Minimum diameter of bolt

Shear governs $\quad d_{min} = 0.309\text{ in.} \longleftarrow$

1.8-4 Tube hoisted by a crane



T = tensile force in cable

W = weight of steel tube

d = diameter of pin

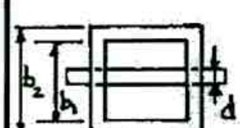
b_1 = inner dimension of tube $= 210\text{ mm}$

b_2 = outer dimension of tube $= 250\text{ mm}$

L = length of tube $= 6.0\text{ m}$

$\tau_{allow} = 60\text{ MPa}$

$\sigma_b = 90\text{ MPa}$



Weight of tube

γ_s = weight density of steel $= 77.0\text{ kN/m}^3$

A = area of tube $= b_2^2 - b_1^2 = 18{,}400\text{ mm}^2$

CONT.

1.8-4 CONT.

$W = \gamma_s A L = (77.0\ \text{kN/m}^3)(18{,}400\ \text{mm}^2)(6.0\ \text{m})$
$= 8.501\ \text{kN}$

Diameter of pin based upon shear

Double shear. $2\tau_{allow} A_{pin} = W$

$2(60\ \text{MPa})\left(\frac{\pi}{4}\right)d^2 = 8.501\ \text{kN}$ $\quad d^2 = 90.20\ \text{mm}^2$
$d_1 = 9.497\ \text{mm}$

Diameter of pin based upon bearing

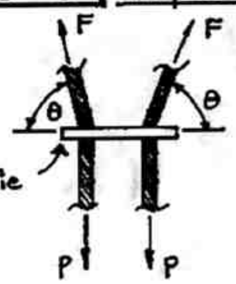
$\sigma_b(b_2 - b_1)d = W$

$(90\ \text{MPa})(40\ \text{mm})d = 8.501\ \text{kN}$ $\quad d_2 = 2.361\ \text{mm}$

Minimum diameter of pin

Shear governs. $\quad d_{min} = 9.50\ \text{mm}$ ⟵

1.8-5 Suspender tie on a suspension bridge



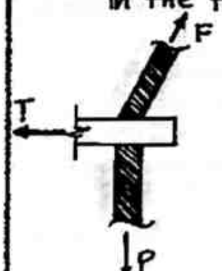
F = tensile force in cable above tie

P = tensile force in cable below tie

σ_{allow} = allowable tensile stress in the tie

Free-body diagram of half the tie

Note: Include a small amount of the cable in the free-body diagram



T = tensile force in the tie

Force triangle

$\cot\theta = \frac{T}{P}$

$T = P\cot\theta$

(a) Minimum required area of tie

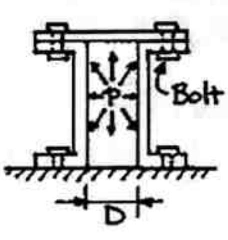
$$A_{min} = \frac{T}{\sigma_{allow}} = \frac{P\cot\theta}{\sigma_{allow}} \longleftarrow$$

(b) Substitute numerical values:

$P = 30\ \text{k} \quad \theta = 75° \quad \sigma_{allow} = 12{,}000\ \text{psi}$

$A_{min} = 0.670\ \text{in.}^2$ ⟵

1.8-6 Pressurized cylinder



$p = 1900\ \text{kPa}$
$D = 250\ \text{mm}$
$d_B = 12\ \text{mm}$
$\sigma_{allow} = 70\ \text{MPa}$
n = number of bolts

CONT.

1.8-6 CONT.

F = total force acting on the cover plate from the internal pressure

$F = p\left(\frac{\pi D^2}{4}\right)$

Number of bolts

P = tensile force in one bolt

$P = \frac{F}{n} = \frac{\pi p D^2}{4n}$

A_B = area of one bolt $= \frac{\pi}{4} d_B^2$

σ_B = tensile stress in a bolt

$\sigma_B = \frac{P}{A_B} = \frac{\pi p D^2}{4n}\left(\frac{4}{\pi d_B^2}\right) = \frac{pD^2}{n d_B^2}$

$$n = \frac{pD^2}{d_B^2\sigma_{allow}} \longleftarrow$$

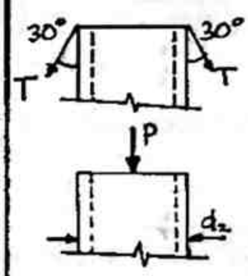
Substitute numerical values:

$p = 1900\ \text{kPa} \quad D = 250\ \text{mm} \quad d_B = 12\ \text{mm}$

$\sigma_{allow} = 70\ \text{MPa}$

$n = 11.8 \quad \therefore\ n_{min} = 12$ bolts ⟵

1.8-7 Tubular post with guy cables



d_2 = outer diameter
d_1 = inner diameter
t = wall thickness
$= 0.5$ in.
T = tensile force in a cable
$= 32\ \text{k}$
$\sigma_{allow} = 6000\ \text{psi}$
P = compressive force in post
$= 2T\cos 30°$

Required area of post

$$A = \frac{P}{\sigma_{allow}} = \frac{2T\cos 30°}{\sigma_{allow}}$$

Area of post

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}\left[d_2^2 - (d_2 - 2t)^2\right]$$
$$= \pi t(d_2 - t)$$

Equate areas and solve for d_2:

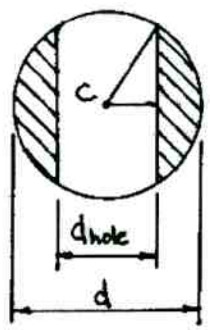
$$\frac{2T\cos 30°}{\sigma_{allow}} = \pi t(d_2 - t)$$

$$d_2 = \frac{2T\cos 30°}{\pi t\,\sigma_{allow}} + t \longleftarrow$$

Substitute numerical values:

$(d_2)_{min} = 6.38$ in. ⟵

1.8-8 Bar with a hole



$d_{hole} = 28$ mm
$P = 125$ kN
$\sigma_Y = 250$ MPa
$n = 2.0$
$\sigma_{allow} = 125$ MPa

Case 15, Appendix D

$a = d_{hole}/2 = 14$ mm $\qquad r = d/2$

$\alpha = \arccos \frac{a}{r} = \arccos \frac{14 \text{ mm}}{d/2} = \arccos \frac{28}{d}$ (d = millimeters)

$b = \sqrt{r^2 - a^2} = \sqrt{\left(\frac{d}{2}\right)^2 - 196}$

$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$

$= 2\left(\frac{d}{2}\right)^2\left[\arccos \frac{28}{d} - \frac{14}{d^2/4}\sqrt{\left(\frac{d}{2}\right)^2 - 196}\right]$

$A = \frac{d^2}{2} \arccos \frac{28}{d} - 28\sqrt{\left(\frac{d}{2}\right)^2 - 196}$ (Eq. 1)

Required area

$A = \frac{P}{\sigma_{allow}} = \frac{125 \text{ kN}}{125 \text{ MPa}} = 1\,000 \text{ mm}^2$ (Eq. 2)

Equate (1) and (2):

$\frac{d^2}{2} \arccos \frac{28}{d} - 28\sqrt{\left(\frac{d}{2}\right)^2 - 196} = 1000 \text{ mm}^2$

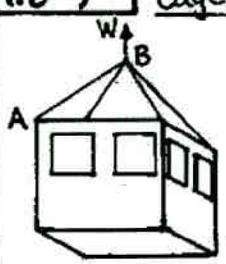
Rearrange the equation:

$d^2 \arccos \frac{28}{d} - 28\sqrt{d^2 - 784} - 2000 = 0$ (d = millimeters)

Solve for the minimum diameter d:

$d_{min} = 56.6$ mm ◄

1.8-9 Cage hoisted by a crane



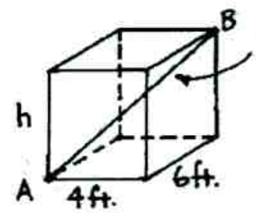
Dimensions of cage: 8 ft × 12 ft
Length of a cable: 14 ft
Weight of cage and contents:
$W = 10{,}800$ lb
Breaking strength of a cable:
$\sigma_{ult} = 84$ ksi
Factor of safety $n = 3.5$

$\sigma_{allow} = \frac{\sigma_{ult}}{n} = \frac{84 \text{ ksi}}{3.5} = 24{,}000$ psi

CONT.

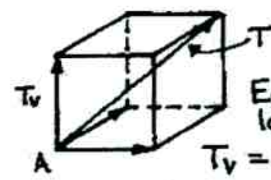
1.8-9 CONT.

Geometry of a cable



length = 14 ft.
h = height
$h^2 + (4 \text{ ft})^2 + (6 \text{ ft})^2 = (14 \text{ ft})^2$
$h = 12$ ft

Force in a cable



T = force in one cable
T_V = vertical component of T
Each cable carries the same load.

$T_V = \frac{W}{4} = \frac{10{,}800 \text{ lb}}{4} = 2700$ lb

$\frac{T}{T_V} = \frac{14 \text{ ft}}{h} = \frac{14 \text{ ft}}{12 \text{ ft}} = \frac{7}{6}$

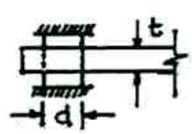
$T = \frac{7}{6} T_V = 3150$ lb

Required area of cable

$A_C = \frac{T}{\sigma_{allow}} = \frac{3150 \text{ lb}}{24{,}000 \text{ psi}} = 0.131 \text{ in.}^2$ ◄

(Note: The diameter of the cable can not be calculated from this area, because a cable does not have a solid circular cross section. A cable consists of several strands wound together. For details, see Section 2.2)

1.8-10 Bar with pin connection



Width of bar: $b = 60$ mm
$t = 10$ mm
$\sigma_{allow} = 140$ MPa
$\tau_{allow} = 80$ MPa
d = diameter of pin

Allowable load based upon tension in bar

$P_1 = \sigma_{allow} A_{net} = \sigma_{allow}(b - d)t$

$= (140 \text{ MPa})(60 \text{ mm} - d)(10 \text{ mm}) = 1400(60 - d)$

P_1 = Newtons $\quad$ d = millimeters

$= 84000 - 1400d$ (Eq. 1)

Allowable load based upon shear in pin

Double shear

$P_2 = 2\tau_{allow}\left(\frac{\pi d^2}{4}\right) = \tau_{allow}\left(\frac{\pi d^2}{2}\right)$

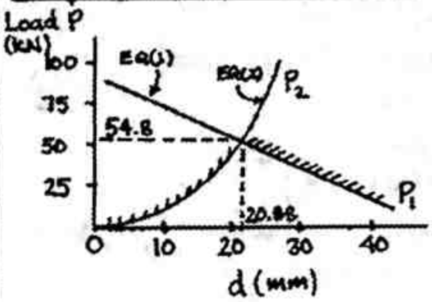
$= (80 \text{ MPa})\left(\frac{\pi}{2}\right) d^2$ $\qquad P_2$ = newtons $\quad$ d = millimeters

$= 40\pi d^2$ (Eq. 2)

CONT.

1.8-10 CONT.

Graph of Equations (1) and (2)



(a) Maximum load occurs when $P_1 = P_2$:

$84{,}000 - 1400d = 40\pi d^2$

Solve quadratic equation: $d = 20.88$ mm

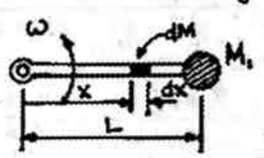
$d_m = 20.9$ mm ⟵

(b) Maximum load

$P_{max} = 84000 - 1400d = 40\pi d^2$

$= 54{,}800\text{ N} = 54.8\text{ kN}$ ⟵

1.8-11 Rotating arm



ρ = mass density of arm

σ_t = allowable stress in tension

Find required area A.

F_1 = inertia force of mass M_1 $(M_1 L\omega^2)$

F_2 = inertia force in arm $\left(\int_0^L (dM)\, x\, \omega^2\right)$

But $dM = \rho A\, dx$

$F_2 = \rho A\omega^2 \int_0^L x\, dx = \frac{1}{2}\rho A\omega^2 L^2$

Total inertia force at the pivot $= F_1 + F_2$

$$A = \frac{F_1 + F_2}{\sigma_t} = \frac{M_1 L\omega^2}{\sigma_t} + \frac{\rho A\omega^2 L^2}{2\sigma_t}$$

Solve for A:

$$A = \frac{2M_1 L\omega^2}{2\sigma_t - \rho L^2\omega^2}$$ ⟵

1.8-12 Pipe column in compression

$P = 240$ kN

$\sigma_{allow} = 80$ MPa

d = outer diameter

t = wall thickness $\frac{d}{t} \le 12$

Column to have the least weight.

Area of column for least weight

$$A = \frac{P}{\sigma_{allow}} = \frac{240\text{ kN}}{80\text{ MPa}} = 3000\text{ mm}^2$$

CONT.

1.8-12 CONT.

Area of column

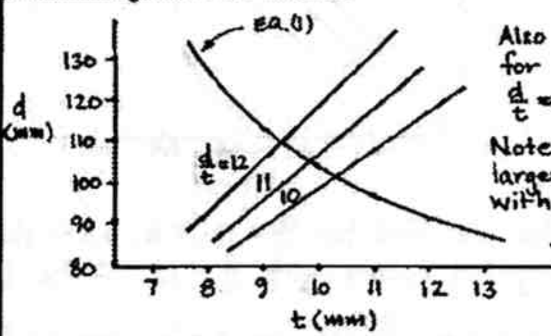
$$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = \pi t(d-t)$$

Equate areas and solve for d:

$\pi t(d-t) = 3000\text{ mm}^2$

$d = \frac{3000}{\pi t} + t$ (Eq. 1) UNITS: millimeters

Plot a graph of Eq. (1):



Also plot lines for $\frac{d}{t} = 12, 11, 10$.

Note that the largest d occurs with $\frac{d}{t} = 12$.

$\therefore$ Use $\frac{d}{t} = 12$

Substitute $\frac{d}{t} = 12$, or $t = \frac{d}{12}$, into Eq. (1) and Solve for d:

$d^2 = \frac{432{,}000}{11\pi} = 12{,}500\text{ mm}^2$ $d_{max} = 111.8$ mm

SAY, $d_{max} = 112$ mm ⟵

$t = \frac{d}{12} = 9.317$ mm

$A = \pi t(d-t) = 3000\text{ mm}^2$ (Check)

$\sigma = \frac{P}{A} = \frac{240\text{ kN}}{3000\text{ mm}^2} = 80$ MPa (Check)

1.8-13 Pipe column in compression

$P = 60$ k

$\sigma_{allow} = 12$ ksi

d = outer diameter

t = wall thickness $\frac{d}{t} \le 12$

Column to have the least weight.

Area of column for least weight

$$A = \frac{P}{\sigma_{allow}} = \frac{60\text{ k}}{12\text{ ksi}} = 5.0\text{ in.}^2$$

Area of column

$$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = \pi t(d-t)$$

Equate areas and solve for d:

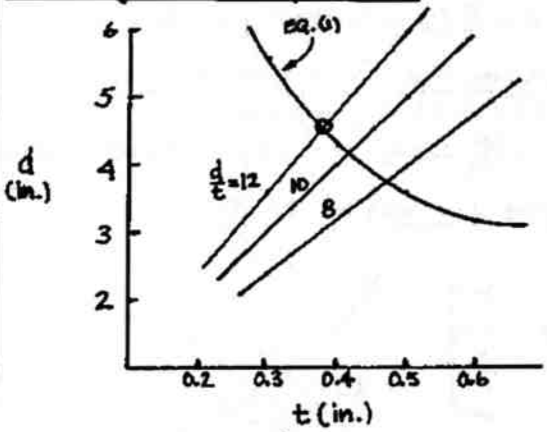
$\pi t(d-t) = 5.0\text{ in.}^2$

$d = \frac{5}{\pi t} + t$ (Eq. 1) Units: inches

CONT.

1.8-13 CONT.

Plot a graph of Eq. (1):



Also plot lines for $\frac{d}{t} = 12, 10, 8$. Note that the largest d occurs with $\frac{d}{t} = 12$. $\therefore$ Use $\frac{d}{t} = 12$

Substitute $\frac{d}{t} = 12$, or $t = \frac{d}{12}$, into Eq. (1) and solve for d:

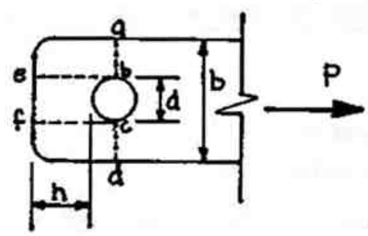
$d^2 = \frac{720}{11\pi}$ $\quad d^2 = 20.835 \text{ in.}^2$ $\quad d_{max} = 4.56$ in. ⟵

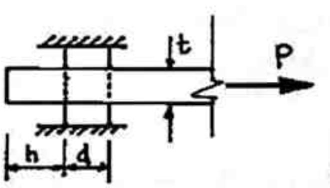
$t = \frac{d}{12} = 0.3804$ in.

$A = \pi t(d-t) = 5.000 \text{ in.}^2$ (CHECK)

$\sigma = \frac{P}{A} = \frac{60\text{k}}{5.00 \text{ in.}^2} = 12$ ksi (CHECK)

1.8-14 Bar with a pin connection





$b = 50$ mm
$t = 6$ mm
d = diameter of hole and pin
$h = 25$ mm
$\sigma_1 = 112$ MPa (tension on net cross section of bar)
$\tau_2 = 64$ MPa (shear stress in pin)
$\tau_3 = 43$ MPa (along planes eb and fc)

Units used in the following calculations:

P is in kN
σ and τ are in N/mm^2 (same as MPa)
t, b, h, and d are in mm

Tension in bar

$P_1 = \sigma_1 t(b-d) = (112 \text{ MPa})(6 \text{ mm})(50 \text{ mm} - d)\left(\frac{1}{1000}\right)$

$= 0.6720(50-d)$ (Eq. 1)

CONT.

1.8-14 CONT.

Shear in pin

$P_2 = \tau_2(2)(\pi d^2/4)$

$= (64 \text{ MPa})(2)(\pi d^2/4)\left(\frac{1}{1000}\right)$

$= 0.10053\, d^2$ (Eq. 2)

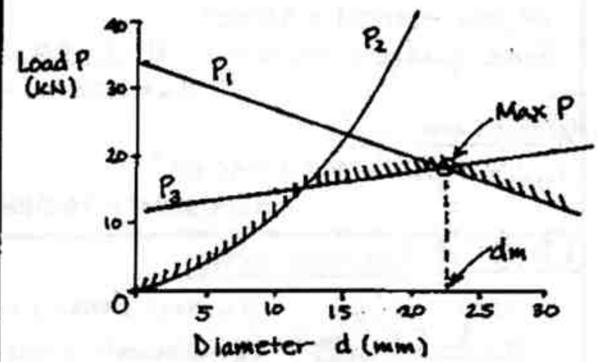
Shear on planes eb and fc

$P_3 = \tau_3(2)(t)\left(h + \frac{d}{2}\right)$

$= (43 \text{ MPa})(2)(6 \text{ mm})\left(25 \text{ mm} + \frac{d}{2}\right)\left(\frac{1}{1000}\right)$

$= 0.2580(50+d)$ (Eq. 3)

Graph of Eqs. (1), (2), and (3)



(a) Maximum P when $P_1 = P_3$

$0.6720(50-d) = 0.2580(50+d)$

Solving, $d = 22.26$ mm

$d_m = 22.3$ mm ⟵

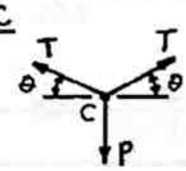
(b) Load P_{max}

Substitute d_m into Eq. (1) or (3):

$P_{max} = 18.6$ kN ⟵

1.8-15 Two bars supporting a load P

Joint C



T = tensile force in each bar

$\Sigma F_{vert} = 0$

$T = \frac{P}{2 \sin\theta}$

Area of bars

$$A = \frac{T}{\sigma_{allow}} = \frac{P}{2\sigma_{allow} \sin\theta}$$

Weight of truss

γ = weight density of material

$L_{AC} = L_{BC} = \frac{L}{2\cos\theta}$

$W = 2\gamma A L_{AC} = 2\gamma\left(\frac{P}{2\sigma_{allow}\sin\theta}\right)\left(\frac{L}{2\cos\theta}\right)$

$W = \frac{\gamma P L}{2\sigma_{allow}\sin\theta\cos\theta}$ (Eq. 1)

CONT.

1.8-15 CONT.

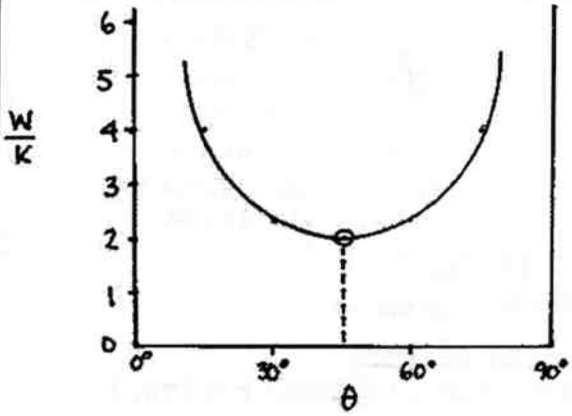
γ, P, L, and σ_{allow} are constants

W varies only with θ

Let $k = \dfrac{\gamma P L}{2\sigma_{allow}}$ (k has units of force)

$$\frac{W}{k} = \frac{1}{\sin\theta\cos\theta} \qquad (\text{Eq. } 2)$$

Graph of Eq. (2)



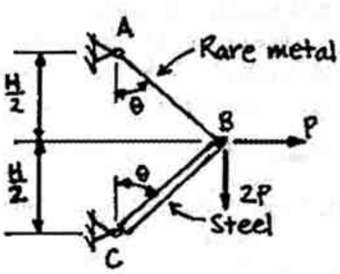
W is minimum when $\theta = 45°$ ←

Alternate solution for θ

$$\frac{d}{d\theta}\left(\frac{W}{k}\right) = \frac{(\sin\theta)(\cos\theta)(0) - 1(-\sin^2\theta + \cos^2\theta)}{\sin^2\theta\cos^2\theta} = 0$$

$$\sin^2\theta = \cos^2\theta$$
$$\sin\theta = \cos\theta$$
$$\therefore\ \theta = 45° \leftarrow$$

1.8-16 Two bars supporting two loads



γ = weight density of the rare metal

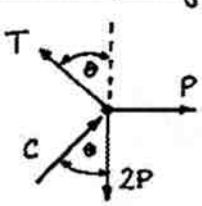
σ_t = allowable stress

T = tensile force in the bar AB

C = compressive force in the bar BC

L = length of bar AB $= \dfrac{H}{2\cos\theta}$

Free-body diagram of joint B



$\Sigma F_{VERT} = 0$:

$T\cos\theta + C\cos\theta - 2P = 0$

$\Sigma F_{HORIZ} = 0$:

$-T\sin\theta + C\cos\theta + P = 0$

CONT.

1.8-16 CONT.

Solve the equations: $T = \dfrac{P}{2}\left(\dfrac{1}{\sin\theta} + \dfrac{2}{\cos\theta}\right)$ (Eq. 1)

Cross-sectional area of bar AB

$$A_{ab} = \frac{T}{\sigma_t} = \frac{P}{2\sigma_t}\left(\frac{1}{\sin\theta} + \frac{2}{\cos\theta}\right)$$

Volume of bar AB

$$V = A_{ab}L = \frac{PH}{4\sigma_t\cos\theta}\left(\frac{1}{\sin\theta} + \frac{2}{\cos\theta}\right)$$

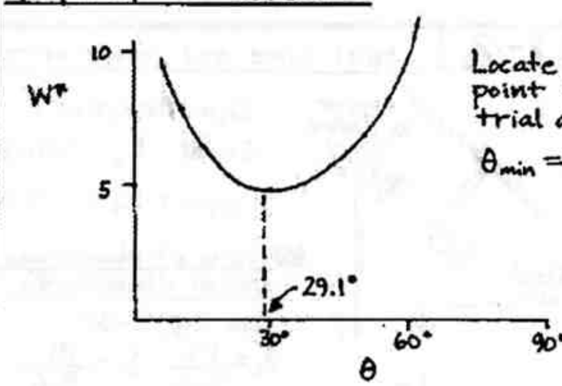
Weight of bar AB

$$W = \gamma V = \frac{PH\gamma}{4\sigma_t}\left(\frac{1}{\sin\theta\cos\theta} + \frac{2}{\cos^2\theta}\right) \qquad (\text{Eq. } 2)$$

Let $W^* = \dfrac{W}{PH\gamma/4\sigma_t}$ (NON-DIMENSIONAL WEIGHT)

$$W^* = \frac{1}{\sin\theta\cos\theta} + \frac{2}{\cos^2\theta} \qquad (\text{Eq. } 3)$$

Graph of W^* versus θ



Locate minimum point by trial and error

$\theta_{min} = 29.1°$

$\therefore\ \theta_{min} = 29.1°$ ←

Weight of bar AB

Substitute $\theta_{min} = 29.1°$ in (Eq. 3):

$W^* = 4.97$

$$W_{min} = W^*\left(\frac{PH\gamma}{4\sigma_t}\right) = 1.24\left(\frac{PH\gamma}{\sigma_t}\right)$$

Alternate Solution for θ_{min}

Determine minimum value of θ by differentation

From Eq. (3):

$$\frac{dW^*}{d\theta} = \frac{(\sin\theta\cos\theta)(0) - 1(-\sin\theta\sin\theta + \cos\theta\cos\theta)}{(\sin\theta\cos\theta)^2} + (4\cos\theta\sin\theta)/\cos^4\theta$$

Combine terms and arrange:

$$\frac{dW^*}{d\theta} = \frac{\cos\theta - 2\cos^3\theta + 4\sin^3\theta}{\cos^3\theta\sin^2\theta}$$

Set $\dfrac{dW^*}{d\theta}$ equal to zero:

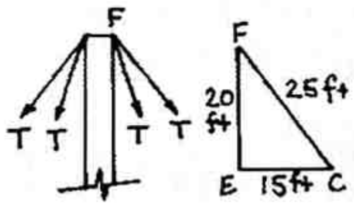
$$\cos\theta - 2\cos^3\theta + 4\sin^3\theta = 0$$

Find the root of this equation:

$\theta_{min} = 29.104°$ SAY, $\theta_{min} = 29.1°$ ←

— END OF CHAPTER 1 —

2.2-1 Pole with guy wires



$E = 10{,}000$ ksi

$A = 18.0$ in.^{2}

T = tension in guy wire $= 3000$ lb

V = vertical component of T $= \frac{20}{25} T = 2400$ lb

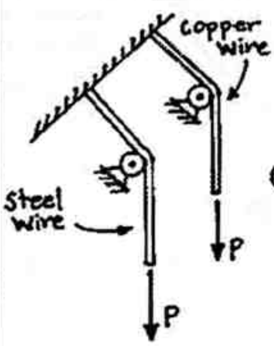
P = compressive force in pole $= 4V = 9600$ lb

δ = shortening of pole

$$= \frac{PL}{EA} = \frac{(9600 \text{ lb})(20 \text{ ft})(12 \text{ in./ft})}{(10{,}000 \text{ ksi})(18.0 \text{ in.}^2)}$$

$$= 0.0128 \text{ in.} \longleftarrow$$

2.2-2 Steel wire and copper wire



Equal lengths

Steel: $E_s = 206$ GPa

Copper: $E_c = 115$ GPa

(a) Ratio of elongations (equal diameters)

Use Eq. (2-3):

$$\delta_c = \frac{PL}{E_c A} \qquad \delta_s = \frac{PL}{E_s A}$$

$$\frac{\delta_c}{\delta_s} = \frac{E_s}{E_c} = 1.79 \longleftarrow$$

(b) Ratio of diameters (equal elongations)

$$\delta_c = \delta_s \qquad \frac{PL}{E_c A_c} = \frac{PL}{E_s A_s} \qquad E_c A_c = E_s A_s$$

$$E_c\left(\frac{\pi}{4}\right) d_c^2 = E_s\left(\frac{\pi}{4}\right) d_s^2$$

$$\frac{d_c^2}{d_s^2} = \frac{E_s}{E_c} \qquad \frac{d_c}{d_s} = \sqrt{\frac{E_s}{E_c}} = 1.34 \longleftarrow$$

2.2-3 Bridge section lifted by a cable

$A = 0.268$ in.^{2} (from Table 2-1)

$W = 7$ tons $= 14{,}000$ lb

$E = 20 \times 10^6$ psi

$L = 40$ ft $= 480$ in.

(a) Stretch of cable

$$\delta = \frac{PL}{EA} = \frac{(14{,}000 \text{ lb})(480 \text{ in.})}{(20 \times 10^6 \text{ psi})(0.268 \text{ in.}^2)}$$

$$= 1.25 \text{ in.} \longleftarrow$$

CONT.

2.2-3 CONT.

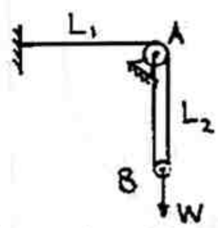
(b) Factor of safety

$P_{ult} = 51{,}900$ lb (from Table 2-1)

$P_{max} = 10$ tons $= 20{,}000$ lb

$$n = \frac{P_{ult}}{P_{max}} = \frac{51{,}900 \text{ lb}}{20{,}000 \text{ lb}} = 2.60 \longleftarrow$$

2.2-4 Cage supported by a cable



$d_A = 300$ mm

$d_B = 150$ mm

$L_1 = 4.6$ m

$L_2 = 10.5$ m

$EA = 10{,}600$ kN

$W = 32$ kN

Force in cable

$$F = \frac{W}{2} = 16 \text{ kN}$$

Length of cable

$$L = L_1 + 2L_2 + \frac{1}{4}(\pi d_A) + \frac{1}{2}(\pi d_B)$$

$$= 4600 \text{ mm} + 21{,}000 \text{ mm} + 236 \text{ mm} + 236 \text{ mm}$$

$$= 26{,}072 \text{ mm}$$

Elongation of cable

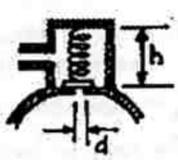
$$\delta = \frac{FL}{EA} = \frac{WL}{2EA} = \frac{(32 \text{ kN})(26{,}072 \text{ mm})}{(2)(10{,}600 \text{ kN})} = 39.4 \text{ mm}$$

Lowering of the cage

h = distance the cage moves downward

$$h = \frac{1}{2}\delta = 19.7 \text{ mm} \longleftarrow$$

2.2-5 Safety valve



h = height of valve (compressed length of the spring)

d = diameter of discharge hole

p = pressure in tank

p_{max} = pressure when valve opens

L = natural length of spring ($L > h$)

k = stiffness of spring

Force in compressed spring

$F = k(L - h)$ (From Eq. 2-1a)

Pressure force on spring

$$P = p_{max}\left(\frac{\pi d^2}{4}\right)$$

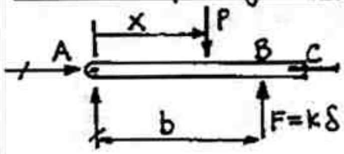
Equate forces and solve for h:

$$F = P \qquad k(L - h) = \frac{\pi p_{max} d^2}{4}$$

$$h = L - \frac{\pi p_{max} d^2}{4k} \longleftarrow$$

2.2-6 Pointer supported by a spring

Free-body diagram of pointer



$P = 12$ N
$k = 1400$ N/m
$b = 200$ mm
δ = displacement of spring
F = force in spring $= k\delta$

$\Sigma M_A = 0$ ↺+ ↻−

$-Px + k\delta b = 0 \quad$ or $\quad \delta = \dfrac{Px}{kb}$

Let α = angle of rotation of pointer ABC

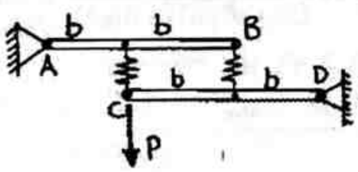
$\tan\alpha = \dfrac{\delta}{b} = \dfrac{Px}{kb^2} \qquad x = \dfrac{kb^2}{P}\tan\alpha$ ←

Substitute numerical values:

$\alpha = 2°$

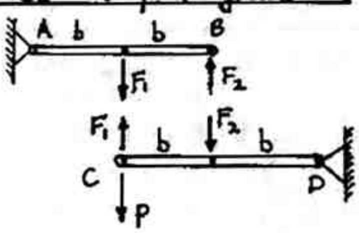
$x = \dfrac{(1400\ \text{N/m})(200\ \text{mm})^2}{12\ \text{N}}\tan 2° = 163\ \text{mm}$ ←

2.2-7 Two bars connected by springs



k = stiffness of springs
δ_c = displacement at point C

Free-body diagrams



F_1 = tensile force in first spring
F_2 = compressive force in second spring

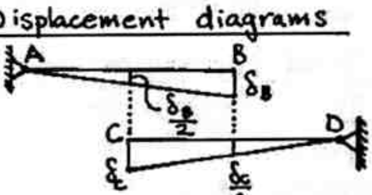
Equilibrium

$\Sigma M_A = 0 \qquad -bF_1 + 2bF_2 = 0 \qquad F_1 = 2F_2$

$\Sigma M_D = 0 \qquad 2bP - 2bF_1 + bF_2 = 0 \qquad F_2 = 2F_1 - 2P$

Solving, $\quad F_1 = \dfrac{4P}{3} \qquad F_2 = \dfrac{2P}{3}$

Displacement diagrams



Δ_1 = elongation of first spring $= \delta_c - \dfrac{\delta_B}{2}$

CONT.

2.2-7 CONT.

Δ_2 = shortening of second spring
$= \delta_B - \dfrac{\delta_c}{2}$

Also, $\quad \Delta_1 = \dfrac{F_1}{k} = \dfrac{4P}{3k}$

$\Delta_2 = \dfrac{F_2}{k} = \dfrac{2P}{3k}$

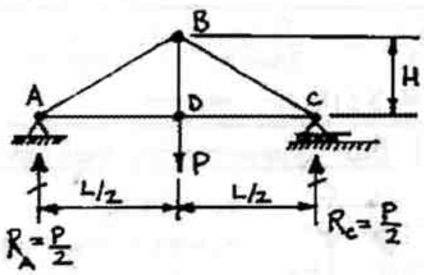
Solve the equations

$\Delta_1 = \Delta_1 \qquad \delta_c - \dfrac{\delta_B}{2} = \dfrac{4P}{3k}$

$\Delta_2 = \Delta_2 \qquad \delta_B - \dfrac{\delta_c}{2} = \dfrac{2P}{3k}$

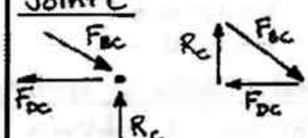
Eliminate δ_B and obtain δ_c: $\quad \delta_c = \dfrac{20P}{9k}$ ←

2.2-8 Truss with vertical load



$L = 6.0$ m
$H = 1.5$ m
$A = 3000\ \text{mm}^2$
$E = 200$ GPa

Joint C



$\dfrac{F_{DC}}{R_C} = \dfrac{L/2}{H}$

$F_{DC} = \dfrac{PL}{4H}$ (Tension)

By symmetry,
$F_{AD} = F_{DC} = \dfrac{PL}{4H}$. Because bars AD and DC are in tension, joint C displaces to the right.

(a) Elongation of ADC

$\delta_c = \dfrac{F_{DC}L}{EA} = \dfrac{PL^2}{4EAH}$ ←

Substitute numerical values:

$P = 120$ kN

$\delta_c = \dfrac{PL^2}{4EAH} = \dfrac{(120\ \text{kN})(6.0\ \text{m})^2}{4(200\ \text{GPa})(3000\ \text{mm}^2)(1.5\ \text{m})}$

$= 1.20$ mm ←

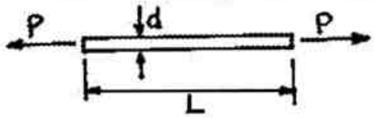
(b) Maximum permissible load P_{max}

$(\delta_c)_{MAX} = 2.0$ mm

$\dfrac{120\ \text{kN}}{1.20\ \text{mm}} = \dfrac{P_{max}}{2.0\ \text{mm}}$

$P_{max} = 200$ kN ←

2.2-9 Steel rod in tension



$d = 0.375$ in.
$E = 30 \times 10^6$ psi
$P = 3400$ lb
$A = \frac{\pi d^2}{4} = 0.11045 \text{ in.}^2$
$L = 20$ in.

(a) Final length of rod

$$\delta = \frac{PL}{EA} = \frac{(3400 \text{ lb})(20 \text{ in.})}{(30 \times 10^6 \text{ psi})(0.11045 \text{ in.}^2)} = 0.02052$$

$L_{final} = L + \delta = 20.02$ in. ⟵

(b) Ratio of length to elongation

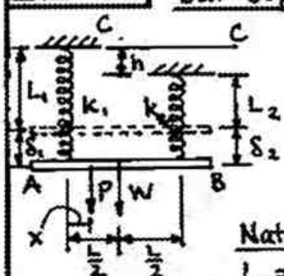
$$\text{Ratio} = \frac{20 \text{ in.}}{0.02052 \text{ in.}} = 975 \longleftarrow$$

(c) Maximum load P_{max} if $\delta_{max} = 0.02$ in.

$$\frac{P_{max}}{P} = \frac{\delta_{max}}{\delta} \qquad \frac{P_{max}}{3400 \text{ lb}} = \frac{0.02 \text{ in.}}{0.02052 \text{ in.}}$$

$\therefore P_{max} = 3310$ lb ⟵

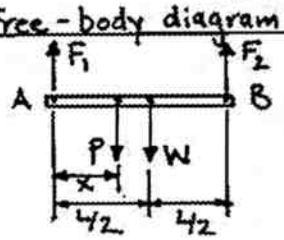
2.2-10 Bar supported by two springs



$W = 24$ N
$k_1 = 4200$ N/m $\quad k_2 = 1200$ N/m
$L = 300$ mm
$h = 10$ mm
$P = 15$ N

Natural lengths of springs
$L_1 = 80$ mm $\quad L_2 = 60$ mm
Find distance x for bar AB to be horizontal.

Free-body diagram of bar AB



$\Sigma M_A = 0$ ↶+ ↷−

$$F_2 L - Px - \frac{WL}{2} = 0 \qquad \text{(Eq. 1)}$$

$\Sigma F_{vert} = 0$ ↑+ ↓−

$$F_1 + F_2 - P - W = 0 \qquad \text{(Eq. 2)}$$

Solve Eqs. (1) and (2):

$$F_1 = P\left(1 - \frac{x}{L}\right) + \frac{W}{2} \qquad F_2 = \frac{Px}{L} + \frac{W}{2} \qquad \text{(Eq. 3 \& Eq. 4)}$$

Elongations of springs

$$\delta_1 = \frac{F_1}{k_1} = \frac{P}{k_1}\left(1 - \frac{x}{L}\right) + \frac{W}{2k_1} \qquad \text{(Eq. 5)}$$

$$\delta_2 = \frac{F_2}{k_2} = \frac{Px}{k_2 L} + \frac{W}{2k_2} \qquad \text{(Eq. 6)}$$

Bar AB remains horizontal

∴ Points A & B are the same distance below line C-C (see figure above).

$$\therefore L_1 + \delta_1 = h + L_2 + \delta_2 \qquad \text{(Eq. 7)}$$

CONT.

2.2-10 CONT.

Substitute from Eqs. (5) and (6):

$$L_1 + \frac{P}{k_1}\left(1 - \frac{x}{L}\right) + \frac{W}{2k_1} = h + L_2 + \frac{Px}{k_2 L} + \frac{W}{2k_2} \qquad \text{(Eq. 8)}$$

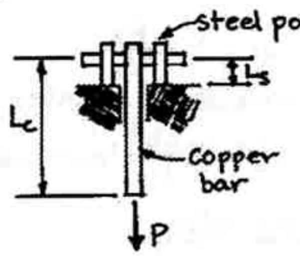
Solve for x:

$$x = \frac{L(k_1 k_2)}{P(k_1 + k_2)}\left[L_1 - L_2 - h + \frac{W}{2}\left(\frac{1}{k_1} - \frac{1}{k_2}\right) + \frac{P}{k_1}\right] \qquad \text{(Eq. 9)}$$

Substitute numerical values into Eq. (9):

$x = 120$ mm ⟵

2.2-11 Copper bar in tension



$L_c = 25$ ft $= 300$ in.
$A_c = 7.5 \text{ in.}^2 \quad E_c = 15 \times 10^6$ psi
$L_s = 42$ in. $\quad E_s = 30 \times 10^6$ psi
$A_s = 7.0 \text{ in.}^2$ (one bar)

(a) Downward displacements ($P = 90$ k)

$$\delta_c = \frac{PL_c}{E_c A_c} = \frac{(90 \text{ k})(300 \text{ in.})}{(15 \times 10^6 \text{ psi})(7.5 \text{ in.}^2)} = 0.240 \text{ in.}$$
(elongation)

$$\delta_s = \frac{(P/2)(L_s)}{E_s A_s} = \frac{(45 \text{ k})(42 \text{ in.})}{(30 \times 10^6 \text{ psi})(7.0 \text{ in.}^2)} = 0.009 \text{ in.}$$
(shortening)

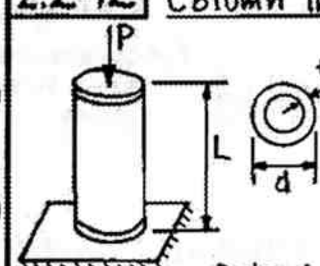
$\delta = \delta_c + \delta_s = 0.249$ in. ⟵

(b) Maximum load P_{max} if $\delta_{max} = 0.30$ in.

$$\frac{P_{max}}{P} = \frac{\delta_{max}}{\delta} \qquad \frac{P_{max}}{90 \text{ k}} = \frac{0.300 \text{ in.}}{0.249 \text{ in.}}$$

$P_{max} = 108$ k ⟵

2.2-12 Column in compression



$P = 500$ kN
$E = 210$ GPa
$L = 2.5$ m
$d = 200$ mm
$\sigma_{allow} = 55$ MPa
$\delta_{allow} = 0.60$ mm

Determine the required thickness t_{min}

Required Area Based upon allowable stress

$$\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma_{allow}} = \frac{500 \text{ kN}}{55 \text{ MPa}} = 9091 \text{ mm}^2$$

Required Area Based upon allowable shortening

$$\delta = \frac{PL}{EA} \qquad A = \frac{PL}{E\delta_{allow}} = \frac{(500 \text{ kN})(2.5 \text{ m})}{(210 \text{ GPa})(0.60 \text{ mm})} = 9921 \text{ mm}^2$$

Shortening governs $\quad A_{min} = 9921 \text{ mm}^2$

CONT.

2.2-12 CONT.

Minimum thickness

$A = \frac{\pi}{4}[d^2 - (d-2t)^2]$ OR $\frac{4A}{\pi} - d^2 = -(d-2t)^2$

$(d-2t)^2 = d^2 - \frac{4A}{\pi}$ OR $d - 2t = \sqrt{d^2 - \frac{4A}{\pi}}$

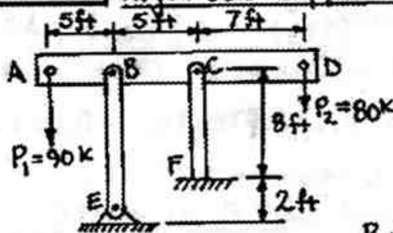
$t = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \frac{A}{\pi}}$ OR $t_{min} = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \frac{A_{min}}{\pi}}$

Substitute numerical values:

$$t_{min} = \frac{200\text{ mm}}{2} - \sqrt{\left(\frac{200\text{mm}}{2}\right)^2 - \frac{9921\text{mm}^2}{\pi}}$$

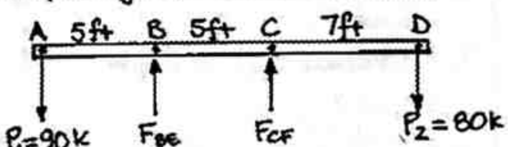
$t_{min} = 17.3$ mm ←

2.2-13 Rigid beam supported by vertical bars



$A_{BE} = 19.5\text{ in.}^2$
$A_{CF} = 16.8\text{ in.}^2$
$E = 29 \times 10^6$ psi
$L_{BE} = 10\text{ft} = 120\text{in.}$
$L_{CF} = 8\text{ ft} = 96\text{ in.}$
$P_1 = 90\text{k}$ $P_2 = 80\text{K}$

Free-body diagram of bar ABCD



$\Sigma M_B = 0$ (+ ↶) (− ↷)

$(90\text{k})(5\text{ft}) + F_{CF}(5\text{ft}) - (80\text{k})(12\text{ft}) = 0$

$F_{CF} = 102\text{k}$ (compression)

$\Sigma M_C = 0$ (+ ↶) (− ↷)

$(90\text{k})(10\text{ft}) - F_{BE}(5\text{ft}) - (80\text{k})(7\text{ft}) = 0$

$F_{BE} = 68\text{ k}$ (compression)

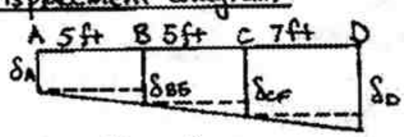
Shortening of bar BE:

$$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA_{BE}} = \frac{(68\text{k})(120\text{in.})}{(29\times10^6\text{psi})(19.5\text{ in}^2)} = 0.01443\text{ in.}$$

Shortening of bar CF:

$$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA_{CF}} = \frac{(102\text{k})(96\text{ in.})}{(29\times10^6\text{psi})(16.8\text{ in.}^2)} = 0.02010\text{ in.}$$

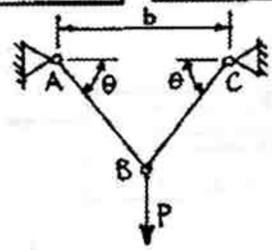
Displacement diagram



$\delta_A = \delta_{BE} - (\delta_{CF} - \delta_{BE}) = 0.00876$ in. ←
(Downward)

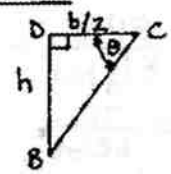
$\delta_D = \delta_{CF} + \left(\frac{7}{5}\right)(\delta_{CF} - \delta_{BE}) = 0.0280$ in. ←
(Downward)

2.2-14 Two wires supporting a load



$b = 1.0$ m
$\theta = 55°$
$P = 225$ N
$EA = 165$ kN

Dimensions

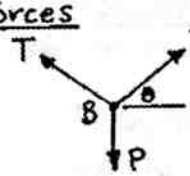


$L_{DC} = \frac{b}{2} = 500$ mm

$h = \frac{b}{2}\tan\theta = 714.074$ mm

$L_{BC} = \frac{b}{2\cos\theta} = 871.723$ mm

Forces

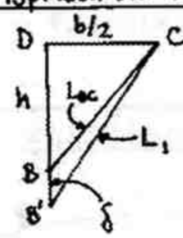


$$T = \frac{P}{2\sin\theta} = 137.337\text{ N}$$

Elongation of wire BC

$$\delta_{BC} = \frac{TL_{BC}}{EA} = \frac{Pb}{4EA\sin\theta\cos\theta} = 0.725576\text{ mm}$$

Displacement diagram



$L_1 = L_{BC} + \delta_{BC} = 872.449$ mm

Triangle DB'C:

$(h+\delta)^2 + \left(\frac{b}{2}\right)^2 = L_1^2$

$(h+\delta)^2 = L_1^2 - \left(\frac{b}{2}\right)^2 = 511{,}167\text{ mm}^2$

$h + \delta = 714.960$ mm

$\delta = 714.960\text{ mm} - h$

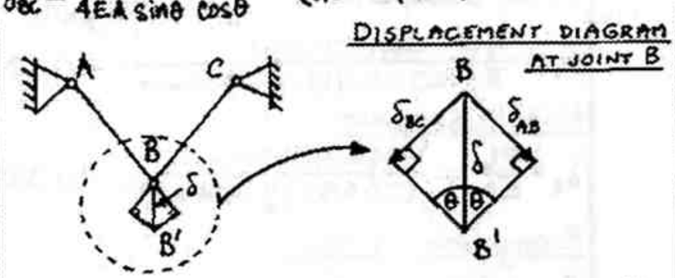
$= 714.960\text{ mm} - 714.074\text{ mm}$

$\delta = 0.886$ mm ←

Alternate solution

$T = \frac{P}{2\sin\theta}$ (as before)

$\delta_{BC} = \frac{Pb}{4EA\sin\theta\cos\theta}$ (as before)



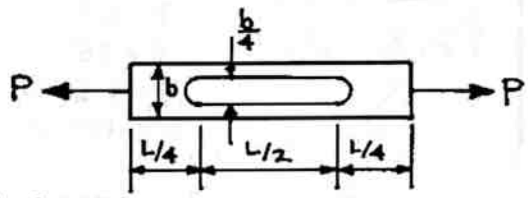
Assume that the displacements are very small.

$$\delta = \frac{\delta_{AB}}{\sin\theta} = \frac{\delta_{BC}}{\sin\theta} = \frac{Pb}{4EA\sin^2\theta\cos\theta}$$

Substitute numerical values:

$\delta = 0.886$ mm (CHECK) ←

2.3-1 Bar with a slot



t = thickness

(a) Elongation

$$\delta = \sum \frac{NL}{EA} = \frac{P(L/4)}{E(bt)} + \frac{P(L/2)}{E(\frac{3}{4}bt)} + \frac{P(L/4)}{E(bt)}$$

$$= \frac{PL}{Ebt}\left(\frac{1}{4} + \frac{4}{6} + \frac{1}{4}\right) = \frac{7PL}{6Ebt} \longleftarrow$$

(b) Substitute numerical values:

σ = stress in middle part of bar $= 24{,}000$ psi

$L = 30$ in. $\quad E = 30 \times 10^6$ psi

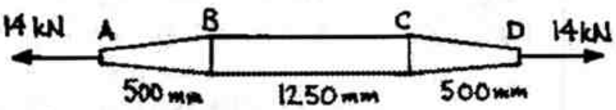
A = area of middle part of bar $= \frac{3}{4}bt$

$$\sigma = \frac{P}{A} = \frac{4P}{3bt} \qquad \frac{P}{bt} = \frac{3\sigma}{4}$$

$$\delta = \frac{7PL}{6Ebt} = \frac{7\sigma L}{8E} = \frac{7(24{,}000\text{ psi})(30\text{ in.})}{(8)(30 \times 10^6\text{ psi})}$$

$$\delta = 0.021 \text{ in.} \longleftarrow$$

2.3-2 Bar with tapered ends



$d_A = d_D = 12$ mm

$d_B = d_C = 24$ mm

$E = 120$ GPa

End segment

From Example 2-4: $\quad \delta = \frac{4PL}{\pi E d_A d_B}$

$$\delta_1 = \frac{4(14\text{ kN})(500\text{ mm})}{\pi(120\text{ GPa})(12\text{ mm})(24\text{ mm})} = 0.2579 \text{ mm}$$

Middle segment

$$\delta_2 = \frac{PL}{EA} = \frac{(14\text{ kN})(1250\text{ mm})}{(120\text{ GPa})(\frac{\pi}{4})(24\text{ mm})^2} = 0.3224 \text{ mm}$$

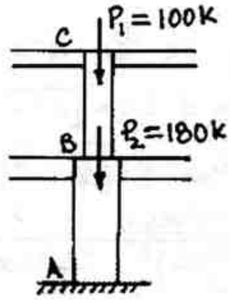
Elongation of bar

$$\delta = \sum \frac{NL}{EA} = 2\delta_1 + \delta_2$$

$$= 2(0.2579\text{ mm}) + (0.3224\text{ mm})$$

$$= 0.838 \text{ mm} \longleftarrow$$

2.3-3 Steel columns in a building



L = length of each column $= 12$ ft $= 144$ in.

$E = 30 \times 10^6$ psi

$A_{AB} = 17.1$ in.^{2}

$A_{BC} = 6.1$ in.^{2}

(a) Downward displacement of point C

$$\delta_c = \sum \frac{N_i L_i}{E_i A_i} = \frac{N_{AB} L}{E A_{AB}} + \frac{N_{BC} L}{E A_{BC}}$$

$$= \frac{(280\text{ k})(144\text{ in.})}{(30 \times 10^6\text{ psi})(17.1\text{ in.}^2)} + \frac{(100\text{ k})(144\text{ in.})}{(30 \times 10^6\text{ psi})(6.1\text{ in.}^2)}$$

$$= 0.07860 \text{ in.} + 0.07869 \text{ in.} = 0.15729 \text{ in.}$$

$$\delta_c = 0.157 \text{ in.} \longleftarrow$$

(b) Additional load P_0 at point C

$(\delta_c)_{max} = 0.2$ in.

δ_0 = additional displacement of point C due to load P_0

$$\delta_0 = (\delta_c)_{max} - \delta_c = 0.2 \text{ in.} - 0.15729 \text{ in.}$$

$$= 0.04271 \text{ in.}$$

$$\delta_0 = \sum \frac{N_i L_i}{E_i A_i} = \frac{N_{AB} L}{E A_{AB}} + \frac{N_{BC} L}{E A_{BC}}$$

$$= \frac{P_0(144\text{ in.})}{(30 \times 10^6\text{ psi})(17.1\text{ in.}^2)} + \frac{P_0(144\text{ in.})}{(30 \times 10^6\text{ psi})(6.1\text{ in.}^2)}$$

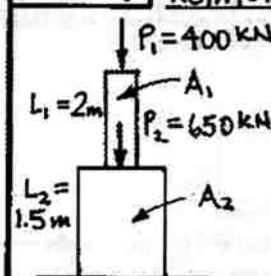
$$0.04271 \text{ in.} = P_0(0.2807 \times 10^{-6} \text{ in./lb}) + P_0(0.78689 \times 10^{-6} \text{ in./lb}) = P_0(1.0676 \times 10^{-6} \text{ in./lb})$$

$P_0 = 40{,}010$ lb

or $P_0 = 40.0$ k $\longleftarrow$

(An additional load $P_0 = 40.0$k will bring the downward displacement of point C to 0.2 in.)

2.3-4 Reinforced concrete pedestal



$E = 25$ GPa

$\delta_{max} = 1.0$ mm

(a) $A_2 = 3A_1$

$$\delta = \sum \frac{N_i L_i}{E_i A_i} = \frac{(400\text{ kN})(2\text{ m})}{(25\text{ GPa})(A_1)} + \frac{(1050\text{ kN})(1.5\text{ m})}{(25\text{ GPa})(3A_1)}$$

CONT.

2.3-4 CONT.

$$1.0\text{ mm} = 0.001\text{ m} = \frac{32\times10^{-6}\text{m}^3}{A_1} + \frac{21\times10^{-6}\text{m}^3}{A_1}$$

$$0.001\text{m} = \frac{53\times10^{-6}\text{m}^3}{A_1}$$

$$A_{1\,min} = 0.053\text{ m}^2 = 53{,}000\text{ mm}^2 \longleftarrow$$

(b) Compressive stresses are equal

$$\sigma = \frac{P_1}{A_1} = \frac{P_1+P_2}{A_2}$$

$$\frac{A_2}{A_1} = \frac{P_1+P_2}{P_1} = 1 + \frac{P_2}{P_1} = 2.625$$

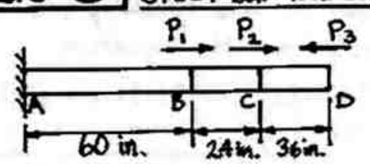
$$\delta = \sum \frac{N_iL_i}{E_iA_i} = \frac{(400\text{kN})(2\text{m})}{(25\text{GPa})(A_1)} + \frac{(1050\text{kN})(1.5\text{m})}{(25\text{GPa})(2.625A_1)}$$

$$0.001\text{ m} = \frac{32\times10^{-6}\text{m}^3}{A_1} + \frac{24\times10^{-6}\text{m}^3}{A_1}$$

$$0.001\text{m} = \frac{56\times10^{-6}\text{m}^3}{A_1}$$

$$A_{1\,min} = 0.056\text{ m}^2 = 56{,}000\text{ mm}^2 \longleftarrow$$

2.3-5 Steel bar loaded by three forces



$A = 0.40\text{ in.}^2$

$P_1 = 2700\text{ lb}$

$P_2 = 1800\text{ lb}$

$P_3 = 1300\text{ lb}$

$E = 30\times10^6\text{ psi}$

Axial forces (+ = tension)

$N_{AB} = P_1 + P_2 - P_3 = 3200\text{ lb}$

$N_{BC} = P_2 - P_3 = 500\text{ lb}$

$N_{CD} = -P_3 = -1300\text{ lb}$

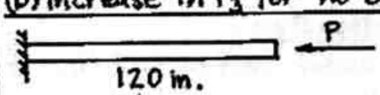
(a) Change in length

$$\delta = \sum\frac{N_iL_i}{E_iA_i} = \frac{1}{EA}(N_{AB}L_{AB} + N_{BC}L_{BC} + N_{CD}L_{CD})$$

$$= \frac{1}{(30\times10^6\text{psi})(0.40\text{ in.}^2)}[(3200\text{ lb})(60\text{ in.}) + (500\text{ lb})(24\text{ in.}) - (1300\text{ lb})(36\text{ in.})]$$

$$= 0.0131\text{ in. (elongation)} \longleftarrow$$

(b) Increase in P_3 for no change in length



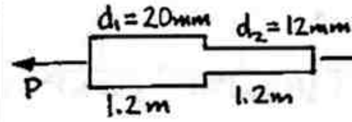
P = increase in force P_3

The force P must produce a shortening equal to 0.0131 in. in order to have no change in length.

$$\therefore\ 0.0131\text{ in.} = \delta = \frac{PL}{EA} = \frac{P(120\text{ in.})}{(30\times10^6\text{psi})(0.40\text{ in.}^2)}$$

$$P = 1310\text{ lb} \longleftarrow$$

2.3-6 Bar with two prismatic segments



$P = 22\text{ kN}$

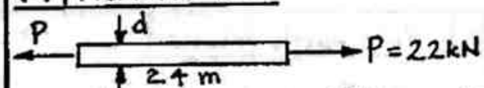
$E = 205\text{ GPa}$

(a) Elongation of bar

$$\delta = \sum\frac{N_iL_i}{E_iA_i} = \frac{(22\text{kN})(1.2\text{m})}{(205\text{ GPa})}\left[\frac{1}{\frac{\pi}{4}(20\text{ mm})^2} + \frac{1}{\frac{\pi}{4}(12\text{mm})^2}\right]$$

$$= 1.55\text{ mm} \longleftarrow$$

(b) Prismatic bar



Original bar: $V_o = (1.2\text{m})(\frac{\pi}{4})(20\text{mm})^2 + (1.2\text{m})(\frac{\pi}{4})(12\text{mm})^2$

$$= \frac{\pi}{4}(652{,}800\text{ mm}^3)$$

Prismatic bar: $V_P = (2.4\text{ m})(\frac{\pi}{4})d^2$

Equate volumes: $V_o = V_P$

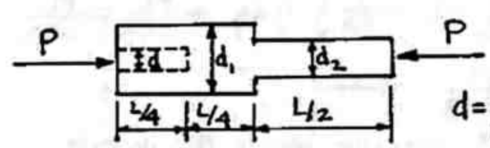
$$652{,}800\text{ mm}^3 = (2400\text{ mm})\,d^2$$

$$d^2 = 272\text{ mm}^2 \quad (d = 16.49\text{ mm})$$

$$\delta = \frac{PL}{EA} = \frac{(22\text{kN})(2.4\text{ m})}{(205\text{ GPa})(\frac{\pi}{4})(272\text{ mm}^2)} = 1.21\text{mm} \longleftarrow$$

Note: A prismatic bar of the same volume will always have a smaller change in length than will a nonprismatic bar, provided the constant axial load P, modulus E, and total length L are the same.

2.3-7 Bar with a hole



d = diameter of hole

Shortening (δ) of the bar

$$\delta = \sum\frac{N_iL_i}{E_iA_i} = \frac{P}{E}\sum\frac{L_i}{A_i}$$

$$= \frac{P}{E}\left[\frac{L/4}{\frac{\pi}{4}(d_1^2-d^2)} + \frac{L/4}{\frac{\pi}{4}d_1^2} + \frac{L/2}{\frac{\pi}{4}d_2^2}\right]$$

$$= \frac{PL}{\pi E}\left(\frac{1}{d_1^2-d^2} + \frac{1}{d_1^2} + \frac{2}{d_2^2}\right) \qquad (\text{Eq. } 1)$$

Numerical values

δ = maximum allowable shortening of the bar = 0.3 in.

$P = 25\text{ k}$ $\quad L = 48\text{ in.}$ $\quad E = 600\text{ ksi}$

$d_1 = 4.0\text{ in.}$ $\quad d_{max}$ = maximum allowable diameter of the hole

$d_2 = 2.5\text{ in.}$

CONT.

2.3-7 CONT.

Substitute numerical values into Eq. (1) and solve for d:

$$0.3\text{ in} = \frac{(25\text{k})(48\text{ in.})}{\pi(600\text{ ksi})}\left[\frac{1}{(4.0\text{ in.})^2 - d^2} + \frac{1}{(4.0\text{ in.})^2} + \frac{2}{(2.5\text{ in.})^2}\right]$$

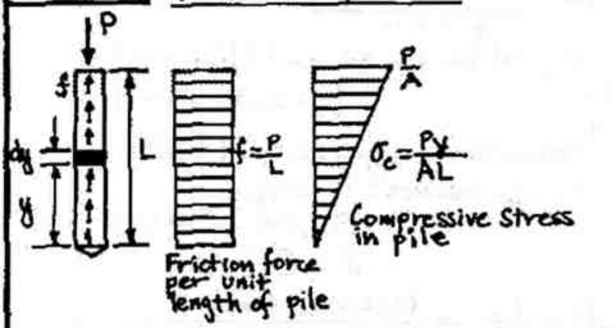
$$0.47124\ \frac{1}{\text{in.}^2} = \frac{1}{16\text{ in.}^2 - d^2} + 0.38250\ \frac{1}{\text{in.}^2}$$

$$0.088739 = \frac{1}{16 - d^2} \qquad 16 - d^2 = 11.2690$$

$$d^2 = 4.7310\text{ in.}^2$$

$$d_{max} = 2.18\text{ in.} \longleftarrow$$

2.3-8 Pile with friction



From free-body diagram of pile:

$$\sum F_{VERT} = 0 \quad \uparrow + \ \downarrow - \quad fL - P = 0 \qquad f = \frac{P}{L}$$

(a) Shortening (δ) of pile

At distance y from the base:

$N(y)$ = axial force $\qquad N(y) = fy$ (compression)

$$d\delta = \frac{N(y)\,dy}{EA} = \frac{fy\,dy}{EA}$$

$$\delta = \int_0^L d\delta = \frac{f}{EA}\int_0^L y\,dy = \frac{fL^2}{2EA} = \frac{PL}{2EA}$$

$$\delta = \frac{PL}{2EA} \longleftarrow$$

(b) Compressive stress σ_c in pile

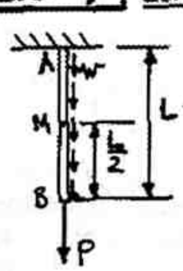
$$\sigma_c = \frac{N(y)}{A} = \frac{fy}{A} = \frac{Py}{AL} \longleftarrow$$

At the base ($y = 0$): $\sigma_c = 0$

At the top ($y = L$): $\sigma_c = \frac{P}{A}$

See the diagram above.

2.3-9 Long steel cable



$P = 5000$ lb

$L = 1500$ ft

$w = 0.42$ lb/ft

$\epsilon_M = 0.00235$

N = axial force

CONT.

2.3-9 CONT.

At A: $N_A = P + wL \qquad \sigma_A = \frac{N_A}{A}$

$$\epsilon_A = \frac{\sigma_A}{E} = \frac{N_A}{EA} = \frac{P + wL}{EA} \qquad \text{(Eq. 1)}$$

At M: $N_M = P + \frac{wL}{2} \qquad \sigma_M = \frac{N_M}{A}$

$$\epsilon_M = \frac{\sigma_M}{E} = \frac{N_M}{EA} = \frac{P + wL/2}{EA} \qquad \text{(Eq. 2)}$$

At B: $N_B = P \quad \sigma_B = \frac{N_B}{A} \quad \epsilon_B = \frac{\sigma_B}{E} = \frac{N_B}{EA} = \frac{P}{EA}$ (Eq. 3)

From (2): $EA = \frac{P + wL/2}{\epsilon_M}$

(a) Substitute into (1):

$$\epsilon_A = \frac{P + wL}{P + \frac{wL}{2}}\,\epsilon_M$$

$$= \frac{5630\text{ lb}}{5315\text{ lb}}(0.00235) = 0.00249 \longleftarrow$$

(b) Substitute into (3):

$$\epsilon_B = \frac{P}{P + \frac{wL}{2}}\,\epsilon_M$$

$$= \frac{5000\text{ lb}}{5315\text{ lb}}(0.00235) = 0.00221 \longleftarrow$$

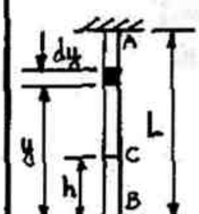
(c) Elongation of cable

Since the strain varies linearly from B to A, we can obtain δ from the equation

$$\delta = \epsilon_M L = (0.00235)(1500\text{ ft})(12\text{ in./ft})$$

$$= 42.3\text{ in.} \longleftarrow$$

2.3-10 Prismatic bar hanging vertically



W = Weight of bar

(a) Downward displacement δ_C

Consider an element at distance y from the lower end.

$$N(y) = \frac{Wy}{L} \qquad d\delta = \frac{N(y)\,dy}{EA} = \frac{Wy\,dy}{EAL}$$

$$\delta_C = \int_h^L d\delta = \int_h^L \frac{Wy\,dy}{EAL} = \frac{W}{EAL}\left[\frac{y^2}{2}\right]_h^L = \frac{W}{2EAL}(L^2 - h^2)$$

$$\delta_C = \frac{W}{2EAL}(L^2 - h^2) \longleftarrow$$

(b) Elongation of bar ($h = 0$)

$$\delta_B = \frac{WL}{2EA} \longleftarrow$$

(c) Ratio of elongations

Elongation of upper half of bar ($h = \frac{L}{2}$):

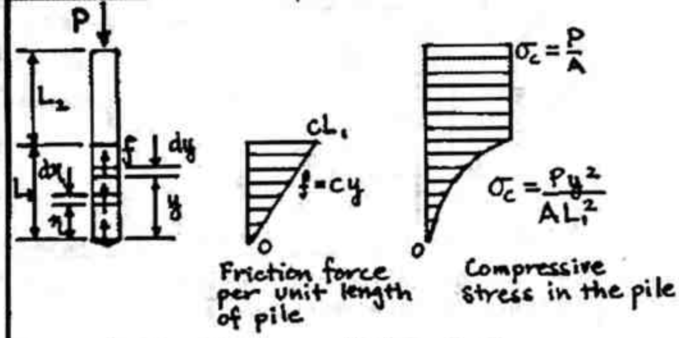
$$\delta_{UPPER} = \frac{3WL}{8EA}$$

Elongation of lower half of bar:

$$\delta_{LOWER} = \delta_B - \delta_{UPPER} = \frac{WL}{2EA} - \frac{3WL}{8EA} = \frac{WL}{8EA}$$

$$r = \frac{\delta_{UPPER}}{\delta_{LOWER}} = \frac{3/8}{1/8} = 3 \longleftarrow$$

2.3-11 Timber pile with friction



Free-body diagram of the pile:

$\Sigma F_{vert} = 0 \quad \uparrow + \ \downarrow -$

$\int_0^{L_1} f\,dy - P = 0 \qquad P = \int_0^{L_1} cy\,dy = \frac{1}{2}cL_1^2$

$c = \frac{2P}{L_1^2} \qquad f = cy = \frac{2Py}{L_1^2}$

Axial force at distance y from the base

To obtain the compressive axial force at distance y, we must first find the friction force acting on an element at distance η and then integrate from 0 to y.

dF = friction force on element $d\eta$

$dF = f\,d\eta = \frac{2P\eta}{L_1^2}\,d\eta$

F = total friction force from 0 to y

$F = \int dF = \int_0^y \frac{2P\eta}{L_1^2}\,d\eta = \frac{2P}{L_1^2}\int_0^y \eta\,d\eta$

$= \frac{2P}{L_1^2}\left[\frac{\eta^2}{2}\right]\Big|_0^y = \frac{Py^2}{L_1^2}$

$N(y)$ = axial force (compression) in pile at distance y

$N(y) = F = \frac{Py^2}{L_1^2} \qquad (0 \le y \le L_1)$

Shortening δ_1 of lower part of pile

$d\delta = \frac{N(y)\,dy}{EA} = \frac{Py^2}{EAL_1^2}\,dy$

$\delta_1 = \int d\delta = \frac{P}{EAL_1^2}\int_0^{L_1} y^2\,dy = \frac{PL_1}{3EA}$

Shortening δ_2 of upper part of pile

The axial force is constant and equal to P.

$\delta_2 = \frac{PL_2}{EA}$

(a) Shortening of Pile

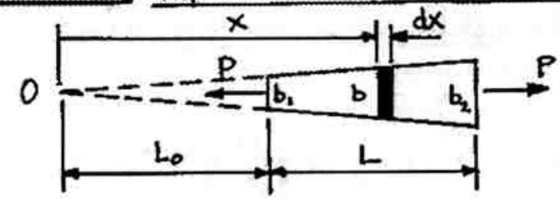
$\delta = \delta_1 + \delta_2 = \frac{PL_1}{3EA} + \frac{PL_2}{EA} = \frac{P}{3EA}(L_1 + 3L_2) \longleftarrow$

(b) Compressive Stress σ_c (SEE DIAGRAM ABOVE)

Lower part of pile: $\sigma_c = \frac{N(y)}{A} \quad \frac{Py^2}{AL_1^2} \longleftarrow$

Upper part of pile: $\sigma_c = \frac{P}{A} \longleftarrow$

2.3-12 Tapered bar (rectangular cross section)



t = thickness (constant)

$b = b_1\left(\frac{x}{L_0}\right)$

$A(x) = bt = b_1 t\left(\frac{x}{L_0}\right)$

(a) Elongation of the bar

$d\delta = \frac{P\,dx}{EA(x)} = \frac{PL_0\,dx}{Eb_1 t\,x}$

$\delta = \int_{L_0}^{L_0+L} d\delta = \frac{PL_0}{Eb_1 t}\int_{L_0}^{L_0+L}\frac{dx}{x}$

$= \frac{PL_0}{Eb_1 t}\ln x\Big|_{L_0}^{L_0+L} = \frac{PL_0}{Eb_1 t}\ln\frac{L_0+L}{L_0} \qquad$ (Eq. 1)

From similar triangles:

$\frac{b_1}{L_0} = \frac{b_2}{L_0+L} \qquad \frac{L_0+L}{L_0} = \frac{b_2}{b_1} \qquad$ (Eq. 2)

Substitute Eq. (2) into Eq. (1):

$\delta = \frac{PL_0}{Eb_1 t}\ln\frac{b_2}{b_1} \qquad$ (Eq. 3)

Solve Eq. (2) for L_0: $\quad L_0 = L\left(\frac{b_1}{b_2 - b_1}\right) \qquad$ (Eq. 4)

Substitute into Eq. (3):

$\delta = \frac{PL}{Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \longleftarrow$

(b) Substitute numerical values:

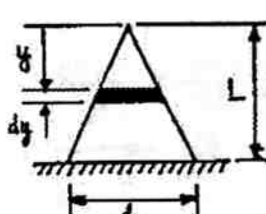
$b_1 = 100$ mm $\quad b_2 = 150$ mm $\quad L = 1.5$ m

$t = 25$ mm $\quad P = 125$ kN $\quad E = 200$ GPa

$\delta = 0.304$ mm $\longleftarrow$

2.3-13 Conical bar and prismatic bar

(a) Conical bar



(ASSUME ANGLE OF TAPER IS SMALL.)

Let y = distance from vertex to element dy

Let d_c = diameter at base

Then: V_c = volume of entire cone

$= \frac{1}{3}$(area of base)(length)

$= \frac{1}{12}(\pi d_c^2 L)$

W_c = weight of entire cone

$= \gamma V_c$

$= \frac{\pi\gamma d_c^2 L}{12}$

CONT.

2.3-13 CONT.

At distance y from the vertex

$d(y)$ = diameter

$= \frac{y}{L} d_c$

$A(y) = \frac{\pi}{4}[d(y)]^2 = \frac{\pi d_c^2}{4L^2} y^2$

$V(y)$ = volume of cone above the element dy

$= \frac{1}{3}$(area of base)(length)

$= \frac{1}{3}\left(\frac{\pi d_c^2}{4L^2}\right)(y^2)y = \frac{\pi d_c^2}{12L^2} y^3$

$W(y)$ = weight of cone above the element dy

$= \gamma V(y) = \frac{\pi \gamma d_c^2}{12L^2} y^3$

Shortening of the conical bar

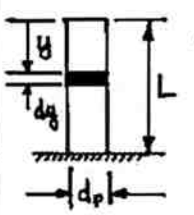
$d\delta_c$ = shortening of element dy

$= \frac{W(y)\,dy}{EA(y)} = \frac{1}{E}\left(\frac{\pi\gamma d_c^2 y^3 dy}{12L^2}\right)\left(\frac{4L^2}{\pi d_c^2 y^2}\right)$

$= \frac{\gamma y\, dy}{3E}$

$\delta_c = \int_0^L d\delta_c = \frac{\gamma}{3E}\int_0^L y\,dy = \frac{\gamma L^2}{6E}$ ←

(b) Prismatic bar



Let d_p = diameter of prismatic bar

Then: V_p = volume of entire bar

$= \frac{\pi d_p^2 L}{4}$

W_p = weight of entire bar

$= \gamma V_p = \frac{\pi\gamma d_p^2 L}{4}$

Note: $d_p = \frac{d_c}{\sqrt{3}} = 0.5774\, d_c$

At distance y from the top

$V(y)$ = volume of bar above the element dy

$= \frac{\pi}{4} d_p^2 y$

$W(y)$ = weight of bar above the element dy

$= \gamma V(y) = \frac{\pi\gamma d_p^2}{4} y$

Shortening of bar

$d\delta_p$ = Shortening of element

$= \frac{W(y)\,dy}{EA} = \frac{\pi\gamma d_p^2 y\,dy}{4E\left(\frac{\pi}{4}\right)d_p^2} = \frac{\gamma y\,dy}{E}$

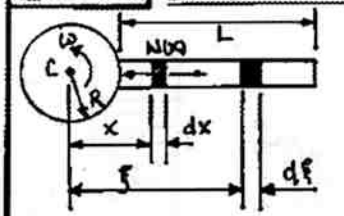
$\delta_p = \int_0^L d\delta_p = \frac{\gamma}{E}\int_0^L y\,dy = \frac{\gamma L^2}{2E}$ ←

(c) Ratio of shortenings

$\frac{\delta_p}{\delta_c} = 3$ ←

(Under the action of its own weight, a prismatic bar has three times the change in length of a conical bar having the same length and made of the same material.)

2.3-14 Circular disc with prismatic blades



ρ = mass density

E = modulus of elasticity

A = cross-sectional area

Axial force $N(x)$ at distance x from center

Consider an element of length dx located at distance x from the center C.

$N(x)$ = axial force acting on this element.

To find $N(x)$, we must find the inertia force of the part of the blade from the element (at distance x) to the free end of the blade (at distance $R+L$).

Consider an element of length $d\xi$ located at distance ξ from the center ($\xi \geq x$).

Mass of element $= \rho A\, d\xi$

Acceleration of element $= \xi\omega^2$

Centrifugal force of element = (mass)(acceleration)

$= \rho A \omega^2 \xi\, d\xi$

$N(x) = \int_x^{R+L} \rho A \omega^2 \xi\, d\xi$

Integrating this expression, we get

$N(x) = \frac{\rho A \omega^2}{2}\left[(R+L)^2 - x^2\right]$

Elongation of blade

$\delta = \int \frac{N(x)\,dx}{EA} = \int_R^{R+L} \frac{\rho A\omega^2\left[(R+L)^2 - x^2\right]}{2EA}\,dx$

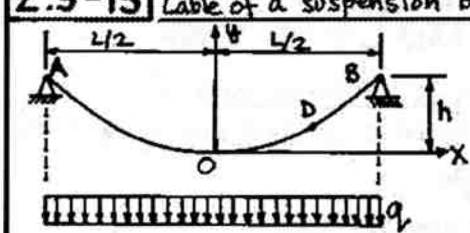
$= \frac{\rho\omega^2}{2E}\int_R^{R+L}\left[(R+L)^2 - x^2\right]dx$

Integrating this expression, combining terms, and simplifying the result, we get

$\delta = \frac{\rho\omega^2 L^2}{6E}(3R + 2L)$ ←

(Note that the cross-sectional area of the blade does not affect the elongation.)

2.3-15 Cable of a suspension bridge



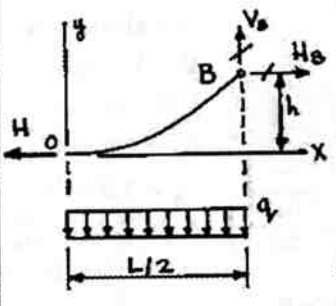
Equation of parabolic curve: $y = \frac{4hx^2}{L^2}$

$\frac{dy}{dx} = \frac{8hx}{L^2}$

CONT.

2.3-15 CONT.

Free-body diagram of half of cable



$\Sigma M_B = 0$

$-Hh + \frac{qL}{2}\left(\frac{L}{4}\right) = 0$

$H = \frac{qL^2}{8h}$

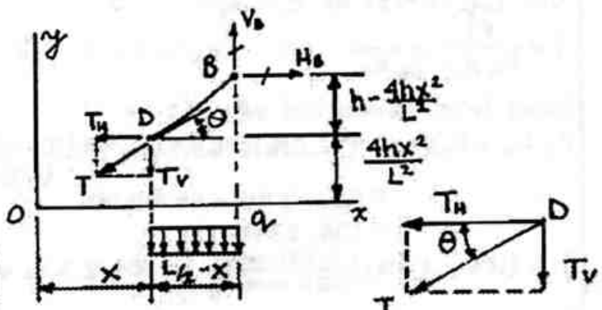
$\Sigma F_{HORIZONTAL} = 0$

$H_B = H = \frac{qL^2}{8h}$ (Eq. 1)

$\Sigma F_{VERTICAL} = 0$

$V_B = \frac{qL}{2}$ (Eq. 2)

Free-body diagram of segment DB of cable



$\Sigma F_{HORIZ} = 0 \quad T_H = H_B$

$= \frac{qL^2}{8h}$ (Eq. 3)

$\Sigma F_{VERT} = 0 \quad V_B - T_V - q\left(\frac{L}{2} - x\right) = 0$

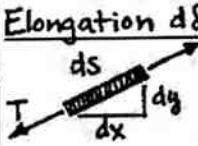
$T_V = V_B - q\left(\frac{L}{2} - x\right) = \frac{qL}{2} - \frac{qL}{2} + qx$

$= qx$ (Eq. 4)

Tensile force T in cable

$T = \sqrt{T_H^2 + T_V^2} = \sqrt{\left(\frac{qL^2}{8h}\right)^2 + (qx)^2} = \frac{qL^2}{8h}\sqrt{1 + \frac{64h^2x^2}{L^4}}$ (Eq. 5)

Elongation $d\delta$ of an element of length ds



$d\delta = \frac{T\,ds}{EA}$

$ds = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$= dx\sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}$

$= dx\sqrt{1 + \frac{64h^2x^2}{L^4}}$ (Eq. 6)

(a) Elongation δ of cable AOB

$\delta = \int d\delta = \int \frac{T\,ds}{EA}$

Substitute for T from Eq. (5) and for ds from Eq. (6):

$\delta = \frac{1}{EA}\int \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$

For both halves of cable:

$\delta = \frac{2}{EA}\int_0^{L/2} \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$

CONT.

2.3-15 CONT.

$\delta = \frac{qL^3}{8hEA}\left(1 + \frac{16h^2}{3L^2}\right)$ ←

(b) Golden Gate Bridge cable

$L = 4200$ ft $\quad h = 470$ ft $\quad q = 12{,}700$ lb/ft

$E = 28{,}800{,}000$ psi

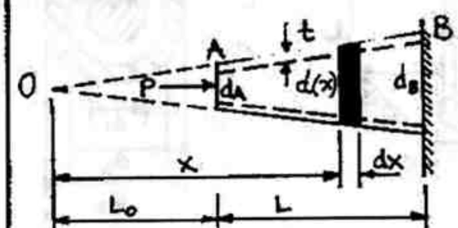
27,572 wires of diameter $d = 0.196$ in.

$A = (27{,}572)\left(\frac{\pi}{4}\right)(0.196 \text{ in.})^2 = 831.90 \text{ in.}^2$

Substitute into Eq. (7)

$\delta = 133.7$ in $= 11.14$ ft ←

2.3-16 Hollow post in compression



t = wall thickness

L = length of bar (post)

d_A = outside diameter at end A

d_B = outside diameter at end B

$d(x)$ = outside diameter at distance x from origin O

L_o = distance from origin to end A of the bar

From similar triangles:

$\frac{L_o}{L + L_o} = \frac{d_A}{d_B}$ or $L_o = \frac{d_A L}{d_B - d_A}$ (Eq. 1)

$\frac{d(x)}{x} = \frac{d_A}{L_o}$ or $d(x) = \frac{d_A x}{L_o}$ (Eq. 2)

Cross-sectional area of bar at distance x

$A(x) = \frac{\pi}{4}\left\{[d(x)]^2 - [d(x) - 2t]^2\right\} = \frac{\pi(4t)}{4}[d(x) - t]$

$= \frac{\pi t}{L_o}(d_A x - L_o t)$ (Eq. 3)

Shortening of element dx

$d\delta = \frac{P\,dx}{EA(x)} = \frac{PL_o\,dx}{\pi E t(d_A x - L_o t)}$

Shortening δ of bar AB

$\delta = \int d\delta = \frac{PL_o}{\pi E t}\int_{L_o}^{L+L_o} \frac{dx}{d_A x - L_o t}$

From Appendix C:

$\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

$\therefore \delta = \frac{PL_o}{\pi E t}\left(\frac{1}{d_A}\right)\left[\ln(d_A x - L_o t)\right]\Big|_{L_o}^{L+L_o}$

$= \frac{PL_o}{\pi E d_A t}\left\{\ln[d_A(L + L_o) - L_o t] - \ln[d_A L_o - L_o t]\right\}$

CONT.

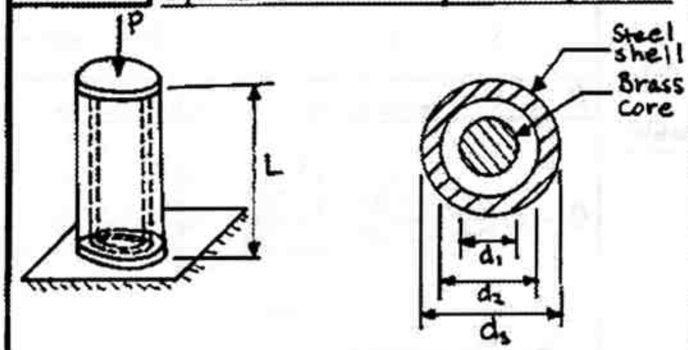
2.3-16 CONT.

$$\delta = \frac{PL_0}{\pi E d_A t} \ln\left(\frac{d_A(L+L_0) - L_0 t}{d_A L_0 - L_0 t}\right)$$

Substitute for L_0 from Eq. (1) and simplify:

$$\delta = \frac{PL}{\pi E t (d_B - d_A)} \ln\left(\frac{d_B - t}{d_A - t}\right) \longleftarrow$$

2.4-1 Cylindrical assembly in compression



$d_1 = 0.25$ in.	$E_b = 15 \times 10^6$ psi
$d_2 = 0.28$ in.	$E_s = 30 \times 10^6$ psi
$d_3 = 0.35$ in.	$A_s = \frac{\pi}{4}(d_3^2 - d_2^2) = 0.03464$ in.^{2}
$L = 4.0$ in.	$A_b = \frac{\pi}{4} d_1^2 = 0.04909$ in.^{2}

(a) Decrease in length ($\delta = 0.003$ in.)

Use Eq. (2-13) of Example 2-5.

$$\delta = \frac{PL}{E_s A_s + E_b A_b} \quad \text{OR} \quad P = (E_s A_s + E_b A_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$$E_s A_s + E_b A_b = (30 \times 10^6 \text{ psi})(0.03464 \text{ in.}^2) + (15 \times 10^6 \text{ psi})(0.04909 \text{ in.}^2)$$

$$= 1.776 \times 10^6 \text{ lb}$$

$$P = (1.776 \times 10^6 \text{ lb})\left(\frac{0.003 \text{ in.}}{4.0 \text{ in.}}\right) = 1330 \text{ lb} \longleftarrow$$

(b) Allowable load

$\sigma_s = 22$ ksi $\qquad \sigma_b = 16$ ksi

Use Eqs. (2-12 a and b) of Example 2-5.

For steel:

$$\sigma_s = \frac{PE_s}{E_s A_s + E_b A_b} \qquad P_s = (E_s A_s + E_b A_b)\frac{\sigma_s}{E_s}$$

$$P_s = (1.776 \times 10^6 \text{ lb})\left(\frac{22 \text{ ksi}}{30 \times 10^6 \text{ psi}}\right) = 1300 \text{ lb}$$

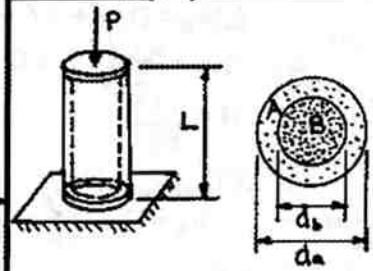
For brass:

$$\sigma_b = \frac{PE_b}{E_s A_s + E_b A_b} \qquad P_s = (E_s A_s + E_b A_b)\frac{\sigma_b}{E_b}$$

$$P_s = (1.776 \times 10^6 \text{ lb})\left(\frac{16 \text{ ksi}}{15 \times 10^6 \text{ psi}}\right) = 1890 \text{ lb}$$

Steel governs. $P_{allow} = 1300$ lb $\longleftarrow$

2.4-2 Cylindrical assembly in compression



A = aluminum

B = brass

$L = 350$ mm

$d_A = 40$ mm

$d_b = 25$ mm

$A_A = \frac{\pi}{4}(d_A^2 - d_b^2)$

$= 765.8$ mm^2

$E_a = 72$ GPa $\qquad E_b = 100$ GPa $\qquad A_b = \frac{\pi}{4} d_b^2 = 490.9$ mm^2

(a) Decrease in length ($\delta = 0.1\%$ of $L = 0.350$ mm)

Use Eq. (2-13) of Example 2-5.

$$\delta = \frac{PL}{E_A A_A + E_b A_b} \quad \text{OR} \quad P = (E_A A_A + E_b A_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$$E_A A_A + E_b A_b = (72 \text{ GPa})(765.8 \text{ mm}^2) + (100 \text{ GPa})(490.9 \text{ mm}^2)$$

$$= 55.135 \text{ MN} + 49.090 \text{ MN}$$

$$= 104.23 \text{ MN}$$

$$P = (104.23 \text{ MN})\left(\frac{0.350 \text{ mm}}{350 \text{ mm}}\right) = 104.2 \text{ kN} \longleftarrow$$

(b) Allowable load

$\sigma_A = 80$ MPa $\qquad \sigma_b = 120$ MPa

Use Eqs. (2-12a and b) of Example 2-5.

For aluminum:

$$\sigma_A = \frac{PE_A}{E_A A_A + E_b A_b} \qquad P_A = (E_A A_A + E_b A_b)\left(\frac{\sigma_A}{E_A}\right)$$

$$P_A = (104.23 \text{ MN})\left(\frac{80 \text{ MPa}}{72 \text{ GPa}}\right) = 115.8 \text{ kN}$$

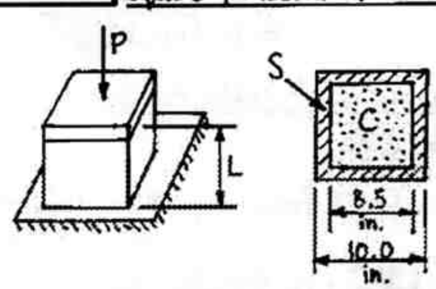
For brass:

$$\sigma_b = \frac{PE_b}{E_A A_A + E_b A_b} \qquad P_b = (E_A A_A + E_b A_b)\left(\frac{\sigma_b}{E_b}\right)$$

$$P_b = (104.23 \text{ MN})\left(\frac{120 \text{ MPa}}{100 \text{ GPa}}\right) = 125.1 \text{ kN}$$

Aluminum governs. $P_{max} = 116$ kN $\longleftarrow$

2.4-3 Square pedestal of steel and concrete



$L = 26$ in.

$E_s = 30 \times 10^3$ ksi

$E_c = 4.0 \times 10^3$ ksi

$P = 100$ k

$$A_s = (10.0 \text{ in.})^2 - (8.5 \text{ in.})^2 = 27.75 \text{ in.}^2$$

$$A_c = (8.5 \text{ in.})^2 = 72.25 \text{ in.}^2$$

CONT.

2.4-3 CONT.

$$E_sA_s + E_cA_c = (30\times10^3\ \text{ksi})(27.75\ \text{in.}^2) + (4.0\times10^3\ \text{ksi})(72.25\ \text{in.}^2)$$
$$= 1.1215\times10^6\ \text{k}$$

(a) Fraction of the load carried by the steel casing

Use Eq. (2-11a) of Example 2-5.

$$\frac{P_s}{P} = \frac{E_sA_s}{E_sA_s + E_cA_c} = \frac{1}{1+\frac{E_cA_c}{E_sA_s}} = \frac{1}{1+\left(\frac{4}{30}\right)\left(\frac{72.25}{27.75}\right)}$$

$= 0.7423$ OR $74.2\ \%$ ←

(b) Stresses in steel and concrete ($P = 100\text{k}$)

Use Eqs. (2-12a and b) of Example 2-5.

$$\sigma_s = \frac{PE_s}{E_sA_s + E_cA_c} = \frac{(100\text{k})(30\times10^3\ \text{ksi})}{(1.1215\times10^6\ \text{k})}$$
$$= 2{,}670\ \text{psi}\ \leftarrow$$

$$\frac{\sigma_c}{\sigma_s} = \frac{E_c}{E_s} = \frac{4.0}{30} \qquad \sigma_c = \frac{4.0}{30}(2670\ \text{psi})$$
$$= 357\ \text{psi}\ \leftarrow$$

(c) Shortening of the pedestal ($P = 100\text{k}$)

Use Eq. (2-13) of Example 2-5.

$$\delta = \frac{PL}{E_sA_s + E_cA_c} = \frac{(100\text{k})(26\ \text{in.})}{1.1215\times10^6\ \text{k}} = 0.00232\ \text{in.}\ \leftarrow$$

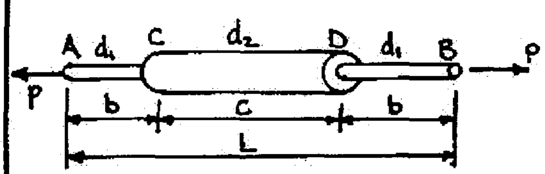
(d) Stiffness and flexibility

$\delta = 0.002318$ in.

$$k = \frac{P}{\delta} = \frac{100\text{k}}{0.002318\ \text{in.}} = 43{,}100\ \text{k/in.}\ \leftarrow$$

$$f = \frac{\delta}{P} = 23.2\times10^{-6}\ \text{in./k}\ \leftarrow$$

2.4-4 Plastic rod with sleeve



$P = 15$ kN $\quad d_1 = 30$ mm $\quad b = 100$ mm

$L = 500$ mm $\quad d_2 = 45$ mm $\quad c = 300$ mm

Rod: $E_1 = 3.1$ GPa

Sleeve: $E_2 = 2.5$ GPa

Rod: $A_1 = \frac{\pi d_1^2}{4} = 706.86\ \text{mm}^2$

Sleeve: $A_2 = \frac{\pi}{4}(d_2^2 - d_1^2) = 883.57\ \text{mm}^2$

$E_1A_1 + E_2A_2 = 4.400$ MN

(a) Elongation of rod

Part AC: $\delta_{AC} = \frac{Pb}{E_1A_1} = 0.6845$ mm

CONT.

2.4-4 CONT.

Part CD: $\delta_{CD} = \frac{Pc}{E_1A_1 + E_2A_2}$ (From Eq. 2-13 of Example 2-5)

$= 1.0227$ mm

$\delta = 2\delta_{AC} + \delta_{CD} = 2.39$ mm ←

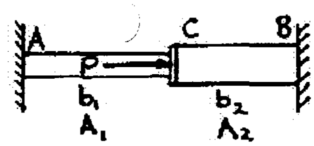
(b) Sleeve at full length

$$\delta = \delta_{CD}\left(\frac{L}{c}\right) = (1.0227\ \text{mm})\left(\frac{500\ \text{mm}}{300\ \text{mm}}\right) = 1.70\ \text{mm}\ \leftarrow$$

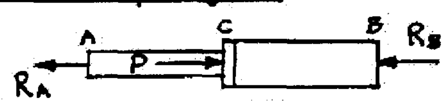
(c) Sleeve removed

$$\delta = \frac{PL}{E_1A_1} = 3.42\ \text{mm}\ \leftarrow$$

2.4-5 Bar with intermediate load



Free-body diagram



Equation of Equilibrium

$\Sigma F_{HORIZ} = 0 \qquad R_A + R_B = P \qquad$ (Eq. 1)

Equation of compatibility

δ_{AC} = elongation of AC

δ_{CB} = shortening of CB

$\delta_{AC} = \delta_{CB} \qquad$ (Eq. 2)

Force displacement relations

$$\delta_{AC} = \frac{R_A b_1}{EA_1} \qquad \delta_{CB} = \frac{R_B b_2}{EA_2} \qquad \text{(Eqs. 3\&4)}$$

(a) Solution of equations

Substitute Eq.(3) and Eq.(4) into Eq.(2):

$$\frac{R_A b_1}{EA_1} = \frac{R_B b_2}{EA_2} \qquad \text{(Eq. 5)}$$

Solve Eq.(1) and Eq.(5) simultaneously:

$$R_A = \frac{b_2A_1P}{b_1A_2 + b_2A_1} \qquad R_B = \frac{b_1A_2P}{b_1A_2 + b_2A_1}\ \leftarrow$$

(b) Displacement of point C

$$\delta_C = \delta_{AC} = \frac{R_A b_1}{EA_1} = \frac{b_1b_2P}{E(b_1A_2 + b_2A_1)}\ \leftarrow$$

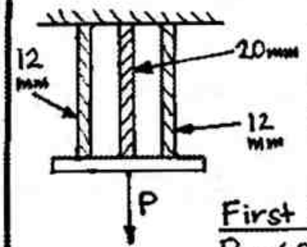
(c) Ratio of stresses

$$\sigma_1 = \frac{R_A}{A_1}\ \text{(tension)} \qquad \sigma_2 = \frac{R_B}{A_2}\ \text{(compession)}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{b_2}{b_1}\ \leftarrow$$

(Note that if $b_1 = b_2$, the stresses are numerically equal regardless of the areas A_1 and A_2.)

2.4-6 Three cables in tension



Areas of cable (from Table 2-1)

Middle cable: $A_M = 173\ mm^2$

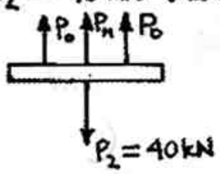
Outer cable: $A_O = 76.7\ mm^2$ (each cable)

First loading

$P_1 = 60$ kN (Each cable carries $\frac{P_1}{3}$ or 20 kN)

Second loading

$P_2 = 40$ kN (additional load)



Equation of equilibrium

$\Sigma F_{VERT} = 0 \quad 2P_o + P_M - P_2 = 0$ (Eq. 1)

Equation of compatibility

$\delta_M = \delta_o$ (Eq. 2)

Force-displacement relations

$$\delta_M = \frac{P_M L}{EA_M} \qquad \delta_o = \frac{P_o L}{EA_o} \qquad \text{(Eqs. 3 \& 4)}$$

Solution of equations

Substitute Eq. (3) and Eq. (4) into compatibility Eq. (2):

$$\frac{P_M L}{EA_M} = \frac{P_o L}{EA_o} \quad \text{OR} \quad \frac{P_M}{A_M} = \frac{P_o}{A_o} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_M = P_2\left(\frac{A_M}{A_M + 2A_o}\right) = (40\text{ kN})\left(\frac{173\ mm^2}{326.4\ mm^2}\right) = 21.20\text{ kN}$$

$$P_o = P_2\left(\frac{A_o}{A_M + 2A_o}\right) = (40\text{ kN})\left(\frac{76.7\ mm^2}{326.4\ mm^2}\right) = 9.40\text{ kN}$$

Forces in cables

Middle cable: Force = 20 kN + 21.20 kN = 41.20 kN

Outer cable: Force = 20 kN + 9.40 kN = 29.40 kN (each cable)

(a) Percent of total load carried by middle cable

$$\text{Percent of total load} = \frac{41.20\text{ kN}}{100\text{ kN}}(100\%)$$

$$= 41.2\% \longleftarrow$$

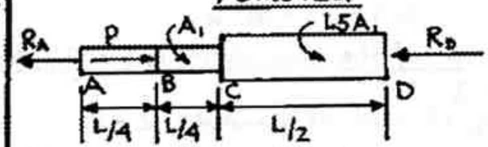
(b) Stresses in cables ($\sigma = P/A$)

Middle cable: $\sigma_M = \dfrac{41.20\text{ kN}}{173\ mm^2} = 238$ MPa $\longleftarrow$

Outer cables: $\sigma_o = \dfrac{29.40\text{ kN}}{76.7\ mm^2} = 383$ MPa $\longleftarrow$

2.4-7 Bar with fixed ends

Free-body diagram of bar with supports removed



Equation of equilibrium

$\Sigma F_{HORIZ} = 0 \qquad R_A + R_D = P$ (Eq. 1)

Equation of compatibility

$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0$ (Eq. 2)

δ is an algebraic quantity. (A positive value means elongation.)

Force-displacement equations

$$\delta_{AB} = \frac{R_A(L/4)}{EA_1} \qquad \delta_{BC} = \frac{(R_A - P)(L/4)}{EA_1} \qquad \text{(Eqs. 3 \& 4)}$$

$$\delta_{CD} = -\frac{R_D(L/2)}{E(1.5A_1)} \qquad \text{(Eq. 5)}$$

Solution of equations

Substitute Eqs. (3), (4), and (5) into Eq. (2):

$$\frac{R_A L}{4EA_1} + \frac{(R_A - P)(L)}{4EA_1} - \frac{R_D L}{3EA_1} = 0 \qquad \text{(Eq. 6)}$$

(a) Reactions

Solve simultaneously Eqs. (1) and (6):

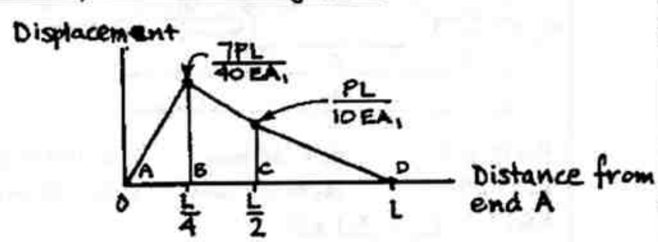
$$R_A = \frac{7P}{10} \qquad R_D = \frac{3P}{10} \longleftarrow$$

(b) Displacements at points B and C

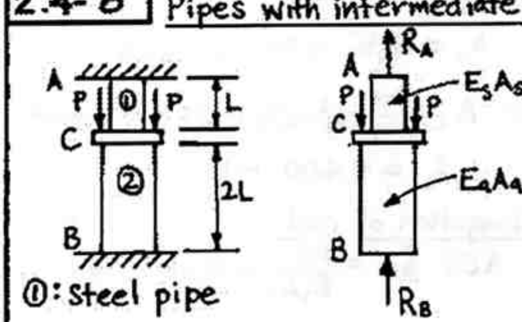
$$\delta_B = \delta_{AB} = \frac{R_A L}{4EA_1} = \frac{7PL}{40EA_1} \text{ (To the right)} \longleftarrow$$

$$\delta_C = |\delta_{CD}| = \frac{R_D L}{3EA_1} = \frac{PL}{10EA_1} \text{ (To the right)} \longleftarrow$$

(c) Displacement diagram



2.4-8 Pipes with intermediate loads



①: steel pipe

②: aluminum pipe

CONT.

2.4-8 CONT.

Equation of equilibrium

$\Sigma F_{VERT} = 0 \qquad R_A + R_B = 2P \qquad$ (Eq. 1)

Equation of compatibility

$\delta_{AB} = \delta_{AC} + \delta_{CB} = 0 \qquad$ (Eq. 2)

δ is an algebraic quantity. (A positive value means elongation.)

Force-displacement relations

$\delta_{AC} = \dfrac{R_A L}{E_s A_s} \qquad \delta_{BC} = -\dfrac{R_B(2L)}{E_a A_a} \qquad$ (Eqs. 3&4)

Solution of equations

Substitute Eqs. (3) and (4) into Eq. (2):

$\dfrac{R_A L}{E_s A_s} - \dfrac{R_B(2L)}{E_a A_a} = 0 \qquad$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$R_A = \dfrac{4E_sA_sP}{E_aA_a + 2E_sA_s} \qquad R_B = \dfrac{2E_aA_aP}{E_aA_a + 2E_sA_s} \qquad$ (Eqs. 6&7)

(a) Axial stresses

Aluminum: $\sigma_A = \dfrac{R_B}{A_a} = \dfrac{2E_aP}{E_aA_a + 2E_sA_s}$ (compression) (Eq. 8)

Steel: $\sigma_s = \dfrac{R_A}{A_s} = \dfrac{4E_sP}{E_aA_a + 2E_sA_s}$ (tension) (Eq. 9)

(b) Numerical results

$P = 50$ kN $\quad A_a = 6000$ mm² $\quad A_s = 600$ mm²

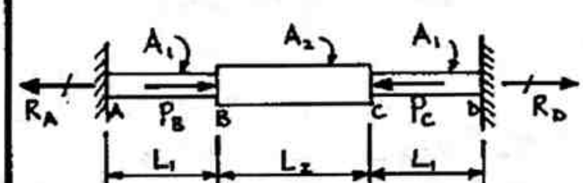
$E_a = 70$ GPa $\quad E_s = 200$ GPa

Substitute into Eqs. (8) and (9):

$\sigma_a = 10.6$ MPa (Compression) ←

$\sigma_s = 60.6$ MPa (Tension) ←

2.4-9 Nonprismatic bar with fixed ends

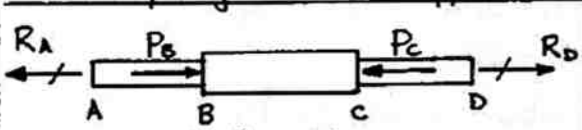


$P_B = 5100$ lb $\qquad P_C = 3400$ lb

$L_1 = 8.0$ in. $\qquad L_2 = 10.0$ in.

$A_1 = 1.2$ in.² $\qquad A_2 = 1.8$ in.²

Free-body diagram with supports removed



Equation of Equilibrium

$\Sigma F_{HORIZ} = 0 \qquad R_A - R_D = P_B - P_C = 1700$ lb (Eq. 1)

CONT.

2.4-9 CONT.

Equation of compatibility

δ_{AB} = elongation of segment AB

δ_{BC} = elongation of segment BC

δ_{CD} = elongation of segment CD

δ = elongation of entire bar

$\delta = \delta_{AB} + \delta_{BC} + \delta_{CD} = 0 \qquad$ (Eq. 2)

Note: Units in all expressions are pounds and inches.

Force-displacement relations

$\delta_{AB} = \dfrac{R_A L_1}{EA_1} \qquad \delta_{BC} = \dfrac{(R_A - P_B)L_2}{EA_2} \qquad \delta_{CD} = \dfrac{R_D L_1}{EA_1}$

$= \dfrac{R_A}{E}(6.667) \qquad = \dfrac{R_A}{E}(5.556) - \dfrac{P_B}{E}(5.556) \qquad = \dfrac{R_D}{E}(6.667) \qquad$ (Eqs. 3, 4, 5)

Solution of equations

Substitute (3), (4), and (5) into Eq. (2):

$\dfrac{R_A}{E}(6.667) + \dfrac{R_A}{E}(5.556) - \dfrac{(5100\text{ lb})}{E}(5.556) + \dfrac{R_D}{E}(6.667) = 0$

OR

$R_A(12.222) + R_D(6.667) = 28{,}333 \qquad$ (Eq. 6)

(a) Reactions R_A and R_D

Solve simultaneously Eqs. (1) and (6):

From (1): $R_D = R_A - 1700$

Substitute into Eq. (6):

$R_A(12.222) + (R_A - 1700)(6.667) = 28{,}333$

$18.889\, R_A = 39{,}666$

$R_A = 2100$ lb (to the left) ←

$R_D = R_A - 1700 = 400$ lb (to the right) ←

(b) Compressive Axial force F in middle segment

From the free-body diagram:

$F = P_B - R_A = P_C - R_D = 3000$ lb (Compression)

$= 3000$ lb (Compression) ←

(c) Lengths and area are doubled in value

No change in Eq. (1)

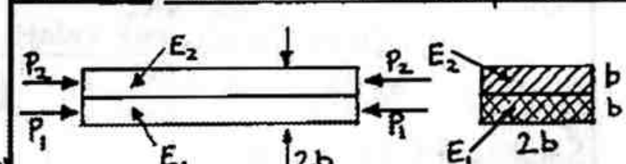
No change in Eq. (2)

No change in Eqs. (3), (4), and (5)

No change in Eq. (6)

∴ No change in R_A, R_D, and F ←

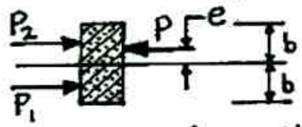
2.4-10 Bimetallic bar in compression



CONT.

2.4-10 CONT.

Free-body diagram of end plate



Equation of equilibrium

$\Sigma F = 0 \quad P_1 + P_2 = P$ (Eq. 1)

$\Sigma M = 0 \quad \curvearrowleft + \quad Pe + P_1\left(\frac{b}{2}\right) - P_2\left(\frac{b}{2}\right) = 0$ (Eq. 2)

Equation of compatibility

$\delta_2 = \delta_1$

$$\frac{P_2 L}{E_2 A} = \frac{P_1 L}{E_1 A} \quad \text{OR} \quad \frac{P_2}{E_2} = \frac{P_1}{E_1} \qquad \text{(Eq. 3)}$$

(a) Axial forces

Solve simultaneously Eqs. (1) and (3):

$$P_1 = \frac{PE_1}{E_1 + E_2} \qquad P_2 = \frac{PE_2}{E_1 + E_2} \quad \longleftarrow$$

(b) Eccentricity of load P

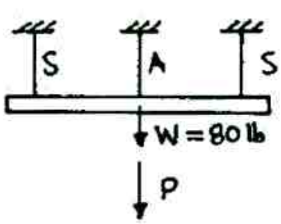
Substitute P_1 and P_2 into Eq. (2) and solve for e:

$$e = \frac{b(E_2 - E_1)}{2(E_2 + E_1)} \quad \longleftarrow$$

(c) Ratio of stresses

$$\sigma_1 = \frac{P_1}{A} \qquad \sigma_2 = \frac{P_2}{A} \qquad \frac{\sigma_1}{\sigma_2} = \frac{P_1}{P_2} = \frac{E_1}{E_2} \quad \longleftarrow$$

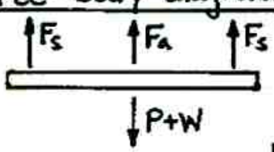
2.4-11 Rigid bar hanging from three wires



Steel wires:
$d_s = 1/8$ in. $\sigma_s = 20{,}000$ psi
$E_s = 30 \times 10^6$ psi

Aluminum wires:
$d_A = 3/16$ in. $\sigma_A = 12{,}000$ psi
$E_A = 10 \times 10^6$ psi

Free-body diagram of rigid bar



Equation of Equilibrium

$\Sigma F_{vert} = 0$

$2F_s + F_A - P - W = 0$ (Eq. 1)

Equation of compatibility

$\delta_s = \delta_A$ (Eq. 2)

Force displacement relations

$$\delta_s = \frac{F_s L}{E_s A_s} \qquad \delta_A = \frac{F_A L}{E_A A_A} \qquad \text{(Eqs. 3\&4)}$$

Solution of equations

Substitute (3) and (4) into Eq. (2):

$$\frac{F_s L}{E_s A_s} = \frac{F_A L}{E_A A_A} \qquad \text{(Eq. 5)}$$

CONT.

2.4-11 CONT.

Solve simultaneously Eqs. (1) and (5):

$$F_A = (P+W)\left(\frac{E_A A_A}{E_A A_A + 2E_s A_s}\right) \qquad \text{(Eq. 6)}$$

$$F_s = (P+W)\left(\frac{E_s A_s}{E_A A_A + 2E_s A_s}\right) \qquad \text{(Eq. 7)}$$

Stresses in the wires

$$\sigma_A = \frac{F_A}{A_A} = \frac{(P+W)E_A}{E_A A_A + 2E_s A_s} \qquad \text{(Eq. 8)}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{(P+W)E_s}{E_A A_A + 2E_s A_s} \qquad \text{(Eq. 9)}$$

Allowable loads (from Eqs. (8) and (9))

$$P_A = \frac{\sigma_A}{E_A}(E_A A_A + 2E_s A_s) - W \qquad \text{(Eq. 10)}$$

$$P_s = \frac{\sigma_s}{E_s}(E_A A_A + 2E_s A_s) - W \qquad \text{(Eq. 11)}$$

Substitute numerical values into Eqs. (10) and (11):

$A_s = \frac{\pi}{4}\left(\frac{1}{8}\text{ in.}\right)^2 = 0.012272$ in.^{2}

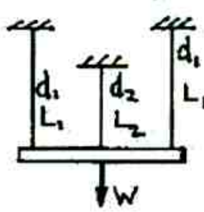
$A_A = \frac{\pi}{4}\left(\frac{3}{16}\text{ in.}\right)^2 = 0.027612$ in.^{2}

$P_A = 1215$ lb $- 80$ lb $= 1135$ lb

$P_s = 675$ lb $- 80$ lb $= 595$ lb

Steel governs. $P_{allow} = 595$ lb $\longleftarrow$

2.4-12 Rigid bar supported by three rods

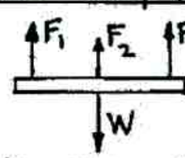


W = 30 kN

BRASS:
$E_1 = 100$ GPa
$d_1 = 8$ mm
$L_1 = 900$ mm
$\sigma_1 = 140$ MPa

COPPER:
$E_2 = 120$ GPa
$d_2 = ?$ $L_2 = ?$
$\sigma_2 = 210$ MPa

Free-body diagram of rigid bar



Equation of equilibrium

$\Sigma F_{vert} = 0$

$2F_1 + F_2 - W = 0$ (Eq. 1)

Fully stressed rods

$F_1 = \sigma_1 A_1 \qquad F_2 = \sigma_2 A_2$

$$A_1 = \frac{\pi d_1^2}{4} \qquad A_2 = \frac{\pi d_2^2}{4}$$

Substitute into Eq. (1):

$$2\sigma_1\left(\frac{\pi d_1^2}{4}\right) + \sigma_2\left(\frac{\pi d_2^2}{4}\right) = W$$

Diameter d_1 is known; solve for d_2:

$$d_2^2 = \frac{4W}{\pi\sigma_2} - \frac{2\sigma_1 d_1^2}{\sigma_2} \quad \longleftarrow \qquad \text{(Eq. 2)}$$

CONT.

2.4-12 CONT.

Substitute numerical values:

$$d_2^2 = \frac{4(30\text{ kN})}{\pi(210\text{ MPa})} - \frac{2(140\text{ MPa})(8\text{ mm})^2}{210\text{ MPa}}$$

$$= 0.00018189 - 0.00008533 = 0.00009656\text{ m}^2$$

$d_2 = 9.83$ mm ←

Equation of compatibility

$$\delta_1 = \delta_2 \qquad (\text{Eq. } 3)$$

Force-displacement relations

$$\delta_1 = \frac{F_1 L_1}{E_1 A_1} = \sigma_1\left(\frac{L_1}{E_1}\right) \qquad (\text{Eq. } 4)$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2} = \sigma_2\left(\frac{L_2}{E_2}\right) \qquad (\text{Eq. } 5)$$

Substitute (4) and (5) into Eq. (3):

$$\sigma_1\left(\frac{L_1}{E_1}\right) = \sigma_2\left(\frac{L_2}{E_2}\right)$$

Length L_1 is known; solve for L_2:

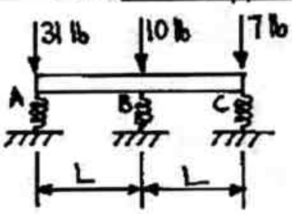
$$L_2 = L_1\left(\frac{\sigma_1 E_2}{\sigma_2 E_1}\right) \leftarrow \qquad (\text{Eq. } 6)$$

Substitute numerical values:

$$L_2 = (900\text{ mm})\left(\frac{140\text{ MPa}}{210\text{ MPa}}\right)\left(\frac{120\text{ GPa}}{100\text{ GPa}}\right)$$

$$= (900\text{ mm})(0.8) = 720\text{ mm} \leftarrow$$

2.4-13 Bar supported by three springs



Rigid bar ABC

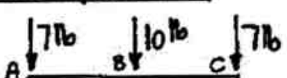
$L = 10$ in.

k = spring stiffness

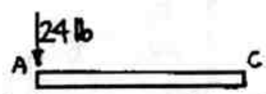
$= 80$ lb/in.

Separate the loading into 2 parts:

Load system #1

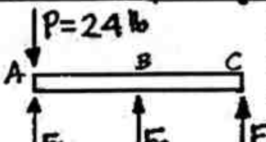


Load system #2



Because load system #1 is symmetrical, the rigid bar will displace downward but not rotate. Therefore, we need only to determine the rotation caused by load system #2.

Free-body diagram for load system #2



F_A, F_B, F_C are assumed to be compressive forces.

CONT.

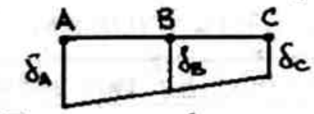
2.4-13 CONT.

Equations of Equilibrium (System #2)

$$\Sigma F_{vert} = 0 \qquad F_A + F_B + F_C = P \qquad (\text{Eq. } 1)$$

$$\Sigma M_A = 0 \qquad F_B L + F_C(2L) = 0 \qquad (\text{Eq. } 2)$$

Displacement diagram



DISPLACEMENTS ARE ASSUMED TO BE POSITIVE WHEN DOWNWARD.

Equation of compatibility

$$\delta_B = \frac{\delta_A + \delta_C}{2} \quad \text{or} \quad \delta_A + \delta_C - 2\delta_B = 0 \qquad (\text{Eq. } 3)$$

Force-displacement relations

$$\delta_A = \frac{F_A}{k} \qquad \delta_B = \frac{F_B}{k} \qquad \delta_C = \frac{F_C}{k} \qquad (\text{Eqs. } 4, 5, 6)$$

Solution of equations

Substitute (4), (5), and (6) into Eq. (3):

$$F_A - 2F_B + F_C = 0 \qquad (\text{Eq. } 7)$$

Solve simultaneously Eqs. (1), (2), and (7):

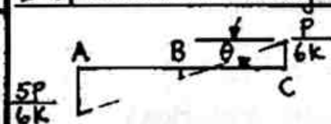
$$F_A = \frac{5P}{6} \qquad F_B = \frac{P}{3} \qquad F_C = -\frac{P}{6} \text{ (tension)}$$

Displacements (from Eqs. 4, 5 and 6)

$$\delta_A = \frac{5P}{6k} \qquad \delta_B = \frac{P}{3k} \qquad \delta_C = -\frac{P}{6k}$$

(Downward) (Downward) (Upward)

Displacement diagram for load system #2



$$\theta = \frac{\frac{5P}{6k} + \frac{P}{6k}}{2L}$$

$$\theta = \frac{P}{2kL} \quad \text{(counter-clockwise)}$$

Substitute numerical values:

$$\theta = \frac{24\text{ lb}}{2(80\text{ lb/in.})(10\text{ in.})} = 0.015\text{ rad} = 0.859° \leftarrow$$

Note: The displacements for load system #2 are as follows

$$\delta_A = \frac{5P}{6k} = \frac{5(24\text{ lb})}{6(80\text{ lb/in.})} = 0.250\text{ in. (Downward)}$$

$$\delta_B = \frac{P}{3k} = \frac{24\text{ lb}}{3(80\text{ lb/in.})} = 0.100\text{ in. (Downward)}$$

$$\delta_C = \frac{P}{6k} = 0.050\text{ in. (Upward)}$$

The downward displacement of the bar under load system #1 is as follows:

$$\delta_1 = \frac{7\text{ lb} + 10\text{ lb} + 7\text{ lb}}{3k} = \frac{24\text{ lb}}{3(80\text{ lb/in.})} = 0.1\text{ in.}$$

CONT.

2.4-13 CONT.

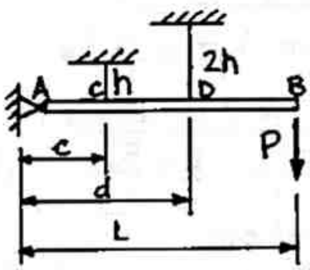
The downward displacements under the three original loads are as follows:

$\delta_A = 0.250 \text{ in.} + 0.1 \text{ in.} = 0.350 \text{ in.}$

$\delta_B = 0.100 \text{ in.} + 0.1 \text{ in.} = 0.200 \text{ in.}$

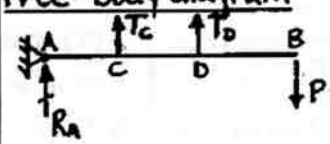
$\delta_C = -0.050 \text{ in.} + 0.1 \text{ in.} = 0.050 \text{ in.}$

2.4-14 Bar supported by two wires

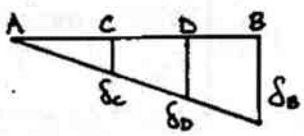


$h = 400$ mm
$2h = 800$ mm
$c = 500$ mm
$d = 1200$ mm
$L = 1600$ mm
$E = 200$ GPa
$A = 16 \text{ mm}^2$
$P = 970$ N

Free-body diagram



Displacement diagram



Equation of equilibrium

$\Sigma M_A = 0 \quad T_C(c) + T_D(d) = PL$ (Eq. 1)

Equation of compatibility

$$\frac{\delta_c}{c} = \frac{\delta_d}{d} \quad \text{(Eq. 2)}$$

Force-displacement relations

$$\delta_C = \frac{T_C h}{EA} \qquad \delta_D = \frac{T_D(2h)}{EA} \quad \text{(Eqs. 3, 4)}$$

Solution of equations

Substitute (3) and (4) into Eq. (2):

$$\frac{T_C h}{cEA} = \frac{T_D(2h)}{dEA} \quad \text{or} \quad \frac{T_C}{c} = \frac{2T_D}{d} \quad \text{(Eq. 5)}$$

Tensile forces in the wires

Solve simultaneously Eqs. (1) and (5):

$$T_C = \frac{2cPL}{2c^2 + d^2} \qquad T_D = \frac{dPL}{2c^2 + d^2}$$

Tensile stresses in the wires

$$\sigma_C = \frac{2cPL}{A(2c^2 + d^2)} \qquad \sigma_d = \frac{dPL}{A(2c^2 + d^2)}$$

Substitute numerical values:

$$A(2c^2 + d^2) = (16 \text{ mm}^2)[2(500 \text{ mm})^2 + (1200 \text{ mm})^2] = 31.04 \times 10^6 \text{ mm}^4$$

CONT.

2.4-14 CONT.

$$\sigma_C = \frac{2(500 \text{ mm})(970 \text{ N})(1600 \text{ mm})}{31.04 \times 10^6 \text{ mm}^4} = 50.0 \text{ MPa}$$

$$\sigma_D = \frac{(1200 \text{ mm})(970 \text{ N})(1600 \text{ mm})}{31.04 \times 10^6 \text{ mm}^4} = 60.0 \text{ MPa}$$

Displacement at end of bar

$$\delta_B = \delta_D\left(\frac{L}{d}\right) = \frac{T_D(2h)}{EA}\left(\frac{L}{d}\right) = \frac{2hPL^2}{EA(2c^2 + d^2)}$$

Substitute numerical values:

$$\delta_B = \frac{2(400 \text{ mm})(970 \text{ N})(1600 \text{ mm})^2}{(200 \text{ GPa})(31.04 \times 10^6 \text{ mm}^4)} = 0.320 \text{ mm}$$

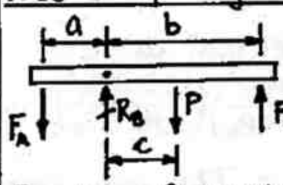
$\delta_B = 0.320$ mm

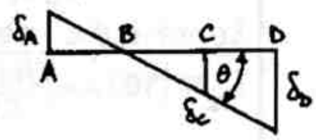
2.4-15 Rigid bar supported by springs

Numerical data

$a = 18.0$ in. $\quad b = 36.0$ in.
$c = 14.4$ in. $\quad k_1 = 64$ lb/in.
$k_2 = 210$ lb/in.
$\theta_{max} = 2° = \frac{\pi}{90}$ radians

Free-body diagram and displacement diagram





Equation of equilibrium

$\Sigma M_B = 0 \quad F_A(a) - P(c) + F_D(b) = 0$ (Eq. 1)

Equation of compatibility

$$\frac{\delta_A}{a} = \frac{\delta_D}{b} \quad \text{(Eq. 2)}$$

Force-displacement relations

$$\delta_A = \frac{F_A}{k_1} \qquad \delta_D = \frac{F_D}{k_2} \quad \text{(Eqs. 3, 4)}$$

Solution of equations

Substitute (3) and (4) into Eq. (2):

$$\frac{F_A}{ak_1} = \frac{F_D}{bk_2} \quad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$F_A = \frac{ack_1P}{a^2k_1 + b^2k_2} \qquad F_D = \frac{bck_2P}{a^2k_1 + b^2k_2}$$

Angle of rotation

$$\delta_D = \frac{F_D}{k_2} = \frac{bcP}{a^2k_1 + b^2k_2} \qquad \theta = \frac{\delta_D}{b} = \frac{cP}{a^2k_1 + b^2k_2}$$

Maximum load

$$P = \frac{\theta}{c}(a^2k_1 + b^2k_2) \qquad P_{max} = \frac{\theta_{max}}{c}(a^2k_1 + b^2k_2)$$

CONT.

2.4-15 CONT.

Substitute numerical values:

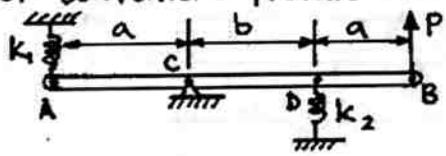
$\theta_{max} = \frac{\pi}{90}$ rad

$$P_{max} = \frac{\pi/90 \text{ rad}}{14.4 \text{ in.}}\left[(18.0 \text{ in.})^2\left(64\frac{\text{lb}}{\text{in.}}\right)+(36.0 \text{ in.})^2\left(210\frac{\text{lb}}{\text{in.}}\right)\right]$$

$P_{max} = 710$ lb ⟵

2.4-16 Bar supported by two Springs

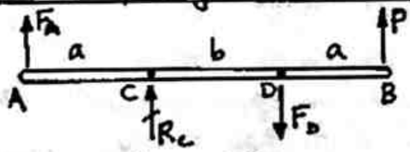
For convenience, rotate the bar 90°.



$k_1 = 15$ kN/m
$k_2 = 25$ kN/m
$a = 250$ mm
$b = 350$ mm
$\theta_{max} = 2.5°$
$= \frac{\pi}{72}$ rad

Find P_{max}.

Free-body diagram of the bar



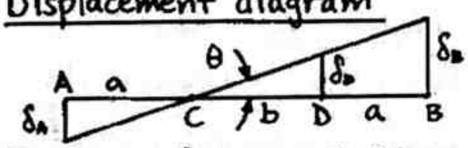
Equation of equilibrium

$\Sigma M_c = 0$ ↺+ ↻−

$-F_A(a) - F_D(b) + P(a+b) = 0$

$F_A a + F_D b = P(a+b)$ (Eq. 1)

Displacement diagram



Equation of compatibility

$$\frac{\delta_A}{a} = \frac{\delta_D}{b} \quad \text{(Eq. 2)}$$

Force-displacement relations

$$\delta_A = \frac{F_A}{k_1} \qquad \delta_D = \frac{F_D}{k_2} \quad \text{(Eqs. 3, 4)}$$

Solution of equations

Substitute (3) and (4) into Eq. (2):

$$\frac{F_A}{ak_1} = \frac{F_D}{bk_2} \quad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$F_A = \frac{ak_1P(a+b)}{a^2k_1+b^2k_2} \qquad F_D = \frac{bk_2P(a+b)}{a^2k_1+b^2k_2}$$

Displacements

$$\delta_A = \frac{F_A}{k_1} = \frac{aP(a+b)}{a^2k_1+b^2k_2}$$

CONT.

2.4-16 CONT.

$$\delta_D = \frac{F_D}{k_2} = \frac{bP(a+b)}{a^2k_1+b^2k_2}$$

Angle of rotation

$$\theta = \frac{\delta_D}{b} = \frac{P(a+b)}{a^2k_1+b^2k_2} \qquad (\theta = \text{radians})$$

Maximum load P

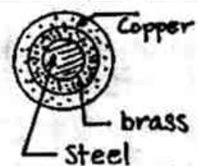
$$P_{max} = \frac{\theta_{max}(a^2k_1+b^2k_2)}{a+b} \quad ⟵$$

Substitute numerical values:

$$P_{max} = \frac{(\pi/72 \text{ rad})\left[(250 \text{ mm})^2(15 \text{ kN/m})+(350 \text{ mm})^2(25 \text{ kN/m})\right]}{250 \text{ mm} + 350 \text{ mm}}$$

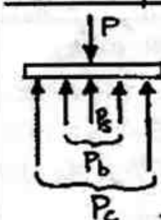
$$= \left(\frac{\pi}{72}\right)\frac{4000 \text{ N}\cdot\text{m}}{0.6 \text{ m}} = 291 \text{ N} \quad ⟵$$

2.4-17 Trimetallic bar in compression



P_s = compressive force in steel core
P_b = compressive force in brass tube
P_c = compressive force in copper tube

Free-body diagram of rigid end plate



Equation of equilibrium

$\Sigma F_{vert} = 0 \quad P_s + P_b + P_c = P$ (Eq. 1)

Equations of compatibility

$\delta_s = \delta_b \qquad \delta_c = \delta_s$ (Eqs. 2)

Force-displacement relations

$$\delta_s = \frac{P_sL}{E_sA_s} \qquad \delta_b = \frac{P_bL}{E_bA_b} \qquad \delta_c = \frac{P_cL}{E_cA_c} \quad \text{(Eqs. 3, 4, 5)}$$

Solution of equations

Substitute (3), (4), and (5) into Eqs. (2):

$$P_b = P_s\frac{E_bA_b}{E_sA_s} \qquad P_c = P_s\frac{E_cA_c}{E_sA_s} \quad \text{(Eqs. 6, 7)}$$

Solve simultaneously Eqs. (1), (6), and (7):

$$P_s = P\frac{E_sA_s}{E_sA_s+E_bA_b+E_cA_c} \qquad P_b = P\frac{E_bA_b}{E_sA_s+E_bA_b+E_cA_c}$$

$$P_c = P\frac{E_cA_c}{E_sA_s+E_bA_b+E_cA_c}$$

Compressive stresses

Let $\Sigma EA = E_sA_s + E_bA_b + E_cA_c$

$$\sigma_s = \frac{P_s}{A_s} = \frac{PE_s}{\Sigma EA} \qquad \sigma_b = \frac{P_b}{A_b} = \frac{PE_b}{\Sigma EA} \qquad \sigma_c = \frac{P_c}{A_c} = \frac{PE_c}{\Sigma EA}$$

Substitute numerical values:

$E_s = 30 \times 10^6$ psi $\quad E_b = 15 \times 10^6$ psi $\quad E_c = 18 \times 10^6$ psi

$A_s = \frac{\pi}{4}(0.4 \text{ in.})^2 = 0.1257 \text{ in.}^2$

$A_b = \frac{\pi}{4}[(0.6 \text{ in.})^2 - (0.4 \text{ in.})^2] = 0.1571 \text{ in.}^2$

$A_c = \frac{\pi}{4}[(0.8 \text{ in.})^2 - (0.6 \text{ in.})^2] = 0.2199 \text{ in.}^2$

CONT.

2.4-17 CONT.

$P = 2000\text{ lb} \quad \Sigma EA = 10.085 \times 10^6 \text{ lb}$

$\sigma_s = \dfrac{PE_s}{\Sigma EA} = 5950 \text{ psi} \longleftarrow$

$\sigma_b = \dfrac{PE_b}{\Sigma EA} = 2970 \text{ psi} \longleftarrow$

$\sigma_c = \dfrac{PE_c}{\Sigma EA} = 3570 \text{ psi} \longleftarrow$

2.5-1 Expansion of railroad rails

The rails are prevented from expanding because of their great length and lack of expansion joints.

Therefore, each rail is in the same condition as a bar with fixed ends (see Example 2-7).

The compressive stress in the rails may be calculated from Eq. (2-18).

$\sigma = E\alpha(\Delta T) = (30 \times 10^6 \text{ psi})(6.5 \times 10^{-6}/°\text{F})(75°\text{F})$

$\sigma = 14{,}600 \text{ psi} \longleftarrow$

2.5-2 Aluminum and steel pipes

Initial conditions: $L_a = 60 \text{ m} \quad T_0 = 18°\text{C}$

$L_s = 60.005 \text{ m} \quad T_0 = 18°\text{C}$

$\alpha_a = 23 \times 10^{-6}/°\text{C} \quad \alpha_s = 12 \times 10^{-6}/°\text{C}$

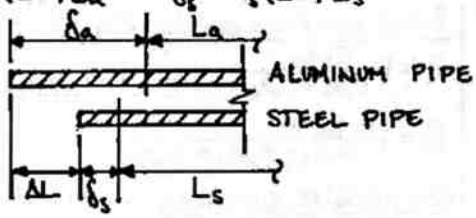
$\alpha_a L_a - \alpha_s L_s = 659.9 \times 10^{-6} \text{ m}/°\text{C}$

Final Conditions:

Aluminum pipe is longer than the steel pipe by the amount $\Delta L = 15 \text{ mm}$

ΔT = increase in temperature

$\delta_a = \alpha_a(\Delta T)L_a \quad \delta_s = \alpha_s(\Delta T)L_s$



$\delta_a - \delta_s - (L_s - L_a) = \Delta L$

$\alpha_a(\Delta T)L_a - \alpha_s(\Delta T)L_s - L_s + L_a = \Delta L$

$\Delta T = \dfrac{\Delta L + (L_s - L_a)}{\alpha_a L_a - \alpha_s L_s} \longleftarrow$

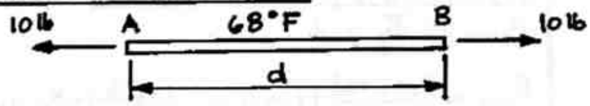
Substitute numerical values:

$\Delta T = \dfrac{15 \text{ mm} + 5 \text{ mm}}{659.9 \times 10^{-6} \text{ m}/°\text{C}} = 30.31°\text{C}$

$T = T_0 + \Delta T = 18°\text{C} + 30.31°\text{C} = 48.3°\text{C} \longleftarrow$

2.5-3 Steel measuring tape

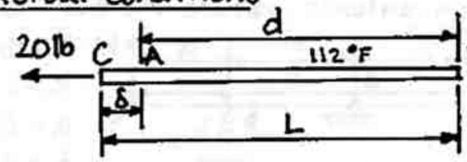
Calibration conditions



d = true distance between A and B (Same as the reading on the tape)

$EA = (30 \times 10^6 \text{ psi})(0.3 \text{ in.})(0.014 \text{ in.}) = 126{,}000 \text{ lb}$

Actual conditions



P = increase in tensile force = 20 lb − 10 lb = 10 lb

ΔT = increase in temperature = 112°F − 68°F = 44°F

δ = increase in length of the tape

$= \alpha(\Delta T)d + \dfrac{Pd}{EA}$

L = true length of tape $= d + \delta$

R_A = reading on tape at point A (85.49 ft)

R_C = reading on tape at end C (Numerically the same as the distance d)

$\dfrac{R_A}{R_C} = \dfrac{d}{L} \quad \text{or} \quad \dfrac{R_A}{d} = \dfrac{d}{d+\delta} = \dfrac{d}{d + \alpha(\Delta T)d + \frac{Pd}{EA}}$

$\dfrac{R_A}{d} = \dfrac{1}{1 + \alpha(\Delta T) + \frac{P}{EA}}$

$d = R_A\left[1 + \alpha(\Delta T) + \dfrac{P}{EA}\right] \longleftarrow$

$d = (85.49 \text{ ft})\left[1 + (6.5 \times 10^{-6}/°\text{F})(44°\text{F}) + \dfrac{10 \text{ lb}}{126{,}000 \text{ lb}}\right]$

$d = (85.49 \text{ ft})(1.0003654) = 85.52 \text{ ft} \longleftarrow$

2.5-4 Expansion of a steam pipe

Diameter $d = 120 \text{ mm}$

Initial temperature $T_0 = 18°\text{C}$

Final temperature $T_1 = 120°\text{C}$

$\Delta T = T_1 - T_0 = 102°\text{C}$

$E = 200 \text{ GPa} \quad \alpha = 12 \times 10^{-6}/°\text{C}$

(a) Increase in diameter

$\Delta d = \alpha(\Delta T)d = (12 \times 10^{-6}/°\text{C})(102°\text{C})(120 \text{ mm})$

$= 0.147 \text{ mm} \longleftarrow$

(b) Axial stress in pipe

Free expansion: $\delta_T = \alpha(\Delta T)L$

Restrained expansion: $\delta_R = \frac{1}{2}\delta_T$

The axial force P shortens the pipe by an amount $\delta = \delta_T/2$

CONT.

2.5-4 CONT

$$\delta = \frac{\delta_T}{2} = \frac{\alpha(\Delta T)L}{2} \qquad P = \frac{EA\delta}{L} = \frac{EA\alpha(\Delta T)}{2}$$

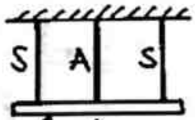
$$\sigma = \frac{P}{A} = \frac{E\alpha(\Delta T)}{2}$$

Substitute numerical values:

$$\sigma = (\tfrac{1}{2})(200\text{ GPa})(12\times10^{-6}/^\circ\text{C})(102^\circ\text{C})$$

$$\sigma = 122\text{ MPa} \longleftarrow$$

2.5-5 Beam supported by three wires



L = INITIAL LENGTH OF WIRES

S = steel A = aluminum

$W = 800$ lb

$d = \frac{1}{8}$ in.

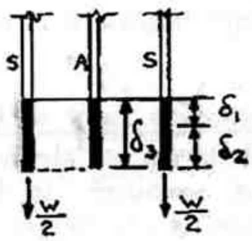
$A_s = \frac{\pi d^2}{4} = 0.012272\text{ in.}^2$

$E_s = 30\times10^6$ psi

$E_sA_s = 368{,}155$ lb

$\alpha_s = 6.5\times10^{-6}/^\circ\text{F}$

$\alpha_a = 12\times10^{-6}/^\circ\text{F}$



δ_1 = increase in length of a steel wire due to temperature increase ΔT
$= \alpha_s(\Delta T)L$

δ_2 = increase in length of a steel wire due to load $W/2$
$= \frac{WL}{2E_sA_s}$

δ_3 = increase in length of aluminum wire due to temperature increase ΔT
$= \alpha_a(\Delta T)L$

For no load in the aluminum wire:

$$\delta_1 + \delta_2 = \delta_3$$

$$\alpha_s(\Delta T)L + \frac{WL}{2E_sA_s} = \alpha_a(\Delta T)L$$

OR

$$\Delta T = \frac{W}{2E_sA_s(\alpha_a - \alpha_s)} \longleftarrow$$

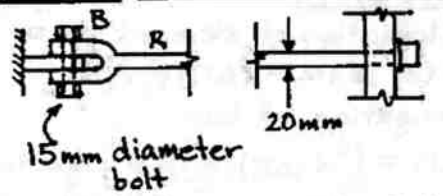
Substitute numerical values:

$$\Delta T = \frac{800\text{ lb}}{(2)(368{,}155\text{ lb})(5.5\times10^{-6}/^\circ\text{F})}$$

$$= 198^\circ\text{F} \longleftarrow$$

Note: If the temperature increase is larger than ΔT, the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than ΔT, the aluminum wire will be in tension and carry part of the load.

2.5-6 Steel rod with bolted connection



R = rod
B = bolt

P = tensile force in steel rod due to temperature drop ΔT

A_R = cross-sectional area of steel rod

From Eq. (2-17) of Example 2-7: $P = EA_R\alpha(\Delta T)$

Bolt is in double shear.

V = shear force acting over one cross section of the bolt

$V = P/2 = \frac{1}{2}EA_R\alpha(\Delta T)$

τ = average shear stress on cross section of the bolt

A_B = cross-sectional area of bolt

$$\tau = \frac{V}{A_B} = \frac{EA_R\alpha(\Delta T)}{2A_B}$$

Solve for ΔT: $\Delta T = \frac{2\tau A_B}{EA_R\alpha}$

$A_B = \frac{\pi d_B^2}{4}$ where d_B = diameter of bolt

$A_R = \frac{\pi d_R^2}{4}$ where d_R = diameter of steel rod

$$\Delta T = \frac{2\tau d_B^2}{E\alpha d_R^2} \longleftarrow$$

Substitute numerical values:

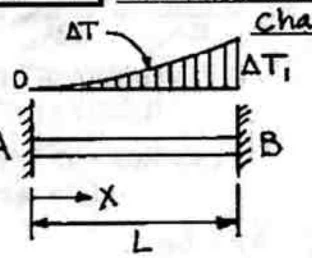
$\tau = 50$ MPa $d_B = 15$ mm $d_R = 20$ mm

$\alpha = 12\times10^{-6}/^\circ\text{C}$ $E = 200$ GPa

$$\Delta T = \frac{2(50\text{ MPa})(15\text{ mm})^2}{(200\text{ GPa})(12\times10^{-6}/^\circ\text{C})(20\text{ mm})^2}$$

$$\Delta T = 23.4^\circ\text{C} \longleftarrow$$

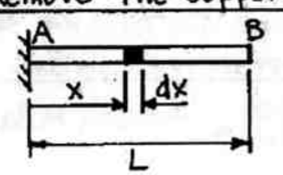
2.5-7 Bar with nonuniform temperature change



At distance x:

$$\Delta T = \Delta T_1\left(\frac{x^2}{L^2}\right)$$

Remove the support at end B of the bar:



Consider an element dx at a distance x from end A.

CONT.

2.5-7 CONT.

$d\delta$ = Elongation of element dx

$d\delta = \alpha(\Delta T)dx = \alpha(\Delta T_1)\left(\frac{x^2}{L^2}\right)dx$

δ = elongation of bar

$\delta = \int_0^L d\delta = \int_0^L \alpha(\Delta T_1)\left(\frac{x^2}{L^2}\right)dx = \frac{1}{3}\alpha(\Delta T_1)L$

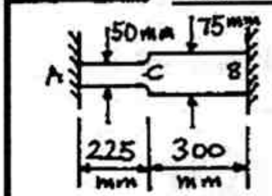
Compressive force P required to shorten the bar by the amount δ

$P = \frac{EA\delta}{L} = \frac{1}{3}EA\alpha(\Delta T_1)$

Compressive stress in the bar

$\sigma_c = \frac{P}{A} = \frac{E\alpha(\Delta T_1)}{3}$ ←

2.5-8 Bar with rigid supports



$\Delta T = 30°C$

$E = 6.0\,\text{GPa}$ $\alpha = 100 \times 10^{-6}/°C$

Left-hand part:

$L_1 = 225\,\text{mm}$ $d_1 = 50\,\text{mm}$

$A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50\,\text{mm})^2$
$= 1963.5\,\text{mm}^2$

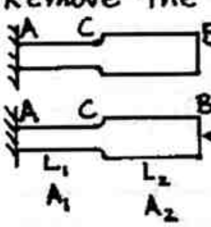
Right-hand part:

$L_2 = 300\,\text{mm}$ $d_2 = 75\,\text{mm}$

$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(75\,\text{mm})^2 = 4417.9\,\text{mm}^2$

(a) Compressive force P

Remove the support at end B.



δ_T = elongation due to temperature

$= \alpha(\Delta T)(L_1 + L_2)$

$= 1.5750\,\text{mm}$

δ_P = Shortening due to P

$= \frac{PL_1}{EA_1} + \frac{PL_2}{EA_2}$

$= P(19.0986 \times 10^{-9}\,\text{m/N} + 11.3177 \times 10^{-9}\,\text{m/N})$

$= (30.4163 \times 10^{-9}\,\text{m/N})P$
(P = newtons)

Compatibility $\delta_T = \delta_P$

$1.5750 \times 10^{3}\,\text{m} = (30.4163 \times 10^{-9}\,\text{m/N})P$

$P = 51{,}781\,\text{N}$ or $P = 51.8\,\text{kN}$ ←

(b) Maximum compressive stress

$\sigma_c = \frac{P}{A_1} = \frac{51.78\,\text{kN}}{1963.5\,\text{mm}^2} = 26.4\,\text{MPa}$ ←

CONT.

2.5-8 CONT.

(C) Displacement of point C

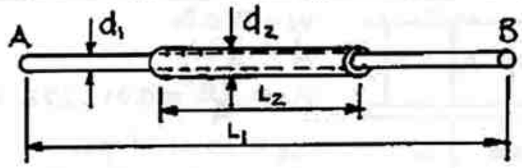
δ_C = Shortening of AC

$\delta_C = \frac{PL_1}{EA_1} - \alpha(\Delta T)L_1 = 0.9890\,\text{mm} - 0.6750\,\text{mm}$

$\delta_C = 0.314\,\text{mm}$ ←

(Positive means AC shortens and point C displaces to the left.)

2.5-9 Steel rod with bronze sleeve



$L_1 = 3.0\,\text{ft}$
$L_2 = 1.0\,\text{ft}$

Elongation of the two outer parts of the bar

$\delta_1 = \alpha_s(\Delta T)(L_1 - L_2)$

$= (6.5 \times 10^{-6}/°F)(600°F)(36\,\text{in.} - 12\,\text{in.})$

$= 0.09360\,\text{in.}$

Elongation of the middle part of the bar

The steel rod and bronze sleeve lengthen the same amount, so they are in the same condition as the bolt and sleeve of Example 2-8. Thus, we can calculate the elongation from Eq. (2-21):

$\delta_2 = \frac{(\alpha_s E_s A_s + \alpha_b E_b A_b)(\Delta T)L_2}{E_s A_s + E_b A_b}$

Substitute numerical values:

$\alpha_s = 6.5 \times 10^{-6}/°F$ $\alpha_b = 11 \times 10^{-6}/°F$

$E_s = 30 \times 10^6\,\text{psi}$ $E_b = 15 \times 10^6\,\text{psi}$

$d_1 = 1.0\,\text{in.}$ $A_s = \frac{\pi}{4}d_1^2 = 0.78540\,\text{in.}^2$

$d_2 = 1.25\,\text{in.}$ $A_b = \frac{\pi}{4}(d_2^2 - d_1^2) = 0.44179\,\text{in.}^2$

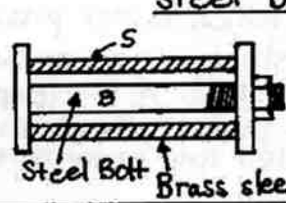
$\Delta T = 600°F$ $L_2 = 12.0\,\text{in.}$

$\delta_2 = 0.05391\,\text{in.}$

Total elongation

$\delta = \delta_1 + \delta_2 = 0.148\,\text{in.}$ ←

2.5-10 Brass sleeve fitted over a Steel bolt



Subscript S means "sleeve"

Subscript B means "bolt"

CONT.

2.5-10 CONT.

Use the results of Example 2-8.

σ_S = compressive force in sleeve S

Equation (2-20a):

$$\sigma_S = \frac{(\alpha_S - \alpha_B)(\Delta T) E_S E_B A_B}{E_S A_S + E_B A_B} \quad \text{(Compression)}$$

Solve for ΔT:

$$\Delta T = \frac{\sigma_S (E_S A_S + E_B A_B)}{(\alpha_S - \alpha_B) E_S E_B A_B} \longleftarrow$$

Substitute numerical values:

Bolt diameter $d_B = 25\text{ mm}$; $A_B = 490.87\text{ mm}^2$

Sleeve outside diameter $d_2 = 36\text{ mm}$

Sleeve inside diameter $d_1 = 26\text{ mm}$

$A_S = 486.95\text{ mm}^2$

$\sigma_S = 25\text{ MPa}$ (compressive stress in sleeve)

Brass sleeve: $E_S = 100\text{ GPa}$ $\alpha_S = 20 \times 10^{-6}/°C$

Steel bolt: $E_B = 200\text{ GPa}$ $\alpha_B = 12 \times 10^{-6}/°C$

$\Delta T = 46.75°C \longleftarrow$

(Increase in temperature)

2.5-11 Rectangular bars held by pi

Diameter of pin $d_P = \frac{7}{16}\text{ in.} = 0.4375\text{ in.}$

Area of pin $A_P = \frac{\pi}{4} d_P^2 = 0.15033\text{ in.}^2$

Area of two copper bars $A_c = 2.0\text{ in.}^2$

Area of aluminum bar $A_a = 2.0\text{ in.}^2$

$\Delta T = 100°F$

Copper: $E_c = 18{,}000\text{ ksi}$ $\alpha_c = 9.5 \times 10^{-6}/°F$

Aluminum: $E_a = 10{,}000\text{ ksi}$ $\alpha_a = 13 \times 10^{-6}/°F$

Use the results of Example 2-8.

Find the forces P_a and P_c in the aluminum bar and copper bar, respectively, from Eq. (2-19).

Replace the subscript "S" in that equation by "a" (for aluminum) and replace the subscript "B" by "c" (for copper).

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T) E_a A_a E_c A_c}{E_a A_a + E_c A_c}$$

Note that P_a is the compressive force in the aluminum bar and P_c is the combined tensile force in the two copper bars.

CONT.

2.5-11 CONT.

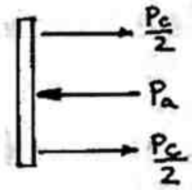
Substitute numerical values:

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T) E_c A_c}{1 + \frac{E_c A_c}{E_a A_a}}$$

$$= \frac{(3.5 \times 10^{-6}/°F)(100°F)(18{,}000\text{ ksi})(2\text{ in.}^2)}{1 + \left(\frac{18}{10}\right)\left(\frac{2.0}{2.0}\right)}$$

$= 4500\text{ lb}$

Free-body diagram of pin



V = shear force in pin

$= P_c/2$

$= 2250\text{ lb}$

τ = average shear stress on cross section of pin

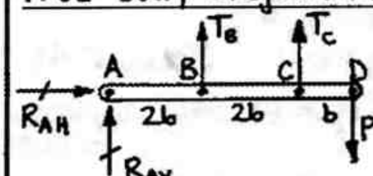
$$\tau = \frac{V}{A_P}$$

$$\tau = \frac{2250\text{ lb}}{0.15033\text{ in.}^2}$$

$\tau = 15.0\text{ ksi} \longleftarrow$

2.5-12 Rigid bar supported by two cables

Free-body diagram of bar ABCD



T_B = force in cable B

T_C = force in cable C

$d_B = 12\text{ mm}$

$d_C = 20\text{ mm}$

From Table 2-1:

$A_B = 76.7\text{ mm}^2$ $A_C = 173\text{ mm}^2$

$E = 140\text{ GPa}$ $\alpha = 12 \times 10^{-6}/°C$

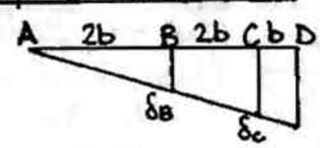
$\Delta T = 60°C$

Equation of equilibrium

$\Sigma M_A = 0$ ↺+ ↻− $T_B(2b) + T_C(4b) - P(5b) = 0$

or $2T_B + 4T_C = 5P$ (Eq. 1)

Displacement diagram



Compatibility:

$\delta_C = 2\delta_B$ (Eq. 2)

Force-displacement and temperature-displacement relations

$$\delta_B = \frac{T_B L}{E A_B} + \alpha(\Delta T) L \quad \text{(Eq. 3)}$$

$$\delta_C = \frac{T_C L}{E A_C} + \alpha(\Delta T) L \quad \text{(Eq. 4)}$$

CONT.

2.5-12 CONT.

Substitute Eqs.(3) and (4) into Eq. (2):

$$\frac{T_C L}{EA_C} + \alpha(\Delta T)L = \frac{2T_B L}{EA_B} + 2\alpha(\Delta T)L$$

OR

$$2T_B A_C - T_C A_B = -E\alpha(\Delta T)A_B A_C \qquad \text{(Eq. 5)}$$

Substitute numerical values into Eq. (5):

$$T_B(346) - T_C(76.7) = -1{,}338{,}000 \qquad \text{(Eq. 6)}$$

in which T_B and T_C have units of newtons.

Solve simultaneously Eqs. (1) and (6):

$$T_B = 0.2494\ P - 3{,}480 \qquad \text{(Eq. 7)}$$

$$T_C = 1.1253\ P + 1{,}740 \qquad \text{(Eq. 8)}$$

in which P has units of newtons.

Solve Eqs. (7) and (8) for the load P:

$$P_B = 4.0096\, T_B + 13{,}953 \qquad \text{(Eq. 9)}$$

$$P_C = 0.8887\, T_C - 1{,}546 \qquad \text{(Eq. 10)}$$

Allowable loads

From Table 2-1:

$(T_B)_{ULT} = 102{,}000\ N$

$(T_C)_{ULT} = 231{,}000\ N$

Factor of safety $= 5$

$(T_B)_{ALLOW} = 20{,}400\ N \qquad (T_C)_{ALLOW} = 46{,}200\ N$

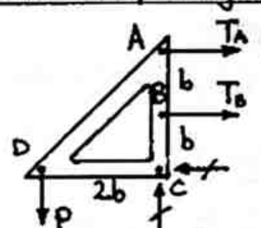
From Eq. (9): $P_B = (4.0096)(20{,}400\ N) + 13{,}953\ N = 95{,}700\ N$

From Eq. (10): $P_C = (0.8887)(46{,}200\ N) - 1546\ N = 39{,}500\ N$

Cable C governs. $P_{allow} = 39.5\ kN$ ←

2.5-13 Triangular frame held by two wires

Free-body diagram of frame



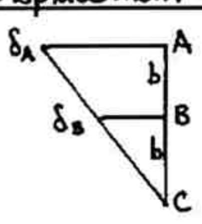
Equation of equilibrium

$\Sigma M_C = 0$

$$P(2b) - T_A(2b) - T_B(b) = 0$$

$$\text{OR } 2T_A + T_B = 2P \qquad \text{(Eq. 1)}$$

Displacement diagram



Equation of Compatibility

$$\delta_A = 2\delta_B \qquad \text{(Eq. 2)}$$

CONT.

2.5-13 CONT.

(a) Load P only

Force-displacement relations:

$$\delta_A = \frac{T_A L}{EA} \qquad \delta_B = \frac{T_B L}{EA} \qquad \text{(Eqs. 3, 4)}$$

(L = length of wires at A and B.)

Substitute (3) and (4) into Eq. (2):

$$\frac{T_A L}{EA} = \frac{2T_B L}{EA} \quad \text{OR} \quad T_A = 2T_B \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$T_A = \frac{4P}{5} \qquad T_B = \frac{2P}{5} \qquad \text{(Eqs. 6, 7)}$$

Numerical values:

$P = 500\ lb$

$\therefore T_A = 400\ lb \qquad T_B = 200\ lb$ ←

(b) Load P and temperature increase ΔT

Force-displacement and temperature-displacement relations:

$$\delta_A = \frac{T_A L}{EA} + \alpha(\Delta T)L \qquad \text{(Eq. 8)}$$

$$\delta_B = \frac{T_B L}{EA} + \alpha(\Delta T)L \qquad \text{(Eq. 9)}$$

Substitute (8) and (9) into Eq. (2):

$$\frac{T_A L}{EA} + \alpha(\Delta T)L = \frac{2T_B L}{EA} + 2\alpha(\Delta T)L$$

$$\text{OR } T_A - 2T_B = EA\alpha(\Delta T) \qquad \text{(Eq. 10)}$$

Solve simultaneously Eqs. (1) and (10):

$$T_A = \frac{1}{5}[4P + EA\alpha(\Delta T)] \qquad \text{(Eq. 11)}$$

$$T_B = \frac{2}{5}[P - EA\alpha(\Delta T)] \qquad \text{(Eq. 12)}$$

Substitute numerical values:

$P = 500\ lb \qquad EA = 120{,}000\ lb \qquad \Delta T = 180°F$

$\alpha = 12.5 \times 10^{-6}/°F$

$T_A = \frac{1}{5}(2000\ lb + 270\ lb) = 454\ lb$ ←

$T_B = \frac{2}{5}(500\ lb - 270\ lb) = 92\ lb$ ←

(c) Wire B becomes slack

Set $T_B = 0$ in Eq. (12):

$P = EA\alpha(\Delta T)$

OR

$$\Delta T = \frac{P}{EA\alpha} = \frac{500\ lb}{(120{,}000\ lb)(12.5 \times 10^{-6}/°F)} = 333.3°F$$

Further increase in temperature

$= 333.3°F - 180°F$

$= 153°F$ ←

2.5-14 Steel wire with initial prestress

Initial prestress $\sigma_1 = 40$ MPa
Initial temperature $T_1 = 20°C$
$E = 210$ GPa
$\alpha = 14 \times 10^{-6}/°C$

(a) Stress σ when temperature drops to 0°C

$T_2 = 0°C$ $\Delta T = 20°C$ (Positive means decrease in temperature)

Stress σ equals the initial stress σ_1 plus the additional stress σ_2 due to the temperature drop.

From Eq. (2-18): $\sigma_2 = E\alpha(\Delta T)$

$$\sigma = \sigma_1 + \sigma_2 = \sigma_1 + E\alpha(\Delta T)$$
$$= 40\text{ MPa} + (210\text{ GPa})(14 \times 10^{-6}/°C)(20°C)$$
$$= 40\text{ MPa} + 58.8\text{ MPa} = 98.8\text{ MPa} \longleftarrow$$

(b) Temperature when stress equals zero

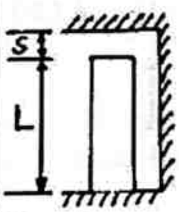
$$\sigma = \sigma_1 + \sigma_2 = 0 \qquad \sigma_1 + E\alpha(\Delta T) = 0$$

$\Delta T = -\dfrac{\sigma_1}{E\alpha}$ (Minus means an <u>increase</u> in temperature)

$$\Delta T = -\frac{40\text{ MPa}}{(210\text{ GPa})(14 \times 10^{-6}/°C)} = -13.6°C$$

$$T = 20°C + 13.6°C = 33.6°C \longleftarrow$$

2.5-15 Bar with a gap (temperature change)



$L = 40$ in. $S = 0.008$ in.
$\Delta T = 90°F$ (increase)
$\alpha = 9.8 \times 10^{-6}/°F$
$E = 16 \times 10^6$ psi

δ = elongation of the bar if it is free to expand
$= \alpha(\Delta T)L$

δ_C = elongation that is prevented by the support
$= \alpha(\Delta T)L - S$

ϵ_c = strain in the bar due to the restraint
$= \delta_c / L$

σ_c = stress in the bar

$$= E\epsilon_c = \frac{E\delta_c}{L} = \frac{E}{L}[\alpha(\Delta T)L - S] \longleftarrow$$

Note: This result is valid only if $\alpha(\Delta T)L \geq S$

CONT.

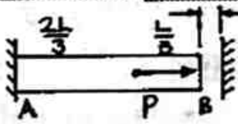
2.5-15 CONT.

Substitute numerical values:

$$\sigma_c = \frac{16 \times 10^6\text{ psi}}{40\text{ in.}}\left[(9.8 \times 10^{-6}/°F)(90°F)(40\text{ in.}) - 0.008\text{ in.}\right]$$

$$= 10{,}900\text{ psi} \longleftarrow$$

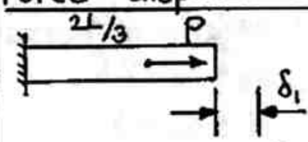
2.5-16 Bar with a gap (load P)



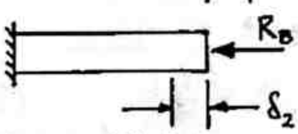
L = length of bar
S = size of gap
EA = axial rigidity

Reactions must be equal; find S.

Force - displacement relations



$$\delta_1 = \frac{P\left(\frac{2L}{3}\right)}{EA}$$



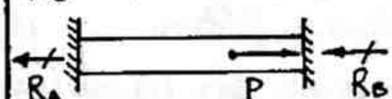
$$\delta_2 = \frac{R_B L}{EA}$$

Compatibility equation

$$\delta_1 - \delta_2 = S \quad \text{OR} \quad \frac{2PL}{3EA} - \frac{R_B L}{EA} = S \qquad \text{(Eq. 1)}$$

Equilibrium equation

R_A = reaction at end A (to the left)
R_B = reaction at end B (to the left)



$P = R_A + R_B$

Reactions must be equal. $\therefore R_A = R_B$

$$P = 2R_B \qquad R_B = \frac{P}{2}$$

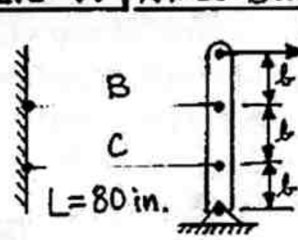
Substitute for R_B in Eq. (1):

$$\frac{2PL}{3EA} - \frac{PL}{2EA} = S \quad \text{OR} \quad S = \frac{PL}{6EA} \longleftarrow$$

<u>Note</u>: This solution is valid provided the load P is large enough to close the gap; that is, $\delta_1 \geq S$,

or $\dfrac{2PL}{3EA} \geq S$, or $P \geq \dfrac{3EAS}{2L}$.

2.5-17 Wires B and C attached to a bar

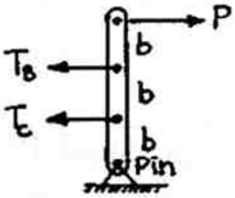


$A = 0.03$ in.^{2}
$E = 30 \times 10^6$ psi
$L_B = 79.98$ in.
$L_C = 79.95$ in.

CONT.

2.5-17 CONT.

Equilibrium Equation

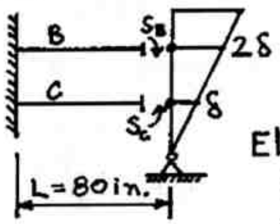


$\Sigma M_{Pin} = 0$ (+) (−)

$T_C(b) + T_B(2b) = P(3b)$

$2T_B + T_C = 3P$ (Eq. 1)

Displacement diagram



$S_B = 80 \text{ in.} - L_B = 0.02 \text{ in.}$

$S_C = 80 \text{ in.} - L_C = 0.05 \text{ in.}$

Elongation of wires:

$\delta_B = S_B + 2\delta$ (Eq. 2)

$\delta_C = S_C + \delta$ (Eq. 3)

Force-displacement relations

$\delta_B = \dfrac{T_B L}{EA}$ $\qquad \delta_C = \dfrac{T_C L}{EA}$ (Eq. 4, Eq. 5)

Solution of equations

Combine Eqs. (2) and (4): $\dfrac{T_B L}{EA} = S_B + 2\delta$ (Eq. 6)

Combine Eqs. (3) and (5): $\dfrac{T_C L}{EA} = S_C + \delta$ (Eq. 7)

Eliminate δ between Eqs. (6) and (7):

$T_B - 2T_C = \dfrac{EAS_B}{L} - \dfrac{2EAS_C}{L}$ (Eq. 8)

Solve simultaneously Eqs. (1) and (8):

$T_B = \dfrac{6P}{5} + \dfrac{EAS_B}{5L} - \dfrac{2EAS_C}{5L}$ ←

$T_C = \dfrac{3P}{5} - \dfrac{2EAS_B}{5L} + \dfrac{4EAS_C}{5L}$ ←

Substitute numerical values:

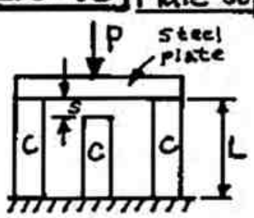
$\dfrac{EA}{5L} = 2250 \text{ lb/in.}$

$T_B = 840 \text{ lb} + 45 \text{ lb} - 225 \text{ lb} = 660 \text{ lb}$ ←

$T_C = 420 \text{ lb} - 90 \text{ lb} + 450 \text{ lb} = 780 \text{ lb}$ ←

(Both forces are positive, which means tension, as required for wires.)

2.5-18 Plate supported by three posts



C = CONCRETE POST

S = size of gap = 1.0 mm

L = length of posts = 2.0 m

$A = 40{,}000 \text{ mm}^2$

$\sigma_{allow} = 18 \text{ MPa}$

$E = 30 \text{ GPa}$

CONT.

2.5-18 CONT.

Does the gap close?

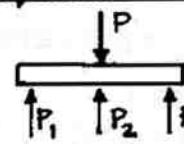
Stress in the two outer posts when the gap is just closed:

$\sigma = E\epsilon = E\left(\dfrac{S}{L}\right) = (30 \text{ GPa})\left(\dfrac{1.0 \text{ mm}}{2.0 \text{ m}}\right)$

$= 15 \text{ MPa}$

Since this stress is less than the allowable stress, the allowable force P will close the gap.

Equilibrium equation



$2P_1 + P_2 = P$ (Eq. 1)

Compatibility equation

δ_1 = shortening of outer posts

δ_2 = shortening of inner post

$\delta_1 = \delta_2 + S$ (Eq. 2)

Force-displacement relations

$\delta_1 = \dfrac{P_1 L}{EA}$ $\qquad \delta_2 = \dfrac{P_2 L}{EA}$ (Eq. 3, Eq. 4)

Solution of equations

Substitute (3) and (4) into Eq. (2):

$\dfrac{P_1 L}{EA} = \dfrac{P_2 L}{EA} + S$ or $P_1 - P_2 = \dfrac{EAS}{L}$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$P = 3P_1 - \dfrac{EAS}{L}$

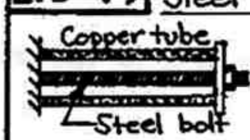
By inspection, we know that P_1 is larger than P_2. Therefore, P_1 will control and will be equal to $\sigma_{allow}A$.

$P_{allow} = 3\sigma_{allow}A - \dfrac{EAS}{L}$

$= 2160 \text{ kN} - 600 \text{ kN} = 1560 \text{ kN}$

$= 1.56 \text{ MN}$ ←

2.5-19 Steel bolt and copper tube



$L = 20 \text{ in.}$

$p = 1/8 \text{ in.}$

$n = 1/4$ (See Eq. 2-22)

Steel bolt: $A_S = 1.0 \text{ in.}^2$

$E_S = 30 \times 10^6 \text{ psi}$

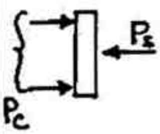
Copper tube: $A_C = 2.0 \text{ in.}^2$

$E_C = 16 \times 10^6 \text{ psi}$

CONT.

2.5-19 CONT.

Equilibrium equation



P_s = tensile force in steel bolt

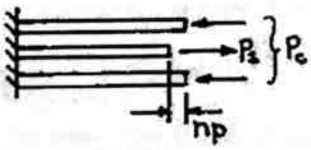
P_c = compressive force in copper tube

$$P_c = P_s \qquad (\text{Eq. } 1)$$

Compatibility equation

δ_c = shortening of copper tube

δ_s = elongation of steel bolt



$$\delta_c + \delta_s = np \qquad (\text{Eq. } 2)$$

Force-displacement relations

$$\delta_c = \frac{P_c L}{E_c A_c} \qquad \delta_s = \frac{P_s L}{E_s A_s} \qquad (\text{Eq. } 3,\ \text{Eq. } 4)$$

Solution of equations

Substitute (3) and (4) into Eq. (2):

$$\frac{P_c L}{E_c A_c} + \frac{P_s L}{E_s A_s} = np \qquad (\text{Eq. } 5)$$

Solve simultaneously Eqs. (1) and (5):

$$P_s = P_c = \frac{np E_s A_s E_c A_c}{L(E_s A_s + E_c A_c)} = 24{,}190 \text{ lb}$$

Stresses

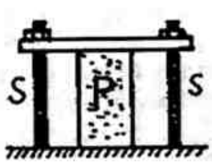
Steel bolt: $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{24{,}190 \text{ lb}}{1.0 \text{ in.}^2}$

$= 24{,}200$ psi ←

Copper: $\sigma_c = \dfrac{P_c}{A_c} = \dfrac{24{,}190 \text{ lb}}{2.0 \text{ in.}^2}$

$= 12{,}100$ psi ←

2.5-20 Plastic cylinder and two steel bolts



$L = 250$ mm

$p = 1.2$ mm

$E_s = 200$ GPa

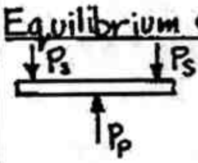
$A_s = 36.0 \text{ mm}^2$ (for one bolt)

$E_p = 7.5$ GPa

$A_p = 960 \text{ mm}^2$

$n = 1$ (See Eq. 2-22)

Equilibrium equation



P_s = tensile force in one steel bolt

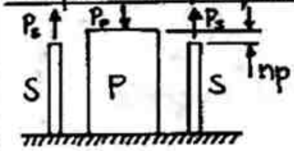
P_p = compressive force in plastic cylinder

$$P_p = 2P_s \qquad (\text{Eq. } 1)$$

CONT.

2.5-20 CONT.

Compatibility equation



δ_s = elongation of steel bolt

δ_p = shortening of plastic cylinder

$$\delta_s + \delta_p = np \qquad (\text{Eq. } 2)$$

Force-displacement relations

$$\delta_s = \frac{P_s L}{E_s A_s} \qquad \delta_p = \frac{P_p L}{E_p A_p} \qquad (\text{Eq. } 3,\ \text{Eq. } 4)$$

Solution of equations

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \qquad (\text{Eq. } 5)$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2np E_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

Stress in the plastic cylinder

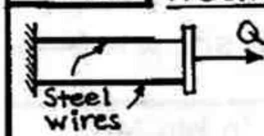
$$\sigma_p = \frac{P_p}{A_p} = \frac{2np E_s A_s E_p}{L(E_p A_p + 2E_s A_s)} \leftarrow$$

Substitute numerical values:

$$\sigma_p = \frac{2(1)(1.2\text{ mm})(200\text{ GPa})(36.0\text{ mm}^2)(7.5\text{ GPa})}{(250\text{ mm})[(7.5\text{ GPa})(960\text{ mm}^2) + 2(200\text{ GPa})(36.0\text{ mm}^2)]}$$

$= 24.0$ MPa ←

2.5-21 Prestressed concrete beam

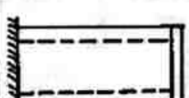


L = length

σ_0 = initial stress in wires $= \dfrac{Q}{A_s}$

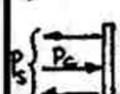
A_s = total area of steel wires

A_c = area of concrete $= 30 A_s$



$E_s = 8E_c$

P_s = total tensile force in steel wires



P_c = compressive force in concrete

Equilibrium equation

$$P_s = P_c \qquad (\text{Eq. } 1)$$

Compatibility equation and force-displacement relations

δ_1 = initial elongation of steel wires

$$= \frac{QL}{E_s A_s} = \frac{\sigma_0 L}{E_s}$$

CONT.

2.5-21 CONT.

δ_2 = final elongation of steel wires

$= \frac{P_s L}{E_s A_s}$

δ_3 = shortening of concrete

$= \frac{P_c L}{E_c A_c}$

$\delta_1 - \delta_2 = \delta_3$ or $\frac{\sigma_0 L}{E_s} - \frac{P_s L}{E_s A_s} = \frac{P_c L}{E_c A_c}$ (Eq. 2, Eq. 3)

Solve simultaneously Eqs. (1) and (3):

$$P_s = P_c = \frac{\sigma_0 A_s}{1 + \frac{E_s A_s}{E_c A_c}}$$

Stresses

$$\sigma_s = \frac{P_s}{A_s} = \frac{\sigma_0}{1 + \frac{E_s A_s}{E_c A_c}} \longleftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{\sigma_0}{\frac{A_c}{A_s} + \frac{E_s}{E_c}} \longleftarrow$$

Substitute numerical values:

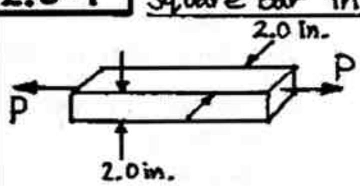
$\frac{E_s}{E_c} = 8 \quad \frac{A_s}{A_c} = \frac{1}{30} \quad \frac{E_s A_s}{E_c A_c} = \frac{4}{15}$

$\sigma_0 = 95{,}000$ psi

$$\sigma_s = \frac{95{,}000 \text{ psi}}{1 + \frac{4}{15}} = 75{,}000 \text{ psi} \longleftarrow$$

$$\sigma_c = \frac{95{,}000 \text{ psi}}{30 + 8} = 2{,}500 \text{ psi} \longleftarrow$$

2.6-1 Square bar in tension



$A = 2.0 \text{ in.} \times 2.0 \text{ in.} = 4.0 \text{ in.}^2$

Maximum Normal Stress $\sigma_x = \frac{P}{A}$

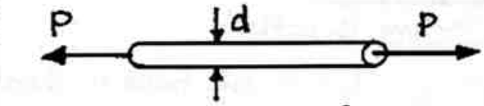
Maximum shear stress: $\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$

$\sigma_{allow} = 16{,}000$ psi $\quad \tau_{allow} = 9{,}000$ psi

Because τ_{allow} is greater than one-half of σ_{allow}, the normal stress governs.

$$P_{max} = \sigma_{allow} A = (16{,}000 \text{ psi})(4.0 \text{ in.}^2) = 64{,}000 \text{ lb} \longleftarrow$$

2.6-2 Steel rod in tension



$P = 90$ kN $\qquad A = \frac{\pi d^2}{4}$

Maximum normal stress: $\sigma_x = \frac{P}{A}$

Maximum shear stress: $\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$

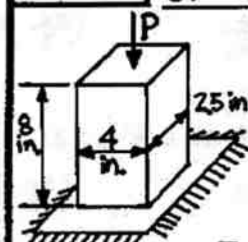
$\sigma_{allow} = 110$ MPa $\quad \tau_{allow} = 50$ MPa

Because τ_{allow} is less than one-half of σ_{allow}, the shear stress governs.

$\tau_{max} = \frac{P}{2A}$ or $50 \text{ MPa} = \frac{90 \text{ kN}}{(2)(\frac{\pi d^2}{4})}$

Solve for d: $d_{min} = 33.9$ mm $\longleftarrow$

2.6-3 Standard brick in compression



$A = 2.5 \text{ in.} \times 4.0 \text{ in.} = 10.0 \text{ in.}^2$

Maximum normal stress: $\sigma_x = \frac{P}{A}$

Maximum shear stress: $\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$

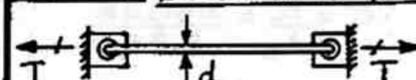
$\sigma_{ult} = 3700$ psi $\quad \tau_{ult} = 1050$ psi

Because τ_{ult} is less than one-half of σ_{ult}, the shear stress governs.

$\tau_{max} = \frac{P}{2A}$ or $P_{max} = 2A\,\tau_{ult}$

$P_{max} = 2(10.0 \text{ in.}^2)(1050 \text{ psi}) = 21{,}000 \text{ lb} \longleftarrow$

2.6-4 Brass wire in tension



$d = 2.42$ mm

$A = \frac{\pi d^2}{4} = 4.60 \text{ mm}^2$

$\alpha = 20 \times 10^{-6}/°C \quad E = 100$ GPa $\quad \tau_{allow} = 80$ MPa

Initial tensile force: $T = 138$ N

Stress due to initial tension: $\sigma_x = \frac{T}{A}$

Stress due to temperature drop: $\sigma_x = E\alpha(\Delta T)$
(See Eq. 2-18 of Section 2.5)

Total stress: $\sigma_x = \frac{T}{A} + E\alpha(\Delta T)$

Maximum shear stress

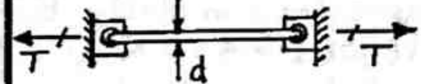
$$\tau_{max} = \frac{\sigma_x}{2} = \frac{1}{2}\left[\frac{T}{A} + E\alpha(\Delta T)\right]$$

Solve for temperature drop ΔT:

$\Delta T = \frac{2\tau_{max} - T/A}{E\alpha} \qquad \tau_{max} = \tau_{allow}$

Substitute numerical values: $\Delta T = 65.0°C \longleftarrow$

2.6-5 Brass wire in tension



$d = 1/16$ in.

$A = \dfrac{\pi d^2}{4} = 0.003068 \text{ in.}^2$

$\alpha = 10.6 \times 10^{-6}/°F$

$E = 15 \times 10^6$ psi

Initial tensile force: $T = 32$ lb

Stress due to initial tension: $\sigma_x = \dfrac{T}{A}$

Stress due to temperature drop: $\sigma_x = E\alpha(\Delta T)$
(See Eq. 2-18 of Section 2.5)

Total stress: $\sigma_x = \dfrac{T}{A} + E\alpha(\Delta T)$

(a) Maximum shear stress when temperature drops 50°F

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{1}{2}\left[\frac{T}{A} + E\alpha(\Delta T)\right] \qquad \text{(Eq. 1)}$$

Substitute numerical values:

$\tau_{max} = 9{,}190$ psi ⟵

(b) Maximum permissible temperature drop if $\tau_{allow} = 10{,}000$ psi

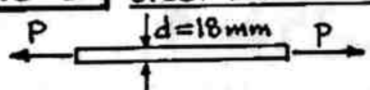
Solve Eq. (1) for ΔT:

$$\Delta T = \frac{2\tau_{max} - T/A}{E\alpha} \qquad \tau_{max} = \tau_{allow}$$

Substitute numerical values:

$\Delta T = 60.2°F$ ⟵

2.6-6 Steel bar in tension



$P = 20$ kN

(a) Maximum normal stress

$$\sigma_x = \frac{P}{A} = \frac{20 \text{ kN}}{\frac{\pi}{4}(18 \text{ mm})^2} = 78.60 \text{ MPa}$$

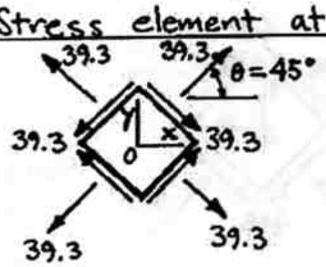
$\sigma_{max} = 78.6$ MPa ⟵

(b) Maximum shear stress

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

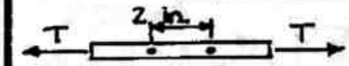
$\tau_{max} = \dfrac{\sigma_x}{2} = 39.3$ MPa ⟵

(c) Stress element at 45°



Note:
All stresses have units of MPa

2.6-7 Tension test



Elongation $\delta = 0.00140$ in.
(2 in. gage length)

Strain $\epsilon = \dfrac{\delta}{L} = \dfrac{0.00140 \text{ in.}}{2 \text{ in.}} = 0.00070$

Hooke's law: $\sigma_x = E\epsilon = (30 \times 10^6 \text{ psi})(0.00070) = 21{,}000$ psi

(a) Maximum normal stress

σ_x is the maximum normal stress

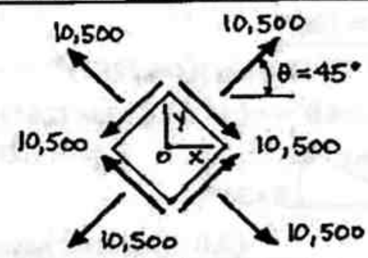
$\sigma_{max} = 21{,}000$ psi ⟵

(b) Maximum shear stress

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

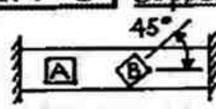
$\tau_{max} = \dfrac{\sigma_x}{2} = 10{,}500$ psi ⟵

(c) Stress element at $\theta = 45°$



Note:
All stresses have units of psi

2.6-8 Copper bar with rigid supports



$\Delta T = 60°C$ (Increase)

$\alpha = 17.5 \times 10^{-6}/°C$

$E = 120$ GPa

Stress due to temperature increase

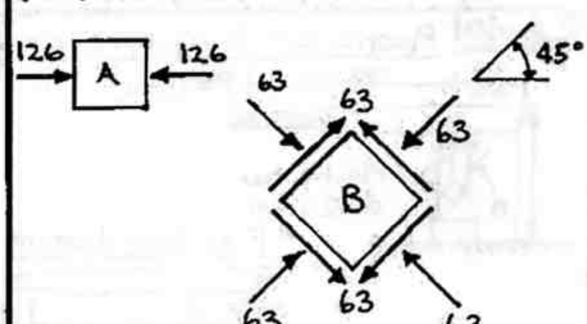
$\sigma_x = E\alpha(\Delta T)$ (See Eq. 2-18 of Section 2.5)
$= 126$ MPa (Compression)

Maximum shear stress

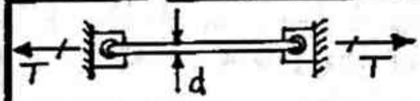
$\tau_{max} = \dfrac{\sigma_x}{2} = 63$ MPa

Stresses on elements A and B

(All stresses have units of MPa)



2.6-5 Brass wire in tension



$d = 1/16$ in.

$A = \frac{\pi d^2}{4} = 0.003068 \text{ in.}^2$

$\alpha = 10.6 \times 10^{-6}/°F$

$E = 15 \times 10^6$ psi

Initial tensile force: $T = 32$ lb

Stress due to initial tension: $\sigma_x = \frac{T}{A}$

Stress due to temperature drop: $\sigma_x = E\alpha(\Delta T)$

(See Eq. 2-18 of Section 2.5)

Total stress: $\sigma_x = \frac{T}{A} + E\alpha(\Delta T)$

(a) Maximum shear stress when temperature drops 50°F

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{1}{2}\left[\frac{T}{A} + E\alpha(\Delta T)\right] \qquad \text{(Eq. 1)}$$

Substitute numerical values:

$$\tau_{max} = 9{,}190 \text{ psi} \longleftarrow$$

(b) Maximum permissible temperature drop if $\tau_{allow} = 10{,}000$ psi

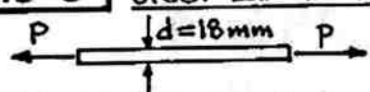
Solve Eq. (1) for ΔT:

$$\Delta T = \frac{2\tau_{max} - T/A}{E\alpha} \qquad \tau_{max} = \tau_{allow}$$

Substitute numerical values:

$$\Delta T = 60.2°F \longleftarrow$$

2.6-6 Steel bar in tension



$P = 20$ kN

(a) Maximum normal stress

$$\sigma_x = \frac{P}{A} = \frac{20 \text{ kN}}{\frac{\pi}{4}(18 \text{ mm})^2} = 78.60 \text{ MPa}$$

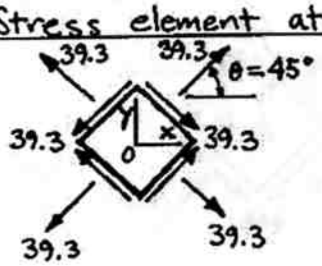
$$\sigma_{max} = 78.6 \text{ MPa} \longleftarrow$$

(b) Maximum shear stress

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

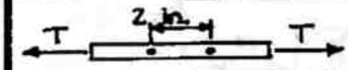
$$\tau_{max} = \frac{\sigma_x}{2} = 39.3 \text{ MPa} \longleftarrow$$

(c) Stress element at 45°



Note: All stresses have units of MPa

2.6-7 Tension test



Elongation $\delta = 0.00140$ in. (2 in. gage length)

Strain $\epsilon = \frac{\delta}{L} = \frac{0.00140 \text{ in.}}{2 \text{ in.}} = 0.00070$

Hooke's law: $\sigma_x = E\epsilon = (30 \times 10^6 \text{ psi})(0.00070) = 21{,}000$ psi

(a) Maximum normal stress

σ_x is the maximum normal stress

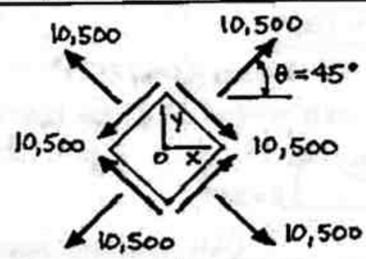
$$\sigma_{max} = 21{,}000 \text{ psi} \longleftarrow$$

(b) Maximum shear stress

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

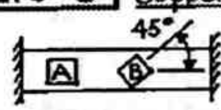
$$\tau_{max} = \frac{\sigma_x}{2} = 10{,}500 \text{ psi} \longleftarrow$$

(c) Stress element at $\theta = 45°$



Note: All stresses have units of psi

2.6-8 Copper bar with rigid supports



$\Delta T = 60°C$ (Increase)

$\alpha = 17.5 \times 10^{-6}/°C$

$E = 120$ GPa

Stress due to temperature increase

$\sigma_x = E\alpha(\Delta T)$ (See Eq. 2-18 of Section 2.5)

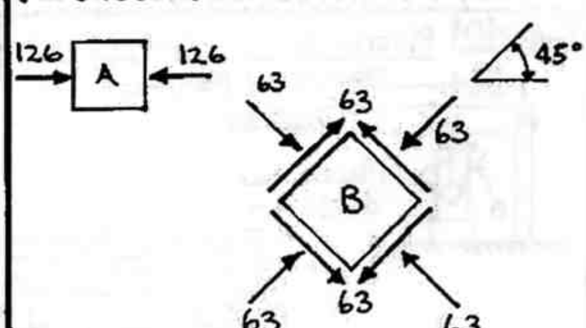
$= 126$ MPa (Compression)

Maximum shear stress

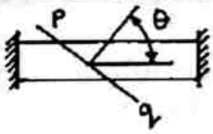
$$\tau_{max} = \frac{\sigma_x}{2} = 63 \text{ MPa}$$

Stresses on elements A and B

(All stresses have units of MPa)



2.6-11 Plastic bar between rigid supports



$\alpha = 60 \times 10^{-6}/°F \quad E = 450 \times 10^3$ psi

Temperature increase:

$\Delta T = 160°F - 68°F = 92°F$

<u>Normal stress σ_x in the bar</u>

$\sigma_x = -E\alpha(\Delta T)$ (See Eq. 2-18 in Section 2.5)

$\sigma_x = -(450 \times 10^3 \text{ psi})(60 \times 10^{-6}/°F)(92°F)$

$= -2484$ psi (Compression)

<u>Angle θ to plane pq</u>

$\sigma_\theta = \sigma_x \cos^2\theta$ For plane pq: $\sigma_\theta = -1700$ psi

Therefore, $-1700 \text{ psi} = (-2484 \text{ psi})(\cos^2\theta)$

$$\cos^2\theta = \frac{-1700 \text{ psi}}{-2484 \text{ psi}} = 0.6844$$

$\cos\theta = 0.8273 \qquad \theta = 34.18°$

<u>(a) Shear stress on plane pq</u>

$\tau_\theta = -\sigma_x \sin\theta\cos\theta$

$= -(-2484 \text{ psi})(\sin 34.18°)(\cos 34.18°)$

$= 1150$ psi (Counter clockwise) ←

<u>(b) Stress element oriented to plane pq</u>

$\theta = 34.18° \quad \sigma_\theta = -1700 \text{ psi} \quad \tau_\theta = 1150 \text{ psi}$

$\theta = 34.18° + 90° = 124.18°$

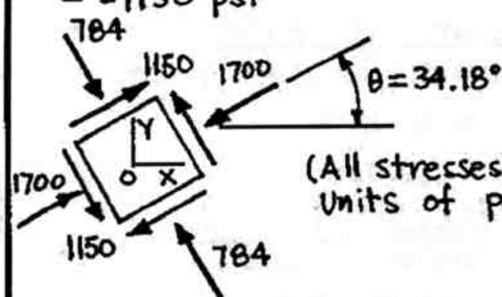
$\sigma_\theta = \sigma_x\cos^2\theta = (-2484 \text{ psi})(\cos 124.18°)^2$

$= -784$ psi

$\tau_\theta = -\sigma_x \sin\theta\cos\theta$

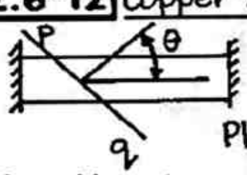
$= -(-2484 \text{ psi})(\sin 124.18°)(\cos 124.18°)$

$= -1150$ psi



(All stresses have units of psi)

2.6-12 Copper bar between rigid supports



$\alpha = 17 \times 10^{-6}/°C$

$E = 120$ GPa

Plane pq: $\theta = 55°$

Allowable stresses on plane pq:

$\sigma_{allow} = 60$ MPa (Compression)

$\tau_{allow} = 30$ MPa (Shear)

CONT.

2.6-12 CONT.

<u>(a) Maximum permissible temperature rise ΔT</u>

$\sigma_\theta = \sigma_x\cos^2\theta \qquad -60 \text{ MPa} = \sigma_x(\cos 55°)^2$

$\sigma_x = -182.4$ MPa

$\tau_\theta = -\sigma_x\sin\theta\cos\theta$

$30 \text{ MPa} = -\sigma_x(\sin 55°)(\cos 55°)$

$\sigma_x = -63.85$ MPa

Shear stress governs. $\sigma_x = -63.85$ MPa

Due to temperature increase ΔT:

$\sigma_x = -E\alpha(\Delta T)$ (See Eq. 2-18 in Section 2.5)

$-63.85 \text{ MPa} = -(120 \text{ GPa})(17 \times 10^{-6}/°C)(\Delta T)$

$\Delta T = 31.3°C$ ←

<u>(b) Stresses on plane pq</u>

$\sigma_x = -63.85$ MPa

$\sigma_\theta = \sigma_x\cos^2\theta = (-63.85 \text{ MPa})(\cos 55°)^2$

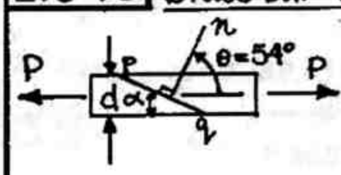
$= -21.0$ MPa (Compression) ←

$\tau_\theta = -\sigma_x\sin\theta\cos\theta$

$= -(-63.85 \text{ MPa})(\sin 55°)(\cos 55°)$

$= 30.0$ MPa (Counter clockwise) ←

2.6-13 Brass bar in tension



$\alpha = 36°$

$\theta = 90° - \alpha = 54°$

$P = 6500$ lb

$$A = \frac{\pi d^2}{4}$$

<u>Stress σ_x based upon allowable stresses in the brass</u>

Tensile stress ($\theta = 0°$): $\sigma_{allow} = 13{,}000$ psi

$\sigma_x = 13{,}000$ psi (1)

Shear stress ($\theta = 45°$): $\tau_{allow} = 7{,}000$ psi

$$\tau_{max} = \frac{\sigma_x}{2}$$

$\sigma_x = 2\tau_{allow}$

$= 14{,}000$ psi (2)

<u>Stress σ_x based upon allowable stresses on the brazed joint ($\theta = 54°$)</u>

$\sigma_{allow} = 6000$ psi (tension)

$\tau_{allow} = 3000$ psi (shear)

Tensile stress: $\sigma_\theta = \sigma_x\cos^2\theta$

$$\sigma_x = \frac{\sigma_{allow}}{\cos^2\theta} = \frac{6000 \text{ psi}}{(\cos 54°)^2}$$

$= 17{,}370$ psi (3)

CONT.

2.6-13 CONT.

Shear stress: $\tau_\theta = -\sigma_x \sin\theta\cos\theta$

$$\sigma_x = \left|\frac{\tau_{allow}}{\sin\theta\cos\theta}\right| = \frac{3{,}000\text{ psi}}{(\sin 54°)(\cos 54°)} = 6{,}310\text{ psi} \qquad (4)$$

Allowable stress

Compare (1), (2), (3), and (4).

Shear stress on the brazed joint governs.

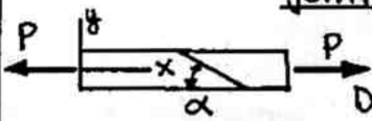
$\sigma_x = 6310$ psi

Diameter of bar

$$A = \frac{P}{\sigma_x} = \frac{6500\text{ lb}}{6310\text{ psi}} = 1.030\text{ in.}^2$$

$$A = \frac{\pi d^2}{4} \quad d^2 = \frac{4A}{\pi} \quad d_{min} = \sqrt{\frac{4A}{\pi}} = 1.15\text{ in.} \longleftarrow$$

2.6-14 Two boards joined by a scarf joint



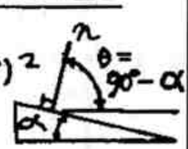
$10° \le \alpha \le 40°$

Due to load P:

$\sigma_x = 4.9$ MPa

(a) Stresses on joint when $\alpha = 20°$

$\theta = 90° - \alpha = 70°$



$\sigma_\theta = \sigma_x \cos^2\theta = (4.9\text{ MPa})(\cos 70°)^2$

$= 0.57$ MPa $\longleftarrow$

$\tau_\theta = -\sigma_x \sin\theta\cos\theta$

$= (-4.9\text{ MPa})(\sin 70°)(\cos 70°)$

$= -1.58$ MPa $\longleftarrow$

(b) Largest angle α if $\tau_{allow} = 2.25$ MPa

$\tau_{allow} = -\sigma_x \sin\theta\cos\theta$

The shear stress on the joint has a negative sign. Its numerical value can not exceed $\tau_{allow} = 2.25$ MPa. Therefore,

$-2.25\text{ MPa} = -(4.9\text{ MPa})(\sin\theta)(\cos\theta)$

or $\sin\theta\cos\theta = 0.4592$

From trigonometry: $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

Therefore: $\sin 2\theta = 2(0.4592) = 0.9184$

Solving: $2\theta = 66.69°$ or $113.31°$

$\theta = 33.34°$ or $56.66°$

$\alpha = 90° - \theta \qquad \alpha = 56.66°$ or $33.34°$

Since α must be between 10° and 40°, we select $\alpha = 33.3°$ $\longleftarrow$

CONT.

2.6-14 CONT.

Note: If α is between 10° and 33.3°,

$|\tau_\theta| < 2.25$ MPa.

If α is between 33.3° and 40°,

$|\tau_\theta| > 2.25$ MPa.

(c) What is α if $\tau_\theta = 2\sigma_\theta$?

Numerical values only:

$|\tau_\theta| = \sigma_x \sin\theta\cos\theta \qquad |\sigma_\theta| = \sigma_x\cos^2\theta$

$\left|\frac{\tau_\theta}{\sigma_\theta}\right| = 2 \qquad \sigma_x \sin\theta\cos\theta = 2\sigma_x\cos^2\theta$

$\sin\theta = 2\cos\theta$ or $\tan\theta = 2$

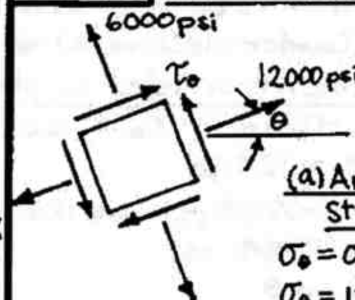
$\theta = 63.43° \qquad \alpha = 90° - \theta$

$= 26.6°$ $\longleftarrow$

Note: For $\alpha = 26.6°$ and $\theta = 63.4°$, we find $\sigma_\theta = 0.98$ MPa and $\tau_\theta = -1.96$ MPa.

Thus, $\left|\frac{\tau_\theta}{\sigma_\theta}\right| = 2$ as required.

2.6-15 Bar in uniaxial stress



(a) Angle θ and shear stress τ_θ

$\sigma_\theta = \sigma_x\cos^2\theta$

$\sigma_\theta = 12{,}000$ psi

$$\sigma_x = \frac{\sigma_\theta}{\cos^2\theta} = \frac{12{,}000\text{ psi}}{\cos^2\theta} \qquad (1)$$

Plane at angle $\theta + 90°$

$\sigma_{\theta+90°} = \sigma_x[\cos(\theta+90°)]^2 = \sigma_x[-\sin\theta]^2$

$= \sigma_x\sin^2\theta$

$\sigma_{\theta+90°} = 6{,}000$ psi

$$\sigma_x = \frac{\sigma_{\theta+90°}}{\sin^2\theta} = \frac{6{,}000\text{ psi}}{\sin^2\theta} \qquad (2)$$

Equate (1) and (2):

$$\frac{12{,}000\text{ psi}}{\cos^2\theta} = \frac{6{,}000\text{ psi}}{\sin^2\theta}$$

$\tan^2\theta = \frac{1}{2} \qquad \tan\theta = \frac{1}{\sqrt{2}} \qquad \theta = 35.26°$ $\longleftarrow$

From Eq. (1) or (2):

$\sigma_x = 18{,}000$ psi

$\tau_\theta = -\sigma_x\sin\theta\cos\theta$

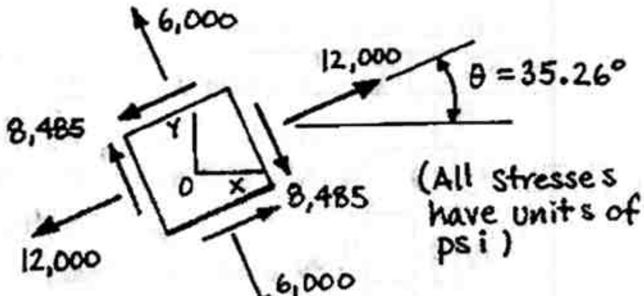
$= (-18{,}000\text{ psi})(\sin 35.26°)(\cos 35.26°)$

$= -8{,}485$ psi $\longleftarrow$

CONT.

2.6-15 CONT.

Minus sign means that τ_θ acts clockwise on the plane for which $\theta = 35.26°$.



(All stresses have units of psi)

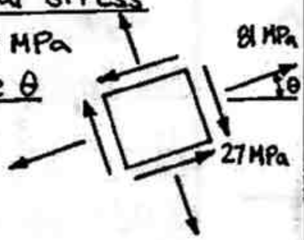
(b) Maximum normal and shear stress

$\sigma_{max} = \sigma_x = 18{,}000$ psi ←

$\tau_{max} = \dfrac{\sigma_x}{2} = 9{,}000$ psi ←

2.6-16 Bar in uniaxial stress

$\sigma_\theta = 81$ MPa $\quad \tau_\theta = -27$ MPa



Inclined plane at angle θ

$\sigma_\theta = \sigma_x \cos^2\theta$

$81 \text{ MPa} = \sigma_x \cos^2\theta$

$$\sigma_x = \frac{81 \text{ MPa}}{\cos^2\theta} \qquad (1)$$

$\tau_\theta = -\sigma_x \sin\theta \cos\theta \qquad -27\text{MPa} = -\sigma_x \sin\theta\cos\theta$

$$\sigma_x = \frac{27 \text{ MPa}}{\sin\theta\cos\theta} \qquad (2)$$

Equate (1) and (2):

$$\frac{81 \text{ MPa}}{\cos^2\theta} = \frac{27 \text{ MPa}}{\sin\theta\cos\theta}$$

or $\tan\theta = \dfrac{27}{81} = \dfrac{1}{3} \qquad \theta = 18.43°$

From (1) or (2): $\sigma_x = 90.0$ MPa (tension)

Stress element at $\theta = 30°$

$\sigma_\theta = \sigma_x \cos^2\theta = (90 \text{ MPa})(\cos 30°)^2 = 67.5 \text{ MPa}$

$\tau_\theta = -\sigma_x \sin\theta\cos\theta = (-90.0\text{MPa})(\sin 30°)(\cos 30°)$

$= -39.0$ MPa

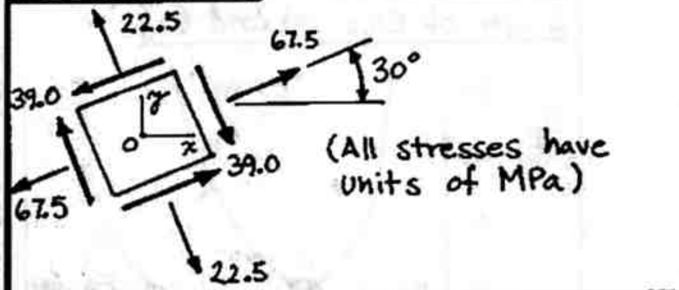
Plane at $\theta = 30° + 90° = 120°$

$\sigma_\theta = (90 \text{ MPa})(\cos 120°)^2 = 22.5$ MPa

$\tau_\theta = (-90\text{MPa})(\sin 120°)(\cos 120°) = 39.0$ MPa

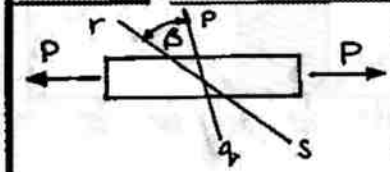
CONT.

2.6-16 CONT.



(All stresses have units of MPa)

2.6-17 Bar in tension



$\sigma_\theta = \sigma_x \cos^2\theta$

Plane pq:

$\sigma_1 = 8220$ psi $\quad \theta = \theta_1$

$\sigma_1 = \sigma_x \cos^2\theta_1 \qquad (1)$

Plane rs: $\sigma_2 = 3290$ psi $\qquad \theta = \theta_1 + \beta = \theta_1 + 30°$

$\sigma_2 = \sigma_x \cos^2(\theta_1 + 30°) \qquad (2)$

From (1) and (2): $\sigma_x = \dfrac{\sigma_1}{\cos^2\theta_1} = \dfrac{\sigma_2}{\cos^2(\theta_1 + 30°)} \quad (3)$

From (3):

$$\left[\frac{\cos\theta_1}{\cos(\theta_1 + 30°)}\right]^2 = \frac{\sigma_1}{\sigma_2}$$

or $\dfrac{\cos\theta_1}{\cos(\theta_1 + 30°)} = \sqrt{\dfrac{\sigma_1}{\sigma_2}} = \sqrt{\dfrac{8220}{3290}} = 1.5807$

Solve by iteration or use a computer program:

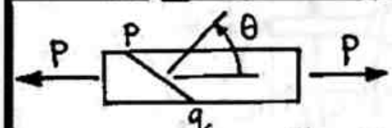
$\theta_1 = 25.02°$

From (1) or (2): $\sigma_x = 10{,}000$ psi

$\sigma_{max} = \sigma_x = 10{,}000$ psi ←

$\tau_{max} = \dfrac{\sigma_x}{2} = 5{,}000$ psi ←

2.6-18 Bar in tension with glued joint



$25° < \theta < 45°$

$A = 900 \text{ mm}^2$

$\left.\begin{array}{l}\sigma_{allow} = 15 \text{ MPa} \\ \tau_{allow} = 9 \text{ MPa}\end{array}\right\}$ on glued joint

Allowable stress σ_x in tension

$\sigma_\theta = \sigma_x \cos^2\theta \qquad \sigma_x = \dfrac{\sigma_\theta}{\cos^2\theta} = \dfrac{15 \text{ MPa}}{\cos^2\theta} \quad (1)$

$\tau_\theta = -\sigma_x \sin\theta\cos\theta$

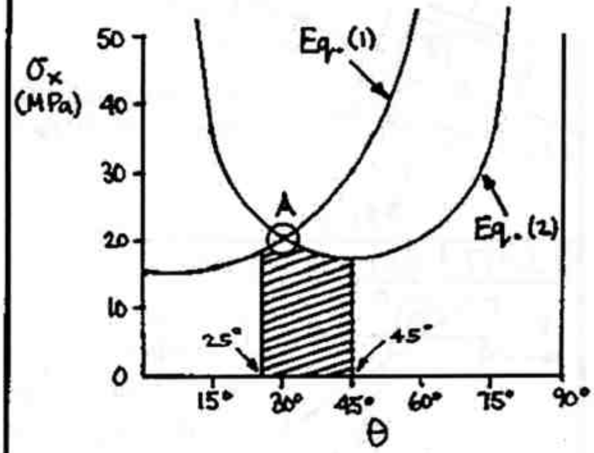
Since the direction of τ_θ is immaterial, we can write: $|\tau_\theta| = \sigma_x \sin\theta\cos\theta$

or $\sigma_x = \dfrac{|\tau_\theta|}{\sin\theta\cos\theta} = \dfrac{9\text{MPa}}{\sin\theta\cos\theta} \quad (2)$

CONT.

2.6-18 CONT.

Graph of Eqs. (1) and (2)



(a) Determine angle θ for largest load

Point A gives the largest value of σ_x and hence the largest load. To determine the angle θ corresponding to point A, we equate Eqs. (1) and (2).

$$\frac{15\text{ MPa}}{\cos^2\theta} = \frac{9\text{ MPa}}{\sin\theta\cos\theta}$$

$$\tan\theta = \frac{9}{15} = \frac{3}{5} \qquad \theta = 30.96° \longleftarrow$$

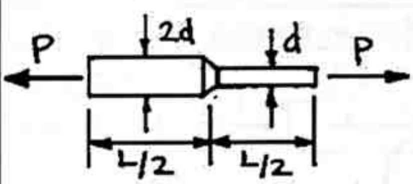
(b) Determine the maximum load

From Eq. (1) or Eq. (2):

$$\sigma_x = \frac{15\text{ MPa}}{\cos^2\theta} = \frac{9\text{ MPa}}{\sin\theta\cos\theta} = 20.4\text{ MPa}$$

$$P_{max} = \sigma_x A = (20.4\text{ MPa})(900\text{ mm}^2)$$
$$= 18.4\text{ kN} \longleftarrow$$

2.7-1 Bar with two segments



(a) Strain energy of the bar

Add the strain energies of the two segments of the bar (see Eq. 2-40).

$$U = \sum_{i=1}^{N} \frac{N_i^2 L_i}{2E_i A_i} = \frac{P^2(L/2)}{2E}\left[\frac{1}{\frac{\pi}{4}(2d)^2} + \frac{1}{\frac{\pi}{4}(d^2)}\right]$$

$$= \frac{P^2 L}{\pi E}\left(\frac{1}{4d^2} + \frac{1}{d^2}\right) = \frac{5P^2 L}{4\pi E d^2} \longleftarrow$$

(b) Substitute numerical values:

$P = 6000$ lb $\qquad L = 24$ in.

CONT.

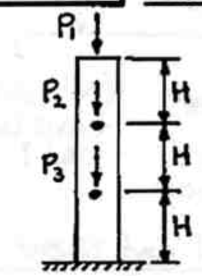
2.7-1 CONT.

$E = 15 \times 10^6$ psi $\qquad d = 1.5$ in.

$$U = \frac{5(6000\text{ lb})^2(24\text{ in.})}{4\pi(15\times 10^6\text{ psi})(1.5\text{ in.})^2}$$

$$= 10.2\text{ in.-lb} \longleftarrow$$

2.7-2 Three-story column



$H = 3$ m $\qquad E = 200$ GPa

$A = 7500\text{ mm}^2 \qquad P_1 = 150$ kN

$P_2 = P_3 = 300$ kN

To find the strain energy of the column, add the strain energies of the three segments (see Eq. 2-40).

$$U = \sum_{i=1}^{n} \frac{N_i^2 L_i}{2E_i A_i} = \frac{H}{2EA}\sum_{i=1}^{3} N_i^2$$

$$= \frac{H}{2EA}\left[P_1^2 + (P_1+P_2)^2 + (P_1+P_2+P_3)^2\right] \longleftarrow$$

Substitute numerical values:

$$\frac{H}{2EA} = \frac{3\text{ m}}{2(200\text{ GPa})(7500\text{ mm}^2)} = 1\times 10^{-9}\ \frac{\text{m}}{\text{N}}$$

$$\sum_{i=1}^{3} N_i^2 = (150\text{ kN})^2 + (450\text{ kN})^2 + (750\text{ kN})^2$$
$$= 787.5 \times 10^9\text{ N}^2$$

$$U = \frac{H}{2EA}\sum_{i=1}^{3} N_i^2 = (1\times 10^{-9}\ \tfrac{\text{m}}{\text{N}})(787.5\times 10^9\text{ N}^2)$$

$$U = 787.5\text{ N}\cdot\text{m} = 788\text{ J} \longleftarrow$$

2.7-3 Strain-energy densities for materials

Material	Specific Weight (lb/in.³)	Modulus of Elasticity (ksi)	Proportional Limit (psi)
Mild Steel	0.284	30,000	36,000
Tool Steel	0.284	30,000	120,000
Aluminum	0.0984	10,500	50,000
Rubber (soft)	0.0405	0.300	200

Strain energy per unit volume

$$U = \frac{P^2 L}{2EA} \qquad \text{Volume } V = AL \qquad \mu = \frac{U}{V} = \frac{\sigma^2}{2E}$$

At the proportional limit:

$\mu = \mu_R$ = modulus of resistance

$$\mu_R = \frac{\sigma_{PL}^2}{2E} \qquad \text{(Eq. 1)}$$

CONT.

2.7-3 CONT.

Strain energy per unit weight

$U = \frac{P^2L}{2EA}$ Weight $W = \gamma AL$

$\mu = \frac{U}{W} = \frac{\sigma^2}{2\gamma E} = \frac{\sigma^2}{2\gamma E}$

At the proportional limit:

$\mu = \mu_W = \frac{\sigma_{PL}^2}{2\gamma E}$ Eq. (2)

	Eq. (1) μ_R (psi)	Eq. (2) μ_W (in.)
Mild steel	22	76
Tool steel	240	845
Aluminum	119	1210
Rubber (soft)	67	1650

2.7-4 Bar with three loads

$P = 21$ kN
$L = 1.5$ m
$A = 882$ mm²
$E = 200$ GPa

$N_{AB} = 2P \quad N_{BC} = -P \quad N_{CD} = P$

(a) Strain energy of the bar (Eq. 2-40)

$U = \sum_{i=1}^{3} \frac{N_i^2 L_i}{2E_iA_i} = \frac{L/3}{2EA}\left[(2P)^2 + (-P)^2 + (P)^2\right]$

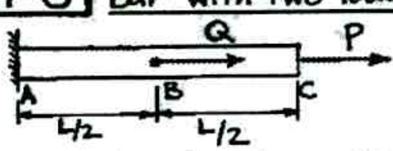
$= \frac{P^2L}{EA}$ ←

(b) Substitute numerical values:

$U = \frac{(21 \text{ kN})^2(1.5\text{m})}{(200 \text{ GPa})(882 \text{ mm}^2)} = 3.75 \text{ N·m}$

$= 3.75$ J ←

2.7-5 Bar with two loads



(a) Force P acts alone (Q = 0)

$U_1 = \frac{P^2L}{2EA}$ ←

(b) Force Q acts alone (P = 0)

$U_2 = \frac{Q^2(L/2)}{2EA} = \frac{Q^2L}{4EA}$ ←

CONT.

2.7-5 CONT.

(c) Forces P and Q act simultaneously

Segment BC: $U_{BC} = \frac{P^2(L/2)}{2EA} = \frac{P^2L}{4EA}$

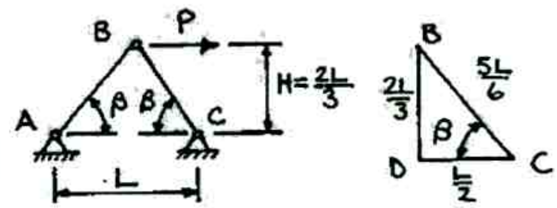
Segment AB: $U_{AB} = \frac{(P+Q)^2(L/2)}{2EA}$

$= \frac{P^2L}{4EA} + \frac{PQL}{2EA} + \frac{Q^2L}{4EA}$

$U = U_{BC} + U_{AB} = \frac{P^2L}{2EA} + \frac{PQL}{2EA} + \frac{Q^2L}{4EA}$ ←

(Note that U is not equal to $U_1 + U_2$)

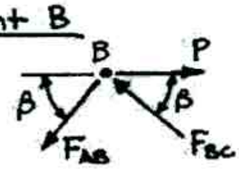
2.7-6 Truss subjected to a load P



$L_{BC} = L_{AB} = \frac{5L}{6}$

$\cos\beta = \frac{L/2}{5L/6} = \frac{3}{5} \qquad \sin\beta = \frac{2L/3}{5L/6} = \frac{4}{5}$

Joint B



$\sum F_{vert} = 0 \quad \uparrow + \; \downarrow -$

$-F_{AB}\sin\beta + F_{BC}\sin\beta = 0$

$F_{AB} = F_{BC}$

$\sum F_{horiz} = 0 \quad \xrightarrow{+} \; \xleftarrow{-}$

$-F_{AB}\cos\beta - F_{BC}\cos\beta + P = 0$

$F_{AB} = F_{BC} = \frac{P}{2\cos\beta} = \frac{5P}{6}$

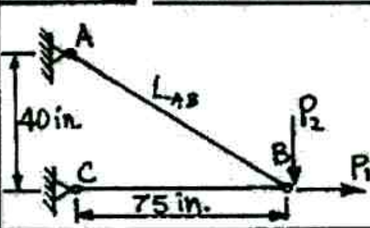
(a) Strain energy of truss (Eq. 2-40)

$U = \sum \frac{N_i^2 L_i}{2E_iA_i} = 2\left[\frac{1}{2EA}\left(\frac{5P}{6}\right)^2\left(\frac{5L}{6}\right)\right] = \frac{125P^2L}{216EA}$ ←

(b) Horizontal displacement of joint B (Eq. 2-42)

$\delta_B = \frac{2U}{P} = \frac{2}{P}\left(\frac{125P^2L}{216EA}\right) = \frac{125PL}{108EA}$ ←

2.7-7 Truss with two loads



$P_1 = 1600$ lb
$P_2 = 3200$ lb
$L_{BC} = 75$ in.

CONT.

2.7-7 CONT.

$A = 2.50 \text{ in.}^2$

$E = 29 \times 10^6$ psi

$L_{AB} = \sqrt{(40 \text{ in.})^2 + (75 \text{ in.})^2} = 85$ in.

Forces in the bars (from equilibrium of joint B)

Bar	P_1 alone	P_2 alone	P_1 and P_2
AB	0	+6800 lb	+6800 lb
BC	+1600 lb	−6000 lb	−4400 lb

(a) Load P_1 acts alone

$$U_1 = \frac{(F_{BC})^2 L_{BC}}{2EA} = \frac{(1600 \text{ lb})^2 (75 \text{ in.})}{2(29 \times 10^6 \text{ psi})(2.50 \text{ in.}^2)}$$

$$= \frac{192 \times 10^6 \text{ lb}^2\text{-in.}}{145 \times 10^6 \text{ lb}} = 1.32 \text{ in.-lb} \longleftarrow$$

(b) Load P_2 acts alone

$$U_2 = \frac{1}{2EA}\left[(F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC}\right]$$

$$= \frac{1}{2EA}\left[(6800 \text{ lb})^2 (85 \text{ in.}) + (-6000 \text{ lb})(75 \text{ in.})\right]$$

$$= \frac{6630.4 \times 10^6 \text{ lb}^2\text{-in.}}{145 \times 10^6 \text{ lb}} = 45.7 \text{ in.-lb} \longleftarrow$$

(c) Loads P_1 and P_2 act simultaneously

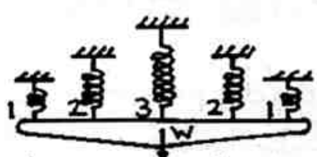
$$U = \frac{1}{2EA}\left[(F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC}\right]$$

$$= \frac{1}{2EA}\left[(6800 \text{ lb})^2 (85 \text{ in.}) + (-4400 \text{ lb})^2 (75 \text{ in.})\right]$$

$$= \frac{5382.4 \times 10^6 \text{ lb}^2\text{-in.}}{145 \times 10^6 \text{ lb}} = 37.1 \text{ in.-lb} \longleftarrow$$

Note: Strain energy U is not equal to $U_1 + U_2$

2.7-8 Rigid bar supported by springs



$k_1 = 3k$

$k_2 = 1.5k$

$k_3 = k$

δ = downward displacement of rigid bar

For a spring: $U = \frac{k\delta^2}{2}$ Eq. (2-38b)

(a) Strain energy U of all springs

$$U = 2\left(\frac{3k\delta^2}{2}\right) + 2\left(\frac{1.5k\delta^2}{2}\right) + \frac{k\delta^2}{2} = 5k\delta^2 \longleftarrow$$

(b) Displacement δ

Work done by the weight W equals $\frac{W\delta}{2}$

CONT.

2.7-8 CONT.

Strain energy of the springs equals $5k\delta^2$

$$\therefore \frac{W\delta}{2} = 5k\delta^2 \quad \text{and} \quad \delta = \frac{W}{10k} \longleftarrow$$

(c) Forces in the springs

$$F_1 = 3k\delta = \frac{3W}{10} \quad F_2 = 1.5k\delta = \frac{3W}{20} \longleftarrow$$

$$F_3 = k\delta = \frac{W}{10} \longleftarrow$$

(d) Numerical values

$W = 400$ N $\quad k = 5.0$ N/mm

$$U = 5k\delta^2 = 5k\left(\frac{W}{10k}\right)^2 = \frac{W^2}{20k} = 1.6 \text{ N·m} = 1.6 \text{ J}$$

$$\delta = \frac{W}{10k} = 8.0 \text{ mm} \longleftarrow$$

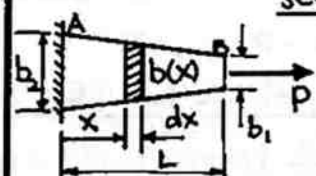
$$F_1 = \frac{3W}{10} = 120 \text{ N} \longleftarrow$$

$$F_2 = \frac{3W}{20} = 60 \text{ N} \longleftarrow$$

$$F_3 = \frac{W}{10} = 40 \text{ N} \longleftarrow$$

Note: $W = 2F_1 + 2F_2 + F_3 = 400$ N (Check)

2.7-9 Tapered bar of rectangular cross section



$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

$$A(x) = t\,b(x) = t\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]$$

(a) Strain energy of the bar

$$U = \int \frac{[N(x)]^2 dx}{2EA(x)} \quad \text{(Eq. 2-41)}$$

$$= \int_0^L \frac{P^2 dx}{2Et\,b(x)} = \frac{P^2}{2Et}\int_0^L \frac{dx}{b_2 - (b_2 - b_1)\frac{x}{L}} \quad (1)$$

From Appendix C: $\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

Apply this integration formula to Eq. (1):

$$U = \frac{P^2}{2Et}\left[\frac{1}{-(b_2 - b_1)\left(\frac{1}{L}\right)} \ln\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]\right]_0^L$$

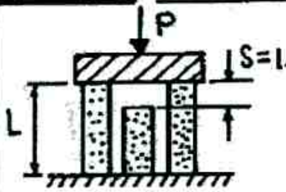
$$= \frac{P^2}{2Et}\left[\frac{-L}{(b_2 - b_1)}\ln b_1 - \frac{-L}{(b_2 - b_1)}\ln b_2\right]$$

$$U = \frac{P^2 L}{2Et(b_2 - b_1)} \ln\frac{b_2}{b_1} \longleftarrow$$

(b) Elongation of the bar (Eq. 2-42)

$$\delta = \frac{2U}{P} = \frac{PL}{Et(b_2 - b_1)} \ln\frac{b_2}{b_1} \longleftarrow$$

2.7-10 Three bars in compression



$S = 1.0\text{ mm}$ $\quad L = 1.0\text{ m}$

For each bar:

$A = 3000\text{ mm}^2$

$E = 45\text{ GPa}$

$\frac{EA}{L} = 135 \times 10^6\text{ N/m}$

(a) Load P_1 required to close the gap

In general, $\delta = \frac{PL}{EA}$ and $P = \frac{EA\delta}{L}$

For two bars, we obtain:

$P_1 = 2\left(\frac{EAS}{L}\right) = 2(135 \times 10^6\text{ N/m})(1.0\text{ mm})$

$P_1 = 270\text{ kN}$ ←

(b) Displacement δ for $P = 400\text{ kN}$

Since $P > P_1$, all three bars are compressed. The force P equals P_1 plus the additional force required to compress all three bars by the amount $\delta - S$.

$P = P_1 + 3\left(\frac{EA}{L}\right)(\delta - S)$

or $400\text{kN} = 270\text{ kN} + 3(135 \times 10^6\text{ N/m})(\delta - 0.001\text{m})$

Solving, we get $\delta = 1.321\text{ mm}$ ←

(c) Strain energy U for $P = 400\text{ kN}$

$U = \sum \frac{EA\delta^2}{2L}$ $\quad$ Outer bars: $\delta = 1.321\text{ mm}$

Middle bar: $\delta = 1.321\text{mm} - S = 0.321\text{ mm}$

$U = \frac{EA}{2L}\left[2(1.321\text{ mm})^2 + (0.321\text{ mm})^2\right]$

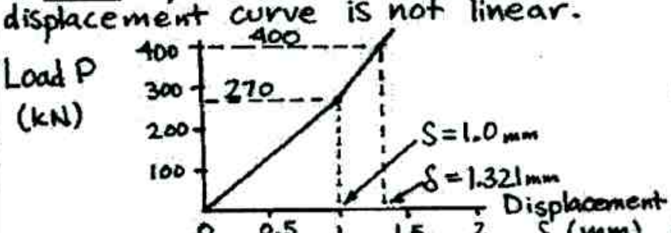
$= \frac{1}{2}(135 \times 10^6\text{ N/m})(3.593\text{ mm}^2)$

$= 243\text{ N}\cdot\text{m} = 243\text{ J}$ ←

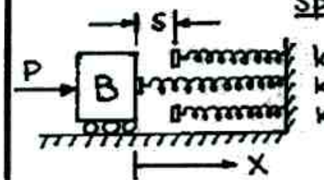
(d) Strain energy U is not equal to $\frac{P\delta}{2}$

$\frac{P\delta}{2} = \frac{1}{2}(400\text{kN})(1.321\text{ mm}) = 264\text{ N}\cdot\text{m}$

(This quantity is greater than the strain energy U.) The strain energy U is equal to the work done by the load P, which is equal to the area below the load-displacement curve. However, that area is NOT equal to $P\delta/2$ because the load-displacement curve is not linear.



2.7-11 Block pushed against three springs



Force P_0 required to close the gap:

$$P_0 = k_1 S \qquad (1)$$

Force-displacement relation before gap is closed

$$P = k_1 x \qquad (0 \le x \le s)(0 \le P \le P_0) \qquad (2)$$

Force-displacement relation after gap is closed

All three springs are compressed. Total stiffness equals $k_1 + 2k_2$. Additional displacement equals $x - s$. Force P equals P_0 plus the force required to compress all three springs by the amount $x - s$.

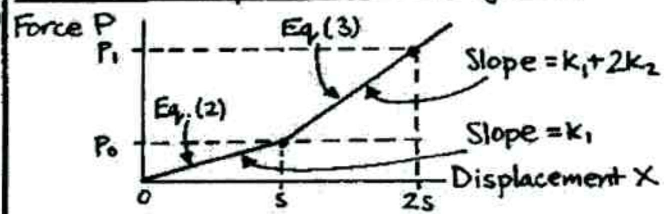
$P = P_0 + (k_1 + 2k_2)(x - s)$

$= k_1 s + (k_1 + 2k_2)x - k_1 s - 2k_2 s$

$$P = (k_1 + 2k_2)x - 2k_2 s \qquad (3)$$

$(x \ge s)$; $(P \ge P_0)$

(a) Force-displacement diagram



P_1 = force P when $x = 2s$

Substitute $x = 2s$ into Eq. (3):

$$P_1 = 2(k_1 + k_2)s \qquad (4)$$

(b) Strain energy U_1 when $x = 2s$

U_1 = Area below force-displacement curve

$= \frac{1}{2}P_0 s + P_0 s + \frac{1}{2}(P_1 - P_0)s = P_0 s + \frac{1}{2}P_1 s$

$= k_1 s^2 + (k_1 + k_2)s^2$

$$U_1 = (2k_1 + k_2)s^2 \quad \leftarrow \qquad (5)$$

(c) Strain energy U_1 is not equal to $\frac{P\delta}{2}$, where $\delta = 2s$

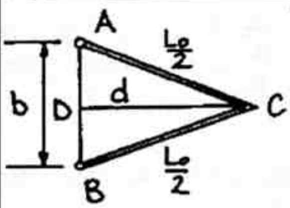
For $\delta = 2s$: $\frac{P\delta}{2} = \frac{1}{2}P_1(2s) = P_1 s = 2(k_1 + k_2)s^2$

(This quantity is greater than U_1.)

The strain energy is equal to the work done by the force P, which is equal to the area below the force-displacement curve. However, that area is NOT equal to $P\delta/2$ because the force-displacement curve is not linear.

2.7-12 Bungee cord subjected to a load P.

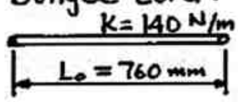
Dimensions before the load P is applied



$L_o = 760$ mm $\quad \frac{L_o}{2} = 380$ mm

$b = 380$ mm

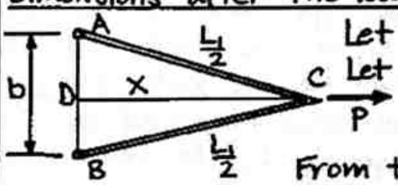
Bungee cord:



From triangle ACD:

$$d = \frac{1}{2}\sqrt{L_o^2 - b^2} = 329.09 \text{ mm} \qquad (1)$$

Dimensions after the load P is applied



Let x = distance CD

Let L_1 = stretched length of bungee cord

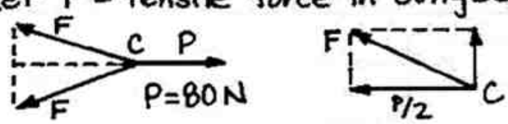
From triangle ACD:

$$\frac{L_1}{2} = \sqrt{\left(\frac{b}{2}\right)^2 + x^2} \qquad (2)$$

$$L_1 = \sqrt{b^2 + 4x^2} \qquad (3)$$

Equilibrium at point C

Let F = tensile force in bungee cord



$$\frac{F}{P/2} = \frac{L_1/2}{x} \qquad F = \left(\frac{P}{2}\right)\left(\frac{L_1}{2}\right)\left(\frac{1}{x}\right) = \frac{P}{2}\sqrt{1 + \left(\frac{b}{2x}\right)^2} \quad (4)$$

Elongation of bungee cord

Let δ = elongation of the entire bungee cord

$$\delta = \frac{F}{k} = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (5)$$

Final length of bungee cord = original length + δ

$$L_1 = L_o + \delta = L_o + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (6)$$

Solution of equations

Combine Eqs. (6) and (3):

$$L_1 = L_o + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = \sqrt{b^2 + 4x^2}$$

$$\text{or } L_1 = L_o + \frac{P}{4kx}\sqrt{b^2 + 4x^2} = \sqrt{b^2 + 4x^2}$$

$$L_o = \left(1 - \frac{P}{4kx}\right)\sqrt{b^2 + 4x^2} \qquad (7)$$

This equation can be solved for x.

Substitute numerical values into Eq. (7):

$$760 \text{ mm} = \left[1 - \frac{(80 \text{ N})(1000 \text{ mm/m})}{4(140 \text{ N/m})\,x}\right]\sqrt{(380 \text{ mm})^2 + 4x^2} \quad (8)$$

$$760 = \left(1 - \frac{142.857}{x}\right)\sqrt{144{,}400 + 4x^2} \qquad (9)$$

Units: x is in millimeters

CONT.

2.7-12 CONT.

Solve for x (Use trial & error or a computer program): $\quad x = 497.88$ mm

(a) Strain energy U of the bungee cord

$$U = \frac{k\delta^2}{2} \qquad k = 140 \text{ N/m} \qquad P = 80 \text{ N}$$

From Eq. (5): $\delta = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = 305.81$ mm

$$U = \frac{1}{2}(140 \text{ N/m})(305.81 \text{ mm})^2 = 6.55 \text{ N}\cdot\text{m}$$

$U = 6.55$ J ←

(b) Displacement δ_c of point C

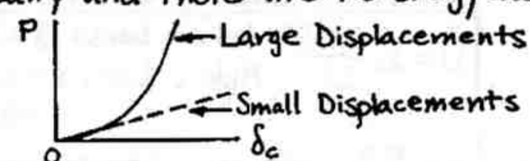
$\delta_c = x - d = 497.88 \text{ mm} - 329.09 \text{ mm} = 168.8$ mm ←

(c) Comparison of strain energy U with the quantity $P\delta_c/2$

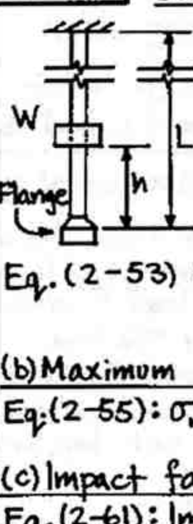
$U = 6.55$ J

$$\frac{P\delta_c}{2} = \frac{1}{2}(80 \text{ N})(168.8 \text{ mm}) = 6.75 \text{ J}$$

The two quantities are not the same. The work done by the load P is NOT equal to $P\delta_c/2$ because the load-displacement relation (see below) is non-linear when the displacements are large. (The work done by the load P is equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.)



2.8-1 Collar falling onto a flange



W = 100 lb $\quad E = 30 \times 10^6$ psi

h = 4.0 in. $\quad$ L = 6.0 ft $\quad A = 0.5 \text{ in.}^2$

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA} = 0.00048 \text{ in.}$$

$$\text{Eq. (2-53)}: \delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.0624 \text{ in.} \leftarrow$$

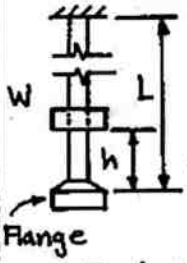
(b) Maximum tensile stress

$$\text{Eq. (2-55)}: \sigma_{max} = \frac{E\delta_{max}}{L} = 26{,}000 \text{ psi} \leftarrow$$

(c) Impact factor

$$\text{Eq. (2-61)}: \text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.0624 \text{ in.}}{0.00048 \text{ in.}} = 130 \leftarrow$$

2.8-2 Collar falling onto a flange



$M = 70\text{ kg}$ $\quad W = Mg = (70\text{ kg})(9.81\text{ m/s}^2) = 686.7\text{ N}$

$h = 50\text{ mm}$ $\quad L = 1.2\text{ m}$ $\quad A = 480\text{ mm}^2$

$E = 206\text{ GPa}$

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA} = 0.008334\text{ mm}$$

$$\text{Eq. (2-53): } \delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.921\text{ mm} \longleftarrow$$

(b) Maximum tensile stress

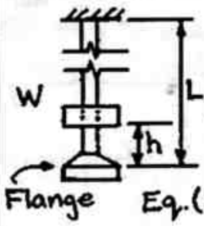
$$\text{Eq. (2-55): } \sigma_{max} = \frac{E\delta_{max}}{L} = 158\text{ MPa} \longleftarrow$$

(c) Impact factor

$$\text{Eq. (2-61): Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.921\text{ mm}}{0.008334\text{ mm}}$$

$$= 111 \longleftarrow$$

2.8-3 Collar falling onto a flange



$W = 175\text{ lb}$ $\quad h = 20\text{ in.}$ $\quad L = 10\text{ ft}$

$A = 0.55\text{ in.}^2$ $\quad E = 28{,}000\text{ ksi}$

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA} = 0.001364\text{ in.}$$

$$\text{Eq. (2-53): } \delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.235\text{ in.} \longleftarrow$$

(b) Maximum tensile stress

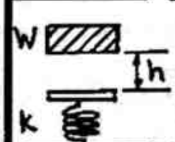
$$\text{Eq. (2-55): } \sigma_{max} = \frac{E\delta_{max}}{L} = 54.8\text{ ksi} \longleftarrow$$

(c) Impact factor

$$\text{Eq. (2-61): Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.235\text{ in.}}{0.001364\text{ in.}}$$

$$= 172 \longleftarrow$$

2.8-4 Block dropping onto a spring



$W = 20\text{ N}$ $\quad h = 440\text{ mm}$ $\quad k = 10.0\text{ kN/m}$

(a) Maximum shortening of the spring

$$\delta_{st} = \frac{W}{k} = \frac{20\text{ N}}{10.0\text{ kN/m}} = 2.0\text{ mm}$$

$$\text{Eq. (2-53): } \delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

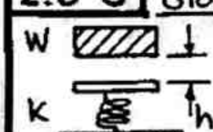
$$= 44.0\text{ mm} \longleftarrow$$

(b) Impact factor

$$\text{Eq. (2-61): Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{44.0\text{ mm}}{2.0\text{ mm}}$$

$$= 22 \longleftarrow$$

2.8-5 Block dropping onto a spring



$W = 4.0\text{ lb}$ $\quad h = 21\text{ in.}$ $\quad k = 80\text{ lb/in.}$

CONT.

2.8-5 CONT.

(a) Maximum shortening of the spring

$$\delta_{st} = \frac{W}{k} = \frac{4.0\text{ lb}}{80\text{ lb/in.}} = 0.050\text{ in.}$$

$$\text{Eq. (2-53): } \delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right] = 1.50\text{ in}$$

(b) Impact factor

$$\text{Eq. (2-61): Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{1.50\text{ in.}}{0.050\text{ in.}}$$

$$= 30 \longleftarrow$$

2.8-6 Rubber ball attached to a paddle

$g = 9.81\text{ m/s}^2$ $\quad E = 2.0\text{ MPa}$

$A = 1.6\text{ mm}^2$ $\quad L_o = 300\text{ mm}$

$L_1 = 1.0\text{ m}$ $\quad W = 300\text{ mN}$

When the ball leaves the paddle

$$KE = \frac{Wv^2}{2g}$$

When the rubber cord is fully stretched:

$$U = \frac{EA\delta^2}{2L_o} = \frac{EA}{2L_o}(L_1 - L_o)^2$$

Conservation of energy

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{EA}{2L_o}(L_1 - L_o)^2$$

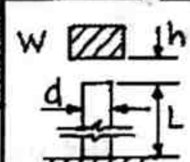
$$v^2 = \frac{gEA}{WL_o}(L_1 - L_o)^2 \qquad v = (L_1 - L_o)\sqrt{\frac{gEA}{WL_o}} \longleftarrow$$

Substitute numerical values:

$$v = 700\text{ mm}\sqrt{\frac{(9.81\text{ m/s}^2)(2.0\text{ MPa})(1.6\text{ mm}^2)}{(300\text{ mN})(300\text{ mm})}}$$

$$= 13.1\text{ m/s} \longleftarrow$$

2.8-7 Weight falling on a wood pole



$W = 4500\text{ lb}$ $\quad d = 12\text{ in.}$

$L = 18\text{ ft} = 216\text{ in.}$

$$A = \frac{\pi d^2}{4} = 113.10\text{ in.}^2$$

$E = 1.5 \times 10^6\text{ psi}$

$\sigma_{allow} = 2500\text{ psi}$

Find h_{max}

Static stress

$$\sigma_{st} = \frac{W}{A} = \frac{4500\text{ lb}}{113.10\text{ in.}^2} = 39.79\text{ psi}$$

Maximum height h_{max}

$$\text{Eq. (2-59): } \sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for h:

$$h = h_{max} = \frac{L\sigma_{max}}{2E}\left(\frac{\sigma_{max}}{\sigma_{st}} - 2\right) \longleftarrow$$

CONT.

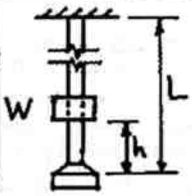
2.8-7 CONT.

Substitute numerical values:

$\sigma_{max} = \sigma_{allow} = 2500$ psi

$h_{max} = 10.9$ in. ←

2.8-8 Slider dropping onto a restrainer



$W = Mg = (40 kg)(9.81 m/s^2) = 392.4$ N

$A = 40.0 mm^2$ $E = 130$ GPa

$h = 1100$ mm $\sigma_{allow} = \sigma_{max} = 500$ MPa

Find minimum length L_{min}

Static stress

$$\sigma_{st} = \frac{W}{A} = \frac{392.4 N}{40.0 mm^2} = 9.81 \text{ MPa}$$

Minimum length L_{min}

Eq. (2-59): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or $\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$

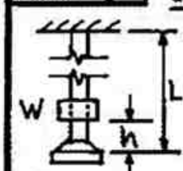
Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \longleftarrow$$

Substitute numerical values:

$L_{min} = 11.7$ m ←

2.8-9 Slider dropping onto a restrainer



$W = 100$ lb $A = 0.065$ in.^{2}

$E = 20 \times 10^6$ psi $h = 45$ in.

$\sigma_{allow} = \sigma_{max} = 70$ ksi

Find minimum length L_{min}

Static stress

$$\sigma_{st} = \frac{W}{A} = 1538 \text{ psi}$$

Minimum length L_{min}

Eq. (2-59): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or $\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$

Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \longleftarrow$$

Substitute numerical values:

$L_{min} = 591$ in. ←

2.8-10 Bumping post for a railway car

$k = 6.1$ MN/m $W = 470$ kN

d = maximum displacement of spring

$d = \delta_{max} = 460$ mm

Find V_{max}

CONT.

2.8-10 CONT.

Kinetic energy before impact

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

Strain energy when spring is compressed to the maximum allowable amount

$$U = \frac{k\delta^2_{max}}{2} = \frac{kd^2}{2}$$

Conservation of energy

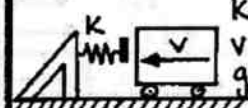
$$KE = U \quad \frac{Wv^2}{2g} = \frac{kd^2}{2} \qquad v^2 = \frac{kgd^2}{W}$$

$$v = v_{max} = d\sqrt{\frac{kg}{W}} \longleftarrow$$

Substitute numerical values:

$v_{max} = 5.19$ m/s ←

2.8-11 Bumper for a mine car



$k = 1000$ lb/in. $W = 3100$ lb

$v = 5$ mph $= 88$ in./sec

$g = 32.2$ ft/sec^2 $= 386.4$ in./sec^2

Find the shortening δ_{max} of the spring.

Kinetic energy just before impact

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

Strain energy when spring is fully compressed

$$U = \frac{k\delta^2_{max}}{2}$$

Conservation of energy

$$KE = U \quad \frac{Wv^2}{2g} = \frac{k\delta^2_{max}}{2}$$

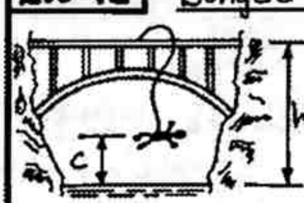
Solve for δ_{max}: $\delta_{max} = \sqrt{\frac{Wv^2}{kg}} \longleftarrow$

Substitute numerical values

$$\delta_{max} = \sqrt{\frac{(3100 \text{ lb})(88 \text{ in./sec})^2}{(1000 \text{ lb/in.})(384.4 \text{ in./sec}^2)}}$$

$= 7.88$ in. ←

2.8-12 Bungee jumper



$W = Mg = (50 kg)(9.81 m/s)$

$= 490.5$ N

$EA = 2.1$ kN

Height: $h = 60$ m

Clearance: $C = 10$ m

Find length L of the bungee cord.

P.E. = Potential energy of the jumper at the top of bridge (with respect to lowest position)

$= W(L + \delta_{max})$

U = strain energy of cord at lowest position

$$= \frac{EA\delta^2_{max}}{2L}$$

CONT.

2.8-12 CONT.

Conservation of energy

P.E. = U $\quad W(L+\delta_{max}) = \frac{EA\delta_{max}^2}{2L}$

or $\delta_{max}^2 - \frac{2WL}{EA}\delta_{max} - \frac{2WL^2}{EA} = 0$

Solve for δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

$$= \frac{WL}{EA}\left[1+\left(1+\frac{2EA}{W}\right)^{1/2}\right]$$

Vertical height

$h = C + L + \delta_{max}$

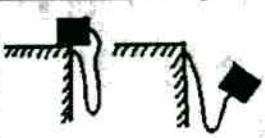
$$h - C = L + \frac{WL}{EA}\left[1+\left(1+\frac{2EA}{W}\right)^{1/2}\right]$$

Solve for L: $\quad L = \dfrac{h-C}{1+\frac{W}{EA}\left[1+\left(1+\frac{2EA}{W}\right)^{1/2}\right]}$ ←

Substitute numerical values:

$L = 25.6\ \text{m}$ ←

2.8-13 Weight falling off a wall



W = Weight

Properties of Elastic cord: E, A, L

(a) Impact Factor

Let δ_{max} = elongation of elastic cord

Let P.E. = potential energy of weight before fall (with respect to the lowest point)

P.E. $= W(L+\delta_{max})$

Let U = strain energy of cord at lowest position

$$= \frac{EA\delta_{max}^2}{2L}$$

Conservation of energy

P.E. = U $\quad W(L+\delta_{max}) = \frac{EA\delta_{max}^2}{2L}$

or $\delta_{max}^2 - \frac{2WL}{EA}\delta_{max} - \frac{2WL^2}{EA} = 0$

Solve for δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \sqrt{\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)} \qquad \delta_{st} = \frac{WL}{EA}$$

Impact factor $= \frac{\delta_{max}}{\delta_{st}} = 1 + \sqrt{1+\frac{2EA}{W}}$ ←

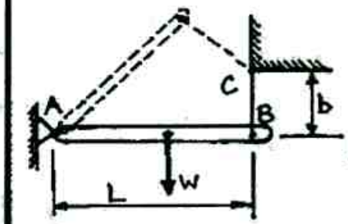
(b) Evaluate the impact factor

The weight W produces a static elongation equal to $1/60^{th}$ of the original length.

$\delta_{st} = \frac{WL}{EA} = \frac{L}{60}$ or $\frac{W}{EA} = \frac{1}{60}$ and $\frac{EA}{W} = 60$

Impact factor $= 1 + \sqrt{1+2(60)} = 12$ ←

2.8-14 Falling bar AB



Rigid bar:

$W = Mg = (1.0\ \text{kg})(9.81\ \text{m/s}^2) = 9.81\ \text{N}$

$L = 0.5\ \text{m}$

Nylon cord:

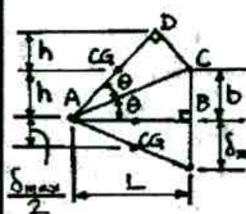
$A = 30\ \text{mm}^2$

$b = 0.25\ \text{m}$

$E = 2.1\ \text{GPa}$

Find maximum stress σ_{max} in cord BC.

Geometry of bar AB and cord BC



$\overline{CD} = \overline{CB} = b$

$\overline{AD} = \overline{AB} = L$

h = height of center of gravity of raised bar AD

δ_{max} = elongation of cord

From triangle ABC: $\sin\theta = \frac{b}{\sqrt{b^2+L^2}}$

$\cos\theta = \frac{L}{\sqrt{b^2+L^2}}$

From line AD: $\sin 2\theta = \frac{2h}{AD} = \frac{2h}{L}$

From Appendix C: $\sin 2\theta = 2\sin\theta\cos\theta$

$\therefore \frac{2h}{L} = 2\left(\frac{b}{\sqrt{b^2+L^2}}\right)\left(\frac{L}{\sqrt{b^2+L^2}}\right) = \frac{2bL}{b^2+L^2}$

and $h = \frac{bL^2}{b^2+L^2}$ $\qquad$ Eq. (1)

Conservation of energy

P.E. = potential energy of raised bar AD

$= W\left(h + \frac{\delta_{max}}{2}\right)$

U = strain energy of stretched cord $= \frac{EA\delta_{max}^2}{2b}$

P.E. = U $\quad W\left(h+\frac{\delta_{max}}{2}\right) = \frac{EA\delta_{max}^2}{2b}$ $\qquad$ Eq. (2)

For the cord: $\delta_{max} = \frac{\sigma_{max}b}{E}$

Substitute into Eq. (2) and rearrange:

$\sigma_{max}^2 - \frac{W}{A}\sigma_{max} - \frac{2WhE}{bA} = 0$ $\qquad$ Eq. (3)

Substitute from Eq. (1) into Eq. (3):

$\sigma_{max}^2 - \frac{W}{A}\sigma_{max} - \frac{2WL^2E}{A(b^2+L^2)} = 0$ $\qquad$ Eq. (4)

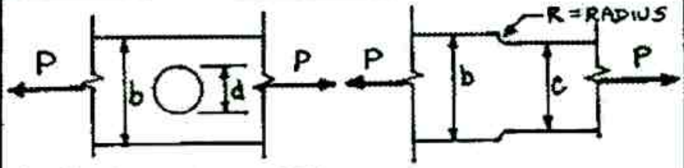
Solve for σ_{max}:

$$\sigma_{max} = \frac{W}{2A}\left[1+\sqrt{1+\frac{8L^2EA}{W(b^2+L^2)}}\right] \leftarrow$$

Substitute numerical values:

$\sigma_{max} = 33.3\ \text{MPa}$ ←

2.10-1 Flat bars in tension



$P = 3.0\text{ k} \qquad t = 0.25\text{ in.}$

(a) Bar with circular hole ($b = 6$ in.)

Obtain K from Fig. 2-63

For $d = 1$ in.: $c = b - d = 5$ in.

$$\sigma_{nom} = \frac{P}{ct} = \frac{3.0\text{ k}}{(5\text{ in.})(0.25\text{ in.})} = 2.40\text{ ksi}$$

$d/b = 1/6 \quad K \approx 2.60 \quad \sigma_{max} = K\sigma_{nom} \approx 6.2\text{ ksi} \longleftarrow$

For $d = 2$ in.: $c = b - d = 4$ in.

$$\sigma_{nom} = \frac{P}{ct} = \frac{3.0\text{ k}}{(4\text{ in.})(0.25\text{ in.})} = 3.00\text{ ksi}$$

$d/b = 1/3 \quad K \approx 2.31 \quad \sigma_{max} = K\sigma_{nom} \approx 6.9\text{ ksi} \longleftarrow$

(b) Stepped bar with shoulder fillets

$b = 4.0$ in. $\quad c = 2.5$ in. ; Obtain K from Fig. 2-64

$$\sigma_{nom} = \frac{P}{ct} = \frac{3.0\text{ k}}{(2.5\text{ in.})(0.25\text{ in.})} = 4.80\text{ ksi}$$

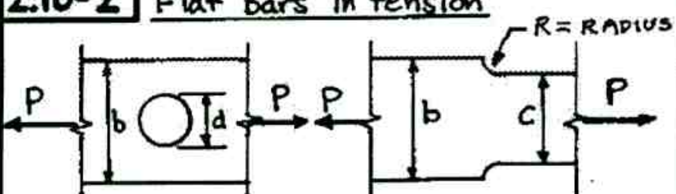
For $R = 0.25$ in.: $R/c = 0.1 \quad b/c = 1.60$

$K \approx 2.30 \quad \sigma_{max} = K\sigma_{nom} \approx 11.0\text{ ksi} \longleftarrow$

For $R = 0.5$ in.: $R/c = 0.2 \quad b/c = 1.60$

$K \approx 1.87 \quad \sigma_{max} = K\sigma_{nom} \approx 9.0\text{ ksi} \longleftarrow$

2.10-2 Flat bars in tension



$P = 2.5\text{ kN} \qquad t = 5.0\text{ mm}$

(a) Bar with circular hole ($b = 60$ mm)

Obtain K from Fig. 2-63

For $d = 12$ mm: $c = b - d = 48$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5\text{ kN}}{(48\text{ mm})(5\text{ mm})} = 10.42\text{ MPa}$$

$d/b = 1/5 \quad K \approx 2.51 \quad \sigma_{max} = K\sigma_{nom} \approx 26\text{ MPa} \longleftarrow$

For $d = 20$ mm: $c = b - d = 40$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5\text{ kN}}{(40\text{ mm})(5\text{ mm})} = 12.50\text{ MPa}$$

$d/b = 1/3 \quad K \approx 2.31 \quad \sigma_{max} = K\sigma_{nom} \approx 29\text{ MPa} \longleftarrow$

(b) Stepped bar with shoulder fillets

$b = 60$ mm $\quad c = 40$ mm ; Obtain K from Fig. 2-64

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5\text{ kN}}{(40\text{ mm})(5\text{ mm})} = 12.50\text{ MPa}$$

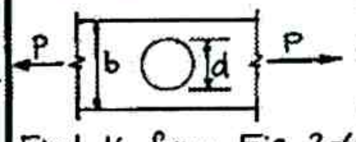
For $R = 6$ mm: $R/c = 0.15 \quad b/c = 1.5$

$K \approx 2.00 \quad \sigma_{max} = K\sigma_{nom} \approx 25\text{ MPa} \longleftarrow$

For $R = 10$ mm: $R/c = 0.25 \quad b/c = 1.5$

$K \approx 1.75 \quad \sigma_{max} = K\sigma_{nom} \approx 22\text{ MPa} \longleftarrow$

2.10-3 Flat bar in tension



t = thickness

σ_t = allowable tensile stress

Find P_{max}

Find K from Fig. 2-64

$$P_{max} = \sigma_{nom}\, ct = \frac{\sigma_{max}}{K} ct = \frac{\sigma_t}{K}(b-d)t = \frac{\sigma_t}{K} bt\left(1 - \frac{d}{b}\right)$$

Because σ_t, b, and t are constants, we write:

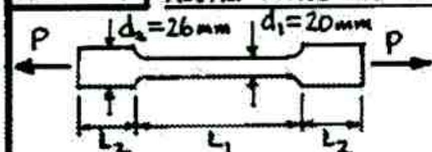
$$P^* = \frac{P_{max}}{\sigma_t bt} = \frac{1}{K}\left(1 - \frac{d}{b}\right)$$

d/b	K	P^*
0	3.00	0.333
0.1	2.73	0.330
0.2	2.50	0.320
0.3	2.35	0.298
0.4	2.24	0.268

We observe that P_{max} decreases as d/b increases. Therefore, the maximum load occurs when the hole becomes very small. ($d/b \to 0$ & $K \to 3$)

$$P_{max} = \frac{\sigma_t bt}{3} \longleftarrow$$

2.10-4 Round brass bar with upset ends



$E = 100$ GPa

$\delta = 0.12$ mm

$L_2 = 0.1$ m

$L_1 = 0.3$ m

$$R = \text{radius of fillets} = \frac{26\text{ mm} - 20\text{ mm}}{2} = 3\text{ mm}$$

$$\delta = 2\left(\frac{PL_2}{EA_2}\right) + \frac{PL_1}{EA_1}$$

Solve for P: $\quad P = \dfrac{\delta E A_1 A_2}{2L_2A_1 + L_1A_2}$

Use Fig. 2-65 for the stress-concentration factor

$$\sigma_{nom} = \frac{P}{A_1} = \frac{\delta E A_2}{2L_2A_1 + L_1A_2} = \frac{\delta E}{2L_2\left(\frac{A_1}{A_2}\right) + L_1} = \frac{\delta E}{2L_2\left(\frac{d_1}{d_2}\right)^2 + L_1}$$

Substitute numerical values:

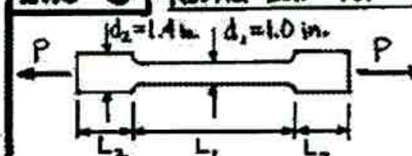
$$\sigma_{nom} = \frac{(0.12\text{ mm})(100\text{ GPa})}{2(0.1\text{ m})\left(\frac{20}{26}\right)^2 + 0.3\text{ m}} = 28.68\text{ MPa}$$

$$\frac{R}{D_1} = \frac{3\text{ mm}}{20\text{ mm}} = 0.15$$

Use the dashed curve in Fig. 2-65. $K \approx 1.6$

$\sigma_{max} = K\sigma_{nom} \approx (1.6)(28.68\text{ MPa}) \approx 46\text{ MPa} \longleftarrow$

2.10-5 Round bar with upset ends



$E = 25 \times 10^6$ psi

$\delta = 0.0040$ in.

$L_1 = 20$ in.

$L_2 = 5$ in.

$$R = \text{radius of fillets} \quad R = \frac{1.4\text{ in.} - 1.0\text{ in.}}{2} = 0.2\text{ in.}$$

$\delta = 2\left(\dfrac{PL_2}{EA_2}\right) + \dfrac{PL_1}{EA_1}$ $\quad$ Solve for P: $P = \dfrac{\delta E A_1 A_2}{2L_2A_1 + L_1A_2}$

Use Fig. 2-65 for the stress-concentration factor.

$$\sigma_{nom} = \frac{P}{A_1} = \frac{\delta E A_2}{2L_2A_1 + L_1A_2} = \frac{\delta E}{2L_2\left(\frac{A_1}{A_2}\right) + L_1} = \frac{\delta E}{2L_2\left(\frac{d_1}{d_2}\right)^2 + L_1}$$

CONT.

2.10-5 CONT.

Substitute numerical values:

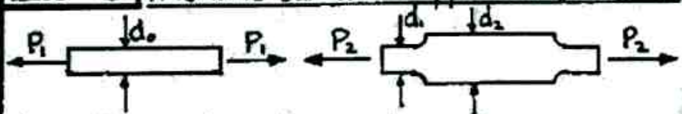
$$\sigma_{nom} = \frac{(0.0040\text{ in.})(25 \times 10^6 \text{ psi})}{2(5\text{ in.})\left(\frac{1.0}{1.4}\right)^2 + 20\text{ in.}} = 3{,}984 \text{ psi}$$

$$\frac{R}{D_1} = \frac{0.2\text{ in.}}{1.0\text{ in.}} = 0.2$$

Use the dashed curve in Fig 2-65. $K \approx 1.53$

$\sigma_{max} = K\sigma_{nom} \approx (1.53)(3984 \text{ psi}) \approx 6100 \text{ psi}$ ←

2.10-6 Prismatic bar and stepped bar (round bars)



$d_0 = 20$ mm $d_1 = 20$ mm $d_2 = 25$ mm

Fillet radius: $R = 2$ mm Allowable stress: $\sigma_t = 80$ MPa

(a) Comparison of bars

Prismatic bar: $P_1 = \sigma_t A_0 = \sigma_t\left(\frac{\pi d_0^2}{4}\right)$

$= (80\text{ MPa})\left(\frac{\pi}{4}\right)(20\text{ mm})^2 = 25.1$ kN ←

Stepped bar: See Fig. 2-65 for the stress-concentration factor.

$R = 2.0$ mm $D_1 = 20$ mm $D_2 = 25$ mm

$R/D_1 = 0.10$ $D_2/D_1 = 1.25$ $K \approx 1.75$

$$\sigma_{nom} = \frac{P_2}{\frac{\pi}{4}d_1^2} = \frac{P_2}{A_1} \qquad \sigma_{nom} = \frac{\sigma_{max}}{K}$$

$$P_2 = \sigma_{nom} A_1 = \frac{\sigma_{max}}{K}A_1 = \frac{\sigma_t}{K}A_1 = \left(\frac{80\text{ MPa}}{1.75}\right)\left(\frac{\pi}{4}\right)(20\text{ mm})^2$$

$= 14.4$ kN ←

Enlarging the bar makes it weaker, not stronger.

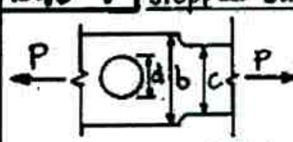
The ratio of loads is $P_1/P_2 = K = 1.75$

(b) Diameter of prismatic bar for the same allowable load

$P_1 = P_2 \quad \sigma_t\left(\frac{\pi d_0^2}{4}\right) = \frac{\sigma_t}{K}\left(\frac{\pi d_1^2}{4}\right) \qquad d_0^2 = \frac{d_1^2}{K}$

$d_0 = \frac{d_1}{\sqrt{K}} = \frac{20\text{ mm}}{\sqrt{1.75}} = 15.1$ mm ←

2.10-7 Stepped bar with a hole (flat bar)



$b = 2.4$ in. $c = 1.6$ in.

Fillet radius: $R = 0.2$ in.

Find d_{max}

Based upon fillets (Use Fig. 2-64)

$b = 2.4$ in. $c = 1.6$ in. $R = 0.2$ in. $R/c = 0.125$

$b/c = 1.5 \quad K \approx 2.10$

$$P_{max} = \sigma_{nom} ct = \frac{\sigma_{max}}{K}ct = \frac{\sigma_{max}}{K}\left(\frac{c}{b}\right)(bt)$$

$\approx 0.317\, bt\sigma_{max}$

Based upon hole (Use Fig. 2-63)

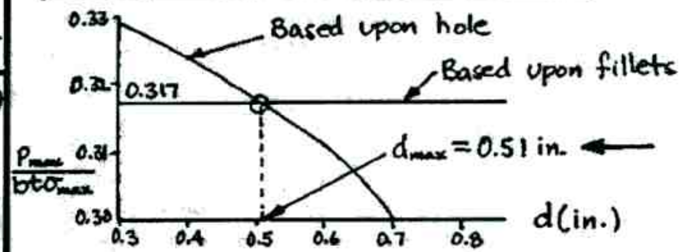
$b = 2.4$ in. d = diameter of hole (in.) $c_1 = b - d$

$$P_{max} = \sigma_{nom} c_1 t = \frac{\sigma_{max}}{K}(b-d)t = \frac{1}{K}\left(1-\frac{d}{b}\right)bt\sigma_{max}$$

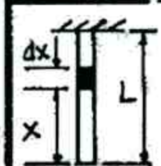
CONT.

2.10-7 CONT.

d (in.)	d/b	K	$P_{max}/bt\sigma_{max}$
0.3	0.125	2.66	0.329
0.4	0.167	2.57	0.324
0.5	0.208	2.49	0.318
0.6	0.250	2.41	0.311
0.7	0.292	2.37	0.299



2.11-1 Bar hanging under its own weight



Let A = cross-sectional area

Let N = axial force at distance x

$N = \gamma A x$

$\sigma = \frac{N}{A} = \gamma x$

Strain at distance x

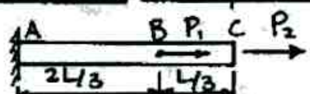
$$\epsilon = \frac{\sigma}{E} + \frac{\sigma_0 \alpha}{E}\left(\frac{\sigma}{\sigma_0}\right)^m = \frac{\gamma x}{E} + \frac{\sigma_0 \alpha}{E}\left(\frac{\gamma x}{\sigma_0}\right)^m$$

Elongation of bar

$$\delta = \int_0^L \epsilon\, dx = \int_0^L \frac{\gamma x}{E}\,dx + \frac{\sigma_0 \alpha}{E}\int_0^L \left(\frac{\gamma x}{\sigma_0}\right)^m dx$$

$$= \frac{\gamma L^2}{2E} + \frac{\sigma_0 \alpha L}{(m+1)E}\left(\frac{\gamma L}{\sigma_0}\right)^m \qquad \text{Q.E.D.}$$

2.11-2 Axially loaded bar



$L = 1.8$ m $A = 480$ mm²

$P_1 = 30$ kN $P_2 = 60$ kN

Ramberg–Osgood Equation: $\epsilon = \frac{\sigma}{45{,}000} + \frac{1}{618}\left(\frac{\sigma}{170}\right)^{10}$

Find displacement at end of bar. (σ = MPa)

(a) P_1 acts alone

AB $\quad \sigma = \frac{P_1}{A} = \frac{30\text{ kN}}{480\text{ mm}^2} = 62.5$ MPa

$\epsilon = 0.001389$

$\delta_c = \epsilon\left(\frac{2L}{3}\right) = 1.67$ mm ←

(b) P_2 acts alone

ABC $\quad \sigma = \frac{P_2}{A} = \frac{60\text{ kN}}{480\text{ mm}^2} = 125$ MPa

$\epsilon = 0.002853$

$\delta_c = \epsilon L = 5.13$ mm ←

(c) Both P_1 and P_2 are acting

AB $\quad \sigma = \frac{P_1 + P_2}{A} = \frac{90\text{ kN}}{480\text{ mm}^2} = 187.5$ MPa

$\epsilon = 0.008477$

$\delta_{AB} = \epsilon\left(\frac{2L}{3}\right) = 10.17$ mm

CONT.

2.11-2 CONT.

BC $\sigma = \frac{P_2}{A} = \frac{60\text{ kN}}{480\text{ mm}^2} = 125\text{ MPa}$

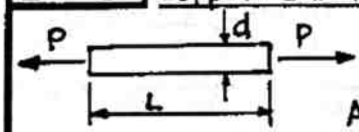
$\epsilon = 0.002853$

$\delta_{BC} = \epsilon\left(\frac{L}{3}\right) = 1.71\text{ mm}$

$\delta_C = \delta_{AB} + \delta_{BC} = 11.88\text{ mm}$ ←

(Note that the displacement when both loads act simultaneously is NOT equal to the sum of the displacements when the loads act separately.)

2.11-3 Copper bar in tension



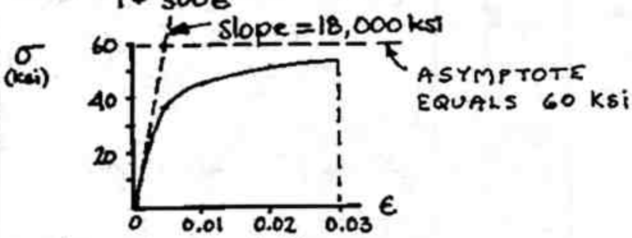
$L = 32\text{ in.}$

$d = 0.75\text{ in.}$

$A = \frac{\pi d^2}{4} = 0.4418\text{ in.}^2$

(a) Stress-strain diagram

$\sigma = \frac{18{,}000\epsilon}{1 + 300\epsilon} \qquad 0 \le \epsilon \le 0.03 \qquad (\sigma = \text{ksi})$



(b) Allowable load P

Max. elongation $\delta_{max} = 0.25\text{ in.}$

Max. stress $\sigma_{max} = 40\text{ ksi}$

Based upon elongation:

$\epsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.25\text{ in.}}{32\text{ in.}} = 0.007813$

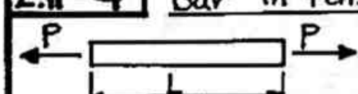
$\sigma_{max} = \frac{18000\,\epsilon_{max}}{1 + 300\,\epsilon_{max}} = 42.06\text{ ksi}$

Based upon stress:

$\sigma_{max} = 40\text{ ksi}$

Stress governs. $P = \sigma_{max}A = (40\text{ ksi})(0.4418\text{ in.}^2)$
$= 17.7\text{ k}$ ←

2.11-4 Bar in tension



$L = 2.0\text{ m}$

$A = 249\text{ mm}^2$

Stress-strain diagram

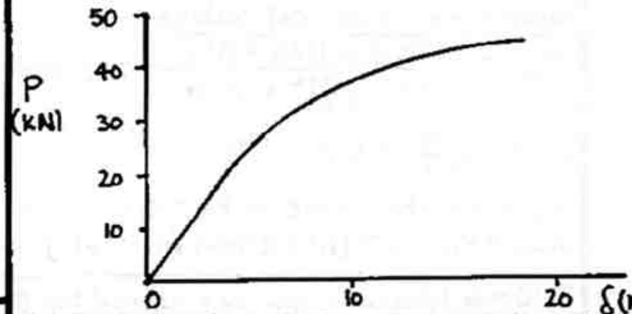
(See the problem statement for the diagram)

Load-displacement diagram

P (kN)	σ=P/A (MPa)	ϵ (from diagram)	$\delta=\epsilon L$ (mm)
10	40	0.0009	1.8
20	80	0.0018	3.6
30	120	0.0031	6.2
40	161	0.0060	12.0
45	181	0.0081	16.2

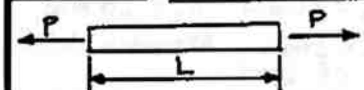
CONT.

2.11-4 CONT.



Note: The load-displacement curve has the same shape as the stress-strain curve.

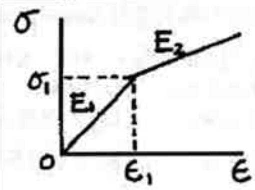
2.11-5 Aluminum bar in tension



$L = 150\text{ in.}$

$A = 2.0\text{ in.}^2$

Stress-strain diagram



$E_1 = 10 \times 10^6\text{ psi}$

$E_2 = 2.4 \times 10^6\text{ psi}$

$\sigma_1 = 12{,}000\text{ psi}$

$\epsilon_1 = \frac{\sigma_1}{E_1} = \frac{12{,}000\text{ psi}}{10\times10^6\text{ psi}}$

$= 0.0012$

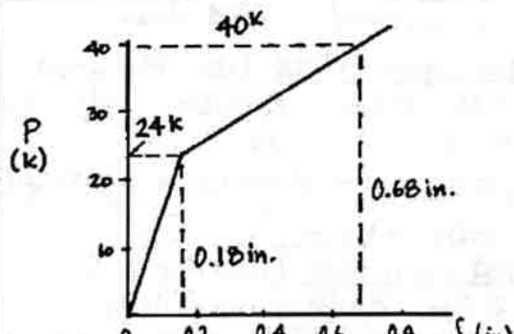
For $0 \le \sigma \le \sigma_1$: $\epsilon = \frac{\sigma}{E_1} = \frac{\sigma}{10\times10^6\text{ psi}} \quad (\sigma = \text{psi})$ Eq. (1)

For $\sigma \ge \sigma_1$: $\epsilon = \epsilon_1 + \frac{\sigma - \sigma_1}{E_2} = 0.0012 + \frac{\sigma - 12000}{2.4\times10^6}$

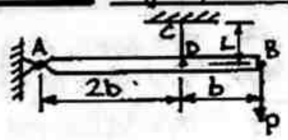
$= \frac{\sigma}{2.4\times10^6} - 0.0038 \quad (\sigma = \text{psi})$ Eq. (2)

Load-displacement diagram

P (k)	σ=P/A (psi)	ϵ (from Eq. 1 or Eq. 2)	$\delta=\epsilon L$ (in.)
8	4,000	0.00040	0.060
16	8,000	0.00080	0.120
24	12,000	0.00120	0.180
32	16,000	0.00287	0.430
40	20,000	0.00453	0.680



2.11-6 Rigid bar supported by a wire



Wire: $E = 210$ GPa
$\sigma_Y = 820$ MPa
$L = 1.0$ m
$d = 3$ mm
$A = \frac{\pi d^2}{4} = 7.0686\ mm^2$

Stress - strain diagram

$\sigma = E\epsilon \quad (0 \le \sigma \le \sigma_Y)$ (1)

$\sigma = \sigma_Y \left(\frac{E\epsilon}{\sigma_Y}\right)^n \quad (\sigma \ge \sigma_Y) \quad (n = 0.2)$ (2)

(a) Displacement δ_B at end of bar

δ = elongation of wire $\quad \delta_B = \frac{3}{2}\delta = \frac{3}{2}\epsilon L$ (3)

Obtain ϵ from stress-strain equations:

From Eq. (1): $\epsilon = \frac{\sigma}{E} \quad (0 \le \sigma \le \sigma_Y)$ (4)

From Eq. (2): $\epsilon = \frac{\sigma_Y}{E}\left(\frac{\sigma}{\sigma_Y}\right)^{\frac{1}{n}}$ (5)

Axial force in wire: $F = \frac{3P}{2}$

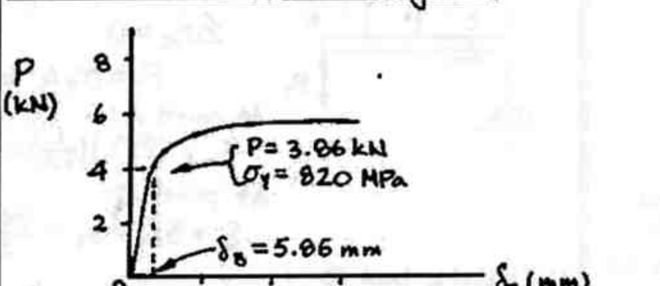
Stress in wire: $\sigma = \frac{F}{A} = \frac{3P}{2A}$ (6)

Procedure: Assume a value of P
Calculate σ from Eq. (6)
Calculate ϵ from Eq. (4) or (5)
Calculate δ_B from Eq. (3)

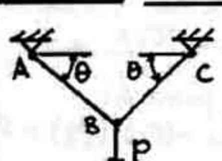
P (kN)	σ (MPa) Eq. (6)	ϵ Eq. (4) or (5)	δ_B (mm) Eq. (3)
2.4	509.3	0.002425	3.64
3.2	679.1	0.003234	4.85
4.0	848.8	0.004640	6.96
4.8	1018.6	0.01155	17.3
5.6	1188.4	0.02497	37.5

For $\sigma = \sigma_Y = 820$ MPa:
$\epsilon = 0.0039048 \quad P = 3.864$ kN $\quad \delta_B = 5.86$ mm

(b) Load - displacement diagram



2.12-1 Two bars supporting a load P



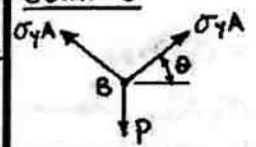
Structure is statically determinate.
The yield load P_Y and the plastic load P_P occur at the same time,

CONT.

2.12-1 CONT.

namely, when both bars reach the yield stress.

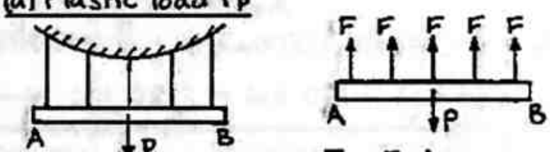
Joint B



$\Sigma F_{vert} = 0$
$(2\sigma_Y A)\sin\theta = P$
$P_Y = P_P = 2\sigma_Y A \sin\theta$ ⟵

2.12-2 Rigid bar supported by five wires

(a) Plastic load P_P



$F = \sigma_Y A$

At the plastic load, each wire is stressed to the yield stress. $\therefore P_P = 5\sigma_Y A$ ⟵

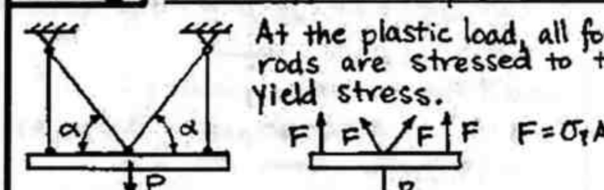
(b) Bar AB is flexible

At the plastic load, each wire is stressed to the yield stress, so the plastic load is not changed. ⟵

(c) Radius R is increased

Again, the forces in the wires are not changed, so the plastic load is not changed.

2.12-3 Beam supported by four rods



At the plastic load, all four rods are stressed to the yield stress.

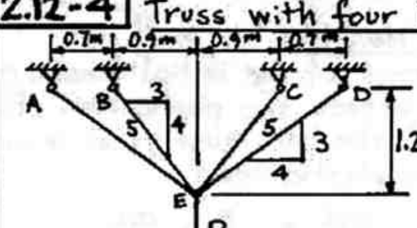
$F = \sigma_Y A$

Sum forces in the vertical direction and solve for the load:

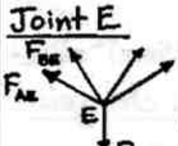
$P_P = 2F + 2F\sin\alpha$

$P_P = 2\sigma_Y A(1 + \sin\alpha)$ ⟵

2.12-4 Truss with four bars



Joint E



Equilibrium:
$2F_{AE}\left(\frac{3}{5}\right) + 2F_{BE}\left(\frac{4}{5}\right) = P$
or
$P = \frac{6}{5}F_{AE} + \frac{8}{5}F_{BE}$

CONT.

2.12-4 CONT.

Plastic load P_P

At the plastic load, all bars are stressed to the yield stress.

$F_{AE} = \sigma_Y A_{AE}$ $\quad F_{BE} = \sigma_Y A_{BE}$

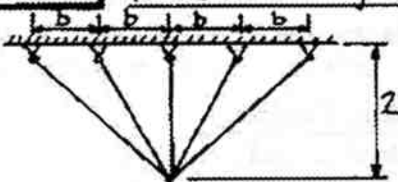
$P_P = \frac{6}{5}\sigma_Y A_{AE} + \frac{8}{5}\sigma_Y A_{BE}$ ←

Substitute numerical values:

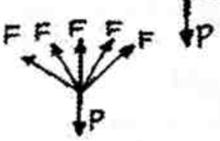
$\sigma_Y = 250 \text{ MPa}$ $\quad A_{AE} = 200 \text{ mm}^2$
$\quad A_{BE} = 400 \text{ mm}^2$

$P_P = \frac{6}{5}(250 \text{ MPa})(200 \text{ mm}^2) + \frac{8}{5}(250 \text{ MPa})(400 \text{ mm}^2)$
$= 60 \text{ kN} + 160 \text{ kN} = 220 \text{ kN}$ ←

2.12-5 Truss consisting of five bars



$d = 0.40 \text{ in.}$
$A = \frac{\pi d^2}{4} = 0.1257 \text{ in.}^2$
$\sigma_Y = 36{,}000 \text{ psi}$



At the plastic load, all five bars are stressed to the yield stress

$F = \sigma_Y A$

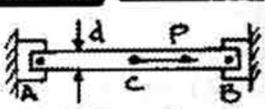
Sum forces in the vertical direction and solve for the load:

$P_P = 2F\left(\frac{1}{\sqrt{2}}\right) + 2F\left(\frac{2}{\sqrt{5}}\right) + F = \frac{\sigma_Y A}{5}(5\sqrt{2} + 4\sqrt{5} + 5)$
$= 4.2031\,\sigma_Y A$ ←

Substitute numerical values:

$P_P = (4.2031)(36{,}000 \text{ psi})(0.1257 \text{ in.}^2)$
$= 19{,}000 \text{ lb}$ ←

2.12-6 Bar held between rigid supports



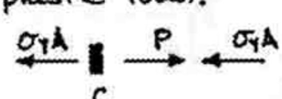
$d = 15 \text{ mm}$
$\sigma_Y = 290 \text{ MPa}$

Initial tensile stress $= 60 \text{ MPa}$

(a) Plastic load P_P

The presence of the initial tensile stress does not affect the plastic load. Both parts of the bar must yield in order to reach the plastic load.

Point C:

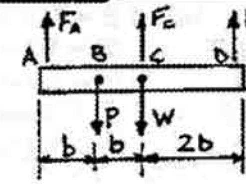


$P_P = 2\sigma_Y A = (2)(290 \text{ MPa})\left(\frac{\pi}{4}\right)(15 \text{ mm})^2 = 102 \text{ kN}$ ←

(b) Initial tensile stress is changed

P_P is not changed ←

2.12-7 Bar supported by three wires



$W = 300 \text{ lb}$
$d = 0.121 \text{ in.}^2$
$\sigma_Y = 30{,}000 \text{ psi}$
$A = \frac{\pi d^2}{4} = 0.01150 \text{ in.}^2$

By inspection, collapse occurs when wires A and C yield. (Note that the wires cannot be in compression, and therefore F_D must be zero or positive, but not negative.)

$\Sigma M_D = 0$

$-F_A(4b) - F_C(2b) + P(3b) + W(2b) = 0$

$P = \frac{1}{3}(4F_A + 2F_C - 2W)$

Substitute $F_A = F_C = \sigma_Y A$:

$P_P = \frac{2}{3}(3\sigma_Y A - W)$ ←

Verify that F_D is zero or positive:

$\Sigma F_{VERT} = 0 \quad F_A + F_C + F_D = P + W$

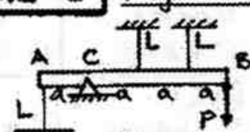
$F_D = P + W - 2\sigma_Y A$

Substitute P_P for P: $F_D = \frac{2}{3}(3\sigma_Y A - W) + W - 2\sigma_Y A$
$= \frac{W}{3}$ (ok)

Substitute numerical values:

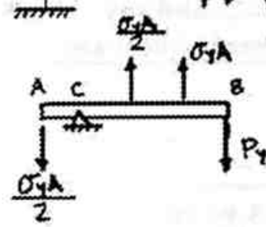
$P_P = \frac{2}{3}(3\sigma_Y A - W) = 490 \text{ lb}$ ←

2.12-8 Rigid bar supported by wires



(a) Yield load P_Y

Yielding occurs when the most highly stressed wire reaches the yield stress σ_Y.



$\Sigma M_C = 0$

$P_Y = \sigma_Y A$ ←

At point A:

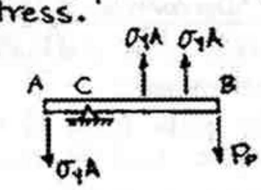
$\delta_A = \left(\frac{\sigma_Y A}{2}\right)\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{2E}$

At point B:

$\delta_B = 3\delta_A = \delta_Y = \frac{3\sigma_Y L}{2E}$ ←

(b) Plastic load P_P

At the plastic load, all wires reach the yield stress.



$\Sigma M_C = 0$

$P_P = \frac{4\sigma_Y A}{3}$ ←

At point A:

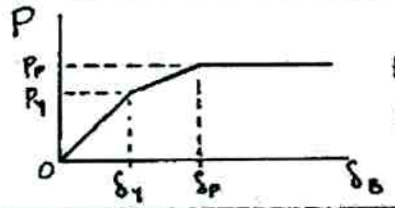
$\delta_A = (\sigma_Y A)\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{E}$

CONT.

2.12-8 CONT.

At point B: $\delta_B = 3\delta_A = \delta_P = \frac{3\sigma_Y L}{E}$ ←

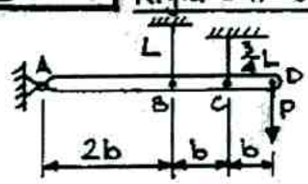
(c) Load-displacement diagram



$P_P = \frac{4}{3} P_Y$

$\delta_P = 2\delta_Y$

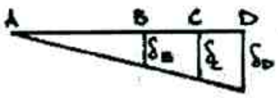
2.12-9 Rigid bar supported by two wires



A = cross-sectional area

σ_Y = yield stress

E = modulus of elasticity

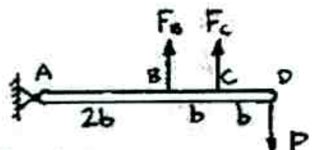


Displacement diagram

Compatibility:

$$\delta_C = \frac{3}{2}\delta_B \qquad (1)$$

$$\delta_D = 2\delta_B \qquad (2)$$



Free-body diagram

Equilibrium:

$\Sigma M_A = 0$ ↺+ $\quad F_B(2b) + F_C(3b) = P(4b)$

$$2F_B + 3F_C = 4P \qquad (3)$$

Force-displacement relations

$$\delta_B = \frac{F_B L}{EA} \qquad \delta_C = \frac{F_C(\frac{3}{4}L)}{EA} \qquad (4,5)$$

Substitute into Eq. (1): $\frac{3F_C L}{4EA} = \frac{3F_B L}{2EA}$

$$F_C = 2F_B \qquad (6)$$

Stresses $\quad \sigma_B = \frac{F_B}{A} \quad \sigma_C = \frac{F_C}{A} \quad \therefore \sigma_C = 2\sigma_B \quad (7)$

Wire C has the larger stress. Therefore, it will yield first.

(a) Yield load

$\sigma_C = \sigma_Y \qquad \sigma_B = \frac{\sigma_C}{2} = \frac{\sigma_Y}{2}$ (From Eq. 7)

$F_C = \sigma_Y A \qquad F_B = \frac{1}{2}\sigma_Y A$

From Eq. (3): $2(\frac{1}{2}\sigma_Y A) + 3(\sigma_Y A) = 4P$

$$P = P_Y = \sigma_Y A \leftarrow$$

From Eq. (4): $\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{2E}$

From Eq. (2): $\delta_D = \delta_Y = 2\delta_B = \frac{\sigma_Y L}{E}$ ←

CONT.

2.12-9 CONT.

(b) Plastic load

At the plastic load, both wires yield.

$\sigma_B = \sigma_Y = \sigma_C \qquad F_B = F_C = \sigma_Y A$

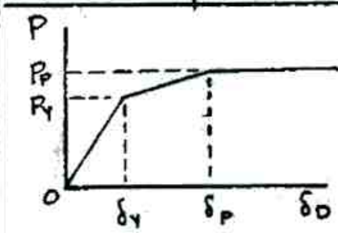
From Eq. (3): $2(\sigma_Y A) + 3(\sigma_Y A) = 4P$

$$P = P_P = \frac{5}{4}\sigma_Y A \leftarrow$$

From Eq. (4): $\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{E}$

From Eq. (2): $\delta_D = \delta_P = 2\delta_B = \frac{2\sigma_Y L}{E}$ ←

(c) Load-displacement diagram



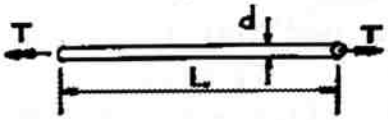
$P_P = \frac{5}{4} P_Y$

$\delta_P = 2\delta_Y$

— END OF CHAPTER 2 —

3.2-1

COPPER ROD IN TORSION



$L = 24$ in.

$\phi = 4.0° = 4\left(\frac{\pi}{180}\right)$ rad $= 0.06981$ rad

$\gamma_{allow} = 0.0008$ rad

FIND d_{max}

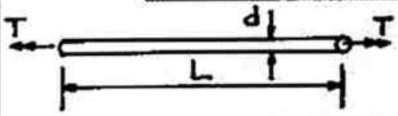
FROM EQ. (3-3): $\gamma_{max} = \frac{r\phi}{L} = \frac{d\phi}{2L}$

$$d_{max} = \frac{2L\gamma_{allow}}{\phi} = \frac{2(24\text{ in})(0.0008\text{ rad})}{0.06981\text{ rad}}$$

$d_{max} = 0.55$ in. ⟵

3.2-2

PLASTIC BAR IN TORSION



$d = 75$ mm

$\phi = 10° = 10\left(\frac{\pi}{180}\right)$ rad $= 0.1745$ rad

$\gamma_{allow} = 0.01$ rad

FIND L_{min}

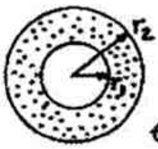
FROM EQ. (3-3): $\gamma_{max} = \frac{r\phi}{L} = \frac{d\phi}{2L}$

$$L_{min} = \frac{d\phi}{2\gamma_{allow}} = \frac{(75\text{ mm})(0.1745\text{ rad})}{2(0.01\text{ rad})}$$

$L_{min} = 654$ mm ⟵

3.2-3

CIRCULAR ALUMINUM TUBE



$r_2 = 2r_1$

$\gamma_{max} = 350 \times 10^{-6}$ rad

$\theta_{allow} = 0.20\,°/\text{ft}$
$= (0.20\,°/\text{ft})\left(\frac{\pi}{180}\frac{\text{rad}}{\text{degree}}\right)\left(\frac{1}{12}\frac{\text{ft}}{\text{in.}}\right)$
$= 290.9 \times 10^{-6}$ rad/in.

(a) SHEAR STRAIN AT INNER SURFACE

FROM EQ. (3-5b):

$\gamma_1 = \frac{1}{2}\gamma_2 = \frac{1}{2}(350 \times 10^{-6}\text{ rad})$

$\gamma_1 = 175 \times 10^{-6}$ rad ⟵

(b) MINIMUM OUTER RADIUS

FROM EQ. (3-5a):

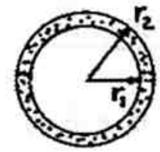
$\gamma_{max} = r_2 \frac{\phi}{L} = r_2\theta$

$$(r_2)_{min} = \frac{\gamma_{max}}{\theta_{allow}} = \frac{350 \times 10^{-6}\text{ rad}}{290.9 \times 10^{-6}\text{ rad/in}}$$

$(r_2)_{min} = 1.20$ in. ⟵

3.2-4

CIRCULAR STEEL TUBE



$L = 1.25$ m

$r_1 = 38$ mm

$\phi = 0.6° = 0.6\left(\frac{\pi}{180}\right)$ rad $= 0.01047$ rad

$\gamma_{max} = 0.0004$ rad

(a) SHEAR STRAIN AT INNER SURFACE

FROM EQ. (3-5b):

$$\gamma_{min} = \gamma_1 = r_1\frac{\phi}{L} = \frac{(38\text{ mm})(0.01047\text{ rad})}{1250\text{ mm}}$$

$\gamma_1 = 318 \times 10^{-6}$ rad ⟵

(b) MAXIMUM OUTER RADIUS

FROM EQ. (3-5a):

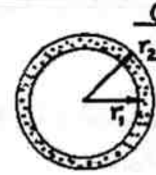
$\gamma_{max} = \gamma_2 = r_2\frac{\phi}{L}$; $r_2 = \frac{\gamma_{max}L}{\phi}$

$$(r_2)_{max} = \frac{(0.0004\text{ rad})(1250\text{ mm})}{0.01047\text{ rad}}$$

$(r_2)_{max} = 47.8$ mm ⟵

3.2-5

CIRCULAR STEEL TUBE



$L = 35$ in.

$r_1 = 1.6$ in.

$\phi = 0.5° = 0.5\left(\frac{\pi}{180}\right)$ rad $= 0.008727$ rad

$\gamma_{max} = 0.0005$ rad

(a) SHEAR STRAIN AT INNER SURFACE

FROM EQ. (3-5b):

$$\gamma_{min} = \gamma_1 = r_1\frac{\phi}{L} = \frac{(1.6\text{ in})(0.008727\text{ rad})}{35\text{ in}}$$

$\gamma_1 = 399 \times 10^{-6}$ rad ⟵

(b) MAXIMUM OUTER RADIUS

FROM EQ. (3-5a):

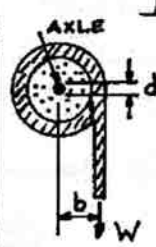
$\gamma_{max} = \gamma_2 = r_2\frac{\phi}{L}$; $r_2 = \frac{\gamma_{max}L}{\phi}$

$$(r_2)_{max} = \frac{(0.0005\text{ rad})(35\text{ in.})}{0.008727\text{ rad}}$$

$(r_2)_{max} = 2.01$ in. ⟵

3.3-1

HAND-POWERED WINCH



$d = 0.5$ in.

$b = 4.0$ in.

$W = 90$ lb

TORQUE T APPLIED TO THE AXLE:

$T = Wb = 360$ lb-in.

CONT.

3.3 — 1 CONT.

MAXIMUM SHEAR STRESS IN THE AXLE

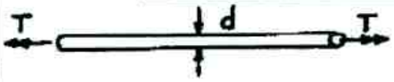
FROM EQ. (3-12): $\tau_{max} = \dfrac{16T}{\pi d^3}$

$$\tau_{max} = \frac{16\,(360\text{ lb-in})}{\pi\,(0.5\text{ in})^3}$$

$$\tau_{max} = 14{,}700\text{ psi} \longleftarrow$$

3.3 — 2

TORSION OF A DRILL BIT



$d = 5$ mm
$T = 0.3$ N·m
$G = 75$ GPa

MAXIMUM SHEAR STRESS

FROM EQ. (3-12): $\tau_{max} = \dfrac{16T}{\pi d^3}$

$$\tau_{max} = \frac{16\,(0.3\text{ N·m})}{\pi\,(5\text{ mm})^3}$$

$$\tau_{max} = 12.2\text{ MPa} \longleftarrow$$

RATE OF TWIST

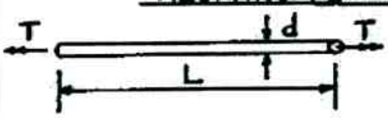
FROM EQ. (3-14): $\theta = \dfrac{T}{GI_p}$

$$\theta = \frac{0.3\text{ N·m}}{(75\text{ GPa})(\frac{\pi}{32})(5\text{ mm})^4}$$

$$\theta = 0.06519\text{ rad/m} = 3.74\,°/\text{m} \longleftarrow$$

3.3 — 3

ALUMINUM BAR IN TORSION



$L = 4.0$ ft $= 48$ in.
$d = 1.0$ in.
$G = 3.8 \times 10^6$ psi
$\phi = 5°$

(a) TORSIONAL STIFFNESS

$$k_T = \frac{GI_p}{L} = \frac{G\pi d^4}{32L} = \frac{(3.8\times10^6\text{ psi})(\pi)(1.0\text{ in.})^4}{32\,(48\text{ in.})}$$

$$k_T = 7770\text{ lb-in.} \longleftarrow$$

(b) MAXIMUM SHEAR STRESS

$$\phi = 5° = (5)\left(\frac{\pi}{180}\right)\text{rad} = 0.087266\text{ rad}$$

FROM EQ. (3-15): $\phi = \dfrac{TL}{GI_p}$ $\quad T = \dfrac{GI_p\phi}{L}$

FROM EQ. (3-11): $\tau_{max} = \dfrac{Tr}{I_p} = \dfrac{Td}{2I_p} = \left(\dfrac{GI_p\phi}{L}\right)\left(\dfrac{d}{2I_p}\right)$

$$\tau_{max} = \frac{Gd\phi}{2L}$$

$$\tau_{max} = \frac{(3.8\times10^6\text{ psi})(1.0\text{ in.})(0.087266\text{ rad})}{2\,(48\text{ in.})}$$

$$\tau_{max} = 3454\text{ psi}$$

SAY, $\tau_{max} = 3450\text{ psi} \longleftarrow$

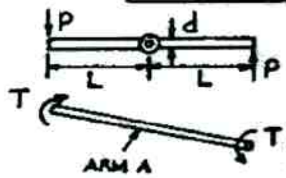
MAXIMUM SHEAR STRAIN

HOOKE'S LAW: $\gamma_{max} = \dfrac{\tau_{max}}{G} = \dfrac{3454\text{ psi}}{3.8\times10^6\text{ psi}}$

$$\gamma_{max} = 909\times10^{-6}\text{ rad} \longleftarrow$$

3.3 — 4

LUG WRENCH



$P = 100$ N $\quad L = 225$ mm
$d = 12$ mm $\quad G = 78$ GPa

$$T = 2PL = 2(100\text{ N})(0.225\text{ m}) = 45\text{ N·m}$$

(a) MAXIMUM SHEAR STRESS

FROM EQ. (3-12): $\tau_{max} = \dfrac{16T}{\pi d^3} = \dfrac{16\,(45\text{ N·m})}{\pi\,(0.012\text{ m})^3}$

$$\tau_{max} = 133\text{ MPa} \longleftarrow$$

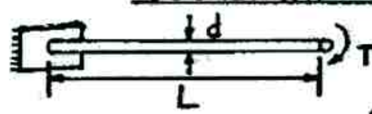
(b) ANGLE OF TWIST

FROM EQ. (3-15): $\phi = \dfrac{TL}{GI_p} = \dfrac{(45\text{ N·m})(0.225\text{ m})}{(78\text{ GPa})(\frac{\pi}{32})(0.012\text{ m})^4}$

$$\phi = 0.06376\text{ rad} = 3.65° \longleftarrow$$

3.3 — 5

SOCKET WRENCH



$d = 0.5$ in.
$L = 18.0$ in.
$\tau_{allow} = 9000$ psi
$G = 11.8\times10^6$ psi

MAXIMUM PERMISSIBLE TORQUE

FROM EQ. (3-12): $\tau_{max} = \dfrac{16T}{\pi d^3}$

$$T_{max} = \frac{\pi d^3 \tau_{max}}{16}$$

$$T_{max} = \frac{\pi\,(0.5\text{ in.})^3(9000\text{ psi})}{16}$$

$$T_{max} = 221\text{ lb-in.} \longleftarrow$$

ANGLE OF TWIST

FROM EQ. (3-15): $\phi = \dfrac{T_{max}L}{GI_p}$

FROM EQ. (3-12): $T_{max} = \dfrac{\pi d^3 \tau_{max}}{16}$

$$\phi = \left(\frac{\pi d^3 \tau_{max}}{16}\right)\left(\frac{L}{GI_p}\right) \qquad I_p = \frac{\pi d^4}{32}$$

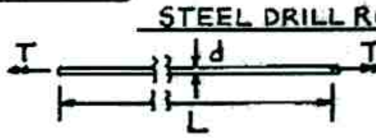
$$\phi = \frac{\pi d^3 \tau_{max} L\,(32)}{16G(\pi d^4)} = \frac{2\tau_{max}L}{Gd}$$

$$\phi = \frac{2\,(9000\text{ psi})(18.0\text{ in.})}{(11.8\times10^6\text{ psi})(0.5\text{ in.})} = 0.05492\text{ rad}$$

$$\phi = 3.15° \longleftarrow$$

3.3 — 6

STEEL DRILL ROD



$G = 80$ GPa
$d = 12$ mm
$\phi = 22.5°$
$\tau_{allow} = 300$ MPa

MINIMUM LENGTH

FROM EQ. (3-12): $\tau_{max} = \dfrac{16T}{\pi d^3} \qquad (1)$

CONT.

3.3-6 CONT.

FROM EQ. (3-15): $\phi = \dfrac{TL}{GI_p} = \dfrac{32TL}{G\pi d^4}$

$T = \dfrac{G\pi d^4 \phi}{32L}$ SUBSTITUTE T INTO EQ. (1):

$$\tau_{max} = \left(\frac{16}{\pi d^3}\right)\left(\frac{G\pi d^4 \phi}{32L}\right) = \frac{Gd\phi}{2L}$$

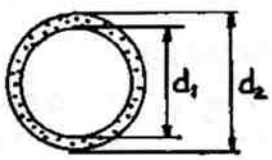
$$L = \frac{Gd\phi}{2\tau_{max}}$$

$$L_{min} = \frac{(80\text{ GPa})(12\text{ mm})(22.5^\circ)(\pi/180\text{ rad}/^\circ)}{2(300\text{ MPa})}$$

$$L_{min} = 0.628\text{ m} = 628\text{ mm} \longleftarrow$$

3.3-7

ALUMINUM TUBE IN TORSION



$L = 20$ in.

$d_1 = 1.2$ in.

$d_2 = 1.6$ in.

$T = 5300$ lb-in.

$\phi = 3.63^\circ = 0.063355$ rad

$I_p = \dfrac{\pi}{32}(d_2^4 - d_1^4) = 0.43982\text{ in}^4$

MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{Tr}{I_p} = \frac{(5300\text{ lb-in})(0.8\text{ in})}{0.43982\text{ in}^4}$$

$$\tau_{max} = 9640\text{ psi} \longleftarrow$$

SHEAR MODULUS OF ELASTICITY

$$\phi = \frac{TL}{GI_p}, \quad G = \frac{TL}{\phi I_p}$$

$$G = \frac{(5300\text{ lb-in})(20\text{ in})}{(0.063355\text{ rad})(0.43982\text{ in}^4)}$$

$$G = 3.80 \times 10^6\text{ psi} \longleftarrow$$

MAXIMUM SHEAR STRAIN

$$\gamma_{max} = \frac{\tau_{max}}{G}$$

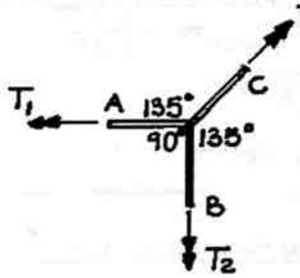
$$\gamma_{max} = \left(\frac{Tr}{I_p}\right)\left(\frac{\phi I_p}{TL}\right) = \frac{r\phi}{L}$$

$$\gamma_{max} = \frac{(0.8\text{ in})(0.063355\text{ rad})}{20\text{ in.}}$$

$$\gamma_{max} = 0.00253\text{ rad} \longleftarrow$$

3.3-8

THREE CIRCULAR BARS



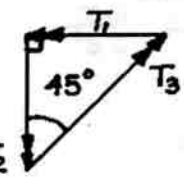
d_1 = DIAMETER OF BARS = 10 mm

d_2 = DIAMETER OF DISKS = 75 mm

$P_1 = 100$ N

$T_1 = P_1 d_2 \quad T_2 = P_2 d_2 \quad T_3 = P_3 d_2$

THE THREE TORQUES MUST BE IN EQUILIBRIUM



T_3 IS THE LARGEST TORQUE

$$T_3 = T_1\sqrt{2} = P_1 d_2 \sqrt{2}$$

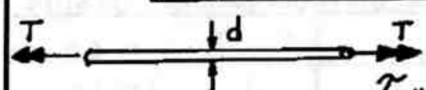
MAXIMUM SHEAR STRESS (EQ. 3-12)

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16T_3}{\pi d_1^3} = \frac{16 P_1 d_2 \sqrt{2}}{\pi d_1^3}$$

$$\tau_{max} = \frac{16(100\text{ N})(0.075\text{ m})\sqrt{2}}{\pi(0.010\text{ m})^3} = 54.0\text{ MPa} \longleftarrow$$

3.3-9

PROPELLER SHAFT



$G = 11.2 \times 10^6$ psi

$d = 3.5$ in.

$\tau_{allow} = 5800$ psi

$\theta = 0.2^\circ/\text{ft} = 0.01667^\circ/\text{in} = 0.0002909\text{ rad/in}$

MAX. TORQUE BASED UPON SHEAR STRESS

$$\tau = \frac{16T}{\pi d^3} \quad T_1 = \frac{\pi d^3 \tau_{allow}}{16} = \frac{\pi(3.5\text{ in})^3(5800\text{ psi})}{16}$$

$$T_1 = 48{,}800\text{ lb-in.}$$

MAX. TORQUE BASED UPON RATE OF TWIST

$$\theta = \frac{T}{GI_p}$$

$$T_2 = GI_p\theta = (11.2 \times 10^6\text{ psi})(\pi/32)(3.5\text{ in})^4(0.0002909\text{ rad/in.})$$

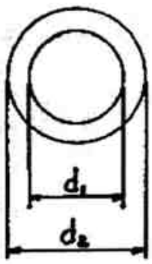
$$T_2 = 48{,}000\text{ lb-in.}$$

RATE OF TWIST GOVERNS

$$T_{max} = 48{,}000\text{ lb-in.} \longleftarrow$$

3.3-10

CONSTRUCTION AUGER



$d_2 = 150\ mm \quad r_2 = 75\ mm$

$d_1 = 100\ mm \quad r_1 = 50\ mm$

$G = 75\ GPa$

$T = 16\ kN \cdot m$

$I_p = \frac{\pi}{32}\left(d_2^4 - d_1^4\right) = 39.88 \times 10^6\ mm^4$

(a) SHEAR STRESS AT OUTER SURFACE

$$\tau_2 = \frac{T r_2}{I_p} = \frac{(16\ kN\cdot m)(75\ mm)}{39.88 \times 10^6\ mm^4} = 30.1\ MPa \longleftarrow$$

(b) SHEAR STRESS AT INNER SURFACE

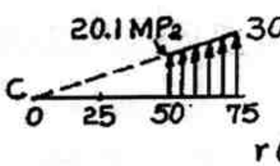
$$\tau_1 = \frac{T r_1}{I_p} = \frac{r_1}{r_2}\tau_2 = 20.1\ MPa \longleftarrow$$

(c) RATE OF TWIST

$$\theta = \frac{T}{G I_p} = \frac{16\ kN\cdot m}{(75\ GPa)(39.88 \times 10^6\ mm^4)}$$

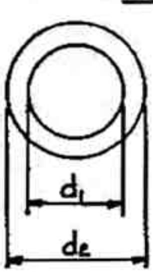
$$\theta = 0.005349\ rad/m = 0.306\ °/m \longleftarrow$$

(d) SHEAR STRESS DIAGRAM



3.3-11

CONSTRUCTION AUGER



$d_2 = 6.0\ in. \quad r_2 = 3.0\ in.$

$d_1 = 4.5\ in. \quad r_1 = 2.25\ in.$

$G = 11 \times 10^6\ psi$

$T = 150\ k\text{-}in.$

$I_p = \frac{\pi}{32}\left(d_2^4 - d_1^4\right) = 86.98\ in.^4$

(a) SHEAR STRESS AT OUTER SURFACE

$$\tau_2 = \frac{T r_2}{I_p} = \frac{(150\ k\text{-}in.)(3.0\ in.)}{86.98\ in.^4} = 5170\ psi \longleftarrow$$

(b) SHEAR STRESS AT INNER SURFACE

$$\tau_1 = \frac{T r_1}{I_p} = \frac{r_1}{r_2}\tau_2 = 3880\ psi \longleftarrow$$

(c) RATE OF TWIST

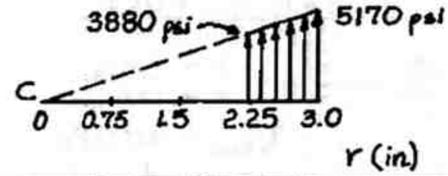
$$\theta = \frac{T}{G I_p} = \frac{(150\ k\text{-}in.)}{(11 \times 10^6\ psi)(86.98\ in.^4)}$$

$$\theta = 157 \times 10^{-6}\ rad/in. = 0.00898\ °/in. \longleftarrow$$

CONT.

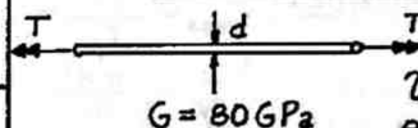
3.3-11 CONT.

(d) SHEAR STRESS DIAGRAM



3.3-12

AXLE OF A WINCH



$G = 80\ GPa$

$T = 1.5\ kN \cdot m$

$\tau_{allow} = 50\ MPa$

$\theta_{allow} = 0.8°/m$
$= 0.01396\ rad/m$

MIN. DIAMETER BASED UPON SHEAR STRESS

$$\tau = \frac{16T}{\pi d^3} \qquad d^3 = \frac{16T}{\pi \tau_{allow}}$$

$$d^3 = \frac{16\,(1.5\ kN\cdot m)}{\pi\,(50\ MPa)} = 0.0001528\ m^3$$

$$d = 0.05346\ m \qquad d_{min} = 53.5\ mm$$

MIN. DIAMETER BASED UPON RATE OF TWIST

$$\theta = \frac{T}{G I_p} = \frac{32T}{G \pi d^4} \qquad d^4 = \frac{32T}{\pi G\, \theta_{allow}}$$

$$d^4 = \frac{32\,(1.5\ kN\cdot m)}{\pi\,(80\ GPa)(0.01396\ rad/m)} = 0.00001368\ m^4$$

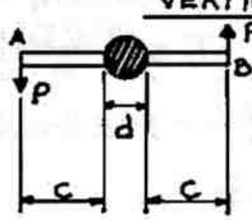
$$d = 0.06082\ m \qquad d_{min} = 60.8\ mm$$

RATE OF TWIST GOVERNS

$$d_{min} = 60.8\ mm \longleftarrow$$

3.3-13

VERTICAL POLE



$P = 1100\ lb$

$c = 5.0\ in.$

$\tau_{allow} = 4500\ psi$

FIND d_{min}

TORSION FORMULA $\quad \tau_{max} = \frac{Tr}{I_p} = \frac{Td}{2 I_p}$

$$T = P(2c + d) \qquad I_p = \frac{\pi d^4}{32}$$

$$\tau_{max} = \frac{P(2c+d)\,d}{\pi d^4/16} = \frac{16P(2c+d)}{\pi d^3}$$

$$(\pi \tau_{max})\,d^3 - (16P)\,d - 32Pc = 0$$

SUBSTITUTE NUMERICAL VALUES:

$$(\pi)(4500\ psi)d^3 - (16)(1100\ lb)\,d - 32(1100\ lb)(5.0\ in.) = 0$$

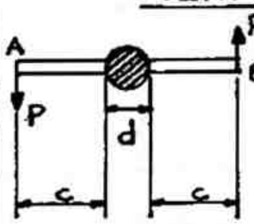
OR $d^3 - 1.24495\,d - 12.4495 = 0$, UNITS: d = INCHES

SOLVE NUMERICALLY: $d = 2.496\ in.$

$$d_{min} = 2.50\ in. \longleftarrow$$

3.3-14

VERTICAL POLE



$P = 5.0$ kN
$c = 125$ mm
$\tau_{allow} = 30$ MPa
FIND d_{min}

TORSION FORMULA $\tau_{max} = \dfrac{Tr}{I_p} = \dfrac{Td}{2I_p}$

$T = P(2c + d) \qquad I_p = \dfrac{\pi d^4}{32}$

$$\tau_{max} = \frac{P(2c+d)d}{\pi d^4/16} = \frac{16P(2c+d)}{\pi d^3}$$

$$(\pi \tau_{max})d^3 - (16P)d - 32Pc = 0$$

SUBSTITUTE NUMERICAL VALUES:

$$(\pi)(30\text{ MPa})d^3 - (16)(5.0\text{ kN})d - 32(5.0\text{ kN})(125\text{ mm}) = 0$$

OR $d^3 - 848.826\,d - 212{,}207 = 0$

UNITS: d = MILLIMETERS

SOLVE NUMERICALLY: $d = 64.38$ mm

$d_{min} = 64.4$ mm ⟵

3.3-15

BRASS BAR IN TORSION

(a) SOLID BAR

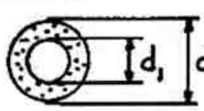
$d = 1.2$ in.
$\tau_{allow} = 12$ ksi

FIND MAX. TORQUE T_1

$$\tau_{max} = \frac{16T}{\pi d^3} \qquad T_1 = \frac{\pi d^3 \tau_{allow}}{16}$$

$$T_1 = \frac{\pi(1.2\text{ in})^3(12\text{ ksi})}{16} = 4072\text{ lb-in}$$ ⟵

(b) BAR WITH A HOLE



$d_2 = d = 1.2$ in.
$d_1 = 0.6$ in.

$$\tau_{max} = \frac{Tr}{I_p} = \frac{Td/2}{\frac{\pi}{32}(d_2^4 - d_1^4)} = \frac{16Td_2}{\pi(d_2^4 - d_1^4)}$$

$$T_2 = \frac{\pi(d_2^4 - d_1^4)\tau_{allow}}{16\,d_2}$$

$$T_2 = \frac{\pi[(1.2\text{ in})^4 - (0.6\text{ in})^4](12\text{ ksi})}{16(1.2\text{ in})}$$

$T_2 = 3817$ lb-in. ⟵

CONT.

3.3-15 CONT.

(c) PERCENT DECREASE IN TORQUE

$$\frac{T_2}{T_1} = \frac{\pi(d_2^4 - d_1^4)\tau_{allow}}{16d_2} \cdot \frac{16}{\pi d_2^3 \tau_{allow}} = 1 - \left(\frac{d_1}{d_2}\right)^4$$

$$\frac{d_1}{d_2} = \frac{1}{2} \qquad \frac{T_2}{T_1} = 0.9375$$

% DECREASE = 6.25 % ⟵

PERCENT DECREASE IN WEIGHT

$$\frac{W_2}{W_1} = \frac{A_2}{A_1} = \frac{d_2^2 - d_1^2}{d_2^2} = 1 - \left(\frac{d_1}{d_2}\right)^2$$

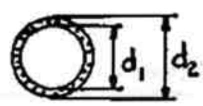
$$\frac{d_1}{d_2} = \frac{1}{2} \qquad \frac{W_2}{W_1} = \frac{3}{4}$$

% DECREASE = 25 % ⟵

NOTE: THE HOLLOW BAR WEIGHS 25% LESS THAN THE SOLID BAR WITH ONLY A 6.25% DECREASE IN STRENGTH.

3.3-16

HOLLOW ALUMINUM TUBE



$d_2 = 100$ mm
$d_1 = 80$ mm
$L = 2.5$ m
$G = 28$ GPa
$\tau_{max} = 50$ MPa

(a) ANGLE OF TWIST FOR THE TUBE

$$\tau_{max} = \frac{Tr}{I_p} = \frac{Td_2}{2I_p}, \quad T = \frac{2I_p\tau_{max}}{d_2}$$

$$\phi = \frac{TL}{GI_p} = \left(\frac{2I_p\tau_{max}}{d_2}\right)\left(\frac{L}{GI_p}\right)$$

$$\phi = \frac{2\tau_{max}L}{Gd_2}$$

$$\phi = \frac{2(50\text{ MPa})(2.5\text{ m})}{(28\text{ GPa})(100\text{ mm})} = 0.08929\text{ rad}$$

$\phi = 5.12°$ ⟵

(b) DIAMETER OF A SOLID SHAFT

τ_{max} IS THE SAME AS FOR TUBE.
TORQUE IS THE SAME.

FOR THE TUBE: $T = \dfrac{2I_p\tau_{max}}{d_2}$

$$T = \frac{2\tau_{max}}{d_2}\left(\frac{\pi}{32}\right)(d_2^4 - d_1^4)$$

CONT.

3.3-16 CONT.

FOR THE SOLID SHAFT:

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3}\left(\frac{2\tau_{max}}{d_2}\right)\left(\frac{\pi}{32}\right)\left(d_2^4 - d_1^4\right)$$

SOLVE FOR d^3: $d^3 = \dfrac{d_2^4 - d_1^4}{d_2}$

$$d^3 = \frac{(100\text{ mm})^4 - (80\text{ mm})^4}{100\text{ mm}} = 590{,}400\text{ mm}^3$$

$d = 83.9$ mm ⟵

(c) RATIO OF WEIGHTS

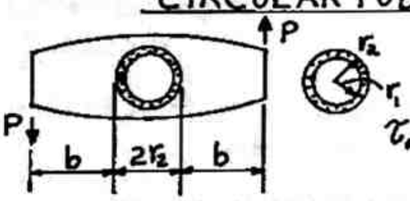
$$\frac{W_{TUBE}}{W_{SOLID}} = \frac{A_{TUBE}}{A_{SOLID}} = \frac{d_2^2 - d_1^2}{d^2}$$

$$\frac{W_{TUBE}}{W_{SOLID}} = \frac{(100_{mm})^2 - (80_{mm})^2}{(83.9_{mm})^2} = 0.51 \longleftarrow$$

THE WEIGHT OF THE TUBE IS 51% OF THE WEIGHT OF THE SOLID SHAFT, BUT THEY RESIST THE SAME TORQUE.

3.3-17

CIRCULAR TUBE IN TORSION



$P = 900$ lb
$b = 5.5$ in.
$\tau_{allow} = 6300$ psi
$r_1 = 1.2$ in.

FIND MINIMUM PERMISSIBLE RADIUS r_2

TORSION FORMULA

$T = 2P(b + r_2)$

$I_p = \frac{\pi}{2}(r_2^4 - r_1^4)$

$$\tau_{max} = \frac{Tr_2}{I_p} = \frac{2P(b+r_2)r_2}{\frac{\pi}{2}(r_2^4 - r_1^4)} = \frac{4P(b+r_2)r_2}{\pi(r_2^4 - r_1^4)}$$

ALL TERMS IN THIS EQUATION ARE KNOWN EXCEPT r_2.

SOLUTION OF EQUATION

SUBSTITUTE NUMERICAL VALUES:

$$6300\text{ psi} = \frac{4(900\text{ lb})(5.5\text{ in.} + r_2)(r_2)}{\pi[(r_2^4) - (1.2\text{ in})^4]}$$

OR

$$\frac{r_2^4 - 2.07360}{r_2(r_2 + 5.5)} - 0.181891 = 0$$

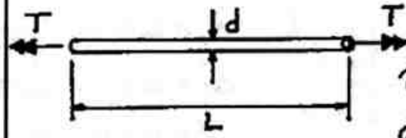
SOLVE NUMERICALLY:

$r_2 = 1.40$ in. ⟵

(MINIMUM PERMISSIBLE RADIUS)

3.3-18

RANGE OF VALUES OF TORQUE T



$L = 800$ mm
$\tau_{allow} = 60$ MPa
$\phi_{allow} = 3°$
$= 0.052360$ rad
$G = 75$ GPa

SHEAR STRESS

FROM EQ. 3-12: $\tau_{max} = \dfrac{16T}{\pi d^3}$

$$d_s^3 = \frac{16T}{\pi\tau_{allow}} = \frac{16T}{\pi(60\text{ MPa})}$$

UNITS: d = METERS, T = N·m

$$d_s = \frac{T^{1/3}}{227.541} \quad \text{(EQ. 1)}$$

ANGLE OF ROTATION

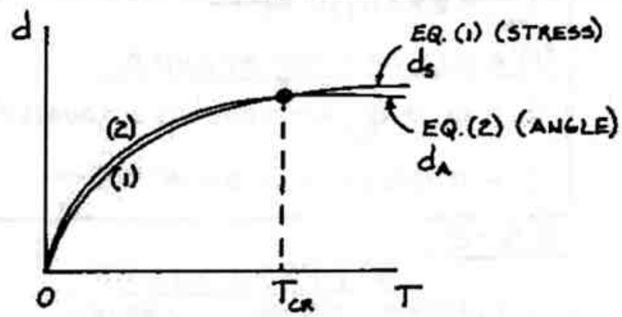
FROM EQ. 3-15: $\phi = \dfrac{TL}{GI_p} = \dfrac{32TL}{\pi G d^4}$

$$d_A^4 = \frac{32TL}{\pi G\phi} = \frac{32T(0.8\text{ m})}{\pi(75\text{ GPa})(0.052360\text{ rad})}$$

UNITS: d = METERS, T = N·m

$$d_A = \frac{T^{1/4}}{148.164} \quad \text{(EQ. 2)}$$

DIAGRAM OF DIAMETER d VERSUS TORQUE T



SHEAR STRESS GOVERNS WHEN T IS GREATER THAN T_{CR}. ANGLE OF ROTATION GOVERNS WHEN T IS LESS THAN T_{CR}. TO FIND T_{CR}, EQUATE d_s AND d_A FROM EQS. (1) AND (2):

$$d_s = \frac{T^{1/3}}{227.541} = d_A = \frac{T^{1/4}}{148.164}$$

$$\frac{T^{1/3}}{T^{1/4}} = T^{1/12} = \frac{227.541}{148.164} = 1.53574$$

$T_{CR} = 172.1$ N·m

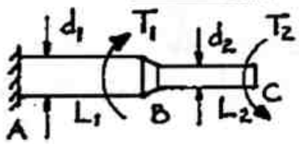
SHEAR STRESS GOVERNS: $T > 172$ N·m ⟵

ANGLE OF TWIST GOVERNS: $T < 172$ N·m ⟵

NOTE: WHEN $T = T_{CR}$, $d = 24.4$ mm

3.4-1

STEPPED SHAFT



$d_1 = 2.5$ in. $L_1 = 25$ in.
$d_2 = 2.0$ in. $L_2 = 18$ in.
$G = 11 \times 10^6$ psi
$T_1 = 9{,}000$ lb-in.
$T_2 = 4{,}000$ lb-in.

SEGMENT AB

$T_{AB} = T_2 - T_1 = -5{,}000$ lb-in.

$$\tau_{AB} = \left|\frac{16\,T_{AB}}{\pi d_1^3}\right| = \frac{16\,(5{,}000 \text{ lb-in.})}{\pi\,(2.5 \text{ in.})^3} = 1630 \text{ psi}$$

$$\phi_{AB} = \frac{T_{AB}L_1}{G(I_p)_{AB}} = \frac{(-5{,}000 \text{ lb-in.})(25 \text{ in.})}{(11\times10^6 \text{ psi})(\frac{\pi}{32})(2.5 \text{ in.})^4} = -0.002963 \text{ rad}$$

SEGMENT BC $T_{BC} = +T_2 = 4{,}000$ lb-in.

$$\tau_{BC} = \frac{16\,T_{BC}}{\pi d_2^3} = \frac{16\,(4{,}000 \text{ lb-in.})}{\pi\,(2.0 \text{ in.})^3} = 2546 \text{ psi}$$

$$\phi_{BC} = \frac{T_{BC}L_2}{G(I_p)_{BC}} = \frac{(4{,}000 \text{ lb-in.})(18 \text{ in.})}{(11\times10^6 \text{ psi})(\frac{\pi}{32})(2.0 \text{ in.})^4} = 0.004167 \text{ rad}$$

(a) MAXIMUM SHEAR STRESS

SEGMENT BC HAS THE MAXIMUM STRESS

$\tau_{max} = 2550$ psi ⟵

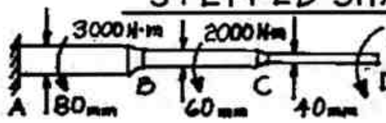
(b) ANGLE OF TWIST AT END C

$$\phi_C = \phi_{AB} + \phi_{BC} = (-0.002963 + 0.004167) \text{ rad}$$

$\phi_C = 0.001204$ rad $= 0.069°$ ⟵

3.4-2

STEPPED SHAFT



$G = 80$ GPa
$r_{AB} = 40$ mm

$L_{AB} = L_{BC} = L_{CD} = 0.5$ m $r_{BC} = 30$ mm $r_{CD} = 20$ mm

TORQUES $T_{AB} = (3000 + 2000 + 800)$ N·m $= 5800$ N·m
$T_{BC} = (2000 + 800)$ N·m $= 2800$ N·m
$T_{CD} = 800$ N·m

POLAR MOMENTS OF INERTIA

$(I_p)_{AB} = \frac{\pi}{32}(80 \text{ mm})^4 = 4.021 \times 10^6 \text{ mm}^4$

$(I_p)_{BC} = \frac{\pi}{32}(60 \text{ mm})^4 = 1.272 \times 10^6 \text{ mm}^4$

$(I_p)_{CD} = \frac{\pi}{32}(40 \text{ mm})^4 = 0.2513 \times 10^6 \text{ mm}^4$

(a) SHEAR STRESSES

$$\tau_{AB} = \frac{T_{AB}\,r_{AB}}{(I_p)_{AB}} = 57.7 \text{ MPa}$$

CONT.

3.4-2 CONT.

$$\tau_{BC} = \frac{T_{BC}\,r_{BC}}{(I_p)_{BC}} = 66.0 \text{ MPa}$$

$$\tau_{CD} = \frac{T_{CD}\,r_{CD}}{(I_p)_{CD}} = 63.7 \text{ MPa}$$

$\tau_{max} = 66.0$ MPa ⟵

(b) ANGLE OF TWIST AT END D

$$\phi_{AB} = \frac{T_{AB}L_{AB}}{G(I_p)_{AB}} = 0.00902 \text{ rad}$$

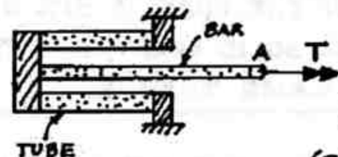
$$\phi_{BC} = \frac{T_{BC}L_{BC}}{G(I_p)_{BC}} = 0.01376 \text{ rad}$$

$$\phi_{CD} = \frac{T_{CD}L_{CD}}{G(I_p)_{CD}} = 0.01990 \text{ rad}$$

$\phi_D = \phi_{AB} + \phi_{BC} + \phi_{CD} = 0.04268$ rad $= 2.45°$ ⟵

3.4-3

BAR AND TUBE



TUBE

$d_3 = 2.75$ in. $d_2 = 2.35$ in.

$L_{TUBE} = 20$ in. $G = 3.9 \times 10^6$ psi

$(I_p)_{TUBE} = \frac{\pi}{32}(d_3^4 - d_2^4)$
$= 2.621 \text{ in.}^4$

BAR

$d_1 = 1.60$ in. $L_{BAR} = 40$ in. $G = 3.9 \times 10^6$ psi

$(I_p)_{BAR} = \frac{\pi}{32} d_1^4 = 0.6434 \text{ in.}^4$

TORQUE $T = 10{,}000$ lb-in.

(a) MAXIMUM SHEAR STRESSES

BAR: $\tau_{BAR} = \frac{16\,T}{\pi d_1^3} = 12{,}430$ psi ⟵

TUBE: $\tau_{TUBE} = \frac{T(d_3/2)}{(I_p)_{TUBE}} = 5{,}250$ psi ⟵

(b) ANGLE OF TWIST AT END A

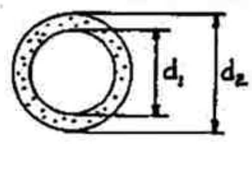
BAR: $\phi_{BAR} = \frac{T\,L_{BAR}}{G(I_p)_{BAR}} = 0.1594$ rad

TUBE: $\phi_{TUBE} = \frac{T\,L_{TUBE}}{G(I_p)_{TUBE}} = 0.0196$ rad

$\phi_A = \phi_{BAR} + \phi_{TUBE} = 0.1790$ rad $= 10.3°$ ⟵

3.4-4

HOLLOW TUBE OF MONEL METAL



$d_2 = 25$ mm $\tau_{allow} = 80$ MPa
$\theta_{allow} = 6°/\text{m} = 0.1047$ rad/m
$G = 66$ GPa (FROM TABLE H-2, APPENDIX H)

CONT.

3.4-4 CONT.

TORQUES

$T_1 = 100\ \text{N·m} \quad T_2 = 50\ \text{N·m} \quad T_3 = 80\ \text{N·m}$

$T_4 = 50\ \text{N·m} \quad T_5 = 80\ \text{N·m}$

$T_{AB} = -T_1 = -100\ \text{N·m} \quad T_{BC} = -T_1 + T_2 = -50\ \text{N·m}$

$T_{CD} = -T_1 + T_2 - T_3 = -130\ \text{N·m}$

$T_{DE} = -T_1 + T_2 - T_3 + T_4 = -80\ \text{N·m}$

LARGEST TORQUE (ABSOLUTE VALUE ONLY):

$T_{max} = 130\ \text{N·m}$

REQUIRED POLAR MOMENT OF INERTIA BASED UPON ALLOWABLE SHEAR STRESS

$$\tau_{max} = \frac{T_{max}\, r}{I_P} \qquad I_P = \frac{T_{max}(d_2/2)}{\tau_{allow}} = 20{,}310\ \text{mm}^4$$

REQUIRED POLAR MOMENT OF INERTIA BASED UPON ALLOWABLE ANGLE OF TWIST

$$\theta = \frac{T_{max}}{G I_P} \qquad I_P = \frac{T_{max}}{G\,\theta_{allow}} = 18{,}810\ \text{mm}^4$$

SHEAR STRESS GOVERNS

REQUIRED $I_P = 20{,}310\ \text{mm}^4$

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4)$$

$$d_1^4 = d_2^4 - \frac{32 I_P}{\pi} = (25\,\text{mm})^4 - \frac{32(20{,}310\ \text{mm}^4)}{\pi}$$

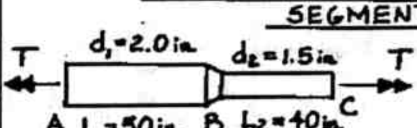
$$= 183{,}700\ \text{mm}^4$$

$d_1 = 20.7\ \text{mm}$ ⟵

(MAXIMUM PERMISSIBLE INSIDE DIAMETER)

3.4-5

BAR CONSISTING OF TWO SEGMENTS



$\tau_{allow} = 4000\ \text{psi}$

$\phi_{allow} = 1.0^\circ = 0.01745\ \text{rad}$

$G = 12 \times 10^6\ \text{psi}$

ALLOWABLE TORQUE BASED UPON SHEAR STRESS

SEGMENT BC HAS THE SMALLER DIAMETER AND HENCE THE LARGER STRESS $\tau_{max} = \dfrac{16\,T}{\pi d^3}$

$$T_{allow} = \frac{\pi d_2^3\, \tau_{allow}}{16} = 2650\ \text{lb-in.}$$

ALLOWABLE TORQUE BASED UPON ANGLE OF TWIST

$$\phi = \sum \frac{T_i L_i}{G I_{P_i}} = \frac{T L_1}{G I_{P_1}} + \frac{T L_2}{G I_{P_2}} = \frac{T}{G}\left(\frac{L_1}{I_{P_1}} + \frac{L_2}{I_{P_2}}\right)$$

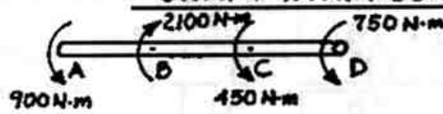
$$\phi = \frac{32\,T}{\pi G}\left(\frac{L_1}{d_1^4} + \frac{L_2}{d_2^4}\right)$$

$$T_{allow} = \frac{\pi\, \phi_{allow}\, G}{32\left(\frac{L_1}{d_1^4} + \frac{L_2}{d_2^4}\right)} = 1860\ \text{lb-in.}$$

ANGLE OF TWIST GOVERNS $\quad T_{allow} = 1860\ \text{lb-in.}$ ⟵

3.4-6

SHAFT WITH FOUR GEARS



$\tau_{allow} = 70\ \text{MPa}$

$T_{AB} = -900\ \text{N·m};\ T_{BC} = 1200\ \text{N·m};\ T_{CD} = 750\ \text{N·m}$

(a) SOLID SHAFT $\quad \tau_{max} = \dfrac{16\,T}{\pi d^3}$

$$d^3 = \frac{16\,T_{max}}{\pi\,\tau_{allow}} = \frac{16(1200\ \text{N·m})}{\pi(70\ \text{MPa})} = 87.308 \times 10^{-6}\ \text{m}^3$$

REQUIRED $d = 44.4\ \text{mm}$ ⟵

(b) HOLLOW SHAFT

INSIDE DIAMETER $d_0 = 40\ \text{mm}$

$$\tau_{max} = \frac{T r}{I_P} \qquad \tau_{allow} = \frac{T_{max}(d/2)}{I_P}$$

$$70\ \text{MPa} = \frac{(1200\ \text{N·m})(d/2)}{\left(\frac{\pi}{32}\right)\left[d^4 - (0.040\,\text{m})^4\right]} \qquad \text{OR}$$

$$70{,}000{,}000 = \frac{6{,}111.55\, d}{d^4 - 0.00000256} \qquad (d = \text{METERS})$$

OR

$$d^4 - 0.000\,087308\, d - 0.00000256 = 0$$

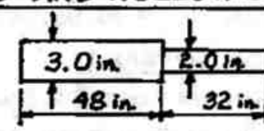
SOLVING, $d = 0.051548\ \text{m}$

REQUIRED $d = 51.5\ \text{mm}$ ⟵

3.4-7

SOLID AND HOLLOW SHAFTS

SOLID SHAFT

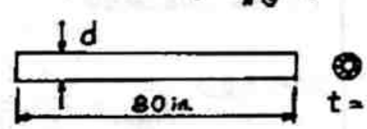


$$\phi_1 = \sum \frac{T L_i}{G I_{P_i}}$$

$$= \frac{T(48\ \text{in.})}{G\left(\frac{\pi}{32}\right)(3.0\ \text{in.})^4} + \frac{T(32\ \text{in.})}{G\left(\frac{\pi}{32}\right)(2.0\ \text{in.})^4}$$

$$= \frac{32\,T}{\pi G}\left(0.59259\ \text{in.}^{-3} + 2.0\ \text{in.}^{-3}\right) = \frac{32\,T}{\pi G}\left(2.5926\ \text{in.}^{-3}\right)$$

HOLLOW SHAFT



$t = \dfrac{d}{12}$

INSIDE DIAMETER

$d_0 = \frac{5}{6} d$

$$\phi_2 = \frac{T L}{G I_P}$$

$$= \frac{T(80\ \text{in.})}{G\left(\frac{\pi}{32}\right)\left[d^4 - \left(\frac{5d}{6}\right)^4\right]} = \frac{32\,T}{\pi G}\left(\frac{154.52\ \text{in.}}{d^4}\right) \quad (d = \text{INCHES})$$

TORSIONAL STIFFNESS $\quad k_T = \dfrac{T}{\phi}$

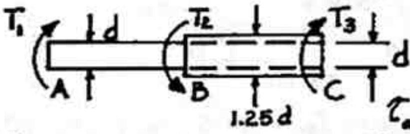
FOR EQUAL STIFFNESSES: $\phi_1 = \phi_2$ (BECAUSE T IS THE SAME)

$$\frac{32\,T}{\pi G}\left(2.5926\ \text{in.}^{-3}\right) = \frac{32\,T}{\pi G}\left(\frac{154.52\ \text{in.}}{d^4}\right)$$

$$d^4 = \frac{154.52\ \text{in.}}{2.5926\ \text{in.}^{-3}} = 59.599\ \text{in.}^4$$

$d = 2.78\ \text{in.}$ ⟵

3.4-8 SOLID BAR AND HOLLOW BAR



$L_{AB} = L_{BC} = 0.6\ m$

$G = 80\ GPa$

$\tau_{allow} = 80\ MPa$

$T_1 = 240\ N\cdot m$; $T_2 = 540\ N\cdot m$; $T_3 = 300\ N\cdot m$

$\phi_{allow} = 4° = 0.06981\ rad$ (BETWEEN ANY TWO GEARS)

TORQUES $T_{AB} = +T_1 = 240\ N\cdot m$

$T_{BC} = +T_1 - T_2 = -300\ N\cdot m$

POLAR MOMENTS OF INERTIA

$$(I_P)_{AB} = \frac{\pi d^4}{32} = 0.09817\ d^4$$

$$(I_P)_{BC} = \frac{\pi}{32}\left[(1.25d)^4 - d^4\right] = 0.14151\ d^4$$

(a) MINIMUM DIAMETER BASED UPON SHEAR STRESS

$$\tau_{max} = \frac{Tr}{I_P}$$

BAR AB: $\tau_{allow} = \dfrac{T_{AB}\left(\frac{d}{2}\right)}{(I_P)_{AB}}$ $\quad 80\ MPa = \dfrac{(240\ N\cdot m)\ d}{2(0.09817\ d^4)}$

SOLVING, $d^3 = 15.280 \times 10^{-6}\ m^3$ $\quad d = 0.0248\ m = 24.8\ mm$

BAR BC: $\tau_{allow} = \dfrac{T_{BC}\ (1.25\ d/2)}{(I_P)_{BC}}$

$$80\ MPa = \frac{(300\ N\cdot m)(0.625\ d)}{0.14151\ d^4}$$

SOLVING, $d^3 = 16.562 \times 10^{-6}\ m^3$; $d = 0.0255\ m = 25.5\ mm$

BAR BC GOVERNS

MINIMUM $d = 25.5\ mm$ ←

(b) MINIMUM DIAMETER BASED UPON ANGLE OF TWIST

BARS AB AND BC TWIST IN OPPOSITE DIRECTIONS. THEREFORE, THE ANGLE OF TWIST ϕ_{AC} BETWEEN GEARS A AND C IS LESS THAN THE LARGER OF ϕ_{AB} AND ϕ_{BC}. CONSEQUENTLY, ONLY ϕ_{AB} AND ϕ_{BC} NEED TO BE CHECKED.

$$\phi = \frac{TL}{GI_P}$$

BAR AB: $\phi_{allow} = \dfrac{T_{AB}\ L_{AB}}{G\ (I_P)_{AB}}$

$$0.06981\ rad = \frac{(240\ N\cdot m)(0.6\ m)}{(80\ GPa)(0.09817\ d^4)}$$

SOLVING, $d^4 = 0.2626 \times 10^{-6}\ m^4$; $d = 0.0226\ m = 22.6\ mm$

BAR BC: $\phi_{allow} = \dfrac{T_{BC}\ L_{BC}}{G\ (I_P)_{BC}}$

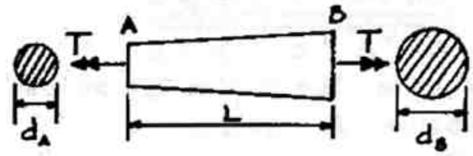
$$0.06981\ rad = \frac{(300\ N\cdot m)(0.6\ m)}{(80\ GPa)(0.14151\ d^4)}$$

SOLVING, $d^4 = 0.2278 \times 10^{-6}\ m^4$; $d = 0.0218\ m = 21.8\ mm$

BAR AB GOVERNS

MINIMUM $d = 22.6\ mm$ ←

3.4-9 TAPERED BAR



$d_B = 1.5\ d_A$ $\quad \beta = 1.5$

$T = 18{,}000\ lb\text{-}in.$

$L = 8.0\ ft = 96\ in.$

$G = 3.9 \times 10^6\ psi$

$\tau_{allow} = 7500\ psi$

$\phi_{allow} = 3.0° = 0.0523599\ rad$

FIND d_A

DIAMETER BASED UPON ALLOWABLE SHEAR STRESS

$$\tau_{max} = \frac{16T}{\pi d_A^3} \quad d_A^3 = \frac{16T}{\pi \tau_{allow}} = \frac{16(18{,}000\ lb\text{-}in.)}{\pi(7500\ psi)}$$

$$= 12.2231\ in.^3$$

$d_A = 2.30\ in.$

DIAMETER BASED UPON ALLOWABLE ANGLE OF TWIST

(FROM EQ. 3-27)

$$\phi = \frac{TL}{G(I_P)_A}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right) = \frac{TL}{G(I_P)_A}(0.469136)$$

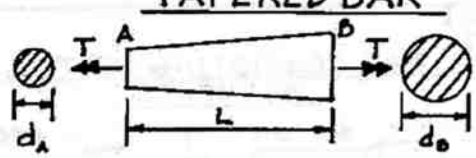
$$= \frac{(18{,}000\ lb\text{-}in.)(96\ in.)}{(3.9 \times 10^6\ psi)\left(\frac{\pi}{32}\right)d_A^4}(0.469136) = \frac{2.11728\ in.^4}{d_A^4}$$

$$d_A^4 = \frac{2.11728\ in.^4}{\phi_{allow}} = \frac{2.11728\ in.^4}{0.0523599\ rad} = 40.4370\ in.^4$$

$d_A = 2.52\ in.$

ANGLE OF TWIST GOVERNS $\quad d_A = 2.52\ in.$ ←

3.4-10 TAPERED BAR



$d_A = 25\ mm$

$G = 82\ GPa$

$T = 90\ N\cdot m$

$\phi_{allow} = 0.5°/m$

FIND d_B

DIAMETER BASED UPON ALLOWABLE ANGLE OF TWIST

(FROM EQ. 3-27) $\quad \theta = \dfrac{\phi}{L} \quad \beta = \dfrac{d_B}{d_A}$

$$\theta = \frac{T}{G(I_P)_A}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right) \quad (I_P)_A = \frac{\pi}{32}d_A^4$$

$$(0.5°/m)\left(\frac{\pi}{180}\ \frac{rad}{degrees}\right) = \frac{90\ N\cdot m}{(82\ GPa)\left(\frac{\pi}{32}\right)(25\ mm)^4}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right)$$

$$0.304915 = \frac{\beta^2+\beta+1}{3\beta^3}$$

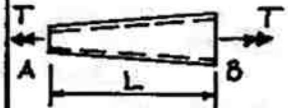
$$0.914745\beta^3 - \beta^2 - \beta - 1 = 0$$

SOLVING, $\beta = 1.94452$

$d_B = \beta\ d_A = 48.6\ mm$ ←

3.4-11

TAPERED TUBE

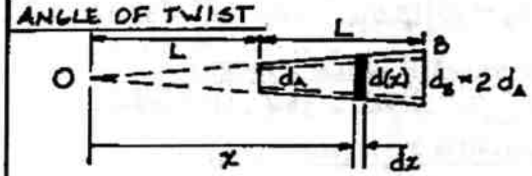


t = THICKNESS (CONSTANT)

d_A, d_B = AVERAGE DIAMETERS AT THE ENDS

$d_B = 2d_A \quad I_P = \dfrac{\pi d^3 t}{4}$ (APPROXIMATE FORMULA)

ANGLE OF TWIST



TAKE THE ORIGIN OF COORDINATES AT POINT O.

$$d(x) = \frac{x}{2L}(d_B) = \frac{x}{L}d_A$$

$$I_P(x) = \frac{\pi[d(x)]^3 t}{4} = \frac{\pi t d_A^3}{4L^3}x^3$$

FOR ELEMENT OF LENGTH dx:

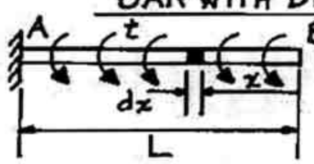
$$d\phi = \frac{T\,dx}{GI_P(x)} = \frac{T\,dx}{G\left(\frac{\pi t d_A^3}{4L^3}\right)x^3} = \frac{4TL^3}{\pi G t d_A^3}\cdot\frac{dx}{x^3}$$

FOR ENTIRE BAR:

$$\phi = \int_L^{2L} d\phi = \frac{4TL^3}{\pi G t d_A^3}\int_L^{2L}\frac{dx}{x^3} = \frac{3TL}{2\pi G t d_A^3} \longleftarrow$$

3.4-12

BAR WITH DISTRIBUTED TORQUE



t = INTENSITY OF DISTRIBUTED TORQUE

d = DIAMETER

G = SHEAR MODULUS OF ELASTICITY

(a) MAXIMUM SHEAR STRESS

$$T_{max} = tL \quad \tau_{max} = \frac{16T_{max}}{\pi d^3} = \frac{16tL}{\pi d^3} \longleftarrow$$

(b) ANGLE OF TWIST

$$T(x) = tx \quad I_P = \frac{\pi d^4}{32}$$

$$d\phi = \frac{T(x)dx}{GI_P} = \frac{32\,tx\,dx}{\pi G d^4}$$

$$\phi = \int_0^L d\phi = \frac{32t}{\pi G d^4}\int_0^L x\,dx = \frac{16tL^2}{\pi G d^4} \longleftarrow$$

3.4-13

BAR WITH LINEARLY VARYING TORQUE

$t(x)$ = INTENSITY OF DISTRIBUTED TORQUE

t_A = MAXIMUM INTENSITY OF TORQUE

CONT.

3.4-13 CONT.

d = DIAMETER

G = SHEAR MODULUS

T_A = MAXIMUM TORQUE $= \dfrac{t_A L}{2}$

(a) MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{16T_{max}}{\pi d^3} = \frac{16T_A}{\pi d^3} = \frac{8t_A L}{\pi d^3} \longleftarrow$$

(b) ANGLE OF TWIST

$T(x)$ = TORQUE AT DISTANCE x FROM END B

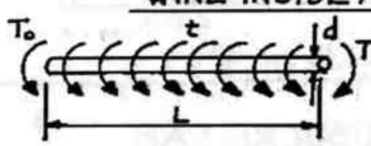
$$T(x) = \frac{t(x)x}{2} = \frac{t_A x^2}{2L} \quad I_P = \frac{\pi d^4}{32}$$

$$d\phi = \frac{T(x)dx}{GI_P} = \frac{16t_A x^2 dx}{\pi G L d^4}$$

$$\phi = \int_0^L d\phi = \frac{16t_A}{\pi G L d^4}\int_0^L x^2 dx = \frac{16t_A L^2}{3\pi G d^4} \longleftarrow$$

3.4-14

WIRE INSIDE A FLEXIBLE TUBE



$d = 4$ mm

$T_0 = 0.2$ N·m

$t = 0.04$ N·m/m

(a) MAXIMUM LENGTH L_{max}

$$\tau_{allow} = 30 \text{ MPa}$$

EQUILIBRIUM: $T = tL + T_0$

FROM EQ. (3-12): $\tau_{max} = \dfrac{16T}{\pi d^3} \quad T = \dfrac{\pi d^3 \tau_{max}}{16}$

$$tL + T_0 = \frac{\pi d^3 \tau_{max}}{16}$$

$$L = \frac{1}{16t}(\pi d^3 \tau_{max} - 16T_0)$$

$$L_{max} = \frac{1}{16t}(\pi d^3 \tau_{allow} - 16T_0) \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES: $L_{max} = 4.42$ m $\longleftarrow$

(b) ANGLE OF TWIST ϕ $\quad L = 4$ m $\quad G = 15$ GPa

ϕ_1 = ANGLE OF TWIST DUE TO DISTRIBUTED TORQUE t

$= \dfrac{16tL^2}{\pi G d^4}$ (FROM PROBLEM 3.4-12)

ϕ_2 = ANGLE OF TWIST DUE TO TORQUE T_0

$= \dfrac{T_0 L}{GI_P} = \dfrac{32T_0 L}{\pi G d^4}$ (FROM EQ. 3-15)

ϕ = TOTAL ANGLE OF TWIST

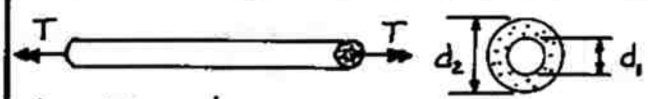
$= \phi_1 + \phi_2$

$$\phi = \frac{16L}{\pi G d^4}(tL + 2T_0) \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\phi = 2.971 \text{ rad} = 170° \longleftarrow$$

3.5-1 HOLLOW ALUMINUM SHAFT



$d_2 = 4.0$ in. $\quad d_1 = 2.0$ in. $\quad \theta = 0.62°/\text{ft}$

$G = 4.0 \times 10^6$ psi

(a) MAXIMUM TENSILE STRESS

$\tau_{max} = G r \theta$ (FROM EQ. 3-7a)

$= (4.0 \times 10^6 \text{psi})\left(\frac{4.0 \text{ in.}}{2}\right)(0.62°/\text{ft})\left(\frac{1}{12}\frac{\text{ft}}{\text{in.}}\right)\left(\frac{\pi}{180}\frac{\text{rad}}{\text{degrees}}\right)$

$= 7214$ psi

$\sigma_{max} = \tau_{max} = 7210$ psi ←

(b) APPLIED TORQUE

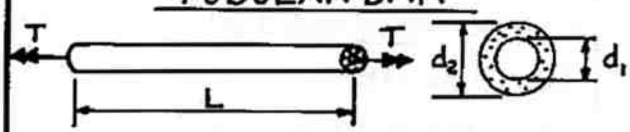
$\tau_{max} = \frac{Tr}{I_p} \quad r = \frac{d_2}{2} = 2.0$ in.

$I_p = \frac{\pi}{32}\left[(4.0 \text{in.})^4 - (2.0 \text{in.})^4\right] = 23.562 \text{ in.}^4$

$T = \frac{\tau_{max} I_p}{r} = \frac{(7214 \text{psi})(23.562 \text{in.}^4)}{2.0 \text{ in.}}$

$T = 85{,}000$ lb-in. ←

3.5-2 TUBULAR BAR



$d_2 = 100$ mm $\quad T = 8.0$ kN·m $\quad \sigma_{max} = 46.8$ MPa

(a) INSIDE DIAMETER d_1

$\sigma_{max} = \tau_{max} = 46.8$ MPa

$\tau_{max} = \frac{Tr}{I_p} \quad r = \frac{d_2}{2} = 50$ mm

$I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = \frac{\pi}{32}\left[(0.100 \text{m})^4 - d_1^4\right]$

$I_p = \frac{Tr}{\tau_{max}} = \frac{(8000 \text{ N·m})(0.05 \text{m})}{46.8 \text{ MPa}} = 8.5470 \times 10^{-6} \text{ m}^4$

$\frac{\pi}{32}\left[(0.100 \text{m})^4 - d_1^4\right] = 8.5470 \times 10^{-6} \text{ m}^4$

SOLVE FOR d_1: $d_1 = 0.05998$ m $= 60.0$ mm ←

(b) ANGLE OF TWIST AND MAX. SHEAR STRAIN

$L = 1.2$ m $\quad G = 28$ GPa

$\tau_{max} = G r \theta = G r \frac{\phi}{L}$ (FROM EQ. 3-7a)

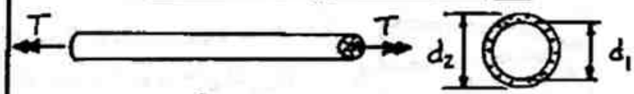
$\phi = \frac{\tau_{max} L}{G r} = \frac{(46.8 \text{ MPa})(1.2 \text{m})}{(28 \text{ GPa})(0.050 \text{m})} = 0.040114$ rad

$\phi = 2.30°$ ←

$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{46.8 \text{ MPa}}{28 \text{ GPa}}$

$\gamma_{max} = 1670 \times 10^{-6}$ rad ←

3.5-3 HOLLOW STEEL BAR



$G = 11 \times 10^6$ psi $\quad \gamma_{max} = 668 \times 10^{-6}$ rad

$d_2 = 3.0$ in. $\quad d_1 = 2.4$ in.

$I_p = \frac{\pi}{32}\left[(3.0 \text{in.})^4 - (2.4 \text{in.})^4\right] = 4.6950 \text{ in.}^4$

MAXIMUM TENSILE STRAIN

$\epsilon_{max} = \frac{\gamma_{max}}{2} = 334 \times 10^{-6}$ ←

MAXIMUM TENSILE STRESS

$\tau_{max} = G\gamma_{max} = (11 \times 10^6 \text{ psi})(668 \times 10^{-6} \text{rad})$

$= 7348$ psi

$\sigma_{max} = \tau_{max} = 7350$ psi ←

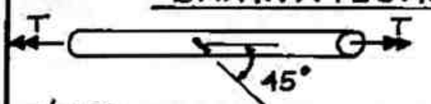
APPLIED TORQUE

$\tau_{max} = \frac{T r_2}{I_p} = \frac{T d_2/2}{I_p}$

$T = \frac{2\tau_{max} I_p}{d_2} = \frac{2(7348 \text{psi})(4.6950 \text{in.}^4)}{3.0 \text{ in.}}$

$T = 23{,}000$ lb-in. ←

3.5-4 BAR IN A TESTING MACHINE



STRAIN GAGE AT 45°: $\epsilon_{max} = 331 \times 10^{-6}$

$d = 50$ mm

$T = 1300$ N·m

SHEAR STRAIN (FROM EQ. 3-33)

$\gamma_{max} = 2\epsilon_{max} = 662 \times 10^{-6}$

SHEAR STRESS

$\tau_{max} = G\gamma_{max} = \frac{16T}{\pi d^3}$

SHEAR MODULUS

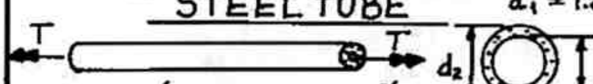
$G = \frac{16T}{\pi d^3 \gamma_{max}} = \frac{16(1300 \text{ N·m})}{\pi(0.050 \text{m})^3(662 \times 10^{-6})}$

$G = 80.0$ GPa ←

3.5-5 STEEL TUBE

$d_2 = 1.6$ in.

$d_1 = 1.2$ in.



$G = 11.2 \times 10^6$ psi $\quad \epsilon_{max} = 203 \times 10^{-6}$

$I_p = \frac{\pi}{32}\left[d_2^4 - d_1^4\right] = 0.43982 \text{ in.}^4$ $\quad$ FIND TORQUE T

SHEAR STRAIN (FROM EQ. 3-33)

$\gamma_{max} = 2\epsilon_{max} = 406 \times 10^{-6}$

SHEAR STRESS

$\tau_{max} = G\gamma_{max} = \frac{T(d_2/2)}{I_p}$

TORQUE

$T = \frac{2G\gamma_{max} I_p}{d_2} = \frac{2(11.2 \times 10^6 \text{psi})(406 \times 10^{-6})(0.43982 \text{in.}^4)}{1.6 \text{ in.}}$

$T = 2500$ lb-in. ←

3.5-6 SOLID CIRCULAR BAR OF STEEL

$T = 450\ N\cdot m \quad G = 78\ GPa$

ALLOWABLE STRESS

TENSION: 80 MPa COMPRESSION: 60 MPa

SHEAR: 50 MPa

ALLOWABLE TENSILE STRAIN: $\epsilon_{max} = 275 \times 10^{-6}$

DIAMETER BASED UPON ALLOWABLE STRESS

THE MAXIMUM TENSILE, COMPRESSIVE, AND SHEAR STRESSES IN A BAR IN PURE TORSION ARE NUMERICALLY EQUAL. THEREFORE, THE LOWEST ALLOWABLE STRESS (SHEAR STRESS) GOVERNS.

$\tau_{allow} = 50\ MPa$

$$\tau_{max} = \frac{16T}{\pi d^3} \quad d^3 = \frac{16T}{\pi \tau_{allow}} = \frac{16(450\ N\cdot m)}{\pi(50\ MPa)}$$

$$d^3 = 45.837 \times 10^{-6}\ m^3$$

$$d = 0.0358\ m = 35.8\ mm$$

DIAMETER BASED UPON ALLOWABLE TENSILE STRAIN

$$\gamma_{max} = 2\epsilon_{max} \quad \tau_{max} = G\gamma_{max} = 2G\epsilon_{max}$$

$$\tau_{max} = \frac{16T}{\pi d^3} \quad d^3 = \frac{16T}{\pi \tau_{max}} = \frac{16T}{2\pi G \epsilon_{max}}$$

$$d^3 = \frac{16(450\ N\cdot m)}{2\pi(78\ GPa)(275 \times 10^{-6})}$$

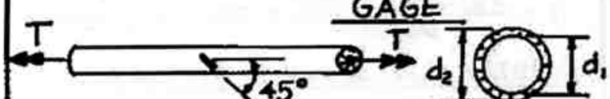
$$d^3 = 53.423 \times 10^{-6}\ m^3$$

$$d = 0.0377\ m = 37.7\ mm$$

TENSILE STRAIN GOVERNS

MINIMUM DIAMETER $d = 37.7\ mm$ ←

3.5-7 CIRCULAR TUBE WITH STRAIN GAGE



$d_2 = 0.80$ in. $T = 1730$ lb-in. $G = 6.8 \times 10^6$ psi

STRAIN GAGE AT 45°: $\epsilon_{max} = 1860 \times 10^{-6}$

MAXIMUM SHEAR STRAIN

$$\gamma_{max} = 2\epsilon_{max}$$

MAXIMUM SHEAR STRESS

$$\tau_{max} = G\gamma_{max} = 2G\epsilon_{max}$$

$$\tau_{max} = \frac{T(d_2/2)}{I_p} \quad I_p = \frac{Td_2}{2\tau_{max}} = \frac{Td_2}{4G\epsilon_{max}}$$

$$I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = \frac{Td_2}{4G\epsilon_{max}}$$

$$d_2^4 - d_1^4 = \frac{8Td_2}{\pi G\epsilon_{max}} \quad d_1^4 = d_2^4 - \frac{8Td_2}{\pi G\epsilon_{max}}$$

SUBSTITUTE NUMERICAL VALUES: $d_1^4 = (0.80\ in)^4 - \dfrac{8(1730\ lb\text{-}in)(0.8\ in.)}{\pi(6.8 \times 10^6\ psi)(1860 \times 10^{-6})}$

$$= 0.4096\ in^4 - 0.278647\ in^4$$

$$= 0.13095\ in^4$$

$$d_1 = 0.60\ in$$ ←

3.5-8 ALUMINUM TUBE

$d_1 = 50\ mm \quad G = 27\ GPa$

$T = 4.0\ kN\cdot m \quad \tau_{allow} = 50\ MPa \quad \epsilon_{allow} = 900 \times 10^{-6}$

DETERMINE THE REQUIRED DIAMETER d_2

ALLOWABLE SHEAR STRESS BASED ON SHEAR

$$(\tau_{allow})_1 = 50\ MPa$$

ALLOWABLE SHEAR STRESS BASED ON NORMAL STRAIN

$$\epsilon_{max} = \frac{\gamma}{2} = \frac{\tau}{2G} \quad \tau = 2G\epsilon_{max}$$

$$(\tau_{allow})_2 = 2G\epsilon_{allow} = 2(27\ GPa)(900 \times 10^{-6})$$

$$= 48.6\ MPa$$

NORMAL STRAIN GOVERNS $\tau_{allow} = 48.6\ MPa$

REQUIRED DIAMETER

$$\tau = \frac{Tr}{I_p} \quad 48.6\ MPa = \frac{(4000\ N\cdot m)(d_2/2)}{\frac{\pi}{32}\left[d_2^4 - (0.050\ m)^4\right]}$$

REARRANGE AND SIMPLIFY:

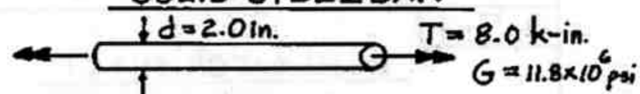
$$d_2^4 - (419.174 \times 10^{-6})d_2 - 6.25 \times 10^{-6} = 0$$

SOLVE:

$$d_2 = 0.07927\ m$$

$$d_2 = 79.3\ mm$$ ←

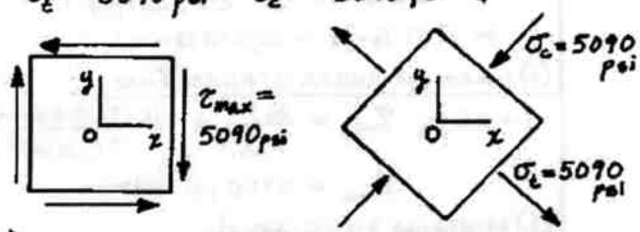
3.5-9 SOLID STEEL BAR



(a) MAXIMUM STRESSES

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(8000\ lb\text{-}in.)}{\pi(2.0\ in.)^3} = 5093\ psi$$ ←

$\sigma_t = 5090\ psi \quad \sigma_c = -5090\ psi$ ←

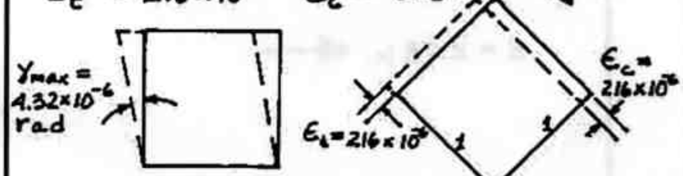


(b) MAXIMUM STRAINS

$$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{5093\ psi}{11.8 \times 10^6\ psi} = 432 \times 10^{-6}\ rad$$ ←

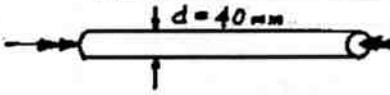
$$\epsilon_{max} = \frac{\gamma_{max}}{2} = 216 \times 10^{-6}$$

$\epsilon_t = 216 \times 10^{-6} \quad \epsilon_c = -216 \times 10^{-6}$ ←



3.5-10

SOLID ALUMINUM BAR

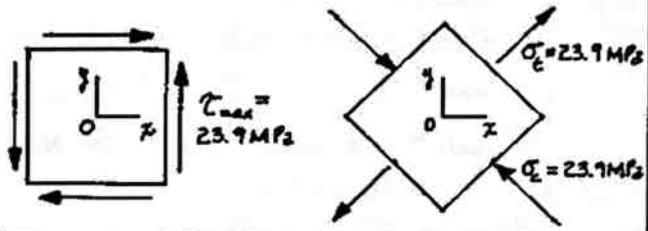


$G = 27\ GPa$

(a) MAXIMUM STRESSES

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(300\ N\cdot m)}{\pi(0.040\ m)^3} = 23.87\ MPa \longleftarrow$$

$$\sigma_t = 23.9\ MPa \quad \sigma_c = -23.9\ MPa \longleftarrow$$

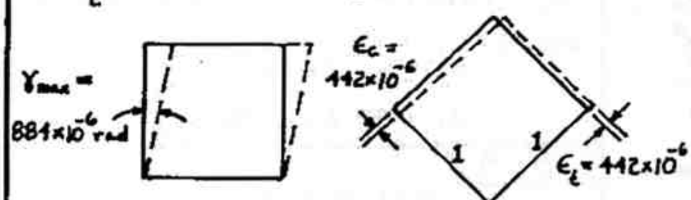


(b) MAXIMUM STRAINS

$$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{23.87\ MPa}{27\ GPa} = 884 \times 10^{-6}\ rad \longleftarrow$$

$$\epsilon_{max} = \frac{\gamma_{max}}{2} = 442 \times 10^{-6}$$

$$\epsilon_t = 442 \times 10^{-6} \quad \epsilon_c = -442 \times 10^{-6} \longleftarrow$$



3.7-1

GENERATOR SHAFT

$n = 120\ rpm \quad H = 40\ hp \quad d = \text{DIAMETER}$

TORQUE $\quad H = \dfrac{2\pi n T}{33{,}000} \quad H = hp \quad n = rpm \quad T = lb\text{-}ft$

$$T = \frac{33{,}000\ H}{2\pi n} = \frac{(33{,}000)(40\ hp)}{2\pi(120\ rpm)}$$

$$= 1751\ lb\text{-}ft = 21{,}010\ lb\text{-}in.$$

(a) MAXIMUM SHEAR STRESS τ_{max}

$$d = 3.0\ in. \quad \tau_{max} = \frac{16T}{\pi d^3} = \frac{16(21{,}010\ lb\text{-}in)}{\pi(3.0\ in)^3}$$

$$\tau_{max} = 3960\ psi \longleftarrow$$

(b) MINIMUM DIAMETER d

$\tau_{allow} = 5000\ psi$

$$d^3 = \frac{16T}{\pi \tau_{allow}} = \frac{16(21{,}010\ lb\text{-}in.)}{\pi(5000\ psi)}$$

$$d^3 = 21.40\ in^3$$

$$d = 2.78\ in. \longleftarrow$$

3.7-2

MOTOR-DRIVEN SHAFT

$f = 12\ Hz \quad P = 18\ kW = 18{,}000\ N\cdot m/s$

TORQUE

$P = 2\pi f T \quad P = \text{WATTS} \quad f = Hz = s^{-1}$

$T = \text{NEWTON METERS}$

$$T = \frac{P}{2\pi f} = \frac{18{,}000\ W}{2\pi(12\ Hz)} = 238.7\ N\cdot m$$

(a) MAXIMUM SHEAR STRESS τ_{max}

$d = 30\ mm$

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(238.7\ N\cdot m)}{\pi(0.030\ m)^3} = 45.0\ MPa \longleftarrow$$

(b) MINIMUM DIAMETER d $\quad \tau_{allow} = 40\ MPa$

$$d^3 = \frac{16T}{\pi \tau_{allow}} = \frac{16(238.7\ N\cdot m)}{\pi(40\ MPa)}$$

$$d^3 = 30.39 \times 10^{-6}\ m^3$$

$$d = 0.03121\ m = 31.2\ mm \longleftarrow$$

3.7-3

HOLLOW PROPELLER SHAFT

$d_2 = 14\ in. \quad d_1 = 10\ in. \quad \tau_{allow} = 9000\ psi$

$$I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = 2790\ in.^4$$

TORQUE $\quad \tau_{max} = \dfrac{T(d_2/2)}{I_p} \quad T = \dfrac{2\tau_{allow} I_p}{d_2}$

$$T = \frac{2(9000\ psi)(2790\ in.^4)}{14\ in.} = 3.587 \times 10^6\ lb\text{-}in.$$

$$= 298{,}900\ lb\text{-}ft$$

(a) HORSEPOWER

$n = 600\ rpm \quad H = \dfrac{2\pi n T}{33{,}000} \quad n = rpm,\ T = lb\text{-}ft,\ H = hp$

$$H = \frac{2\pi(600)(298{,}900)}{33{,}000} = 34{,}100\ hp \longleftarrow$$

(b) ROTATIONAL SPEED IS DOUBLED

$$H = \frac{2\pi n T}{33{,}000}$$

IF n IS DOUBLED BUT H REMAINS THE SAME, THEN T IS HALVED.

IF T IS HALVED, SO IS THE MAXIMUM SHEAR STRESS.

SHEAR STRESS IS HALVED $\longleftarrow$

3.7-4

DRIVE SHAFT FOR A TRUCK

$d_2 = 60\ mm \quad d_1 = 40\ mm \quad n = 2500\ rpm$

$$I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = 1.0210 \times 10^{-6}\ m^4$$

(a) MAXIMUM SHEAR STRESS

$P = 180\ kW \quad P = \dfrac{2\pi n T}{60} \quad T = \text{NEWTON METERS},\ P = \text{WATTS}$

$$T = \frac{60P}{2\pi n} = \frac{60(180{,}000\ W)}{2\pi(2500\ rpm)}$$

$$T = 687.5\ N\cdot m$$

$$\tau_{max} = \frac{T(d_2/2)}{I_p} = \frac{(687.5\ N\cdot m)(30\ mm)}{1.0210 \times 10^{-6}\ m^4} = 20.2\ MPa \longleftarrow$$

CONT.

3.7-4 CONT.

(b) MAXIMUM POWER

$\tau_{allow} = 35\text{ MPa}$ $\quad \tau_{allow} = \dfrac{T(d_2/2)}{I_p}$

$$T_{allow} = \frac{\tau_{allow} I_p}{d_2/2} = \frac{(35\text{ MPa})(1.0210\times10^{-6}\text{ m}^4)}{30\text{ mm}}$$

$$= 1191\text{ N}\cdot\text{m}$$

$$P_{max} = \frac{2\pi n T_{allow}}{60} = \frac{2\pi(2500\text{ rpm})(1191\text{ N}\cdot\text{m})}{60}$$

$$P_{max} = 312\text{ kW} \longleftarrow$$

3.7-5

HOLLOW SHAFT

$d_0 = 0.8d$ $\quad d_0$ = INSIDE DIAMETER

d = OUTSIDE DIAMETER $\quad H = 500\text{ hp}$ $\quad n = 800\text{ rpm}$

$\tau_{allow} = 6000\text{ psi}$ $\quad I_p = \frac{\pi}{32}(d^4 - d_0^4) = \frac{\pi}{32}[d^4 - (0.8d)^4]$

$I_p = 0.057962\,d^4$

$H = \dfrac{2\pi n T}{33{,}000}$ $\quad H = \text{hp}$ $\quad n = \text{rpm}$ $\quad T = \text{lb-ft}$

$$T = \frac{33{,}000\,H}{2\pi n} = \frac{(33{,}000)(500\text{ hp})}{2\pi(800\text{ rpm})}$$

$$T = 3283\text{ lb-ft} = 39{,}390\text{ lb-in.}$$

MINIMUM OUTSIDE DIAMETER

$\tau_{max} = \dfrac{T(d/2)}{I_p}$ $\quad I_p = \dfrac{T(d/2)}{\tau_{allow}}$

$$0.057962\,d^4 = \frac{(39{,}390\text{ lb-in.})(d)}{2(6000\text{ psi})}$$

$$d^3 = 56.63\text{ in.}^3$$

$$d = 3.84\text{ in.} \longleftarrow$$

3.7-6

HOLLOW SHAFT

$P = 120\text{ kW}$ $\quad f = 15\text{ Hz}$ $\quad \tau_{allow} = 45\text{ MPa}$ $\quad d_0 = 0.75d$

d = OUTSIDE DIAMETER $\quad d_0$ = INSIDE DIAMETER

$I_p = \frac{\pi}{32}(d^4 - d_0^4) = \frac{\pi}{32}[d^4 - (0.75d)^4] = 0.067112\,d^4$

$P = 2\pi f T$ $\quad P$ = WATTS $\quad f$ = Hz $\quad T$ = NEWTON METERS

$$T = \frac{P}{2\pi f} = \frac{120{,}000\text{ W}}{2\pi(15\text{ Hz})} = 1273\text{ N}\cdot\text{m}$$

MINIMUM OUTSIDE DIAMETER

$\tau_{max} = \dfrac{T(d/2)}{I_p}$ $\quad I_p = \dfrac{T(d/2)}{\tau_{allow}}$

$$0.067112\,d^4 = \frac{(1273\text{ N}\cdot\text{m})(d)}{2(45\text{ MPa})}$$

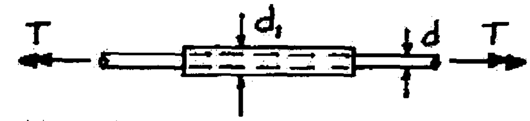
$$d^3 = 0.0002108\text{ m}^3$$

$$d = 0.05951\text{ m}$$

$$d = 59.5\text{ mm} \longleftarrow$$

3.7-7

SPLICE IN A PROPELLER SHAFT



SOLID SHAFT $\quad \tau_{max} = \dfrac{16T_1}{\pi d^3}$ $\quad T_1 = \dfrac{\pi d^3 \tau_{max}}{16}$

HOLLOW COLLAR

$I_p = \frac{\pi}{32}(d_1^4 - d^4)$ $\quad \tau_{max} = \dfrac{T_2 r}{I_p} = \dfrac{T_2(d_1/2)}{I_p}$

$$T_2 = \frac{2\tau_{max} I_p}{d_1} = \frac{2\tau_{max}}{d_1}\left(\frac{\pi}{32}\right)(d_1^4 - d^4)$$

$$= \frac{\pi\tau_{max}}{16 d_1}(d_1^4 - d^4)$$

EQUATE TORQUES

FOR THE SAME POWER, THE TORQUES MUST BE THE SAME. FOR THE SAME MATERIAL, BOTH PARTS CAN BE STRESSED TO THE SAME MAXIMUM STRESS.

$\therefore\ T_1 = T_2$ $\quad \dfrac{\pi d^3 \tau_{max}}{16} = \dfrac{\pi\tau_{max}}{16 d_1}(d_1^4 - d^4)$

OR $\left(\dfrac{d_1}{d}\right)^4 - \dfrac{d_1}{d} - 1 = 0$

SOLVING, $d_1 = 1.221\,d \longleftarrow$

3.7-8

HOLLOW PROPELLER SHAFT

$d_2 = 50\text{ mm}$ $\quad d_1 = 40\text{ mm}$ $\quad G = 80\text{ GPa}$ $\quad n = 600\text{ rpm}$

$\tau_{allow} = 80\text{ MPa}$ $\quad \theta_{allow} = 2.5°/\text{m}$

$$I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = 362.3\times10^{-9}\text{ m}^4$$

BASED UPON ALLOWABLE SHEAR STRESS

$\tau_{max} = \dfrac{T_1(d_2/2)}{I_p}$ $\quad T_1 = \dfrac{2\tau_{allow} I_p}{d_2}$

$$T_1 = \frac{2(80\text{ MPa})(362.3\times10^{-9}\text{ m}^4)}{0.050\text{ m}} = 1159\text{ N}\cdot\text{m}$$

BASED UPON ALLOWABLE RATE OF TWIST

$\theta = \dfrac{T_2}{G I_p}$ $\quad T_2 = G I_p \theta_{allow}$

$$T_2 = (80\text{ GPa})(362.3\times10^{-9}\text{ m}^4)(2.5°/\text{m})\left(\frac{\pi}{180}\text{ rad/°}\right)$$

$$T_2 = 1265\text{ N}\cdot\text{m}$$

SHEAR STRESS GOVERNS

$$T_{allow} = T_1 = 1159\text{ N}\cdot\text{m}$$

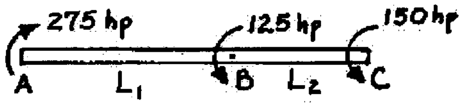
MAXIMUM POWER

$$P = \frac{2\pi n T}{60} = \frac{2\pi(600\text{ rpm})(1159\text{ N}\cdot\text{m})}{60}$$

$$P = 72{,}820\text{ W} = 72.8\text{ kW} \longleftarrow$$

3.7-9

MOTOR-DRIVEN SHAFT



$L_1 = 6$ ft
$L_2 = 4$ ft

d = DIAMETER $n = 1000$ rpm $\tau_{allow} = 7500$ psi

$(\phi_{AC})_{allow} = 1.5° = 0.02618$ rad $G = 11.5 \times 10^6$ psi

TORQUES ACTING ON THE SHAFT

$$H = \frac{2\pi n T}{33{,}000} \quad H = \text{hp} \quad n = \text{rpm} \quad T = \text{lb-ft}$$

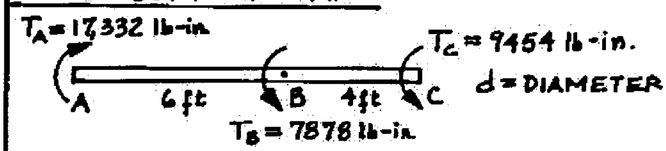
$$T = \frac{33{,}000\, H}{2\pi n}$$

AT POINT A: $T_A = \dfrac{33{,}000(275\text{ hp})}{2\pi(1000\text{ rpm})} = 1444$ lb-ft $= 17{,}332$ lb-in

AT POINT B: $T_B = \dfrac{125}{275} T_A = 7878$ lb-in.

AT POINT C: $T_C = \dfrac{150}{275} T_A = 9454$ lb-in.

FREE-BODY DIAGRAM



INTERNAL TORQUES $T_{AB} = 17{,}332$ lb-in.

$T_{BC} = 9454$ lb-in.

DIAMETER BASED UPON ALLOWABLE SHEAR STRESS

THE LARGER TORQUE OCCURS IN SEGMENT AB

$$\tau_{max} = \frac{16\, T_{AB}}{\pi d^3} \quad d^3 = \frac{16\, T_{AB}}{\pi \tau_{allow}} = \frac{16(17{,}332\text{ lb-in})}{\pi(7500\text{ psi})} = 11.77\text{ in.}^3$$

$d = 2.27$ in.

DIAMETER BASED UPON ALLOWABLE ANGLE OF TWIST

$$I_P = \frac{\pi d^4}{32} \quad \phi = \frac{TL}{GI_P} = \frac{32TL}{\pi G d^4}$$

SEGMENT AB:

$$\phi_{AB} = \frac{32\, T_{AB} L_{AB}}{\pi G d^4} = \frac{32(17{,}330\text{ lb-in.})(6\text{ ft})(12\text{ in/ft})}{\pi(11.5\times10^6\text{ psi})\, d^4}$$

$$\phi_{AB} = \frac{1.1052}{d^4}$$

SEGMENT BC:

$$\phi_{BC} = \frac{32\, T_{BC} L_{BC}}{\pi G d^4} = \frac{32(9450\text{ lb-in.})(4\text{ ft})(12\text{ in./ft})}{\pi(11.5\times10^6\text{ psi})\, d^4}$$

$$\phi_{BC} = \frac{0.4018}{d^4}$$

FROM A TO C: $\phi_{AC} = \phi_{AB} + \phi_{BC} = \dfrac{1.5070}{d^4}$

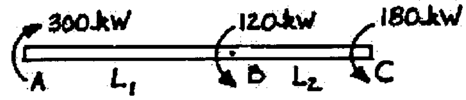
$(\phi_{AC})_{allow} = 0.02618$ rad

$\therefore\ 0.02618 = \dfrac{1.5070}{d^4}$ AND $d = 2.75$ in.

ANGLE OF TWIST GOVERNS $d = 2.75$ in. ⟵

3.7-10

MOTOR-DRIVEN SHAFT



$L_1 = 1.5$ m
$L_2 = 0.9$ m

d = DIAMETER $f = 32$ Hz $\tau_{allow} = 50$ MPa $G = 75$ GPa

$(\phi_{AC})_{allow} = 4° = 0.06981$ rad

TORQUES ACTING ON THE SHAFT

$P = 2\pi f T$ P = WATTS f = Hz T = NEWTON METERS

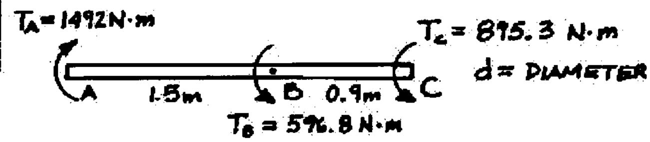
$$T = \frac{P}{2\pi f}$$

AT POINT A: $T_A = \dfrac{300{,}000\text{ W}}{2\pi(32\text{ Hz})} = 1492$ N·m

AT POINT B: $T_B = \dfrac{120}{300} T_A = 596.8$ N·m

AT POINT C: $T_C = \dfrac{180}{300} T_A = 895.3$ N·m

FREE-BODY DIAGRAM



INTERNAL TORQUES $T_{AB} = 1492$ N·m

$T_{BC} = 895.3$ N·m

DIAMETER BASED UPON ALLOWABLE SHEAR STRESS

THE LARGER TORQUE OCCURS IN SEGMENT AB

$$\tau_{max} = \frac{16\, T_{AB}}{\pi d^3} \quad d^3 = \frac{16\, T_{AB}}{\pi \tau_{allow}} = \frac{16(1492\text{ N·m})}{\pi(50\text{ MPa})}$$

$d^3 = 0.0001520\text{ m}^3$ $d = 0.0534$ m $= 53.4$ mm

DIAMETER BASED UPON ALLOWABLE ANGLE OF TWIST

$$I_P = \frac{\pi d^4}{32} \quad \phi = \frac{TL}{GI_P} = \frac{32TL}{\pi G d^4}$$

SEGMENT AB:

$$\phi_{AB} = \frac{32\, T_{AB} L_{AB}}{\pi G d^4} = \frac{32(1492\text{ N·m})(1.5\text{ m})}{\pi(75\text{ GPa})\, d^4}$$

$$\phi_{AB} = \frac{0.3039\times10^{-6}}{d^4}$$

SEGMENT BC:

$$\phi_{BC} = \frac{32\, T_{BC} L_{BC}}{\pi G d^4} = \frac{32(895.3\text{ N·m})(0.9\text{ m})}{\pi(75\text{ GPa})\, d^4}$$

$$\phi_{BC} = \frac{0.1094\times10^{-6}}{d^4}$$

FROM A TO C:

$$\phi_{AC} = \phi_{AB} + \phi_{BC} = \frac{0.4133\times10^{-6}}{d^4}$$

$(\phi_{AC})_{allow} = 0.06981$ rad

$\therefore\ 0.06981 = \dfrac{0.4133\times10^{-6}}{d^4}$

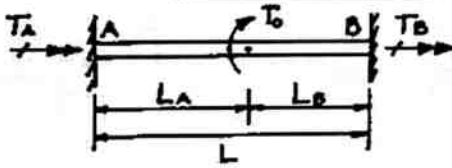
AND $d = 0.04933$ m $= 49.3$ mm

SHEAR STRESS GOVERNS

$d = 53.4$ mm ⟵

3.8-1

CIRCULAR BAR WITH FIXED ENDS

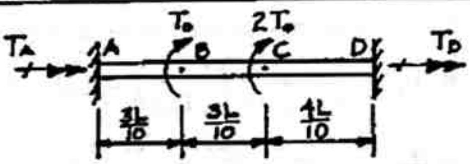


FROM EQS. (3-46 a AND b):

$$T_A = \frac{T_0 L_B}{L}$$

$$T_B = \frac{T_0 L_A}{L}$$

APPLY THE ABOVE FORMULAS TO THE GIVEN BAR:



$$T_A = T_0\left(\frac{7}{10}\right) + 2T_0\left(\frac{4}{10}\right) = \frac{15 T_0}{10}$$

$$T_D = T_0\left(\frac{3}{10}\right) + 2T_0\left(\frac{6}{10}\right) = \frac{15 T_0}{10}$$

ANGLE OF TWIST AT SECTION B

$$\phi_B = \phi_{AB} = \frac{T_A(3L/10)}{G I_p} = \frac{9 T_0 L}{20 G I_p}$$

ANGLE OF TWIST AT SECTION C

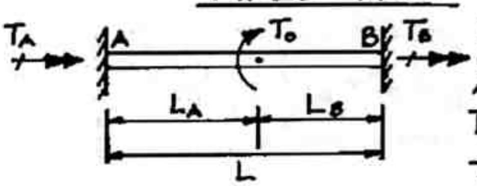
$$\phi_C = \phi_{CD} = \frac{T_D(4L/10)}{G I_p} = \frac{3 T_0 L}{5 G I_p}$$

MAXIMUM ANGLE OF TWIST

$$\phi_{max} = \phi_C = \frac{3 T_0 L}{5 G I_p} \longleftarrow$$

3.8-2

CIRCULAR BAR WITH FIXED ENDS

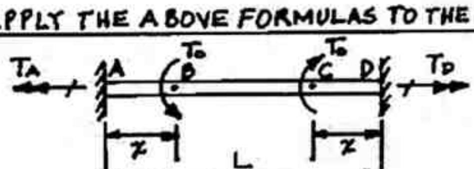


FROM EQS. (3-46 a AND b):

$$T_A = \frac{T_0 L_B}{L}$$

$$T_B = \frac{T_0 L_A}{L}$$

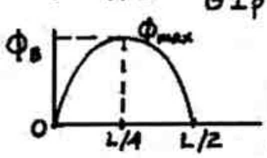
APPLY THE ABOVE FORMULAS TO THE GIVEN BAR:



$$T_A = \frac{T_0(L-x)}{L} - \frac{T_0 x}{L} = \frac{T_0}{L}(L-2x) \quad T_D = T_A$$

(a) ANGLE OF TWIST AT SECTIONS B AND C

$$\phi_B = \phi_{AB} = \frac{T_A x}{G I_p} = \frac{T_0}{G I_p L}(L-2x)(x)$$



$$\frac{d\phi_B}{dx} = \frac{T_0}{G I_p L}(L-4x)$$

$$\frac{d\phi_B}{dx} = 0; \; L - 4x = 0$$

$$\text{OR} \quad x = \frac{L}{4} \longleftarrow$$

CONT.

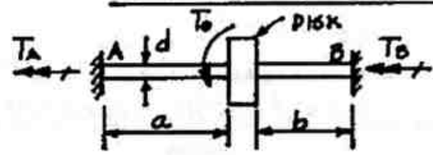
3.8-2 CONT.

(b) MAXIMUM ANGLE OF TWIST

$$\phi_{max} = (\phi_B)_{max} = (\phi_B)_{x=\frac{L}{4}} = \frac{T_0 L}{8 G I_p} \longleftarrow$$

3.8-3

SHAFT FIXED AT BOTH ENDS



$L = a + b$

$a > b$

ASSUME THAT A TORQUE T_0 ACTS AT THE DISK.

THE REACTIVE TORQUES CAN BE OBTAINED FROM EQS. (3-46 a AND b): $T_A = \frac{T_0 b}{L}$ $\quad T_B = \frac{T_0 a}{L}$

SINCE $a > b$, THE LARGER TORQUE (AND HENCE THE LARGER STRESS) IS IN THE RIGHT HAND SEGMENT.

$$\tau_{max} = \frac{T_B(d/2)}{I_p} = \frac{T_0 a d}{2 L I_p}$$

$$T_0 = \frac{2 L I_p \tau_{max}}{a d} \qquad (T_0)_{max} = \frac{2 L I_p \tau_{allow}}{a d}$$

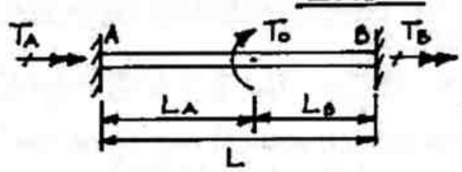
ANGLE OF ROTATION OF THE DISK (FROM EQ 3-49)

$$\phi = \frac{T_0 a b}{G L I_p}$$

$$\phi_{max} = \frac{(T_0)_{max}\, a b}{G L I_p} = \frac{2 b \tau_{allow}}{G d} \longleftarrow$$

3.8-4

HOLLOW SHAFT WITH FIXED ENDS

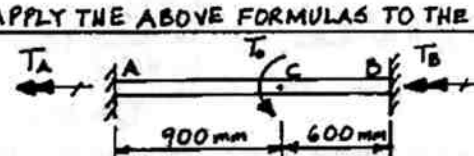


FROM EQS. (3-46 a AND b):

$$T_A = \frac{T_0 L_B}{L}$$

$$T_B = \frac{T_0 L_A}{L}$$

APPLY THE ABOVE FORMULAS TO THE GIVEN SHAFT:



$T_0 = P(400\text{ mm})$

$L_B = 600\text{ mm}$

$L_A = 900\text{ mm}$

$d_2 = 50\text{ mm} \quad d_1 = 40\text{ mm}$ $\qquad L = L_A + L_B = 1500\text{ mm}$

$\tau_{allow} = 55\text{ MPa}$

$$T_A = \frac{T_0 L_B}{L} = \frac{P(0.4\text{m})(0.6\text{m})}{1.5\text{m}} = 0.16P$$

$$T_B = \frac{T_0 L_A}{L} = \frac{P(0.4\text{m})(0.9\text{m})}{1.5\text{m}} = 0.24P$$

UNITS: P = NEWTONS $\quad T$ = NEWTON METERS

THE LARGER TORQUE, AND HENCE THE LARGER SHEAR STRESS, OCCURS IN PART CB OF THE SHAFT.

$$T_{max} = T_B = 0.24P$$

CONT.

3.8-4 CONT.

SHEAR STRESS IN PART CB

$\tau_{max} = \frac{T_{max}(d/2)}{I_p}$ $T_{max} = \frac{2\tau_{max}I_p}{d}$ (1)

UNITS: NEWTONS AND METERS

$\tau_{max} = 55\times10^6\ N/m^2$ $I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = 362.26\times10^{-9}\ m^4$

$d = d_2 = 0.05\,m$

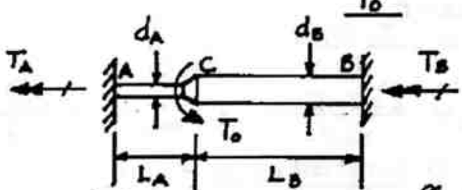
SUBSTITUTE NUMERICAL VALUES INTO EQ. (1):

$$0.24P = \frac{2(55\times10^6\ N/m^2)(362.26\times10^{-9}\ m^4)}{0.05\,m} = 796.98\ N\cdot m$$

$P = 3320\ N$ ⟵

3.8-5

STEPPED SHAFT WITH TORQUE T_0



$d_A = 0.8$ in.
$d_B = 1.6$ in.
$L_A = 8.0$ in.
$L_B = 20.0$ in.
$\tau_{allow} = 8000$ psi

FIND $(T_0)_{max}$

REACTIVE TORQUES (FROM EQS. 3-45a AND b)

$$T_A = T_0\left(\frac{L_B I_{PA}}{L_B I_{PA} + L_A I_{PB}}\right) \quad (1)$$

$$T_B = T_0\left(\frac{L_A I_{PB}}{L_B I_{PA} + L_A I_{PB}}\right) \quad (2)$$

ALLOWABLE TORQUE BASED UPON SHEAR STRESS IN SEGMENT AC

$$\tau_{AC} = \frac{16T_A}{\pi d_A^3} \quad T_A = \frac{1}{16}\pi d_A^3 \tau_{AC} = \frac{1}{16}\pi d_A^3 \tau_{allow} \quad (3)$$

COMBINE EQS. (1) AND (3) AND SOLVE FOR T_0:

$$T_0 = \frac{1}{16}\pi d_A^3 \tau_{allow}\left(1 + \frac{L_A I_{PB}}{L_B I_{PA}}\right)$$
$$= \frac{1}{16}\pi d_A^3 \tau_{allow}\left(1 + \frac{L_A d_B^4}{L_B d_A^4}\right) \quad (4)$$

SUBSTITUTE NUMERICAL VALUES:

$T_0 = 5950$ lb-in.

ALLOWABLE TORQUE BASED UPON SHEAR STRESS IN SEGMENT CB

$$\tau_{CB} = \frac{16T_B}{\pi d_B^3} \quad T_B = \frac{1}{16}\pi d_B^3 \tau_{CB} = \frac{1}{16}\pi d_B^3 \tau_{allow} \quad (5)$$

COMBINE EQS. (2) AND (5) AND SOLVE FOR T_0:

$$T_0 = \frac{1}{16}\pi d_B^3 \tau_{allow}\left(1 + \frac{L_B I_{PA}}{L_A I_{PB}}\right)$$
$$= \frac{1}{16}\pi d_B^3 \tau_{allow}\left(1 + \frac{L_B d_A^4}{L_A d_B^4}\right) \quad (6)$$

SUBSTITUTE NUMERICAL VALUES:

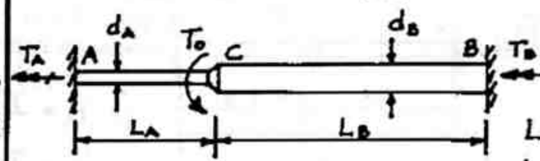
$T_0 = 7440$ lb-in.

SEGMENT AC GOVERNS

$(T_0)_{max} = 5950$ lb-in ⟵

3.8-6

STEPPED SHAFT WITH TORQUE T_0



$d_A = 16$ mm
$d_B = 20$ mm
$L_A = 125$ mm
$L_B = 250$ mm
$\tau_{allow} = 60$ MPa

FIND $(T_0)_{max}$

REACTIVE TORQUES (FROM EQS. 3-45a AND b)

$$T_A = T_0\left(\frac{L_B I_{PA}}{L_B I_{PA} + L_A I_{PB}}\right) \quad (1)$$

$$T_B = T_0\left(\frac{L_A I_{PB}}{L_B I_{PA} + L_A I_{PB}}\right) \quad (2)$$

ALLOWABLE TORQUE BASED UPON SHEAR STRESS IN SEGMENT AC

$$\tau_{AC} = \frac{16T_A}{\pi d_A^3} \quad T_A = \frac{1}{16}\pi d_A^3 \tau_{AC} = \frac{1}{16}\pi d_A^3 \tau_{allow} \quad (3)$$

COMBINE EQS. (1) AND (3) AND SOLVE FOR T_0:

$$T_0 = \frac{1}{16}\pi d_A^3 \tau_{allow}\left(1 + \frac{L_A I_{PB}}{L_B I_{PA}}\right)$$
$$= \frac{1}{16}\pi d_A^3 \tau_{allow}\left(1 + \frac{L_A d_B^4}{L_B d_A^4}\right) \quad (4)$$

SUBSTITUTE NUMERICAL VALUES:

$T_0 = 107\ N\cdot m$

ALLOWABLE TORQUE BASED UPON SHEAR STRESS IN SEGMENT CB

$$\tau_{CB} = \frac{16T_B}{\pi d_B^3} \quad T_B = \frac{1}{16}\pi d_B^3 \tau_{CB} = \frac{1}{16}\pi d_B^3 \tau_{allow} \quad (5)$$

COMBINE EQS. (2) AND (5) AND SOLVE FOR T_0:

$$T_0 = \frac{1}{16}\pi d_B^3 \tau_{allow}\left(1 + \frac{L_B I_{PA}}{L_A I_{PB}}\right)$$
$$= \frac{1}{16}\pi d_B^3 \tau_{allow}\left(1 + \frac{L_B d_A^4}{L_A d_B^4}\right) \quad (6)$$

SUBSTITUTE NUMERICAL VALUES:

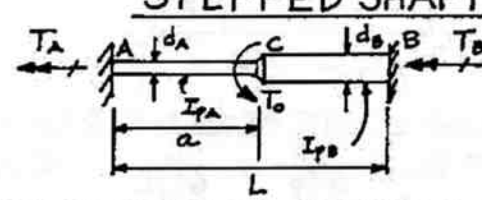
$T_0 = 171\ N\cdot m$

SEGMENT AC GOVERNS

$(T_0)_{max} = 107\ N\cdot m$ ⟵

3.8-7

STEPPED SHAFT



REACTIVE TORQUES (FROM EQS. 3-45a AND b)

$$T_A = T_0\left(\frac{L_B I_{PA}}{L_B I_{PA} + L_A I_{PB}}\right); \quad T_B = T_0\left(\frac{L_A I_{PB}}{L_B I_{PA} + L_A I_{PB}}\right)$$

IN WHICH $L_A = a$ AND $L_B = L - a$

CONT.

3.8-7 CONT.

(a) EQUAL SHEAR STRESSES

$$\tau_{AC} = \frac{T_A(d_A/2)}{I_{PA}} \qquad \tau_{CB} = \frac{T_B(d_B/2)}{I_{PB}}$$

$$\tau_{AC} = \tau_{CB} \quad \text{OR} \quad \frac{T_A d_A}{I_{PA}} = \frac{T_B d_B}{I_{PB}}$$

SUBSTITUTE FOR T_A AND T_B AND SIMPLIFY:

$$\frac{L_B I_{PA} d_A}{I_{PA}} = \frac{L_A I_{PB} d_B}{I_{PB}} \quad \text{OR} \quad (L-a)\,d_A = a\,d_B$$

SOLVE FOR a/L: $\quad \dfrac{a}{L} = \dfrac{d_A}{d_A + d_B}$ ⟵

(b) EQUAL TORQUES

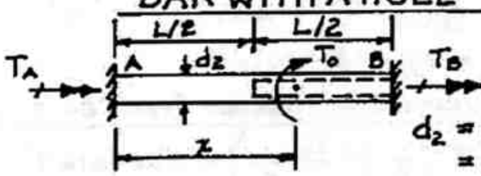
$$T_A = T_B \quad \text{OR} \quad L_B I_{PA} = L_A I_{PB}$$

OR $(L-a)\,I_{PA} = a\,I_{PB}$

SOLVE FOR a/L: $\quad \dfrac{a}{L} = \dfrac{I_{PA}}{I_{PA}+I_{PB}}$ ⟵

3.8-8

BAR WITH A HOLE



$L = 1250$ mm

$L/2 = 625$ mm

d_2 = OUTER DIAMETER = 100 mm

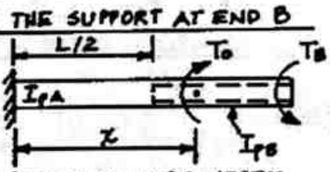
d_1 = DIAMETER OF HOLE = 80 mm

T_0 = TORQUE APPLIED AT DISTANCE x

FIND x SO THAT $T_A = T_B$

EQUILIBRIUM $\quad T_A + T_B = T_0 \quad \therefore \quad T_A = T_B = \dfrac{T_0}{2} \qquad (1)$

REMOVE THE SUPPORT AT END B



ϕ_B = ANGLE OF TWIST AT B

I_{PA} = POLAR MOMENT OF INERTIA AT LEFT-HAND END

I_{PB} = POLAR MOMENT OF INERTIA AT RIGHT-HAND END

$$\phi_B = \frac{T_B(L/2)}{G I_{PB}} + \frac{T_B(L/2)}{G I_{PA}} - \frac{T_0(x-L/2)}{G I_{PB}} - \frac{T_0(L/2)}{G I_{PA}} \qquad (2)$$

COMPATIBILITY $\quad \phi_B = 0 \qquad (3)$

SUBSTITUTE EQS. (1) AND (2) INTO EQ. (3) AND SIMPLIFY:

$$\frac{L}{4I_{PB}} + \frac{L}{4I_{PA}} - \frac{x}{I_{PB}} + \frac{L}{2I_{PB}} - \frac{L}{2I_{PA}} = 0$$

OR

$$\frac{x}{I_{PB}} = \frac{3L}{4I_{PB}} - \frac{L}{4I_{PA}} \qquad (4)$$

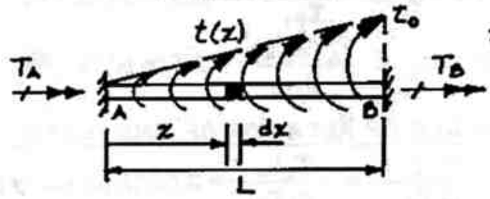
SOLVE FOR x: $\quad x = \dfrac{L}{4}\left(3 - \dfrac{I_{PB}}{I_{PA}}\right) \qquad (5)$

$$\frac{I_{PB}}{I_{PA}} = \frac{d_2^4 - d_1^4}{d_2^4} = 1 - \left(\frac{d_1}{d_2}\right)^4 = 1 - \left(\frac{80}{100}\right)^4 = 0.5904$$

$$x = \frac{1250\text{ mm}}{4}(3 - 0.5904) = 753\text{ mm}$$ ⟵

3.8-9

FIXED-END BAR WITH TRIANGULAR LOAD



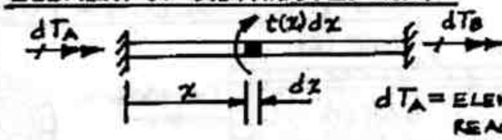
$$t(x) = \frac{t_0 x}{L}$$

T_0 = RESULTANT OF DISTRIBUTED TORQUE

$$T_0 = \int_0^L t(x)\,dx = \int_0^L \frac{t_0 x}{L}\,dx = \frac{t_0 L}{2}$$

EQUILIBRIUM $\quad T_A + T_B = T_0 = \dfrac{t_0 L}{2}$

ELEMENT OF DISTRIBUTED LOAD



dT_A = ELEMENTAL REACTIVE TORQUE

dT_B = ELEMENTAL REACTIVE TORQUE

FROM EQS. (3-46 a AND b):

$$dT_A = t(x)\,dx\left(\frac{L-x}{L}\right) \qquad dT_B = t(x)\,dx\left(\frac{x}{L}\right)$$

REACTIVE TORQUES (FIXED-END TORQUES)

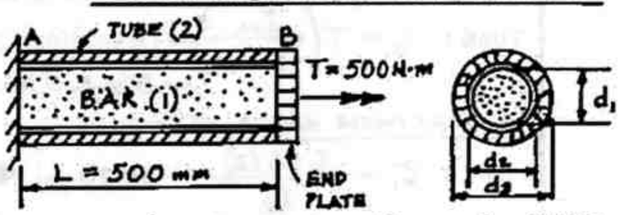
$$T_A = \int dT_A = \int_0^L \left(t_0\frac{x}{L}\right)\left(\frac{L-x}{L}\right)dx = \frac{t_0 L}{6}$$ ⟵

$$T_B = \int dT_B = \int_0^L \left(t_0\frac{x}{L}\right)\left(\frac{x}{L}\right)dx = \frac{t_0 L}{3}$$ ⟵

NOTE: $\quad T_A + T_B = \dfrac{t_0 L}{2} \quad$ (CHECK)

3.8-10

BAR ENCLOSED IN A TUBE



$d_1 = 30$ mm $\quad d_2 = 36$ mm $\quad d_3 = 45$ mm $\quad G = 80$ GPa

POLAR MOMENT OF INERTIA

BAR: $\quad I_{P1} = \dfrac{\pi}{32} d_1^4 = 79.522 \times 10^{-9}\ \text{m}^4$

TUBE: $\quad I_{P2} = \dfrac{\pi}{32}(d_3^4 - d_2^4) = 237.68 \times 10^{-9}\ \text{m}^4$

TORQUES IN THE BAR (1) AND TUBE (2) FROM EQS. (3-44 a AND b)

$$\text{BAR: } T_1 = T\left(\frac{I_{P1}}{I_{P1}+I_{P2}}\right) = (500\text{ N·m})(0.25070) = 125.35\text{ N·m}$$

$$\text{TUBE: } T_2 = T\left(\frac{I_{P2}}{I_{P1}+I_{P2}}\right) = (500\text{ N·m})(0.74930) = 374.65\text{ N·m}$$

CONT.

3.8-10 CONT.

(a) MAXIMUM SHEAR STRESSES

BAR: $\tau_1 = \dfrac{T_1(d_1/2)}{I_{P_1}} = 23.6\ \text{MPa}$ ←

TUBE: $\tau_2 = \dfrac{T_2(d_3/2)}{I_{P_2}} = 35.5\ \text{MPa}$ ←

(b) ANGLE OF ROTATION OF END PLATE

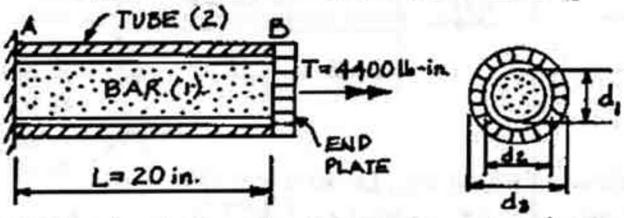
$$\phi = \frac{T_1 L}{G I_{P_1}} = \frac{T_2 L}{G I_{P_2}} = 0.009852\ \text{rad} = 0.564^\circ$$ ←

(c) TORSIONAL STIFFNESS

$$k_T = \frac{T}{\phi} = \frac{500\ \text{N}\cdot\text{m}}{0.009852\ \text{rad}} = 50.8\ \text{kN}\cdot\text{m}$$ ←

3.8-11

BAR ENCLOSED IN A TUBE



$d_1 = 1.2$ in. $d_2 = 1.4$ in. $d_3 = 1.8$ in. $G = 11.6\times10^6$ psi

POLAR MOMENTS OF INERTIA

BAR: $I_{P_1} = \frac{\pi}{32} d_1^4 = 0.20358\ \text{in.}^4$

TUBE: $I_{P_2} = \frac{\pi}{32}(d_3^4 - d_2^4) = 0.65345\ \text{in.}^4$

TORQUES IN THE BAR (1) AND TUBE (2) FROM EQS. (3-44 a AND b)

BAR: $T_1 = T\left(\dfrac{I_{P_1}}{I_{P_1}+I_{P_2}}\right) = (4400)(0.23754)$ lb-in. $= 1045.2$ lb-in.

TUBE: $T_2 = T\left(\dfrac{I_{P_2}}{I_{P_1}+I_{P_2}}\right) = (4400)(0.76246)$ lb-in. $= 3354.8$ lb-in.

(a) MAXIMUM SHEAR STRESSES

BAR: $\tau_1 = \dfrac{T_1(d_1/2)}{I_{P_1}} = 3080$ psi ←

TUBE: $\tau_2 = \dfrac{T_2(d_3/2)}{I_{P_2}} = 4620$ psi ←

(b) ANGLE OF ROTATION OF END PLATE

$$\phi = \frac{T_1 L}{G I_{P_1}} = \frac{T_2 L}{G I_{P_2}} = 0.008852\ \text{rad}$$

$\phi = 0.507^\circ$ ←

(c) TORSIONAL STIFFNESS

$$k_T = \frac{T}{\phi} = 497\ \text{k-in.}$$ ←

3.8-12

COMPOSITE SHAFT

STEEL SLEEVE, BRASS CORE

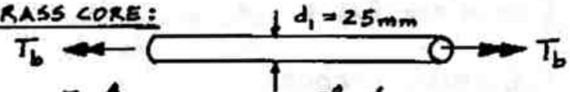
$d_1 = 25$ mm
$d_2 = 40$ mm

$G_b = 39$ GPa $G_s = 75$ GPa

ALLOWABLE STRESSES: $\tau_b = 30$ MPa $\tau_s = 50$ MPa

ISOLATE THE BRASS CORE AND THE STEEL SLEEVE

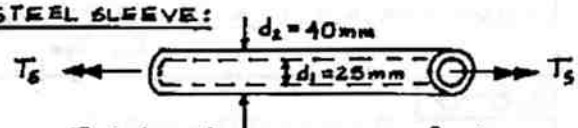
BRASS CORE:



$I_{Pb} = \frac{\pi}{32} d_1^4 = 38.350\times10^{-9}\ \text{m}^4$

$G_b I_{Pb} = 1495.63\ \text{N}\cdot\text{m}^2$

STEEL SLEEVE:



$I_{Ps} = \frac{\pi}{32}(d_2^4 - d_1^4) = 212.98\times10^{-9}\ \text{m}^4$

$G_s I_{Ps} = 15{,}973.3\ \text{N}\cdot\text{m}^2$

CALCULATION OF TORQUES (FROM EQ. 3-44a)

$T_b = T\left(\dfrac{G_b I_{Pb}}{G_b I_{Pb} + G_s I_{Ps}}\right) = 0.085616\,T$

$T_s = T - T_b = 0.914384\,T$

ALLOWABLE TORQUES BASED UPON SHEAR STRESSES

BRASS CORE: $\tau_b = \dfrac{T_b(d_1/2)}{I_{Pb}}$ $T_b = \dfrac{(\tau_b)_{allow} I_{Pb}}{d_1/2}$

$0.085616\,T = \dfrac{(30\ \text{MPa})(38.350\times10^{-9}\ \text{m}^4)}{(25\ \text{mm})/2} = 92.039\ \text{N}\cdot\text{m}$

$T = 1075\ \text{N}\cdot\text{m}$

STEEL SLEEVE: $\tau_s = \dfrac{T_s(d_2/2)}{I_{Ps}}$ $T_s = \dfrac{(\tau_s)_{allow} I_{Ps}}{d_2/2}$

$0.914384\,T = \dfrac{(50\ \text{MPa})(212.98\times10^{-9}\ \text{m}^4)}{(40\ \text{mm})/2} = 532.44\ \text{N}\cdot\text{m}$

$T = 582\ \text{N}\cdot\text{m}$

STEEL SLEEVE GOVERNS

$T_{max} = 582\ \text{N}\cdot\text{m}$ ←

3.8-13

COMPOSITE SHAFT

STEEL SLEEVE, BRASS CORE

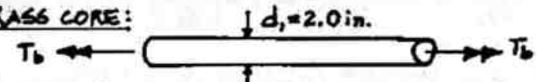
$d_1 = 2.0$ in.
$d_2 = 2.4$ in.

$G_b = 5600$ ksi
$G_s = 11{,}500$ ksi

ALLOWABLE STRESSES:
$\tau_b = 4500$ psi
$\tau_s = 7500$ psi

ISOLATE THE BRASS CORE AND THE STEEL SLEEVE

BRASS CORE:



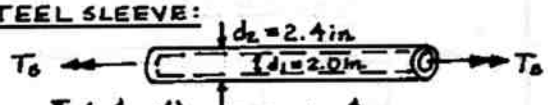
$I_{Pb} = \frac{\pi}{32} d_1^4 = 1.5708\ \text{in.}^4$

$G_b I_{Pb} = 8.7965\times10^6\ \text{lb-in.}^2$

CONT.

3.8-13 CONT.

STEEL SLEEVE:



$I_{ps} = \frac{\pi}{32}(d_2^4 - d_1^4) = 1.6864 \text{ in}^4$

$G_s I_{ps} = 19.394 \times 10^6 \text{ lb-in}^2$

CALCULATION OF TORQUES (FROM EQ. 3-44a)

$$T_b = T\left(\frac{G_b I_{pb}}{G_b I_{pb} + G_s I_{ps}}\right) = 0.31204\,T$$

$$T_s = T - T_b = 0.68796\,T$$

ALLOWABLE TORQUES BASED UPON SHEAR STRESSES

BRASS CORE: $\tau_b = \frac{T_b(d_1/2)}{I_{pb}}$ $\quad T_b = \frac{(\tau_b)_{allow} I_{pb}}{d_1/2}$

$$0.31204\,T = \frac{(4500 \text{ psi})(1.5708 \text{ in}^4)}{1.0 \text{ in.}} = 7068.6 \text{ lb-in.}$$

$$T = 22.7 \text{ k-in.}$$

STEEL SLEEVE: $\tau_s = \frac{T_s(d_2/2)}{I_{ps}}$ $\quad T_s = \frac{(\tau_s)_{allow} I_{ps}}{d_2/2}$

$$0.68796\,T = \frac{(7500 \text{ psi})(1.6864 \text{ in}^4)}{1.2 \text{ in.}} = 10{,}540 \text{ lb-in.}$$

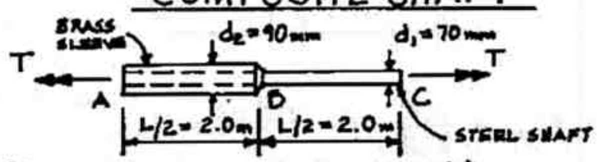
$$T = 15.3 \text{ k-in.}$$

STEEL SLEEVE GOVERNS

$$T_{max} = 15.3 \text{ k-in.}$$

3.8-14

COMPOSITE SHAFT



PROPERTIES OF THE STEEL SHAFT (s)

$G_s = 80$ GPa $\quad d_1 = 70$ mm

ALLOWABLE SHEAR STRESS: $\tau_s = 110$ MPa

$I_{ps} = \frac{\pi}{32} d_1^4 = 2.3572 \times 10^{-6} \text{ m}^4$

$G_s I_{ps} = 188.574 \times 10^3 \text{ N}\cdot\text{m}^2$

PROPERTIES OF THE BRASS SLEEVE (b)

$G_b = 40$ GPa $\quad d_2 = 90$ mm $\quad d_1 = 70$ mm

ALLOWABLE SHEAR STRESS: $\tau_b = 70$ MPa

$I_{pb} = \frac{\pi}{32}(d_2^4 - d_1^4) = 4.0841 \times 10^{-6} \text{ m}^4$

$G_b I_{pb} = 163.363 \times 10^3 \text{ N}\cdot\text{m}^2$

TORQUES IN THE COMPOSITE BAR AB

T_s = TORQUE IN THE STEEL SHAFT AB

T_b = TORQUE IN THE BRASS SLEEVE AB

FROM EQ. (3-44a): $T_s = T\left(\frac{G_s I_{ps}}{G_s I_{ps} + G_b I_{pb}}\right)$

$$T_s = T\,(0.53582) \qquad \text{EQ. (1)}$$

$$T_b = T - T_s = T\,(0.46418) \qquad \text{EQ. (2)}$$

CONT.

3.8-14 CONT.

ANGLE OF TWIST OF THE COMPOSITE BAR AB

$$\phi_{AB} = \frac{T_s(L/2)}{G_s I_{ps}} = \frac{T_b(L/2)}{G_b I_{pb}} = (5.6828 \times 10^{-6})\,T \qquad \text{EQ. (3)}$$

UNITS: $T = \text{N}\cdot\text{m}$ $\quad \phi = \text{rad}$

ANGLE OF TWIST OF PART BC OF THE STEEL SHAFT

$$\phi_{BC} = \frac{T(L/2)}{G_s I_{ps}} = (10.6059 \times 10^{-6})\,T \qquad \text{EQ. (4)}$$

ANGLE OF TWIST OF THE ENTIRE SHAFT ABC

$$\phi = \phi_{AB} + \phi_{BC} \quad \text{(EQS. 3 AND 4)}$$

$$\phi = (16.2887 \times 10^{-6})\,T \quad \text{UNITS: } \phi = \text{rad} \quad T = \text{N}\cdot\text{m}$$

(a) ALLOWABLE TORQUE T_1 BASED UPON ANGLE OF TWIST

$$\phi_{allow} = 8.0° = 0.13963 \text{ rad}$$

$$\phi = (16.2887 \times 10^{-6})\,T = 0.13963 \text{ rad}$$

$$T_1 = 8.57 \text{ kN}\cdot\text{m}$$

(b) ALLOWABLE TORQUE T_2 BASED UPON SHEAR STRESS IN THE BRASS SLEEVE

$\tau_b = \frac{T_b(d_2/2)}{I_{pb}}$ $\quad \tau_b = 70$ MPa $\quad T_b = 0.46418\,T$ (FROM EQ. 2)

$$70 \text{ MPa} = \frac{(0.46418\,T)(0.045 \text{ m})}{4.0841 \times 10^{-6} \text{ m}^4}$$

SOLVE FOR T (EQUAL TO T_2): $\quad T_2 = 13.69 \text{ kN}\cdot\text{m}$

(c) ALLOWABLE TORQUE T_3 BASED UPON SHEAR STRESS IN THE STEEL SHAFT BC

$\tau_s = \frac{T(d_1/2)}{I_{ps}}$ $\quad \tau_s = 110$ MPa

$$110 \text{ MPa} = \frac{T(0.035 \text{ m})}{2.3572 \times 10^{-6} \text{ m}^4}$$

SOLVE FOR T (EQUAL TO T_3): $\quad T_3 = 7.41 \text{ kN}\cdot\text{m}$

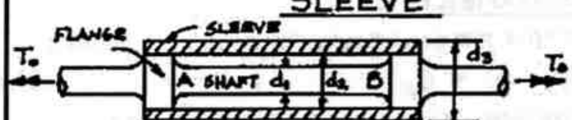
(d) MAXIMUM ALLOWABLE TORQUE

SHEAR STRESS IN STEEL GOVERNS

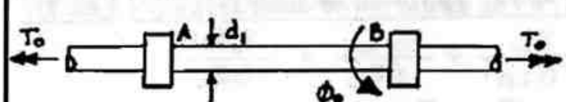
$$T_{max} = 7.41 \text{ kN}\cdot\text{m}$$

3.8-15

FLANGED SHAFT AB WITH SLEEVE



ANGLE OF TWIST OF SHAFT AB WHEN TORQUES T_0 ARE ACTING AND SLEEVE IS NOT YET IN PLACE



L = LENGTH OF SHAFT AB $\quad$ G = SHEAR MODULUS

$$\phi_0 = \frac{T_0 L}{G I_{p1}} \qquad \text{EQ. (1)}$$

$$I_{p1} = \frac{\pi}{32} d_1^4 \qquad \text{EQ. (2)}$$

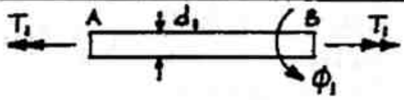
SLEEVE IS NOW ATTACHED

THEN THE TORQUES T_0 ARE REMOVED AND THE SHAFT PARTIALLY UNWINDS. THE SYSTEM REACHES EQUILIBRIUM WITH NO EXTERNAL TORQUES.

CONT.

3.8-15 CONT.

FREE-BODY DIAGRAM OF SHAFT AB

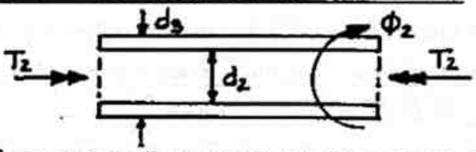


T_1 = RESIDUAL TORQUE ACTING ON THE SHAFT

ϕ_1 = RESIDUAL ANGLE OF TWIST OF THE SHAFT

$$\phi_1 = \frac{T_1 L}{G I_{p1}} \qquad \text{EQ. (3)}$$

FREE-BODY DIAGRAM OF SLEEVE



T_2 = TORQUE ACTING ON THE SLEEVE

ϕ_2 = ANGLE OF TWIST OF THE SLEEVE

NOTE THAT THE DIRECTION OF ϕ_2 IS OPPOSITE TO THE DIRECTION OF ϕ_1, AND THE DIRECTION OF T_2 IS OPPOSITE TO THE DIRECTION OF T_1.

$$\phi_2 = \frac{T_2 L}{G I_{p2}} \qquad \text{EQ. (4)}$$

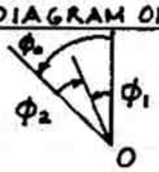
$$I_{p2} = \frac{\pi}{32}(d_3^4 - d_2^4) \qquad \text{EQ. (5)}$$

EQUILIBRIUM

SINCE THERE ARE NO EXTERNAL TORQUES, THE TORQUES T_1 AND T_2 ARE EQUAL IN MAGNITUDE AND OPPOSITE IN DIRECTION.

$$T_1 = T_2 \qquad \text{EQ. (6)}$$

DIAGRAM OF ANGLES OF TWIST



ϕ_o = INITIAL TWIST OF THE SHAFT DUE TO TORQUES T_o (EQ. 1)

ϕ_2 = TWIST OF THE SLEEVE AS THE SYSTEM PARTIALLY UNWINDS (EQ. 4)

ϕ_1 = RESIDUAL TWIST OF THE SHAFT (EQ. 3)

COMPATIBILITY

FROM THE DIAGRAM OF ANGLES OF TWIST:

$$\phi_1 + \phi_2 = \phi_o \qquad \text{EQ. (7)}$$

TORQUE-DISPLACEMENT RELATIONS

SEE EQS. (1), (3), AND (4)

SUBSTITUTE THE TORQUE-DISPLACEMENT RELATIONS INTO THE EQUATION OF COMPATIBILITY (EQ. 7):

$$\frac{T_1 L}{G I_{p1}} + \frac{T_2 L}{G I_{p2}} = \frac{T_o L}{G I_{p1}}$$

OR $$\frac{T_1}{I_{p1}} + \frac{T_2}{I_{p2}} = \frac{T_o}{I_{p1}} \qquad \text{EQ. (8)}$$

SOLVE SIMULTANEOUSLY EQS. (6) AND (8):

$$T_1 = T_2 = \frac{T_o I_{p2}}{I_{p1} + I_{p2}} \qquad \text{EQ. (9)}$$

MAXIMUM SHEAR STRESS REMAINING IN THE SHAFT

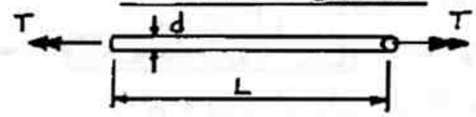
$$\tau_{max} = \frac{T_1 (d_1/2)}{I_{p1}} = \frac{T_o I_{p2} d_1}{2(I_{p1} + I_{p2}) I_{p1}}$$

SUBSTITUTE FROM EQS. (2) AND (5):

$$\tau_{max} = \frac{16 T_o (d_3^4 - d_2^4)}{\pi d_1^3 (d_1^4 + d_3^4 - d_2^4)} \qquad \text{EQ. (10)}$$

3.9-1

COPPER BAR



$G = 6.0 \times 10^6$ psi

$L = 30$ in.

$d = 1.5$ in.

$\tau_{max} = 4000$ psi

$$\tau_{max} = \frac{16T}{\pi d^3} \qquad T = \frac{\pi d^3 \tau_{max}}{16} \qquad \text{EQ. (1)}$$

$$I_p = \frac{\pi d^4}{32}$$

(a) STRAIN ENERGY

$$U = \frac{T^2 L}{2 G I_p} = \left(\frac{\pi d^3 \tau_{max}}{16}\right)^2 \left(\frac{L}{2G}\right)\left(\frac{32}{\pi d^4}\right)$$

$$= \frac{\pi d^2 L \tau_{max}^2}{16 G} \qquad \text{EQ. (2)}$$

SUBSTITUTE NUMERICAL VALUES:

$U = 35.3$ in.-lb ←

(b) ANGLE OF TWIST

$$U = \frac{T\phi}{2} \qquad \phi = \frac{2U}{T}$$

SUBSTITUTE FOR T AND U FROM EQS. (1) AND (2):

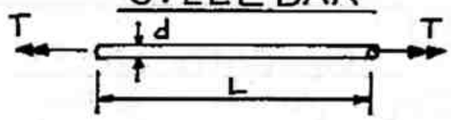
$$\phi = \frac{2 L \tau_{max}}{G d}$$

SUBSTITUTE NUMERICAL VALUES:

$\phi = 0.02667$ rad $= 1.53°$ ←

3.9-2

STEEL BAR



$G = 80$ GPa

$L = 1.5$ m

$d = 75$ mm

$\tau_{max} = 45$ MPa

$$\tau_{max} = \frac{16T}{\pi d^3} \qquad T = \frac{\pi d^3 \tau_{max}}{16} \qquad \text{EQ. (1)}$$

$$I_p = \frac{\pi d^4}{32}$$

(a) STRAIN ENERGY

$$U = \frac{T^2 L}{2 G I_p} = \left(\frac{\pi d^3 \tau_{max}}{16}\right)^2 \left(\frac{L}{2G}\right)\left(\frac{32}{\pi d^4}\right)$$

$$= \frac{\pi d^2 L \tau_{max}^2}{16 G} \qquad \text{EQ. (2)}$$

SUBSTITUTE NUMERICAL VALUES:

$U = 41.9$ J ←

(b) ANGLE OF TWIST

$$U = \frac{T\phi}{2} \qquad \phi = \frac{2U}{T}$$

SUBSTITUTE FOR T AND U FROM EQS. (1) AND (2):

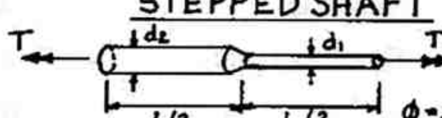
$$\phi = \frac{2 L \tau_{max}}{G d}$$

SUBSTITUTE NUMERICAL VALUES:

$\phi = 0.0225$ rad $= 1.29°$ ←

3.9-3

STEPPED SHAFT



$d_1 = 1.5$ in. $d_2 = 2.0$ in.

$L = 80$ in.

$G = 11.8 \times 10^6$ psi

$\phi = 2.5°$

CONT.

3.9-3 CONT.

STRAIN ENERGY

$$U = \sum \frac{T^2 L}{2GI_p} = \frac{16T^2(L/2)}{\pi G d_2^4} + \frac{16T^2(L/2)}{\pi G d_1^4}$$

$$= \frac{8T^2L}{\pi G}\left(\frac{1}{d_2^4} + \frac{1}{d_1^4}\right) \quad \text{EQ. (1)}$$

ALSO, $U = \frac{T\phi}{2}$ EQ. (2)

EQUATE U FROM EQS. (1) AND (2) AND SOLVE FOR T:

$$T = \frac{\pi G d_1^4 d_2^4 \phi}{16L(d_1^4 + d_2^4)}$$

$$U = \frac{T\phi}{2} = \frac{\pi G \phi^2}{32L}\left(\frac{d_1^4 d_2^4}{d_1^4 + d_2^4}\right)$$

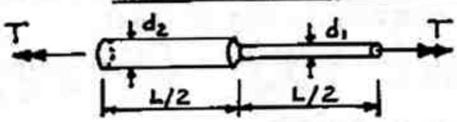
SUBSTITUTE NUMERICAL VALUES:

(WHERE $\phi = 2.5° = 0.043633$ rad)

$U = 106.0$ in.-lb ⟵

3.9-4

STEPPED SHAFT



$d_1 = 25$ mm
$d_2 = 30$ mm
$L = 1.1$ m

$G = 40$ GPa $\quad \phi = 3.5° = 0.061087$ rad

STRAIN ENERGY

$$U = \sum \frac{T^2L}{2GI_p} = \frac{16T^2(L/2)}{\pi G d_2^4} + \frac{16T^2(L/2)}{\pi G d_1^4}$$

$$= \frac{8T^2L}{\pi G}\left(\frac{1}{d_2^4} + \frac{1}{d_1^4}\right) \quad \text{EQ. (1)}$$

ALSO, $U = \frac{T\phi}{2}$ EQ. (2)

EQUATE U FROM EQS. (1) AND (2) AND SOLVE FOR T:

$$T = \frac{\pi G d_1^4 d_2^4 \phi}{16L(d_1^4 + d_2^4)}$$

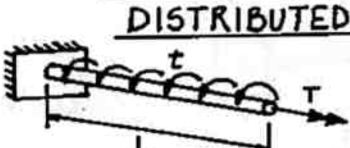
$$U = \frac{T\phi}{2} = \frac{\pi G \phi^2}{32L}\left(\frac{d_1^4 d_2^4}{d_1^4 + d_2^4}\right)$$

SUBSTITUTE NUMERICAL VALUES:

$U = 3.51$ J ⟵

3.9-5

CANTILEVER BAR WITH DISTRIBUTED TORQUE



G = SHEAR MODULUS
I_p = POLAR MOMENT OF INERTIA
T = TORQUE ACTING AT FREE END
t = TORQUE PER UNIT DISTANCE

(a) LOAD T ACTS ALONE

$$U_1 = \frac{T^2L}{2GI_p} \longleftarrow$$

(b) LOAD t ACTS ALONE

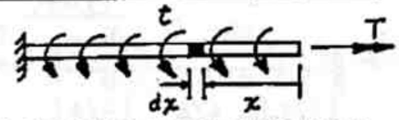
FROM EQ. (3-56) OF EXAMPLE 3-11:

$$U_2 = \frac{t^2L^3}{6GI_p} \longleftarrow$$

CONT.

3.9-5 CONT.

(c) BOTH LOADS ACT SIMULTANEOUSLY



AT DISTANCE x FROM THE FREE END:

$$T(x) = T + tx$$

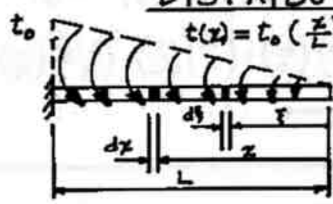
$$U_3 = \int_0^L \frac{[T(x)]^2}{2GI_p}dx = \frac{1}{2GI_p}\int_0^L (T+tx)^2 dx$$

$$= \frac{T^2L}{2GI_p} + \frac{TtL^2}{2GI_p} + \frac{t^2L^3}{6GI_p} \longleftarrow$$

NOTE: U_3 IS NOT THE SUM OF U_1 AND U_2.

3.9-6

CANTILEVER BAR WITH DISTRIBUTED TORQUE



x = DISTANCE FROM RIGHT-HAND END OF THE BAR

ELEMENT $d\xi$

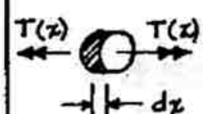
CONSIDER A DIFFERENTIAL ELEMENT $d\xi$ AT DISTANCE ξ FROM THE RIGHT-HAND END.

dT = EXTERNAL TORQUE ACTING ON THIS ELEMENT

$$dT = t(\xi)\,d\xi$$
$$= t_0\left(\frac{\xi}{L}\right)d\xi$$

ELEMENT dx AT DISTANCE x

$T(x)$ = INTERNAL TORQUE ACTING ON THIS ELEMENT



$T(x)$ = TOTAL TORQUE FROM $x = 0$ TO $x = x$

$$T(x) = \int_0^x dT = \int_0^x t_0\left(\frac{\xi}{L}\right)d\xi$$
$$= \frac{t_0 x^2}{2L}$$

STRAIN ENERGY OF ELEMENT dx

$$dU = \frac{[T(x)]^2 dx}{2GI_p} = \frac{1}{2GI_p}\left(\frac{t_0}{2L}\right)^2 x^4 dx$$
$$= \frac{t_0^2}{8L^2GI_p}x^4 dx$$

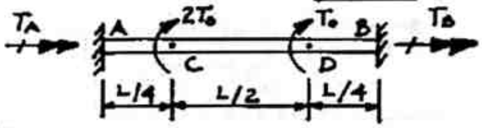
STRAIN ENERGY OF ENTIRE BAR

$$U = \int_0^L dU = \frac{t_0^2}{8L^2GI_p}\int_0^L x^4 dx$$
$$= \frac{t_0^2}{8L^2GI_p}\left(\frac{L^5}{5}\right)$$

$$U = \frac{t_0^2L^3}{40GI_p} \longleftarrow$$

3.9-7

STATICALLY INDETERMINATE BAR



REACTIVE TORQUES

FROM EQ. (3-46 a): $T_A = \frac{(2T_0)\left(\frac{3L}{4}\right)}{L} + \frac{T_0\left(\frac{L}{4}\right)}{L} = \frac{7T_0}{4}$

$$T_B = 3T_0 - T_A = \frac{5T_0}{4}$$

INTERNAL TORQUES

$$T_{AC} = -\frac{7T_0}{4} \quad T_{CD} = \frac{T_0}{4} \quad T_{DB} = \frac{5T_0}{4}$$

STRAIN ENERGY (FROM EQ. 3-53)

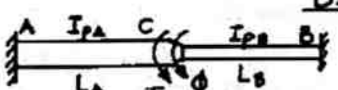
$$U = \sum_{i=1}^{n} \frac{T_i^2 L_i}{2G_i I_{P_i}} = \frac{1}{2GI_P}\left[T_{AC}^2\left(\frac{L}{4}\right) + T_{CD}^2\left(\frac{L}{2}\right) + T_{DB}^2\left(\frac{L}{4}\right)\right]$$

$$= \frac{1}{2GI_P}\left[\left(-\frac{7T_0}{4}\right)^2\left(\frac{L}{4}\right) + \left(\frac{T_0}{4}\right)^2\left(\frac{L}{2}\right) + \left(\frac{5T_0}{4}\right)^2\left(\frac{L}{4}\right)\right]$$

$$U = \frac{19T_0^2 L}{32GI_P} \longleftarrow$$

3.9-8

STATICALLY INDETERMINATE BAR



STRAIN ENERGY (FROM EQ. 3-51b)

$$U = \sum_{i=1}^{n} \frac{GI_{Pi}\phi_i^2}{2L_i} = \frac{GI_{PA}\phi^2}{2L_A} + \frac{GI_{PB}\phi^2}{2L_B} = \frac{G\phi^2}{2}\left(\frac{I_{PA}}{L_A} + \frac{I_{PB}}{L_B}\right)$$

WORK DONE BY THE TORQUE T_0

$$W = \frac{T_0\phi}{2}$$

EQUATE U AND W AND SOLVE FOR ϕ

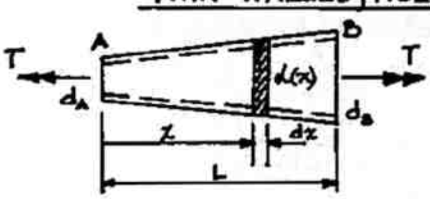
$$\frac{G\phi^2}{2}\left(\frac{I_{PA}}{L_A} + \frac{I_{PB}}{L_B}\right) = \frac{T_0\phi}{2}$$

$$\phi = \frac{T_0 L_A L_B}{G(L_B I_{PA} + L_A I_{PB})} \longleftarrow$$

(THIS RESULT AGREES WITH EQ. (3-48) OF EXAMPLE 3-9, SECTION 3.8.)

3.9-9

THIN-WALLED, HOLLOW TUBE



t = THICKNESS

d_A = AVERAGE DIAMETER AT END A

d_B = AVERAGE DIAMETER AT END B

$d(x)$ = AVERAGE DIAMETER AT DISTANCE x FROM END A

$$d(x) = d_A + \left(\frac{d_B - d_A}{L}\right)x$$

CONT.

3.9-9 CONT.

POLAR MOMENT OF INERTIA $\quad I_P = \frac{\pi d^3 t}{4}$

$$I_P(x) = \frac{\pi[d(x)]^3 t}{4} = \frac{\pi t}{4}\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^3$$

(a) STRAIN ENERGY (FROM EQ. 3-54)

$$U = \int_0^L \frac{T^2 dx}{2GI_P(x)} = \frac{2T^2}{\pi G t}\int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^3} \qquad \text{EQ.(1)}$$

FROM APPENDIX C:

$$\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}$$

THEREFORE,

$$\int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^3} = -\frac{1}{\frac{2(d_B - d_A)}{L}\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^2}\Bigg|_0^L$$

$$= -\frac{L}{2(d_B - d_A)(d_B)^2} + \frac{L}{2(d_B - d_A)(d_A)^2} = \frac{L(d_A + d_B)}{2d_A^2 d_B^2}$$

SUBSTITUTE THIS EXPRESSION FOR THE INTEGRAL INTO THE EQUATION FOR U (EQ. 1):

$$U = \frac{2T^2}{\pi G t} \cdot \frac{L(d_A + d_B)}{2d_A^2 d_B^2} = \frac{T^2 L}{\pi G t}\left(\frac{d_A + d_B}{d_A^2 d_B^2}\right) \longleftarrow$$

(b) ANGLE OF TWIST

WORK OF THE TORQUE T: $W = \frac{T\phi}{2}$

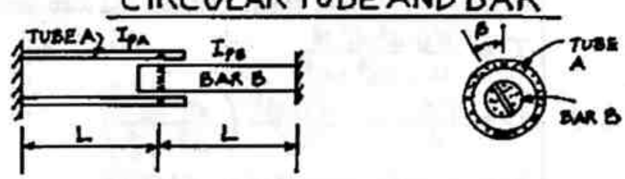
$$W = U \quad \frac{T\phi}{2} = \frac{T^2 L(d_A + d_B)}{\pi G t d_A^2 d_B^2}$$

SOLVE FOR ϕ:

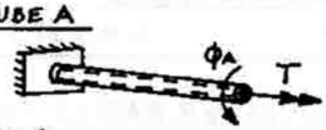
$$\phi = \frac{2TL(d_A + d_B)}{\pi G t d_A^2 d_B^2} \longleftarrow$$

3.9-10

CIRCULAR TUBE AND BAR



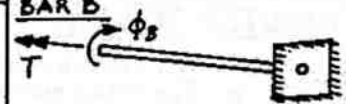
TUBE A



T = TORQUE ACTING ON THE TUBE

ϕ_A = ANGLE OF TWIST

BAR B



T = TORQUE ACTING ON THE BAR

ϕ_B = ANGLE OF TWIST

COMPATIBILITY

$$\phi_A + \phi_B = \beta$$

FORCE – DISPLACEMENT RELATIONS

$$\phi_A = \frac{TL}{GI_{PA}} \qquad \phi_B = \frac{TL}{GI_{PB}}$$

SUBSTITUTE INTO THE EQUATION OF COMPATIBILITY AND SOLVE FOR T: $\quad T = \frac{\beta G}{L}\left(\frac{I_{PA}I_{PB}}{I_{PA} + I_{PB}}\right)$

CONT.

3.9-10 CONT.

STRAIN ENERGY

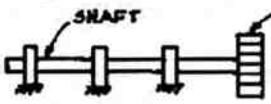
$$U = \sum \frac{T^2L}{2GI_p} = \frac{T^2L}{2GI_{pA}} + \frac{T^2L}{2GI_{pB}} = \frac{T^2L}{2G}\left(\frac{1}{I_{pA}} + \frac{1}{I_{pB}}\right)$$

SUBSTITUTE FOR T AND SIMPLIFY:

$$U = \frac{\beta^2 G}{2L}\left(\frac{I_{pA}I_{pB}}{I_{pA}+I_{pB}}\right) \longleftarrow$$

3.9-11

ROTATING FLYWHEEL



d = DIAMETER
n = rpm

KINETIC ENERGY OF FLYWHEEL

$$K.E. = \frac{1}{2} I_m \omega^2$$

UNITS:
I_m = (FORCE)(LENGTH)(SECOND)^2

$$\omega = \frac{2\pi n}{60} \quad n = \text{rpm}$$

ω = RADIANS PER SECOND
K.E = (LENGTH)(FORCE)

$$K.E. = \frac{1}{2} I_m\left(\frac{2\pi n}{60}\right)^2 = \frac{\pi^2 n^2 I_m}{1800}$$

STRAIN ENERGY OF SHAFT (FROM EQ. 3-51b)

$$U = \frac{GI_p\phi^2}{2L}$$

$$I_p = \frac{\pi}{32} d^4$$

d = DIAMETER OF SHAFT

$$U = \frac{\pi G d^4 \phi^2}{64L}$$

UNITS:
G = (FORCE)/(LENGTH)^2
I_p = (LENGTH)^4
ϕ = RADIANS L = LENGTH
U = (LENGTH)(FORCE)

EQUATE KINETIC ENERGY AND STRAIN ENERGY

$$K.E. = U \qquad \frac{\pi^2 n^2 I_m}{1800} = \frac{\pi G d^4 \phi^2}{64L}$$

SOLVE FOR ϕ:

$$\phi = \frac{2n}{15d^2}\sqrt{\frac{2\pi I_m L}{G}} \longleftarrow$$

MAXIMUM SHEAR STRESS

$$\tau = \frac{T(d/2)}{I_p} \qquad \phi = \frac{TL}{GI_p}$$

ELIMINATE T:

$$\tau = \frac{Gd\phi}{2L}$$

$$\tau_{max} = \frac{Gd}{2L}\cdot\frac{2n}{15d^2}\sqrt{\frac{2\pi I_m L}{G}}$$

$$\tau_{max} = \frac{n}{15d}\sqrt{\frac{2\pi G I_m}{L}} \longleftarrow$$

3.10-1

HOLLOW CIRCULAR TUBE

T = 1500 k-in. t = 1.0 in.
r = RADIUS TO MEDIAN LINE
r = 5.0 in.
d_2 = OUTSIDE DIAMETER = 11.0 in.
d_1 = INSIDE DIAMETER = 9.0 in.

CONT.

3.10-1 CONT.

(a) APPROXIMATE THEORY (FROM EQ. 3-63)

$$\tau = \frac{T}{2\pi r^2 t} = \frac{1500\text{ k-in.}}{2\pi(5.0\text{ in.})^2(1.0\text{ in.})} = 9549\text{ psi}$$

$$\tau_{APPROX} = 9550\text{ psi} \longleftarrow$$

(b) EXACT THEORY (FROM EQ. 3-11)

$$\tau = \frac{T(d_2/2)}{I_p} \qquad I_p = \frac{\pi r t}{2}(4r^2 + t^2) \quad \text{(EQ. 3-17)}$$

$$I_p = \frac{\pi(5.0\text{ in.})(1.0\text{ in.})}{2}\left[4(5.0\text{ in.})^2 + (1.0\text{ in.})^2\right] = 793.25\text{ in.}^4$$

$$\tau_{EXACT} = \frac{(1500\text{ k-in.})(5.5\text{ in.})}{793.25\text{ in.}^4} = 10{,}400\text{ psi} \longleftarrow$$

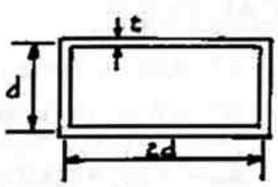
BECAUSE THE APPROXIMATE THEORY GIVES STRESSES THAT ARE TOO LOW, IT IS UNCONSERVATIVE. THEREFORE, THE APPROXIMATE THEORY SHOULD ONLY BE USED FOR VERY THIN TUBES.

3.10-2

SOLID BAR

$$\tau_{max} = \frac{16T}{\pi d^3} \quad \text{(EQ. 3-12)}$$

RECTANGULAR TUBE



$$A_m = (d)(2d) = 2d^2 \quad \text{(EQ. 3-64)}$$

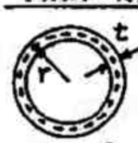
$$\tau_{max} = \frac{T}{2tA_m} = \frac{T}{4td^2} \quad \text{(EQ. 3-61)}$$

EQUATE THE MAXIMUM SHEAR STRESSES AND SOLVE FOR t

$$\frac{16T}{\pi d^3} = \frac{T}{4td^2} \qquad t_{min} = \frac{\pi d}{64} \longleftarrow$$

3.10-3

THIN-WALLED TUBE (1)



$$A_m = \pi r^2 \quad J = 2\pi r^3 t \quad A = 2\pi r t$$

$$\tau_{max} = \frac{T}{2tA_m} = \frac{T}{2\pi r^2 t}$$

$$T = 2\pi r^2 t\,\tau_{max}$$

$$U_1 = \frac{T^2L}{2GJ} \qquad U_1 = \frac{(2\pi r^2 t\,\tau_{max})^2 L}{2G(2\pi r^3 t)}$$

$$U_1 = \frac{\pi r t\,\tau_{max}^2 L}{G} \qquad \text{BUT } rt = \frac{A}{2\pi}$$

$$U_1 = \frac{A\tau_{max}^2 L}{2G}$$

SOLID BAR (2)

$$A = \pi r_2^2 \quad I_p = \frac{\pi}{2} r_2^4$$

$$\tau_{max} = \frac{Tr_2}{I_p} = \frac{2T}{\pi r_2^3} \qquad T = \frac{\pi r_2^3 \tau_{max}}{2}$$

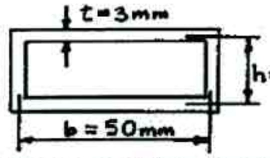
$$U_2 = \frac{T^2L}{2GI_p} = \frac{(\pi r_2^3 \tau_{max})^2 L}{8G\left(\frac{\pi}{2} r_2^4\right)} = \frac{\pi r_2^2 \tau_{max}^2 L}{4G}$$

$$\text{BUT } \pi r_2^2 = A \qquad U_2 = \frac{A\tau_{max}^2 L}{4G}$$

RATIO $\quad \frac{U_1}{U_2} = 2 \longleftarrow$

3.10-4

THIN-WALLED TUBE



$T = 90\text{ N·m}$ $L = 0.25\text{ m}$

$G = 26\text{ GPa}$

$A_m = (50\text{mm})(20\text{mm}) = 1000\text{ mm}^2$

(a) SHEAR STRESS (FROM EQ. 3-61)

$$\tau = \frac{T}{2tA_m} = \frac{90\text{ N·m}}{2(3\text{mm})(1000\text{mm}^2)} = 15.0\text{ MPa} \longleftarrow$$

(b) ANGLE OF TWIST (FROM EQ. 3-72)

$$\phi = \frac{TL}{GJ} \quad \text{EQ. (3-71)}: \; J = \frac{2b^2h^2t_1t_2}{bt_1 + ht_2} \quad t_1 = t_2 = t = 3\text{mm}$$

$$J = \frac{2b^2h^2t}{b+h} = \frac{2(50\text{mm})^2(20\text{mm})^2(3\text{mm})}{50\text{mm} + 20\text{mm}}$$

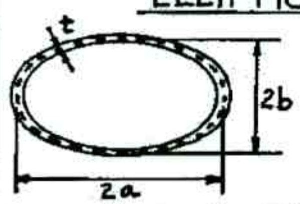
$$= 85{,}714\text{ mm}^4 = 85.714 \times 10^{-9}\text{ m}^4$$

$$\phi = \frac{(90\text{ N·m})(0.25\text{m})}{(26\text{ GPa})(85.714 \times 10^{-9}\text{m}^4)}$$

$$= 0.01010\text{ rad} = 0.578° \longleftarrow$$

3.10-5

ELLIPTICAL TUBE



$T = 30\text{ k-in.}$ $G = 12 \times 10^6\text{ psi}$

$t = 0.2\text{ in.}$ $a = 3.0\text{in.}$ $b = 2.0\text{in.}$

FROM APPENDIX D, CASE 16:

$$A_m = \pi ab = \pi(3.0\text{in.})(2.0\text{in.}) = 18.850\text{ in.}^2$$

$$L_m \approx \pi\left[1.5(a+b) - \sqrt{ab}\right]$$

$$= \pi\left[1.5(5.0\text{in.}) - \sqrt{6.0\text{in.}^2}\right] = 15.867\text{ in.}$$

SHEAR STRESS

$$\text{FROM EQ. (3-69)}: \; J = \frac{4tA_m^2}{L_m} = \frac{4(0.2\text{in.})(18.850\text{in.}^2)^2}{15.867\text{ in.}} = 17.92\text{ in.}^4$$

$$\tau = \frac{T}{2tA_m} = \frac{30\text{k-in.}}{2(0.2\text{in.})(18.850\text{in.}^2)} = 3980\text{ psi} \longleftarrow$$

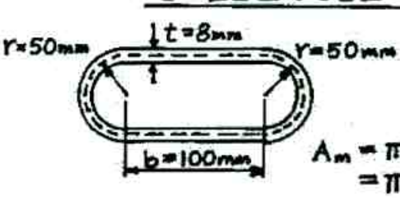
ANGLE OF TWIST PER UNIT LENGTH (RATE OF TWIST)

$$\theta = \frac{\phi}{L} = \frac{T}{GJ} = \frac{30\text{ k-in.}}{(12 \times 10^6\text{psi})(17.92\text{in.}^4)}$$

$$\theta = 139.5 \times 10^{-6}\text{ rad/in.} = 0.0080°/\text{in.} \longleftarrow$$

3.10-6

STEEL TUBE



$G = 76\text{ GPa}$

$L = 1.5\text{m}$

$T = 15.0\text{ kN·m}$

$$A_m = \pi r^2 + 2br = \pi(50\text{mm})^2 + 2(100\text{mm})(50\text{mm}) = 17{,}850\text{ mm}^2$$

$$L_m = 2b + 2\pi r = 2(100\text{mm}) + 2\pi(50\text{mm}) = 514.2\text{ mm}$$

$$J = \frac{4tA_m^2}{L_m} = \frac{4(8\text{mm})(17{,}850\text{mm}^2)^2}{514.2\text{ mm}} = 19.83 \times 10^6\text{ mm}^4$$

CONT.

3.10-6 CONT.

SHEAR STRESS

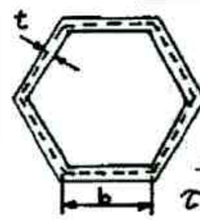
$$\tau = \frac{T}{2tA_m} = \frac{15\text{ kN·m}}{2(8\text{mm})(17{,}850\text{mm}^2)} = 52.5\text{ MPa} \longleftarrow$$

ANGLE OF TWIST

$$\phi = \frac{TL}{GJ} = \frac{(15\text{kN·m})(1.5\text{m})}{(76\text{ GPa})(19.83 \times 10^6\text{mm}^4)} = 0.01493\text{ rad} = 0.856° \longleftarrow$$

3.10-7

REGULAR HEXAGON



b = LENGTH OF SIDE

t = THICKNESS

$$A_m = \frac{3\sqrt{3}}{2}b^2 \qquad L_m = 6b$$

SHEAR STRESS

$$\tau = \frac{T}{2tA_m} = \frac{T\sqrt{3}}{9b^2t} \longleftarrow$$

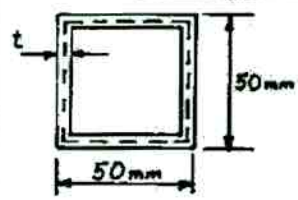
ANGLE OF TWIST PER UNIT LENGTH (RATE OF TWIST)

$$J = \frac{4A_m^2}{\int_0^{L_m}\frac{ds}{t}} = \frac{4A_m^2t}{L_m} = \frac{9b^3t}{2}$$

$$\theta = \frac{T}{GJ} = \frac{2T}{G(9b^3t)} = \frac{2T}{9Gb^3t} \longleftarrow$$

3.10-8

SQUARE ALUMINUM TUBE



OUTER DIMENSIONS: 50mm x 50mm

$G = 28\text{ GPa}$ $T = 300\text{ N·m}$

$\tau_{allow} = 20\text{ MPa}$ $\theta_{allow} = 0.025\text{ rad/m}$

LET $b = 50\text{mm}$

$A_m = (b-t)^2$ $L_m = 4(b-t)$

$$J = \frac{4tA_m^2}{L_m} = \frac{4t(b-t)^4}{4(b-t)} = t(b-t)^3$$

THICKNESS t BASED UPON SHEAR STRESS

$$\tau = \frac{T}{2tA_m} \qquad tA_m = \frac{T}{2\tau} \qquad t(b-t)^2 = \frac{T}{2\tau}$$

UNITS: t = METERS b = METERS T = NEWTON METERS

τ = NEWTONS PER SQUARE METER

$$t(0.050\text{m} - t)^2 = \frac{300\text{ N·m}}{2(20 \times 10^6\text{ Pa})} = 7.5 \times 10^{-6}\text{ m}^3$$

SOLVE FOR t:

$$t = 0.00346\text{ m} = 3.46\text{ mm}$$

THICKNESS t BASED UPON RATE OF TWIST

$$\theta = \frac{T}{GJ} = \frac{T}{Gt(b-t)^3} \qquad t(b-t)^3 = \frac{T}{G\theta}$$

UNITS: t = METERS b = METERS T = NEWTON METERS

G = NEWTONS PER SQUARE METER

θ = RADIANS PER METER

$$t(0.050\text{m} - t)^3 = \frac{300\text{N·m}}{(28\text{GPa})(0.025\text{rad/m})} = \frac{3}{7 \times 10^6}$$

SOLVE FOR t: $\quad t = 0.004571\text{ m} = 4.57\text{ mm}$

ANGLE OF TWIST GOVERNS

$$t_{min} = 4.57\text{ mm} \longleftarrow$$

3.10-9

THIN-WALLED TUBE

APPROXIMATE THEORY

$$\phi_1 = \frac{TL}{GJ} \quad J = 2\pi r^3 t \quad \phi_1 = \frac{TL}{2\pi G r^3 t}$$

EXACT THEORY

$$\phi_2 = \frac{TL}{GI_p} \quad \text{FROM EQ. (3-17): } I_p = \frac{\pi r t}{2}(4r^2 + t^2)$$

$$\phi_2 = \frac{TL}{GI_p} = \frac{2TL}{\pi G r t (4r^2 + t^2)}$$

RATIO

$$\frac{\phi_1}{\phi_2} = \frac{4r^2 + t^2}{4r^2} = 1 + \frac{t^2}{4r^2}$$

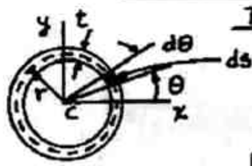
LET $\beta = \dfrac{r}{t}$ $\quad \dfrac{\phi_1}{\phi_2} = 1 + \dfrac{1}{4\beta^2}$ ⟵

β	ϕ_1/ϕ_2
5	1.0100
10	1.0025
20	1.0006

AS THE TUBE BECOMES THINNER AND β BECOMES LARGER, THE RATIO ϕ_1/ϕ_2 APPROACHES UNITY. THUS, THE THINNER THE TUBE, THE MORE ACCURATE THE APPROXIMATE THEORY BECOMES.

3.10-10

THIN TUBE WITH VARIABLE THICKNESS



PROPERTIES OF THE CROSS SECTION

$$t = t_0\left(1 + \sin\frac{\theta}{2}\right)$$

r = RADIUS TO THE MEDIAN LINE

$$L_m = 2\pi r \quad A_m = \pi r^2$$

$$J = \frac{4A_m^2}{\int_0^{L_m} \frac{ds}{t}} \qquad ds = r\,d\theta$$

$$\int_0^{L_m} \frac{ds}{t} = \int_0^{2\pi} \frac{r\,d\theta}{t_0\left(1 + \sin\frac{\theta}{2}\right)} = \frac{r}{t_0}\int_0^{2\pi} \frac{d\theta}{1 + \sin\frac{\theta}{2}}$$

FROM A TABLE OF INTEGRALS (SEE APPENDIX C):

$$\int \frac{dx}{1 + \sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)$$

$$\int_0^{2\pi} \frac{d\theta}{1 + \sin\frac{\theta}{2}} = \left[-2\tan\left(\frac{\pi}{4} - \frac{\theta}{4}\right)\right]_0^{2\pi}$$

$$= -2\tan\left(\frac{\pi}{4} - \frac{\pi}{2}\right) + 2\tan\left(\frac{\pi}{4}\right) = 4$$

$$\therefore \int_0^{L_m} \frac{ds}{t} = \frac{4r}{t_0}$$

$$J = \frac{4A_m^2}{\int_0^{L_m} \frac{ds}{t}} = \frac{4(\pi r^2)^2}{\frac{4r}{t_0}} = \pi^2 r^3 t_0$$

SHEAR STRESSES

$$\tau = \frac{T}{2tA_m} = \frac{T}{2\pi r^2 t_0 \left(1 + \sin\frac{\theta}{2}\right)}$$

$$\theta = 0: \ \tau_{max} = \frac{T}{2\pi r^2 t_0} \longleftarrow$$

$$\theta = \pi: \ \tau_{min} = \frac{T}{4\pi r^2 t_0} \longleftarrow$$

ANGLE OF TWIST

$$\phi = \frac{TL}{GJ} = \frac{TL}{G\pi^2 r^3 t_0} \longleftarrow$$

3.10-11

THIN TUBE

$T = 45{,}000$ lb-in.

$\tau_{allow} = 6000$ psi

t IS IN INCHES

r = AVERAGE RADIUS $= 2 \text{ in.} + \dfrac{t}{2}$

r_1 = INNER RADIUS $= 2$ in.

r_2 = OUTER RADIUS $= 2 \text{ in.} + t$

(a) APPROXIMATE THEORY

$$\tau = \frac{T}{2tA_m} = \frac{T}{2t(\pi r^2)} = \frac{T}{2\pi r^2 t}$$

$$6000 \text{ psi} = \frac{45{,}000 \text{ lb-in.}}{2\pi\left(2 + \frac{t}{2}\right)^2 t}$$

$$\text{OR } t\left(2 + \frac{t}{2}\right)^2 = \frac{45{,}000}{2\pi(6000)} = \frac{15}{4\pi}$$

SOLVE FOR t:

$$t = 0.263 \text{ in.} \longleftarrow$$

(b) EXACT THEORY

$$\tau = \frac{Tr_2}{I_p} \quad I_p = \frac{\pi}{2}(r_2^4 - r_1^4)$$

$$= \frac{\pi}{2}\left[(2+t)^4 - (2)^4\right]$$

$$= \frac{\pi t}{2}(t^3 + 8t^2 + 24t + 32)$$

$$6000 \text{ psi} = \frac{(45{,}000 \text{ lb-in.})(2+t)}{\frac{\pi t}{2}(t^3 + 8t^2 + 24t + 32)}$$

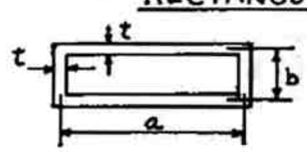
SOLVE FOR t:

$$t = 0.277 \text{ in.} \longleftarrow$$

THE APPROXIMATE RESULT IS 5% LESS THAN THE EXACT RESULT. THUS, THE APPROXIMATE THEORY IS NONCONSERVATIVE AND SHOULD ONLY BE USED FOR THIN-WALLED TUBES.

3.10-12

RECTANGULAR TUBE



t = THICKNESS

a, b = DIMENSIONS OF THE TUBE

$\beta = \dfrac{a}{b}$

$L_m = 2(a + b)$ = CONSTANT

T = CONSTANT

SHEAR STRESS

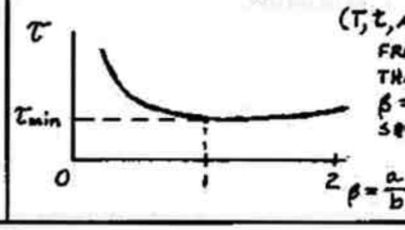
$$\tau = \frac{T}{2tA_m} \quad A_m = ab = \beta b^2$$

$$L_m = 2b(1 + \beta) = \text{CONSTANT}$$

$$b = \frac{L_m}{2(1+\beta)} \quad A_m = \beta\left[\frac{L_m}{2(1+\beta)}\right]^2 = \frac{\beta L_m^2}{4(1+\beta)^2}$$

$$\tau = \frac{T}{2tA_m} = \frac{T(4)(1+\beta)^2}{2t\beta L_m^2} = \frac{2T(1+\beta)^2}{tL_m^2\beta} \longleftarrow$$

(T, t, AND L_m ARE CONSTANTS)



FROM THE GRAPH, WE SEE THAT τ IS MINIMUM WHEN $\beta = 1$ AND THE TUBE IS SQUARE. ⟵

$$\tau_{min} = \frac{8T}{tL_m^2}$$

CONT.

3.10-12 CONT.

ALTERNATE SOLUTION

$$\tau = \frac{2T}{tL_m^2} \cdot \frac{(1+\beta)^2}{\beta}$$

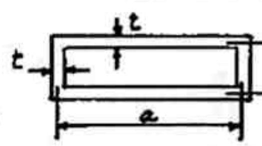
$$\frac{d\tau}{d\beta} = \frac{2T}{tL_m^2}\left[\frac{\beta(2)(1+\beta)-(1+\beta)^2(1)}{\beta^2}\right] = 0$$

OR $2\beta(1+\beta)-(1+\beta)^2 = 0 \therefore \beta = 1$

THUS, THE TUBE IS SQUARE AND τ IS EITHER A MINIMUM OR A MAXIMUM. FROM THE GRAPH, WE SEE THAT τ IS A MINIMUM.

3.10-13

RECTANGULAR TUBE



t = THICKNESS

a, b = DIMENSIONS TO MEDIAN LINES OF THE TUBE

$\beta = \frac{a}{b}$

$L_m = 2(a+b)$ = CONSTANT

T = CONSTANT

ANGLE OF TWIST

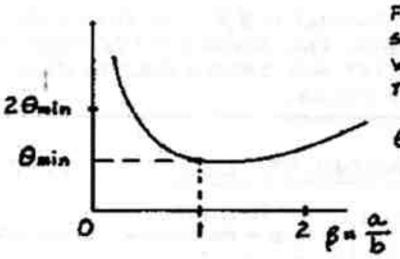
$$\theta = \frac{T}{GJ} \quad J = \frac{4tA_m^2}{L_m} \quad A_m = ab = \beta b^2$$

$L_m = 2b(1+\beta)$ = CONSTANT $\quad b = \frac{L_m}{2(1+\beta)}$

$$A_m = \frac{\beta L_m^2}{4(1+\beta)^2} \quad J = \frac{4t\beta^2 L_m^4}{L_m(16)(1+\beta)^4} = \frac{tL_m^3\beta^2}{4(1+\beta)^4}$$

$$\theta = \frac{T}{GJ} = \frac{4T(1+\beta)^4}{GtL_m^3\beta^2} \longleftarrow$$

(T, G, t, AND L_m ARE CONSTANTS)



FROM THE GRAPH, WE SEE THAT θ IS MINIMUM WHEN $\beta = 1$ AND THE TUBE IS SQUARE.

$$\theta_{min} = \frac{64T}{GtL_m^3} \longleftarrow$$

ALTERNATE SOLUTION

$$\theta = \frac{4T}{GtL_m^3} \cdot \frac{(1+\beta)^4}{\beta^2}$$

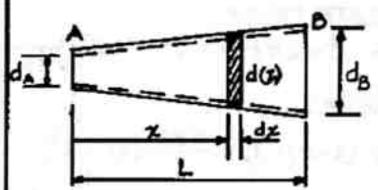
$$\frac{d\theta}{d\beta} = \frac{4T}{GtL_m^3}\left[\frac{\beta^2(4)(1+\beta)^3-(1+\beta)^4(2\beta)}{\beta^4}\right] = 0$$

OR $4\beta^2(1+\beta)^3 - 2\beta(1+\beta)^4 = 0 \quad \therefore \beta = 1$

THUS, THE TUBE IS SQUARE AND θ IS EITHER A MINIMUM OR A MAXIMUM. FROM THE GRAPH, WE SEE THAT θ IS A MINIMUM.

3.10-14

THIN-WALLED TAPERED TUBE



t = THICKNESS

d_A = AVERAGE DIAMETER AT END A

d_B = AVERAGE DIAMETER AT END B

T = TORQUE

$d(x)$ = AVERAGE DIAMETER AT DISTANCE x FROM END A.

$$d(x) = d_A + \left(\frac{d_B - d_A}{L}\right)x$$

$$J = 2\pi r^3 t = \frac{\pi d^3 t}{4}$$

$$J(x) = \frac{\pi t}{4}\left[d(x)\right]^3 = \frac{\pi t}{4}\left[d_A + \left(\frac{d_B-d_A}{L}\right)x\right]^3$$

FOR ELEMENT OF LENGTH dx:

$$d\phi = \frac{T\,dx}{GJ(x)} = \frac{4T\,dx}{G\pi t\left[d_A + \left(\frac{d_B-d_A}{L}\right)x\right]^3}$$

FOR ENTIRE TUBE:

$$\phi = \frac{4T}{\pi Gt}\int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B-d_A}{L}\right)x\right]^3}$$

FROM TABLE OF INTEGRALS (SEE APPENDIX C):

$$\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}$$

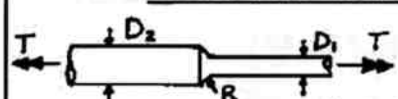
$$\phi = \frac{4T}{\pi Gt}\left[-\frac{1}{2\left(\frac{d_B-d_A}{L}\right)\left(d_A + \frac{d_B-d_A}{L}\cdot x\right)^2}\right]_0^L$$

$$= \frac{4T}{\pi Gt}\left[-\frac{L}{2(d_B-d_A)d_B^2} + \frac{L}{2(d_B-d_A)d_A^2}\right]$$

$$\phi = \frac{2TL}{\pi Gt}\left(\frac{d_A+d_B}{d_A^2\,d_B^2}\right) \longleftarrow$$

3.11-1

STEPPED SHAFT IN TORSION



$D_1 = 2.0$ in.

$D_2 = 2.4$ in.

$R = 0.1$ in. $\tau_{allow} = 9000$ psi

USE FIG. 3-48 FOR THE STRESS-CONCENTRATION FACTOR

$$\frac{R}{D_1} = \frac{0.1\text{ in.}}{2.0\text{ in.}} = 0.05 \quad \frac{D_2}{D_1} = \frac{2.4\text{ in.}}{2.0\text{ in.}} = 1.2$$

$$K \approx 1.52 \quad \tau_{max} = K\tau_{nom} = K\left(\frac{16\,T_{max}}{\pi D_1^3}\right)$$

$$T_{max} = \frac{\pi D_1^3 \tau_{max}}{16K}$$

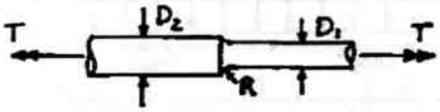
$$= \frac{\pi(2.0\text{ in.})^3(9000\text{ psi})}{16(1.52)}$$

$T_{max} = 9300$ lb-in.

$\therefore T_{max} = 9300$ lb-in. $\longleftarrow$

3.11-2

STEPPED SHAFT IN TORSION



$D_1 = 40$ mm $D_2 = 60$ mm $T = 1100$ N·m $\tau_{allow} = 120$ MPa

USE FIG. 3-48 FOR THE STRESS-CONCENTRATION FACTOR

$$\tau_{max} = K\,\tau_{nom} = K\left(\frac{16T}{\pi D_1^3}\right)$$

$$K = \frac{\pi D_1^3 \tau_{max}}{16T} = \frac{\pi(40\text{mm})^3(120\text{MPa})}{16(1100\text{N·m})} = 1.37$$

$$\frac{D_2}{D_1} = \frac{60\text{mm}}{40\text{mm}} = 1.5$$

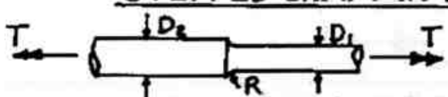
FROM FIG. (3-48) WITH $\frac{D_2}{D_1} = 1.5$ AND $K = 1.37$,

WE GET $\frac{R}{D_1} \approx 0.10$

$$\therefore\ R_{min} \approx 0.10(40\text{mm}) = 4.0\text{mm}$$

3.11-3

STEPPED SHAFT IN TORSION



$D_2 = 1.0$ in. $T = 500$ lb-in. $D_1 = 0.7, 0.8,$ AND 0.9 in.

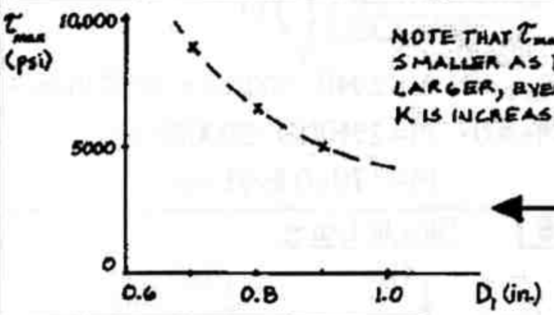
FULL QUARTER-CIRCULAR FILLET $(D_2 = D_1 + 2R)$

$$R = \frac{D_2 - D_1}{2} = 0.5\text{in.} - \frac{D_1}{2}$$

USE FIG. 3-48 FOR THE STRESS-CONCENTRATION FACTOR

$$\tau_{max} = K\,\tau_{nom} = K\left(\frac{16T}{\pi D_1^3}\right) = K\,\frac{16(500\text{lb-in})}{\pi D_1^3} = 2546\frac{K}{D_1^3}$$

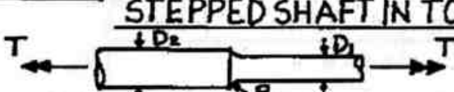
D_1 (in.)	D_2/D_1	R (in.)	R/D_1	K	τ_{max} (psi)
0.7	1.43	0.15	0.214	1.20	8900
0.8	1.25	0.10	0.125	1.29	6400
0.9	1.11	0.05	0.056	1.41	4900



NOTE THAT τ_{max} GETS SMALLER AS D_1 GETS LARGER, EVEN THOUGH K IS INCREASING.

3.11-4

STEPPED SHAFT IN TORSION



$P = 600$ kW $n = 400$ rpm $D_1 = 100$ mm $\tau_{allow} = 100$ MPa

FULL QUARTER-CIRCULAR FILLET

CONT.

3.11-4 CONT.

POWER $P = \frac{2\pi n T}{60}$ (EQ. 3-42 OF SECTION 3.7)

P = WATTS n = rpm T = NEWTON METERS

$$T = \frac{60P}{2\pi n} = \frac{60(600\times10^3\text{W})}{2\pi(400\text{rpm})} = 14{,}320\text{ N·m}$$

USE FIG. 3-48 FOR THE STRESS-CONCENTRATION FACTOR

$$\tau_{max} = K\,\tau_{nom} = K\left(\frac{16T}{\pi D_1^3}\right)$$

$$K = \frac{\tau_{max}(\pi D_1^3)}{16T} = \frac{(100\text{MPa})(\pi)(100\text{mm})^3}{16(14{,}320\text{ N·m})} = 1.37$$

USE THE DASHED LINE FOR A FULL QUARTER-CIRCULAR FILLET.

$$\frac{R}{D_1} \approx 0.075 \quad R \approx 0.075 D_1 = 0.075(100\text{mm}) = 7.5\text{mm}$$

$$D_2 = D_1 + 2R = 100\text{mm} + 2(7.5\text{mm}) = 115\text{ mm}$$

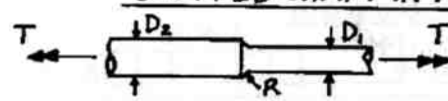
$$\therefore\ D_2 = 115\text{ mm}$$

THIS VALUE OF D_2 IS A LOWER LIMIT

(IF D_2 IS LESS THAN 115mm, R/D_1 IS SMALLER, K IS LARGER, AND τ_{max} IS LARGER, WHICH MEANS THAT THE ALLOWABLE STRESS IS EXCEEDED.)

3.11-5

STEPPED SHAFT IN TORSION



$D_2 = 1.5$ in. $\tau_{allow} = 15{,}000$ psi $T = 4800$ lb-in.

FULL QUARTER-CIRCULAR FILLET $D_2 = D_1 + 2R$

$$R = \frac{D_2 - D_1}{2} = 0.75\text{in.} - \frac{D_1}{2}$$

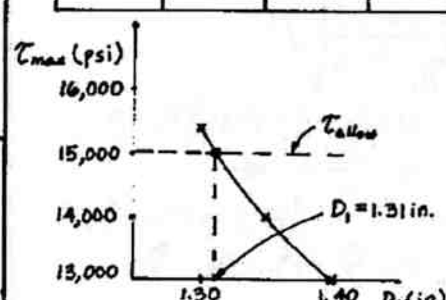
USE FIG. 3-48 FOR THE STRESS-CONCENTRATION FACTOR

$$\tau_{max} = K\,\tau_{nom} = K\left(\frac{16T}{\pi D_1^3}\right) = \frac{K}{D_1^3}\left[\frac{16(4800\text{lb-in})}{\pi}\right]$$

$$= 24{,}450\,\frac{K}{D_1^3}$$

USE TRIAL-AND-ERROR. SELECT TRIAL VALUES OF D_1.

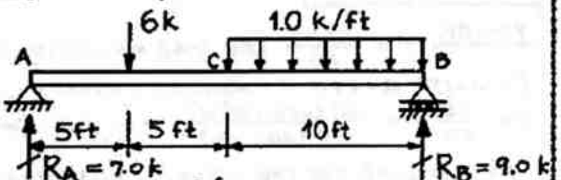
D_1 (in.)	R (in.)	R/D_1	K	τ_{max} (psi)
1.30	0.100	0.077	1.38	15,400
1.35	0.075	0.056	1.41	14,000
1.40	0.050	0.036	1.46	13,000



– END OF CHAP. 3 –

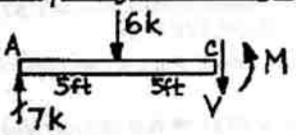
FROM THE GRAPH, MINIMUM $D_1 \approx 1.31$ in.

4.3-1 Simple beam



$$\Sigma M_B = 0:\ R_A = \frac{1}{20}[(6)(15)+(1)(10)(5)] = 7\,\text{k}$$

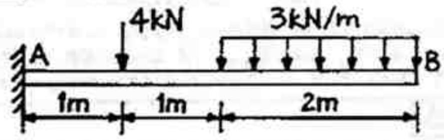
Free-body diagram of segment AC



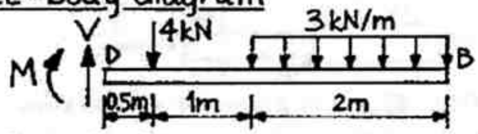
$$\Sigma F_{vert} = 0:\quad V = 7 - 6 = 1.0\,\text{k} \leftarrow$$

$$\Sigma M_C = 0:\quad M = 7(10) - 6(5) = 40\,\text{k-ft} \leftarrow$$

4.3-2 Cantilever beam



Free-body diagram

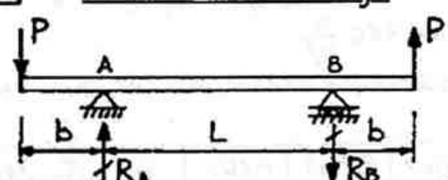


$$\Sigma F_{vert} = 0:\quad V = 4 + 3(2) = 10\,\text{kN} \leftarrow$$

$$\Sigma M_D = 0:\quad M = -4(0.5) - 3(2)(2.5)$$

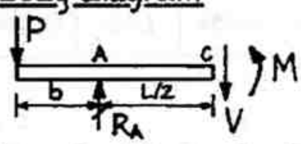
$$= -17\,\text{kN·m} \leftarrow$$

4.3-3 Beam with overhangs



$$\Sigma M_B = 0:\ R_A = \frac{1}{L}[P(L+b+b)] = P\left(1+\frac{2b}{L}\right)$$

Free-body diagram



$$\Sigma F_{vert} = 0:\quad V = R_A - P = P\left(1+\frac{2b}{L}\right) - P$$

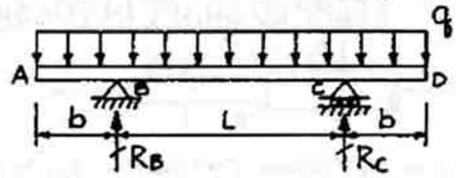
$$V = \frac{2b}{L}P \leftarrow$$

$$\Sigma M_C = 0:\quad M = P\left(1+\frac{2b}{L}\right)\left(\frac{L}{2}\right) - P\left(b+\frac{L}{2}\right)$$

$$M = \frac{PL}{2} + Pb - Pb - \frac{PL}{2}$$

$$M = 0 \leftarrow$$

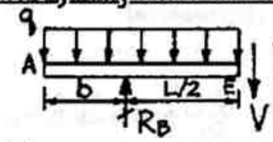
4.3-4 Beam with overhangs



From symmetry and equilibrium of vertical forces:

$$R_B = R_C = q\left(b+\frac{L}{2}\right)$$

Free-body diagram of left-hand half of beam:



$$\Sigma M_E = 0$$

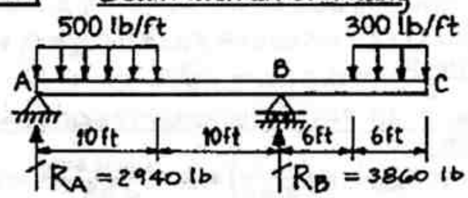
$$-R_B\left(\frac{L}{2}\right) + q\left(\frac{1}{2}\right)\left(b+\frac{L}{2}\right)^2 = 0$$

$$-q\left(b+\frac{L}{2}\right)\left(\frac{L}{2}\right) + q\left(\frac{1}{2}\right)\left(b+\frac{L}{2}\right)^2 = 0$$

Solve for b/L:

$$\frac{b}{L} = \frac{1}{2} \leftarrow$$

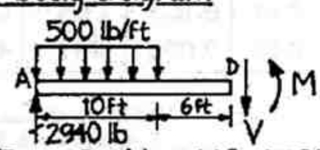
4.3-5 Beam with an overhang



$$\Sigma M_B = 0:\ R_A = \frac{1}{20}[500(10)(15) - 300(6)(9)]$$

$$= 2940\ \text{lb}$$

Free-body diagram

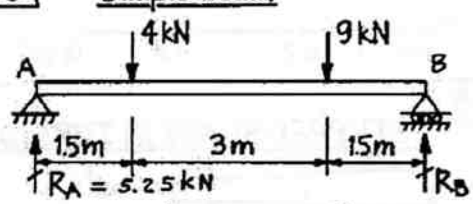


$$\Sigma F_{vert} = 0:\ V = 2940 - 500(10) = -2060\ \text{lb} \leftarrow$$

$$\Sigma M_D = 0:\ M = 2940(16) - 5000(5+6)$$

$$M = -7960\ \text{lb-ft} \leftarrow$$

4.3-6 Simple beam

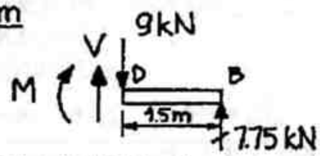


$$\Sigma M_A = 0:\ R_B = \frac{1}{6}[9(4.5) + 4(1.5)] = 7.75\ \text{kN}$$

CONT.

4.3-6 CONT.

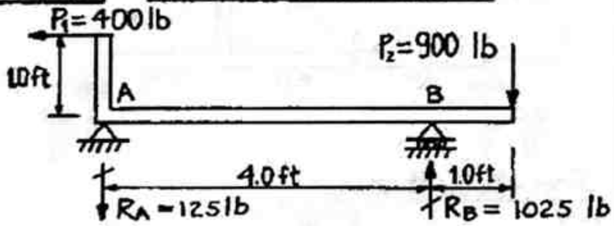
Free-body diagram



$\Sigma F_{vert} = 0$: $V = 9 - 7.75 = 1.25$ kN ⟵

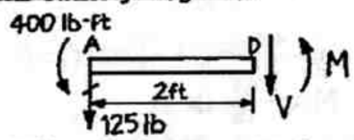
$\Sigma M_D = 0$: $M = 7.75(1.5) = 11.62$ kN·m ⟵

4.3-7 Beam with vertical arm



$\Sigma M_B = 0$: $R_A = \frac{1}{4}[(900)(1) - (400)(1)] = 125$ lb

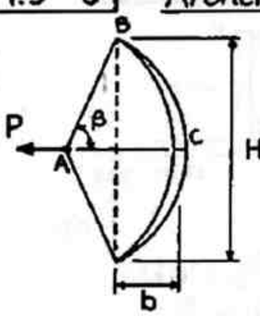
Free-body diagram



$\Sigma F_{vert} = 0$: $V = -125$ lb ⟵

$\Sigma M_D = 0$: $M = -400 - 125(2) = -650$ lb-ft ⟵

4.3-8 Archer's bow



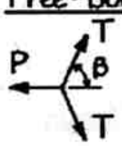
$P = 150$ N

$\beta = 70°$

$H = 1400$ mm
$= 1.4$ m

$b = 350$ mm
$= 0.35$ m

Free-body diagram of point A

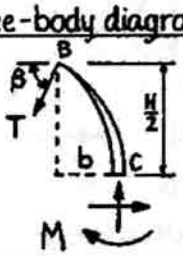


T = tensile force in the bowstring

$\Sigma F_{horiz} = 0$: $2T\cos\beta - P = 0$

$$T = \frac{P}{2\cos\beta}$$

Free-body diagram of segment BC



$\Sigma M_C = 0$ ↶+ ↷−

$T(\cos\beta)(\frac{H}{2}) + T(\sin\beta)(b) - M = 0$

$M = T(\frac{H}{2}\cos\beta + b\sin\beta)$

$= \frac{P}{2}(\frac{H}{2} + b\tan\beta)$

CONT.

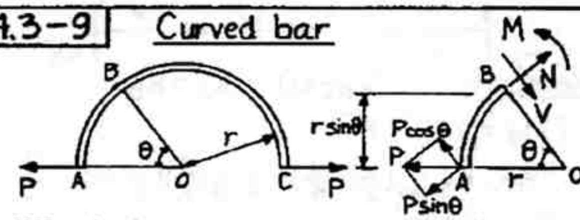
4.3-8 CONT.

Substitute numerical values:

$$M = \frac{150\text{N}}{2}\left[\frac{1.4\text{m}}{2} + (0.35\text{m})(\tan 70°)\right]$$

$M = 125$ N·m ⟵

4.3-9 Curved bar

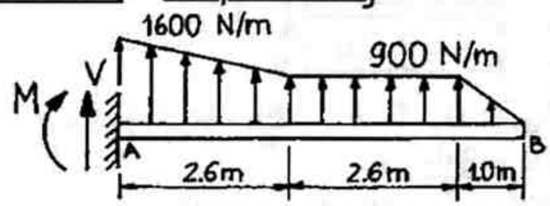


$\Sigma F_N = 0$ ↗+ $N - P\sin\theta = 0$ $N = P\sin\theta$ ⟵

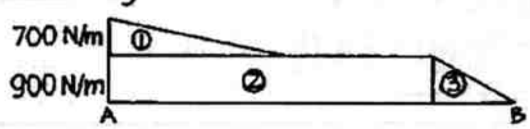
$\Sigma F_V = 0$ ↘+ $V - P\cos\theta = 0$ $V = P\cos\theta$ ⟵

$\Sigma M_O = 0$ ↶+ $M - Nr = 0$ $M = Nr = Pr\sin\theta$ ⟵

4.3-10 Airplane wing



Loading



Shear Force

$\Sigma F_{vert} = 0$ ↑+ ↓−

$V + \frac{1}{2}(700\text{ N/m})(2.6\text{m}) + (900\text{ N/m})(5.2\text{ m})$
$+ \frac{1}{2}(900\text{ N/m})(1.0\text{ m}) = 0$

$V = -6040\text{N} = -6.04$ kN ⟵

(Minus means the shear force acts downward.)

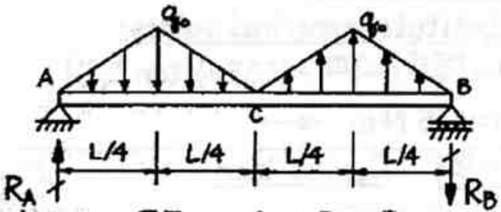
Bending Moment

$\Sigma M_A = 0$ ↶+ ↷−

$-M + \frac{1}{2}(700\text{ N/m})(2.6\text{ m})(\frac{2.6\text{ m}}{3})$
$+ (900\text{ N/m})(5.2\text{ m})(2.6\text{ m})$
$+ \frac{1}{2}(900\text{ N/m})(1.0\text{ m})(5.2\text{ m} + \frac{1.0\text{ m}}{3}) = 0$

$M = 788.67$ N·m + 12,168 N·m + 2490 N·m
$= 15{,}450$ N·m
$= 15.45$ kN·m ⟵

4.3-11 Simple beam with triangular loads



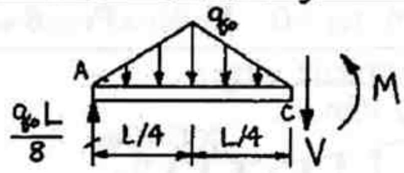
Reactions $\Sigma F_{vert} = 0 \quad \therefore R_A = R_B$

$\Sigma M_B = 0$

$$-R_A L + \frac{1}{2} q_0 \left(\frac{L}{2}\right)\left(\frac{3L}{4}\right) - \frac{1}{2} q_0 \left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{q_0 L}{8}$$

Shear force and bending moment at midpoint



$\Sigma F_{vert} = 0 \quad \frac{q_0 L}{8} - \frac{1}{2} q_0 \left(\frac{L}{2}\right) - V = 0$

$$V = -\frac{q_0 L}{8} \longleftarrow$$

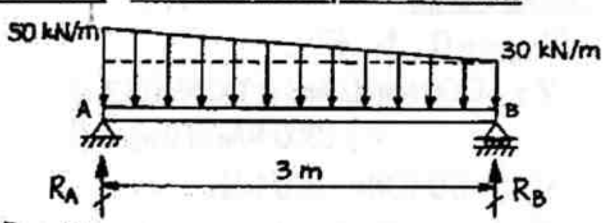
(Minus means V acts upward)

$\Sigma M_C = 0$

$$M - \frac{q_0 L}{8}\left(\frac{L}{2}\right) + \frac{1}{2} q_0 \left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$M = 0 \longleftarrow$$

4.3-12 Beam with trapezoidal load



Reactions

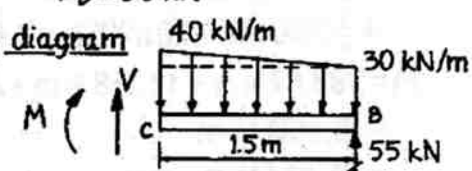
$\Sigma M_B = 0 \quad -R_A(3\text{m}) + (30\text{ kN/m})(3\text{m})(1.5\text{m}) + (20\text{ kN/m})(3\text{m})(1/2)(2\text{m}) = 0$

$R_A = 65\text{ kN}$

$\Sigma F_{vert} = 0 \quad R_A + R_B - \frac{1}{2}(50\text{ kN/m} + 30\text{ kN/m})(3\text{m}) = 0$

$R_B = 55\text{ kN}$

Free-body diagram



$\Sigma F_{vert} = 0 \quad V - (30\text{ kN/m})(1.5\text{m}) - \frac{1}{2}(10\text{ kN/m})(1.5\text{m}) + 55\text{ kN} = 0 \quad V = -2.5\text{ kN} \longleftarrow$

CONT.

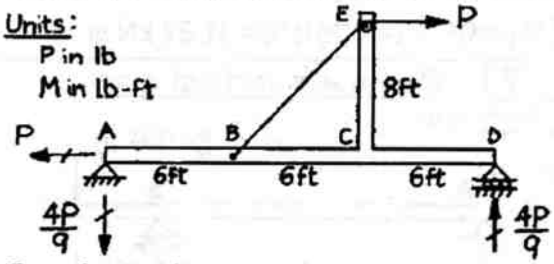
4.3-12 CONT.

$\Sigma M_C = 0 \quad -M - (30\text{ kN/m})(1.5\text{m})(0.75\text{m}) - \frac{1}{2}(10\text{ kN/m})(1.5\text{m})(0.5\text{m}) + (55\text{ kN})(1.5\text{m}) = 0$

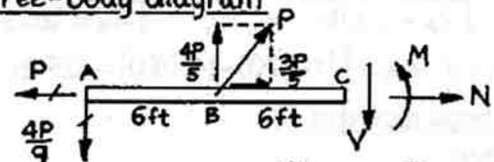
$M = 45.0\text{ kN·m} \longleftarrow$

4.3-13 Beam with a cable

Units:
P in lb
M in lb-ft



Free-body diagram



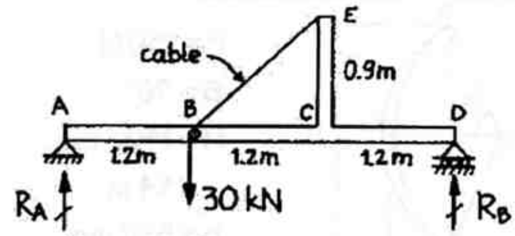
$\Sigma M_C = 0 \quad M - \frac{4P}{5}(6\text{ft}) + \frac{4P}{9}(12\text{ft}) = 0$

$M = -\frac{8P}{15}\text{ lb-ft}$

Numerical value of M equals 800 lb-ft.

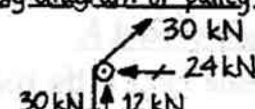
$\therefore \frac{8P}{15}\text{ lb-ft} = 800\text{ lb-ft} \quad P = 1500\text{ lb} \longleftarrow$

4.3-14 Beam with cable and weight

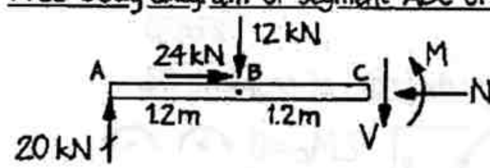


$R_A = 20\text{ kN} \qquad R_B = 10\text{ kN}$

Free-body diagram of pulley at B



Free-body diagram of segment ABC of beam

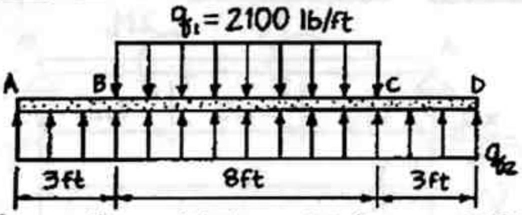


$\Sigma F_{horiz} = 0 \qquad N = 24\text{ kN} \longleftarrow$

$\Sigma F_{vert} = 0 \qquad V = 8.0\text{ kN} \longleftarrow$

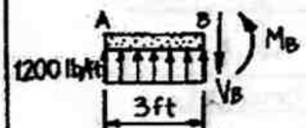
$\Sigma M_C = 0 \qquad M = 33.6\text{ kN·m} \longleftarrow$

4.3-15 Concrete foundation beam



$\Sigma F_{VERT} = 0 \quad q_2(14\text{ ft}) = q_1(8\text{ ft}) \quad q_2 = 1200\text{ lb/ft}$

(a) V and M at point B



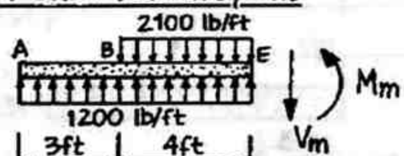
$\Sigma F_{VERT} = 0$

$V_B = 3600\text{ lb} \longleftarrow$

$\Sigma M_B = 0 \quad M_B = (1200\text{ lb/ft})(3\text{ ft})(1.5\text{ ft})$

$= 5400\text{ lb-ft} \longleftarrow$

(b) V and M at midpoint



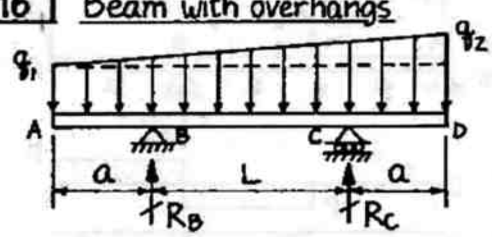
$\Sigma F_{VERT} = 0 \quad V_m = (1200\text{ lb/ft})(7\text{ft}) - (2100\text{ lb/ft})(4\text{ft})$

$V_m = 0 \longleftarrow$

$\Sigma M_E = 0 \quad M_m = (1200\text{ lb/ft})(7\text{ft})(3.5\text{ ft}) - (2100\text{ lb/ft})(4\text{ft})(2\text{ ft})$

$M_m = 12{,}600\text{ lb-ft} \longleftarrow$

4.3-16 Beam with overhangs



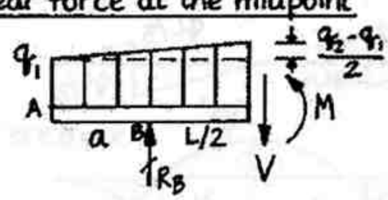
Reaction at B $\quad \Sigma M_C = 0$

$$-R_B L + q_1(L+2a)\left(\frac{L}{2}\right) + \frac{1}{2}(q_2 - q_1)(L+2a)\left(\frac{L+2a}{3} - a\right) = 0$$

Solve for R_B:

$$R_B = \frac{L+2a}{6}\left[q_1\left(2+\frac{a}{L}\right) + q_2\left(1+\frac{a}{L}\right)\right]$$

Shear force at the midpoint



CONT.

4.3-16 CONT.

$\Sigma F_{VERT} = 0$

$$V = R_B - q_1\left(a+\frac{L}{2}\right) - \left(\frac{q_2-q_1}{2}\right)\left(\frac{1}{2}\right)\left(a+\frac{L}{2}\right)$$

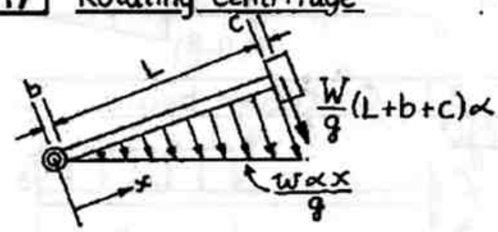
Substitute for R_B and simplify:

$$V = \frac{1}{24}(L+2a)(q_1 - q_2)\left(\frac{4a}{L} - 1\right)$$

V is always zero when $\frac{4a}{L} - 1 = 0$ and

$$\frac{a}{L} = \frac{1}{4} \longleftarrow$$

4.3-17 Rotating centrifuge



Tangential acceleration $= r\alpha$

Inertial force $= Mr\alpha = \frac{W}{g}r\alpha$

Maximum V and M occur at $x = b$

$$V_{max} = \frac{W}{g}(L+b+c)\alpha + \int_b^{L+b}\frac{w\alpha}{g}x\,dx$$

$$= \frac{W\alpha}{g}(L+b+c) + \frac{wL\alpha}{2g}(L+2b) \longleftarrow$$

$$M_{max} = \frac{W\alpha}{g}(L+b+c)(L+c) + \int_b^{L+b}\frac{w\alpha}{g}x(x-b)\,dx$$

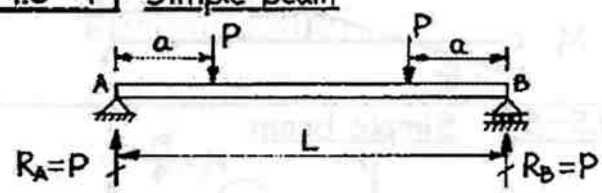
$$= \frac{W\alpha}{g}(L+b+c)(L+c) + \frac{wL^2\alpha}{6g}(2L+3b) \longleftarrow$$

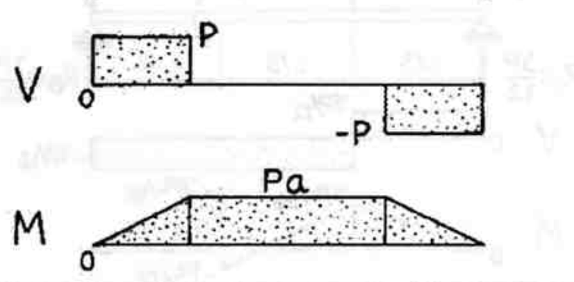
If $W = 2.5wL$, $b = L/9$, and $c = L/10$:

$$V_{max} = \frac{131\,wL^2\alpha}{36\,g} \longleftarrow$$

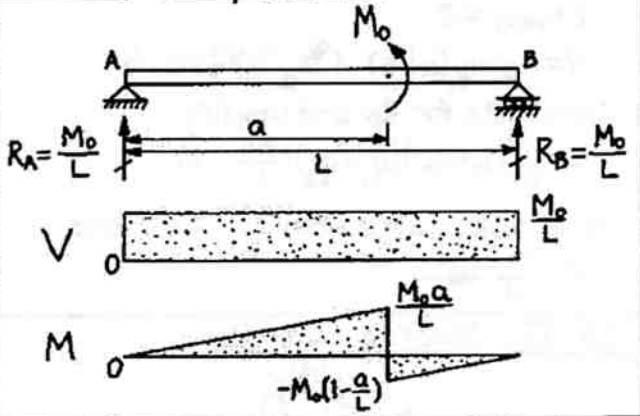
$$M_{max} = \frac{1339\,wL^3\alpha}{360\,g} \longleftarrow$$

4.5-1 Simple beam

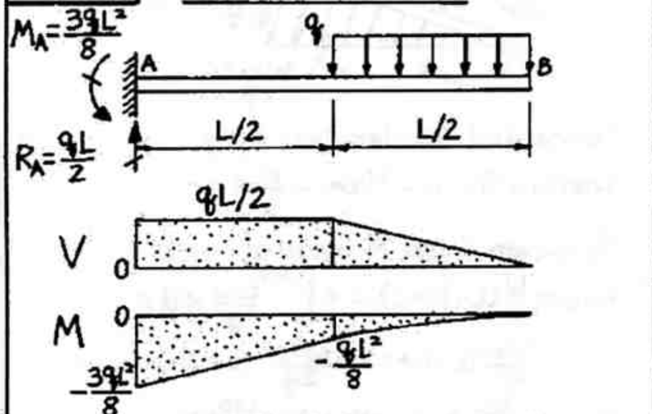




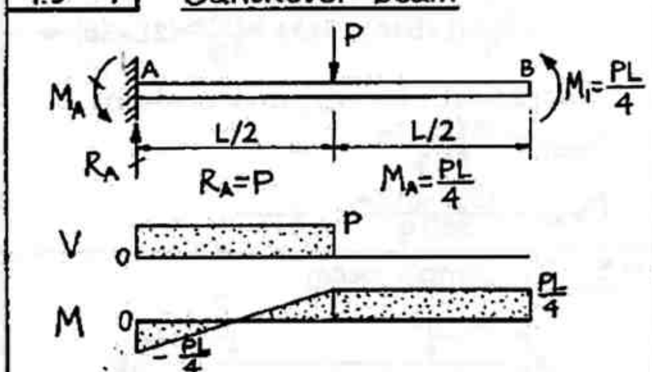
4.5-2 Simple beam



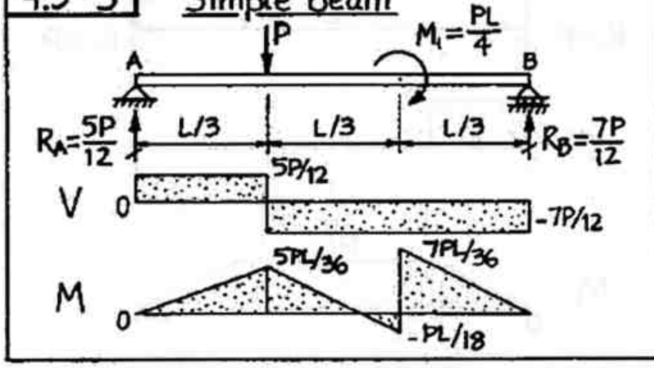
4.5-3 Cantilever beam



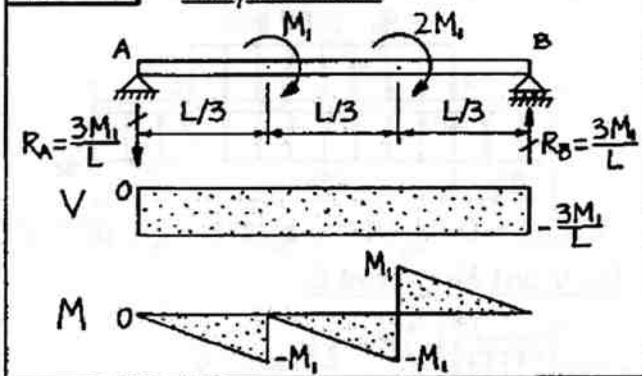
4.5-4 Cantilever beam



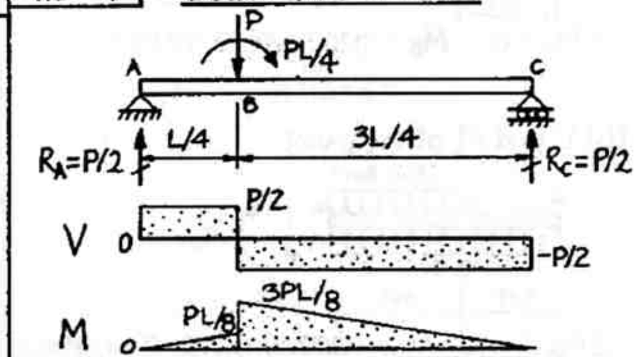
4.5-5 Simple beam



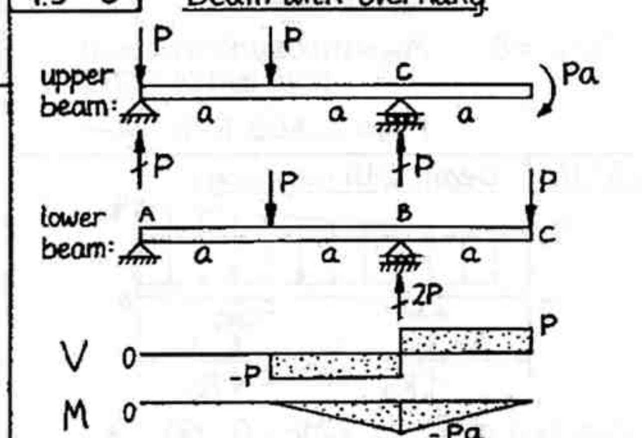
4.5-6 Simple beam



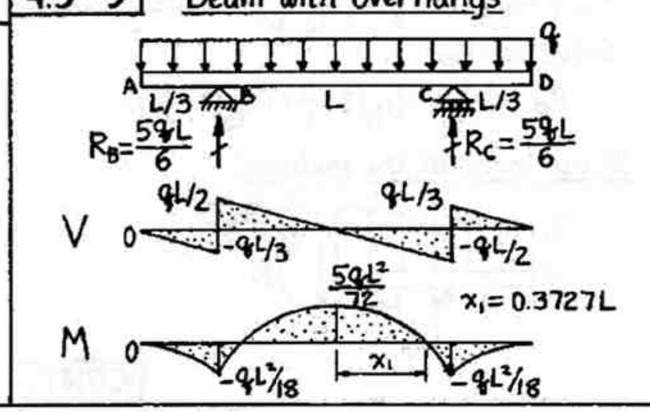
4.5-7 Beam with bracket



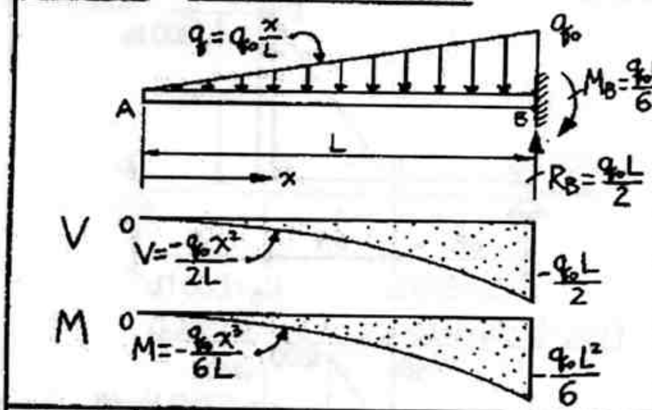
4.5-8 Beam with overhang



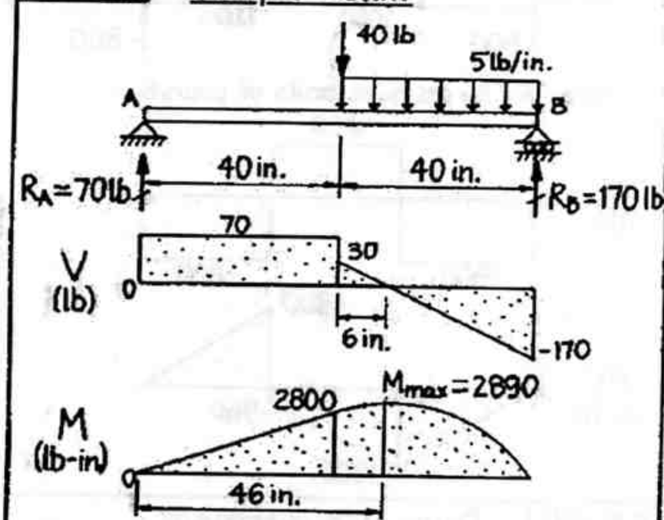
4.5-9 Beam with overhangs



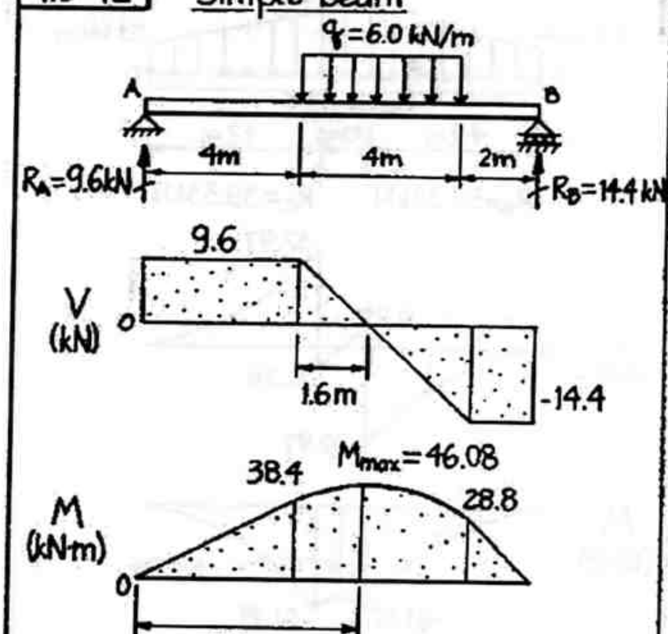
4.5-10 Cantilever beam



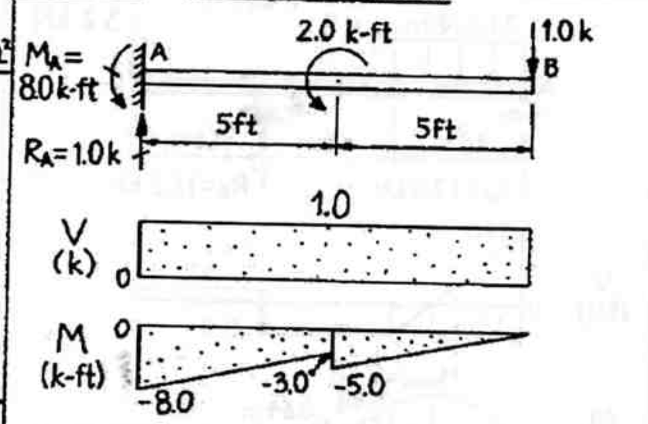
4.5-11 Simple beam



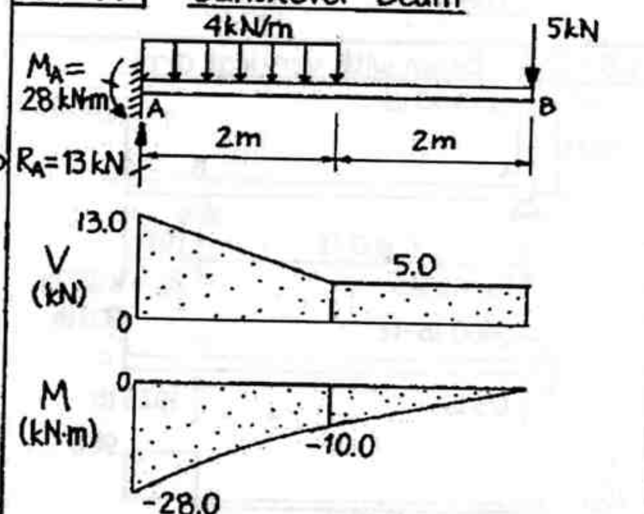
4.5-12 Simple beam



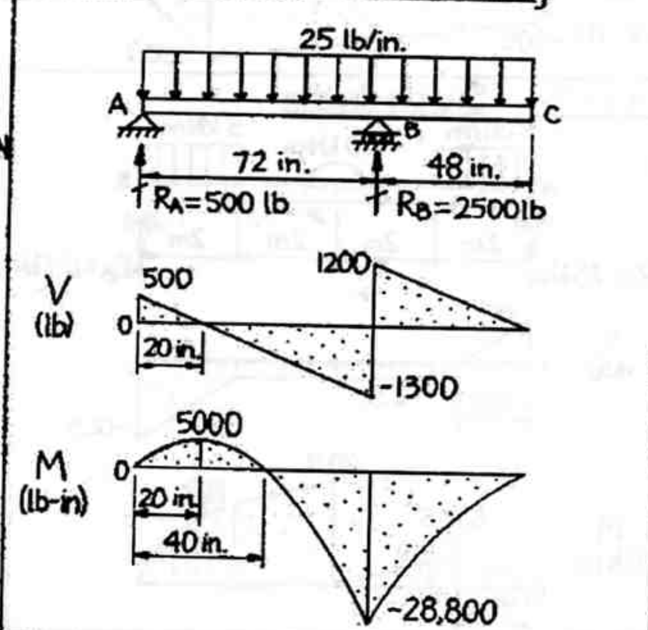
4.5-13 Cantilever beam



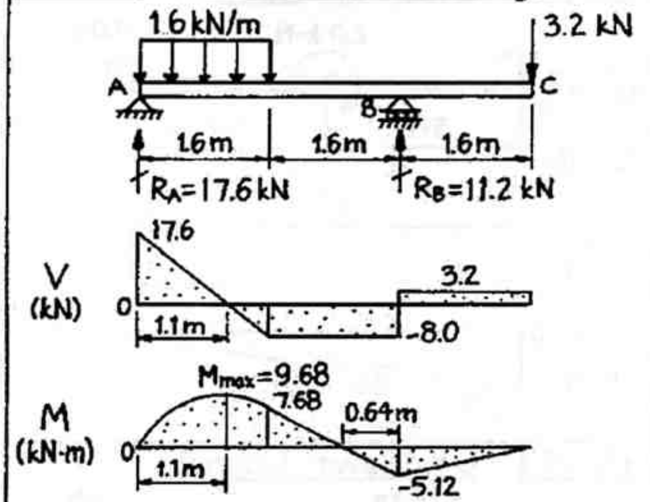
4.5-14 Cantilever beam



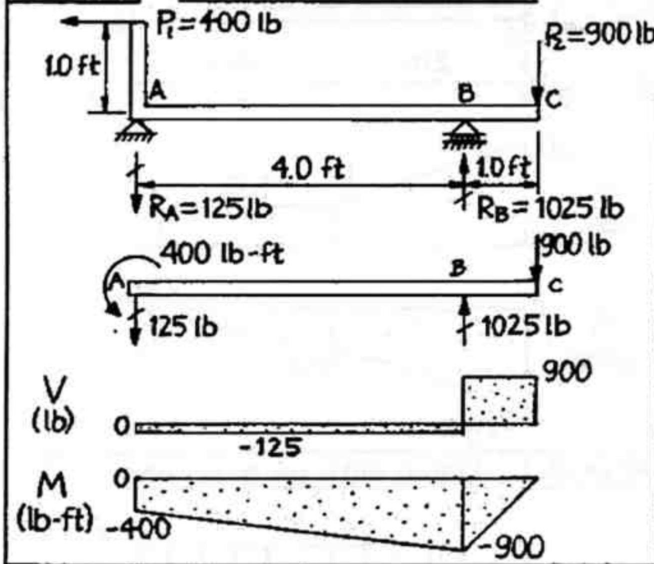
4.5-15 Beam with an overhang



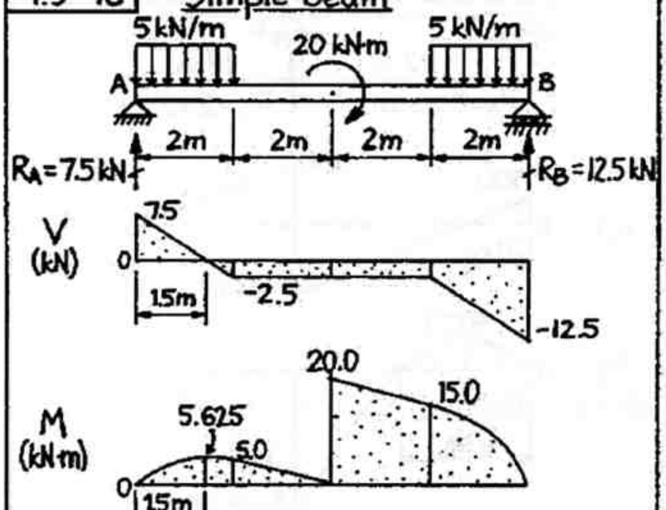
4.5-16 Beam with an overhang



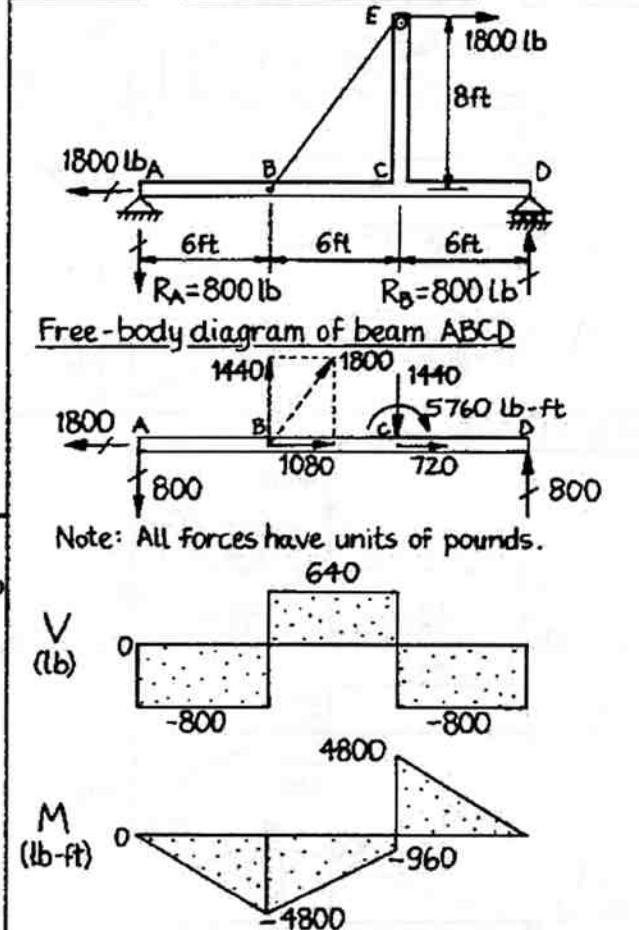
4.5-17 Beam with vertical arm



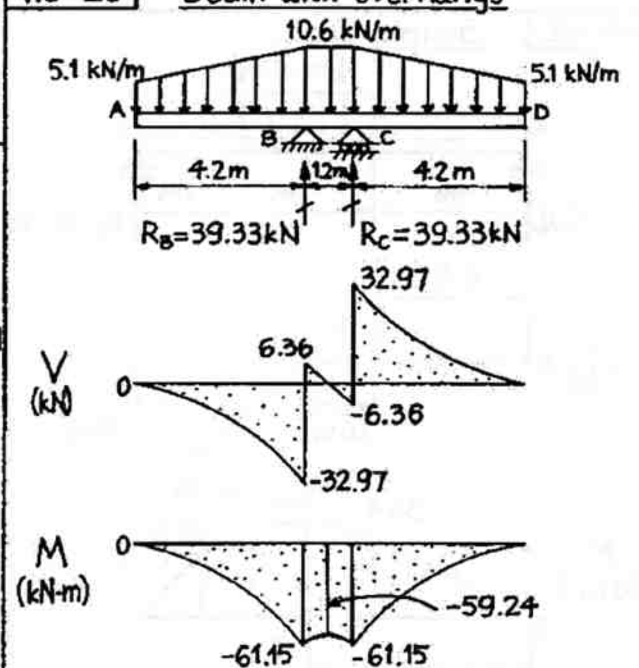
4.5-18 Simple beam



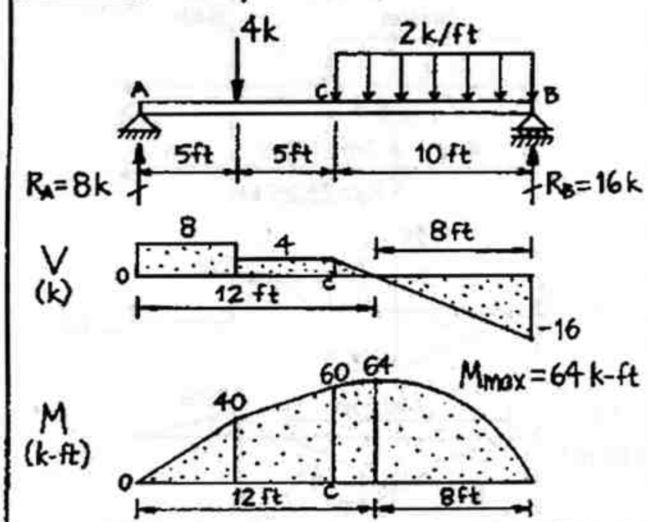
4.5-19 Beam with a cable



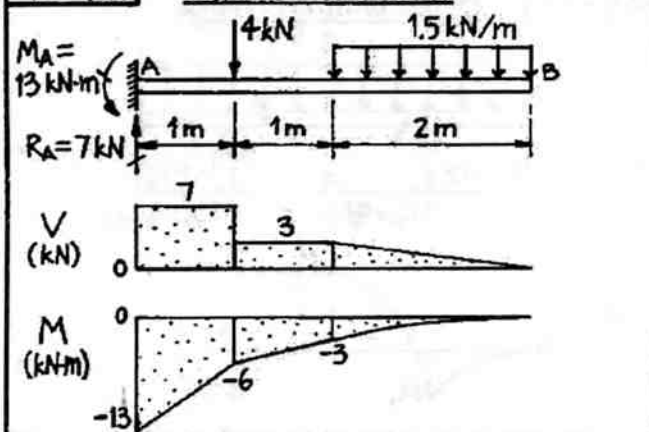
4.5-20 Beam with overhangs



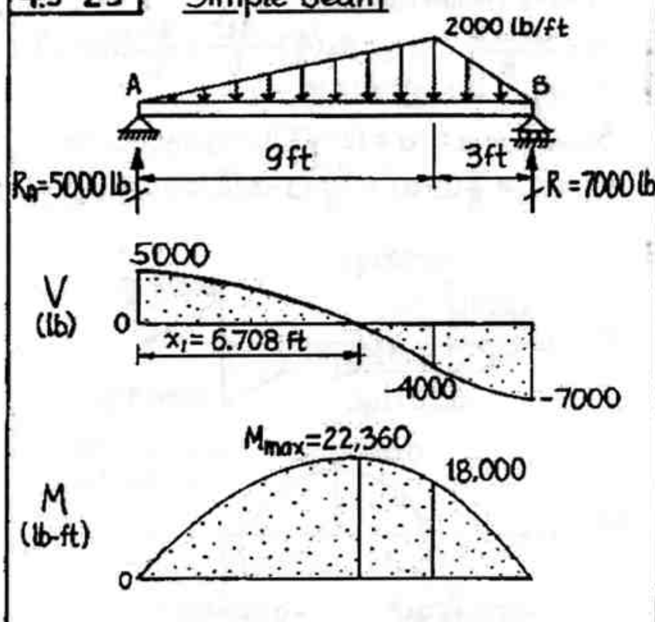
4.5-21 Simple beam



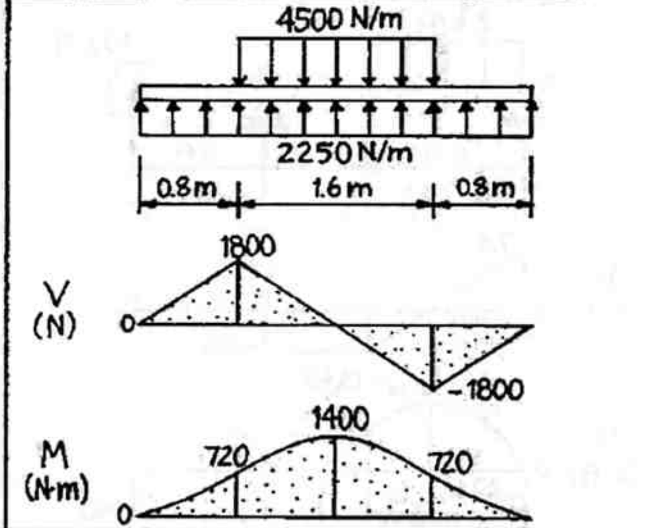
4.5-22 Cantilever beam



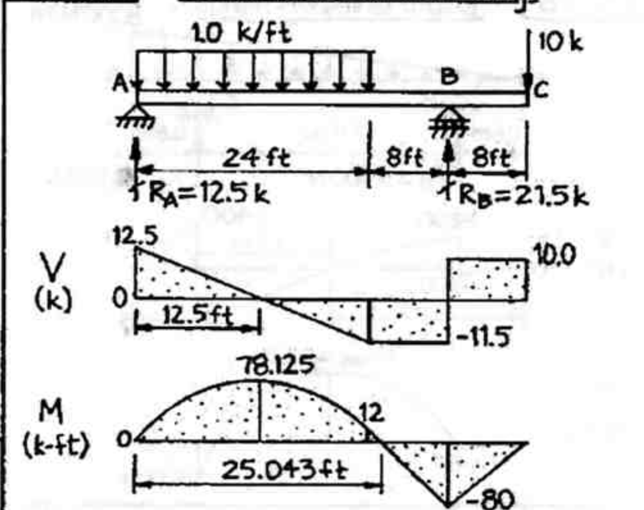
4.5-23 Simple beam



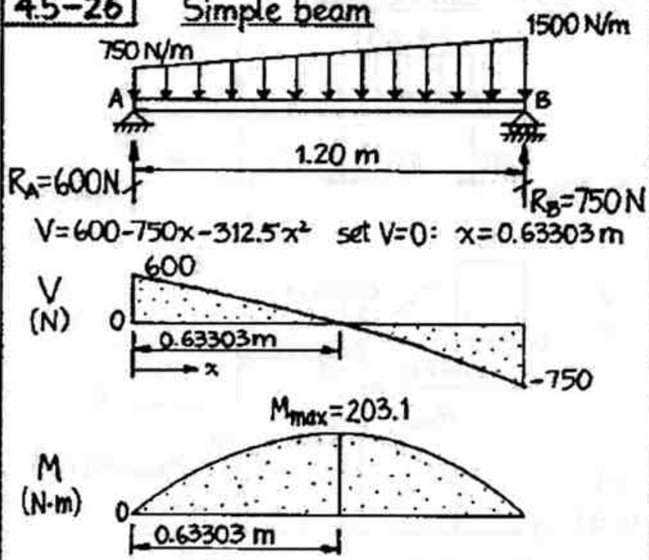
4.5-24 Beam with distributed loads



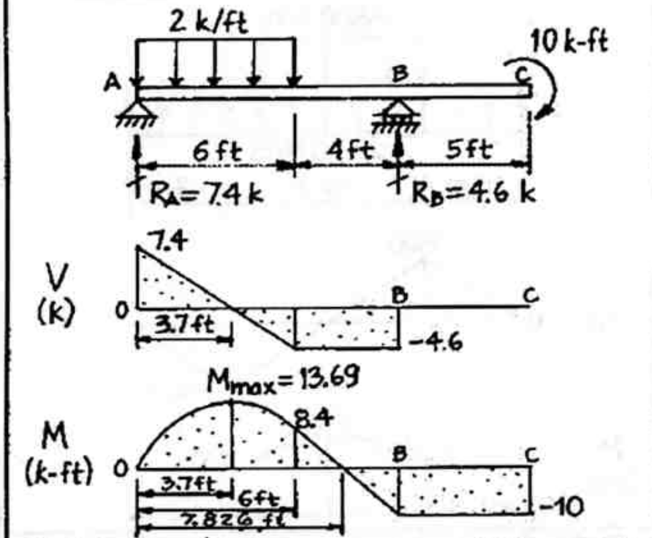
4.5-25 Beam with an overhang



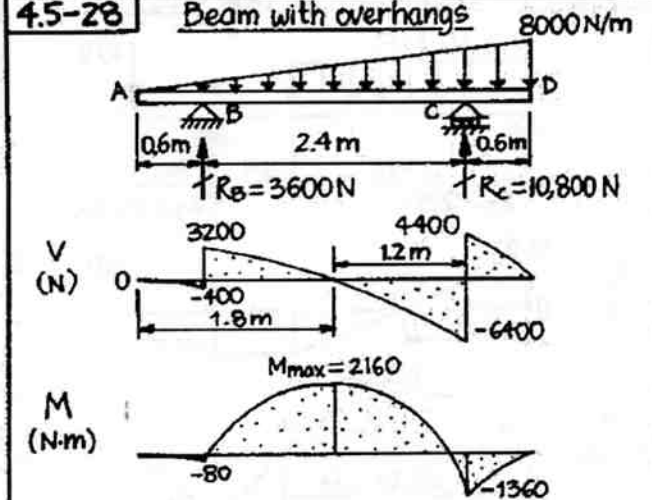
4.5-26 Simple beam



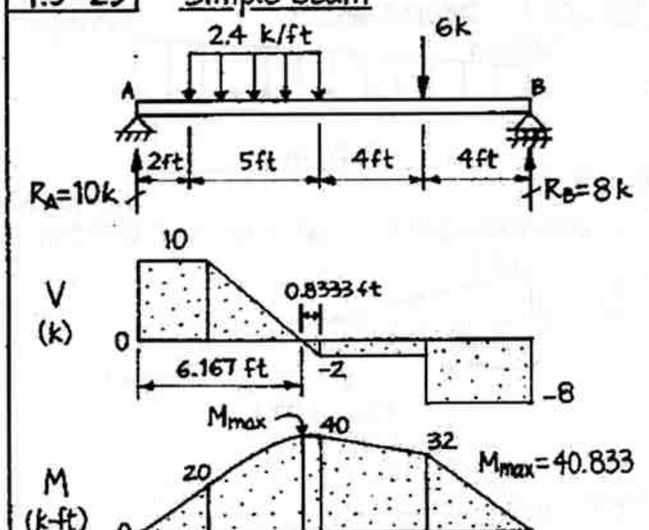
4.5-27 Beam with an overhang



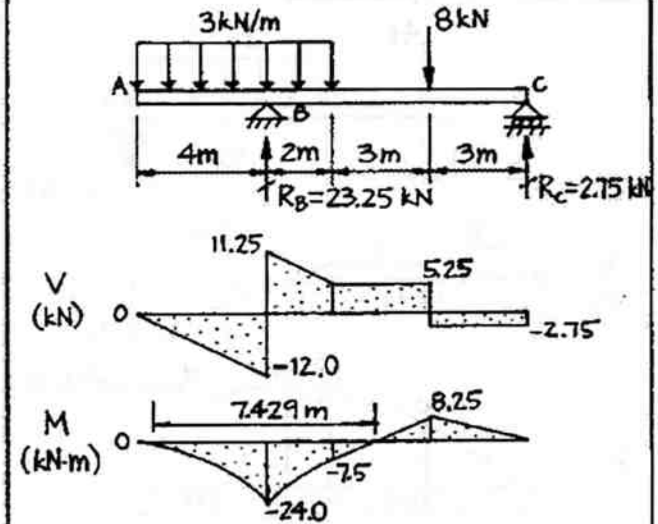
4.5-28 Beam with overhangs



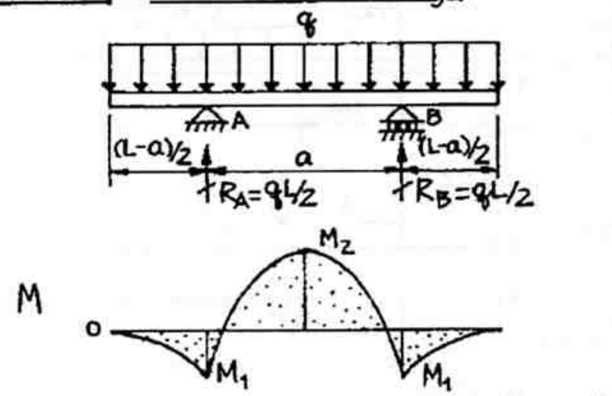
4.5-29 Simple beam



4.5-30 Beam with an overhang



4.5-31 Beam with overhangs



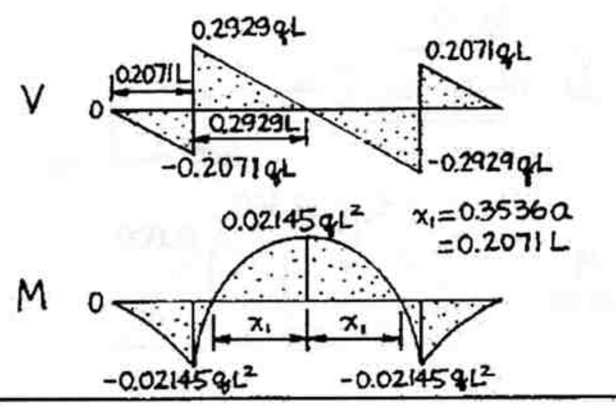
The maximum bending moment is smallest when $M_1 = M_2$ (numerically).

$$M_1 = \frac{q(L-a)^2}{8} \qquad M_2 = R_A\left(\frac{a}{2}\right) - \frac{qL^2}{8} = \frac{qL}{8}(2a - L)$$

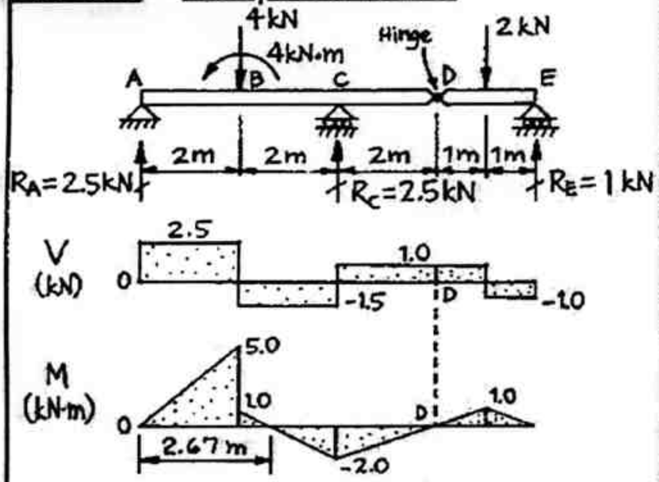
$$M_1 = M_2 \qquad (L-a)^2 = L(2a - L)$$

Solve for a: $a = (2 - \sqrt{2})L = 0.5858L$

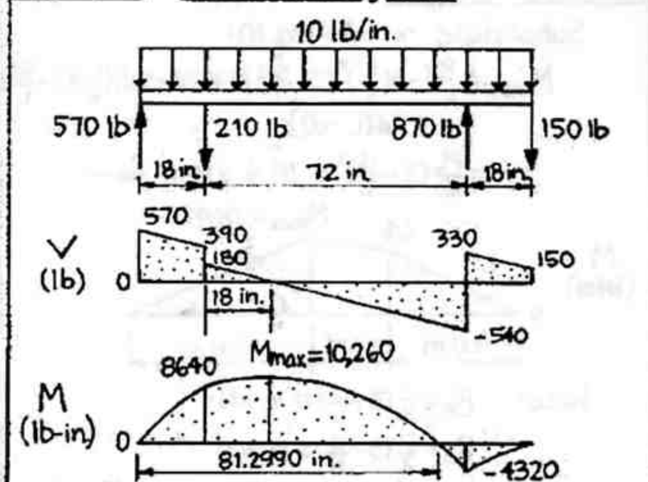
$$M_1 = M_2 = \frac{q}{8}(L-a)^2 = \frac{qL^2}{8}(3 - 2\sqrt{2}) = 0.02145qL^2$$



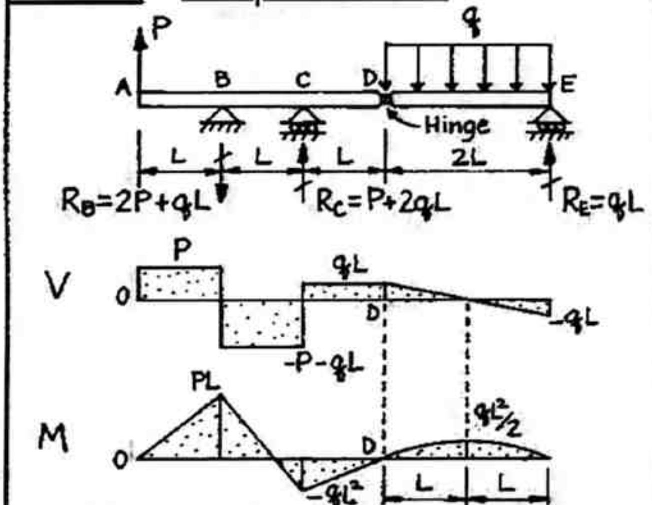
4.5-32 Compound beam



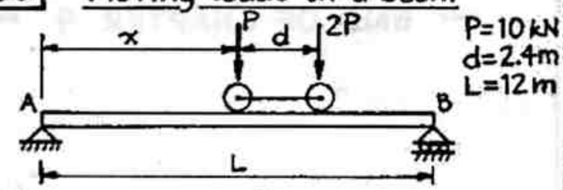
4.5-35 Beam (V is given)



4.5-33 Compound beam

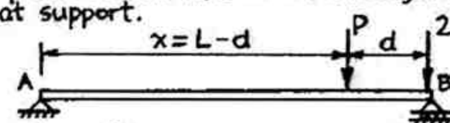


4.5-36 Moving loads on a beam



(a) Maximum shear force

By inspection, the maximum shear force occurs at support B when the larger load is placed close to, but not directly over, that support.



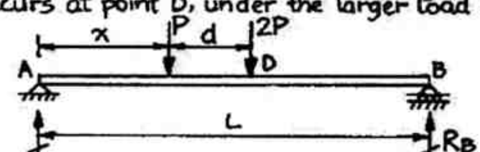
$R_A = \frac{Pd}{L}$ $\qquad R_B = P\left(3 - \frac{d}{L}\right)$

$x = L - d = 9.6$ m

$V_{max} = R_B = P\left(3 - \frac{d}{L}\right) = 28$ kN

(b) Maximum bending moment

By inspection, the maximum bending moment occurs at point D, under the larger load 2P.



Reaction at support B:

$$R_B = \frac{P}{L}x + \frac{2P}{L}(x+d) = \frac{P}{L}(2d+3x)$$

Bending moment at D:

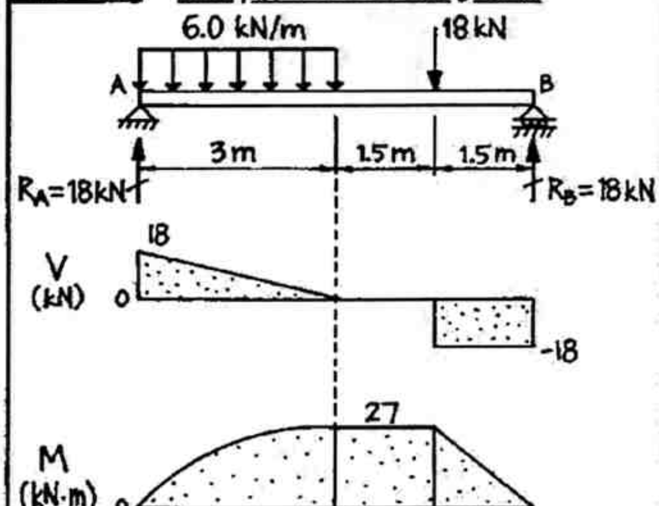
$$M_D = R_B(L-x-d) = \frac{P}{L}(2d+3x)(L-x-d)$$

$$= \frac{P}{L}\left[-3x^2 + (3L-5d)x + 2d(L-d)\right] \qquad \text{Eq.(1)}$$

$$\frac{dM_D}{dx} = \frac{P}{L}(-6x + 3L - 5d) = 0$$

Solve for x: $x = \frac{L}{6}\left(3 - \frac{5d}{L}\right) = 4.0$ m

4.5-34 Simple beam (V is given)



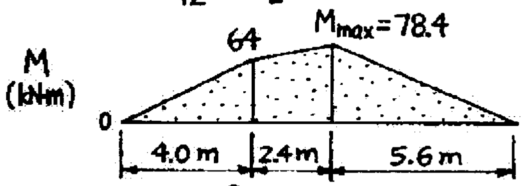
CONT.

4.5-36 CONT.

Substitute x into Eq. (1):

$$M_{max} = \frac{P}{L}\left[-3\left(\frac{L}{6}\right)^2\left(3-\frac{5d}{L}\right)^2 + (3L-5d)\left(\frac{L}{6}\right)\left(3-\frac{5d}{L}\right) + 2d(L-d)\right]$$

$$= \frac{PL}{12}\left(3-\frac{d}{L}\right)^2 = 78.4 \text{ kN·m}$$



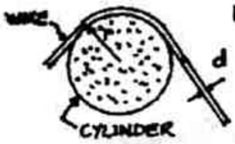
Note: $R_A = \frac{P}{2}\left(3+\frac{d}{L}\right) = 16$ kN

$R_B = \frac{P}{2}\left(3-\frac{d}{L}\right) = 14$ kN

— END OF CHAPTER 4 —

5.4-1

STEEL WIRE



$r = 20$ in. $\quad d = \frac{1}{16}$ in.

From Eq. (5-4):

$$\epsilon_{max} = \frac{y}{\rho} = \frac{d/2}{r + d/2} = \frac{d}{2r + d}$$

Substitute numerical values:

$\epsilon_{max} = 1560 \times 10^{-6}$ ⟵

5.4-2

COPPER WIRE

$d = 3$ mm $\quad \epsilon_{max} = 0.004$

$L = 2\pi\rho \quad \rho = \frac{L}{2\pi}$

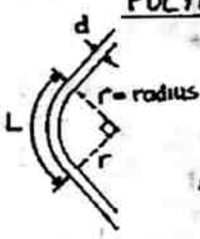
From Eq. (5-4):

$$\epsilon_{max} = \frac{y}{\rho} = \frac{d/2}{L/2\pi} = \frac{\pi d}{L}$$

$L_{min} = \frac{\pi d}{\epsilon_{max}} = \frac{\pi(3\text{ mm})}{0.004} = 2.36$ m ⟵

5.4-3

POLYETHYLENE PIPE



L = length of 90° bend

$L = 45$ ft. $= 540$ in. $\quad d = 4$ in.

$L = \frac{2\pi r}{4} = \frac{\pi r}{2}$

Angle equals 90° or $\pi/2$ radians.

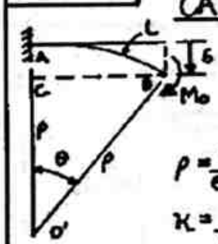
$r = \rho$ = radius of curvature

$$\rho = \frac{L}{\pi/2} = \frac{2L}{\pi} \quad \epsilon_{max} = \frac{y}{\rho} = \frac{d/2}{2L/\pi}$$

$\epsilon_{max} = \frac{\pi d}{4L} = \frac{\pi}{4}\left(\frac{4\text{ in.}}{540\text{ in.}}\right) = 5820 \times 10^{-6}$ ⟵

5.4-4

CANTILEVER BEAM



L = length of beam

$L = 1.2$ m $\quad \epsilon_{max} = 0.0008$

$y = 50$ mm $\quad \epsilon_{max} = \frac{y}{\rho}$

$\rho = \frac{y}{\epsilon_{max}} = \frac{50\text{ mm}}{0.0008} = 62.5$ m ⟵

$\kappa = \frac{1}{\rho} = 0.0160$ m^{-1} ⟵

Assume that the deflection curve is nearly flat. Then the distance BC is the same as the length L of the beam.

$\therefore \sin\theta = \frac{L}{\rho} = \frac{1.2\text{ m}}{62.5\text{ m}} = 0.0192$

$\theta = \arcsin 0.0192 = 0.0192$ rad

$\delta = \rho(1 - \cos\theta) = (62.5\text{ m})(1 - \cos(0.0192\text{ rad})) = 11.5$ mm ⟵

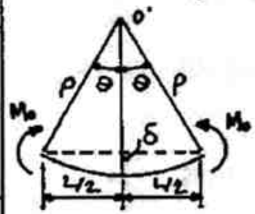
Note: $\frac{L}{\delta} = 104$, which confirms that the deflection curve is nearly flat.

5.4-5

STRIP OF STEEL

$L = 36$ in. $\quad t = 0.4$ in. $\quad \delta = 0.29$ in.

Note that the deflection curve is nearly flat ($L/\delta = 124$) and θ is a very small angle.



$\sin\theta = \frac{L/2}{\rho}$

$\theta = \frac{L/2}{\rho}$ (radians)

$\delta = \rho(1 - \cos\theta)$

$= \rho\left(1 - \cos\frac{L}{2\rho}\right)$

Substitute numerical values (ρ = inches)

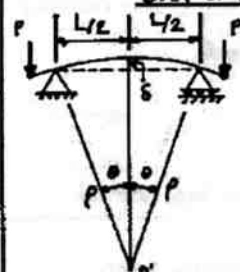
$0.29\text{ in.} = \rho\left(1 - \cos\frac{36\text{ in.}}{2\rho}\right)$. Solve numerically: $\rho = 559$ in.

$\epsilon = \frac{y}{\rho} = \frac{t/2}{\rho} = \frac{0.2\text{ in.}}{559\text{ in.}} = 358 \times 10^{-6}$ ⟵

(Shortening at the top surface)

5.4-6

BAR OF RECTANGULAR CROSS SECTION



$L = 1.5$ m $\quad h = 120$ mm $\quad \delta = 2.5$ mm

Note that the deflection curve is nearly flat ($L/\delta = 600$) and θ is a very small angle.

$\sin\theta = \frac{L/2}{\rho}$

$\theta = \frac{L/2}{\rho}$ (radians)

$\delta = \rho(1 - \cos\theta) = \rho\left(1 - \cos\frac{L}{2\rho}\right)$

Substitute numerical values:

$0.0025\text{ m} = \rho\left(1 - \cos\frac{1.5\text{ m}}{2\rho}\right)$ (ρ = meters)

Solve numerically: $\rho = 112.5$ m

$\epsilon = \frac{y}{\rho} = \frac{h/2}{\rho} = \frac{60\text{ mm}}{112.5\text{ m}} = 533 \times 10^{-6}$ ⟵

5.5-1

STEEL WIRE BENT AROUND A PULLEY

$E = 29 \times 10^6$ psi $\quad d = \frac{1}{32}$ in. $\quad R_0 = 11\frac{5}{16}$ in.

(a) Maximum stress in wire

$\rho = R_0 + \frac{d}{2} = \frac{181}{16} + \frac{1}{64} = \frac{725}{64}$ in.

$y = \frac{d}{2} = \frac{1}{64}$ in.

From Eq. (5-7):

$\sigma_{max} = \frac{Ey}{\rho} = \frac{(29 \times 10^6\text{ psi})(1/64\text{ in.})}{\frac{725}{64}\text{ in.}} = 40,000$ psi ⟵

(b) Change in stress

If the radius increases, the stress σ_{max} decreases. ⟵

5.5-2

COPPER STRIP BENT INTO A CIRCLE

$E = 113$ GPa $\quad L = 2$ m $\quad t = 2$ mm

(a) Maximum stress in strip

$L = 2\pi r = 2\pi\rho \quad \rho = \frac{L}{2\pi}$

CONT.

5.5-2 CONT.

From Eq. (5-7):

$$\sigma = \frac{Ey}{\rho} = \frac{2\pi E y}{L}$$

$$\sigma_{max} = \frac{2\pi E(t/2)}{L} = \frac{\pi E t}{L}$$

Substitute numerical values:

$$\sigma_{max} = \frac{\pi (113\text{ GPa})(2\text{ mm})}{2\text{ m}} = 355\text{ MPa} \longleftarrow$$

(b) Change in stress

If the thickness t increases, the stress σ_{max} increases. $\longleftarrow$

5.5-3

STEEL RULE BENT INTO AN ARC

$E = 29 \times 10^6$ psi $t = 0.10$ in. $L = 30$ in. $\alpha = 60° = 1.0472$ rad

(a) Maximum bending stress

$\alpha\rho = L$ $\rho = \frac{L}{\alpha}$

$$\sigma_{max} = \frac{Ey}{\rho} = \frac{E(t/2)}{L/\alpha} = \frac{Et\alpha}{2L} \quad (\alpha = \text{radians})$$

$$\sigma_{max} = \frac{(29 \times 10^6\text{ psi})(0.10\text{ in})(1.0472\text{ rad})}{2(30\text{ in})}$$

$$= 50{,}600\text{ psi} \longleftarrow$$

(b) Change in stress $\longleftarrow$

If the angle α increases, the stress σ_{max} increases.

5.5-4

SIMPLE BEAM WITH UNIFORM LOAD

$L = 3.75$ m $q = 6.4$ kN/m $b = 150$ mm $h = 300$ mm

$M_{max} = \frac{qL^2}{8}$ $S = \frac{bh^2}{6}$

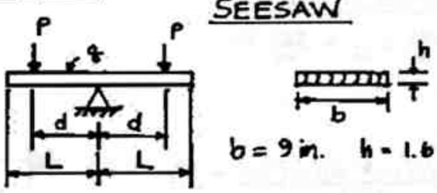
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{6M_{max}}{bh^2} = \frac{3qL^2}{4bh^2}$$

Substitute numerical values:

$$\sigma_{max} = \frac{3(6.4\text{ kN/m})(3.75\text{ m})^2}{4(150\text{ mm})(300\text{ mm})^2} = 5.0\text{ MPa} \longleftarrow$$

5.5-5

SEESAW



$b = 9$ in. $h = 1.6$ in.

$q = 4$ lb/ft $P = 80$ lb $d = 8.5$ ft $L = 10$ ft

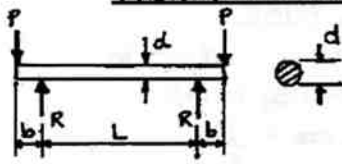
$$M_{max} = Pd + \frac{qL^2}{2} = 680\text{ lb-ft} + 200\text{ lb-ft}$$
$$= 880\text{ lb-ft} = 10{,}560\text{ lb-in.}$$

$$S = \frac{bh^2}{6} = 3.84\text{ in}^3$$

$$\sigma_{max} = \frac{M}{S} = \frac{10{,}560\text{ lb-in.}}{3.84\text{ in}^3} = 2750\text{ psi} \longleftarrow$$

5.5-6

FREIGHT-CAR AXLE



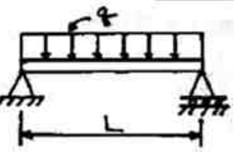
$d = 80$ mm $L = 1.45$ m $b = 200$ mm $P = 46.5$ kN

$M_{max} = Pb$ $S = \frac{\pi d^3}{32}$ $\sigma_{max} = \frac{M_{max}}{S}$

$$\sigma_{max} = \frac{32Pb}{\pi d^3} = \frac{32(46.5\text{ kN})(200\text{ mm})}{\pi(80\text{ mm})^3} = 185.0\text{ MPa} \longleftarrow$$

5.5-7

BRIDGE GIRDER



$L = 170$ ft $q = 1.2$ k/ft

$S = 2890\text{ in}^3$

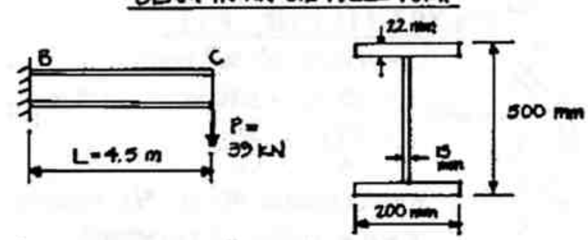
$M_{max} = \frac{qL^2}{8}$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{qL^2}{8S}$$

$$\sigma_{max} = \frac{(1.2\text{ k/ft})(170\text{ ft})^2(12\text{ in/ft})}{8(2890\text{ in}^3)} = 18\text{ ksi} \longleftarrow$$

5.5-8

BEAM IN AN OIL-WELL PUMP



$$M_{max} = PL = (39\text{ kN})(4.5\text{ m}) = 175.5\text{ kN·m}$$

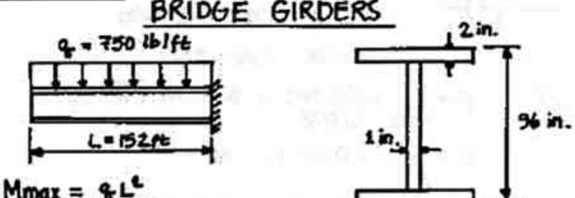
$$I = \frac{1}{12}(200\text{ mm})(500\text{ mm})^3 - \frac{1}{12}(185\text{ mm})(456\text{ mm})^3$$

$$= 621.5 \times 10^6\text{ mm}^4 \quad c = \frac{500\text{ mm}}{2} = 250\text{ mm}$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{(175.5\text{ kN·m})(250\text{ mm})}{621.5 \times 10^6\text{ mm}^4} = 70.6\text{ MPa} \longleftarrow$$

5.5-9

BRIDGE GIRDERS



$$M_{max} = \frac{qL^2}{2}$$
$$= \frac{(750\text{ lb/ft})(152\text{ ft})^2}{2} = 8.664 \times 10^6\text{ lb-ft}$$
$$= 103.97 \times 10^6\text{ lb-in.}$$

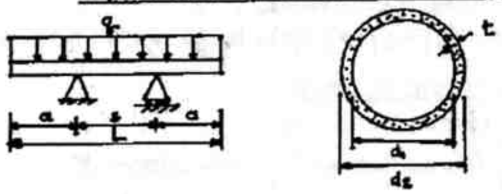
$$I = \frac{1}{12}(24\text{ in})(96\text{ in})^3 - \frac{1}{12}(23\text{ in})(92\text{ in})^3 = 277.0 \times 10^3\text{ in}^4$$

$$c = \frac{96\text{ in.}}{2} = 48\text{ in.}$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{(103.97 \times 10^6\text{ lb-in})(48\text{ in})}{277.0 \times 10^3\text{ in}^4} = 18{,}020\text{ psi} \longleftarrow$$

5.5-10

PIPE HOISTED BY A CRANE



$d_2 = 150$ mm $\quad t = 6$ mm $\quad d_1 = 138$ mm $\quad s = 4$ m

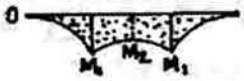
$\gamma = 18\ \text{kN/m}^3 \quad L = 13$ m $\quad a = (L-s)/2 = 4.5$ m

$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2714\ \text{mm}^2 \quad c = \frac{d_2}{2} = 75$ mm

$q = \gamma A = (18\ \text{kN/m}^3)(2714\ \text{mm}^2) = 48.85$ N/m

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 7048 \times 10^3\ \text{mm}^4$

Bending-moment diagram



$M_1 = -\frac{qa^2}{2} = -494.6$ N·m

$M_2 = -\frac{qL}{4}\left(\frac{L}{2} - s\right) = -396.9$ N·m

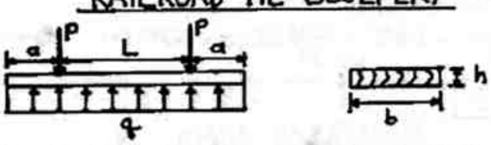
$M_{max} = 494.6$ N·m

Maximum stress

$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{(494.6\ \text{N·m})(75\ \text{mm})}{7048 \times 10^3\ \text{mm}^4} = 5.26$ MPa ←

5.5-11

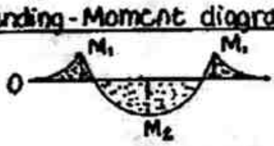
RAILROAD TIE (SLEEPER)



$P = 36$ k $\quad b = 12$ in. $\quad h = 10$ in. $\quad L = 57$ in. $\quad a = 19.5$ in.

$S = \frac{bh^2}{6} = 200\ \text{in}^3 \quad q = \frac{2P}{L + 2a} = 750$ lb/in.

Bending-Moment diagram



$M_1 = \frac{qa^2}{2} = 142.6 \times 10^3$ lb-in.

$M_2 = -\frac{q}{8}(L^2 - 4a^2)$

$= -162 \times 10^3$ lb-in.

$M_{max} = 162 \times 10^3$ lb-in.

Maximum bending stress

$\sigma_{max} = \frac{M_{max}}{S} = \frac{162 \times 10^3\ \text{lb-in.}}{200\ \text{in}^3} = 810$ psi ←

5.5-12

VERTICAL WOOD BEAM

$h = 2.4$ m $\quad t = 150$ mm $\quad \gamma = 9.81\ \text{kN/m}^3$ (water)

Let b = width of beam perpendicular to the paper.

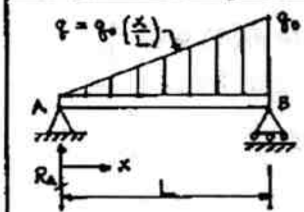
Let q_0 = maximum intensity of distributed load.

$q_0 = \gamma b h \quad S = \frac{bt^2}{6}$

CONT.

5.5-12 CONT.

Maximum bending moment



$R_A = \frac{q_0 L}{6}$

$M = R_A x - \frac{q_0 x^3}{6L}$

$= \frac{q_0 L x}{6} - \frac{q_0 x^3}{6L}$

$\frac{dM}{dx} = \frac{q_0 L}{6} - \frac{q_0 x^2}{2L} = 0 \qquad x = \frac{L}{\sqrt{3}}$

Substitute $x = L/\sqrt{3}$ into the equation for M:

$M_{max} = \frac{q_0 L}{6}\left(\frac{L}{\sqrt{3}}\right) - \frac{q_0}{6L}\left(\frac{L^3}{3\sqrt{3}}\right) = \frac{q_0 L^2}{9\sqrt{3}}$

For the vertical wood beam: $L = h$; $M_{max} = \frac{q_0 h^2}{9\sqrt{3}}$

Maximum bending stress

$\sigma_{max} = \frac{M_{max}}{S} = \frac{2q_0 h^2}{3\sqrt{3}\,bt^2} = \frac{2\gamma h^3}{3\sqrt{3}\,t^2}$

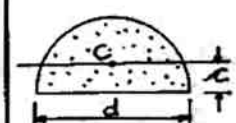
Substitute numerical values:

$\sigma_{max} = 2.32$ MPa ←

5.5-13

MAXIMUM TENSILE STRESS

(a) Semicircle



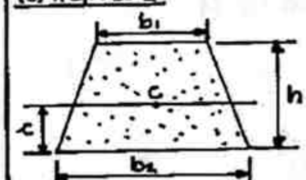
From Appendix D, Case 10:

$I_c = \frac{(9\pi^2 - 64)r^4}{72\pi} = \frac{(9\pi^2 - 64)d^4}{1152\pi}$

$c = \frac{4r}{3\pi} = \frac{2d}{3\pi}$

$\sigma_t = \frac{Mc}{I_c} = \frac{768M}{(9\pi^2 - 64)d^3} = 30.93\frac{M}{d^3}$ ←

(b) Trapezoid



$b_1 = b \quad b_2 = \frac{4b}{3}$

From Appendix D, Case 8:

$I_c = \frac{h^3(b_1^2 + 4b_1b_2 + b_2^2)}{36(b_1 + b_2)}$

$= \frac{73bh^3}{756}$

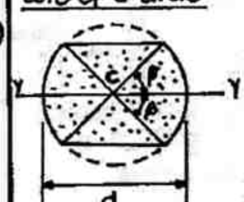
$c = \frac{h(2b_1 + b_2)}{3(b_1 + b_2)} = \frac{10h}{21}$

$\sigma_t = \frac{Mc}{I_c} = \frac{360M}{73bh^2}$ ←

5.5-14

MAXIMUM STRESS

Core of a circle



From Appendix D, Cases 9 and 15:

$I_Y = \frac{\pi r^4}{4} - \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right)$

$r = \frac{d}{2} \quad \alpha = 90° - \beta = \frac{\pi}{2} - \beta$

β = radians

$a = r\sin\beta \quad b = r\cos\beta$

CONT.

5.5-14 CONT.

$$I_y = \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta - \sin\beta\cos\beta + 2\sin\beta\cos^3\beta\right)$$
$$= \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta - (\sin\beta\cos\beta)(1 - 2\cos^2\beta)\right)$$
$$= \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta - (\tfrac{1}{2}\sin 2\beta)(-\cos 2\beta)\right)$$
$$= \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta + \frac{1}{4}\sin 4\beta\right)$$
$$= \frac{d^4}{128}(4\beta - \sin 4\beta)$$

$$c = r\sin\beta = \frac{d}{2}\sin\beta$$

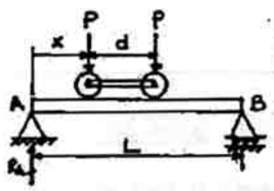
$$\sigma_{max} = \frac{Mc}{I_Y} = \frac{64M\sin\beta}{d^3(4\beta - \sin 4\beta)}$$

For $\beta = 60° = \pi/3$ rad:

$$\sigma_{max} = \frac{576\,M}{(8\pi\sqrt{3} + 9)\,d^3} = 10.965\,\frac{M}{d^3} \longleftarrow$$

5.5-15

WHEEL LOADS ON A BEAM



$L = 30$ ft $= 360$ in.
$d = 6$ ft $= 72$ in.
$P = 2$ K
$S = 16.2$ in.^{3}

Maximum bending moment

$$R_A = \frac{P}{L}(L - x) + \frac{P}{L}(L - x - d) = \frac{P}{L}(2L - d - 2x)$$

$$M = R_A x = \frac{P}{L}(2Lx - dx - 2x^2)$$

$$\frac{dM}{dx} = \frac{P}{L}(2L - d - 4x) = 0 \qquad x = \frac{L}{2} - \frac{d}{4}$$

Substitute x into the equation for M:

$$M_{max} = \frac{P}{2L}\left(L - \frac{d}{2}\right)^2$$

Maximum bending stress

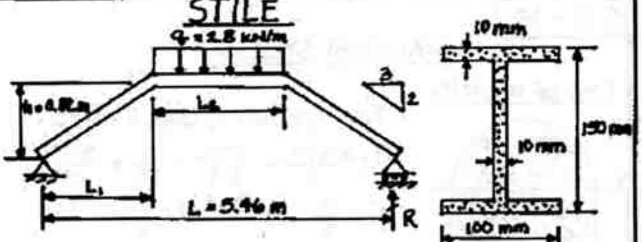
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{P}{2LS}\left(L - \frac{d}{2}\right)^2$$

Substitute numerical values:

$$\sigma_{max} = \frac{2K}{2(360\text{ in.})(16.2\text{ in.}^3)}(360\text{ in.} - 36\text{ in.})^2 = 18.0\text{ ksi} \longleftarrow$$

5.5-16

STILE



$$\frac{L_1}{h} = \frac{3}{2} \qquad L_1 = \frac{3h}{2} = 1.23\text{ m} \qquad L_2 = L - 2L_1 = 3.0\text{ m}$$

Reaction $\quad R = \frac{1}{2}qL_2 = 4200$ N

CONT.

5.5-16 CONT.

Maximum bending moment

$$M_{max} = R\left(\frac{L}{2}\right) - q\left(\frac{L_2}{2}\right)\left(\frac{L_2}{4}\right) = 11{,}466 - 3{,}150 = 8{,}316\text{ N}\cdot\text{m}$$

Maximum bending stress

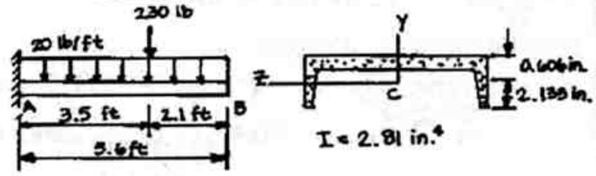
$$c = \frac{1}{2}(150\text{ mm}) = 75\text{ mm}$$

$$I = \frac{1}{12}(100\text{ mm})(150\text{ mm})^3 - \frac{1}{12}(90\text{ mm})(130\text{ mm})^3$$
$$= 11.648 \times 10^6\text{ mm}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{(8316\text{ N}\cdot\text{m})(75\text{ mm})}{11.648 \times 10^6\text{ mm}^4} = 53.5\text{ MPa} \longleftarrow$$

5.5-17

CANTILEVER BEAM



$I = 2.81$ in.^{4}

$$M_{max} = (230\text{ lb})(3.5\text{ ft}) + (20\text{ lb/ft})(5.6\text{ ft})\left(\frac{5.6\text{ ft}}{2}\right)$$
$$= 805\text{ lb-ft} + 313.6\text{ lb-ft} = 118.6\text{ lb-ft} = 13{,}423\text{ lb-in.}$$

Maximum tensile stress

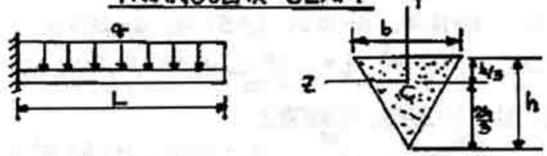
$$\sigma_t = \frac{Mc_1}{I} = \frac{(13{,}423\text{ lb-in.})(0.606\text{ in.})}{2.81\text{ in.}^4} = 2890\text{ psi} \longleftarrow$$

Maximum compressive stress

$$\sigma_c = \frac{Mc_2}{I} = \frac{(13{,}423\text{ lb-in.})(2.133\text{ in.})}{2.81\text{ in.}^4} = 10{,}190\text{ psi} \longleftarrow$$

5.5-18

TRIANGULAR BEAM



$L = 0.8$ m $\quad b = 54$ mm $\quad h = 80$ mm $\quad \gamma = 85$ kN/m^3

(a) Maximum stresses

$$q = \gamma A = \gamma\left(\frac{bh}{2}\right) \qquad M_{max} = \frac{qL^2}{2} = \frac{\gamma bhL^2}{4}$$

$$I_z = I_C = \frac{bh^3}{36} \qquad c_1 = \frac{h}{3} \qquad c_2 = \frac{2h}{3}$$

Tensile stress: $\sigma_t = \dfrac{Mc_1}{I_z} = \dfrac{3\gamma L^2}{h}$

Compressive stress: $\sigma_c = 2\sigma_t$

Substitute numerical values: $\sigma_t = 2.04$ MPa $\longleftarrow$

$\sigma_c = 4.08$ MPa $\longleftarrow$

(b) Width b is doubled

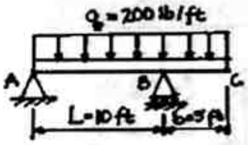
Stresses stay the same. $\longleftarrow$

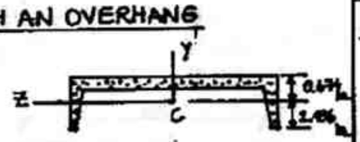
(c) Height h is doubled

Stresses are halved. $\longleftarrow$

5.5-19

BEAM WITH AN OVERHANG

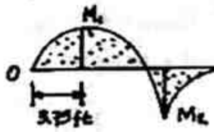




$I_z = 5.14 \text{ in.}^4$

$c_1 = 0.674$ in. $\quad c_2 = 2.496$ in.

$R_A = 750$ lb $\quad R_B = 2250$ lb



$M_1 = 1406.2$ lb-ft $= 16{,}875$ lb-in.

$M_2 = 2500$ lb-ft $= 30{,}000$ lb-in.

At cross section of maximum positive bending moment

$$\sigma_t = \frac{M_1 c_2}{I_z} = \frac{(16{,}875 \text{ lb-in})(2.496 \text{ in})}{5.14 \text{ in}^4} = 8190 \text{ psi}$$

$$\sigma_c = \frac{M_1 c_1}{I_z} = \frac{(16{,}875 \text{ lb-in})(0.674 \text{ in})}{5.14 \text{ in}^4} = 2210 \text{ psi}$$

At cross section of maximum negative bending moment

$$\sigma_t = \frac{M_2 c_1}{I_z} = \frac{(30{,}000 \text{ lb-in})(0.674 \text{ in})}{5.14 \text{ in}^4} = 3930 \text{ psi}$$

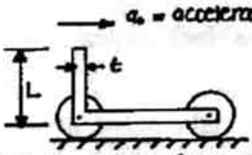
$$\sigma_c = \frac{M_2 c_2}{I_z} = \frac{(30{,}000 \text{ lb-in})(2.496 \text{ in})}{5.14 \text{ in}^4} = 14{,}570 \text{ psi}$$

Maximum stresses

$\sigma_t = 8190$ psi $\quad \sigma_c = 14{,}570$ psi ←

5.5-20

FRAME TRAVELING HORIZONTALLY

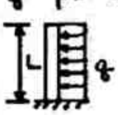


ρ = mass density

Let b = width of vertical arm (perpendicular to the figure)

Let q = inertia force per unit length of vertical arm

$q = \rho b t a_0$



$$M_{max} = \frac{qL^2}{2} = \frac{\rho b t a_0 L^2}{2}$$

$$S = \frac{bt^2}{6}$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{3\rho L^2 a_0}{t} \quad \leftarrow$$

Note: Typical units for use in the preceding equation:

SI units $\quad \rho = \text{kg/m}^3$ (or $\text{N}\cdot\text{s}^2/\text{m}^4$)

L = meters (m) $\quad a_0 = \text{m/s}^2$

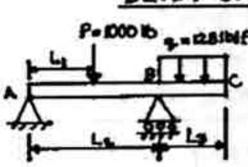
t = meters (m) $\quad \sigma = \text{N/m}^2$ (pascal)

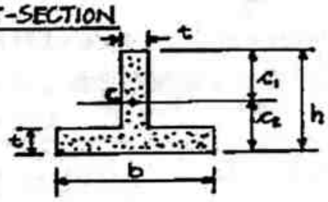
USCS units $\quad \rho = \text{slug/ft}^3$ (or $\text{lb-s}^2/\text{ft}^4$)

L = ft $\quad a_0 = \text{ft/s}^2 \quad$ t = ft $\quad \sigma = \text{lb/ft}^2$

5.5-21

BEAM OF T-SECTION





$L_1 = 4$ ft $\quad L_2 = 8$ ft $\quad L_3 = 5$ ft

CONT.

5.5-21 CONT.

Properties of the cross section

$t = 0.5$ in. $\quad b = 2.5$ in. $\quad h = 3.0$ in.

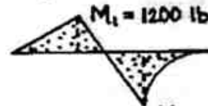
$A = 2.5 \text{ in.}^2 \quad c_1 = 2.0$ in. $\quad c_2 = 1.0$ in.

$I_c = \frac{25}{12} \text{ in.}^4 = 2.0833 \text{ in.}^4$

Reactions $\quad R_A = 300$ lb (upward)

$R_B = 1340$ lb (upward)

Bending-moment diagram



$M_1 = 1200$ lb-ft $= 14{,}400$ lb-in.

$M_2 = -1600$ lb-ft $= -19{,}200$ lb-in.

At cross section of maximum positive bending moment

$\sigma_t = \frac{M_1 c_2}{I_c} = 6{,}910$ psi $\quad \sigma_c = \frac{M_1 c_1}{I_c} = 13{,}820$ psi

At cross section of maximum negative bending moment

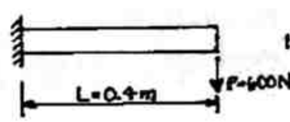
$\sigma_t = \frac{M_2 c_1}{I_c} = 18{,}430$ psi $\quad \sigma_c = \frac{M_2 c_2}{I_c} = 9{,}220$ psi

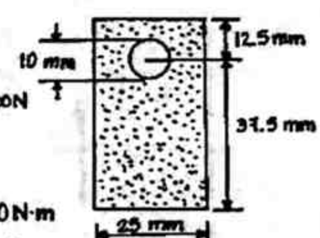
Maximum stresses

$\sigma_t = 18{,}430$ psi $\quad \sigma_c = 13{,}820$ psi ←

5.5-22

CANTILEVER BEAM WITH A LONGITUDINAL HOLE

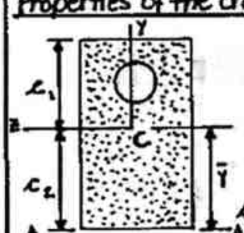




Bending moment

$M = PL = (600 \text{ N})(0.4 \text{ m}) = 240 \text{ N}\cdot\text{m}$

Properties of the cross section



$A = (25 \text{ mm})(50 \text{ mm}) - \frac{\pi}{4}(10 \text{ mm})^2 = 1171.5 \text{ mm}^2$

$\Sigma A_i \bar{y}_i = (25 \text{ mm})(50 \text{ mm})(25 \text{ mm})$

$-\left(\frac{\pi}{4}\right)(10 \text{ mm})^2(37.5 \text{ mm}) = 28{,}305 \text{ mm}^3$

$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{A} = \frac{28{,}305 \text{ mm}^3}{1171.5 \text{ mm}^2} = 24.162 \text{ mm}$

$c_2 = \bar{y} = 24.16$ mm

$c_1 = 50 \text{ mm} - c_2 = 25.84$ mm

Moment of inertia about the neutral axis

Rectangle: $I_z = I_c + Ad^2$

$= \left(\frac{1}{12}\right)(25)(50)^3 + (25)(50)(25 - 24.16)^2$

$= 260{,}420 + 880 = 261{,}300 \text{ mm}^4$

Hole: $I_z = I_c + Ad^2 = \frac{\pi}{64}(10)^4 + \frac{\pi}{4}(10)^2(37.5 - 24.162)^2$

$= 490.9 + 13{,}972 = 14{,}460 \text{ mm}^4$

Cross section: $I = 261{,}300 - 14{,}460 = 246{,}800 \text{ mm}^4$

Stresses

Top of the beam: $\sigma = \frac{Mc_1}{I} = \frac{(240 \text{ N}\cdot\text{m})(25.84 \text{ mm})}{246{,}800 \text{ mm}^4}$

$= 25.1$ MPa (tension) ←

CONT.

5.5-22 CONT.

Top of the hole: $y = 37.5 + 5 - 24.162 = 18.338$ mm

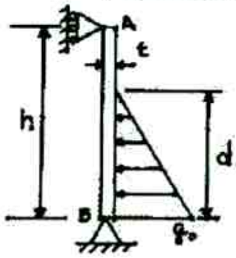
$$\sigma = \frac{My}{I} = \frac{(240\ \text{N}\cdot\text{m})(18.338\ \text{mm})}{246{,}800\ \text{mm}^4} = 17.8\ \text{MPa (tension)}$$

Bottom of the beam: $$\sigma = -\frac{Mc_L}{I} = -\frac{(240\ \text{N}\cdot\text{m})(24.16\ \text{mm})}{246{,}800\ \text{mm}^4}$$

$$= -23.5\ \text{MPa (compression)}$$

5.5-23

VERTICAL WOOD BEAM



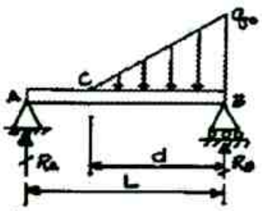
$h = 6$ ft

$t = 2.5$ in.

$\gamma = 62.4$ lb/ft^3

Let b = width of beam (perpendicular to the figure)

Let q_0 = intensity of load at depth d



$q_0 = \gamma bd$

$L = h = 6$ ft

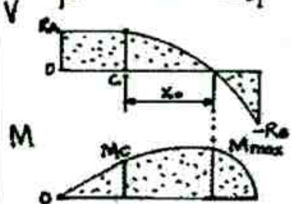
$R_A = \dfrac{q_0 d^2}{6L}$

$R_B = \dfrac{q_0 d}{6}\left(3 - \dfrac{d}{L}\right)$

$x_0 = d\sqrt{\dfrac{d}{3L}}$

$M_C = R_A(L-d)$

$M_{max} = \dfrac{q_0 d^2}{6}\left(1 - \dfrac{d}{L} + \dfrac{2d}{3L}\sqrt{\dfrac{d}{3L}}\right)$



Maximum stress

Section modulus: $S = \frac{1}{6}bt^2$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{6}{bt^2}\left[\frac{q_0 d^2}{6}\left(1 - \frac{d}{L} + \frac{2d}{3L}\sqrt{\frac{d}{3L}}\right)\right]$$

$q_0 = \gamma bd$

$$\sigma_{max} = \frac{\gamma d^3}{t^2}\left(1 - \frac{d}{L} + \frac{2d}{3L}\sqrt{\frac{d}{3L}}\right)$$

Substitute numerical values:

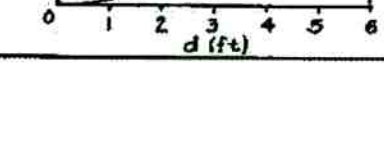
d = depth of water (ft)

$L = h = 6$ ft $\gamma = 62.4$ lb/ft^3 $t = 2.5$ in.

σ_{max} = psi

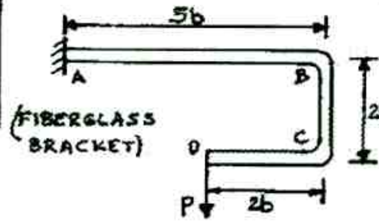
$$\sigma_{max} = \frac{(62.4)d^3}{(2.5)^2}\left(1 - \frac{d}{6} + \frac{d}{9}\sqrt{\frac{d}{18}}\right) = 0.1849d^3\left(54 - 9d + d\sqrt{2d}\right)$$

d (ft)	σ_{max} (psi)
0	0
1	9
2	59
3	171
4	347
5	573
6	830



5.6-1

BRACKET ABCD



$P = 7.2$ lb

$b = 1.25$ in.

$\sigma_{allow} = 3200$ psi

d = diameter

$I = \dfrac{\pi d^4}{64}$ $M_{max} = P(3b)$

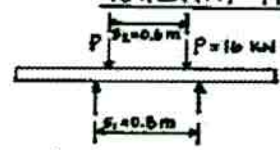
$\sigma_{max} = \dfrac{M_{max}(d/2)}{I} = \dfrac{96bP}{\pi d^3}$ $d^3_{min} = \dfrac{96bP}{\pi\sigma_{allow}}$

Substitute numerical values:

$d^3_{min} = 0.08594$ in.^{3} $d_{min} = 0.441$ in.

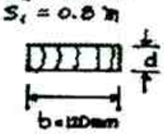
5.6-2

RAILWAY TIE



$s_2 = 0.6$ m

$s_1 = 0.8$ m



$\sigma_{allow} = 8$ MPa

$M_{max} = \dfrac{P(s_1 - s_2)}{2} = 1600$ N·m

$S = \dfrac{bd^2}{6} = \dfrac{1}{6}(0.120\ \text{m})(d^2) = 0.020d^2\ (\text{m}^3)$ d = meters

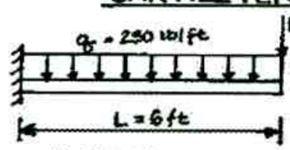
$M_{max} = \sigma_{allow}S$

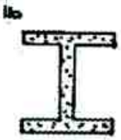
$1600\ \text{N}\cdot\text{m} = (8\times10^6\ \text{N/m}^2)(0.020d^2)$ $d^2 = 0.0100\ \text{m}^2$

$d = 0.100$ m $d_{min} = 100$ mm

5.6-3

CANTILEVER BEAM





$\sigma_{allow} = 16{,}000$ psi

$M_{max} = PL + \dfrac{qL^2}{2} = 18{,}000 + 4{,}500 = 22{,}500$ lb-ft $= 270{,}000$ lb-in.

$S = \dfrac{M_{max}}{\sigma_{allow}} = \dfrac{270{,}000\ \text{lb-in.}}{16{,}000\ \text{psi}} = 16.88$ in.^{3}

Try W 8×21 $S = 18.2$ in.^{3} $q_0 = 21$ lb/ft

$M_0 = \dfrac{q_0 L^2}{2} = 378$ lb-ft $= 4536$ lb-in.

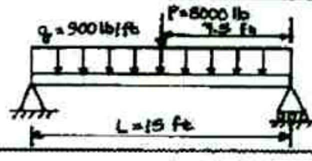
$M_{max} = 270{,}000 + 4{,}536 = 274{,}500$ lb-in.

$S = \dfrac{M_{max}}{\sigma_{allow}} = \dfrac{274{,}500\ \text{lb-in.}}{16{,}000\ \text{psi}} = 17.16$ in^3 (O.K.)

USE W 8×21

5.6-4

SIMPLE BEAM



Use a 12-inch wide-flange beam

CONT.

5.6-4 CONT.

$\sigma_{allow} = 18{,}000$ psi

$M_{max} = \frac{qL^2}{8} + \frac{PL}{4} = 25{,}310 + 30{,}000 = 55{,}310$ lb-ft $= 663{,}700$ lb-in.

$S = \frac{M_{max}}{\sigma_{allow}} = \frac{663{,}700 \text{ lb-in.}}{18{,}000 \text{ psi}} = 36.87 \text{ in.}^3$

Try W 12 X 35 $\quad S = 45.6 \text{ in.}^3 \quad q_0 = 35$ lb/ft

$M_0 = \frac{q_0 L^2}{8} = 984$ lb-ft $= 11{,}810$ lb-in.

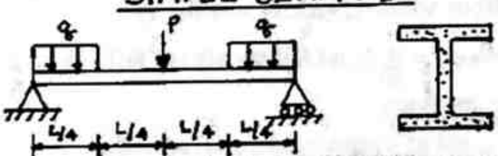
$M_{max} = 663{,}700 + 11{,}810 = 675{,}500$ lb-in.

$S = \frac{M_{max}}{\sigma_{allow}} = \frac{675{,}500 \text{ lb-in.}}{18{,}000 \text{ psi}} = 37.53 \text{ in.}^3$ (O.K.)

USE W12X35 ←

5.6-5

SIMPLE BEAM AB



$\sigma_{allow} = 16{,}000$ psi $\quad L = 24$ ft $\quad P = 2000$ lb $\quad q = 300$ lb/ft

$M_{max} = \frac{PL}{4} + \frac{qL^2}{32} = 12{,}000 + 5{,}400 = 17{,}400$ lb-ft $= 208{,}800$ lb-in.

$S = \frac{M_{max}}{\sigma_{allow}} = \frac{208{,}800 \text{ lb-in.}}{16{,}000 \text{ psi}} = 13.05 \text{ in.}^3$

Try S 8 X 18.4 $\quad S = 14.4 \text{ in.}^3 \quad q_0 = 18.4$ lb/ft

$M_0 = \frac{q_0 L^2}{8} = 1325$ lb-ft $= 15{,}900$ lb-in.

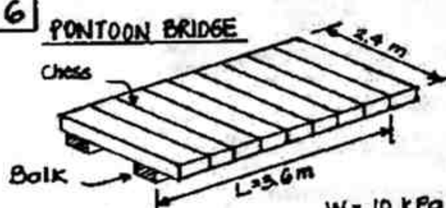
$M_{max} = 208{,}800 + 15{,}900 = 224{,}700$ lb-in.

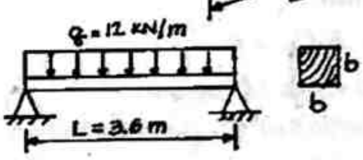
$S = \frac{M_{max}}{\sigma_{allow}} = \frac{224{,}700 \text{ lb-in.}}{16{,}000 \text{ psi}} = 14.04 \text{ in.}^3$ (O.K.)

USE S 8 X 18.4 ←

5.6-6

PONTOON BRIDGE





W = 10 kPa (floor load)

$q = (10 \text{ kPa})\left(\frac{2.4 \text{ m}}{2}\right) = 12$ kN/m

$\sigma_{allow} = 17.5$ MPa

For one balk:

$M_{max} = \frac{qL^2}{8} = \frac{1}{8}(12 \text{ kN/m})(3.6 \text{ m})^2 = 19{,}440$ N·m

$\sigma_{allow} S = M_{max}$

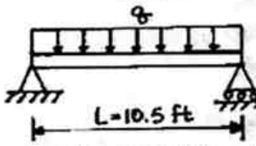
$(17.5 \text{ MPa})\left(\frac{b^3}{6}\right) = 19{,}440$ N·m

$b^3 = 0.006665 \text{ m}^3$

$b_{min} = 0.188 \text{ m} = 188$ mm ←

5.6-7

FLOOR JOISTS



$\sigma_{allow} = 1350$ psi

$L = 10.5$ ft $= 126$ in.

$W = 120 \text{ lb/ft}^2 = 0.8333 \text{ lb/in.}^2$ (floor load)

s = spacing of joists = 16 in.

$q = Ws = (0.8333 \text{ lb/in.}^2)(16 \text{ in.}) = 13.33$ lb/in.

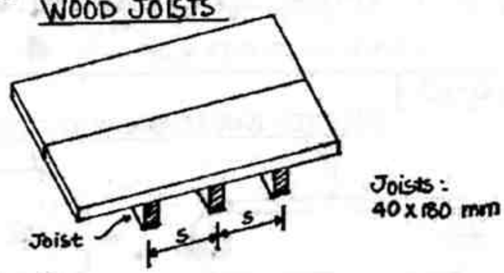
$M_{max} = \frac{qL^2}{8} = \frac{1}{8}(13.33 \text{ lb/in})(126 \text{ in.})^2 = 26{,}460$ lb-in.

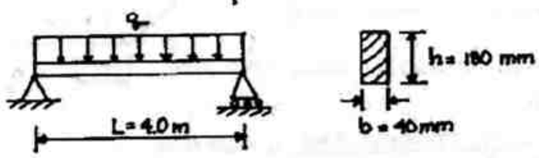
$S = \frac{M_{max}}{\sigma_{allow}} = \frac{26{,}460 \text{ lb-in.}}{1{,}350 \text{ psi}} = 19.6 \text{ in.}^3$ ←

From Appendix F: Select 2X10 in. joists ←

5.6-8

WOOD JOISTS





$L = 4.0$ m $\quad s$ = spacing of joists $\quad W = 3.6$ kPa (floor load) $\quad \sigma_{allow} = 15$ MPa

$q = Ws \quad S = \frac{bh^2}{6} \quad M_{max} = \frac{qL^2}{8} = \frac{WsL^2}{8}$

$M_{max} = \sigma_{allow} S$

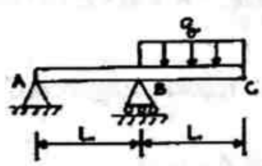
$\frac{WsL^2}{8} = \sigma_{allow}\left(\frac{bh^2}{6}\right)$ or $s_{max} = \frac{4\sigma_{allow} b h^2}{3WL^2}$

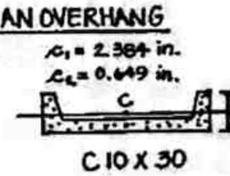
Substitute numerical data:

$s_{max} = \frac{4(15 \text{ MPa})(40 \text{ mm})(180 \text{ mm}^2)}{3(3.6 \text{ kPa})(4.0 \text{ m})^2} = 0.450 \text{ m} = 450 \text{ mm}$ ←

5.6-9

BEAM WITH AN OVERHANG





C 10 X 30

$L = 4$ ft $\quad q_0 = 30$ lb/ft (weight of beam)

Allowable stresses: $\sigma_t = 20$ ksi $\quad \sigma_c = 12$ ksi

From Table E-3, Appendix E: $I = 3.94 \text{ in.}^4$

Maximum bending moment occurs at support B (tension on top, compression on bottom).

CONT.

5.6-9 CONT.

Allowable bending moment

Based upon tension at the top of the beam:

$M_t = \dfrac{\sigma_t I}{c_1} = \dfrac{(20\text{ ksi})(3.94\text{ in.}^4)}{2.384\text{ in.}}$

$= 33{,}050$ lb-in. $= 2{,}754$ lb-ft

Based upon compression at the bottom of the beam:

$M_c = \dfrac{\sigma_c I}{c_2} = \dfrac{(12\text{ ksi})(3.94\text{ in.}^4)}{0.649\text{ in.}}$

$= 72{,}850$ lb-in. $= 6{,}071$ lb-ft

Tension governs. $M_{allow} = 2{,}754$ lb-ft

Allowable uniform load q

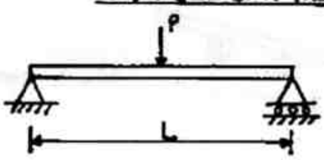
$M_{allow} = (q + q_0)\dfrac{L^2}{2} \qquad q + q_0 = \dfrac{2M_{allow}}{L^2}$

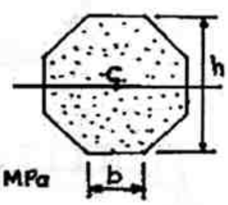
$q_{allow} = \dfrac{2M_{allow}}{L^2} - q_0 = \dfrac{2(2754\text{ lb-ft})}{(4\text{ ft})^2} - 30$ lb/ft

$= 344$ lb/ft $- 30$ lb/ft $= 314$ lb/ft ⟵

5.6-10

TRAPEZE BAR (REGULAR OCTAGON)



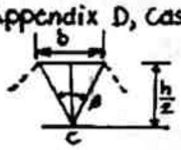


$L = 2.4$ m $\quad P = 1100$ N $\quad \sigma_{allow} = 180$ MPa

Find h.

$M_{max} = \dfrac{PL}{4} = \dfrac{(1100\text{ N})(2.4\text{ m})}{4} = 660$ N·m

Appendix D, Case 25:



b = length of a side

$I_c = \dfrac{nb^4}{192}\left(\cot\dfrac{\beta}{2}\right)\left(3\cot^2\dfrac{\beta}{2} + 1\right)$

$n = 8; \beta = \dfrac{360°}{n} = \dfrac{360°}{8} = 45°; \; I_c = 1.8595\,b^4$

$\dfrac{h}{2} = \dfrac{b}{2}\cot\dfrac{\beta}{2} \qquad b = h\tan\dfrac{\beta}{2} = 0.41421\,h$

$I_c = 1.8595\,b^4 = 0.054737\,h^4$

$S = \dfrac{I_c}{h/2} = \dfrac{0.054737\,h^4}{h/2} = 0.10947\,h^3$

$\sigma = \dfrac{M}{S} \qquad 180\text{ MPa} = \dfrac{660\text{ N·m}}{0.10947\,h^3} \qquad h^3 = 33.495 \times 10^{-6}\text{ m}^3$

$h_{min} = 32.2$ mm ⟵

Note: Exact formulas for a regular octagon are as follows:

$\beta = 45° \qquad \tan\dfrac{\beta}{2} = \sqrt{2} - 1 \qquad \cot\dfrac{\beta}{2} = 1 + \sqrt{2}$

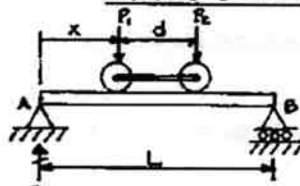
$b = (\sqrt{2} - 1)h \qquad h = (1 + \sqrt{2})b$

$I_c = \left(\dfrac{11 + 8\sqrt{2}}{12}\right)b^4 = \left(\dfrac{4\sqrt{2} - 5}{12}\right)h^4$

$S = \left(\dfrac{4\sqrt{2} - 5}{6}\right)h^3$

5.6-11

MOVING CARRIAGE ON A SIMPLE BEAM



$P_1 = 4.8$ k

$P_2 = 2.4$ k

$d = 5$ ft

$L = 16$ ft

$\sigma_{allow} = 18$ ksi

Let x = distance from support A to load P_1 (feet)

$R_A = P_1\left(\dfrac{L - x}{L}\right) + P_2\left(\dfrac{L - x - d}{L}\right) = \dfrac{3}{20}(43 - 3x) \quad (R_A = \text{kips})$

$M = R_A x = \dfrac{3x}{20}(43 - 3x) \quad (M = \text{k-ft})$

Maximum bending moment

To find distance x_1 for maximum moment:

$\dfrac{dM}{dx} = 0$ from which $x_1 = \dfrac{43}{6}$ ft $= 7.167$ ft

$M_{max} = (M)_{x = x_1} = \dfrac{3}{20}(7.167\text{ ft})[43 - 3(7.167\text{ ft})] = 23.113$ k-ft

(a) Section modulus

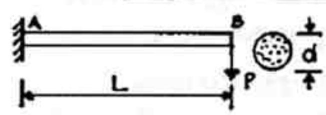
$S = \dfrac{M_{max}}{\sigma_{allow}} = \dfrac{(23.113\text{ k-ft})(12\text{ in/ft})}{18\text{ ksi}} = 15.41\text{ in}^3$ ⟵

(b) Select an I-beam (S shape)

Select S8×23 ⟵

5.6-12

CANTILEVER BEAM



$L = 500$ mm

$P = 200$ N

$\sigma_{allow} = 80$ MPa

$\gamma = 77.0$ kN/m³

q = weight of beam per unit length

$q = \gamma\left(\dfrac{\pi d^2}{4}\right) \qquad S = \dfrac{\pi d^3}{32}$

$M_{max} = PL + \dfrac{qL^2}{2} = PL + \dfrac{\pi\gamma d^2 L^2}{8}$

$M_{max} = \sigma_{allow} S$

$PL + \dfrac{\pi\gamma d^2 L^2}{8} = \sigma_{allow}\left(\dfrac{\pi d^3}{32}\right)$

Rearrange the equation:

$\sigma_{allow} d^3 - 4\gamma L^2 d^2 - \dfrac{32PL}{\pi} = 0$

Substitute numerical values (d = meters):

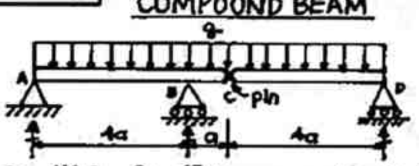
$(80\text{ MPa})d^3 - 4(77.0\text{ kN/m}^3)(0.5\text{ m})^2 d^2 - \dfrac{32}{\pi}(200\text{ N})(0.5\text{ m}) = 0$

$80{,}000\,d^3 - 77\,d^2 - 1.01859 = 0$

Solve the equation: $d = 0.0237$ m or $d_{min} = 23.7$ mm ⟵

5.6-13

COMPOUND BEAM

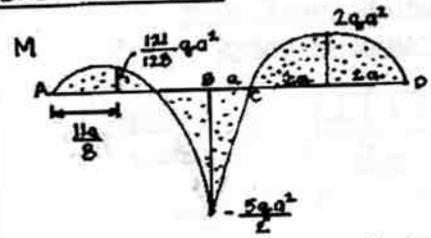


PIN AT POINT C.

$R_A = \dfrac{119qa}{8} \qquad R_B = \dfrac{459qa}{8} \qquad R_D = 2qa$

CONT.

5.6-13 CONT.



$a = 8\text{ ft} = 96\text{ in.}$ W16X57 $S = 92.2\text{ in.}^3$

$\sigma_{allow} = 16{,}000\text{ psi}$ Find q_{allow}

$M_{max} = \frac{5qa^2}{2} = \sigma_{allow} S$

$q_{allow} = \frac{2\sigma_{allow} S}{5a^2} = \frac{(2)(16{,}000\text{ psi})(92.2\text{ in.}^3)}{5(96\text{ in.})^2}$

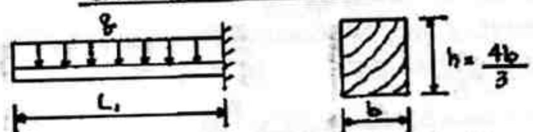
$= 64.03\text{ lb/in.} = 768.3\text{ lb/ft}$

Weight of beam $= 57\text{ lb/ft}$

Allowable load: $q_{allow} = 768.3 - 57 = 711\text{ lb/ft}$ ←

5.6-14

CANTILEVER BEAM FOR A BALCONY



$L_1 = 2.1\text{ m}$ $L_2 = 2.5\text{ m}$ Floor dimensions: $L_1 \times L_2$

Design load $= w = 5.5\text{ kPa}$

$\gamma = 5.5\text{ kN/m}^3$ (weight density of wood beam)

$\sigma_{allow} = 15\text{ MPa}$

Middle beam supports 50% of the load.

$\therefore q = w\left(\frac{L_2}{2}\right) = (5.5\text{ kPa})\left(\frac{2.5\text{ m}}{2}\right) = 6875\text{ N/m}$

Weight of beam:

$q_0 = \gamma bh = \frac{4\gamma b^2}{3} = \frac{4}{3}(5.5\text{ kN/m}^3)b^2$

$= 7333\, b^2\text{ (N/m)}$ (b = meters)

$M_{max} = \frac{(q+q_0)L_1^2}{2} = \frac{(6875 + 7333b^2)(2.1\text{ m})^2}{2} = 15{,}159 + 16{,}170b^2$ (N·m)

$S = \frac{bh^2}{6} = \frac{8b^3}{27}$

$\sigma = \frac{M}{S}$ $15\text{ MPa} = \frac{15{,}159 + 16{,}170b^2\text{ (N·m)}}{8b^3/27}$

Rearrange the equation:

$(120 \times 10^6)b^3 - 436{,}590\, b^2 - 409{,}290 = 0$

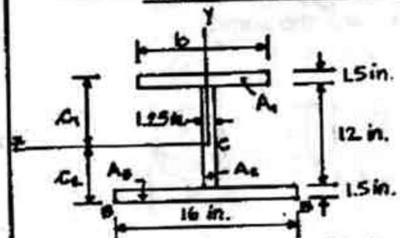
Solve for b:

$b = 0.152\text{ m} = 152\text{ mm}$ ←

$h = \frac{4b}{3} = 202\text{ mm}$ ←

5.6-15

UNSYMMETRIC I-BEAM



Stresses at top and bottom are in the ratio 8:7. Find b.

$\frac{\sigma_{top}}{\sigma_{bottom}} = \frac{8}{7} = \frac{c_1}{c_2}$ $c_1 = \frac{8}{15}(12 + 1.5 + 1.5) = 8.0\text{ in.}$

$c_2 = \frac{7}{15}(12 + 1.5 + 1.5) = 7.0\text{ in.}$

First moment of the area of the cross section about the lower edge BB

$Q_{BB} = \sum \bar{y}_i A_i = (14.25)(b)(1.5) + (7.5)(12)(1.25) + (0.75)(16)(1.5)$

$= 21.375\, b + 112.5 + 18.0$

$= 21.375 b + 130.5\ (\text{in.}^3)$

Area of the cross section

$A = 1.5b + 12(1.25) + 16(1.5) = 1.5b + 39.0\ (\text{in.}^2)$

Distance to the centroid

$c_2 = \frac{Q_{BB}}{A} = \frac{21.375b + 130.5}{1.5b + 39.0} = 7.0\text{ in.}$

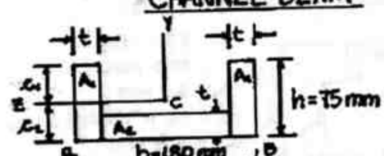
Solve for b

$21.375\, b + 130.5 = 7(1.5\, b) + 7(39.0)$

$10.875\, b = 142.5$ $b = 13.10\text{ in.}$ ←

5.6-16

CHANNEL BEAM



t = thickness (mm)

Stresses at top and bottom in the ratio 7:3

Find t.

$\frac{\sigma_{top}}{\sigma_{bottom}} = \frac{7}{3} = \frac{c_1}{c_2}$ $c_1 = \frac{7}{10}(75\text{ mm}) = 52.5\text{ mm}$

$c_2 = \frac{3}{10}(75\text{ mm}) = 22.5\text{ mm}$

First moment of the area of the cross section about the lower edge BB

$Q_{BB} = \sum \bar{y}_i A_i = (2)\left(\frac{75}{2}\right)(75t) + \left(\frac{t}{2}\right)(180 - 2t)(t)$

$= t(5625 + 90t - t^2)$ (t = mm)

Area of the cross section

$A = 2(75t) + (180 - 2t)(t) = t(330 - 2t)$

Distance to the centroid

$c_2 = \frac{Q_{BB}}{A} = \frac{t(5625 + 90t - t^2)}{t(330 - 2t)} = 22.5\text{ mm}$

Solve for t

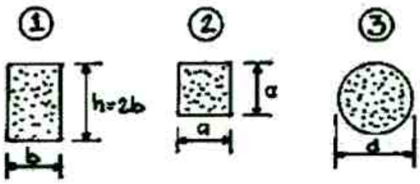
$5625 + 90t - t^2 = 22.5(330 - 2t)$

$t^2 - 135t + 1800 = 0$ $t = 15\text{ mm}$ ←

5.6-17

RATIO OF WEIGHTS OF THREE BEAMS

$L, \gamma, M_{max}, \sigma_{max}$ are the same.



$S = \frac{M}{\sigma}$ Since M and σ are equal in all three cases, the section moduli must be equal also.

Rectangle: $S = \frac{bh^2}{6} = \frac{2b^3}{3}$ $\quad b = \left(\frac{3S}{2}\right)^{1/3}$

$A_1 = 2b^2 = 2\left(\frac{3S}{2}\right)^{2/3} = 2.6207\,S^{2/3}$

Square: $S = \frac{a^3}{6}$ $\quad a = (6S)^{1/3}$

$A_2 = a^2 = (6S)^{2/3} = 3.3019\,S^{2/3}$

Circle: $S = \frac{\pi d^3}{32}$ $\quad d = \left(\frac{32S}{\pi}\right)^{1/3}$

$A_3 = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{32S}{\pi}\right)^{2/3} = 3.6905\,S^{2/3}$

Weights are proportional to cross-sectional areas (since L and γ are equal in all 3 cases).

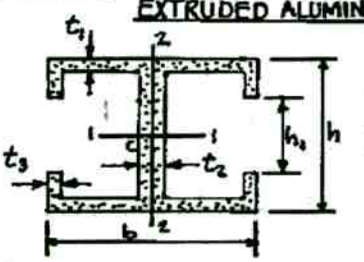
$W_1 : W_2 : W_3 = A_1 : A_2 : A_3$

$A_1 : A_2 : A_3 = 2.6207 : 3.3019 : 3.6905$

$W_1 : W_2 : W_3 = 1 : 1.260 : 1.408$ ←

5.6-18

EXTRUDED ALUMINUM BEAM



$b = 220$ mm
$h = 200$ mm
$h_1 = 80$ mm
$t_1 = t_2 = 10$ mm
$t_3 = 20$ mm

Axis 1-1

$I_1 = \frac{bh^3}{12} - \frac{1}{12}(b - t_2 - 2t_3)(h - 2t_1)^3 - \frac{2}{12}(t_3)(h_1^3)$

$= 62.340 \times 10^6 \text{ mm}^4$

$c_1 = \frac{h}{2} = 100$ mm $\quad S_1 = \frac{I_1}{c_1} = 623.4 \times 10^3 \text{ mm}^3$

Axis 2-2

$I_2 = \frac{hb^3}{12} - \frac{h_1 b^3}{12} - \frac{1}{12}(h - h_1 - 2t_1)(b - 2t_3)^3 + \frac{1}{12}(h - 2t_1)(t_2^3)$

$= 57.895 \times 10^6 \text{ mm}^4$

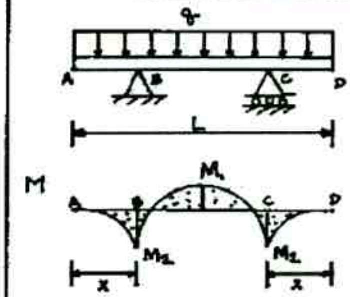
$c_2 = \frac{b}{2} = 110$ mm $\quad S_2 = \frac{I_2}{c_2} = 526.3 \times 10^3 \text{ mm}^3$

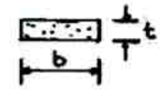
(a) $S_1 > S_2$ $\quad\therefore$ Beam is stronger when bent about axis 1-1 ←

(b) Ratio $= \frac{S_1}{S_2} = 1.184$ $\quad \beta = 1.184$ ←

5.6-19

HORIZONTAL SHELF WITH ADJUSTABLE SUPPORTS





$L = 36$ in.
$b = 12$ in.
$t = 0.75$ in.
$\sigma_{allow} = 750$ psi

For maximum load-carrying capacity, place the supports so that $M_1 = |M_2|$.

Let x = length of overhang

$M_1 = \frac{qL}{8}(L - 4x)$ $\quad |M_2| = \frac{qx^2}{2}$

$\therefore \frac{qL}{8}(L - 4x) = \frac{qx^2}{2}$

Solve for x: $x = \frac{L}{2}(\sqrt{2} - 1)$

Substitute x into the equation for either M_1 or $|M_2|$:

$M_{max} = \frac{qL^2}{8}(3 - 2\sqrt{2})$ $\quad$ (1)

$M_{max} = \sigma_{allow} S = \sigma_{allow}\left(\frac{bt^2}{6}\right)$ $\quad$ (2)

Equate M_{max} from Eqs. (1) and (2) and solve for q:

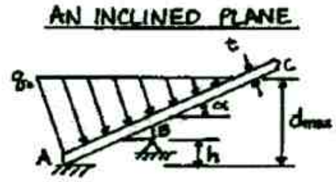
$q_{max} = \frac{4bt^2\sigma_{allow}}{3L^2(3 - 2\sqrt{2})}$

Substitute numerical values:

$q_{max} = 30.4$ lb/in. ←

5.6-20

WATER PRESSURE AGAINST AN INCLINED PLANE



d_{max} = maximum depth of water
(There is no reaction at end A when the water depth equals d_{max}.)

t = thickness of panel

α = angle of inclination

γ = weight density of water

b = width of panel perpendicular to the plane of the figure

q_0 = maximum intensity of distributed load acting on the panel

$q_0 = \gamma d_{max} b$

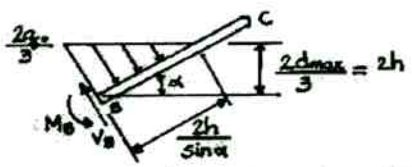
CONT.

5.6-20 CONT.

The resultant of the triangular load acts through the centroid of the triangle. This resultant must also act through point B.

$$\therefore\ h = \frac{d_{max}}{3}$$

Free-body diagram of part BC of the panel



The maximum bending moment occurs at section B.

$$M_{max} = M_B = \left(\frac{1}{2}\right)\left(\frac{2q_0}{3}\right)\left(\frac{2h}{\sin\alpha}\right)\left(\frac{1}{3}\right)\left(\frac{2h}{\sin\alpha}\right) = \frac{4q_0 h^2}{9\sin^2\alpha}$$

$q_0 = \gamma\, d_{max}\, b = \gamma(3h)b$

$$M_{max} = \frac{4\gamma b h^3}{3\sin^2\alpha} \qquad (1)$$

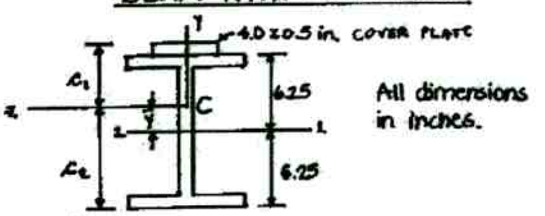
$$M_{max} = S\sigma_{allow} = \left(\frac{bt^2}{6}\right)\sigma_{allow} \qquad (2)$$

Equate M_{max} from Eqs. (1) and (2) and solve for t:

$$t_{min} = \sqrt{\frac{8\gamma h^3}{\sigma_{allow}(\sin^2\alpha)}} \quad \longleftarrow$$

5.6-21

BEAM WITH COVER PLATE



Wide-flange beam alone. (Axis 1-1 is centroidal axis)

W 12 X 35 $\quad d = 12.50$ in.

$A_0 = 10.3$ in.^{2} $\quad I_0 = 285$ in.^{4} $\quad S_0 = 45.6$ in.^{3}

Beam with cover plate (z axis is centroidal axis)

$A_1 = A_0 + (4.0\text{ in.})(0.5\text{ in.}) = 12.3\text{ in.}^2$

First moment with respect to axis 1-1:

$Q_1 = \Sigma \bar{y}_i A_i = (6.25\text{ in.} + 0.25\text{ in.})(4.0\text{ in.})(0.5\text{ in.}) = 13.00\text{ in.}^3$

$$\bar{y} = \frac{Q_1}{A_1} = \frac{13.00\text{ in.}^3}{12.3\text{ in.}^2} = 1.057\text{ in.}$$

$c_1 = 6.25 + 0.5 - \bar{y} = 5.693$ in.

$c_2 = 6.25 + \bar{y} = 7.307$ in.

Moment of inertia about axis 1-1:

$$I_{1-1} = I_0 + \frac{1}{12}(4.0)(0.5)^3 + (4.0)(0.5)(6.25+0.25)^2 = 369.5\text{ in.}^4$$

Moment of inertia about z axis:

$I_{1-1} = I_z + A_1\bar{y}^2 \qquad I_z = I_{1-1} - A_1\bar{y}^2$

$I_z = 369.5\text{ in.}^4 - (12.3\text{ in.}^2)(1.057\text{ in.})^2 = 355.8\text{ in.}^4$

Section modulus: Use the smaller of the two section moduli

$$S_1 = \frac{I_z}{c_2} = \frac{355.8\text{ in.}^4}{7.307\text{ in.}} = 48.69\text{ in.}^3$$

CONT.

5.6-21 CONT.

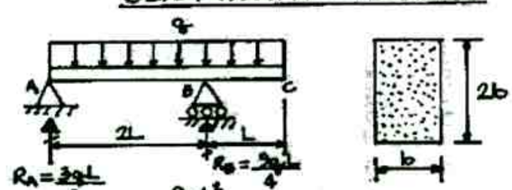
Ratio of section moduli

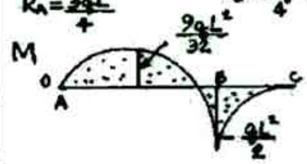
$$\frac{S_1}{S_0} = \frac{48.69}{45.6} = 1.068$$

Percent increase $= 6.8\%$ $\longleftarrow$

5.6-22

BEAM WITH AN OVERHANG





$L = 150$ mm

$q = 3.5$ kN/m

$\sigma_{allow} = 60$ MPa

$\gamma = 77.0$ kN/m^3

$$M_{max} = \frac{qL^2}{2} \qquad S = \frac{bh^2}{6} = \frac{2b^3}{3}$$

(a) Disregard the weight of the beam

$$M_{max} = \sigma_{allow} S \qquad \frac{qL^2}{2} = \sigma_{allow}\left(\frac{2b^3}{3}\right)$$

$$b^3 = \frac{3qL^2}{4\sigma_{allow}}$$

Substitute numerical values:

$$b^3 = \frac{3(3.5\text{ kN/m})(150\text{ mm})^2}{4(60\text{ MPa})} = 0.98438 \times 10^{-6}\text{ m}^3$$

$b = 0.00995$ m $= 9.95$ mm $\longleftarrow$

(b) Include the weight of the beam

q_0 = weight of beam per unit length

$q_0 = \gamma(b)(2b) = 2\gamma b^2$

$$M_{max} = \frac{(q+q_0)L^2}{2} = \frac{1}{2}(q + 2\gamma b^2)L^2$$

$$S = \frac{2b^3}{3} \qquad M_{max} = \sigma_{allow} S$$

$$\frac{1}{2}(q + 2\gamma b^2)L^2 = \sigma_{allow}\left(\frac{2b^3}{3}\right)$$

Rearrange the equation:

$$4\sigma_{allow} b^3 - 6\gamma L^2 b^2 - 3qL^2 = 0$$

Substitute numerical values:

$(240 \times 10^6)b^3 - 10{,}395 b^2 - 236.25 = 0$ (b = meters)

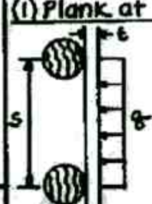
Solve the equation:

$b = 0.00996$ m $= 9.96$ mm $\longleftarrow$

5.6-23

RETAINING WALL

(1) Plank at the bottom



t = thickness of plank $= 3$ in.

b = width of plank (perpendicular to the plane of the figure)

p_0 = maximum soil pressure $= 400$ lb/ft^2 $= 2.778$ lb/in.^{2}

s = spacing of piles

CONT.

5.6-23 CONT.

$q = P_2 b$ $\quad \sigma_{allow} = 1200$ psi $\quad S$ = section modulus

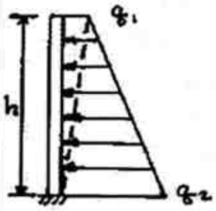
$M_{max} = \frac{q s^2}{8} = \frac{P_2 b s^2}{8}$ $\quad S = \frac{bt^2}{6}$

$M_{max} = \sigma_{allow} S$ or $\frac{P_2 b s^2}{8} = \sigma_{allow}\left(\frac{bt^2}{6}\right)$

Solve for s:

$s = \sqrt{\frac{4\sigma_{allow} t^2}{3P_2}} = 72.0$ in.

(2) Vertical pile



$h = 5$ ft $= 60$ in.

P_1 = soil pressure at the top

$= 100$ lb/ft^2 $= 0.6944$ lb/in.^{2}

$q_1 = P_1 s$

$q_2 = P_2 s$

d = diameter of pile = 12 in.

Divide the trapezoidal load into two triangles (see dashed line).

$M_{max} = \frac{1}{2}(q_1)(h)\left(\frac{2h}{3}\right) + \frac{1}{2}(q_2)(h)\left(\frac{h}{3}\right) = \frac{sh^2}{6}(2p_1 + p_2)$

$S = \frac{\pi d^3}{32}$ $\quad M_{max} = \sigma_{allow} S$ or

$\frac{sh^2}{6}(2p_1 + p_2) = \sigma_{allow}\left(\frac{\pi d^3}{32}\right)$

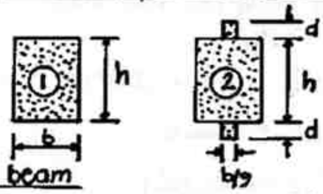
Solve for s:

$s = \frac{3\pi\,\sigma_{allow}\, d^3}{16\, h^2 (2p_1 + p_2)} = 81.4$ in.

Plank governs $\quad s_{max} = 72.0$ in. ←

5.6-24

BEAM WITH PROJECTIONS



① Original beam

$I_1 = \frac{bh^3}{12}$ $\quad c_1 = \frac{h}{2}$ $\quad S_1 = \frac{I_1}{c_1} = \frac{bh^2}{6}$

② Beam with projections

$I_2 = \frac{1}{12}\left(\frac{8b}{9}\right)h^3 + \frac{1}{12}\left(\frac{b}{9}\right)(h+2d)^3$

$= \frac{b}{108}\left[8h^3 + (h+2d)^3\right]$

$c_2 = \frac{h}{2} + d = \frac{1}{2}(h + 2d)$

$S_2 = \frac{I_2}{c_2} = \frac{b\left[8h^3 + (h+2d)^3\right]}{54(h+2d)}$

Ratio of section moduli

$\frac{S_2}{S_1} = \frac{b\left[8h^3 + (h+2d)^3\right]}{9(h+2d)(bh^2)} = \frac{8 + \left(1 + \frac{2d}{h}\right)^3}{9\left(1 + \frac{2d}{h}\right)}$

CONT.

5.6-24 CONT.

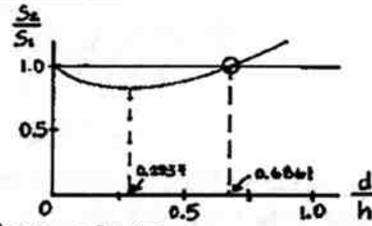
Equal section moduli

Set $\frac{S_2}{S_1} = 1$ and solve numerically for $\frac{d}{h}$.

$\frac{d}{h} = 0.6861$

Plot a graph of $\frac{S_2}{S_1}$ versus $\frac{d}{h}$

$\frac{d}{h}$	$\frac{S_2}{S_1}$
0	1.000
0.25	0.8426
0.50	0.8889
0.75	1.0500
1.00	1.2963



Moment capacity is increased when $\frac{d}{h} > 0.6861$ ←

Moment capacity is decreased when $\frac{d}{h} < 0.6861$ ←

Note:

$\frac{S_2}{S_1} = 1$ when $\left(1 + \frac{2d}{h}\right)^3 - 9\left(1 + \frac{2d}{h}\right) + 8 = 0$

or $\frac{d}{h} = 0.6861$

$\frac{S_2}{S_1}$ is minimum when $\frac{d}{h} = \frac{\sqrt[3]{4} - 1}{2} = 0.2937$

$\left(\frac{S_2}{S_1}\right)_{minimum} = 0.8399$

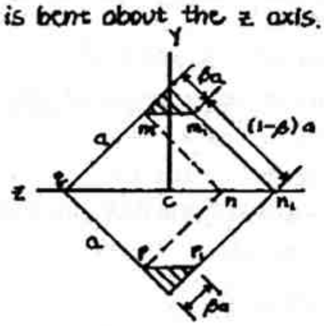
5.6-25

BEAM OF SQUARE CROSS SECTION WITH CORNERS REMOVED

a = length of each side

βa = amount removed

Beam is bent about the z axis.



Entire cross section

$I_0 = \frac{a^4}{12}$ $\quad c_0 = \frac{a}{\sqrt{2}}$ $\quad S_0 = \frac{I_0}{c_0} = \frac{a^3\sqrt{2}}{12}$

Square mnpq: $\quad I_1 = \frac{(1-\beta)^4 a^4}{12}$

Parallelogram mm_1n_1n

$I_2 = \frac{1}{3}(\text{base})(\text{height})^3$

$I_2 = \frac{1}{3}(\beta a\sqrt{2})\left[\frac{(1-\beta)a}{\sqrt{2}}\right]^3 = \frac{\beta a^4}{6}(1-\beta)^3$

CONT.

5.6-25 CONT.

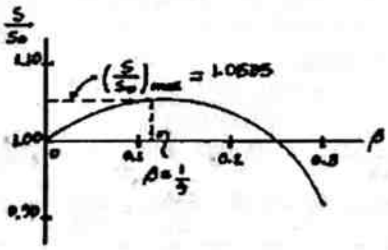
Reduced cross section

$I = I_1 + 2I_2 = \frac{a^4}{12}(1+3\beta)(1-\beta)^3$

$c = \frac{(1-\beta)a}{\sqrt{2}}$ $\quad S = \frac{I}{c} = \frac{\sqrt{2}\,a^3}{12}(1+3\beta)(1-\beta)^2$

Ratio of section moduli

$\frac{S}{S_0} = (1+3\beta)(1-\beta)^2$ $\qquad$ (1)



(a) Value of β for a maximum value of S/S_0

$\frac{d}{d\beta}\left(\frac{S}{S_0}\right) = 0$

Take the derivative and solve this equation for β.

$\beta = \frac{1}{9}$ ⟵

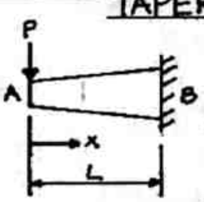
(b) Maximum value of S/S_0

Substitute $\beta = 1/9$ into Eq. (1). $(S/S_0)_{max} = 1.0535$

The section modulus is increased by 5.35% when the triangular areas are removed. ⟵

5.7-1

TAPERED CANTILEVER BEAM



Solid circular cross sections

d_A = diameter at small end

d_B = diameter at large end

d_x = diameter at distance x

$\frac{d_B}{d_A} = 2.5$

$d_x = d_A + (d_B - d_A)\frac{x}{L} = d_A\left(1 + \frac{3x}{2L}\right)$

$S_x = \frac{\pi d_x^3}{32} = \frac{\pi d_A^3}{32}\left(1 + \frac{3x}{2L}\right)^3$

$\sigma_1 = \frac{M_x}{S_x} = \frac{32Px}{\pi d_A^3\left(1 + \frac{3x}{2L}\right)^3}$

At point B (x = L):

$\sigma_B = \frac{256PL}{125\pi d_A^3}$

Cross section of maximum stress

$\frac{d\sigma_1}{dx} = 0$ Evaluate the derivative, set it equal to zero, and solve for x: $x = \frac{L}{3}$ ⟵

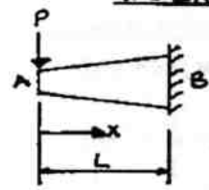
Maximum stress

$\sigma_{max} = (\sigma_1)_{x = \frac{L}{3}}$ $\qquad \sigma_{max} = \frac{256PL}{81\pi d_A^3}$ ⟵

$\frac{\sigma_{max}}{\sigma_B} = \frac{125}{81} = 1.543$ ⟵

5.7-2

TAPERED CANTILEVER BEAM



Square cross sections

h_A = height and width at smaller end

h_B = height and width at larger end

h_x = height and width at distance x

$\frac{h_B}{h_A} = 1.75$

$h_x = h_A + (h_B - h_A)\frac{x}{L} = h_A\left(1 + \frac{3x}{4L}\right)$

$S_x = \frac{h_x^3}{6} = \frac{h_A^3}{6}\left(1 + \frac{3x}{4L}\right)^3$

$\sigma_1 = \frac{M_x}{S_x} = \frac{6Px}{h_A^3\left(1 + \frac{3x}{4L}\right)^3}$

At support B (x = L):

$\sigma_B = \frac{384PL}{343h_A^3}$

Cross section of maximum stress

$\frac{d\sigma_1}{dx} = 0$ Evaluate the derivative, set it equal to zero, and solve for x: $x = \frac{2L}{3}$ ⟵

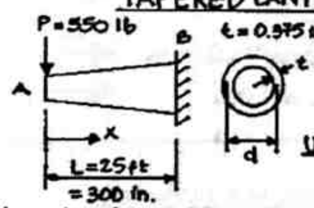
Maximum stress

$\sigma_{max} = (\sigma_1)_{x = \frac{2L}{3}} = \frac{32PL}{27h_A^3}$ ⟵

$\frac{\sigma_{max}}{\sigma_B} = \frac{343}{324} = 1.059$ ⟵

5.7-3

TAPERED CANTILEVER BEAM



$d_A = 3.5$ in.

$d_B = 10.5$ in.

Units: Inches and pounds

$d_x = d_A + (d_B - d_A)\frac{x}{L} = 3.5 + 7\left(\frac{x}{L}\right) = 3.5\left(1 + \frac{2x}{L}\right)$

$S_x = \frac{\pi d_x^2 t}{4} = \frac{\pi}{4}(3.5)^2\left(1 + \frac{2x}{L}\right)^2(0.375)$

$= \frac{147\pi}{128}\left(1 + \frac{2x}{L}\right)^2$ $\qquad M_x = 550x$

$\sigma_1 = \frac{M_x}{S_x} = \frac{70{,}400\,x}{147\pi\left(1 + \frac{2x}{L}\right)^2}$

At support B (x = L = 300 in.):

$\sigma_B = \frac{7{,}040{,}000}{441\pi} = 5081.4$ psi

Cross section of maximum stress

$\frac{d\sigma_1}{dx} = 0$ Evaluate the derivative, set it equal to zero, and solve for x: $x = \frac{L}{2} = 12.5$ ft ⟵

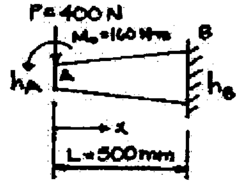
Maximum stress

$\sigma_{max} = (\sigma_1)_{x = \frac{L}{2}} = \frac{880{,}000}{49\pi} = 5717$ psi ⟵

$\frac{\sigma_{max}}{\sigma_B} = \frac{9}{8} = 1.125$ ⟵

5.7-4

TAPERED CANTILEVER BEAM



Rectangular cross sections

h_A = height at small end = 50 mm

h_B = height at large end = 75 mm

b = width of beam = 25 mm

Units: Newtons and millimeters

$$h_x = h_A + (h_B - h_A)\frac{x}{L}$$

$$S_x = \frac{bh_x^2}{6} = \frac{b}{6}\left[h_A + (h_B - h_A)\frac{x}{L}\right]^2$$

$$= \frac{25}{6}\left(50 + 25\frac{x}{L}\right)^2 = \frac{15{,}625}{6}\left(2 + \frac{x}{L}\right)^2$$

$$M_x = Px + M_0 = 400x + 160{,}000 \quad (\text{N}\cdot\text{mm})$$

$$= 400(x + 400)$$

$$\sigma_1 = \frac{M_x}{S_x} = \frac{(400)(x+400)(6)}{15{,}625\left(2+\frac{x}{L}\right)^2} = \left(\frac{24\times10^6}{625}\right)\left[\frac{(400+x)}{(1000+x)^2}\right] \quad (\text{N/mm}^2)$$

At support B (x = L):

$$\sigma_B = \left(\frac{24\times10^6}{625}\right)\left[\frac{900}{(1500)^2}\right] = 15.36 \text{ N/mm}^2 = 15.36 \text{ MPa}$$

Cross section of maximum stress

$\frac{d\sigma_1}{dx} = 0$ Evaluate the derivative, set it equal to zero, and solve for x: x = 200 mm

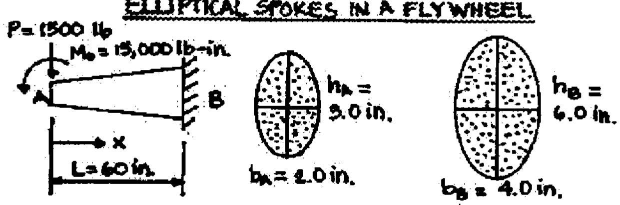
Maximum stress

$$\sigma_{max} = (\sigma_1)_{x=200} = 16 \text{ N/mm}^2 = 16 \text{ MPa}$$

$$\frac{\sigma_{max}}{\sigma_B} = \frac{16 \text{ MPa}}{15.36 \text{ MPa}} = \frac{25}{24} = 1.042$$

5.7-5

ELLIPTICAL SPOKES IN A FLYWHEEL



Units: Pounds and inches

$$b_x = b_A + (b_B - b_A)\frac{x}{L} = 2 + 2\frac{x}{L} = \frac{1}{30}(60+x) \quad (\text{in.})$$

$$h_x = h_A + (h_B - h_A)\frac{x}{L} = 3 + 3\frac{x}{L} = \frac{1}{20}(60+x) \quad (\text{in.})$$

From Case 16, Appendix D: $I = \frac{\pi}{64}bh^3$

$$I_x = \frac{\pi}{64}(b_x)(h_x^3) \qquad S_x = \frac{I_x}{h_x/2} = \frac{\pi}{32}b_x h_x^2$$

$$= \frac{\pi}{384{,}000}(60+x)^3 \quad (\text{in.}^3)$$

$$M_x = M_0 + Px = 15{,}000 + 1500x = 1500(10+x) \quad (\text{lb-in.})$$

$$\sigma_1 = \frac{M_x}{S_x} = \frac{1500(10+x)(384{,}000)}{\pi(60+x)^3} = \frac{(576\times10^6)(10+x)}{\pi(60+x)^3} \quad (\text{psi})$$

CONT.

5.7-5 CONT.

(a) At end A (x = 0)

$$\sigma_A = \frac{(576\times10^6)(10)}{\pi(60)^3} = \frac{80{,}000}{3\pi} = 8488 \text{ psi}$$

(b) At end B (x = L = 60 in.)

$$\sigma_B = \frac{(576\times10^6)(70)}{\pi(120)^3} = \frac{70{,}000}{3\pi} = 7427 \text{ psi}$$

(c) Cross section of maximum stress

$\frac{d\sigma_1}{dx} = 0$ Evaluate the derivative, set it equal to zero, and solve for x: x = 15 in.

(d) Maximum stress

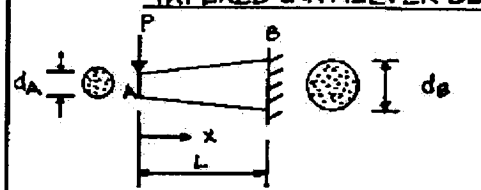
$$\sigma_{max} = (\sigma_1)_{x=15} = \frac{(576\times10^6)(10+15)}{\pi(60+15)^3}$$

$$= \frac{102{,}400}{3\pi} = 10{,}865 \text{ psi}$$

Note: $\frac{\sigma_{max}}{\sigma_A} = \frac{32}{25} = 1.28$; $\frac{\sigma_{max}}{\sigma_B} = \frac{256}{175} = 1.463$

5.7-6

TAPERED CANTILEVER BEAM



From Eq. (5-32), Example 5-9

$$\sigma_1 = \frac{32Px}{\pi\left[d_A + (d_B - d_A)\left(\frac{x}{L}\right)\right]^3} \qquad (1)$$

Find the value of x that makes σ_1 a maximum

Let $\sigma_1 = \frac{u}{v}$ $\quad \frac{d\sigma_1}{dx} = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2} = \frac{N}{D}$

$$N = \pi\left[d_A + (d_B - d_A)\left(\frac{x}{L}\right)\right]^3[32P]$$
$$- [32Px][\pi][3]\left[d_A + (d_B - d_A)\left(\frac{x}{L}\right)\right]^2\left[\frac{1}{L}(d_B - d_A)\right]$$

After simplification:

$$N = 32\pi P\left[d_A + (d_B - d_A)\left(\frac{x}{L}\right)\right]^2\left[d_A - 2(d_B - d_A)\frac{x}{L}\right]$$

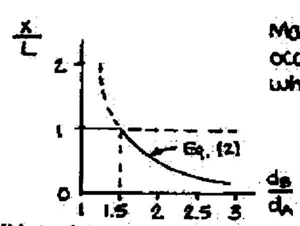
$$D = \pi^2\left[d_A + (d_B - d_A)\left(\frac{x}{L}\right)\right]^6$$

$$\frac{d\sigma_1}{dx} = \frac{N}{D} = \frac{32P\left[d_A - 2(d_B - d_A)\frac{x}{L}\right]}{\pi\left[d_A + (d_B - d_A)\left(\frac{x}{L}\right)\right]^4}$$

$$\frac{d\sigma_1}{dx} = 0 \qquad d_A - 2(d_B - d_A)\left(\frac{x}{L}\right) = 0$$

$$\frac{x}{L} = \frac{d_A}{2(d_B - d_A)} = \frac{1}{2\left(\frac{d_B}{d_A} - 1\right)} \qquad (2)$$

(a) Graph of x/L versus d_B/d_A (Eq. 2)



Maximum bending stress occurs at the support when

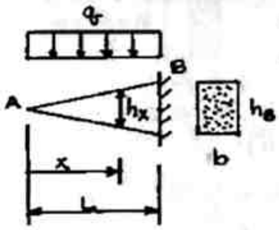
$$1 \le \frac{d_B}{d_A} \le 1.5$$

(b) Maximum stress

Substitute x/L = 1 into Eq. (1): $\sigma_{max} = \frac{3PL}{\pi d_B^3}$

5.7-7

FULLY STRESSED BEAM WITH CONSTANT WIDTH AND VARYING HEIGHT



h_x = height at distance x

h_B = height at end B

b = width (constant)

At distance x: $M = \frac{qx^2}{2}$ $\quad S = \frac{bh_x^2}{6}$

$$\sigma_{allow} = \frac{M}{S} = \frac{3qx^2}{bh_x^2}$$

$$h_x = x\sqrt{\frac{3q}{b\sigma_{allow}}}$$

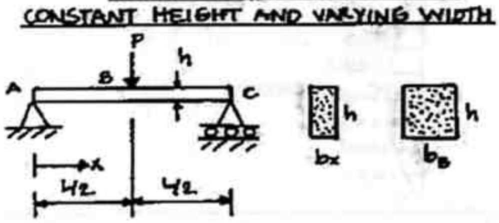
At the fixed end (x = L):

$$h_B = L\sqrt{\frac{3q}{b\sigma_{allow}}}$$

Therefore, $\frac{h_x}{h_B} = \frac{x}{L}$ $\quad h_x = \frac{h_B x}{L}$ ←

5.7-8

FULLY STRESSED BEAM WITH CONSTANT HEIGHT AND VARYING WIDTH



h = height of beam (constant)

b_x = width at distance x from end A $(0 \le x \le \frac{L}{2})$

b_B = width at midpoint B $(x = L/2)$

At distance x $\quad M = \frac{Px}{2}$ $\quad S = \frac{1}{6}b_x h^2$

$$\sigma_{allow} = \frac{M}{S} = \frac{3Px}{b_x h^2} \qquad b_x = \frac{3Px}{\sigma_{allow} h^2}$$

At midpoint B (x = L/2)

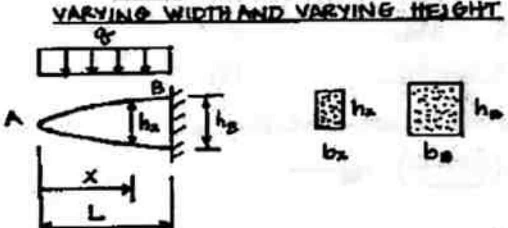
$$b_B = \frac{3PL}{2\sigma_{allow} h^2}$$

Therefore, $\frac{b_x}{b_B} = \frac{2x}{L}$ and $b_x = \frac{2b_B x}{L}$ ←

Note: The equation is valid for $0 \le x \le \frac{L}{2}$ and the beam is symmetrical about the midpoint.

5.7-9

FULLY STRESSED BEAM WITH VARYING WIDTH AND VARYING HEIGHT



CONT.

5.7-9 CONT.

h_x = height at distance x

h_B = height at end B

b_x = width at distance x

b_B = width at end B

$b_x = b_B\left(\frac{x}{L}\right)$

At distance x

$$M = \frac{qx^2}{2} \qquad S = \frac{b_x h_x^2}{6} = \frac{b_B x}{6L}(h_x)^2$$

$$\sigma_{allow} = \frac{M}{S} = \frac{3qLx}{b_B h_x^2}$$

$$h_x = \sqrt{\frac{3qLx}{b_B \sigma_{allow}}}$$

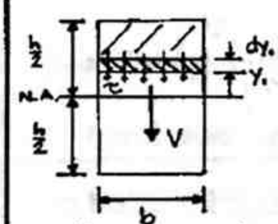
At the fixed end (x = L)

$$h_B = \sqrt{\frac{3qL^2}{b_B \sigma_{allow}}}$$

Therefore, $\frac{h_x}{h_B} = \sqrt{\frac{x}{L}}$ $\quad h_x = h_B\sqrt{\frac{x}{L}}$ ←

5.8-1

RESULTANT R OF THE SHEAR STRESSES



$$I = \frac{bh^3}{12}$$

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

$$R = \int_{-h/2}^{h/2} \tau b\, dy_1 = 2\int_0^{h/2} \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right) b\, dy_1$$

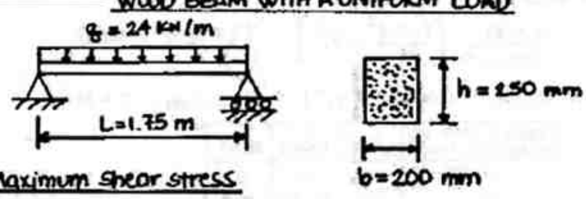
$$= \frac{12V}{bh^3}(b)\int_0^{h/2}\left(\frac{h^2}{4} - y_1^2\right)dy_1$$

$$= \frac{12V}{h^3}\left(\frac{2h^3}{24}\right) = V$$

$\therefore R = V$ Q.E.D. ←

5.8-2

WOOD BEAM WITH A UNIFORM LOAD



Maximum shear stress

$V = \frac{qL}{2}$ $\quad A = bh$

$$\tau_{max} = \frac{3V}{2A} = \frac{3qL}{4bh} = \frac{3(24\text{ kN/m})(1.75\text{ m})}{4(200\text{ mm})(250\text{ mm})}$$

$= 630$ kPa ←

Maximum bending stress

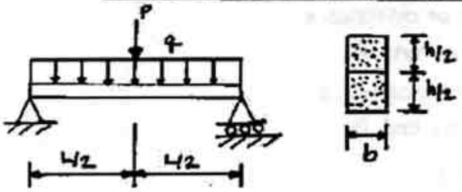
$M = \frac{qL^2}{8}$ $\quad S = \frac{bh^2}{6}$

$$\sigma_{max} = \frac{M}{S} = \frac{3qL^2}{4bh^2} = \frac{3(24\text{ kN/m})(1.75\text{ m})^2}{4(200\text{ mm})(250\text{ mm})^2}$$

$= 4.41$ MPa ←

5.8-3

SIMPLE BEAM WITH A GLUED JOINT



$L = 6\text{ ft} = 72\text{ in.}\quad b = 3.5\text{ in.}\quad h = 7.0\text{ in.}$

$\tau_{allow} = 200\text{ psi}\quad \gamma = 0.02\text{ lb/in.}^3$

q = weight of beam

$= \gamma bh$

Maximum load P_{max}

$V = \frac{P}{2} + \frac{qL}{2}\qquad A = bh$

$\tau_{max} = \frac{3V}{2A} = \frac{3\left(\frac{P}{2}+\frac{qL}{2}\right)}{2bh} = \frac{3}{4bh}(P+qL)$

$P_{max} = \frac{4}{3}bh\tau - qL = \frac{4}{3}bh\tau - \gamma bhL$

$= bh\left(\frac{4}{3}\tau - \gamma L\right)$

Substitute numerical values:

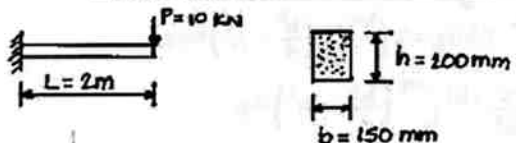
$P_{max} = (3.5\text{ in.})(7.0\text{ in.})\left[\frac{4}{3}(200\text{ psi}) - (0.02\text{ lb/in.}^3)(72\text{ in.})\right]$

$= 6500\text{ lb}$ ←

(This result is based solely on the shear stress)

5.8-4

SHEAR STRESSES IN A CANTILEVER BEAM



Eq. (5-39): $\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$

$V = P = 10\text{ kN} = 10{,}000\text{ N}\qquad I = \frac{bh^3}{12} = 100\times10^6\text{ mm}^4$

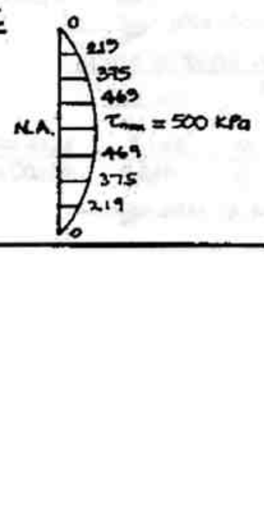
$h = 200\text{ mm}\quad (y_1 = \text{mm})$

$\tau = \frac{10{,}000}{2(100\times10^6)}\left[\frac{(200)^2}{4} - y_1^2\right]\qquad (\tau = \text{N/m}^2 = \text{MPa})$

$\tau = 50\times10^{-6}(10{,}000 - y_1^2)\qquad (y_1 = \text{mm};\ \tau = \text{MPa})$

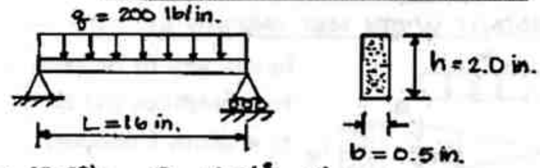
Distance from the top surface (mm)	y_1 (mm)	τ (MPa)	τ (kPa)
0	100	0	0
25	75	0.219	219
50	50	0.375	375
75	25	0.469	469
100 (N.A.)	0	0.500	500

Graph of shear stress τ



5.8-5

SHEAR STRESSES IN A SIMPLE BEAM



Eq. (5-39): $\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$

$V = \frac{qL}{2} = 1600\text{ lb}\qquad I = \frac{bh^3}{12} = \frac{1}{3}\text{ in.}^4$

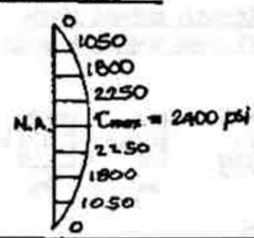
Units: pounds and inches

$\tau = \frac{1600}{2\left(\frac{1}{3}\right)}\left[\frac{(2)^2}{4} - y_1^2\right] = (2400)(1 - y_1^2)$

$(\tau = \text{psi};\ y_1 = \text{in.})$

Distance from the top surface (in.)	y_1 (in.)	τ (psi)
0	1.00	0
0.25	0.75	1050
0.50	0.50	1800
0.75	0.25	2250
1.00 (N.A.)	0	2400

Graph of shear stress τ



5.8-6

BEAM OF RECTANGULAR CROSS SECTION

b = width $\quad h$ = height $\quad L$ = length

Uniform load $\quad q$ = intensity of load

Allowable stresses $\quad \sigma_{allow}$ and τ_{allow}

(a) Simple beam

Bending

$M_{max} = \frac{qL^2}{8}\qquad S = \frac{bh^2}{6}$

$\sigma_{max} = \frac{M_{max}}{S} = \frac{3qL^2}{4bh^2}$

$q_{allow} = \frac{4\sigma_{allow}bh^2}{3L^2}\qquad (1)$

Shear

$V_{max} = \frac{qL}{2}\qquad A = bh$

$\tau_{max} = \frac{3V}{2A} = \frac{3qL}{4bh}$

$q_{allow} = \frac{4\tau_{allow}bh}{3L}\qquad (2)$

Equate (1) and (2) and solve for L_0:

$L_0 = h\left(\frac{\sigma_{allow}}{\tau_{allow}}\right)$ ←

CONT.

5.8-6 CONT.

(b) Cantilever beam

Bending

$M_{max} = \frac{qL^2}{2}$ $\quad S = \frac{bh^2}{6}$

$\sigma_{max} = \frac{M_{max}}{S} = \frac{3qL^2}{bh^2}$ $\quad q_{allow} = \frac{\sigma_{allow} bh^2}{3L^2}$ (3)

Shear

$V_{max} = qL$ $\quad A = bh$

$\tau_{max} = \frac{3V}{2A} = \frac{3qL}{2bh}$

$q_{allow} = \frac{2\tau_{allow} bh}{3L}$ (4)

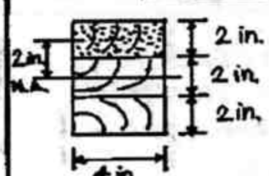
Equate (3) and (4) and solve for L_0:

$$L_0 = \frac{h}{2}\left(\frac{\sigma_{allow}}{\tau_{allow}}\right) \longleftarrow$$

Note: If the actual length is less than L_0, the shear stress governs the design.
If the length is greater than L_0, the bending stress governs.

5.8-7

LAMINATED WOOD BEAM ON SIMPLE SUPPORTS



$L = 6\text{ ft} = 72\text{ in.}$

$\tau_{allow} = 50\text{ psi}$

$\sigma_{allow} = 1500\text{ psi}$

Allowable load based upon shear stress in the glued joints

$\tau = \frac{VQ}{Ib}$ $\quad Q = (4\text{ in.})(2\text{ in.})(2\text{ in.}) = 16\text{ in.}^3$

$V = \frac{P}{2}$ $\quad I = \frac{bh^3}{12} = \frac{1}{12}(4\text{ in.})(6\text{ in.})^3 = 72\text{ in.}^4$

$\tau = \frac{(P/2)(16\text{ in.}^3)}{(72\text{ in.}^4)(4\text{ in.})} = \frac{P}{36}$ $\quad (P = \text{lb};\ \tau = \text{psi})$

$P = 36\,\tau_{allow} = 36(50\text{ psi}) = 1800\text{ lb}$

Allowable load based upon bending stress

$\sigma = \frac{M}{S}$ $\quad M = \frac{PL}{4} = P\left(\frac{72\text{ in.}}{4}\right) = 18P$ (lb-in.)

$S = \frac{bh^2}{6} = \frac{1}{6}(4\text{ in.})(6\text{ in.})^2 = 24\text{ in.}^3$

$\sigma = \frac{(18P\text{ lb-in.})}{24\text{ in.}^3} = \frac{3P}{4}$ $\quad (P = \text{lb};\ \sigma = \text{psi})$

$P = \frac{4}{3}\sigma_{allow} = \frac{4}{3}(1500\text{ psi}) = 2000\text{ lb}$

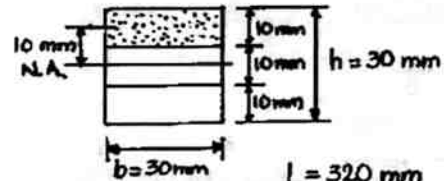
Allowable load

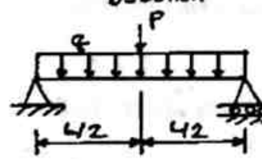
Shear stress in the glued joints governs.

$P_{max} = 1800\text{ lb} \longleftarrow$

5.8-8

LAMINATED PLASTIC BEAM





$L = 320\text{ mm}$

$W = 3.2\text{ N}$

$q = \frac{W}{L} = \frac{3.2\text{ N}}{320\text{ mm}} = 10\text{ N/m}$

$I = \frac{bh^3}{12} = \frac{1}{12}(30\text{ mm})(30\text{ mm})^3 = 67{,}500\text{ mm}^4$

$S = \frac{bh^2}{6} = \frac{1}{6}(30\text{ mm})(30\text{ mm})^2 = 4500\text{ mm}^3$

(a) Allowable load based upon shear in glued joints

$\tau_{allow} = 0.3\text{ MPa}$

$\tau = \frac{VQ}{Ib}$ $\quad V = \frac{P}{2} + \frac{qL}{2} = \frac{P}{2} + 1.6\text{ N}$ $\quad$ (V = newtons; P = newtons)

$Q = (30\text{ mm})(10\text{ mm})(10\text{ mm}) = 3000\text{ mm}^3$

$\frac{Q}{Ib} = \frac{3000\text{ mm}^3}{(67{,}500\text{ mm}^4)(30\text{ mm})} = \frac{1}{675\text{ mm}^2}$

$\tau = \frac{VQ}{Ib} = \frac{P/2 + 1.6\text{ N}}{675\text{ mm}^2}$ $\quad (\tau = \text{N/mm}^2 = \text{MPa})$

Solve for P:

$P = 1350\,\tau_{allow} - 3.2 = 405\text{ N} - 3.2\text{ N} = 402\text{ N} \longleftarrow$

(b) Allowable load based upon bending stresses

$\sigma_{allow} = 8\text{ MPa}$

$\sigma = \frac{M_{max}}{S}$ $\quad M_{max} = \frac{PL}{4} + \frac{qL^2}{8} = 0.08P + 0.128$ (N·m)

(P = newtons; M = N·m)

$\sigma = \frac{(0.08P + 0.128)(\text{N·m})}{4.5\times10^{-6}\text{ m}^3}$ $\quad (\sigma = \text{N/m}^2 = \text{Pa})$

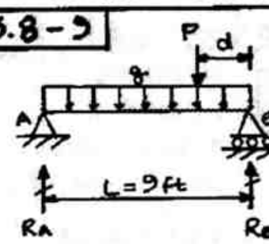
Solve for P:

$P = (56.25\times10^{-6})\,\sigma_{allow} - 1.6$

$= (56.25\times10^{-6})(8\times10^{6}\text{ Pa}) - 1.6$

$= 450 - 1.6 = 448\text{ N} \longleftarrow$

5.8-9



$q = 120\text{ lb/ft}$

$P = 8800\text{ lb}$

$d = 3\text{ ft}$

$\sigma_{allow} = 2500\text{ psi}$

$\tau_{allow} = 150\text{ psi}$

$R_A = \frac{qL}{2} + \frac{P}{3}$ $\quad R_B = \frac{qL}{2} + \frac{2P}{3}$

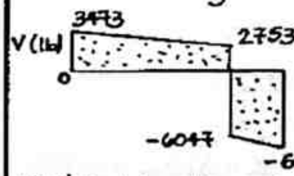
CONT.

5.8-9 CONT.

(a) Disregarding the weight of the beam

$R_A = \frac{(120\ \text{lb/ft})(9\ \text{ft})}{2} + \frac{8800\ \text{lb}}{3} = 3473\ \text{lb}$

$R_B = 540\ \text{lb} + \frac{2}{3}(8800\ \text{lb}) = 6407\ \text{lb}$



$V_{max} = R_B = 6407\ \text{lb}$

Maximum bending moment occurs under the concentrated load.

$M_{max} = R_B d - \frac{qd^2}{2}$

$= (6407\ \text{lb})(3\ \text{ft}) - \frac{1}{2}(120\ \text{lb/ft})(3\ \text{ft})^2$

$= 18{,}680\ \text{lb-ft} = 224{,}200\ \text{lb-in.}$

$\tau_{max} = \frac{3V}{2A}$ $\quad A_{req} = \frac{3V_{max}}{2\tau_{allow}} = \frac{3(6407\ \text{lb})}{2(150\ \text{psi})} = 64.1\ \text{in.}^2$

$\sigma = \frac{M}{S}$ $\quad S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{224{,}200\ \text{lb-in.}}{2500\ \text{psi}} = 89.7\ \text{in.}^3$

From Appendix F: Select 8×10 in. beam (nominal dimensions) ←

$A = 71.25\ \text{in.}^2 \quad S = 112.8\ \text{in.}^3$

(b) Considering the weight of the beam

$q_{beam} = 17.3\ \text{lb/ft}$ (Weight density = 35 lb/ft³)

$R_B = 6407\ \text{lb} + \frac{(17.3\ \text{lb/ft})(9\ \text{ft})}{2} = 6407 + 78 = 6485\ \text{lb}$

$V_{max} = 6485\ \text{lb}$ $\quad A_{req'd} = \frac{3V_{max}}{2\tau_{allow}} = 64.9\ \text{in.}^2$

8×10 beam is still satisfactory for shear.

$q_{total} = 120\ \text{lb/ft} + 17.3\ \text{lb/ft} = 137.3\ \text{lb/ft}$

$M_{max} = R_B d - \frac{qd^2}{2} = (6485\ \text{lb})(3\ \text{ft}) - \frac{1}{2}(137.3\ \tfrac{\text{lb}}{\text{ft}})(3\ \text{ft})^2$

$= 18{,}837\ \text{lb-ft} = 226{,}050\ \text{lb-in.}$

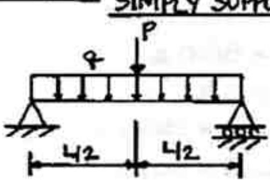
$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{226{,}050\ \text{lb-in.}}{2500\ \text{psi}} = 90.4\ \text{in.}^3$

8×10 beam is still satisfactory for moment.

Use 8×10 in. beam ←

5.8-10

SIMPLY SUPPORTED WOOD BEAM



$b = 140\ \text{mm}$

$h = 240\ \text{mm}$

$A = bh = 33{,}600\ \text{mm}^2$

$S = \frac{bh^2}{6} = 1344 \times 10^3\ \text{mm}^3$

$\gamma = 5.4\ \text{kN/m}^3$

$L = 1.2\ \text{m} \quad q = \gamma bh = 181.44\ \text{N/m}$

CONT.

5.8-10 CONT.

(a) Allowable load P based upon bending stress

$\sigma_{allow} = 8.5\ \text{MPa} \quad \sigma = \frac{M_{max}}{S}$

$M_{max} = \frac{PL}{4} + \frac{qL^2}{8} = \frac{P(1.2\ \text{m})}{4} + \frac{(181.44\ \text{N/m})(1.2\ \text{m})^2}{8}$

$= 0.3P + 32.66\ \text{N·m}$ (P = newtons; M = N·m)

$M_{max} = S\sigma_{allow} = (1344 \times 10^3\ \text{mm}^3)(8.5\ \text{MPa}) = 11{,}424\ \text{N·m}$

Equate values of M_{max} and solve for P:

$0.3P + 32.66 = 11{,}424 \quad P = 37{,}970\ \text{N}$

or $P = 38.0\ \text{kN}$ ←

(b) Allowable load P based upon shear stress

$\tau_{allow} = 0.8\ \text{MPa} \quad \tau = \frac{3V}{2A}$

$V = \frac{P}{2} + \frac{qL}{2} = \frac{P}{2} + \frac{(181.44\ \text{N/m})(1.2\ \text{m})}{2} = \frac{P}{2} + 108.86\ \text{(N)}$

$V = \frac{2A\tau}{3} = \frac{2}{3}(33{,}600\ \text{mm}^2)(0.8\ \text{MPa}) = 17{,}920\ \text{N}$

Equate values of V and solve for P:

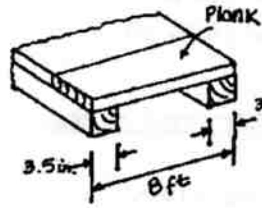
$\frac{P}{2} + 108.86 = 17{,}920 \quad P = 35{,}622\ \text{N}$

or $P = 35.6\ \text{kN}$ ←

Note: the shear stress governs and $P_{allow} = 35.6\ \text{kN}$

5.8-11

WOOD PLATFORM WITH A PLANK DECK



Platform: 8 ft × 8 ft

t = thickness of planks = 1.5 in.

w = uniform load on the deck (lb/ft²)

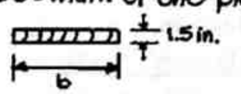
$\sigma_{allow} = 2400\ \text{psi}$

$\tau_{allow} = 100\ \text{psi}$

Find w_{allow} (lb/ft²)

(a) Allowable load based upon bending stress in the planks

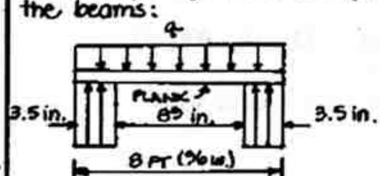
Let b = width of one plank (in.)



$A = 1.5b\ \text{(in.}^2)$

$S = \frac{b}{6}(1.5\ \text{in.})^2 = 0.375b\ \text{(in.}^3)$

Free-body diagram of one plank supported on the beams:



Load on one plank:

$q = \left[\frac{w\ (\text{lb/ft}^2)}{144\ \text{in.}^2/\text{ft}^2}\right](b\ \text{in.}) = \frac{wb}{144}\ \text{(lb/in.)}$

CONT.

5.8-11 CONT.

Reaction $R = q\left(\frac{96\text{ in.}}{2}\right) = \left(\frac{wb}{144}\right)(48) = \frac{wb}{3}$

(R = lb ; w = lb/ft²; b = in.)

M_{max} occurs at midspan.

$M_{max} = R\left(\frac{3.5\text{ in.}}{2} + \frac{89\text{ in.}}{2}\right) - \frac{q(48\text{ in.})^2}{2}$

$= \frac{wb}{3}(46.25) - \frac{wb}{144}(1152) = \frac{89}{12}wb$

(M = lb-in.; w = lb/ft²; b = in.)

Allowable bending moment:

$M_{allow} = \sigma_{allow} S = (2400\text{ psi})(0.375b) = 900b$ (lb-in.)

Equate M_{max} and M_{allow} and solve for w:

$\frac{89}{12}wb = 900b \quad w_1 = 121\text{ lb/ft}^2$ ←

(b) Allowable load based upon shear stress in the planks

See the free-body diagram in part (a).

V_{max} occurs at the inside face of the support.

$V_{max} = q\left(\frac{89\text{ in.}}{2}\right) = 44.5q = (44.5)\left(\frac{wb}{144}\right) = \frac{89wb}{288}$

(V = lb; w = lb/ft²; b = in.)

Allowable shear force:

$\tau = \frac{3V}{2A} \quad V_{allow} = \frac{2A\tau_{allow}}{3} = \frac{2(1.5b)(100\text{ psi})}{3} = 100b$ (lb)

Equate V_{max} and V_{allow} and solve for w:

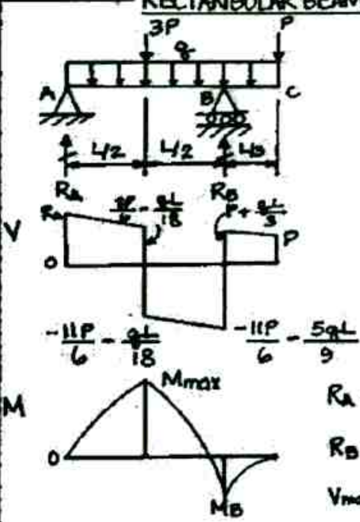
$\frac{89wb}{288} = 100b \quad w_2 = 324\text{ lb/ft}^2$ ←

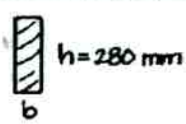
(c) Allowable load

Bending stress governs. $w_{allow} = 121\text{ lb/ft}^2$ ←

5.8-12

RECTANGULAR BEAM WITH AN OVERHANG





$L = 3.6$ m

$P = 5$ kN

$\gamma = 5.5\text{ kN/m}^3$ (FOR THE WOOD)

$q = \gamma bh$

Find b

$R_A = \frac{7P}{6} + \frac{4qL}{9}$

$R_B = \frac{17P}{6} + \frac{8qL}{9}$

$V_{max} = \frac{11P}{6} + \frac{5qL}{9}$

$M_{max} = \frac{7PL}{12} + \frac{7qL^2}{72} \quad M_B = -\frac{PL}{3} - \frac{qL^2}{18}$

CONT.

5.8-12 CONT.

(a) Required width b based upon bending stress

$M_{max} = \frac{7PL}{12} + \frac{7qL^2}{72} = \frac{7}{12}(5000\text{ N})(3.6\text{ m}) + \frac{7}{72}(\gamma bh)(3.6\text{ m})^2$

$= 10{,}500\text{ N·m} + \frac{7}{72}(5500\text{ N/m}^3)(b)(0.280\text{ m})(3.6\text{ m})^2$

$= 10{,}500 + 1940.4b$ (b = meters) (M = newton-meters)

$\sigma = \frac{M_{max}}{S} = \frac{6M_{max}}{bh^2} \quad \sigma_{allow} = 8.2\text{ MPa}$

$M_{max} = \frac{bh^2\sigma_{allow}}{6} = \frac{b}{6}(0.280\text{ m})^2(8.2\times10^6\text{ Pa})$

$= 107{,}150b$

Equate moments and solve for b:

$10{,}500 + 1940.4b = 107{,}150b$

$b = 0.0998\text{ m} = 99.8\text{ mm}$ ←

(b) Required width b based upon shear stress

$V_{max} = \frac{11P}{6} + \frac{5qL}{9}$

$= \frac{11}{6}(5000\text{ N}) + \frac{5}{9}(\gamma bh)(3.6\text{ m})$

$= 9167\text{ N} + \frac{5}{9}(5500\text{ N/m}^3)(b)(0.280\text{ m})(3.6\text{ m})$

$= 9167 + 3080b$ (b = meters)

$\tau = \frac{3V_{max}}{2A} = \frac{3V_{max}}{2bh}$ (V = newtons)

$\tau_{allow} = 0.7\text{ MPa}$

$V_{max} = \frac{2bh\tau_{allow}}{3} = \frac{2b}{3}(0.280\text{ m})(0.7\times10^6\text{ N/m}^2)$

$= 130{,}670b$

Equate shear forces and solve for b:

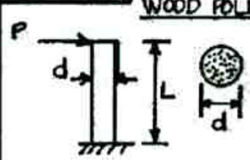
$9167 + 3080b = 130{,}670b$

$b = 0.0718\text{ m} = 71.8\text{ mm}$ ←

Note: Bending stress governs. $b = 99.8$ mm

5.9-1

WOOD POLE OF CIRCULAR CROSS SECTION



$P = 500$ lb $\quad L = 6\text{ ft} = 72$ in.

$\sigma_{allow} = 2100$ psi

$\tau_{allow} = 150$ psi

Find diameter d

(a) Based upon bending stress

$M_{max} = PL = (500\text{ lb})(72\text{ in.}) = 36{,}000$ lb-in.

$\sigma = \frac{M}{S} = \frac{32M}{\pi d^3} \quad d^3 = \frac{32M_{max}}{\pi\sigma_{allow}} = 174.6\text{ in.}^3$

$d_{min} = 5.59$ in. ←

(b) Based upon shear stress

$V_{max} = 500$ lb

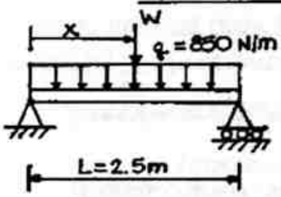
$\tau = \frac{4V}{3A} = \frac{16V}{3\pi d^2} \quad d^2 = \frac{16V_{max}}{3\pi\tau_{allow}} = 5.659\text{ in.}^2$

$d_{min} = 2.38$ in. ←

(Bending stress governs)

5.9-2

LOG BRIDGE



Diameter $d = 300$ mm

$\sigma_{allow} = 7.5$ MPa

$\tau_{allow} = 0.8$ MPa

Find allowable load W

(a) Based upon bending stress

Maximum moment occurs when wheel is at midspan ($x = L/2$).

$$M_{max} = \frac{WL}{4} + \frac{qL^2}{8} = \frac{W}{4}(2.5\text{m}) + \frac{1}{8}(850\text{ N/m})(2.5\text{m})^2$$

$$= 0.625\,W + 664.1 \text{ (N·m)} \quad (W = \text{newtons})$$

$$S = \frac{\pi d^3}{32} = 2.651 \times 10^{-3}\text{ m}^3$$

$$M_{max} = S\sigma_{allow} = (2.651 \times 10^{-3}\text{ m}^3)(7.5\text{ MPa})$$

$$= 19{,}880 \text{ N·m}$$

$$\therefore 0.625\,W + 664.1 = 19{,}880$$

$$W = 30.7 \text{ kN} \longleftarrow$$

(b) Based upon shear stress

Maximum shear force occurs when wheel is adjacent to support ($x = 0$).

$$V_{max} = W + \frac{qL}{2} = W + \frac{1}{2}(850\text{ N/m})(2.5\text{m})$$

$$= W + 1062.5 \text{ N} \quad (W = \text{newtons})$$

$$A = \frac{\pi d^2}{4} = 0.070686 \text{ m}^2$$

$$\tau_{allow} = \frac{4V_{max}}{3A}$$

$$V_{max} = \frac{3A\tau_{allow}}{4} = \frac{3}{4}(0.070686\text{ m}^2)(0.8\text{ MPa})$$

$$= 42{,}410 \text{ N}$$

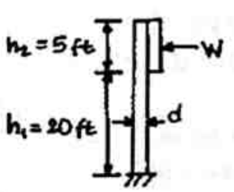
$$\therefore W + 1062.5\text{ N} = 42{,}410 \text{ N}$$

$$W = 41.3 \text{ kN} \longleftarrow$$

(Bending stress governs.)

5.9-3

WIND LOAD ON A SIGN



b = width of sign

$b = 10$ ft

$p = 75$ lb/ft^2

$\sigma_{allow} = 6000$ psi

$\tau_{allow} = 2000$ psi

d = diameter

W = wind force on one pole

$$t = \frac{d}{10} \qquad W = ph_2\left(\frac{b}{2}\right) = 1875 \text{ lb}$$

(a) Required diameter based upon bending stress

$$M_{max} = W\left(h_1 + \frac{h_2}{2}\right) = 506{,}250 \text{ lb-in.}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) \quad d_2 = d \quad d_1 = d - 2t = \frac{4}{5}d$$

CONT.

5.9-3 CONT.

$$I = \frac{\pi}{64}\left[d^4 - \left(\frac{4d}{5}\right)^4\right] = \frac{\pi d^4}{64}\left(\frac{369}{625}\right) = \frac{369\pi d^4}{40{,}000} \text{ (in.}^4\text{)}$$

$$c = \frac{d}{2} \qquad (d = \text{inches})$$

$$\sigma = \frac{Mc}{I} = \frac{M(d/2)}{369\pi d^4/40{,}000} = \frac{17.253\,M}{d^3}$$

$$d^3 = \frac{17.253\,M_{max}}{\sigma_{allow}} = \frac{(17.253)(506{,}250\text{ lb-in.})}{6000\text{ psi}}$$

$$= 1456 \text{ in.}^3 \qquad d = 11.3 \text{ in.} \longleftarrow$$

(b) Required diameter based upon shear stress

$$V_{max} = W = 1875 \text{ lb}$$

$$\tau = \frac{4V}{3A}\left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}\right) \qquad r_2 = \frac{d}{2}$$

$$r_1 = \frac{d}{2} - t = \frac{d}{2} - \frac{d}{10} = \frac{2d}{5}$$

$$\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2} = \frac{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)\left(\frac{2d}{5}\right) + \left(\frac{2d}{5}\right)^2}{\left(\frac{d}{2}\right)^2 + \left(\frac{2d}{5}\right)^2} = \frac{61}{41}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}\left[d^2 - \left(\frac{4d}{5}\right)^2\right] = \frac{9\pi d^2}{100}$$

$$\tau = \frac{4V}{3}\left(\frac{61}{41}\right)\left(\frac{100}{9\pi d^2}\right) = 7.0160\,\frac{V}{d^2}$$

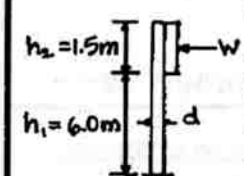
$$d^2 = \frac{7.0160\,V_{max}}{\tau_{allow}} = \frac{(7.0160)(1875\text{ lb})}{2000\text{ psi}} = 6.5775 \text{ in.}^2$$

$$d = 2.56 \text{ in.} \longleftarrow$$

(Bending stress governs.)

5.9-4

WIND LOAD ON A SIGN



b = width of sign

$b = 3.0$ m

$p = 3.6$ kPa

$\sigma_{allow} = 50$ MPa

$\tau_{allow} = 16$ MPa

d = diameter

W = wind force on one pole

$$t = \frac{d}{10} \qquad W = ph_2\left(\frac{b}{2}\right) = 8.1 \text{ kN}$$

(a) Required diameter based upon bending stress

$$M_{max} = W\left(h_1 + \frac{h_2}{2}\right) = 54.675 \text{ kN·m}$$

$$\sigma = \frac{Mc}{I} \qquad I = \frac{\pi}{64}(d_2^4 - d_1^4) \qquad d_2 = d \quad d_1 = d - 2t = \frac{4}{5}d$$

$$I = \frac{\pi}{64}\left[d^4 - \left(\frac{4d}{5}\right)^4\right] = \frac{\pi d^4}{64}\left(\frac{369}{625}\right) = \frac{369\pi d^4}{40{,}000} \text{ (m}^4\text{)}$$

$$c = \frac{d}{2} \qquad (d = \text{meters})$$

$$\sigma = \frac{Mc}{I} = \frac{M(d/2)}{369\pi d^4/40{,}000} = \frac{17.253\,M}{d^3}$$

$$d^3 = \frac{17.253\,M_{max}}{\sigma_{allow}} = \frac{(17.253)(54.675\text{ kN·m})}{50\text{ MPa}}$$

$$= 0.018866 \text{ m}^3$$

$$d = 0.266 \text{ m} = 266 \text{ mm} \longleftarrow$$

CONT.

5.9-4 CONT.

(b) Required diameter based upon shear stress

$V_{max} = W = 8.1\ \text{kN}$

$\tau = \frac{4V}{3A}\left(\frac{r_2^2 + r_1 r_2 + r_1^2}{r_2^2 + r_1^2}\right) \qquad r_2 = \frac{d}{2}$

$r_1 = \frac{d}{2} - t = \frac{d}{2} - \frac{d}{10} = \frac{2d}{5}$

$\frac{r_2^2 + r_1 r_2 + r_1^2}{r_2^2 + r_1^2} = \frac{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)\left(\frac{2d}{5}\right) + \left(\frac{2d}{5}\right)^2}{\left(\frac{d}{2}\right)^2 + \left(\frac{2d}{5}\right)^2} = \frac{61}{41}$

$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}\left[d^2 - \left(\frac{4d}{5}\right)^2\right] = \frac{9\pi d^2}{100}$

$\tau = \frac{4V}{3}\left(\frac{61}{41}\right)\left(\frac{100}{9\pi d^2}\right) = 7.0160\frac{V}{d^2}$

$d^2 = \frac{7.0160\, V_{max}}{\tau_{allow}} = \frac{(7.0160)(8.1\ \text{kN})}{16\ \text{MPa}}$

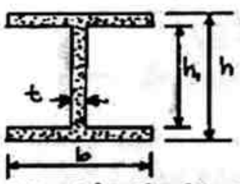
$= 0.003552\ \text{m}^2$

$d = 0.0596\ \text{m} = 59.6\ \text{mm}$

(Bending stress governs)

5.10-1

WIDE FLANGE BEAM



$b = 8$ in.
$t = 0.4$ in.
$h = 12$ in.
$h_1 = 10.7$ in
$V = 20{,}000$ lb

Moment of inertia (Eq. 5-47)

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 376.1\ \text{in.}^4$

(a) Maximum shear stress in the web (Eq. 5-48a)

$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 4680\ \text{psi}$

(b) Minimum shear stress in the web (Eq. 5-48b)

$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 3920\ \text{psi}$

(c) Average shear stress in the web (Eq. 5-50)

$\tau_{aver} = \frac{V}{th_1} = 4670\ \text{psi}$

$\frac{\tau_{max}}{\tau_{aver}} = 1.002$

(d) Shear force in the web (Eq. 5-49)

$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 18{,}960\ \text{lb}$

$\frac{V_{web}}{V} = 0.948$

5.10-2

WIDE FLANGE BEAM

$b = 160$ mm $\quad t = 8$ mm $\quad h = 300$ mm $\quad h_1 = 274$ mm $\quad V = 50$ kN

Moment of inertia (Eq. 5-47)

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 99.44 \times 10^{-6}\ \text{m}^4$

(a) Maximum shear stress in the web (Eq. 5-48a)

$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 23.5\ \text{MPa}$

CONT.

5.10-2 CONT.

(b) Minimum shear stress in the web (Eq. 5-48b)

$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 18.8\ \text{MPa}$

(c) Average shear stress in the web (Eq. 5-50)

$\tau_{aver} = \frac{V}{th_1} = 22.8\ \text{MPa}$

$\frac{\tau_{max}}{\tau_{aver}} = 1.029$

(d) Shear force in the web (Eq. 5-49)

$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 48.0\ \text{kN}$

$\frac{V_{web}}{V} = 0.960$

5.10-3

WIDE-FLANGE BEAM

W8X28 $\quad b = 6.535$ in. $\quad t = 0.285$ in.
$h = 8.06$ in. $\quad h_1 = 7.130$ in.
$V = 10$ K

Moment of inertia (Eq. 5-47)

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 96.36\ \text{in.}^4$

(a) Maximum shear stress in the web (Eq. 5-48a)

$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 4860\ \text{psi}$

(b) Minimum shear stress in the web (Eq. 5-48b)

$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 4200\ \text{psi}$

(c) Average shear stress in the web (Eq. 5-50)

$\tau_{aver} = \frac{V}{th_1} = 4920\ \text{psi}$

$\frac{\tau_{max}}{\tau_{aver}} = 0.988$

(d) Shear force in the web (Eq. 5-49)

$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 9.43\ \text{K}$

$\frac{V_{web}}{V} = 0.943$

5.10-4

WIDE FLANGE BEAM

$b = 220$ mm $\quad t = 12$ mm $\quad h = 600$ mm $\quad h_1 = 570$ mm

$V = 200$ kN

Moment of inertia (Eq. 5-47)

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 750.0 \times 10^{-6}\ \text{m}^4$

(a) Maximum shear stress in the web (Eq. 5-48a)

$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 32.3\ \text{MPa}$

(b) Minimum shear stress in the web (Eq. 5-48b)

$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 21.5\ \text{MPa}$

(c) Average shear stress in the web (Eq. 5-50)

$\tau_{aver} = \frac{V}{th_1} = 29.2\ \text{MPa}$

$\frac{\tau_{max}}{\tau_{aver}} = 1.104$

CONT.

5.10-4 CONT.

(d) Shear force in the web (Eq. 5-49)

$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 196$ kN ←

$\frac{V_{web}}{V} = 0.981$ ←

5.10-5 WIDE-FLANGE BEAM

W 24 X 94

$b = 9.065$ in. $t = 0.515$ in. $h = 24.31$ in.

$h_1 = 22.56$ in. $V = 50$ K

Moment of inertia (Eq. 5-47)

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 2672$ in.^{4}

(a) Maximum shear stress in the web (Eq. 5-48a)

$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 4570$ psi ←

(b) Minimum shear stress in the web (Eq. 5-48b)

$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 3380$ psi ←

(c) Average shear stress in the web (Eq. 5-50)

$\tau_{aver} = \frac{V}{th_1} = 4300$ psi ←

$\frac{\tau_{max}}{\tau_{aver}} = 1.06$ ←

(d) Shear force in the web (Eq. 5-49)

$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 48.5$ K ←

$\frac{V_{web}}{V} = 0.97$ ←

5.10-6 WIDE-FLANGE BEAM

$b = 120$ mm $t = 7$ mm $h = 350$ mm $h_1 = 330$ mm $V = 60$ kN

Moment of inertia (Eq. 5-47)

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 90.34 \times 10^{-6}$ m^4

(a) Maximum shear stress in the web (Eq. 5-48a)

$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 28.4$ MPa ←

(b) Minimum shear stress in the web (Eq. 5-48b)

$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 19.4$ MPa ←

(c) Average shear stress in the web (Eq. 5-50)

$\tau_{aver} = \frac{V}{th_1} = 26.0$ MPa ←

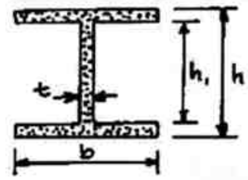
$\frac{\tau_{max}}{\tau_{aver}} = 1.09$ ←

(d) Shear force in the web (Eq. 5-49)

$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 58.6$ kN ←

$\frac{V_{web}}{V} = 0.98$ ←

5.10-7 CANTILEVER BEAM OF WIDE-FLANGE SHAPE



W10X30

From Table E-1:

$b = 5.810$ in.

$t = 0.300$ in.

$h = 10.47$ in.

$h_1 = 10.47$ in. $- 2(0.510$ in.$)$
$= 9.45$ in.

$I = 170$ in.^{4}

$S = 32.4$ in.^{3}

$L = 6$ ft $= 72$ in.

$\sigma_{allow} = 18$ ksi

$\tau_{allow} = 9$ ksi

(a) Allowable load based upon bending stress

$M_{max} = \frac{qL^2}{2} = S\sigma_{allow}$

$q_{allow} = \frac{2S\sigma_{allow}}{L^2} = 225$ lb/in. $= 2700$ lb/ft ←

(b) Allowable load based upon shear stress

$V = qL \quad \tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2)$ (Eq. 5-48a)

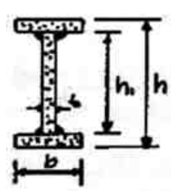
$V_{max} = \frac{\tau_{allow}(8It)}{bh^2 - bh_1^2 + th_1^2}$

$q_{allow} = \frac{V_{max}}{L} = \frac{8\tau_{allow}It}{L(bh^2 - bh_1^2 + th_1^2)}$

$= 352.1$ lb/in. $= 4230$ lb/ft ←

(Bending stress governs)

5.10-8 BRIDGE GIRDER ON A SIMPLE SPAN



$L = 12$ m

$b = 450$ mm

$t = 15$ mm

$h = 1850$ mm

$h_1 = 1800$ mm

$\sigma_{allow} = 90$ MPa

$\tau_{allow} = 50$ MPa

$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3)$ (Eq. 5-47)

$= 26.03 \times 10^{-3}$ m^4

$S = \frac{I}{c} = \frac{26.03 \times 10^{-3} \text{ m}^4}{h/2} = 28.14 \times 10^{-3}$ m^3

(a) Allowable load based upon bending stress

$M_{max} = \frac{qL^2}{8} = S\sigma_{allow}$

$q_{allow} = \frac{8S\sigma_{allow}}{L^2} = 141$ kN/m ←

(b) Allowable load based upon shear stress

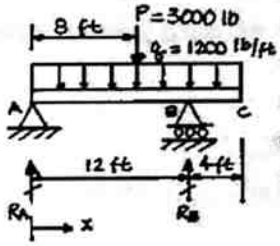
$V = \frac{qL}{2} \quad \tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2)$ (Eq. 5-48a)

$V_{max} = \frac{8\tau_{allow}It}{bh^2 - bh_1^2 + th_1^2}$

$q_{allow} = \frac{2V_{max}}{L} = \frac{16\tau_{allow}It}{L(bh^2 - bh_1^2 + th_1^2)} = 199$ kN/m ←

(Bending stress governs)

5.10-9 BEAM WITH AN OVERHANG



$\sigma_{allow} = 18$ ksi

$\tau_{allow} = 11$ ksi

Select a beam of S shape

$R_A = 7400$ lb

$R_B = 14{,}800$ lb

Maximum shear force: $V_{max} = 10{,}000$ lb at $x = 12$ ft

Maximum bending moment:

$M_{max} = 22{,}820$ lb-ft at $x = 6.167$ ft

Required section modulus

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{(22{,}820 \text{ lb-ft})(12 \text{ in./ft})}{18{,}000 \text{ psi}} = 15.2 \text{ in.}^3$$

From Table E-2:

Lightest beam is S8×23

$I = 64.9$ in.^{4} $\qquad S = 16.2$ in.^{3}

$b = 4.171$ in. $\qquad t = 0.441$ in.

$h = 8.00$ in. $\qquad h_1 = 8.00 - 2(0.426) = 7.148$ in.

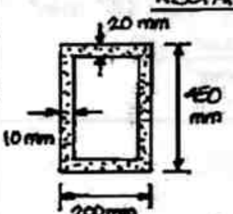
Maximum shear stress (Eq. 5-48a)

$$\tau_{max} = \frac{V_{max}}{8It}(bh^2 - bh_1^2 + th_1^2)$$

$= 3340$ psi $< 11{,}000$ psi $\therefore$ OK for shear

Select S8×23 beam ⟵

5.10-10 RECTANGULAR BOX BEAM



$\tau_{allow} = 40$ MPa

Find V_{allow}

$$\tau = \frac{VQ}{It}$$

$$V_{allow} = \frac{\tau_{allow} It}{Q}$$

$$I = \frac{1}{12}(200)(450)^3 - \frac{1}{12}(180)(410)^3 = 484.9 \times 10^6 \text{ mm}^4$$

$t = 2(10 \text{ mm}) = 20$ mm

$$Q = (200)\left(\frac{450}{2}\right)\left(\frac{450}{4}\right) - (180)\left(\frac{410}{2}\right)\left(\frac{410}{4}\right) = 1.280 \times 10^6 \text{ mm}^3$$

$$V_{allow} = \frac{\tau_{allow} It}{Q} = \frac{(40 \text{ MPa})(484.9 \times 10^6 \text{ mm}^4)(20 \text{ mm})}{1.280 \times 10^6 \text{ mm}^3}$$

$= 303$ kN ⟵

5.10-11 WIDE-FLANGE BEAM WITH COVER PLATES

W16×77

$I = 1110$ in.^{4} $\qquad t_w = 0.455$ in.

$h = 16.52$ in. $\qquad t_f = 0.760$ in.

$h/2 = 8.26$ in.

$b = 10.295$ in. $\qquad$ Find V_{allow}

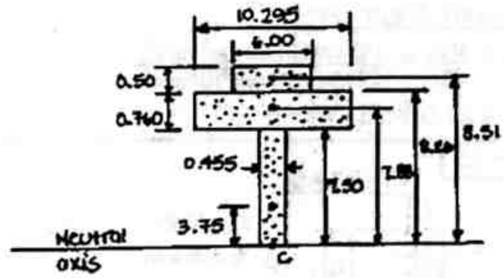
$\tau_{allow} = 6000$ psi

CONT.

5.10-11 CONT.

$$\tau = \frac{VQ}{It} \qquad V_{allow} = \frac{\tau_{allow} It}{Q}$$

All dimensions in inches:



Moment of inertia

$I = I_{beam} + 2I_{plate}$

$= 1110$ in.$^4 + 2A_{plate}d^2$

$= 1110 + 2(6.00)(0.5)(8.51)^2 = 1545$ in.^{4}

First moment Q

$Q = Q_{web} + Q_{flange} + Q_{plate}$

$= (0.455)(7.50)(3.75) + (10.295)(0.760)(7.88) + (6.00)(0.5)(8.51)$

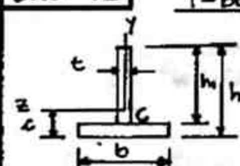
$= 12.80 + 61.65 + 25.53 = 100.0$ in.^{3}

Allowable shear force

$$V_{allow} = \frac{\tau_{allow} I t_w}{Q} = \frac{(6000 \text{ psi})(1545 \text{ in.}^4)(0.455 \text{ in.})}{100.0 \text{ in.}^3}$$

$= 42{,}200$ lb ⟵

5.10-12 T-BEAM



$b = 220$ mm

$t = 15$ mm

$h = 300$ mm $\qquad$ Find τ_{max}

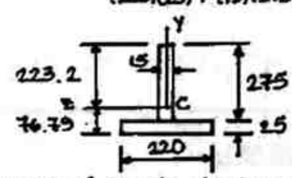
$h_1 = 275$ mm

$V = 70$ kN

Locate neutral axis (all dimensions in millimeters)

$$c = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{b(h-h_1)\left(\frac{h-h_1}{2}\right) + th_1\left(h - \frac{h_1}{2}\right)}{b(h-h_1) + th_1}$$

$$= \frac{(220)(25)\left(\frac{25}{2}\right) + (15)(275)\left(300 - \frac{275}{2}\right)}{(220)(25) + (15)(275)} = 76.79 \text{ mm}$$



Moment of inertia about the z-axis

$$I_{web} = \frac{1}{3}(15)(223.2)^3 + \frac{1}{3}(15)(76.79 - 25)^3$$

$= 56.29 \times 10^6$ mm^4

$$I_{flange} = \frac{1}{12}(220)(25)^3 + (220)(25)\left(76.79 - \frac{25}{2}\right)^2$$

$= 23.02 \times 10^6$ mm^4

$I = I_{web} + I_{flange} = 79.31 \times 10^6$ mm^4

CONT.

5.10-12 CONT.

First moment of area above the z axis

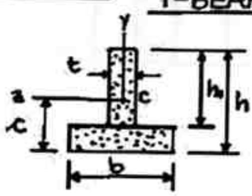
$$Q = (15)(223.2)\left(\frac{223.2}{2}\right) = 373.6 \times 10^3 \text{ mm}^3$$

Maximum shear stress

$$\tau_{max} = \frac{VQ}{It} = \frac{(70 \text{ kN})(373.6 \times 10^3 \text{ mm}^3)}{(79.31 \times 10^6 \text{ mm}^4)(15 \text{ mm})} = 22.0 \text{ MPa} \longleftarrow$$

5.10-13

T-BEAM



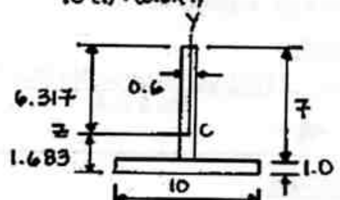
$b = 10$ in.
$t = 0.6$ in.
$h = 8$ in.
$h_1 = 7$ in.
$V = 6000$ lb

Find τ_{max}

Locate neutral axis (all dimensions in inches)

$$c = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{b(h-h_1)\left(\frac{h-h_1}{2}\right) + th_1\left(h - \frac{h_1}{2}\right)}{b(h-h_1) + th_1}$$

$$= \frac{(10)(1)(0.5) + (0.6)(7)(4.5)}{10(1) + (0.6)(7)} = 1.683 \text{ in.}$$



Moment of inertia about the z-axis

$$I_{web} = \frac{1}{3}(0.6)(6.317)^3 + \frac{1}{3}(0.6)(1.683 - 1.0)^3 = 50.48 \text{ in.}^4$$

$$I_{flange} = \frac{1}{12}(10)(1.0)^3 + (10)(1.0)(1.683 - 0.5)^2 = 14.83 \text{ in.}^4$$

$$I = I_{web} + I_{flange} = 65.31 \text{ in.}^4$$

First moment of area above the z axis

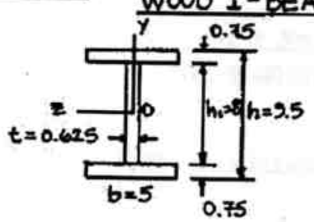
$$Q = (0.6)(6.317)\left(\frac{6.317}{2}\right) = 11.97 \text{ in.}^3$$

Maximum shear stress

$$\tau_{max} = \frac{VQ}{It} = \frac{(6000 \text{ lb})(11.97 \text{ in.}^3)}{(65.31 \text{ in.}^4)(0.6 \text{ in.})} = 1830 \text{ psi} \longleftarrow$$

5.11-1

WOOD I-BEAM



All dimensions in inches.

CONT.

5.11-1 CONT.

Web is glued to the flanges.

Allowable load in shear for the glued joints is 70 lb/in.

$\therefore f_{allow} = 70$ lb/in.

Find V_{max}

$$f = \frac{VQ}{I} \qquad V_{max} = \frac{f_{allow} I}{Q}$$

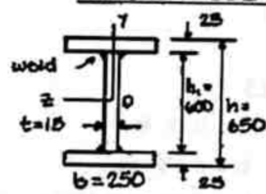
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(5)(9.5)^3 - \frac{1}{12}(4.375)(8)^3 = 170.57 \text{ in.}^4$$

$$Q = Q_{flange} = A_f d_f = (5)(0.75)(4.375) = 16.406 \text{ in.}^3$$

$$V_{max} = \frac{f_{allow} I}{Q} = \frac{(70 \text{ lb/in.})(170.57 \text{ in.}^4)}{16.406 \text{ in.}^3} = 728 \text{ lb} \longleftarrow$$

5.11-2

WELDED STEEL GIRDER



All dimensions in millimeters.

Allowable load in shear for one weld is 500 kN/m.

$\therefore f_{allow} = 2(500) = 1000$ kN/m

$$f = \frac{VQ}{I} \qquad V_{max} = \frac{f_{allow} I}{Q}$$

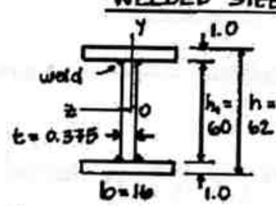
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(250)(650)^3 - \frac{1}{12}(235)(600)^3 = 1491 \times 10^6 \text{ mm}^4$$

$$Q = Q_{flange} = A_f d_f = (250)(25)(312.5) = 1953 \times 10^3 \text{ mm}^3$$

$$V_{max} = \frac{f_{allow} I}{Q} = \frac{(1000 \text{ kN/m})(1491 \times 10^6 \text{ mm}^4)}{1953 \times 10^3 \text{ mm}^3} = 763 \text{ kN} \longleftarrow$$

5.11-3

WELDED STEEL GIRDER



All dimensions in inches.

$V = 250$ K

F = force per inch of length of one weld

f = shear flow $\quad f = 2F = \frac{VQ}{I} \quad F = \frac{VQ}{2I}$

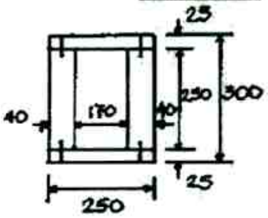
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(16)(62)^3 - \frac{1}{12}(15.625)(60)^3 = 36{,}521 \text{ in.}^4$$

$$Q = Q_{flange} = A_f d_f = (16)(1)(30.5) = 488 \text{ in.}^3$$

$$F = \frac{VQ}{2I} = \frac{(250 \text{ k})(488 \text{ in.}^3)}{2(36{,}521 \text{ in.}^4)} = 1670 \text{ lb/in.} \longleftarrow$$

5.11-4

WOOD BOX BEAM



All dimensions in millimeters.

s = nail spacing = 100 mm

F = allowable shear force for one nail = 750 N

$b = 250 \quad b_1 = 170 \quad h = 300 \quad h_1 = 250$

f = shear flow between a flange and both webs

$$f_{allow} = \frac{2F}{s} = 15 \text{ kN/m}$$

$$f = \frac{VQ}{I} \qquad V_{max} = \frac{f_{allow}\, I}{Q}$$

$$I = \frac{bh^3}{12} - \frac{b_1 h_1^3}{12} = 341.1 \times 10^6 \text{ mm}^4$$

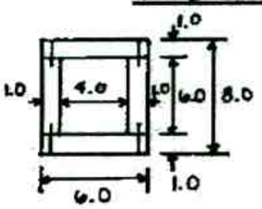
$$Q = Q_{flange} = A_F d_F = (250)(25)(137.5) = 859.4 \times 10^3 \text{ mm}^3$$

$$V_{max} = \frac{f_{allow}\, I}{Q} = \frac{(15 \text{ kN/m})(341.1 \times 10^6 \text{ mm}^4)}{859.4 \times 10^3 \text{ mm}^3}$$

$$= 5.95 \text{ kN} \longleftarrow$$

5.11-5

WOOD BOX BEAM



All dimensions in inches.

F = allowable shear force for one wood screw = 210 lb

V = 1150 lb

$b = 6.0 \quad b_1 = 4.0 \quad h = 8.0 \quad h_1 = 6.0$

s = longitudinal spacing of the screws

f = shear flow between a flange and both webs

$$f = \frac{VQ}{I} = \frac{2F}{s} \qquad \therefore \; s_{max} = \frac{2FI}{VQ}$$

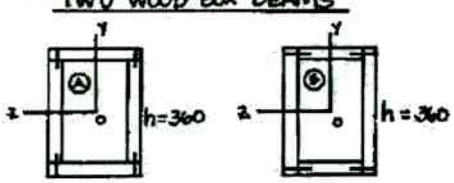
$$I = \frac{bh^3}{12} - \frac{b_1 h_1^3}{12} = 184 \text{ in.}^4$$

$$Q = Q_{flange} = A_F d_F = (6.0)(1.0)(3.5) = 21.0 \text{ in.}^3$$

$$s_{max} = \frac{2FI}{VQ} = \frac{2(210 \text{ lb})(184 \text{ in.}^4)}{(1150 \text{ lb})(21.0 \text{ in.}^3)} = 3.20 \text{ in.} \longleftarrow$$

5.11-6

TWO WOOD BOX BEAMS



All dimensions in millimeters. t = 20 mm

F = allowable load per nail = 250 N

CONT.

5.11-6 CONT.

V = 3.2 kN

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-2t)(h-2t)^3 = 340.7 \times 10^6 \text{ mm}^4$$

f = shear flow between a flange and both webs

$$f = \frac{2F}{s} = \frac{VQ}{I} \qquad \therefore \; s_{max} = \frac{2FI}{VQ}$$

(a) Beam Ⓐ

$$Q = A_F d_F = (bt)\left(\frac{h-t}{2}\right) = (200)(20)\left(\frac{1}{2}\right)(340) = 680 \times 10^3 \text{ mm}^3$$

$$s_A = \frac{2FI}{VQ} = \frac{(2)(250 \text{ N})(340.7 \times 10^6 \text{ mm}^4)}{(3.2 \text{ kN})(680 \times 10^3 \text{ mm}^3)} = 78.3 \text{ mm} \longleftarrow$$

(b) Beam Ⓑ

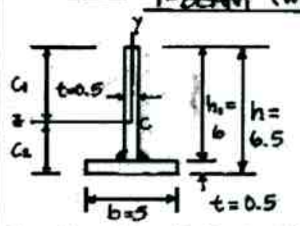
$$Q = A_F d_F = (b-2t)(t)\left(\frac{h-t}{2}\right) = (160)(20)\left(\frac{1}{2}\right)(340) = 544 \times 10^3 \text{ mm}^3$$

$$s_B = \frac{2FI}{VQ} = \frac{(2)(250 \text{ N})(340.7 \times 10^6 \text{ mm}^4)}{(3.2 \text{ kN})(544 \times 10^3 \text{ mm}^3)} = 97.9 \text{ mm} \longleftarrow$$

(c) Beam Ⓑ is more efficient because the shear flow on the contact surfaces is smaller and therefore fewer nails are needed. $\longleftarrow$

5.11-7

T-BEAM (WELDED)



All dimensions in inches.

F = allowable load per weld = 2.4 k/in.

Find V_{max}

Location of neutral axis (z axis)

Use the lower edge of the cross section as a reference axis.

$$Q = (bt)\left(\frac{t}{2}\right) + (h_1 t)\left(h - \frac{h_1}{2}\right)$$

$$= (5)(0.5)(0.25) + (6)(0.5)(3.5) = 11.125 \text{ in.}^3$$

$$A = bt + h_1 t = (5)(0.5) + (6)(0.5) = 5.5 \text{ in.}^2$$

$$c_2 = \frac{Q}{A} = \frac{11.125 \text{ in.}^3}{5.5 \text{ in.}^2} = 2.0227 \text{ in.}$$

$$c_1 = h - c_2 = 4.4773 \text{ in.}$$

Moment of inertia about the neutral axis

$$I = \frac{1}{3}tc_1^3 + \frac{1}{3}t(c_2 - t)^3 + \frac{1}{12}bt^3 + (bt)\left(c_2 - \frac{t}{2}\right)^2$$

$$= \frac{1}{3}(0.5)(4.4773)^3 + \frac{1}{3}(0.5)(1.5227)^3 + \frac{1}{12}(5)(0.5)^3$$

$$+ (5)(0.5)(1.7727)^2 = 23.455 \text{ in.}^4$$

First moment of area of flange

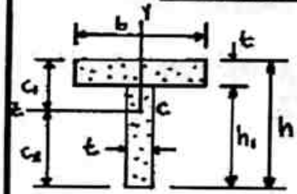
$$Q = bt\left(c_2 - \frac{t}{2}\right) = (5)(0.5)(1.7727) = 4.4318 \text{ in.}^3$$

Shear flow at welds

$$f = 2F = \frac{VQ}{I}$$

$$V_{max} = \frac{2FI}{Q} = \frac{2(2.4 \text{ k/in})(23.455 \text{ in.}^4)}{4.4318 \text{ in.}^3} = 25.4 \text{ k} \longleftarrow$$

5.11-8 T-BEAM (NAILED)



All dimensions in millimeters.

$V = 872$ N

F = allowable load per nail

$F = 400$ N

$b = 120 \quad t = 30 \quad h = 150 \quad h_1 = 120$

s = nail spacing

Find s_{max}

Location of neutral axis (z axis)

Use the lower edge of the cross section as a reference axis.

$$Q = (h_1 t)\left(\frac{h_1}{2}\right) + (bt)\left(h - \frac{t}{2}\right)$$

$$= (120)(30)(60) + (120)(30)(135) = 702 \times 10^3 \text{ mm}^3$$

$$A = h_1 t + bt = (120)(30) + (120)(30) = 7200 \text{ mm}^2$$

$$c_2 = \frac{Q}{A} = \frac{702 \times 10^3 \text{ mm}^3}{7200 \text{ mm}^2} = 97.5 \text{ mm}$$

$$c_1 = h - c_2 = 52.5 \text{ mm}$$

Moment of inertia about the neutral axis

$$I = \frac{1}{3} t c_2^3 + \frac{1}{3} t (h_1 - c_2)^3 + \frac{1}{12} bt^3 + (bt)\left(c_1 - \frac{t}{2}\right)^2$$

$$= \frac{1}{3}(30)(97.5)^3 + \frac{1}{3}(30)(22.5)^3 + \frac{1}{12}(120)(30)^3 + (120)(30)(37.5)^2 = 14.715 \times 10^6 \text{ mm}^4$$

First moment of area of flange

$$Q = bt\left(c_1 - \frac{t}{2}\right) = (120)(30)(37.5) = 135 \times 10^3 \text{ mm}^3$$

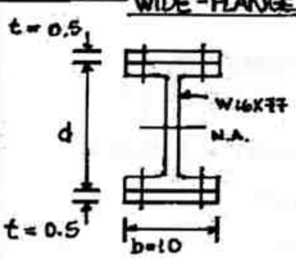
Spacing of nails

$$f = \frac{VQ}{I} = \frac{F}{s}$$

$$\therefore s_{max} = \frac{FI}{VQ} = \frac{(400 \text{ N})(14.715 \times 10^6 \text{ mm}^4)}{(872 \text{ N})(135 \times 10^3 \text{ mm}^3)}$$

$$= 0.050 \text{ m} = 50.0 \text{ mm} \longleftarrow$$

5.11-9 WIDE-FLANGE BEAM WITH COVER PLATES



All dimensions in inches.

Cover plates:

10 in. x 0.5 in.

($b = 10$ in. $\quad t = 0.5$ in.)

F = allowable load per bolt

$F = 2.8$ k $\quad V = 40$ k

Find s_{max}

W16X77

$I_b = 1110$ in.^{4} $\qquad d = 16.52$ in.

CONT.

5.11-9 CONT.

Moment of inertia about the neutral axis

$$I = I_b + 2\left[\frac{1}{12}bt^3 + (bt)\left(\frac{d}{2} + \frac{t}{2}\right)^2\right]$$

$$= 1110 \text{ in.}^4 + 2\left[\frac{1}{12}(10)(0.5)^3 + (10)(0.5)(8.51)^2\right]$$

$$= 1834 \text{ in.}^4$$

First moment of area of a cover plate

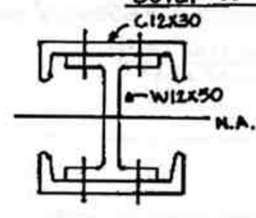
$$Q = bt\left(\frac{d+t}{2}\right) = (10)(0.5)(8.51) = 42.55 \text{ in.}^3$$

Spacing of bolts

$$f = \frac{VQ}{I} = \frac{2F}{s}$$

$$\therefore s_{max} = \frac{2FI}{VQ} = \frac{2(2.8 \text{ k})(1834 \text{ in.}^4)}{(40 \text{ k})(42.55 \text{ in.}^3)} = 6.03 \text{ in.} \longleftarrow$$

5.11-10 BUILT-UP BEAM



All dimensions in inches.

s = longitudinal spacing of bolts

$s = 7.5$ in.

F = allowable load per bolt

$F = 2.4$ k

Find V_{max}

W12X50 $\quad I_b = 394$ in.^{4} $\quad d = 12.19$ in.

C12X30 $\quad I_c = 5.14$ in.^{4} $\quad A_c = 8.82$ in.^{2} $\quad t_w = 0.510$ in.

$c = 0.674$ in.

Moment of inertia about the neutral axis

$$I = I_b + 2\left[I_c + A_c\left(\frac{d}{2} + t_w - c\right)^2\right]$$

$$= 394 \text{ in.}^4 + 2[5.14 \text{ in.}^4 + (8.82 \text{ in.}^2)(6.095 + 0.510 - 0.674)^2]$$

$$= 1025 \text{ in.}^4$$

First moment of area of a channel

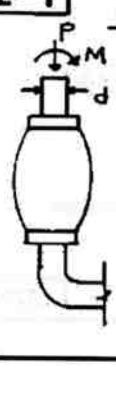
$$Q = A_c\left(\frac{d}{2} + t_w - c\right) = (8.82 \text{ in.}^2)(5.931) = 52.31 \text{ in.}^3$$

Maximum shear force

$$f = \frac{VQ}{I} = \frac{2F}{s}$$

$$V_{max} = \frac{2FI}{Qs} = \frac{2(2.4 \text{ k})(1025 \text{ in.}^4)}{(52.31 \text{ in.}^3)(7.5 \text{ in.})} = 12.5 \text{ k} \longleftarrow$$

5.12-1 BRACE AND BIT



$P = 25$ lb (compression)

$M = Pb = (25 \text{ lb})(4\tfrac{7}{8} \text{ in.})$

$= 121.9$ lb-in.

d = diameter

$d = 7/16$ in.

$$A = \frac{\pi d^2}{4} = 0.1503 \text{ in.}^2$$

$$S = \frac{\pi d^3}{32} = 0.008221 \text{ in.}^3$$

CONT.

5.12-1 CONT.

$$\sigma_t = -\frac{P}{A} + \frac{M}{S} = -\frac{25\ \text{lb}}{0.1503\ \text{in.}^2} + \frac{121.9\ \text{lb-in.}}{0.008221\ \text{in.}^3}$$

$$= -166\ \text{psi} + 14{,}828\ \text{psi} = 14{,}660\ \text{psi} \longleftarrow$$

$$\sigma_c = -\frac{P}{A} - \frac{M}{S} = -166\ \text{psi} - 14{,}828\ \text{psi}$$

$$= -14{,}990\ \text{psi} \longleftarrow$$

5.12-2

ALUMINUM POLE FOR A STREET LIGHT

Weight of pole $= W_1 = 2300$ N

Weight of arm $= W_2 = 330$ N

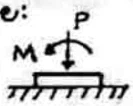
Distance to center of gravity of arm $= b$

$b = 1.2$ m

Diameter (outside) of pole $= d = 225$ mm

Thickness of pole $= t = 18$ mm

Base of pole:



$P = W_1 + W_2$

$= 2630$ N

$M = W_2 b$

$= 396\ \text{N·m}$

$$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = \frac{\pi}{4}\left[(225\ \text{mm})^2 - (189\ \text{mm})^2\right]$$

$$= 11{,}706\ \text{mm}^2$$

$$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = \frac{\pi}{64}\left[(225\ \text{mm})^4 - (189\ \text{mm})^4\right]$$

$$= 63.17 \times 10^6\ \text{mm}^4$$

$$c = \frac{d}{2} = 112.5\ \text{mm}$$

$$\sigma_t = -\frac{P}{A} + \frac{Mc}{I} = -\frac{2630\ \text{N}}{11{,}706\ \text{mm}^2} + \frac{(396\ \text{N·m})(112.5\ \text{mm})}{63.17 \times 10^6\ \text{mm}^4}$$

$$= -0.225\ \text{MPa} + 0.705\ \text{MPa} = 0.480\ \text{MPa}$$

$$= 480\ \text{kPa} \longleftarrow$$

$$\sigma_c = -\frac{P}{A} - \frac{Mc}{I} = -0.225\ \text{MPa} - 0.705\ \text{MPa}$$

$$= -0.930\ \text{MPa} = -930\ \text{kPa} \longleftarrow$$

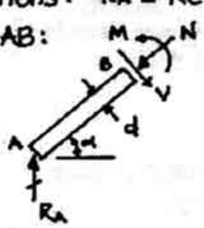
5.12-3

RIGID FRAME

Load P at midpoint

Reactions: $R_A = R_C = \dfrac{P}{2}$

Bar AB:



$\tan\alpha = \dfrac{H}{L}$

$\sin\alpha = \dfrac{H}{\sqrt{H^2 + L^2}}$

d = diameter

Axial force $N = R_A \sin\alpha = \dfrac{P}{2}\sin\alpha$

Bending moment $M = R_A L = \dfrac{PL}{2}$

CONT.

5.12-3 CONT.

$$\sigma_t = -\frac{N}{A} + \frac{Mc}{I} = -\frac{P\sin\alpha}{2A} + \frac{PL(d/2)}{2I}$$

Substitute numerical values:

$A = 16.1\ \text{in.}^2 \quad I = 212\ \text{in.}^4 \quad d = 10.75\ \text{in.}$

$P = 3200\ \text{lb} \quad L = 6\ \text{ft} = 72\ \text{in.} \quad H = 4.5\ \text{ft} = 54\ \text{in.}$

$\sin\alpha = 0.6$

$$\sigma_t = -\frac{(3200\ \text{lb})(0.6)}{2(16.1\ \text{in.}^2)} + \frac{(3200\ \text{lb})(72\ \text{in.})(5.375\ \text{in.})}{2(212\ \text{in.}^4)}$$

$$= -60\ \text{psi} + 2920\ \text{psi} = 2860\ \text{psi} \longleftarrow$$

$$\sigma_c = -\frac{N}{A} - \frac{Mc}{I} = -60\ \text{psi} - 2920\ \text{psi}$$

$$= -2980\ \text{psi} \longleftarrow$$

5.12-4

CURVED BAR (RADIUS = r)

$e = r - r\cos 45°$

$= r\left(1 - \dfrac{1}{\sqrt{2}}\right)$

$M = Pe = \dfrac{Pr}{2}(2 - \sqrt{2})$

h = height of rectangular cross section

t = thickness of rectangular cross section

$$\sigma_t = \frac{P}{A} + \frac{M}{S} = \frac{P}{ht} + \frac{Pr(2-\sqrt{2})}{2(th^2/6)}$$

$$= \frac{P}{ht}\left[1 + \frac{3r(2-\sqrt{2})}{h}\right]$$

$$t_{min} = \frac{P}{h\,\sigma_{allow}}\left[1 + 3(2-\sqrt{2})\frac{r}{h}\right]$$

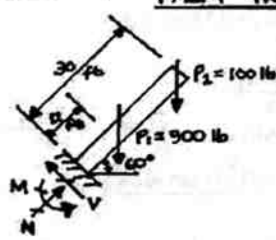
Substitute numerical values:

$P = 1.6$ kN $\quad h = 30$ mm $\quad \sigma_{allow} = 80$ MPa $\quad r = 300$ mm

$t_{min} = 12.38\ \text{mm} \longleftarrow$

5.12-5

PALM TREE



d = 14 in.

$A = \dfrac{\pi d^2}{4} = 153.9\ \text{in.}^2$

$S = \dfrac{\pi d^3}{32} = 269.4\ \text{in.}^3$

$M = (900\ \text{lb})(12\ \text{ft})(\cos 60°) + (100\ \text{lb})(30\ \text{ft})(\cos 60°)$

$= 6900\ \text{lb-ft} = 82{,}800\ \text{lb-in.}$

$N = (P_1 + P_2)\sin 60° = (1000\ \text{lb})(\sin 60°) = 866\ \text{lb}$

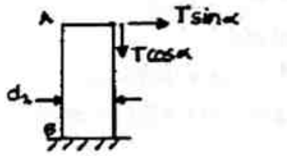
$$\sigma_t = -\frac{N}{A} + \frac{M}{S} = -\frac{866\ \text{lb}}{154\ \text{in.}^2} + \frac{82{,}800\ \text{lb-in.}}{269\ \text{in.}^3}$$

$$= -6\ \text{psi} + 308\ \text{psi} = 302\ \text{psi} \longleftarrow$$

$$\sigma_c = -\frac{N}{A} - \frac{M}{S} = -6\ \text{psi} - 308\ \text{psi}$$

$$= -314\ \text{psi} \longleftarrow$$

5.12-6 ALUMINUM POLE



$L = 2.0\ m$
$\alpha = 28°$
$d_1 = 200\ mm$
$d_2 = 250\ mm$
$(\sigma_c)_{allow} = 80\ MPa$

At the base of the pole

$N = T\cos\alpha = 0.88295\,T \quad (T = \text{newtons})$

$M = (T\cos\alpha)\left(\frac{d_2}{2}\right) + (T\sin\alpha)(L)$

$= 0.11037\,T + 0.93894\,T = 1.0493\,T$

$(M = \text{newton meters})$

$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 17.671 \times 10^3\ mm^2$
$= 17.671 \times 10^{-3}\ m^2$

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 113.21 \times 10^6\ mm^4$
$= 113.21 \times 10^{-6}\ m^4$

Compressive stress

$\sigma_c = \frac{N}{A} + \frac{Mc}{I} = \frac{0.88295\,T}{17.671 \times 10^{-3}\,m^2} + \frac{(1.0493T)(0.125\,m)}{113.21 \times 10^{-6}\,m^4}$

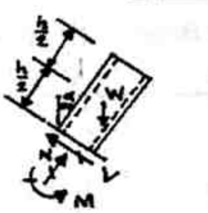
$= 49.966T + 1158.6T = 1208.6T$

$(\sigma_c = \text{pascals})$

$\sigma_c = (\sigma_c)_{allow} \qquad 1208.6\,T = 80 \times 10^6$

$T_{allow} = 66.2\ kN \quad \leftarrow$

5.12-7 LEANING TOWER



d_2 = outer diameter
d_1 = inner diameter

$I = \frac{\pi}{64}(d_2^4 - d_1^4)$
$= \frac{\pi}{64}(d_2^2 - d_1^2)(d_2^2 + d_1^2)$

$A = \frac{\pi(d_2^2 - d_1^2)}{4}$

$N = W\cos\alpha \qquad M = W\left(\frac{h}{2}\right)\sin\alpha \qquad \frac{I}{A} = \frac{d_2^2 + d_1^2}{16}$

$\sigma_t = -\frac{N}{A} + \frac{Mc}{I} = -\frac{W\cos\alpha}{A} + \frac{W\left(\frac{h}{2}\right)(\sin\alpha)\left(\frac{d_2}{2}\right)}{I} = 0$

$\frac{\cos\alpha}{A} = \frac{hd_2\sin\alpha}{4I}$

$\tan\alpha = \frac{4I}{hd_2A} = \frac{d_2^2 + d_1^2}{4hd_2}$

$\alpha = \arctan\frac{d_2^2 + d_1^2}{4hd_2} \quad \leftarrow$

5.12-8 CIRCULAR BAR

$T = 26\ kN \qquad M = 2.7\ kN\cdot m \qquad \sigma_{allow} = 125\ MPa$

Find required diameter d

$A = \frac{\pi d^2}{4} \qquad S = \frac{\pi d^3}{32}$

CONT.

5.12-8 CONT.

$\sigma_t = \frac{T}{A} + \frac{M}{S} \qquad \sigma_{allow} = \frac{4T}{\pi d^2} + \frac{32M}{\pi d^3}$

$\pi\,\sigma_{allow}\,d^3 - 4Td - 32M = 0$

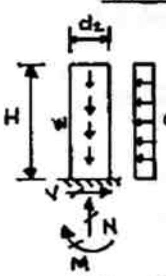
Substitute numerical values:

$(\pi)(125\ MPa)d^3 - 4(26\ kN)d - 32(2.7\ kN\cdot m) = 0$

or $392{,}699{,}000\,d^3 - 104{,}000\,d - 86{,}400 = 0 \quad (d = \text{meters})$

Solving, $d = 0.0618\ m = 61.8\ mm \quad \leftarrow$

5.12-9 BRICK CHIMNEY



P = wind pressure
$q = pd_2$ = intensity of load
d_2 = outer diameter
d_1 = inner diameter
$N = wH$
$M = qH\left(\frac{H}{2}\right) = \frac{1}{2}pd_2H^2$

$A = \frac{\pi}{4}(d_2^2 - d_1^2)$

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = \frac{\pi}{64}(d_2^2 - d_1^2)(d_2^2 + d_1^2)$

$\frac{I}{A} = \frac{1}{16}(d_2^2 + d_1^2)$

$\sigma_t = -\frac{N}{A} + \frac{M(d_2/2)}{I} = 0$

$\frac{M}{N} = \frac{2I}{Ad_2} \qquad \frac{Pd_2H^2}{2wH} = \frac{d_2^2 + d_1^2}{8d_2}$

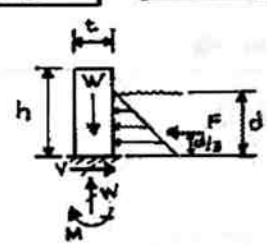
$H = \frac{w(d_2^2 + d_1^2)}{4pd_2^2} \quad \leftarrow$

Substitute numerical values:

$W = 825\ lb/ft \quad d_2 = 4\ ft \quad d_1 = 3\ ft \quad p = 10\ lb/ft^2$

$H_{max} = 32.2\ ft \quad \leftarrow$

5.12-10 CONCRETE WALL



h = height of wall
t = thickness of wall
b = width of wall (perpendicular to the figure)
γ_c = weight density of concrete
γ_w = weight density of water

W = weight of wall

$W = bht\gamma_c$

F = resultant force for the water pressure

$F = \frac{1}{2}(d)(\gamma_w d)(b) = \frac{1}{2}bd^2\gamma_w$

$M = F\left(\frac{d}{3}\right) = \frac{1}{6}bd^3\gamma_w$

$A = bt \qquad S = \frac{1}{6}bt^2$

CONT.

5.12-10 CONT.

$$\sigma_t = -\frac{W}{A} + \frac{M}{S} = -h\gamma_c + \frac{d^3\gamma_w}{t^2} \qquad (1)$$

$$\sigma_c = -\frac{W}{A} - \frac{M}{S} = -h\gamma_c - \frac{d^3\gamma_w}{t^2} \qquad (2)$$

(a) Water level at the top ($d = h$)

$h = 2\text{m} \quad t = 0.3\text{ m} \quad d = 2\text{m}$

$\gamma_c = 23\text{ kN/m}^3 \quad \gamma_w = 9.81\text{ kN/m}^3$

Substitute numerical values into Eqs. (1), (2):

$\sigma_t = -46.0\text{ kPa} + 872.0\text{ kPa} = 826\text{ kPa}$ ←

$\sigma_c = -46.0\text{ kPa} - 872.0\text{ kPa} = -918\text{ kPa}$ ←

(b) Maximum depth for no tension

Solve Eq. (1) for d: $\quad d^3 = ht^2\left(\frac{\gamma_c}{\gamma_w}\right)$

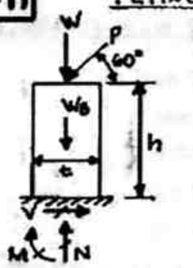
Substitute numerical values:

$d^3 = 0.4220\text{ m}^3$

$d_{max} = 0.750\text{ m}$ ←

5.12-11

FLYING BUTTRESS



b = width of vertical buttress (perpendicular to the figure)

W_B = weight of buttress

$= bth\gamma$

$= (3\text{ ft})(5\text{ ft})(16\text{ ft})(165\text{ lb/ft}^3)$

$= 39{,}600\text{ lb}$

$P = 5{,}400\text{ lb}$

$N = W + W_B + P\sin 60° = W + 39{,}600 + 5400\sin 60°$

$= W + 44{,}277\text{ (lb)}$

$M = hP\cos 60° = (16\text{ ft})(5400\text{ lb})\cos 60°$

$= 43{,}200\text{ (lb-ft)}$

$A = bt = (3\text{ ft})(5\text{ ft}) = 15\text{ ft}^2$

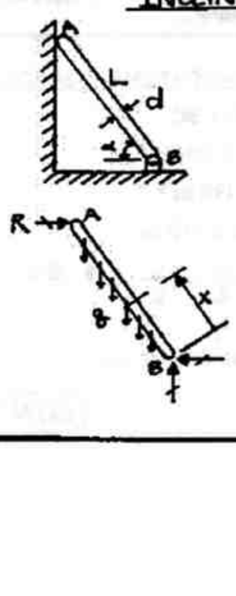
$S = \frac{1}{6}bt^2 = \frac{1}{6}(3\text{ ft})(5\text{ ft})^2 = 12.5\text{ ft}^3$

$$\sigma_t = -\frac{N}{A} + \frac{M}{S} = -\frac{W + 44{,}277\text{ lb}}{15\text{ ft}^2} + \frac{43{,}200\text{ lb-ft}}{12.5\text{ ft}^3} = 0$$

Solve for W: $\quad W = 7560\text{ lb}$ ←

5.12-12

INCLINED BAR



$d = 125\text{ mm}$

$L = 2\text{ m}$

$\alpha = 75°$

$\gamma = 77.0\text{ kN/m}^3$

$A = \frac{\pi d^2}{4} = 12.272 \times 10^{-3}\text{ m}^2$

$S = \frac{\pi d^3}{32} = 191.75 \times 10^{-6}\text{ m}^3$

q = weight of bar per unit length

$q = \gamma A = 944.93\text{ N/m}$

CONT.

5.12-12 CONT.

$\Sigma M_B = 0$ gives

$$R = \frac{qL}{2\tan\alpha} = 253.19\text{ N}$$

At distance x from end B:

x = meters

$N = R\cos\alpha + q(L - x)\sin\alpha$

$= 65.530\text{ N} + (912.73\text{ N/m})(2\text{m} - x)$

$= 1890.99 - 912.73x$ (newtons)

$M = R(L-x)\sin\alpha - q(L-x)\left(\frac{L-x}{2}\right)\cos\alpha$

$= (253.19\text{ N})(2\text{m} - x)\sin 75°$

$\quad - (944.93\text{ N/m})\left(\frac{1}{2}\right)(2\text{m} - x)^2\cos 75°$

$= 244.56x - 122.28x^2$ (newton meters)

Compressive stress at distance x from end B

$$\sigma_c = \frac{N}{A} + \frac{M}{S}$$

$$\sigma_c = \frac{1}{12.272 \times 10^{-3}}(1890.99 - 912.73x) + \frac{1}{191.75 \times 10^{-6}}(244.56x - 122.28x^2)$$

$= 154.090 - 74.375x + 1275.4x - 637.71x^2$

$= 154.09 + 1201.0x - 637.71x^2$ (kPa) $\qquad (1)$

Maximum stress

$\frac{d\sigma_c}{dx} = 0 \qquad 1201.0 - 2(637.71x) = 0$

$x_m = 0.94165\text{ m}$

Substitute x_m into Eq. (1) for σ_c:

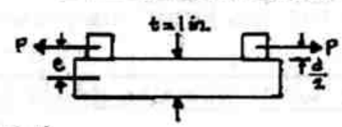
$\sigma_{max} = 154.09 + 1201.0(0.94165) - 637.71(0.94165)^2$

$= 720\text{ kPa}$ ←

Note: The stress at the midpoint of the bar ($x = 1\text{ m}$) is 717 kPa, which is only 0.4% less than the maximum stress.

5.12-13

BLOCK OF STEEL LOADED BY TWO CABLES



$P = 1200\text{ lb}$

d = diameter of cable

$d = 0.25\text{ in.}$

$e = \frac{t}{2} + \frac{d}{2} = 0.625\text{ in.}$

b = width of block

= 3 in.

t = thickness of block

= 1 in.

Normal stress on a cross section

$\sigma = \frac{P}{A} + \frac{Pey}{I} \qquad A = bt = (3\text{ in.})(1\text{ in.}) = 3\text{ in.}^2$

$I = \frac{bt^3}{12} = \frac{1}{12}(3\text{ in.})(1\text{ in.})^3 = 0.25\text{ in.}^4$

CONT.

5.12-13 CONT.

(a) Maximum tensile stress (at top of block)

$$y = \frac{t}{2} = 0.5 \text{ in.}$$

$$\sigma_t = \frac{P}{A} + \frac{Pey}{I} = \frac{1200 \text{ lb}}{3 \text{ in.}^2} + \frac{(1200 \text{ lb})(0.625 \text{ in.})(0.5 \text{ in.})}{0.25 \text{ in.}^4}$$

$$= 400 \text{ psi} + 1500 \text{ psi} = 1900 \text{ psi} \longleftarrow$$

Maximum compressive stress (at bottom of block)

$$y = -\frac{t}{2} = -0.5 \text{ in.}$$

$$\sigma_c = \frac{P}{A} + \frac{Pey}{I} = \frac{1200 \text{ lb}}{3 \text{ in.}^2} + \frac{(1200 \text{ lb})(0.625 \text{ in.})(-0.5 \text{ in.})}{0.25 \text{ in.}^4}$$

$$= 400 \text{ psi} - 1500 \text{ psi} = -1100 \text{ psi} \longleftarrow$$

(b) If d is increased, both stresses increase in magnitude. $\longleftarrow$

5.12-14 CIRCULAR POST

$$A = \frac{\pi d^2}{4} \quad S = \frac{\pi d^3}{32} \quad M = \frac{Pd}{2}$$

Tension: $\sigma_t = -\frac{P}{A} + \frac{M}{S} = -\frac{4P}{\pi d^2} + \frac{16P}{\pi d^2} = \frac{12P}{\pi d^2}$

Compression: $\sigma_c = -\frac{P}{A} - \frac{M}{S} = -\frac{4P}{\pi d^2} - \frac{16P}{\pi d^2} = -\frac{20P}{\pi d^2}$

Rectangular post

$$A = bd \quad S = \frac{bd^2}{6} \quad M = \frac{Pd}{2}$$

Tension: $\sigma_t = -\frac{P}{A} + \frac{M}{S} = -\frac{P}{bd} + \frac{3P}{bd} = \frac{2P}{bd}$

Compression: $\sigma_c = -\frac{P}{A} - \frac{M}{S} = -\frac{P}{bd} - \frac{3P}{bd} = -\frac{4P}{bd}$

For equal maximum tensile stresses

(a) $\frac{12P}{\pi d^2} = \frac{2P}{bd}$ $\quad b = \frac{\pi d}{6} \longleftarrow$

(b) Circular post:

$$\sigma_c = -\frac{20P}{\pi d^2}$$

Rectangular post:

$$\sigma_c = -\frac{4P}{bd} = -\frac{4P}{(\pi d/6)d} = -\frac{24P}{\pi d^2}$$

Rectangular post has the larger compressive stress. $\longleftarrow$

5.12-15 BAR WITH REDUCED CROSS SECTION

(a) Square bar (rectangular cross section mn)

$$A = (b)\left(\frac{b}{2}\right) = \frac{b^2}{2} \quad I = \frac{1}{12}(b)\left(\frac{b}{2}\right)^3 = \frac{b^4}{96}$$

$$M = P\left(\frac{b}{4}\right) \quad c = \frac{b}{4}$$

$$\sigma_t = \frac{P}{A} + \frac{Mc}{I} = \frac{2P}{b^2} + \frac{6P}{b^2} = \frac{8P}{b^2} \longleftarrow$$

$$\sigma_c = \frac{P}{A} - \frac{Mc}{I} = \frac{2P}{b^2} - \frac{6P}{b^2} = -\frac{4P}{b^2} \longleftarrow$$

(b) Circular bar (semicircular cross section mn)

$$A = \frac{1}{2}\left(\frac{\pi b^2}{4}\right) = \frac{\pi b^2}{8} = 0.3927 b^2$$

CONT.

5.12-15 CONT.

From Appendix D, case 10:

$$I = 0.1098\left(\frac{b}{2}\right)^4 = 0.006860\ b^4$$

For tension:

$$c_t = \frac{4r}{3\pi} = \frac{2b}{3\pi} = 0.2122 b$$

For compression:

$$c_c = r - c_t = \frac{b}{2} - \frac{2b}{3\pi} = 0.2878 b$$

$$M = P\left(\frac{2b}{3\pi}\right) = 0.2122 Pb$$

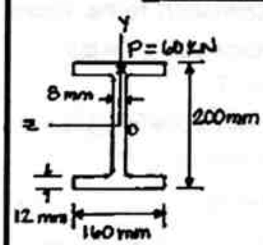
$$\sigma_t = \frac{P}{A} + \frac{Mc_t}{I} = \frac{P}{0.3927 b^2} + \frac{(0.2122 Pb)(0.2122 b)}{0.006860\ b^4}$$

$$= 2.546 \frac{P}{b^2} + 6.564 \frac{P}{b^2} = 9.11 \frac{P}{b^2} \longleftarrow$$

$$\sigma_c = \frac{P}{A} - \frac{Mc_c}{I} = \frac{P}{0.3927 b^2} - \frac{(0.2122 Pb)(0.2878 b)}{0.006860\ b^4}$$

$$= 2.546 \frac{P}{b^2} - 8.903 \frac{P}{b^2} = -6.36 \frac{P}{b^2} \longleftarrow$$

5.12-16 SHORT COLUMN OF WIDE-FLANGE SHAPE



$b = 160$ mm

$t_w = 8$ mm

$h = 200$ mm

$t_f = 12$ mm

$P = 60$ kN

$e = \frac{h}{2} - \frac{t_f}{2} = 94$ mm

$$A = 2bt_f + (h - 2t_f)t_w = 5248 \text{ mm}^2$$

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - t_w)(h - 2t_f)^3 = 37.611 \times 10^6 \text{ mm}^4$$

(a) Maximum stresses

$$\sigma_t = -\frac{P}{A} + \frac{Pe(h/2)}{I}$$

$$= -\frac{60 \text{ kN}}{5248 \text{ mm}^2} + \frac{(60 \text{ kN})(94 \text{ mm})(100 \text{ mm})}{37.611 \times 10^6 \text{ mm}^4}$$

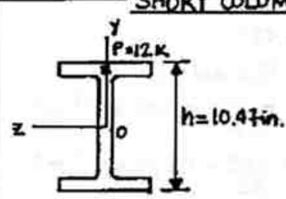
$$\sigma_t = -11.43 \text{ MPa} + 15.00 \text{ MPa} = 3.57 \text{ MPa} \longleftarrow$$

$$\sigma_c = -11.43 \text{ MPa} - 15.00 \text{ MPa} = -26.4 \text{ MPa} \longleftarrow$$

(b) Neutral axis

$$y_0 = -\frac{I}{Ae} = -\frac{37.611 \times 10^6 \text{ mm}^4}{(5248 \text{ mm}^2)(94 \text{ mm})} = -76.2 \text{ mm} \longleftarrow$$

5.12-17 SHORT COLUMN OF WIDE-FLANGE SHAPE



W 10 x 30

$A = 8.84$ in.^{2}

$I = 170$ in.^{4}

$t_f = 0.510$ in.

$e = \frac{h}{2} - \frac{t_f}{2} = 4.98$ in.

(a) Maximum stresses

$$\sigma_t = -\frac{P}{A} + \frac{Pe(h/2)}{I} = -1357 \text{ psi} + 1840 \text{ psi}$$

$$= 480 \text{ psi} \longleftarrow$$

CONT.

5.12-17 CONT.

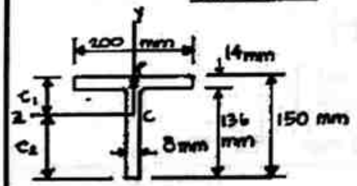
$$\sigma_c = -\frac{P}{A} - \frac{Pe(h/2)}{I} = -1357 \text{ psi} - 1840 \text{ psi}$$

$$= -3200 \text{ psi} \longleftarrow$$

(b) Neutral axis

$$y_0 = -\frac{I}{Ae} = -3.86 \text{ in.} \longleftarrow$$

5.12-18 T-BEAM



$b = 200$ mm

$t_w = 8$ mm

$h = 150$ mm

$h_1 = 136$ mm

$t_f = 14$ mm

$$A = bt_f + h_1 t_w = 3888 \text{ mm}^2$$

Centroid of the cross section

Use the lower edge of the cross section as the reference axis.

$$Q = t_w h_1\left(\frac{h_1}{2}\right) + bt_f\left(h - \frac{t_f}{2}\right)$$

$$= (8)(136)(68) + (200)(14)(143) = 474{,}384 \text{ mm}^3$$

$$c_2 = \frac{Q}{A} = \frac{474{,}384 \text{ mm}^3}{3888 \text{ mm}^2} = 122.01 \text{ mm}$$

$$c_1 = h - c_2 = 27.99 \text{ mm}$$

Moment of inertia about the z-axis

$$I = \frac{1}{12}bt_f^3 + bt_f\left(c_1 - \frac{t_f}{2}\right)^2 + \frac{1}{3}t_w(c_1 - t_f)^3 + \frac{1}{3}t_w c_2^3$$

$$= \frac{1}{12}(200)(14)^3 + (200)(14)(20.99)^2 + \frac{1}{3}(8)(13.99)^3$$

$$+ \frac{1}{3}(8)(122.01)^3 = 6.130 \times 10^6 \text{ mm}^4$$

Eccentricity of the load

$$e = c_1 - \frac{t_f}{2} = 27.99 - 7 = 20.99 \text{ mm}$$

(a) Location of neutral axis

$$y_0 = -\frac{I}{Ae} = -\frac{6.130 \times 10^6 \text{ mm}^4}{(3888 \text{ mm}^2)(20.99 \text{ mm})} = -75.1 \text{ mm} \longleftarrow$$

(b) Maximum load based upon tensile stress

$\sigma_{allow} = 100$ MPa $\quad \sigma_t = -\frac{P}{A} + \frac{Pec_2}{I}$ $\quad$ (P = newtons)

$$100 \text{ MPa} = -\frac{P}{3888 \text{ mm}^2} + \frac{P(20.99 \text{ mm})(122.01 \text{ mm})}{6.130 \times 10^6 \text{ mm}^4}$$

$$100 = -\frac{P}{3888} + \frac{P}{2394} = 160.5 \times 10^{-6} P$$

$$P = 623{,}000 \text{ N} = 623 \text{ kN}$$

Maximum load based upon compressive stress

$\sigma_{allow} = 60$ MPa $\quad \sigma_c = -\frac{P}{A} - \frac{Pec_1}{I}$ $\quad$ (P = newtons)

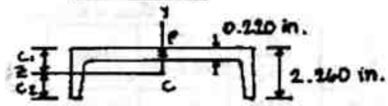
$$-60 \text{ MPa} = -\frac{P}{3888 \text{ mm}^2} - \frac{P(20.99 \text{ mm})(27.99 \text{ mm})}{6.130 \times 10^6 \text{ mm}^4}$$

$$60 = \frac{P}{3888} + \frac{P}{10{,}434} = 353.0 \times 10^{-6} P$$

$$P = 170{,}000 \text{ N} = 170 \text{ kN}$$

Compression governs. $P_{max} = 170$ kN $\longleftarrow$

5.12-19 CHANNEL



C 8 × 11.5

$A = 3.38 \text{ in.}^2 \quad h = 2.260 \text{ in.} \quad t_w = 0.220 \text{ in.}$

$I_z = 1.32 \text{ in.}^4 \quad c_1 = 0.571 \text{ in.} \quad c_2 = 1.689 \text{ in.}$

Eccentricity of the load

$$e = c_1 - \frac{t_w}{2} = 0.571 - 0.110 = 0.461 \text{ in.}$$

(a) Location of the neutral axis

$$y_0 = -\frac{I}{Ae} = -\frac{1.32 \text{ in.}^4}{(3.38 \text{ in.}^2)(0.461 \text{ in.})} = -0.847 \text{ in.} \longleftarrow$$

(b) Maximum load based upon tensile stress

$\sigma_{allow} = 10{,}000$ psi $\quad$ (P = pounds)

$$\sigma_t = -\frac{P}{A} + \frac{Pec_2}{I} = -\frac{P}{3.38 \text{ in}^2} + \frac{P(0.461 \text{ in.})(1.689 \text{ in.})}{1.32 \text{ in.}^4}$$

$$10{,}000 = -\frac{P}{3.38} + \frac{P}{1.695} = 0.2941 P$$

$$P = 34{,}000 \text{ lb} = 34 \text{ K}$$

Maximum load based upon compressive stress

$\sigma_{allow} = 8000$ psi $\quad$ (P = pounds)

$$\sigma_c = -\frac{P}{A} - \frac{Pec_1}{I} = -\frac{P}{3.38 \text{ in}^2} - \frac{P(0.461 \text{ in.})(0.571 \text{ in.})}{1.32 \text{ in.}^4}$$

$$8000 = \frac{P}{3.38} + \frac{P}{5.015} = 0.4953 P$$

$$P = 16{,}200 \text{ lb} = 16.2 \text{ K}$$

Compression governs. $P_{max} = 16.2$ K $\longleftarrow$

5.13-1

$M = 2100$ lb-in. $\quad h = 1.5$ in. $\quad b = 0.375$ in.

(a) Beam with a hole

$$\frac{d}{h} \le \frac{1}{2} \quad \text{Eq. (5-57)}: \ \sigma_C = \frac{6Mh}{b(h^3 - d^3)} = \frac{50{,}400}{3.375 - d^3} \qquad (1)$$

$$\frac{d}{h} \ge \frac{1}{2} \quad \text{Eq. (5-56)}: \ \sigma_B = \frac{12Md}{b(h^3 - d^3)} = \frac{67{,}200d}{3.375 - d^3} \qquad (2)$$

d (in.)	$\frac{d}{h}$	σ_C Eq. (1) (psi)	σ_B Eq. (2) (psi)	σ_{max} (psi)
0.25	0.1667	15,000	—	15,000
0.50	0.3333	15,500	—	15,500
0.75	0.5000	17,100	17,100	17,100
1.00	0.6667	—	28,300	28,300

Note: The larger the hole, the larger the stress.

(b) Beam with notches

$h_1 = 1.25$ in.

$$\frac{h}{h_1} = \frac{1.5 \text{ in.}}{1.25 \text{ in.}} = 1.2$$

Eq. (5-58):

$$\sigma_{nom} = \frac{6M}{bh_1^2} = 21{,}500 \text{ psi}$$

CONT.

5.13-1 CONT.

$\sigma_{max} = K\sigma_{nom}$

R (in.)	$\frac{R}{h_1}$	K (Fig. 5-50)	σ_{max} (psi)
0.05	0.04	3.0	65,000
0.10	0.08	2.3	49,000
0.15	0.12	2.1	45,000
0.20	0.16	1.9	41,000

Note: The larger the notch radius, the smaller the stress.

5.13-2

$M = 250$ N·m $\quad h = 44$ mm $\quad b = 10$ mm

(a) Beam with a hole

$\frac{d}{h} < \frac{1}{2}$ Eq. (5-57): $\sigma_C = \frac{6Mh}{b(h^3 - d^3)} = \frac{6.6 \times 10^6}{85{,}180 - d^3}$ MPa (1)

$\frac{d}{h} \geq \frac{1}{2}$ Eq. (5-56): $\sigma_B = \frac{12Md}{b(h^3 - d^3)} = \frac{300 \times 10^3 d}{85{,}180 - d^3}$ MPa (2)

d (mm)	$\frac{d}{h}$	σ_C Eq. (1) (MPa)	σ_B Eq. (2) (MPa)	σ_{max} (MPa)
10	0.227	78	—	78
16	0.364	81	—	81
22	0.500	89	89	89
28	0.636	—	133	133

Note: The larger the hole, the larger the stress.

(b) Beam with notches

$h_1 = 40$ mm $\qquad \frac{h}{h_1} = \frac{44 \text{ mm}}{40 \text{ mm}} = 1.1$

Eq. (5-58): $\sigma_{nom} = \frac{6M}{bh_1^2} = 93.8$ MPa $\qquad \sigma_{max} = K\sigma_{nom}$

R (mm)	$\frac{R}{h_1}$	K (Fig. 5-50)	σ_{max} (MPa)
2	0.05	2.6	240
4	0.10	2.1	200
6	0.15	1.8	170
8	0.20	1.7	160

Note: The larger the notch radius, the smaller the stress.

5.13-3

BEAM WITH SEMICIRCULAR NOTCHES

$h = 0.88$ in. $\qquad h_1 = 0.80$ in.

$\sigma_{max} = 60$ ksi $\qquad M = 600$ lb-in.

$h = h_1 + 2R \qquad R = \frac{1}{2}(h - h_1) = 0.04$ in.

$\frac{R}{h_1} = \frac{0.04 \text{ in.}}{0.80 \text{ in.}} = 0.05$

From Fig. 5-50: $K \approx 2.57$

$\sigma_{max} = K\sigma_{nom} = K\left(\frac{6M}{bh_1^2}\right)$

$60 \text{ ksi} = 2.57\left[\frac{6(600 \text{ lb-in.})}{b(0.80 \text{ in.})^2}\right]$

Solve for b:

$b_{min} \approx 0.24$ in. ←

5.13-4

BEAM WITH SEMICIRCULAR NOTCHES

$h = 120$ mm $\qquad h_1 = 100$ mm

$\sigma_{max} = 6$ MPa $\qquad M = 150$ N·m

$h = h_1 + 2R \qquad R = \frac{1}{2}(h - h_1) = 10$ mm

$\frac{R}{h_1} = \frac{10 \text{ mm}}{100 \text{ mm}} = 0.10$

From Fig. 5-50: $K \approx 2.20$

$\sigma_{max} = K\sigma_{nom} = K\left(\frac{6M}{bh_1^2}\right)$

$6 \text{ MPa} = (2.20)\left[\frac{6(150 \text{ N·m})}{b(100 \text{ mm})^2}\right]$

Solve for b: $\quad b_{min} \approx 33$ mm ←

5.13-5

BEAM WITH NOTCHES AND A HOLE

$h = 5.5$ in. $\qquad h_1 = 5$ in. $\qquad b = 1.6$ in.

$M = 130$ k-in. $\qquad \sigma_{max} = 42{,}000$ psi

(a) Minimum notch radius

$\frac{h}{h_1} = \frac{5.5 \text{ in.}}{5 \text{ in.}} = 1.1$

$\sigma_{nom} = \frac{6M}{bh_1^2} = 19{,}500$ psi

$K = \frac{\sigma_{max}}{\sigma_{nom}} = \frac{42{,}000 \text{ psi}}{19{,}500 \text{ psi}} = 2.15$

From Fig. 5-50, with $K = 2.15$ and $\frac{h}{h_1} = 1.1$, we get

$\frac{R}{h_1} \approx 0.090$

$\therefore R_{min} \approx 0.090 h_1 = 0.45$ in. ←

(b) Largest hole diameter

Assume $\frac{d}{h} > \frac{1}{2}$ and use Eq. (5-56).

$\sigma_B = \frac{12Md}{b(h^3 - d^3)}$

$42{,}000 \text{ psi} = \frac{12(130 \text{ k-in.})\, d}{(1.6 \text{ in.})[(5.5 \text{ in.})^3 - d^3]}$ or

$d^3 + 23.21\, d - 166.4 = 0$

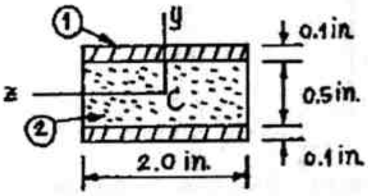
Solve numerically:

$d_{max} = 4.13$ in. ←

- END OF CHAPTER 5 -

6.2-1

Composite Beam



$b = 2$ in.
$h = 0.7$ in.
$h_c = 0.5$ in.
$M = 300$ lb-in.
$E_1 = 4 \times 10^6$ psi
$E_2 = 1.5 \times 10^6$ psi

$$I_1 = \frac{b}{12}(h^3 - h_c^3) = 0.03633 \text{ in.}^4$$

$$I_2 = \frac{bh_c^3}{12} = 0.02083 \text{ in.}^4$$

$$E_1 I_1 + E_2 I_2 = 176{,}600 \text{ lb-in.}^2$$

From Eq. (6-6a): $\sigma_{face} = \pm \frac{M(h/2)E_1}{E_1I_1 + E_2I_2}$

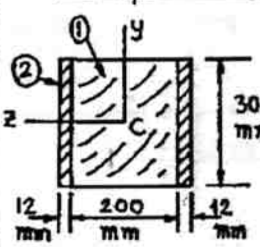
$= \pm 2380$ psi ←

From Eq. (6-6b): $\sigma_{core} = \pm \frac{M(h_c/2)E_2}{E_1I_1 + E_2I_2}$

$= \pm 637$ psi ←

6.2-2

Composite Beam



$b = 200$ mm
$t = 12$ mm
$h = 300$ mm
$E_1 = E_w = 8.5$ GPa
$E_2 = E_s = 204$ GPa
$(\sigma_1)_{allow} = 10$ MPa
$(\sigma_2)_{allow} = 120$ MPa

$$I_1 = \frac{bh^3}{12} = 450 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{2th^3}{12} = 54 \times 10^6 \text{ mm}^4$$

$$E_1I_1 + E_2I_2 = 14.84 \times 10^6 \text{ N·m}^2$$

Maximum moment based upon the wood ①

From Eq. (6-6a):

$$M_{max} = (\sigma_1)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_1}\right] = 116 \text{ kN·m}$$

Maximum moment based upon the steel ②

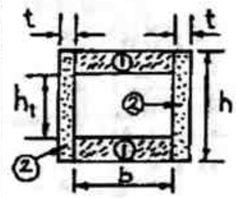
From Eq. (6-6b):

$$M_{max} = (\sigma_2)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_2}\right] = 58.2 \text{ kN·m}$$

Steel governs $M_{max} = 58.2$ kN·m ←

6.2-3

Hollow Box Beam



① Wood Flanges

$b = 4$ in. $h = 12$ in. $h_1 = 8$ in.
$E_1 = 1{,}200{,}000$ psi
$(\sigma_1)_{allow} = 1800$ psi

② Plywood Webs

$t = 1$ in. $h = 12$ in.
$E_2 = 1{,}600{,}000$ psi
$(\sigma_2)_{allow} = 2300$ psi

$$I_1 = \frac{b}{12}(h^3 - h_1^3) = 405.3 \text{ in.}^4$$

$$I_2 = 2\left(\frac{1}{12}\right)(th^3) = 288 \text{ in.}^4$$

$$E_1I_1 + E_2I_2 = 947.2 \times 10^6 \text{ lb-in.}^2$$

Maximum moment based upon the wood ①

From Eq. (6-6a):

$$M_{max} = (\sigma_1)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_1}\right] = 237 \text{ k-in.}$$

Maximum moment based upon the plywood ②

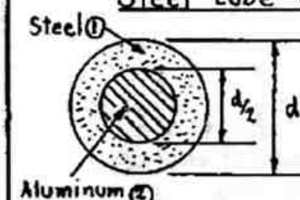
From Eq. (6-6b):

$$M_{max} = (\sigma_2)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_2}\right] = 227 \text{ k-in.}$$

Plywood governs $M_{max} = 227$ k-in. ←

6.2-4

Steel tube with aluminum core



Tube ①: d = outer diameter $\frac{d}{2}$ = inner diameter
E_s = modulus of elasticity
σ_s = allowable stress

$$I_1 = \frac{\pi}{64}\left[d^4 - (d/2)^4\right] = \frac{15\pi d^4}{1024}$$

Core ②: $d/2$ = diameter
E_a = modulus of elasticity

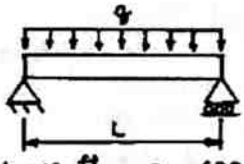
$$I_2 = \frac{\pi}{64}(d/2)^4 = \frac{\pi d^4}{1024}$$

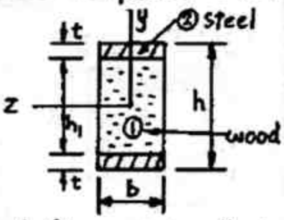
$$E_1I_1 + E_2I_2 = E_sI_1 + E_aI_2 = \frac{\pi d^4}{1024}(15E_s + E_a)$$

From Eq. (6-6a): $M_{allow} = \sigma_s\left[\frac{E_1I_1 + E_2I_2}{(d/2)E_s}\right]$

$$M_{allow} = \frac{\pi d^3 \sigma_s}{512}\left(15 + \frac{E_a}{E_s}\right) \quad ←$$

6.2-5 Simply supported composite beam





$L = 12$ ft $\quad q = 600$ lb/ft

$M_{max} = \dfrac{qL^2}{8} = 10{,}800$ lb-ft $= 129{,}600$ lb-in

Wood beam with steel plates

Wood ① : $b = 4$ in. $\quad h_1 = 11.5$ in. $\quad E_w = 1.5\times10^6$ psi

$I_1 = \dfrac{bh_1^3}{12} = 506.96 \text{ in}^4$

Plate ② : $b = 4$ in. $\quad t = 0.25$ in. $\quad h = 12$ in.

$E_s = 30\times10^6$ psi

$I_2 = \dfrac{b}{12}\left[h^3 - h_1^3\right] = 69.042 \text{ in}^4$

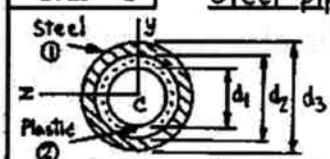
$E_1I_1 + E_2I_2 = E_wI_1 + E_sI_2 = 2{,}832{,}000{,}000 \text{ lb-in}^2$

From Eqs. (6-6a and b):

$\sigma_w = \dfrac{M_{max}(h_1/2)E_w}{E_1I_1 + E_2I_2} = 395 \text{ psi}$ ⟵

$\sigma_s = \dfrac{M_{max}(h/2)E_s}{E_1I_1 + E_2I_2} = 8240 \text{ psi}$ ⟵

6.2-6 Steel pipe with plastic liner



① Pipe : $d_3 = 100$ mm $\quad d_2 = 94$ mm

$E_s = E_1 =$ modulus of elasticity

$(\sigma_1)_{allow} = 38$ MPa

② Liner: $d_2 = 94$ mm $\quad d_1 = 82$ mm

$E_p = E_2 =$ modulus of elasticity

$(\sigma_2)_{allow} = 0.50$ MPa

$E_1 = 75E_2 \quad E_1/E_2 = 75$

$I_1 = \dfrac{\pi}{64}(d_3^4 - d_2^4) = 1.076\times10^{-6} \text{ m}^4$

$I_2 = \dfrac{\pi}{64}(d_2^4 - d_1^4) = 1.613\times10^{-6} \text{ m}^4$

Maximum moment based upon the steel ①

From Eq. (6-6a):

$M_{max} = (\sigma_1)_{allow}\left[\dfrac{E_1I_1 + E_2I_2}{(d_3/2)E_1}\right]$

$= (\sigma_1)_{allow}\dfrac{(E_1/E_2)I_1 + I_2}{(d_3/2)(E_1/E_2)} = 834 \text{ N·m}$

Maximum moment based upon the plastic ②

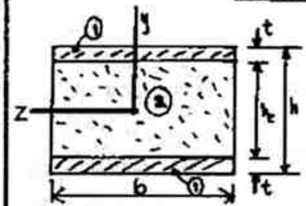
From Eq. (6-6b):

$M_{max} = (\sigma_2)_{allow}\left[\dfrac{E_1I_1 + E_2I_2}{(d_2/2)E_2}\right]$

$= (\sigma_2)_{allow}\left[\dfrac{(E_1/E_2)I_1 + I_2}{(d_2/2)}\right] = 876 \text{ N·m}$

Steel governs $\quad M_{allow} = 834$ N·m ⟵

6.2-7 Sandwich beam



① Aluminum faces: $b = 8.0$ in. $\quad t = 0.25$ in.

$h = 6.0$ in. $\quad E_1 = 10.5\times10^6$ psi

$I_1 = \dfrac{b}{12}(h^3 - h_c^3) = 33.08 \text{ in}^4$

② Foam core: $b = 8.0$ in. $\quad h_c = 5.5$ in.

$E_2 = 12{,}000$ psi

$I_2 = \dfrac{bh_c^3}{12} = 110.92 \text{ in}^4$

$M = 35$ K-in. $\quad E_1I_1 + E_2I_2 = 348.7\times10^6 \text{ lb-in}^2$

(a) General theory (Eqs 6-6 a and b)

$\sigma_{face} = \sigma_1 = \dfrac{M(h/2)E_1}{E_1I_1 + E_2I_2} = 3160$ psi ⟵

$\sigma_{core} = \sigma_2 = \dfrac{M(h_c/2)E_2}{E_1I_1 + E_2I_2} = 3$ psi ⟵

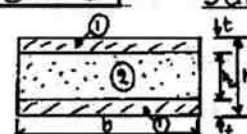
(b) Approximate theory (Eqs. 6-8 and 6-9)

$I_1 = \dfrac{b}{12}(h^3 - h_c^3) = 33.08 \text{ in}^4$

$\sigma_{face} = \dfrac{Mh}{2I_1} = 3170$ psi ⟵

$\sigma_{core} = 0$ ⟵

6.2-8 Sandwich beam



① Fiber glass faces: $b = 50$ mm $\quad t = 4$ mm $\quad h = 100$ mm

$E_1 = 75$ GPa

$I_1 = \dfrac{b}{12}(h^3 - h_c^3) = 0.9221\times10^{-6} \text{ m}^4$

② Plastic Core: $b = 50$ mm $\quad h_c = 92$ mm

$E_2 = 1.2$ GPa

$I_2 = \dfrac{bh_c^3}{12} = 3.245\times10^{-6} \text{ m}^4$

$M = 250$ N·m $\quad E_1I_1 + E_2I_2 = 73{,}050 \text{ N·m}^2$

(a) General theory (Eqs. 6-6a and b)

$\sigma_{face} = \sigma_1 = \dfrac{M(h/2)E_1}{E_1I_1 + E_2I_2} = 12.83$ MPa ⟵

$\sigma_{core} = \sigma_2 = \dfrac{M(h_c/2)E_2}{E_1I_1 + E_2I_2} = 0.19$ MPa ⟵

(b) Approximate theory (Eqs. 6-8 and 6-9)

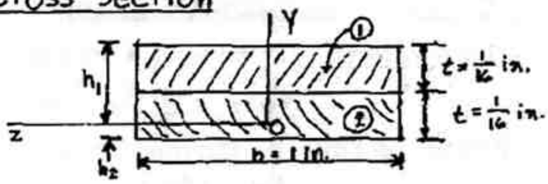
$I_1 = \dfrac{b}{12}(h^3 - h_c^3) = 0.9221\times10^{6} \text{ m}^4$

$\sigma_{face} = \dfrac{Mh}{2I_1} = 13.56$ MPa ⟵

$\sigma_{core} = 0$ ⟵

6.2-9 Bimetallic beam

Cross section



① Aluminum $E_1 = E_a = 10,500,000$ psi

② Copper $E_2 = E_c = 16,800,000$ psi

$M = 10$ lb-in.

Neutral axis (Eq. 6-3)

$\int_1 y\,dA = \bar{y}_1 A_1 = (h_1 - t/2)(bt) = (h_1 - 1/32)(1)(1/16)\text{ in}^3$

$\int_2 y\,dA = \bar{y}_2 A_2 = (h_1 - t - t/2)(bt) = (h_1 - 3/32)(1)(1/16)\text{ in}^3$

$E_1 \int_1 y\,dA + E_2 \int_2 y\,dA = 0$

$(10,500,000)(h_1 - 1/32)(1/16) + (16,800,000)(h_1 - 3/32)(1/16) = 0$

Solve for h_1: $h_1 = 0.06971$ in.

$h_2 = 2(1/16 \text{ in.}) - h_1 = 0.05529$ in.

Moments of inertia (parallel-axis theorem)

$I_1 = \frac{bt^3}{12} + bt(h_1 - t/2)^2 = 0.000128 \text{ in.}^4$

$I_2 = \frac{bt^3}{12} + bt(h_2 - t/2)^2 = 0.00005647 \text{ in.}^4$

$E_1 I_1 + E_2 I_2 = 2133 \text{ lb-in}^2$

Maximum stresses (Eqs. 6-6a and b)

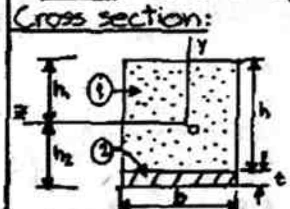
$\sigma_a = \sigma_1 = \frac{M h_1 E_1}{E_1 I_1 + E_2 I_2} = 3430$ psi ←

$\sigma_c = \sigma_2 = \frac{M h_2 E_2}{E_1 I_1 + E_2 I_2} = 4350$ psi ←

6.2-10 Simply supported composite beam

Beam: $L = 3$ m $q = 3.0$ kN/m $M_{max} = \frac{qL^2}{8} = 3375$ N·m

Cross section:



$b = 100$ mm

$h = 150$ mm

$t = 8$ mm

① Wood $E_1 = E_w = 10$ GPa

② Steel $E_2 = E_s = 210$ GPa

Neutral axis

$\int_1 y\,dA = \bar{y}_1 A_1 = (h_1 - h/2)(bh) = (h_1 - 75)(100)(150)\text{ mm}^3$

$\int_2 y\,dA = \bar{y}_2 A_2 = -(h + t/2 - h_1)(bt) = (h_1 - 154)(100)(8)\text{ mm}^3$

$E_1 \int_1 y\,dA + E_2 \int_2 y\,dA = 0$ (EQ. 6-3)

$(10\text{ GPa})(h_1 - 75)(100)(150)(10^{-9}) + (210\text{ GPa})(h_1 - 154)(100)(8)(10^{-9}) = 0$

Solve for h_1: $h_1 = 116.74$ mm

$h_2 = h + t - h_1 = 41.26$ mm

CONT.

6.2-10 CONT.

Moments of inertia (use parallel-axis theorem)

$I_1 = \frac{bh^3}{12} + bh(h_1 - h/2)^2 = 54.26 \times 10^6 \text{ mm}^4$

$I_2 = \frac{bt^3}{12} + bt(h_2 - t/2)^2 = 1.115 \times 10^6 \text{ mm}^4$

$E_1 I_1 + E_2 I_2 = 776,750 \text{ N}\cdot\text{m}^2$

Maximum stresses (Eqs. 6-6a and b)

$\sigma_w = \sigma_1 = \frac{M h_1 E_1}{E_1 I_1 + E_2 I_2} = 5.1$ MPa ← (Compression)

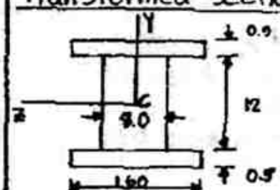
$\sigma_s = \sigma_2 = \frac{M h_2 E_2}{E_1 I_1 + E_2 I_2} = 37.6$ MPa ← (Tension)

6.3-1 Wood beam with steel plates

① Wood beam $b = 8$ in. $h_1 = 12$ in.
$(\sigma_1)_{allow} = 1050$ psi

② Steel plates $b = 8$ in. $h_2 = 13$ in.
$t = 0.5$ in.
$(\sigma_2)_{allow} = 16,000$ psi

Transformed section (wood)



$n = \frac{E_s}{E_w} = 20$

$nb = (20)(8 \text{ in.}) = 160$ in.

All dimensions in inches

$I_T = \frac{1}{12}(160)(13)^3 - \frac{1}{12}(160 - 8)(12)^3 = 7405 \text{ in.}^4$

Maximum moment based upon the wood ① (Eq. 6-15)

$\sigma_1 = \frac{M(h_1/2)}{I_T}$ $M_1 = \frac{(\sigma_1)_{allow} I_T}{h_1/2} = 1300$ k-in.

Maximum moment based upon the steel ② (Eq. 6-17)

$\sigma_2 = \frac{M(h_2/2)n}{I_T}$ $M_2 = \frac{(\sigma_2)_{allow} I_T}{(h_2/2)n} = 911$ k-in.

Steel governs $M_{max} = 911$ k-in. ←

6.3-2 Box beam

$M_{max} = \frac{qL^2}{8} = 61.44$ kN·m

Simple beam: $L = 3.2$ m $q = 48$ kN/m

① Wood flanges: $b = 100$ mm $h = 300$ mm
$h_1 = 150$ mm $E_w = 10$ GPa
(σ_1) allow $= 6.5$ MPa

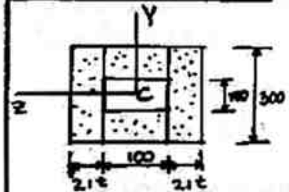
② Steel plates t = thickness $h = 300$ mm
$E_s = 210$ GPa
(σ_2) allow $= 120$ MPa

Transformed section (wood)

(see next page)

CONT.

6.3-2 CONT.



$n = \frac{E_s}{E_w} = 21$

All dimensions in millimeters

$I_T = 1/12\,(100 + 42t)(300)^3 - 1/12\,(100)(150)^3$

$= 196.9 \times 10^6\ mm^4 + 94.5\,t \times 10^6\ mm^4$

Required thickness based upon the wood ① (Eq. 6-15)

$\sigma_1 = \frac{M(h/2)}{I_T} \qquad (I_T)_1 = \frac{M_{max}(h/2)}{(\sigma_1)_{allow}} = 1.418 \times 10^9\ mm^4$

Equate I_T and $(I_T)_1$ and solve for t: $t_1 = 12.92$ mm

Required thickness based upon the steel ② (Eq. 6-17)

$\sigma_2 = \frac{M(h/2)n}{I_T} \quad (I_T)_2 = \frac{M_{max}(h/2)\,n}{(\sigma_2)_{allow}} = 1.612 \times 10^9\ mm^4$

Equate I_T and $(I_T)_2$ and solve for t: $t_2 = 14.97$ mm

Steel governs. $\quad t_{min} = 15.0$ mm

6.3-3 Reinforced I-beam

① Steel beam: S 8 × 18.4 $\quad I_1 = 57.6\ in^4$

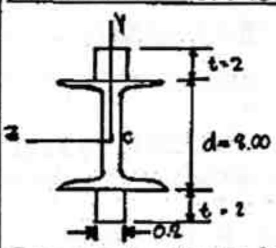
$d = 8.00$ in.

$(\sigma_1)_{allow} = 16{,}000$ psi

② Wood beams: $b = 4$ in. $\quad t = 2$ in.

$(\sigma_2)_{allow} = 1200$ psi

Transformed section (steel)



$n = \frac{E_w}{E_s} = 1/20$

$nb = (1/20)(4) = 0.2$ in.

All dimensions in inches

$I_T = 57.6 + 1/12\,(0.2)(12)^3 - 1/12\,(0.2)(8)^3 = 77.87\ in^4$

Maximum moment based upon the steel ① (Eq. 6-15)

$\sigma_1 = \frac{M(d/2)}{I_T} \qquad M_1 = \frac{(\sigma_1)_{allow} I_T}{d/2} = 311{,}480$ lb-in

Maximum moment based upon the wood ② (Eq. 6-17)

$\sigma_2 = \frac{M(d/2 + t)n}{I_T} \quad M_2 = \frac{(\sigma_2)_{allow} I_T}{(d/2 + t)n} = 311{,}480$ lb-in

By coincidence, $M_1 = M_2$ (exactly)

$M_{max} = 311{,}480$ lb-in.

Simple beam

$L = 16$ ft $\qquad M_{max} = \frac{qL^2}{8}$

$q_{allow} = \frac{8 M_{max}}{L^2} = 811$ lb/ft

6.3-4 Composite beam

Simple beam $\quad L = 5$ m $\quad q = 32$ kN/m

$M_{max} = \frac{qL^2}{8} = 100$ kN·m

① Wood beam: $b = 150$ mm $\quad h_1 = 250$ mm

$E_w = 11$ GPa

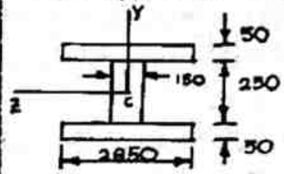
② Steel plates: $b = 150$ mm $\quad t = 50$ mm

$h_2 = 350$ mm $\quad E_s = 209$ GPa

Transformed section (wood)

$n = \frac{E_s}{E_w} = \frac{209}{11} = 19$

$nb = (19)(150\ mm) = 2850$ mm



All dimensions in millimeters.

$I_T = \frac{1}{12}(2850)(350)^3 - \frac{1}{12}(2850 - 150)(250)^3$

$= 6.667 \times 10^9\ mm^4$

Maximum stress in the wood ① (Eq. 6-15)

$\sigma_w = \sigma_1 = \frac{M_{max}(h_1/2)}{I_T} = 1.9$ MPa

Maximum stress in the steel ② (Eq. 6-17)

$\sigma_s = \sigma_2 = \frac{M_{max}(h_2/2)n}{I_T} = 49.9$ MPa

6.3-5 Plastic beam with aluminum strips

① Plastic segments: $b = 3.0$ in. $\quad d = 1.2$ in.

$3d = 3.6$ in.

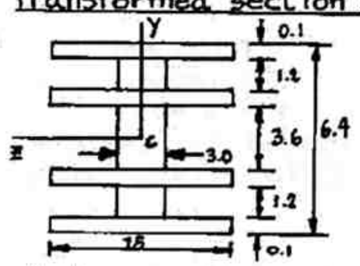
$E_p = 440 \times 10^3$ psi

② Aluminum strips: $b = 3.0$ in. $\quad t = 0.1$ in.

$E_a = 11 \times 10^6$ psi

$h = 4t + 5d = 6.4$ in. $\quad M = 8.0$ k-in.

Transformed section (plastic)



$n = \frac{E_a}{E_p} = 25$

$nb = (25)(3.0\ in.)$

$= 75$ in.

All dimensions in inches

plastic: $I_1 = 2\,[1/12\,(3.0)(1.2)^3 + (3.0)(1.2)(2.50)^2]$

$+ 1/12\,(3.0)(3.6)^3 = 57.528\ in^4$

Aluminum: $I_2 = 2\,[1/12\,(75)(0.1)^3 + (75)(0.1)(3.15)^2$

$+ 1/12\,(75)(0.1)^3 + (75)(0.1)(1.85)^2]$

$= 200.2\ in^4$

$I_T = I_1 + I_2 = 257.73\ in^4$

Maximum stress in the plastic ① (Eq. 6-15)

$\sigma_p = \sigma_1 = \frac{M(h/2 - t)}{I_T} = 96$ psi

Maximum stress in the aluminum ② (Eq. 6-17)

$\sigma_a = \sigma_2 = \frac{M(h/2)n}{I_T} = 2480$ psi

6.3-6

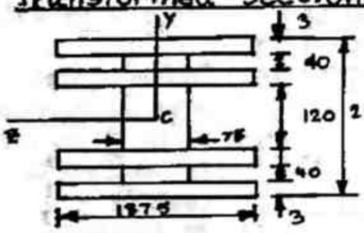
Plastic beam with aluminum strips

① Plastic segments: $b = 75$ mm $d = 40$ mm
$3d = 120$ mm $E_p = 3$ GPa

② Aluminum strips: $b = 75$ mm $t = 3$ mm
$E_a = 75$ GPa

$h = 4t + 5d = 212$ mm $M = 1.5$ kN·m

Transformed section (plastic)



$n = \frac{E_a}{E_p} = 25$

$nb = (25)(75\text{ mm})$
$= 1875$ mm

All dimensions in millimeters

Plastic: $I_1 = 2[1/12\,(75)(40)^3 + (75)(40)(85)^2]$
$+ \frac{1}{12}(75)(120)^3$
$= 52.934 \times 10^6\ \text{mm}^4$

Aluminum: $I_2 = 2[\frac{1}{12}(1875)(3)^3 + (1875)(3)(104.5)^2$
$+ \frac{1}{12}(1875)(3)^3 + (1875)(3)(61.5)^2]$
$= 165.420 \times 10^6\ \text{mm}^4$

$I_T = I_1 + I_2 = 218.35 \times 10^6\ \text{mm}^4$

Maximum stress in the plastic ① (Eq. 6-15)

$\sigma_p = \sigma_1 = \frac{M(\frac{h}{2} - t)}{I_T} = 0.71$ MPa

Maximum stress in the aluminum ② (Eq. 6-17)

$\sigma_a = \sigma_2 = \frac{M(\frac{h}{2})n}{I_T} = 18.20$ MPa

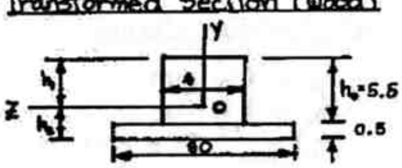
6.3-7

Composite beam

Simple beam: $L = 12$ ft $q = 200$ lb/ft
$M_{max} = \frac{qL^2}{8} = 43{,}200$ lb-in.

① Wood beam: $b = 4$ in. $h_0 = 5.5$ in.
② Steel plate: $b = 4$ in. $t = 0.5$ in.
$h = h_0 + t = 6.0$ in. $\frac{E_s}{E_w} = 20$

Transformed section (wood)



$n = 20$
$nb = (20)(4\text{ in.})$
$= 80$ in.
All dimensions in inches.

Use the base of the cross section as a reference line.

$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(0.25)(80)(0.5) + (3.25)(4)(5.5)}{(80)(0.5) + (4)(5.5)}$
$= 1.3145$ in.

$h_1 = h - h_2 = 4.6855$ in.

$I_T = \frac{1}{12}(4)(5.5)^3 + (4)(5.5)(h_1 - 2.75)^2 + \frac{1}{12}(80)(0.5)^3$
$+ (80)(0.5)(h_2 - 0.25)^2$
$= 184.03\ \text{in}^4$

CONT.

6.3-7 CONT.

Maximum stress in the wood ① (Eq. 6-15)

$\sigma_w = \sigma_1 = \frac{M_{max} h_1}{I_T} = 1100$ psi (compression)

Maximum stress in the steel ② (Eq. 6-17)

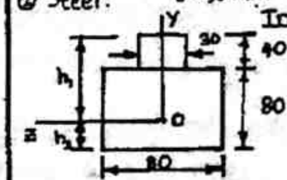
$\sigma_s = \sigma_2 = \frac{M_{max} h_2 n}{I_T} = 6170$ psi (Tension)

6.3-8

Composite beam of aluminum and steel

① Aluminum: $b = 30$ mm $h_a = 40$ mm $E_a = 75$ GPa $\sigma_a = 40$ MPa
② Steel: $b = 30$ mm $h_s = 80$ mm $E_s = 200$ GPa $\sigma_s = ?$

Transformed section (aluminum)



$n = \frac{E_s}{E_a} = \frac{200}{75} = 2.667$

$nb = (2.667)(30\text{ mm})$
$= 80$ mm

All dimensions in millimeters

Use the base of the cross section as a reference line.

$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(40)(80)(80) + (100)(30)(40)}{(80)(80) + (30)(40)}$
$= 49.474$ mm

$h_1 = 120 - h_2 = 70.526$ mm

Maximum stress in the aluminum ① (Eq. 6-15)

$\sigma_a = \sigma_1 = \frac{M h_1}{I_T}$

Maximum stress in the steel ② (Eq. 6-17)

$\sigma_s = \sigma_2 = \frac{M h_2 n}{I_T}$

$\frac{\sigma_s}{\sigma_a} = \frac{h_2 n}{h_1} = \frac{(49.474)(2.667)}{70.526} = 1.8707$

$\sigma_s = 1.8707\ (40\text{ MPa}) = 74.8$ MPa

6.3-9

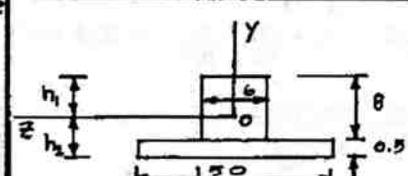
Composite beam of wood and steel

① Wood beam: $b = 6$ in. $h_w = 8$ in.
$E_w = 1.2 \times 10^6$ psi
$(\sigma_w)_{allow} = 1{,}600$ psi

② Steel plate: $b = 6$ in. $t = 0.5$ in.
$E_s = 30 \times 10^6$ psi
$(\sigma_s)_{allow} = 15{,}000$ psi

Transformed section (wood)

$n = \frac{E_s}{E_w} = \frac{30}{1.2} = 25$



$nb = (25)(6\text{ in.})$
$= 150$ in.
All dimensions in inches.

Use the base of the cross section as a reference line.

CONT.

6.3-9 CONT.

$$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(0.25)(150)(0.5)+(4.5)(6)(8)}{(150)(0.5)+(6)(8)}$$

$$= 1.9085 \text{ in.}$$

$$h_1 = 8.5 - h_2 = 6.5915 \text{ in.}$$

$$I_T = \frac{1}{12}(6)(8)^3 + (6)(8)(h_1-4)^2 + \frac{1}{12}(150)(0.5)^3 + (150)(0.5)(h_2-0.25)^2 = 786.22 \text{ in.}^4$$

Maximum moment based upon the wood ① (Eq. 6-15)

$$\sigma_w = \sigma_1 = \frac{Mh_1}{I_T} \qquad M_1 = \frac{(\sigma_w)_{allow} I_T}{h_1} = 191 \text{ k-in.}$$

Maximum moment based upon the steel ② (Eq. 6-17)

$$\sigma_s = \sigma_2 = \frac{Mh_2 n}{I_T} \qquad M_2 = \frac{(\sigma_s)_{allow} I_T}{h_2 n} = 247 \text{ k-in.}$$

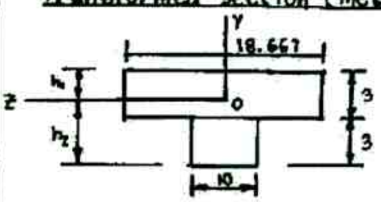
Wood governs $M_{allow} = 191$ k-in. ⟵

6.3-10 Bimetallic strip

Metal Ⓐ: $b = 10$ mm $h_A = 3$ mm $E_A = 196$ GPa

Metal Ⓑ: $b = 10$ mm $h_B = 3$ mm $E_B = 105$ GPa

Transformed section (metal Ⓑ)



$$n = \frac{E_A}{E_B} = \frac{196}{105} = 1.8667$$

$$nb = (1.8667)(10 \text{ mm}) = 18.667 \text{ mm}$$

All dimensions in millimeters.

Use the base of the cross section as a reference line.

$$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(1.5)(10)(3)+(4.5)(18.667)(3)}{(10)(3)+(18.667)(3)} = 3.4535 \text{ mm}$$

$$h_1 = 6 - h_2 = 2.5465 \text{ mm}$$

$$I_T = \frac{1}{12}(10)(3)^3 + (10)(3)(h_2-1.5)^2 + \frac{1}{12}(18.667)(3)^3 + (18.667)(3)(h_1-1.5)^2 = 240.31 \text{ mm}^4$$

Maximum stress in material Ⓑ (Eq. 6-15)

$$\sigma_B = \sigma_1 = \frac{Mh_2}{I_T} \qquad S_B = \frac{M}{\sigma_B} = \frac{I_T}{h_2} = 69.6 \text{ mm}^3$$

Maximum stress in material Ⓐ (Eq. 6-17)

$$\sigma_A = \sigma_2 = \frac{Mh_1 n}{I_T} \qquad S_A = \frac{M}{\sigma_A} = \frac{I_T}{h_1 n} = 50.6 \text{ mm}^3$$

Smaller section modulus

$S_A = 50.6$ mm^3 ⟵

∴ Maximum stress occurs in metal Ⓐ ⟵

6.3-11

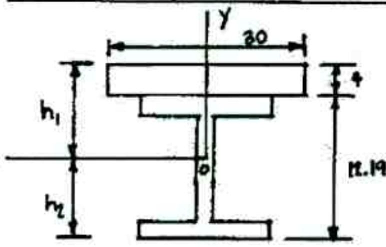
Steel beam and concrete slab

① Concrete: $b = 30$ in. $t = 4$ in.

② Wide-flange beam: W 12 × 50 $d = 12.19$ in $I = 394$ in.^{4} $A = 14.7$ in^2

$M = 100$ k-ft $= 1200$ k-in.

Transformed section (concrete)



$$n = \frac{E_s}{E_c} = \frac{E_2}{E_1} = 12$$

Width of steel beam is increased by the factor n to transform to concrete.

All dimensions in inches.

Use the base of the cross section as a reference line.

$nI = 4728$ in.^{4} $nA = 176.4$ in.^{2}

$$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(\frac{12.19}{2})(176.4)+(14.19)(30)(4)}{176.4+(30)(4)}$$

$$= 9.372 \text{ in.}$$

$$h_1 = 16.19 - h_2 = 6.818 \text{ in.}$$

$$I_T = \frac{1}{12}(30)(4)^3 + (30)(4)(h_1-2)^2 + 4728 + (176.4)\left(h_2 - \frac{12.19}{2}\right)^2 = 9568 \text{ in.}^4$$

Maximum stress in the concrete ① (Eq. 6-15)

$$\sigma_c = \sigma_1 = \frac{Mh_1}{I_T} = 855 \text{ psi}$$ ⟵ (compression)

Maximum stress in the steel ② (Eq. 6-17)

$$\sigma_s = \sigma_2 = \frac{Mh_2 n}{I_T} = 14{,}110 \text{ psi}$$ ⟵ (Tension)

6.3-12

Wood beam and aluminum channel

① Wood beam: $b_w = 150$ mm
$h_w = 250$ mm
$(\sigma_w)_{allow} = 8.5$ MPa

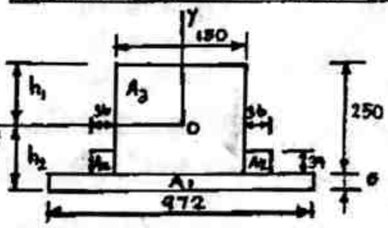
② Aluminum channel: $t = 6$ mm
$b_a = 162$ mm
$h_a = 40$ mm
$(\sigma_a)_{allow} = 40$ MPa

CONT.

6.3-12 CONT.

Wood beam and aluminum channel

Transformed section (wood)



$n = \frac{E_a}{E_w} = 6$

$nb_a = (6)(162\ mm)$
$= 972\ mm$

$nt = (6)(6mm)$
$= 36\ mm$

All dimensions in millimeters.

Use the base of the cross section as a reference line.

$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i}$

Area A_1: $y_1 = 3$ $A_1 = (972)(6) = 5832$
$y_1 A_1 = 17{,}496\ mm^3$

Area A_2: $y_2 = 23$ $A_2 = (36)(34) = 1224$
$y_2 A_2 = 28{,}152\ mm^3$

Area A_3: $y_3 = 131$ $A_3 = (150)(250) = 37{,}500$
$y_3 A_3 = 4{,}912{,}500\ mm^3$

$h_2 = \frac{y_1 A_1 + 2y_2 A_2 + y_3 A_3}{A_1 + 2A_2 + A_3} = \frac{4{,}986{,}300\ mm^3}{45{,}780\ mm^2}$

$= 108.92\ mm$

$h_1 = 256 - h_2 = 147.08\ mm$

Moment of inertia

Area A_1: $I_1 = \frac{1}{12}(972)(6)^3 + (972)(6)(h_2 - 3)^2$
$= 65{,}445{,}000\ mm^4$

Area A_2: $I_2 = \frac{1}{12}(36)(34)^3 + (36)(34)(h_2 - 6 - 17)^2$
$= 9{,}153{,}500\ mm^4$

Area A_3: $I_3 = \frac{1}{12}(150)(250)^3 + (150)(250)(h_1 - 125)^2$
$= 213{,}597{,}000\ mm^4$

$I_T = I_1 + 2I_2 + I_3 = 297.3 \times 10^6\ mm^4$

Maximum moment based upon the wood ①

(Eq. 6-15)

$\sigma_w = \sigma_1 = \frac{Mh_1}{I_T}$ $M_1 = \frac{(\sigma_w)_{allow} I_T}{h_1} = 17.2\ kN \cdot m$

Maximum moment based upon the aluminum ②

(Eq. 6-17)

$\sigma_a = \sigma_2 = \frac{Mh_2 n}{I_T}$ $M_2 = \frac{(\sigma_a)_{allow} I_T}{h_2 n} = 182\ kN \cdot m$

Wood governs $M_{allow} = 17.2\ kN \cdot m$

6.3-13

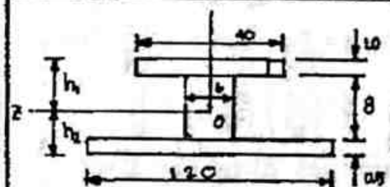
Wood beam with steel on top and bottom

① Wood beam: $b_1 = 6$ in. $h = 8$ in.
$(\sigma_w)_{allow} = 2000$ psi

② Steel plate on the bottom: $b_2 = 6$ in.
$t_2 = 0.5$ in. $(\sigma_s)_{allow} = 18{,}000$ psi

③ Steel bar on the top: $b_3 = 2$ in. $t_3 = 1$ in.

Transformed section (wood)



$n = \frac{E_s}{E_w} = 20$

$nb_2 = 120$ in.

$nb_3 = 40$ in.

All dimensions in inches.

Use the base of the cross section as a reference line.

$h_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(0.25)(120)(0.5) + (4.5)(6)(8) + (9.0)(40)}{(120)(0.5) + (6)(8) + (40)(1)}$

$= 3.493$ in.

$h_1 = 9.5 - h_2 = 5.507$ in.

Moment of inertia

$I_T = \frac{1}{12}(120)(0.5)^3 + (120)(0.5)(h_2 - 0.25)^2$
$+ \frac{1}{12}(6)(8)^3 + (6)(8)(h_1 - 1.0 - 4)^2$
$+ \frac{1}{12}(40)(1.0)^3 + (40)(1.0)(h_1 - 0.5)^2$

$= 2{,}113\ in.^4$

Maximum moment based upon the wood ① (Eq. 6-15)

$\sigma_w = \sigma_1 = \frac{M(h_1 - 1.0)}{I_T}$ $M_1 = \frac{(\sigma_w)_{allow} I_T}{h_1 - 1.0} = 938$ k-in.

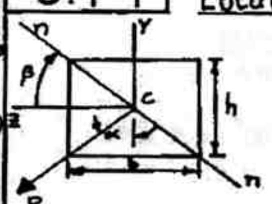
Maximum moment based upon the steel ② (Eq. 6-17)

$\sigma_s = \sigma_2 = \frac{M(h_1)n}{I_T}$ $M_2 = \frac{(\sigma_s)_{allow} I_T}{(h_1)n} = 345$ k-in.

Steel governs $M_{allow} = 345$ k-in.

6.4-1

Location of neutral axis



Load P acts along a diagonal.

$\tan \alpha = \frac{b/2}{h/2} = \frac{b}{h}$

$I_z = \frac{bh^3}{12}$

See Figure 6-15b. $I_y = \frac{hb^3}{12}$

$\frac{I_z}{I_y} = \frac{h^2}{b^2}$

CONT.

6.4-1 CONT.

β = angle between the z axis and the neutral axis nn

θ = angle between the y axis and the load P

$\theta = \alpha + 180°$

$\tan\theta = \tan(\alpha + 180°) = \tan\alpha$

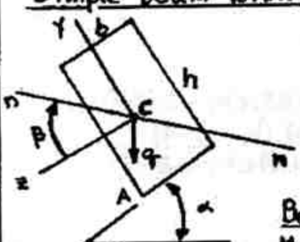
(Eq. 6-23): $\tan\beta = \dfrac{I_z}{I_y}\tan\theta = \dfrac{h^2}{b^2}\tan\theta$

$$= \left(\frac{h^2}{b^2}\right)\left(\frac{b}{h}\right) = \frac{h}{b}$$

∴ The neutral axis lies along the other diagonal. QED

6.4-2

Simple beam with inclined load



$L = 3.0$ m.
$q = 1.6$ kN/m
$b = 75$ mm
$h = 150$ mm
$\alpha = 30°$

Bending moments

$M_y = \dfrac{q_z L^2}{8} = \dfrac{q(\sin\alpha)L^2}{8}$

$= 900$ N·m

$M_z = \dfrac{q_y L^2}{8} = \dfrac{q(\cos\alpha)L^2}{8}$

$= 1559$ N·m

Moments of inertia

$I_y = \dfrac{hb^3}{12} = 5{,}273 \times 10^3\ \text{mm}^4$

$I_z = \dfrac{bh^3}{12} = 21{,}094 \times 10^3\ \text{mm}^4$

Neutral axis nn (Eq. 6-23)

$\tan\beta = \dfrac{I_z}{I_y}\tan\theta = \dfrac{I_z}{I_y}\tan\alpha$

$= \left(\dfrac{h}{b}\right)^2\tan\alpha = 4\tan 30° = 2.3094$

$\beta = 66.6°$

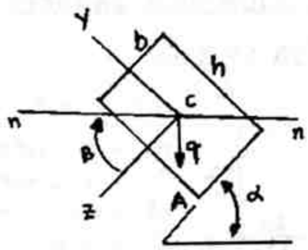
Maximum tensile stress (at point A) (Eq. 6-18)

$\sigma_{max} = \dfrac{M_y\left(\frac{b}{2}\right)}{I_y} - \dfrac{M_z\left(-\frac{h}{2}\right)}{I_z}$

$= \dfrac{(900\ \text{N·m})(37.5\ \text{mm})}{5273 \times 10^3\ \text{mm}^4} + \dfrac{(1559\ \text{N·m})(75\ \text{mm})}{21{,}094 \times 10^3\ \text{mm}^4}$

$\sigma_{max} = 11.9$ MPa

6.4-3 Simple beam with inclined load



$L = 10$ ft
$q = 240$ lb/ft
$b = 6$ in.
$h = 8$ in.
$\tan\alpha = \frac{1}{3}$

Bending moments

$M_y = \dfrac{q_z L^2}{8} = \dfrac{q(\sin\alpha)L^2}{8}$

$= 11{,}380$ lb-in.

$M_z = \dfrac{q_y L^2}{8} = \dfrac{q(\cos\alpha)L^2}{8}$

$= 34{,}150$ lb-in.

Moments of inertia

$I_y = \dfrac{hb^3}{12} = 144\ \text{in.}^4 \qquad I_z = \dfrac{bh^3}{12} = 256\ \text{in}^4$

Neutral axis nn (Eq. 6-23)

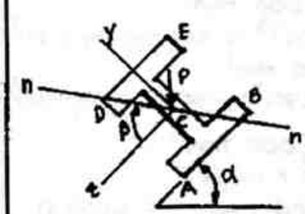
$\tan\beta = \dfrac{I_z}{I_y}\tan\theta = \dfrac{I_z}{I_y}\tan\alpha$

$= \left(\dfrac{h}{b}\right)^2\tan\alpha = 0.5926 \qquad \beta = 30.7°$

Maximum tensile stress (at point A) (Eq. 6-18)

$\sigma_{max} = \dfrac{M_y\left(\frac{b}{2}\right)}{I_y} - \dfrac{M_z\left(-\frac{h}{2}\right)}{I_z} = 771$ psi

6.4-4 Simple beam with inclined load



$L = 8.5$ ft
$P = 5$ K
$\alpha = 26.57°$
Wide-flange beam: W 10×30
$I_y = 16.7\ \text{in.}^4$
$I_z = 170\ \text{in.}^4$
$d = 10.47$ in.
$b = 5.810$ in.

Bending moments

$M_y = \dfrac{P(\sin\alpha)L}{4} = 57{,}030$ lb-in.

$M_z = \dfrac{P(\cos\alpha)L}{4} = 114{,}030$ lb-in.

Neutral axis nn (Eq. 6-23)

$\tan\beta = \dfrac{I_z}{I_y}\tan\theta = \dfrac{I_z}{I_y}\tan\alpha = 5.0909$

$\beta = 78.89°$

Bending stresses (Eq. 6-18)

$\sigma_x = \dfrac{M_y z}{I_y} - \dfrac{M_z y}{I_z} = \dfrac{57{,}030\ \text{lb-in.}}{16.7\ \text{in.}^4}(z) - \dfrac{114{,}030\ \text{lb-in.}}{170\ \text{in.}^4}(y)$

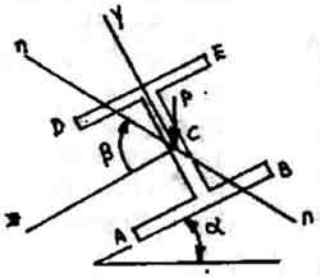
point A: $z_A = \dfrac{b}{2} = 2.905$ in. $\quad y_A = -\dfrac{d}{2} = -5.235$ in.

$\sigma_A = -\sigma_E = 13{,}450$ psi

point B: $z_B = -b/2 = -2.905$ in.

$y_B = -d/2 = -5.235$ in. $\quad \sigma_B = -\sigma_D = -6410$ psi

6.4-5 Simple beam with inclined load



$L = 8\text{ ft} = 96\text{ in.}$
$P = 3.8\text{ k}$
$\alpha = 20°$
Wide-flange beam: W 8×21
$I_Y = 9.77\text{ in}^4$
$I_Z = 75.3\text{ in}^4$
$d = 8.28\text{ in.}$
$b = 5.270\text{ in.}$

Bending moments

$M_Y = \frac{P(\sin\alpha)L}{4} = 31{,}190\text{ lb-in.}$

$M_Z = \frac{P(\cos\alpha)L}{4} = 85{,}700\text{ lb-in.}$

Neutral axis nn (Eq. 6-23)

$\tan\beta = \frac{I_Z}{I_Y}\tan\theta = \frac{I_Z}{I_Y}\tan\alpha = 2.8052$

$\beta = 70.38°$ ⟵

Bending stresses (Eq. 6-18)

$\sigma_x = \frac{M_Y z}{I_Y} - \frac{M_Z y}{I_Z} = \frac{31{,}190\text{ lb-in.}}{9.77\text{ in}^4}(z) - \frac{85{,}700\text{ lb-in.}}{75.3\text{ in}^4}(y)$

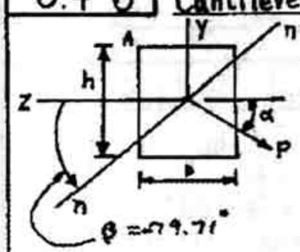
point A: $z_A = \frac{b}{2} = 2.635\text{ in.}$ $y_A = -\frac{d}{2} = -4.140\text{ in.}$

$\sigma_A = -\sigma_B = 13{,}120\text{ psi}$ ⟵

Point B: $z_B = -\frac{b}{2} = -2.635\text{ in.}$ $y_B = -\frac{d}{2} = -4.140\text{ in.}$

$\sigma_B = -\sigma_D = -3{,}700\text{ psi}$ ⟵

6.4-6 Cantilever beam with inclined load



$P = 750\text{ N}$
$L = 1.5\text{ m}$
$\alpha = 36°$
$b = 75\text{ mm}$
$h = 150\text{ mm}$
$I_Y = \frac{hb^3}{12} = 5.273\times10^6\text{ mm}^4$
$I_Z = \frac{bh^3}{12} = 21.094\times10^6\text{ mm}^4$

Bending moments

$M_Y = (P\cos\alpha)L = 910.1\text{ N·m}$
$M_Z = -(P\sin\alpha)L = -661.3\text{ N·m}$

Neutral axis nn (Eq. 6-23)

$\tan\beta = \frac{I_Z}{I_Y}\tan\theta \qquad \theta = \alpha + 90°$

$\tan\beta = \frac{21.094}{5.273}\tan(36° + 90°) = -5.5060$

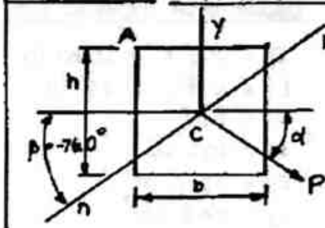
$\beta = -79.71°$ ⟵

Maximum tensile stress (point A) (Eq. 6-18)

$z_A = \frac{b}{2} = 37.5\text{ mm} \qquad y_A = \frac{h}{2} = 75\text{ mm}$

$\sigma_{max} = \sigma_A = \frac{M_Y z_A}{I_Y} - \frac{M_Z y_A}{I_Z} = 8.82\text{ MPa}$ ⟵

6.4-7 Cantilever beam with inclined load



$P = 400\text{ lb}$
$L = 6\text{ ft} = 72\text{ in.}$
$\alpha = 45°$
$b = 4\text{ in.}$
$h = 8\text{ in.}$
$I_Y = \frac{hb^3}{12} = 42.667\text{ in}^4$
$I_Z = \frac{bh^3}{12} = 170.67\text{ in}^4$

Bending moments

$M_Y = (P\cos\alpha)L = 20{,}365\text{ lb-in.}$
$M_Z = -(P\sin\alpha)L = -20{,}365\text{ lb-in.}$

Neutral axis nn (Eq. 6-23)

$\tan\beta = \frac{I_Z}{I_Y}\tan\theta \qquad \theta = \alpha + 90°$

$\tan\beta = \frac{170.67}{42.667}\tan(45° + 90°) = -4.000$

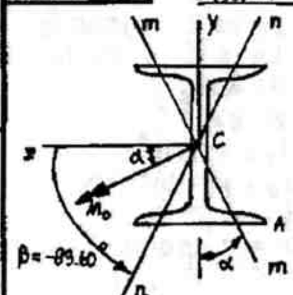
$\beta = -75.96°$ ⟵

Maximum tensile stress (point A) (Eq. 6-18)

$z_A = -\frac{b}{2} = 2\text{ in.} \qquad y_A = \frac{h}{2} = 4\text{ in.}$

$\sigma_{max} = \sigma_A = \frac{M_Y z_A}{I_Y} - \frac{M_Z y_A}{I_Z} = 1430\text{ psi}$ ⟵

6.4-8 Beam in pure bending



$M_0 = 25{,}000\text{ lb-in.}$
$\alpha = 30°$
S 8×18.4
$I_Y = 3.73\text{ in}^4$
$I_Z = 57.6\text{ in}^4$
$d = 8.00\text{ in.}$
$b = 4.001\text{ in.}$
$M_Y = -M_0\sin\alpha = -12{,}500\text{ lb-in.}$
$M_Z = M_0\cos\alpha = 21{,}651\text{ lb-in.}$

Neutral axis nn (Eq. 6-23)

$\theta = -\alpha = -30°$ (See Fig. 6-15)

$\tan\beta = \frac{I_Z}{I_Y}\tan\theta = \frac{57.6}{3.73}\tan(-30°) = -8.9157$

$\beta = -83.60°$ ⟵

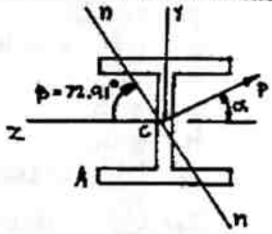
Maximum tensile stress (point A) (Eq. 6-18)

$z_A = -\frac{b}{2} = -2.000\text{ in.}$

$y_A = -\frac{d}{2} = -4.000\text{ in.}$

$\sigma_{max} = \frac{M_Y z_A}{I_Y} - \frac{M_Z y_A}{I_Z} = 8210\text{ psi}$ ⟵

6.4-9 Cantilever beam with inclined load



$P = 2.0\ k = 2000\ \text{lb}$
$L = 6\ \text{ft} = 72\ \text{in.}$
$\alpha = 55°$
$W\ 10 \times 45$
$I_y = 53.4\ \text{in}^4$
$I_z = 248\ \text{in}^4$
$d = 10.10\ \text{in.}$
$b = 8.02\ \text{in.}$

Bending moments

$M_y = (P\cos\alpha)L = 82{,}595\ \text{lb-in.}$
$M_z = (P\sin\alpha)L = 117{,}960\ \text{lb-in.}$

Neutral axis nn (Eq. 6-23)

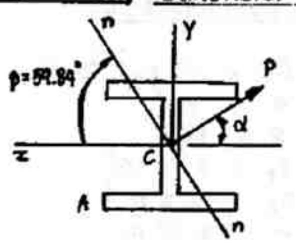
$\theta = 90° - \alpha = 35°$ (see figure 6-15)
$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{248}{53.4}\tan 35° = 3.2519$
$\beta = 72.91°$ ←

Maximum tensile stress (point A) (Eq. 6-18)

$z_A = \frac{b}{2} = 4.01\ \text{in.} \qquad y_A = -\frac{d}{2} = -5.05\ \text{in.}$

$\sigma_{max} = \sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z} = 8{,}600\ \text{psi}$ ←

6.4-10 Cantilever beam with inclined load



$P = 1.5\ K = 1500\ \text{lb}$
$L = 6\ \text{ft} = 72\ \text{in.}$
$\alpha = 60°$
$W\ 8 \times 35$
$I_y = 42.6\ \text{in}^4$
$I_z = 127\ \text{in}^4$
$d = 8.12\ \text{in.}$
$b = 8.020\ \text{in.}$

Bending moments

$M_y = (P\cos\alpha)L = 54{,}000\ \text{lb-in.}$
$M_z = (P\sin\alpha)L = 93{,}530\ \text{lb-in.}$

Neutral axis nn (Eq. 6-23)

$\theta = 90° - \alpha = 30°$ (see figure 6-15)
$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{127}{42.6}\tan 30° = 1.7212$
$\beta = 59.84°$ ←

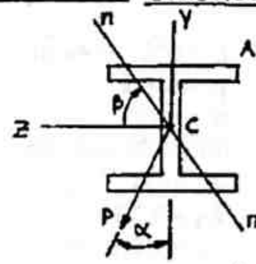
Maximum tensile stress (point A) (Eq. 6-18)

$z_A = \frac{b}{2} = 4.01\ \text{in.} \qquad y_A = -\frac{d}{2} = -4.06\ \text{in.}$

$\sigma_{max} = \sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z}$

$= 8070\ \text{psi}$ ←

6.4-11 Cantilever beam with inclined load



$P = 500\ \text{lb}$
$L = 9\ \text{ft} = 108\ \text{in.}$
$W\ 12 \times 14$
$I_y = 2.36\ \text{in}^4$
$I_z = 88.6\ \text{in}^4$
$d = 11.91\ \text{in.}$
$b = 3.970\ \text{in.}$

Bending moments

$M_y = -(P\sin\alpha)L = -54{,}000\sin\alpha$
$M_z = -(P\cos\alpha)L = -54{,}000\cos\alpha$

(a) Stress at point A (Eq. 6-18)

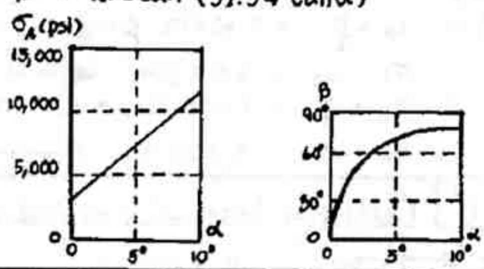
$z_A = -\frac{b}{2} = -1.985\ \text{in.} \qquad y_A = \frac{d}{2} = 5.955\ \text{in.}$

$\sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z}$

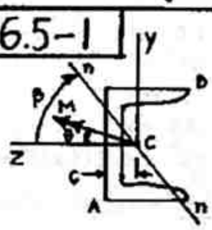
$= 45{,}420\sin\alpha + 3629\cos\alpha\ (\text{psi})$ ←

(b) Neutral axis nn (Eq. 6-23)

$\theta = 180° + \alpha$ (see Figure 6-15)
$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{I_z}{I_y}\tan(180° + \alpha)$
$= \frac{88.6}{2.36}\tan(180° + \alpha) = 37.54\tan\alpha$
$\beta = \arctan(37.54\tan\alpha)$



6.5-1 Channel Section



$M = 10\ \text{k-in.}$
$\tan\theta = \frac{1}{3}$
$\theta = 18.435°$
$C\ 8 \times 11.5$
$c = 0.571\ \text{in.}$
$I_y = 1.32\ \text{in}^4$
$I_z = 32.6\ \text{in}^4$
$d = 8.00\ \text{in.}$
$b = 2.260\ \text{in.}$

Neutral axis nn (Eq. 6-40)

$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{32.6}{1.32}\left(\frac{1}{3}\right) = 8.2323$
$\beta = 83.07°$ ←

Maximum tensile stress (point A) (Eq. 6-38)

$z_A = c = 0.571\ \text{in.} \qquad y_A = -\frac{d}{2} = -4.00\ \text{in.}$

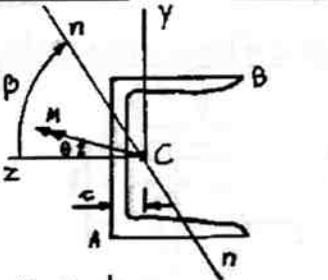
$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z} = 2530\ \text{psi}$ ←

Maximum compressive stress (point B) (Eq. 6-38)

$z_B = -(b - c) = -(2.260 - 0.571) = -1.689\ \text{in.} \quad y_B = \frac{d}{2} = 4.00\ \text{in.}$

$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z} = -5{,}210\ \text{psi}$ ←

6.5-2 Channel section



$M = 6.0$ k-in.
$\theta = 15°$
C 6 × 13
$c = 0.514$ in.
$I_y = 1.05$ in.^{4}
$I_z = 17.4$ in.^{4}
$d = 6.00$ in.
$b = 2.157$ in.

Neutral axis nn (Eq. 6-40)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{17.4}{1.05}\tan 15° = 4.4403$$

$$\beta = 77.31°$$

Maximum tensile stress (point A)(Eq. 6-38)

$z_A = c = 0.514$ in. $\quad y_A = -\frac{d}{2} = -3.00$ in.

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$

$$= 1760 \text{ psi}$$

Maximum compressive stress (point B)(Eq. 6-38)

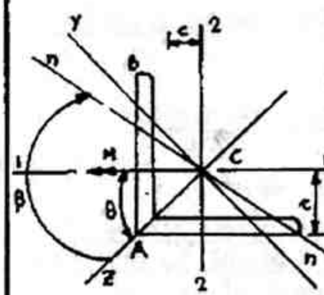
$z_B = -(b-c) = -(2.157-0.514) = -1.643$ in.

$y_B = \frac{d}{2} = 3.00$ in.

$$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z}$$

$$= -3430 \text{ psi}$$

6.5-4 Angle section with equal legs



$M = 5.0$ k-in.
L $4 \times 4 \times \frac{1}{2}$ in.
$h = b = 4.0$ in.
$c = 1.18$ in.
$I_1 = I_2 = 5.56$ in.^{4}
$\theta = 45°$
$A = 3.75$ in.^{2}
$r_{min} = 0.782$ in.
$I_y = A r_{min}^2 = 2.293$ in.^{4}
$I_z = I_1 + I_2 - I_y = 8.827$ in.^{4}

Neutral axis nn (Eq. 6-40)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{8.827}{2.293}\tan 45° = 3.8495$$

$$\beta = 75.44°$$

Maximum tensile stress (point A)(Eq. 6-38)

$z_A = c\sqrt{2} = 1.669$ in. $\quad y_A = 0$

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$

$$= 2570 \text{ psi}$$

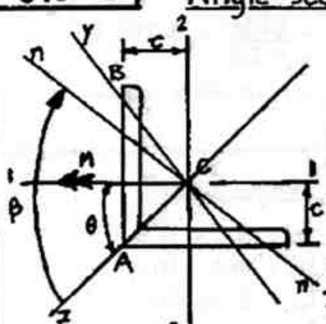
Maximum compressive stress (point B)(Eq. 6-38)

$z_B = c\sqrt{2} - \frac{h}{\sqrt{2}} = -1.160$ in. $\quad y_B = \frac{h}{\sqrt{2}} = 2.828$ in

$$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z}$$

$$= -2920 \text{ psi}$$

6.5-3 Angle section with equal legs



$M = 15.0$ k-in.
L $6 \times 6 \times \frac{3}{4}$ in.
$h = b = 6$ in.
$c = 1.78$ in.
$I_1 = I_2 = 28.2$ in.^{4}
$\theta = 45°$
$A = 8.44$ in.^{2}
$r_{min} = 1.17$ in.
$I_y = A r_{min}^2 = 11.55$ in.^{4}
$I_z = I_1 + I_2 - I_y = 44.85$ in.^{4}

Neutral axis (Eq. 6-40)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{44.85}{11.55}\tan 45° = 3.8831$$

$$\beta = 75.56°$$

Maximum tensile stress (point A)(Eq. 6-38)

$z_A = c\sqrt{2} = 2.517$ in. $\quad y_A = 0$

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$

$$= 2310 \text{ psi}$$

Maximum compressive stress (point B)(Eq. 6-38)

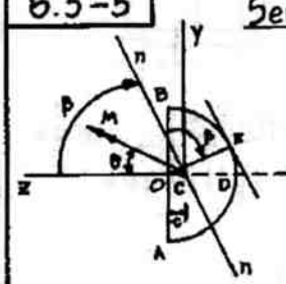
$z_B = c\sqrt{2} - \frac{h}{\sqrt{2}} = -1.725$ in

$y_B = \frac{h}{\sqrt{2}} = 4.243$ in.

$$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z}$$

$$= -2590 \text{ psi}$$

6.5-5 Semicircle



r = radius

$c = \frac{4r}{3\pi} = 0.42441\,r$

$I_y = \frac{(9\pi^2 - 64)\,r^4}{72\pi} = 0.109757\,r^4$

$I_z = \frac{\pi r^4}{8}$

σ_t = maximum tensile stress

σ_c = maximum compressive stress

For $\theta = 0°$: $\sigma_t = \sigma_A = \frac{Mr}{I_z} = \frac{8M}{\pi r^3} = 2.546\frac{M}{r^3}$

$\sigma_c = \sigma_B = -\sigma_A = -\frac{8M}{\pi r^3} = -2.546\frac{M}{r^3}$

For $\theta = 90°$: $\sigma_t = \sigma_O = \frac{Mc}{I_y} = 3.867\frac{M}{r^3}$

$\sigma_c = \sigma_D = \frac{M(r-c)}{I_y} = -5.244\frac{M}{r^3}$

For $\theta = 45°$: Eq. (6-40) : $\tan\beta = \frac{I_z}{I_y}\tan\theta$

$$\tan\beta = \frac{9\pi^2}{9\pi^2 - 64}(1) = 3.577897$$

$\beta = 74.3847° \qquad 90° - \beta = 15.6153°$

CONT.

6.5-5 CONT.

Maximum tensile stress occurs at point A

$z_A = c = 0.42441\, r \qquad y_A = -r$

From (Eq. 6-38):

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$

$$= 4.535\,\frac{M}{r^3} \longleftarrow$$

Maximum compressive stress occurs at point E, where the tangent to the circle is parallel to the neutral axis nn

$z_E = c - r\cos(90° - \beta) = -0.53868\, r$

$y_E = r\sin(90° - \beta) = 0.26918\, r$

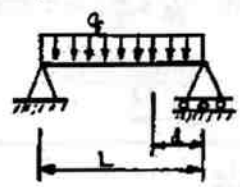
From (Eq. 6-38):

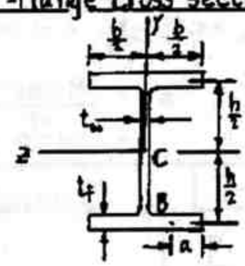
$$\sigma_c = \sigma_E = \frac{(M\sin\theta)z_E}{I_y} - \frac{(M\cos\theta)y_E}{I_z}$$

$$= -3.955\,\frac{M}{r^3} \longleftarrow$$

6.8-1

Simple beam with wide-flange cross section





Simple beam

$q = 3.0$ k/ft $\qquad L = 10$ ft $\qquad R = \frac{qL}{2} = 15.0$ k

$d = 2.0$ ft $\qquad V = |R - qd| = 9.0$ k

Cross section

$h = 10.5$ in. $\qquad b = 7$ in.

$t_f = 0.4$ in. $\qquad t_w = 0.4$ in.

Eq. (6-57): $I_z = \frac{t_w h^3}{12} + \frac{b t_f h^2}{2}$

$= 192.94\ \text{in}^4$

(a) Maximum shear stress (Eq. 6-54)

$$\tau_{max} = \left(\frac{b t_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z}$$

$$= 2360\ \text{psi} \longleftarrow$$

(b) Shear stress at point B

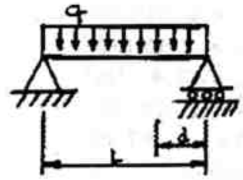
$a = 2.0$ in. $\qquad b/2 = 3.5$ in.

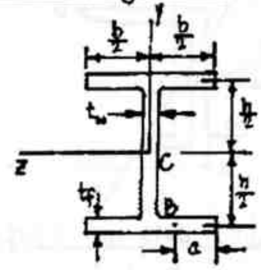
Eq. (6-49): $\tau_1 = \frac{bhV}{4I_z} = 857.1$ psi

$$\tau_B = \frac{a}{b/2}(\tau_1) = 490\ \text{psi} \longleftarrow$$

6.8-2

Simple beam with wide-flange cross section





Simple beam

$q = 40$ kN/m $\qquad L = 3$ m $\qquad R = \frac{qL}{2} = 60$ kN

$d = 0.6$ m $\qquad V = |R - qd| = 36$ kN

Cross section (Figure 6-34)

$h = 260$ mm $\qquad b = 170$ mm

$t_f = 12$ mm $\qquad t_w = 10$ mm

Eq. (6-57): $I_z = \frac{t_w h^3}{12} + \frac{b t_f h^2}{2}$

$= 83.599 \times 10^6\ \text{mm}^4$

(a) Maximum shear stress (Eq. 6-54)

$$\tau_{max} = \left(\frac{b t_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z} = 15.1\ \text{MPa} \longleftarrow$$

(b) Shear stress at point B

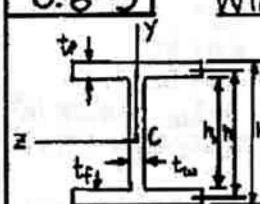
$a = 60$ mm $\qquad b/2 = 85$ mm

Eq. (6-49): $\tau_1 = \frac{bhV}{4I_z} = 4.758$ MPa

$$\tau_B = \frac{a}{b/2}(\tau_1) = 3.4\ \text{MPa} \longleftarrow$$

6.8-3

Wide-flange beam



$b = 5.25$ in.

$h = 7.9$ in.

$t_w = 0.25$ in.

$t_f = 0.4$ in.

$V = 6.0$ k

(a) Calculations based on centerline dimensions (section 6.8)

Moment of inertia (Eq. 6-57):

$$I_z = \frac{t_w h^3}{12} + \frac{b t_f h^2}{2}$$

$= 10.272 + 65.531$

$= 75.803\ \text{in}^4$

Maximum shear stress in the web (Eq. 6-54):

$$\tau_{max} = \left(\frac{b t_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z} = (10.375\ \text{in.})(312.65\ \text{lb/in}^3)$$

$$= 3244\ \text{psi} \longleftarrow$$

CONT.

6.8-3 CONT.

(b) Calculations based on more exact analysis (section 5.10)

See figure 5-38.
Replace h by h_2 and t by t_w.

$h_2 = h + t_f = 8.3$ in.
$h_1 = h - t_f = 7.5$ in.

Moment of inertia (Eq. 5-47):

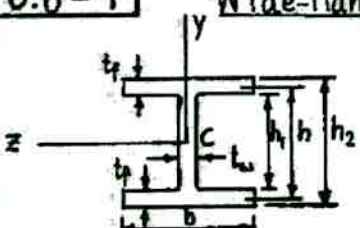
$$I = \frac{1}{12}(bh_2^3 - bh_1^3 + t_w h_1^3)$$
$$= \frac{1}{12}(892.51\ \text{in.}^4) = 74.376\ \text{in.}^4$$

Maximum shear stress in the web (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It_w}(bh_2^2 - bh_1^2 + t_w h_1^2)$$
$$= (40.336\ \text{lb/in.}^5)(80.422\ \text{in.}^3)$$
$$= 3244\ \text{psi} \longleftarrow$$

Note: Within the accuracy of the calculations, the maximum shear stresses are the same.

6.8-4 Wide-flange beam



$b = 145$ mm
$h = 250$ mm
$t_w = 8.0$ mm
$t_f = 14.0$ mm
$V = 30$ kN

(a) Calculations based on centerline dimensions (section 6.8)

Moment of inertia (Eq. 6-57):

$$I_z = \frac{t_w h^3}{12} + \frac{bt_f h^2}{2}$$
$$= 10.417 \times 10^6\ \text{mm}^4 + 63.438 \times 10^6\ \text{mm}^4$$
$$= 73.855 \times 10^6\ \text{mm}^4$$

Maximum shear stress in the web (Eq. 6-54):

$$\tau_{max} = \left(\frac{bt_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z}$$
$$= (316.25\ \text{mm})(0.050775\ \text{N/mm}^3)$$
$$= 16.06\ \text{MPa} \longleftarrow$$

(b) Calculations based on more exact analysis (section 5.10)

See Figure 5-38
Replace h by h_2 and t by t_w

$h_2 = h + t_f = 264$ mm
$h_1 = h - t_f = 236$ mm

Moment of inertia (Eq. 5-47):

$$I = \frac{1}{12}(bh_2^3 - bh_1^3 + t_w h_1^3)$$
$$= \frac{1}{12}(867.20 \times 10^6\ \text{mm}^4)$$
$$= 72.267 \times 10^6\ \text{mm}^4$$

CONT.

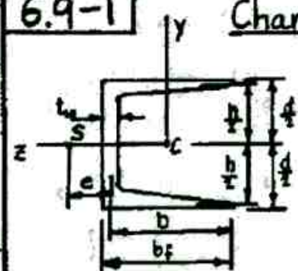
6.8-4 CONT.

Maximum shear stress in the web (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It_w}(bh_2^2 - bh_1^2 + t_w h_1^2)$$
$$= (6.4864 \times 10^{-6}\ \text{N/mm}^5)(2.4756 \times 10^6\ \text{mm}^3)$$
$$= 16.06\ \text{MPa} \longleftarrow$$

Note: Within the accuracy of the calculations the maximum shear stresses are the same.

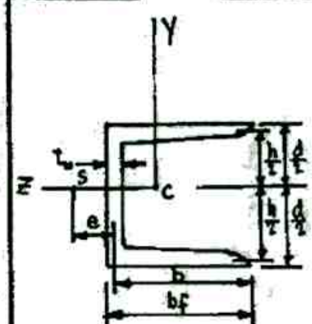
6.9-1 Channel section



C 12 × 20.7
$d = 12.00$ in.
$t_w = 0.282$ in.
$b_f = 2.942$ in.
t_f = average flange thickness
$t_f = 0.501$ in.
$b = b_f - t_w/2$
$= 2.801$ in.
$h = d - t_f$
$= 11.499$ in.

$$\text{Eq. (6-65): } e = \frac{3b^2 t_f}{ht_w + 6bt_f} = 1.011\ \text{in.} \longleftarrow$$

6.9-2 Channel section

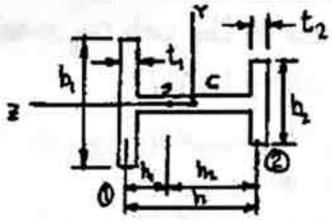


C 8 × 18.75
$d = 8.00$ in.
$t_w = 0.487$ in.
$b_f = 2.527$ in.
t_f = average flange thickness
$t_f = 0.390$ in.
$b = b_f - t_w/2$
$= 2.284$ in.
$h = d - t_f$
$= 7.610$ in.

Eq. (6-65):

$$e = \frac{3b^2 t_f}{ht_w + 6bt_f}$$
$$= 0.674\ \text{in.} \longleftarrow$$

6.9-3 Unbalanced wide-flange beam



Flange ①:

$$\tau_1 = \frac{VQ}{I_z t_1}$$

$$Q = \left(\frac{b_1}{2}\right)(t_1)\left(\frac{b_1}{4}\right) = \frac{t_1 b_1^2}{8}$$

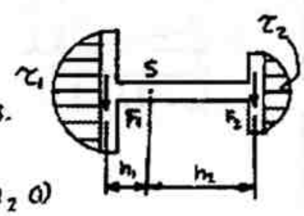
$$\tau_1 = \frac{V b_1^2}{8 I_z}$$

$$F_1 = \frac{2}{3}(\tau_1)(b_1)(t_1) = \frac{V t_1 b_1^3}{12 I_z}$$

Flange ②:

$$F_2 = \frac{V t_2 b_2^3}{12 I_z}$$

Shear force V acts through the shear center S.



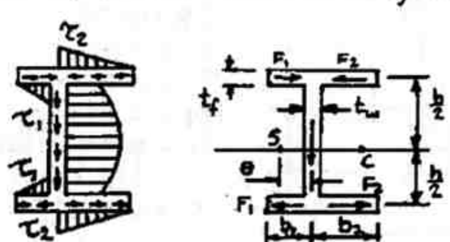
$\therefore \Sigma M_S = F_1 h_1 - F_2 h_2 = 0$

or $(t_1 b_1^3) h_1 = (t_2 b_2^3) h_2$ (1)

$h_1 + h_2 = h$ (2)

Solve Eqs. (1) and (2): $h_1 = \dfrac{t_2 b_2^3 h}{t_1 b_1^3 + t_2 b_2^3}$ ⟵

6.9-4 Unbalanced wide-flange beam



$$\tau_1 = \frac{VQ}{I t_f} = \frac{b_1 h V}{2 I_z} \qquad \tau_2 = \frac{b_2 h V}{2 I_z}$$

$$F_1 = \frac{b_1 \tau_1 t_f}{2} = \frac{b_1^2 h t_f V}{4 I_z} \qquad F_2 = \frac{b_2^2 h t_f V}{4 I_z} \qquad F_3 = V$$

Shear force V acts through the shear center S.

$\therefore \Sigma M_S = -F_3 e - F_1 h + F_2 h = 0$

$$e = \frac{F_2 h - F_1 h}{F_3} \qquad e = \frac{h^2 t_f}{4 I_z}(b_2^2 - b_1^2)$$

$$I_z = \frac{t_w h^3}{12} + 2(b_1 + b_2)(t_f)\left(\frac{h}{2}\right)^2 = \frac{h^2}{12}\left[h t_w + 6 t_f (b_1 + b_2)\right]$$

$$e = \frac{3 t_f (b_2^2 - b_1^2)}{h t_w + 6 t_f (b_1 + b_2)}$$ ⟵

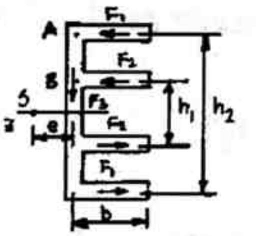
Channel section ($b_1 = 0$, $b_2 = b$)

$e = \dfrac{3 b^2 t_f}{h t_w + 6 b t_f}$ (Eq. 6-65)

Doubly symmetric beam ($b_1 = b_2 = \frac{b}{2}$)

$e = 0$ (shear center coincides with the centroid)

6.9-5 Channel beam with double flanges



t = thickness

$$\tau_A = \frac{VQ_A}{I_z t} = \frac{V(bt)\left(\frac{h_2}{2}\right)}{I_z t} = \frac{b h_2 V}{2 I_z}$$

$$F_1 = \frac{1}{2}\tau_A b t = \frac{b^2 h_2 t V}{4 I_z}$$

$$\tau_B = \frac{b h_1 V}{2 I_z} \qquad F_2 = \frac{b^2 h_1 t V}{4 I_z}$$

$F_3 = V$

Shear force V acts through the shear center S.

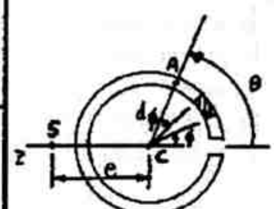
$\therefore \Sigma M_S = -F_3 e + F_1 h_2 + F_2 h_1 = 0$

$$e = \frac{F_2 h_1 + F_1 h_2}{F_3} = \frac{b^2 t}{4 I_z}(h_1^2 + h_2^2)$$

$$I_z = \frac{t h_2^3}{12} + 2\left[bt\left(\frac{h_2}{2}\right)^2 + bt\left(\frac{h_1}{2}\right)^2\right] = \frac{t}{12}\left[h_2^3 + 6b(h_1^2 + h_2^2)\right]$$

$$e = \frac{3b^2(h_1^2 + h_2^2)}{h_2^3 + 6b(h_1^2 + h_2^2)}$$ ⟵

6.9-6 Slit circular tube



$$Q_A = \int y\, dA = \int_0^\theta (r \sin\phi) r t\, d\phi = r^2 t (1 - \cos\theta)$$

r = radius

t = thickness

$$\tau_A = \frac{VQ_A}{I_z t} = \frac{V r^2 (1 - \cos\theta)}{I_z}$$

$$I_z = \pi r^3 t$$

$$\tau_A = \frac{V(1 - \cos\theta)}{\pi r t}$$

At point A: $dA = r t\, d\theta$

T_c = moment of shear stresses about center C

$$T_c = \int \tau_A r\, dA = \int_0^{2\pi} \frac{Vr}{\pi}(1 - \cos\theta)\, d\theta = 2Vr$$

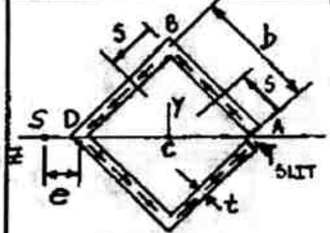
Shear force V acts through the shear center S.

Moment of the shear force V about any point must be equal to the moment of the shear stresses about that same point.

$\therefore \Sigma M_C = Ve = T_c$

$$e = \frac{T_c}{V} = 2r$$ ⟵

6.9-7 Slit square tube

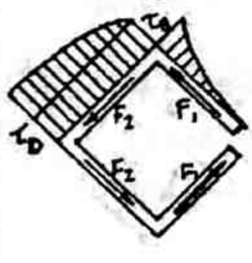


b = length of each side
t = thickness

$\tau = \dfrac{VQ}{I_z t}$

From A to B:

$Q = \dfrac{ts^2}{2\sqrt{2}}$



At A: $Q = 0 \quad \tau_A = 0$

At B: $Q = \dfrac{tb^2}{2\sqrt{2}}$

$\tau_B = \dfrac{b^2 V}{2\sqrt{2}\, I_z}$

$F_1 = \dfrac{\tau_B bt}{3} = \dfrac{b^3 t V}{6\sqrt{2}\, I_z}$

From B to D:

$Q = bt\left(\dfrac{b}{2\sqrt{2}}\right) + st\left(\dfrac{b}{\sqrt{2}} - \dfrac{s}{2\sqrt{2}}\right)$

$= \dfrac{tb^2}{2\sqrt{2}} + \dfrac{ts}{2\sqrt{2}}(2b - s)$

$\tau = \dfrac{VQ}{I_z t} = \dfrac{V}{I_z}\left[\dfrac{b^2}{2\sqrt{2}} + \dfrac{s}{2\sqrt{2}}(2b - s)\right]$

At B: $\tau_B = \dfrac{b^2 V}{2\sqrt{2}\, I_z}$ At D: $\tau_D = \dfrac{b^2 V}{\sqrt{2}\, I_z}$

$F_2 = \tau_B bt + \dfrac{2}{3}(\tau_D - \tau_B)bt = \dfrac{5tb^3 V}{6\sqrt{2}\, I_z}$

Shear force V acts through the shear center S.

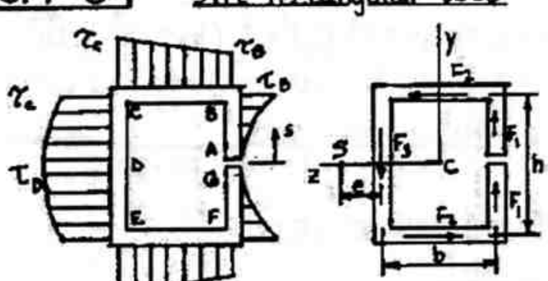
$\therefore \Sigma M_S = 0$

$2\left(\dfrac{F_1}{\sqrt{2}}\right)(b\sqrt{2} + e) + 2\left(\dfrac{F_2}{\sqrt{2}}\right)(e) = 0$

Substitute for F_1 and F_2 and solve for e:

$e = \dfrac{b}{2\sqrt{2}}$ ←

6.9-8 Slit rectangular tube



From A to B: $Q = \dfrac{ts^2}{2}$ t = THICKNESS

$\tau = \dfrac{VQ}{I_z t} = \dfrac{s^2 V}{2 I_z}$

$\tau_A = 0 \qquad \tau_B = \dfrac{h^2 V}{8 I_z}$

$F_1 = \dfrac{\tau_B t}{3}\left(\dfrac{h}{2}\right) = \dfrac{th^3 V}{48 I_z}$

CONT.

6.9-8 CONT.

From B to C: $\tau_B = \dfrac{h^2 V}{8 I_z}$

$Q_C = \dfrac{th}{2}\left(\dfrac{h}{4}\right) + bt\left(\dfrac{h}{2}\right) = \dfrac{th}{8}(h + 4b)$

$\tau_C = \dfrac{h(h + 4b)V}{8 I_z}$

$F_2 = \dfrac{1}{2}(\tau_B + \tau_C)bt = \dfrac{bht(h + 2b)V}{8 I_z}$

$\Sigma F_{VERT} = V \quad F_3 - 2F_1 = V \quad F_3 = V\left(1 + \dfrac{th^3}{24 I_z}\right)$

Shear force V acts through the shear center S.

$\therefore \Sigma M_S = 0 \quad -F_3 e + F_2 h + 2F_1(b + e) = 0$

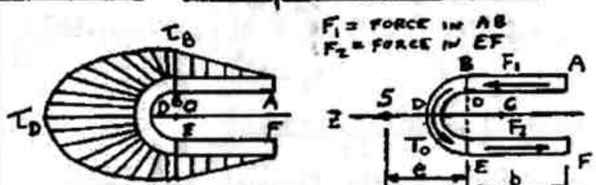
Substitute for F_3, F_2, and F_1 and solve for e:

$e = \dfrac{bh^2 t(2h + 3b)}{12 I_z}$

$I_z = 2\left[\dfrac{1}{12}th^3 + bt\left(\dfrac{h}{2}\right)^2\right] = \dfrac{th^2}{6}(h + 3b)$

$e = \dfrac{b(2h + 3b)}{2(h + 3b)}$ ←

6.9-9 U-shaped cross section



r = radius
t = thickness

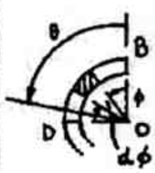
T_0 = MOMENT IN BDE

From A to B: $\tau_A = 0 \quad \tau_B = \dfrac{VQ}{I_z t} = \dfrac{V(btr)}{I_z t} = \dfrac{Vbr}{I_z}$

$F_1 = \dfrac{bt\tau_B}{2} = \dfrac{Vb^2 rt}{2 I_z} \qquad Q_B = btr$

From B to E: $Q_1 = \int y\,dA = \int_0^\theta (r\cos\phi)rt\,d\phi$

$= r^2 t \sin\theta$



$Q_\theta = Q_B + Q_1 = btr + r^2 t\sin\theta$

$\tau_\theta = \dfrac{VQ_\theta}{I_z t} \quad \tau_\theta = \dfrac{Vr(b + r\sin\theta)}{I_z}$

At angle θ: $dA = rt\,d\theta$

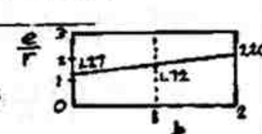
$T_0 = \int \tau_\theta r\,dA = \int_0^\pi \tau_\theta r^2 t\,d\theta$

$= \int_0^\pi \dfrac{Vr^3 t(b + r\sin\theta)d\theta}{I_z}$

$= \dfrac{Vr^3 t}{I_z}(\pi b + 2r)$

Shear force V acts through the shear center S.

Moment of the shear force V about any point must be equal to the moment of the shear stresses about that same point.



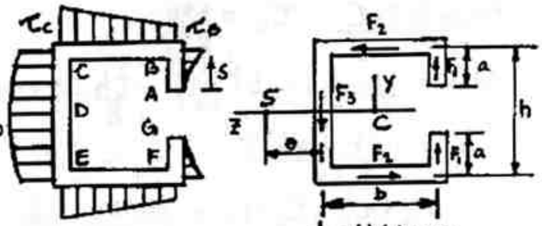
$\therefore \Sigma M_0 = Ve = T_0 + F_1(2r)$

$e = \dfrac{T_0 + 2F_1 r}{V} = \dfrac{r^2 t}{I_z}(\pi br + 2r^2 + b^2)$

$I_z = \dfrac{\pi r^3 t}{2} + 2(btr^2) \qquad \dfrac{e}{r} = \dfrac{2(2 + b^2/r^2 + \pi b/r)}{4b/r + \pi}$

$e = \dfrac{2(2r^2 + b^2 + \pi br)}{4b + \pi r}$ ←

6.9-10 C-section of constant thickness



t = thickness

From A to B:

$Q = st\left(\frac{h}{2} - a + \frac{s}{2}\right) \qquad \tau = \frac{VQ}{I_z t} = s\left(\frac{h}{2} - a + \frac{s}{2}\right)\frac{V}{I_z}$

$\tau_A = 0 \qquad \tau_B = \frac{a}{2}(h-a)\frac{V}{I_z}$

$F_1 = \int_0^a \tau t\, ds = \frac{tV}{I_z}\int_0^a s\left(\frac{h}{2} - a + \frac{s}{2}\right) ds$

$= \frac{a^2 t(3h-4a)V}{12 I_z}$

From B to C:

$\tau_B = \frac{a}{2}(h-a)\frac{V}{I_z} \qquad Q_C = at\left(\frac{h}{2} - \frac{a}{2}\right) + bt\left(\frac{h}{2}\right)$

$= \frac{at}{2}(h-a) + \frac{bht}{2}$

$\tau_C = \left[\frac{a}{2}(h-a) + \frac{bh}{2}\right]\frac{V}{I_z}$

$F_2 = \frac{1}{2}(\tau_B + \tau_C)bt = \frac{bt}{4}\left[2a(h-a) + bh\right]\frac{V}{I_z}$

$\Sigma F_{VERT} = V \qquad F_3 - 2F_1 = V$

$F_3 = V\left[1 + \frac{a^2 t(3h-4a)}{6 I_z}\right]$

Shear force V acts through the shear center S.

$\therefore \Sigma M_S = 0 \qquad -F_3 e + F_2 h + 2F_1(b+e) = 0$

Substitute for F_1, F_2, and F_3 and solve for e:

$e = \frac{bt\left[3h^2(b+2a) - 8a^3\right]}{12 I_z}$

$I_z = 2\left(\frac{1}{12}th^3\right) + 2bt\left(\frac{h}{2}\right)^2 - \frac{t}{12}(h-2a)^3$

$= \frac{t}{12}\left[h^2(h+6b+6a) + 4a^2(2a-3h)\right]$

$e = \frac{3bh^2(b+2a) - 8ba^3}{h^2(h+6b+6a) + 4a^2(2a-3h)}$ ←

Channel section $(a = 0)$

$e = \frac{3b^2}{h+6b}$

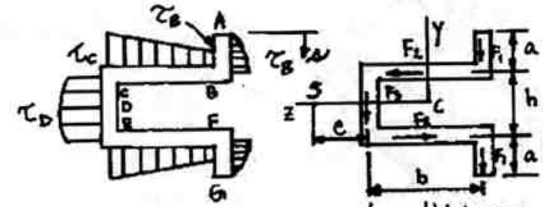
(agrees with Eq. 6-65 when $t_f = t_w$)

Slit rectangular tube $(a = \frac{h}{2})$

$e = \frac{b(2h+3b)}{2(h+3b)}$

(agrees with the result of problem 6.9-8)

6.9-11 Hat section of constant thickness



t = thickness

From A to B:

$Q = st\left(\frac{h}{2} + a - \frac{s}{2}\right) \qquad \tau = \frac{VQ}{I_z t} = s\left(\frac{h}{2} + a - \frac{s}{2}\right)\frac{V}{I_z}$

$\tau_A = 0 \qquad \tau_B = \frac{a}{2}(h+a)\frac{V}{I_z}$

$F_1 = \int_0^a \tau t\, ds = \frac{tV}{I_z}\int_0^a s\left(\frac{h}{2} + a - \frac{s}{2}\right) ds$

$= \frac{a^2 t(3h+4a)V}{12 I_z}$

From B to C:

$\tau_B = \frac{a}{2}(h+a)\frac{V}{I_z}$

$Q_C = at\left(\frac{h}{2} + \frac{a}{2}\right) + bt\left(\frac{h}{2}\right)$

$= \frac{at}{2}(h+a) + \frac{bht}{2}$

$\tau_C = \left[\frac{a}{2}(h+a) + \frac{bh}{2}\right]\frac{V}{I_z}$

$F_2 = \frac{1}{2}(\tau_B + \tau_C)bt = \frac{bt}{4}\left[2a(h+a) + bh\right]\frac{V}{I_z}$

$\Sigma F_{VERT} = V \qquad F_3 + 2F_1 = V$

$F_3 = V\left[1 - \frac{a^2 t(3h+4a)}{6 I_z}\right]$

Shear force V acts through the shear center S.

$\therefore \Sigma M_S = 0 \qquad -F_3 e + F_2 h - 2F_1(b+e) = 0$

Substitute for F_1, F_2, and F_3 and solve for e:

$e = \frac{bt\left[3h^2(b+2a) - 8a^3\right]}{12 I_z}$

$I_z = \frac{1}{12}th^3 + 2bt\left(\frac{h}{2}\right)^2 + \frac{t}{12}(h+2a)^3 - \frac{1}{12}th^3$

$= \frac{t}{12}\left[h^2(h+6b+6a) + 4a^2(2a+3h)\right]$

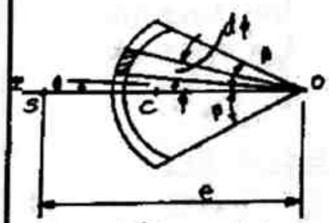
$e = \frac{3bh^2(b+2a) - 8ba^3}{h^2(h+6b+6a) + 4a^2(2a+3h)}$ ←

Channel section $(a = 0)$

$e = \frac{3b^2}{h+6b}$

(agrees with Eq. 6-65 when $t_f = t_w$)

6.9-12 Circular arc



t = thickness

r = radius

At angle θ:

$Q = \int y\,dA$

$= \int_{\theta}^{\beta} (r\sin\phi)\,rt\,d\phi$

$= r^2 t\,(\cos\theta - \cos\beta)$

$$\tau = \frac{VQ}{I_z t} = \frac{Vr^2(\cos\theta - \cos\beta)}{I_z}$$

$$I_z = \int y^2\,dA = \int_{-\beta}^{\beta} (r\sin\phi)^2\,rt\,d\phi$$

$$= r^3 t\,(\beta - \sin\beta\cos\beta)$$

$$\tau = \frac{V(\cos\theta - \cos\beta)}{rt(\beta - \sin\beta\cos\beta)}$$

At angle θ, $dA = rt\,d\theta$ — T_0 = MOMENT OF SHEAR STRESSES

$$T_0 = \int \tau r\,dA = \int_{-\beta}^{\beta} \frac{V(\cos\theta - \cos\beta)}{t(\beta - \sin\beta\cos\beta)}\,rt\,d\theta$$

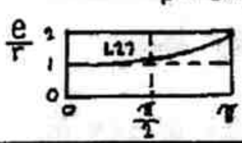
$$= \frac{2Vr(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta}$$

Shear force V acts through the shear center S.

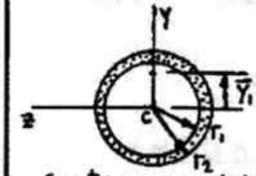
Moment of the shear force V about any point must be equal to the moment of the shear stresses about that same point.

$\therefore \Sigma M_0 = Ve = T_0 \qquad e = \frac{T_0}{V}$

$$e = \frac{2r(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta} \longleftarrow$$



6.10-2 Hollow circular cross section



Neutral axis passes through the centroid C.

Use cases 9 and 10, Appendix D.

Section modulus S

$I_z = \frac{\pi}{4}(r_2^4 - r_1^4) \qquad c = r_2$

$S = \frac{I_z}{c} = \frac{\pi}{4r_2}(r_2^4 - r_1^4)$

Plastic modulus Z (Eq. 6-78)

$A = \pi(r_2^2 - r_1^2)$ For a semicircle, $\bar{y} = \frac{4r}{3\pi}$

$$\bar{y}_1 = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{\left(\frac{4r_2}{3\pi}\right)\left(\frac{\pi r_2^2}{2}\right) - \left(\frac{4r_1}{3\pi}\right)\left(\frac{\pi r_1^2}{2}\right)}{\frac{\pi}{2}(r_2^2 - r_1^2)}$$

$$= \frac{4}{3\pi}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

$\bar{y}_1 = \bar{y}_2 \qquad Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{4}{3}(r_2^3 - r_1^3)$

a) Shape factor f (Eq. 6-79)

$$f = \frac{Z}{S} = \frac{16 r_2(r_2^3 - r_1^3)}{3\pi(r_2^4 - r_1^4)} \longleftarrow$$

b) Thin section

Rewrite the expression for f.

$(r_2^3 - r_1^3) = (r_2 - r_1)(r_2^2 + r_1 r_2 + r_1^2)$

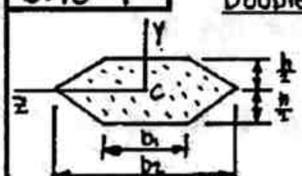
$(r_2^4 - r_1^4) = (r_2 - r_1)(r_2 + r_1)(r_2^2 + r_1^2)$

$$\therefore f = \frac{16 r_2}{3\pi}\left[\frac{r_2^2 + r_1 r_2 + r_1^2}{(r_2 + r_1)(r_2^2 + r_1^2)}\right]$$

$$= \frac{16}{3\pi}\left[\frac{1 + (r_1/r_2) + (r_1/r_2)^2}{(1 + r_1/r_2)(1 + r_1^2/r_2^2)}\right]$$

Let $\frac{r_1}{r_2} \to 1 \quad \therefore f = \frac{4}{\pi} \approx 1.27 \longleftarrow$

6.10-1 Double trapezoid



Neutral axis passes through the centroid C.

Use case 8, Appendix D.

Section modulus S

$c = \frac{h}{2} \qquad I_z = 2\left(\frac{h}{2}\right)^3(3b_1 + b_2)/12$

$S = \frac{I}{c} = \frac{h^2}{24}(3b_1 + b_2) \qquad = \frac{h^3}{48}(3b_1 + b_2)$

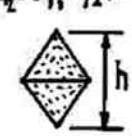
Plastic modulus Z (Eq. 6-78)

$A = 2\left(\frac{h}{2}\right)(b_1 + b_2)/2 = \frac{h}{2}(b_1 + b_2)$

$\bar{y}_1 = \bar{y}_2 = \frac{1}{3}\left(\frac{h}{2}\right)\left(\frac{2b_1 + b_2}{b_1 + b_2}\right) \qquad Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{h^2}{12}(2b_1 + b_2)$

Shape factor f (Eq. 6-79)

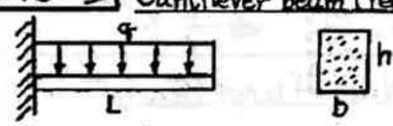
$f = \frac{Z}{S} = \frac{2(2b_1 + b_2)}{3b_1 + b_2} \longleftarrow$



Special case – Rhombus

$b_1 = 0 \qquad f = 2$

6.10-3 Cantilever beam (rectangular cross section)



Maximum bending moment:

$M_{max} = \frac{qL^2}{2}$

Plastic moment: $M_P = \frac{\sigma_Y bh^2}{4}$

$M_{max} = M_P \qquad q = \frac{\sigma_Y bh^2}{2L^2}$

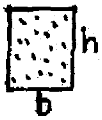
Numerical data:

$\sigma_Y = 40$ ksi $\qquad b = 4.0$ in.

$h = 6.0$ in. $\qquad L = 60$ in.

$\therefore q = 800$ lb/in. $\longleftarrow$

6.10-4 Rectangular cross section



$b = 50$ mm
$h = 80$ mm
$\sigma_y = 210$ MPa

(a) Elastic core ($M = 13.0$ kN·m)

$$M_y = \frac{\sigma_y b h^2}{6} = 11{,}200 \text{ N·m}$$

$$M_p = \frac{\sigma_y b h^2}{4} = 16{,}800 \text{ N·m}$$

M is between M_y and M_p.

Eq. (6-85): $e = h\sqrt{\frac{1}{2}\left(\frac{3}{2} - \frac{M}{M_y}\right)} = 32.950$ mm

Percent of cross-sectional area is

$$\frac{2e}{h}(100) = \frac{65.90(100)}{80}$$

$$= 82.4\,\% \longleftarrow$$

(b) Elastic core ($e = \frac{h}{4} = 20$ mm)

Eq. (6-84): $M = M_y\left(\frac{3}{2} - \frac{2e^2}{h^2}\right)$

$$= 15.4 \text{ kN·m} \longleftarrow$$

6.10-5 Wide-flange beam

$h = 12.0$ in. $b = 6.0$ in.
$t_f = 0.6$ in. $t_w = 0.4$ in.

Section Modulus

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - t_w)(h - 2t_f)^3$$

$$= 276.1 \text{ in.}^4$$

$c = \frac{h}{2} = 6.0$ in. $S = \frac{I}{c} = 46.0$ in.^{3}

Plastic modulus (Eq. 6-86)

$$Z = \frac{1}{4}\left[bh^2 - (b - t_w)(h - 2t_f)^2\right] = 52.7 \text{ in.}^3$$

Shape factor

$$f = \frac{Z}{S} = 1.15 \longleftarrow$$

6.10-6 Wide-flange beam

$h = 400$ mm $b = 160$ mm
$t_f = 12$ mm $t_w = 8$ mm

Section modulus $I = \frac{1}{12}bh^3 - \frac{1}{12}(b - t_w)(h - 2t_f)^3$

$$= 180.0 \times 10^6 \text{ mm}^4$$

$c = \frac{h}{2} = 200$ mm $S = I/c = 900 \times 10^3$ mm^3

Plastic modulus (Eq. 6-86)

$$Z = \frac{1}{4}\left[bh^2 - (b - t_w)(h - 2t_f)^2\right] = 1.028 \times 10^6 \text{ mm}^3$$

Shape factor

$$f = \frac{Z}{S} = 1.14 \longleftarrow$$

6.10-7 Wide-flange beam

W 12 × 87

$h = 12.53$ in. $b = 12.125$ in.
$t_f = 0.810$ in. $t_w = 0.515$ in.
$S = 118$ in.^{3}

Plastic modulus (Eq. 6-86)

$$Z = \frac{1}{4}\left[bh^2 - (b - t_w)(h - 2t_f)^2\right]$$

$$= 130.4 \text{ in.}^3 \longleftarrow$$

Shape factor

$$f = \frac{Z}{S} = 1.11 \longleftarrow$$

6.10-8 Wide-flange beam

W10 × 60

$h = 10.22$ in. $b = 10.080$ in.
$t_f = 0.680$ in. $t_w = 0.420$ in.
$S = 66.7$ in.^{3}

Plastic modulus (Eq. 6-86)

$$Z = \frac{1}{4}\left[bh^2 - (b - t_w)(b - 2t_f)^2\right]$$

$$= 73.63 \text{ in.}^3 \longleftarrow$$

Shape factor

$$f = \frac{Z}{S} = 1.10 \longleftarrow$$

6.10-9 Wide-flange beam

W 16 × 77

$h = 16.52$ in. $b = 10.295$ in.
$t_f = 0.760$ in. $t_w = 0.455$ in.
$\sigma_y = 36$ ksi $S = 134$ in.^{3}

Yield moment

$$M_y = \sigma_y S = 4820 \text{ k-in.} \longleftarrow$$

Plastic modulus (Eq. 6-86)

$$Z = \frac{1}{4}\left[bh^2 - (b - t_w)(h - 2t_f)^2\right]$$

$$= 148.9 \text{ in.}^3$$

Plastic moment

$$M_p = \sigma_y Z = 5360 \text{ k-in.} \longleftarrow$$

6.10-10 Wide-flange beam

W 8×21

$h = 8.28$ in. $b = 5.270$ in.

$t_f = 0.400$ in. $t_w = 0.250$ in.

$\sigma_Y = 36$ ksi $S = 18.2\ \text{in.}^3$

Yield moment

$M_Y = \sigma_Y S = 655$ k-in. ←

Plastic modulus (Eq. 6-86)

$Z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2]$

$= 20.11\ \text{in.}^3$

Plastic moment

$M_P = \sigma_Y Z = 724$ k-in. ←

6.10-11 Hollow box beam

$h = 16$ in. $b = 8$ in.

$t = 0.75$ in. $\sigma_Y = 32$ ksi

Section modulus

$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-2t)(h-2t)^3$

$= 1079\ \text{in.}^4$

$c = \frac{h}{2} = 8.0$ in. $S = \frac{I}{c} = 134.9\ \text{in.}^3$

Yield moment

$M_Y = \sigma_Y S = 4320$ k-in. ←

Plastic modulus

$Z = \frac{1}{4}[bh^2 - (b-2t)(h-2t)^2]$

$= 170.3\ \text{in.}^3$

Plastic moment

$M_P = \sigma_Y Z = 5450$ k-in. ←

6.10-12 Hollow box beam

$h = 0.4$ m $b = 0.2$ m

$t = 20$ mm $\sigma_Y = 230$ MPa

Section modulus

$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-2t)(h-2t)^3$

$= 444.6 \times 10^6\ \text{mm}^4$

$c = \frac{h}{2} = 200$ mm $S = \frac{I}{c} = 2.223 \times 10^6\ \text{mm}^3$

Yield moment

$M_Y = \sigma_Y S = 511$ kN·m ←

Plastic modulus

$Z = \frac{1}{4}[bh^2 - (b-2t)(h-2t)^2] = 2.816 \times 10^6\ \text{mm}^3$

Plastic moment

$M_P = \sigma_Y Z = 648$ kN·m ←

6.10-13 Hollow box beam

$h = 9.0$ in. $b = 5.0$ in.

$h_1 = 7.5$ in. $b_1 = 4.0$ in.

$\sigma_Y = 33$ ksi

Section modulus

$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 163.12\ \text{in.}^4$

$c = \frac{h}{2} = 4.5$ in.

$S = \frac{I}{c} = 36.25\ \text{in.}^3$

Yield moment

$M_Y = \sigma_Y S = 1196$ k-in. ←

Plastic modulus

$Z = \frac{1}{4}(bh^2 - b_1h_1^2) = 45.0\ \text{in.}^3$

Plastic moment

$M_P = \sigma_Y Z = 1485$ k-in. ←

6.10-14 Hollow box beam

$h = 200$ mm $b = 150$ mm

$h_1 = 160$ mm $b_1 = 130$ mm

$\sigma_Y = 220$ MPa

Section modulus

$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 55.63 \times 10^6\ \text{mm}^4$

$c = \frac{h}{2} = 100$ mm

$S = \frac{I}{c} = 556.3 \times 10^3\ \text{mm}^3$

Yield moment

$M_Y = \sigma_Y S = 122$ kN·m ←

Plastic Modulus

$Z = \frac{1}{4}(bh^2 - b_1h_1^2)$

$= 668.0 \times 10^3\ \text{mm}^3$

Plastic moment

$M_P = \sigma_Y Z$

$= 147$ kN·m ←

6.10-15 Hollow box beam

$h = 14$ in. $b = 8$ in.

$h_1 = 12.5$ in. $b_1 = 7$ in.

$\sigma_Y = 42$ ksi

(See figure 6-47, Example 6-9)

CONT.

6.10-15 CONT.

Elastic core

$S_1 = \frac{1}{6}(b-b_1)h_1^2 = 26.04 \text{ in}^3$

$M_1 = \sigma_Y S_1 = 1094 \text{ k-in.}$

Plastic flanges

F = force in one flange

$F = \sigma_Y b(\frac{1}{2})(h-h_1) = 252.0 \text{ k}$

$M_2 = F\left(\frac{h+h_1}{2}\right) = 3339 \text{ k-in.}$

a) Bending moment

$M = M_1 + M_2 = 4430 \text{ k-in.}$ ←

b) Percent due to elastic core

Percent $= \frac{M_1}{M}(100) = 25\%$ ←

6.10-16 Hollow box beam

$h = 400$ mm $b = 200$ mm

$h_1 = 360$ mm $b_1 = 160$ mm

$\sigma_Y = 220$ MPa

(See Figure 6-47, Example 6-9)

Elastic core

$S_1 = \frac{1}{6}(b-b_1)h_1^2 = 864 \times 10^3 \text{ mm}^3$

$M_1 = \sigma_Y S_1 = 190.1 \text{ kN·m}$

Plastic flanges

F = force in one flange

$F = \sigma_Y b(\frac{1}{2})(h-h_1) = 880.0 \text{ kN}$

$M_2 = F\left(\frac{h+h_1}{2}\right) = 334.4 \text{ kN·m}$

(a) Bending moment

$M = M_1 + M_2 = 524 \text{ kN·m}$ ←

(b) Percent due to elastic core

Percent $= \frac{M_1}{M}(100) = 36\%$ ←

6.10-17 Wide-flange beam

W 12 × 50

$h = 12.19$ in. $b = 8.080$ in.

$t_f = 0.640$ in. $t_w = 0.370$ in.

$\sigma_Y = 36$ ksi

Elastic core

$S_1 = \frac{1}{6} t_w (h - 2t_f)^2 = 7.340 \text{ in}^3$

$M_1 = \sigma_Y S_1 = 264.2 \text{ k-in.}$

CONT.

6.10-17 CONT.

Plastic flanges

F = force in one flange

$F = \sigma_Y b t_f = 186.2 \text{ k}$

$M_2 = F(h - t_f) = 2151 \text{ k-in.}$

(a) Bending moment

$M = M_1 + M_2 = 2410 \text{ k-in.}$ ←

(b) Percent due to elastic core

Percent $= \frac{M_1}{M}(100) = 11\%$ ←

6.10-18 Wide-flange beam

$h = 210$ mm $b = 130$ mm

$t_f = 10$ mm $t_w = 6.5$ mm

$\sigma_Y = 200$ MPa

Elastic core

$S_1 = \frac{1}{6} t_w (h - 2t_f)^2 = 39{,}110 \text{ mm}^3$

$M_1 = \sigma_Y S_1 = 7.822 \text{ kN·m}$

Plastic flanges

F = force in one flange

$F = \sigma_Y b t_f = 260.0 \text{ kN}$

$M_2 = F(h - t_f) = 52.0 \text{ kN·m}$

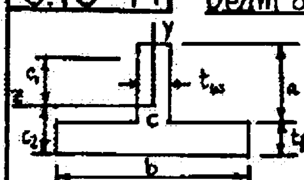
(a) Bending moment

$M = M_1 + M_2 = 59.8 \text{ kN·m}$ ←

(b) Percent due to elastic core

Percent $= \frac{M_1}{M}(100) = 13\%$ ←

6.10-19 Beam of T-section



$b = 6$ in.

$a = 8$ in.

$t_w = 1.5$ in.

$t_f = 1.5$ in.

Elastic bending

$$c_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{(\frac{t_f}{2})(bt_f) + (\frac{a}{2} + t_f)(at_w)}{bt_f + at_w}$$

$= 3.464$ in.

$c_1 = a + t_f - c_2 = 6.036$ in.

$I_z = \frac{1}{3} t_w c_1^3 + \frac{1}{3} b c_2^3 - \frac{1}{3}(b - t_w)(c_2 - t_f)^3$

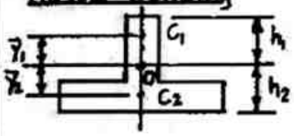
$= 181.72 \text{ in}^4$

$S = \frac{I}{c_1} = 30.11 \text{ in}^3$

CONT.

6.10-19 CONT.

Plastic bending



$A = bt_f + at_w$

$= 21.0 \text{ in.}^2$

$h_1 t_w = \frac{A}{2}$

$h_1 = 7.00 \text{ in.}$

$h_2 = a + t_f - h_1 = 2.5 \text{ in.}$

$\bar{y}_1 = \frac{h_1}{2} = 3.50 \text{ in.}$

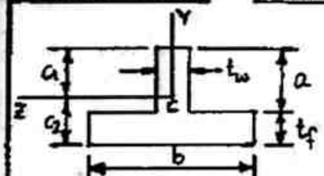
$$\bar{y}_2 = \frac{\Sigma y_i A_i}{\frac{A}{2}} = \frac{\frac{1}{2}bh_2^2 - \frac{1}{2}(b-t_w)(h_2-t_f)^2}{A/2}$$

$= 1.571 \text{ in.}$

$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = 53.25 \text{ in.}^3$ ←

$f = \frac{Z}{S} = 1.77$ ←

6.10-20 Beam of T-section



$b = 140 \text{ mm}$

$a = 200 \text{ mm}$

$t_w = 20 \text{ mm}$

$t_f = 25 \text{ mm}$

Elastic bending

$$c_2 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{\left(\frac{t_f}{2}\right)(bt_f) + \left(\frac{a}{2} + t_f\right)(at_w)}{bt_f + at_w}$$

$= 72.50 \text{ mm}$

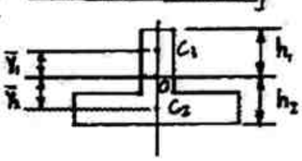
$c_1 = a + t_f - c_2 = 152.50 \text{ mm}$

$I_z = \frac{1}{3}t_w c_1^3 + \frac{1}{3}bc_2^3 - \frac{1}{3}(b-t_w)(c_2-t_f)^3$

$= 37.14 \times 10^6 \text{ mm}^4$

$S = \frac{I}{c_1} = 243.5 \times 10^3 \text{ mm}^3$

Plastic bending



$A = bt_f + at_w = 7500 \text{ mm}^2$

$h_1 t_w = \frac{A}{2}$

$h_1 = 187.5 \text{ mm}$

$h_2 = a + t_f - h_1 = 37.5 \text{ mm}$

$\bar{y}_1 = \frac{h_1}{2} = 93.75 \text{ mm}$

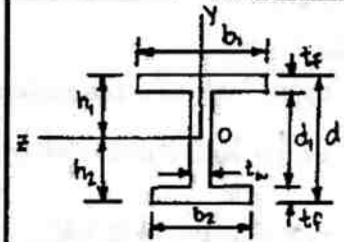
$$\bar{y}_2 = \frac{\Sigma y_i A_i}{A/2} = \frac{\frac{1}{2}bh_2^2 - \frac{1}{2}(b-t_w)(h_2-t_f)^2}{A/2}$$

$= 23.75 \text{ mm}$

$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = 440.6 \times 10^3 \text{ mm}^3$ ←

$f = \frac{Z}{S} = 1.81$ ←

6.10-21 Unbalanced wide-flange beam



$\sigma_Y = 36 \text{ ksi}$

$b_1 = 10 \text{ in.}$

$b_2 = 5 \text{ in.}$

$t_w = 0.5 \text{ in.}$

$d = 8 \text{ in.}$

$d_1 = 7 \text{ in.}$

$t_f = 0.5 \text{ in.}$

$A = b_1 t_f + b_2 t_f + d_1 t_w$

$= 11.0 \text{ in.}^2$

Neutral axis under fully plastic conditions

$\frac{A}{2} = h_1 t_w + (b_1 - t_w)t_f$

from which we get $h_1 = 1.50 \text{ in.}$

$h_2 = d - h_1 = 6.50 \text{ in.}$

Plastic modulus

$$\bar{y}_1 = \frac{\Sigma y_i A_i}{A/2} = \frac{(h_1/2)(t_w)(h_1) + (h_1 - t_f/2)(b_1 - t_w)(t_f)}{A/2}$$

$= 1.182 \text{ in.}$

$$\bar{y}_2 = \frac{\Sigma y_i A_i}{A/2} = \frac{(h_2/2)(t_w)(h_2) + (h_2 - t_f/2)(b_2 - t_w)(t_f)}{A/2}$$

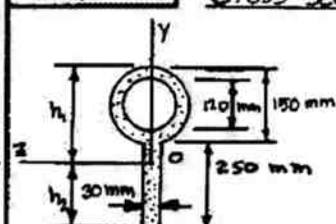
$= 4.477 \text{ in.}$

$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = 31.12 \text{ in.}^3$

Plastic moment

$M_P = \sigma_Y Z = 1120 \text{ k-in.}$ ←

6.10-22 Cross section of beam



$\sigma_Y = 210 \text{ MPa}$

$d_2 = 150 \text{ mm}$

$d_1 = 120 \text{ mm}$

Neutral axis for fully plastic conditions

Cross section is divided into two equal areas.

$A = \frac{\pi}{4}\left[(150 \text{ mm})^2 - (120 \text{ mm})^2\right] + (250 \text{ mm})(30 \text{ mm})$

$= 13{,}862 \text{ mm}^2$

$\frac{A}{2} = 6931 \text{ mm}^2$

$(h_2)(30 \text{ mm}) = \frac{A}{2} = 6931 \text{ mm}^2$

$h_2 = 231.0 \text{ mm}$

$h_1 = 150 \text{ mm} + 250 \text{ mm} - h_2$

$= 169.0 \text{ mm}$

CONT.

6.10-22 CONT.

Plastic modulus

$\bar{y}_1 = \frac{\Sigma y_i A_i}{A/2}$ for upper half of cross section

$\bar{y}_2 = \frac{\Sigma y_i A_i}{A/2}$ for lower half of cross section

$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = (\Sigma y_i A_i)_{upper} + (\Sigma y_i A_i)_{lower}$

(Dimensions are in millimeters)

$$Z = (h_1 - 75)\left(\frac{\pi}{4}\right)(d_2^2 - d_1^2) + \left[\left(\frac{h_1 - 150}{2}\right)(30) \cdot (h_1 - 150)\right] + \left(\frac{h_2}{2}\right)(30)(h_2)$$

$$= 598{,}000 + 5{,}400 + 800{,}400$$

$$= 1404 \times 10^3 \text{ mm}^3$$

Plastic moment

$M_p = \sigma_p Z$

$= (210 \text{ MPa})(1404 \times 10^3 \text{ mm}^3)$

$= 295 \text{ kN·m}$ ←

– END OF CHAPTER 6 –

7.2-1

PLANE STRESS (ANGLE θ)

$\sigma_x = 6800$ psi $\quad \sigma_y = 2000$ psi

$\tau_{xy} = 2750$ psi $\quad \theta = 60°$

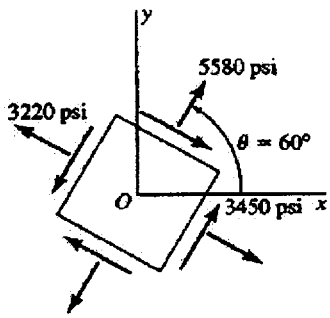
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= 5580$ psi ←

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= -3450$ psi ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 3220$ psi ←



7.2-2

PLANE STRESS (ANGLE θ)

$\sigma_x = 74$ MPa $\quad \sigma_y = 46$ MPa

$\tau_{xy} = 48$ MPa $\quad \theta = 25°$

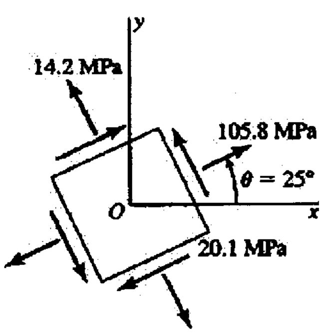
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= 105.8$ MPa ←

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= 20.1$ MPa ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 14.2$ MPa ←



7.2-3

PLANE STRESS (ANGLE θ)

$\sigma_x = -11{,}100$ psi $\quad \sigma_y = -4600$ psi

$\tau_{xy} = 3600$ psi $\quad \theta = 50°$

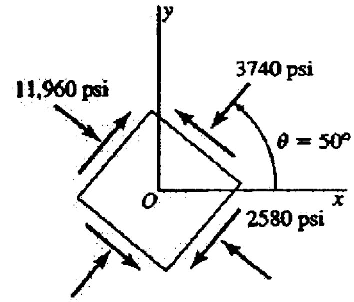
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= -3740$ psi ←

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= 2580$ psi ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -11{,}960$ psi ←



7.2-4

PLANE STRESS (ANGLE θ)

$\sigma_x = 52$ MPa $\quad \sigma_y = -130$ MPa

$\tau_{xy} = -60$ MPa $\quad \theta = 48°$

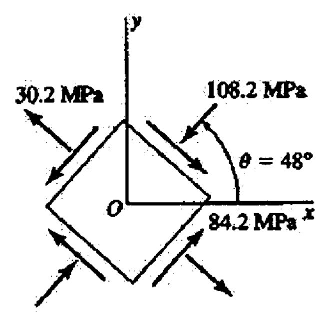
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= -108.2$ MPa ←

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= -84.2$ MPa ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 30.2$ MPa ←



7.2-5

PLANE STRESS (ANGLE θ)

$\sigma_x = 8300$ psi $\quad \sigma_y = -19{,}700$ psi

$\tau_{xy} = -4800$ psi $\quad \theta = 30°$

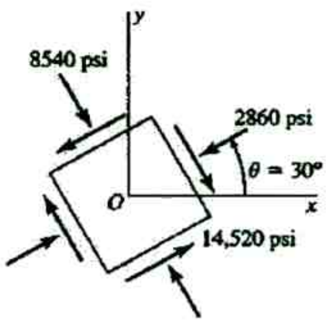
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -2860 \text{ psi} \quad \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -14{,}520 \text{ psi} \quad \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -8540 \text{ psi} \quad \longleftarrow$$



7.2-7

PLANE STRESS (ANGLE θ)

$\sigma_x = -9000$ psi $\quad \sigma_y = -1000$ psi

$\tau_{xy} = -4200$ psi $\quad \theta = 41°$

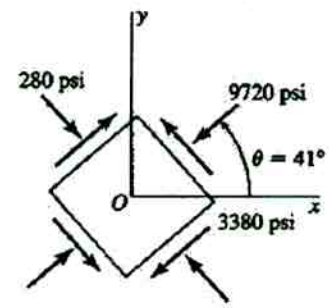
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -9720 \text{ psi} \quad \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= 3380 \text{ psi} \quad \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -280 \text{ psi} \quad \longleftarrow$$



7.2-6

PLANE STRESS (ANGLE θ)

$\sigma_x = -26.5$ MPa $\quad \sigma_y = 5.5$ MPa

$\tau_{xy} = -12.0$ MPa $\quad \theta = -40°$

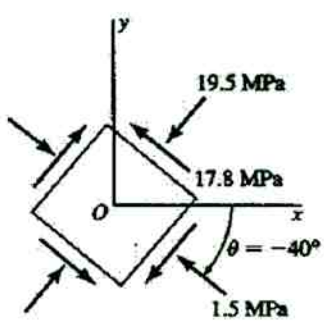
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -1.5 \text{ MPa} \quad \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -17.8 \text{ MPa} \quad \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -19.5 \text{ MPa} \quad \longleftarrow$$



7.2-8

PLANE STRESS (ANGLE θ)

$\sigma_x = -50$ MPa $\quad \sigma_y = -8$ MPa

$\tau_{xy} = 20$ MPa $\quad \theta = -42.5°$

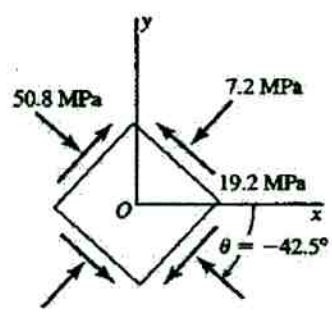
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -50.8 \text{ MPa} \quad \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -19.2 \text{ MPa} \quad \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -7.2 \text{ MPa} \quad \longleftarrow$$



7.2-9

PLANE STRESS (ANGLE θ)

$\sigma_x = 330$ psi $\qquad \sigma_y = 92$ psi

$\tau_{xy} = -120$ psi $\qquad \theta = 30°$

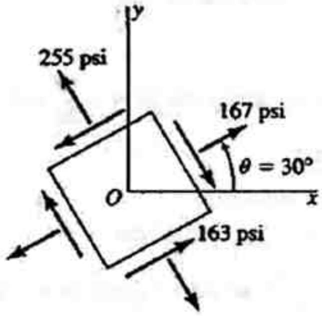
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= 167$ psi ←

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= -163$ psi ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 255$ psi ←



Normal stress on seam equals 167 psi tension. ←

Shear stress on seam equals 163 psi, acting clockwise against the seam. ←

7.2-10

PLANE STRESS (ANGLE θ)

$\sigma_x = 2100$ kPa $\qquad \sigma_y = 300$ kPa

$\tau_{xy} = -560$ kPa $\qquad \theta = 22.5°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= 1440$ kPa ←

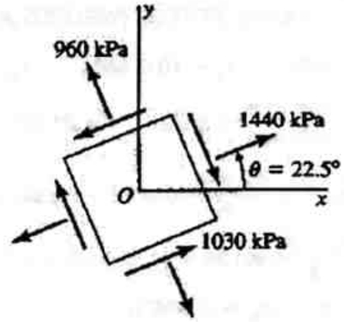
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= -1030$ kPa ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 960$ kPa ←

CONT.

7.2-10 (CONT.)



Normal stress on seam equals 1440 kPa tension. ←

Shear stress on seam equals 1030 kPa, acting clockwise against the seam. ←

7.2-11

BIAXIAL STRESS (WELDED JOINT)

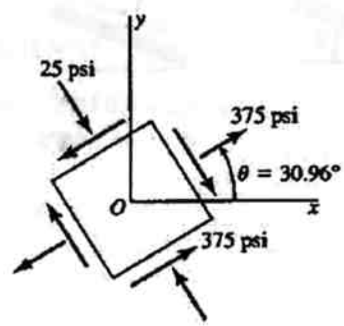
$\sigma_x = 600$ psi $\qquad \sigma_y = -250$ psi $\qquad \tau_{xy} = 0$

$$\theta = \arctan\frac{3 \text{ in.}}{5 \text{ in.}} = \arctan 0.6 = 30.96°$$

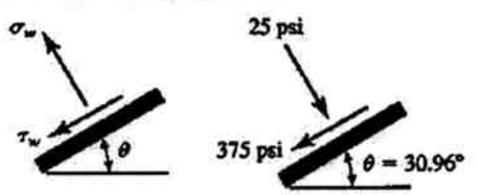
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = 375 \text{ psi}$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -375 \text{ psi}$$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -25$ psi



Stresses acting on the weld



$\sigma_w = -25$ psi ←

$\tau_w = 375$ psi ←

7.2-12

BIAXIAL STRESS (WELDED JOINT)

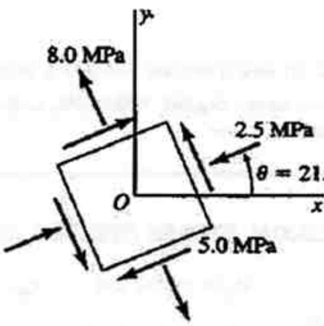
$\sigma_x = -4.5$ MPa $\quad \sigma_y = 10.0$ MPa $\quad \tau_{xy} = 0$

$$\theta = \arctan \frac{100 \text{ mm}}{250 \text{ mm}} = \arctan 0.4 = 21.80°$$

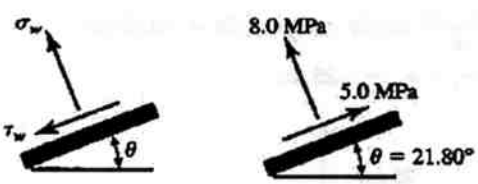
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta = -2.5 \text{ MPa}$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = 5.0 \text{ MPa}$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 8.0 \text{ MPa}$$



Stresses acting on the weld



$\sigma_w = 8.0$ MPa $\leftarrow$

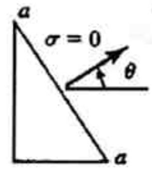
$\tau_w = -5.0$ MPa $\leftarrow$

7.2-13

BIAXIAL STRESS

$\sigma_x = 3600$ psi $\quad \sigma_y = -1600$ psi $\quad \tau_{xy} = 0$

Find angles θ for $\sigma = 0$.



$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= 1000 + 2600 \cos 2\theta \text{ (psi)}$$

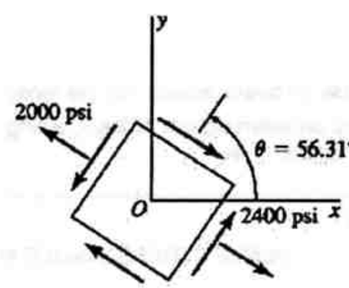
For $\sigma_{x_1} = 0$, we obtain $\cos 2\theta = -\frac{10}{26}$ and $\theta = \pm 56.31° \leftarrow$

Element No. 1

$\sigma_{x_1} = 0 \qquad \theta = 56.31°$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 2000$ psi $\leftarrow$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -2400 \text{ psi} \quad \leftarrow$$

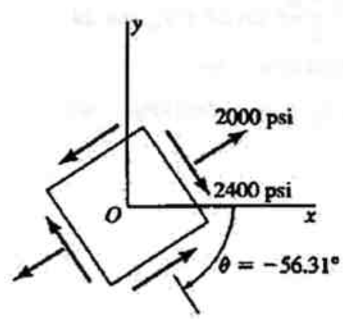


Element No. 2

$\sigma_{x_1} = 0 \qquad \theta = -56.31°$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 2000$ psi $\leftarrow$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = 2400 \text{ psi} \quad \leftarrow$$

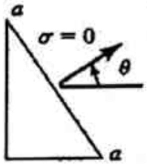


7.2-14

BIAXIAL STRESS

$\sigma_x = 32$ MPa $\qquad \sigma_y = -50$ MPa $\qquad \tau_{xy} = 0$

Find angles θ for $\sigma = 0$.



$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -9 + 41\cos 2\theta \text{ (MPa)}$$

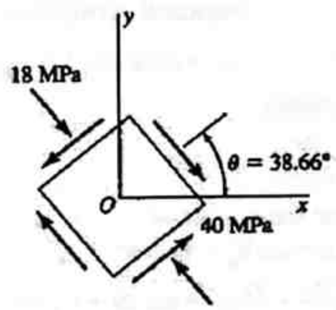
For $\sigma_{x_1} = 0$, we obtain $\cos 2\theta = \dfrac{9}{41}$ and $\theta = \pm 38.66°$ $\longleftarrow$

Element No. 1

$\sigma_{x_1} = 0 \qquad \theta = 38.66°$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -18$ MPa $\longleftarrow$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -40 \text{ MPa} \quad \longleftarrow$$

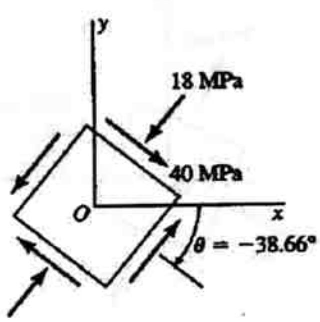


Element No. 2

$\sigma_{x_1} = 0 \qquad \theta = -38.66°$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -18$ MPa $\longleftarrow$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = 40 \text{ MPa} \quad \longleftarrow$$



7.2-15

PLANE STRESS

Transform from $\theta = 30°$ to $\theta = 0°$.

Let: $\sigma_x = -15{,}410$ psi, $\sigma_y = -4450$ psi, $\tau_{xy} = 2470$ psi, and $\theta = -30°$.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -14{,}810 \text{ psi}$$

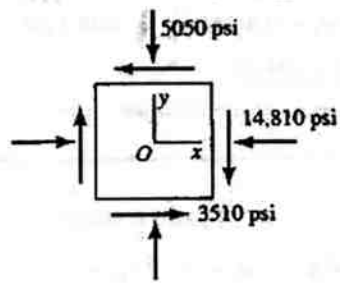
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -3510 \text{ psi}$$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -5050$ psi

For $\theta = 0$:

$\sigma_x = \sigma_{x_1} = -14{,}810$ psi $\longleftarrow$

$\sigma_y = \sigma_{y_1} = -5050$ psi $\longleftarrow \qquad \tau_{xy} = \tau_{x_1y_1} = -3510$ psi $\longleftarrow$



7.2-16

PLANE STRESS

Transform from $\theta = 60°$ to $\theta = 0°$.

Let: $\sigma_x = -21.3$ MPa, $\sigma_y = 64.7$ MPa, $\tau_{xy} = -24.5$ MPa, and $\theta = -60°$.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = 64.4 \text{ MPa}$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -25.0 \text{ MPa}$$

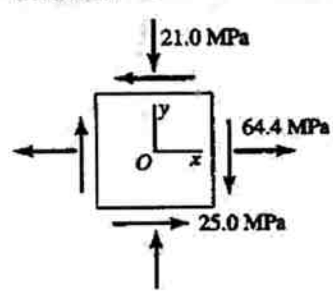
$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -21.0$ MPa

For $\theta = 0$:

$\sigma_x = \sigma_{x_1} = 64.4$ MPa $\longleftarrow$

$\sigma_y = \sigma_{y_1} = -21.0$ MPa $\longleftarrow$

$\tau_{xy} = \tau_{x_1y_1} = -25.0$ MPa $\longleftarrow$



7.2-17

PLANE STRESS

$\sigma_x = 2000$ psi $\quad \sigma_y = ? \quad \tau_{xy} = ?$

At $\theta = 40°$ and $\theta = 80°$: $\sigma_{x_1} = 5000$ psi

Find σ_y and τ_{xy}.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $\theta = 40°$:

$$\sigma_{x_1} = 5000 = \frac{2000 + \sigma_y}{2} + \frac{2000 - \sigma_y}{2}\cos 80° + \tau_{xy}\sin 80°$$

or $0.41318\sigma_y + 0.98481\tau_{xy} = 3826.4$ psi (1)

For $\theta = 80°$:

$$\sigma_{x_1} = 5000 = \frac{2000 + \sigma_y}{2} + \frac{2000 - \sigma_y}{2}\cos 160° + \tau_{xy}\sin 160°$$

or $0.96985\sigma_y + 0.34202\tau_{xy} = 4939.7$ psi (2)

Solve Eqs. (1) and (2):

$\sigma_y = 4370$ psi $\quad \tau_{xy} = 2050$ psi $\leftarrow$

7.2-18

PLANE STRESS

$\sigma_x = 100$ MPa $\quad \sigma_y = ? \quad \tau_{xy} = ?$

At $\theta = 30°$, $\sigma_{x_1} = 35$ MPa

At $\theta = 50°$, $\sigma_{x_1} = -10$ MPa

Find σ_y and τ_{xy}.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $\theta = 30°$:

$$\sigma_{x_1} = 35 = \frac{100 + \sigma_y}{2} + \frac{100 - \sigma_y}{2}\cos 60° + \tau_{xy}\sin 60°$$

or $0.25\sigma_y + 0.86603\tau_{xy} = -40$ MPa (1)

For $\theta = 50°$:

$$\sigma_{x_1} = -10 = \frac{100 + \sigma_y}{2} + \frac{100 - \sigma_y}{2}\cos 100° + \tau_{xy}\sin 100°$$

or $0.58682\sigma_y + 0.98481\tau_{xy} = -51.318$ MPa (2)

Solve Eqs. (1) and (2):

$\sigma_y = -19.3$ MPa $\quad \tau_{xy} = -40.6$ MPa $\leftarrow$

7.2-19

PLANE STRESS

$\sigma_x = -4000$ psi $\quad \sigma_y = 2500$ psi $\quad \tau_{xy} = 2800$ psi

For $\theta = \theta_1$:

$\sigma_{x_1} = 2000$ psi $\quad \sigma_{y_1} = \sigma_b \quad \tau_{xy} = \tau_b$

Find σ_b, τ_b, and θ_1

Stress σ_b

$\sigma_b = \sigma_x + \sigma_y - 2000$ psi $= -3500$ psi $\leftarrow$

Angle θ_1

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

2000 psi $= -750 - 3250\cos 2\theta_1 + 2800\sin 2\theta_1$

or $-65\cos 2\theta_1 + 56\sin 2\theta_1 - 55 = 0$

Solve numerically:

$2\theta_1 = 89.12°$ $\quad \theta_1 = 44.56°$ $\leftarrow$

Shear stress τ_b

$$\tau_b = \tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta_1 + \tau_{xy}\cos 2\theta_1 = 3290 \text{ psi} \leftarrow$$

7.3-1

PRINCIPAL STRESSES

$\sigma_x = 6800$ psi $\quad \sigma_y = 2000$ psi $\quad \tau_{xy} = 2750$ psi

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.146$$

$2\theta_p = 48.89°$ and $\theta_p = 24.44°$

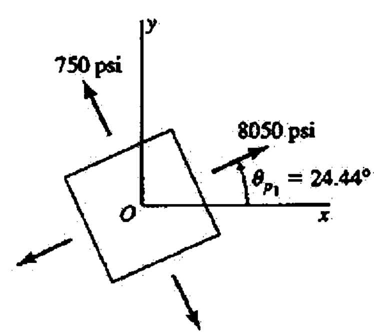
$2\theta_p = 228.89°$ and $\theta_p = 114.44°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 48.89°$: $\sigma_{x_1} = 8050$ psi

For $2\theta_p = 228.89°$: $\sigma_{x_1} = 750$ psi

$$\left.\begin{array}{l}\text{Therefore, } \sigma_1 = 8050 \text{ psi and } \theta_{p_1} = 24.44° \\ \sigma_2 = 750 \text{ psi and } \theta_{p_2} = 114.44°\end{array}\right\} \leftarrow$$



7.3-2

PRINCIPAL STRESSES

$\sigma_x = 74$ MPa $\quad \sigma_y = 46$ MPa $\quad \tau_{xy} = 48$ MPa

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 3.429$$

$2\theta_p = 73.74°$ and $\theta_p = 36.87°$

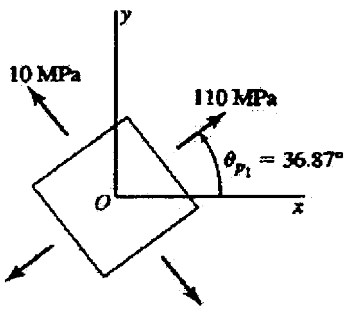
$2\theta_p = 253.74°$ and $\theta_p = 126.87°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 73.74°$: $\sigma_{x_1} = 110$ MPa

For $2\theta_p = 253.74°$: $\sigma_{x_1} = 10$ MPa

$$\text{Therefore, } \left.\begin{array}{l}\sigma_1 = 110 \text{ MPa and } \theta_{p_1} = 36.87° \\ \sigma_2 = 10 \text{ MPa and } \theta_{p_2} = 126.87°\end{array}\right\} \longleftarrow$$



7.3-3

PRINCIPAL STRESSES

$\sigma_x = -11{,}100$ psi $\quad \sigma_y = -4600$ psi $\quad \tau_{xy} = 3800$ psi

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.1077$$

$2\theta_p = -47.92°$ and $\theta_p = -23.96°$

$2\theta_p = 132.08°$ and $\theta_p = 66.04°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

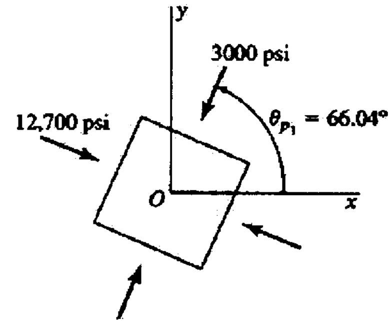
For $2\theta_p = -47.92°$: $\sigma_{x_1} = -12{,}700$ psi

For $2\theta_p = 132.08°$: $\sigma_{x_1} = -3000$ psi

$$\text{Therefore, } \left.\begin{array}{l}\sigma_1 = -3000 \text{ psi and } \theta_{p_1} = 66.04° \\ \sigma_2 = -12{,}700 \text{ psi and } \theta_{p_2} = -23.96°\end{array}\right\} \longleftarrow$$

CONT.

7.3-3 (CONT.)



7.3-4

PRINCIPAL STRESSES

$\sigma_x = 52$ MPa $\quad \sigma_y = -130$ MPa $\quad \tau_{xy} = -60$ MPa

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.6593$$

$2\theta_p = -33.40°$ and $\theta_p = -16.70°$

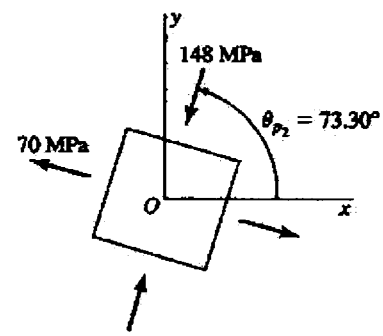
$2\theta_p = 146.60°$ and $\theta_p = 73.30°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -33.40°$: $\sigma_{x_1} = 70.0$ MPa

For $2\theta_p = 146.60°$: $\sigma_{x_1} = -148.0$ MPa

$$\text{Therefore, } \left.\begin{array}{l}\sigma_1 = 70.0 \text{ MPa and } \theta_{p_1} = -16.70° \\ \sigma_2 = -148.0 \text{ MPa and } \theta_{p_2} = 73.30°\end{array}\right\} \longleftarrow$$



7.3-5

MAXIMUM SHEAR STRESSES

$\sigma_x = 8300$ psi $\quad \sigma_y = -19{,}700$ psi $\quad \tau_{xy} = -4800$ psi

Principal angles

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.3429$$

$2\theta_p = -18.92°$ and $\theta_p = -9.46°$

$2\theta_p = 161.08°$ and $\theta_p = 80.54°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -18.92°$: $\sigma_{x_1} = 9100$ psi

For $2\theta_p = 161.08°$: $\sigma_{x_1} = -20{,}500$ psi

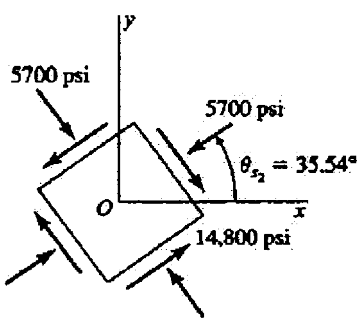
Therefore, $\theta_{p_1} = -9.46°$

Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 14{,}800 \text{ psi}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -54.46° \text{ and } \tau = 14{,}800 \text{ psi} \\ \theta_{s_2} = \theta_{p_1} + 45° = 35.54° \text{ and } \tau = -14{,}800 \text{ psi}\end{array}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -5700 \text{ psi} \quad \leftarrow$$



7.3-6

MAXIMUM SHEAR STRESSES

$\sigma_x = -26.5$ MPa $\quad \sigma_y = 5.5$ MPa $\quad \tau_{xy} = -12.0$ MPa

Principal angles

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.75$$

$2\theta_p = 36.87°$ and $\theta_p = 18.43°$

$2\theta_p = 216.87°$ and $\theta_p = 108.43°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 36.87°$: $\sigma_{x_1} = -30.5$ MPa

For $2\theta_p = 216.87°$: $\sigma_{x_1} = 9.5$ MPa

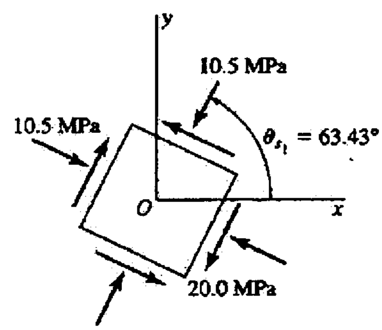
Therefore, $\theta_{p_1} = 108.4°$

Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.0 \text{ MPa}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = 63.43° \text{ and } \tau = 20.0 \text{ MPa} \\ \theta_{s_2} = \theta_{p_1} + 45° = 153.43° \text{ and } \tau = -20.0 \text{ MPa}\end{array}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -10.5 \text{ MPa} \quad \leftarrow$$



7.3-7

MAXIMUM SHEAR STRESSES

$\sigma_x = -9000$ psi $\quad \sigma_y = -1000$ psi $\quad \tau_{xy} = -4200$ psi

Principal angles

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.05$$

$2\theta_p = 46.40°$ and $\theta_p = 23.20°$

$2\theta_p = 226.40°$ and $\theta_p = 113.20°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 46.40°$: $\sigma_{x_1} = -10{,}800$ psi

For $2\theta_p = 226.40°$: $\sigma_{x_1} = 800$ psi

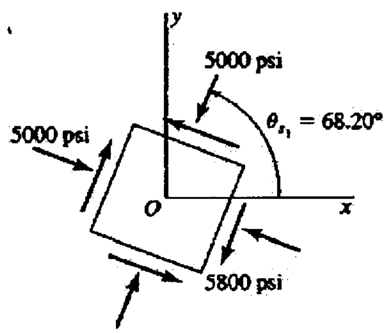
Therefore, $\theta_{p_1} = 113.20°$

Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5800 \text{ psi}$$

$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = 68.20° \text{ and } \tau = 5800 \text{ psi}\\ \theta_{s_2} &= \theta_{p_1} + 45° = 158.20° \text{ and } \tau = -5800 \text{ psi}\end{aligned}\right\} \quad \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -5000 \text{ psi} \quad \leftarrow$$



7.3-8

MAXIMUM SHEAR STRESSES

$\sigma_x = -50$ MPa $\quad \sigma_y = -8$ MPa $\quad \tau_{xy} = 20$ MPa

Principal angles

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.9524$$

$2\theta_p = -43.60°$ and $\theta_p = -21.80°$

$2\theta_p = 136.40°$ and $\theta_p = 68.20°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -43.60°$: $\sigma_{x_1} = -58.0$ MPa

For $2\theta_p = 136.40°$: $\sigma_{x_1} = 0$

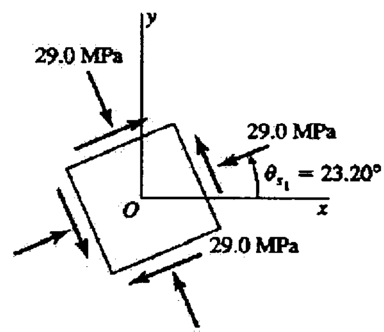
Therefore, $\theta_{p_1} = 68.20°$

Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 29.0 \text{ MPa}$$

$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = 23.20° \text{ and } \tau = 29.0 \text{ MPa}\\ \theta_{s_2} &= \theta_{p_1} + 45° = 113.20° \text{ and } \tau = -29.0 \text{ MPa}\end{aligned}\right\} \quad \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -29.0 \text{ MPa} \quad \leftarrow$$



7.3-9

SHEAR WALL

$\sigma_x = 0 \qquad \sigma_y = -1100 \text{ psi} \qquad \tau_{xy} = -460 \text{ psi}$

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.8364$$

$2\theta_p = -39.91°$ and $\theta_p = -19.95°$

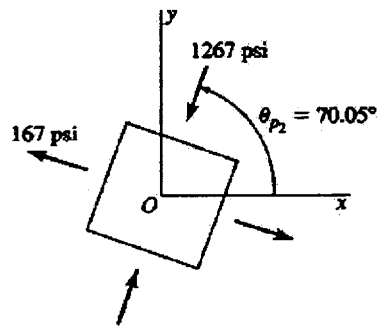
$2\theta_p = 140.09°$ and $\theta_p = 70.05°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -39.91°$: $\sigma_{x_1} = 167.0$ psi

For $2\theta_p = 140.09°$: $\sigma_{x_1} = -1267$ psi

Therefore, $\left.\begin{array}{l}\sigma_1 = 167 \text{ psi and } \theta_{p_1} = -19.95° \\ \sigma_2 = -1267 \text{ psi and } \theta_{p_2} = 70.05°\end{array}\right\} \leftarrow$

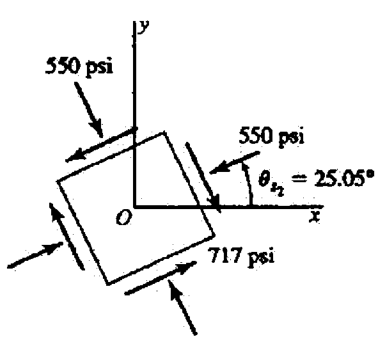


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 717 \text{ psi}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -64.95° \text{ and } \tau = 717 \text{ psi} \\ \theta_{s_2} = \theta_{p_1} + 45° = 25.05° \text{ and } \tau = -717 \text{ psi}\end{array}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -550 \text{ psi} \quad \leftarrow$$



7.3-10

PROPELLER SHAFT

$\sigma_x = -100 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -70 \text{ MPa}$

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.4000$$

$2\theta_p = 54.46°$ and $\theta_p = 27.23°$

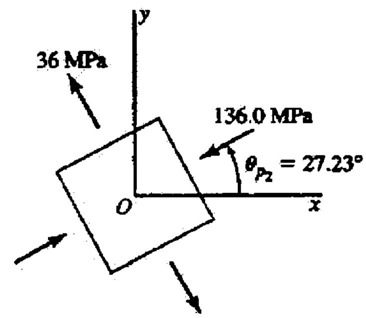
$2\theta_p = 234.46°$ and $\theta_p = 117.23°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 54.46°$: $\sigma_{x_1} = -136.0$ MPa

For $2\theta_p = 234.46°$: $\sigma_{x_1} = 36.0$ MPa

Therefore, $\left.\begin{array}{l}\sigma_1 = 36.0 \text{ MPa and } \theta_{p_1} = 117.23° \\ \sigma_2 = -136.0 \text{ MPa and } \theta_{p_2} = 27.23°\end{array}\right\} \leftarrow$

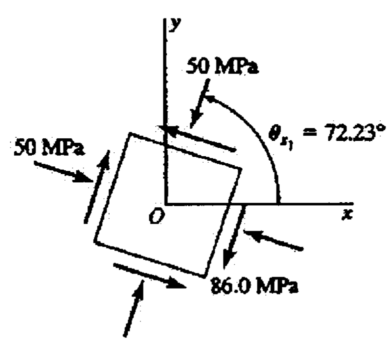


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 86.0 \text{ MPa}$$

$$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = 72.23° \text{ and } \tau = 86.0 \text{ MPa} \\ \theta_{s_2} = \theta_{p_1} + 45° = 162.23° \text{ and } \tau = -86.0 \text{ MPa}\end{array}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa} \quad \leftarrow$$



7.3-11

PLANE STRESS

$\sigma_x = 3300$ psi $\quad \sigma_y = 920$ psi $\quad \tau_{xy} = -1200$ psi

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.0084$$

$2\theta_p = -45.24°$ and $\theta_p = -22.62°$

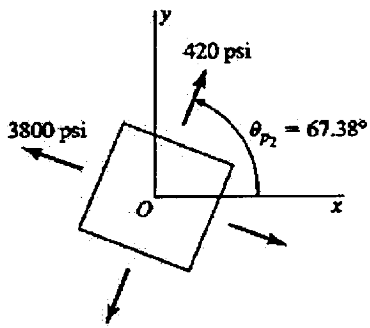
$2\theta_p = 134.76°$ and $\theta_p = 67.38°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -45.24°$: $\sigma_{x_1} = 3800$ psi

For $2\theta_p = 134.76°$: $\sigma_{x_1} = 420$ psi

Therefore, $\sigma_1 = 3800$ psi and $\theta_{p_1} = -22.62°$
$\sigma_2 = 420$ psi and $\theta_{p_2} = 67.38°$ ←

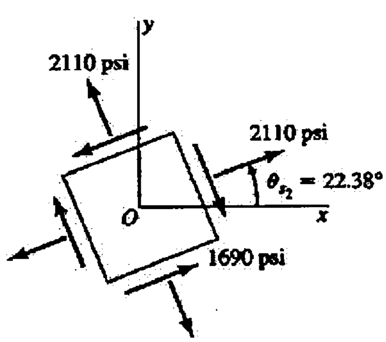


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1690 \text{ psi}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -67.62°$ and $\tau = 1690$ psi
$\theta_{s_2} = \theta_{p_1} + 45° = 22.38°$ and $\tau = -1690$ psi ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = 2110 \text{ psi} \quad \leftarrow$$



7.3-12

PLANE STRESS

$\sigma_x = 2100$ kPa $\quad \sigma_y = 300$ kPa $\quad \tau_{xy} = -560$ kPa

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.6222$$

$2\theta_p = -31.89°$ and $\theta_p = -15.95°$

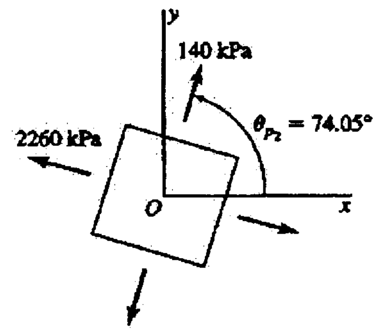
$2\theta_p = 148.11°$ and $\theta_p = 74.05°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -31.89°$: $\sigma_{x_1} = 2260$ kPa

For $2\theta_p = 148.11°$: $\sigma_{x_1} = 140$ kPa

Therefore, $\sigma_1 = 2260$ kPa and $\theta_{p_1} = -15.95°$
$\sigma_2 = 140$ kPa and $\theta_{p_2} = 74.05°$ ←

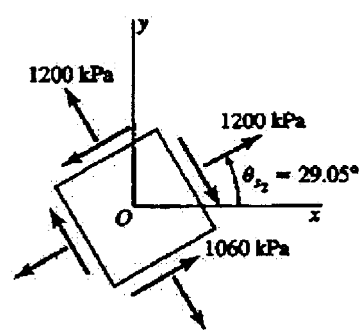


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1060 \text{ kPa}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -60.95°$ and $\tau = 1060$ kPa
$\theta_{s_2} = \theta_{p_1} + 45° = 29.05°$ and $\tau = -1060$ kPa ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = 1200 \text{ kPa} \quad \leftarrow$$



7.3-13

PLANE STRESS

$\sigma_x = 16{,}000$ psi $\quad \sigma_y = 2000$ psi $\quad \tau_{xy} = 1800$ psi

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.2571$$

$2\theta_p = 14.42°$ and $\theta_p = 7.21°$

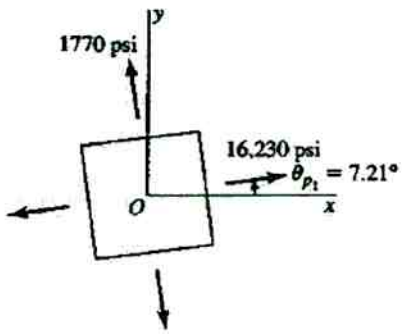
$2\theta_p = 194.42°$ and $\theta_p = 97.21°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 14.42°$: $\sigma_{x_1} = 16{,}230$ psi

For $2\theta_p = 194.42°$: $\sigma_{x_1} = 1770$ psi

Therefore, $\left.\begin{array}{l}\sigma_1 = 16{,}230 \text{ psi and } \theta_{p_1} = 7.21° \\ \sigma_2 = 1770 \text{ psi and } \theta_{p_2} = 97.21°\end{array}\right\} \leftarrow$

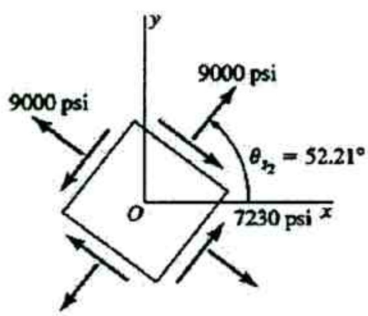


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7230 \text{ psi}$$

$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -37.79° \text{ and } \tau = 7230 \text{ psi} \\ \theta_{s_2} = \theta_{p_1} + 45° = 52.21° \text{ and } \tau = -7230 \text{ psi}\end{array}\right\} \leftarrow$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = 9000 \text{ psi} \quad \leftarrow$$



7.3-14

PLANE STRESS

$\sigma_x = 16$ MPa $\quad \sigma_y = -96$ MPa $\quad \tau_{xy} = -49$ MPa

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.8750$$

$2\theta_p = -41.19°$ and $\theta_p = -20.59°$

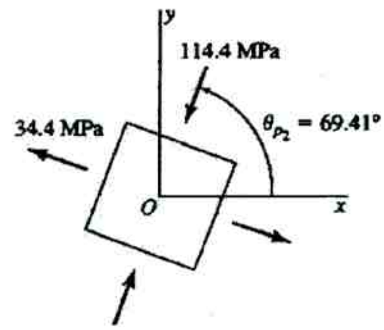
$2\theta_p = 138.81°$ and $\theta_p = 69.41°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -41.19°$: $\sigma_{x_1} = 34.41$ MPa

For $2\theta_p = 138.81°$: $\sigma_{x_1} = -114.41$ MPa

Therefore, $\left.\begin{array}{l}\sigma_1 = 34.4 \text{ MPa and } \theta_{p_1} = -20.59° \\ \sigma_2 = -114.4 \text{ MPa and } \theta_{p_2} = 69.41°\end{array}\right\} \leftarrow$

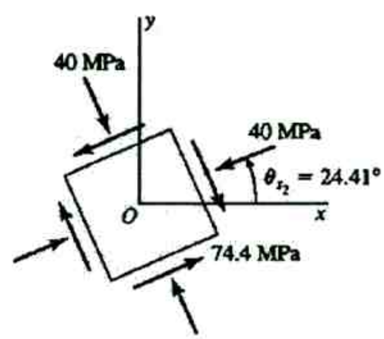


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 74.4 \text{ MPa}$$

$\left.\begin{array}{l}\theta_{s_1} = \theta_{p_1} - 45° = -65.59° \text{ and } \tau = 74.4 \text{ MPa} \\ \theta_{s_2} = \theta_{p_1} + 45° = 24.41° \text{ and } \tau = -74.4 \text{ MPa}\end{array}\right\} \leftarrow$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \quad \leftarrow$$



7.3-15

PLANE STRESS

$\sigma_x = -3000$ psi $\quad \sigma_y = -12{,}000$ psi $\quad \tau_{xy} = 6000$ psi

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.333$$

$2\theta_p = 53.13°$ and $\theta_p = 26.57°$

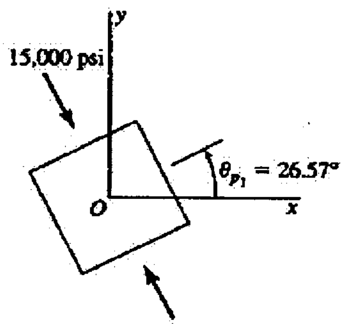
$2\theta_p = 233.13°$ and $\theta_p = 116.57°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 53.13°$: $\sigma_{x_1} = 0$

For $2\theta_p = 233.13°$: $\sigma_{x_1} = -15{,}000$ psi

$$\left.\begin{aligned}\text{Therefore, } \sigma_1 &= 0 \text{ and } \theta_{p_1} = 26.57° \\ \sigma_2 &= -15{,}000 \text{ psi and } \theta_{p_2} = 116.57°\end{aligned}\right\} \leftarrow$$

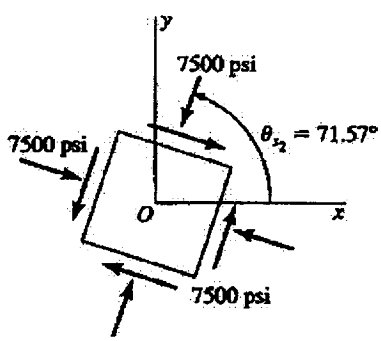


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7500 \text{ psi}$$

$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = -18.43° \text{ and } \tau = 7500 \text{ psi} \\ \theta_{s_2} &= \theta_{p_1} + 45° = 71.57° \text{ and } \tau = -7500 \text{ psi}\end{aligned}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -7500 \text{ psi} \quad \leftarrow$$



7.3-16

PLANE STRESS

$\sigma_x = -100$ MPa $\quad \sigma_y = 50$ MPa $\quad \tau_{xy} = -50$ MPa

(a) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.6667$$

$2\theta_p = 33.69°$ and $\theta_p = 16.85°$

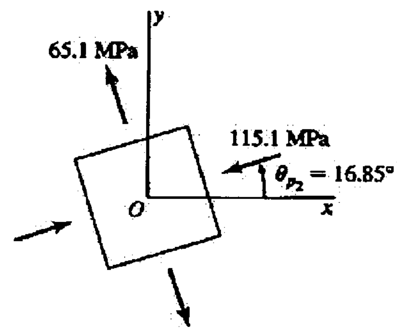
$2\theta_p = 213.69°$ and $\theta_p = 106.85°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 33.69°$: $\sigma_{x_1} = -115.1$ MPa

For $2\theta_p = 213.69°$: $\sigma_{x_1} = 65.1$ MPa

$$\left.\begin{aligned}\text{Therefore, } \sigma_1 &= 65.1 \text{ MPa and } \theta_{p_1} = 106.85° \\ \sigma_2 &= -115.1 \text{ MPa and } \theta_{p_2} = 16.85°\end{aligned}\right\} \leftarrow$$

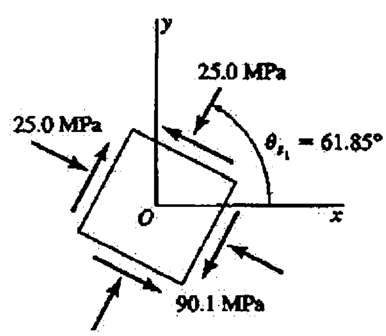


(b) Maximum shear stresses

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 90.1 \text{ MPa}$$

$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = 61.85° \text{ and } \tau = 90.1 \text{ MPa} \\ \theta_{s_2} &= \theta_{p_1} + 45° = 151.85° \text{ and } \tau = -90.1 \text{ MPa}\end{aligned}\right\} \leftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -25.0 \text{ MPa} \quad \leftarrow$$



7.3-17

ALLOWABLE RANGE OF VALUES

$\sigma_x = 6500$ psi $\quad \tau_{xy} = 2100$ psi $\quad \sigma_y = ?$

Find the allowable range of values for σ_y if the maximum allowable shear stress is $\tau_0 = 2900$ psi.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{Eq. (1)}$$

or

$$\tau_{max}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \text{Eq. (2)}$$

<u>Solve for σ_y</u>

$$\sigma_y = \sigma_x \pm 2\sqrt{\tau_{max}^2 - \tau_{xy}^2}$$

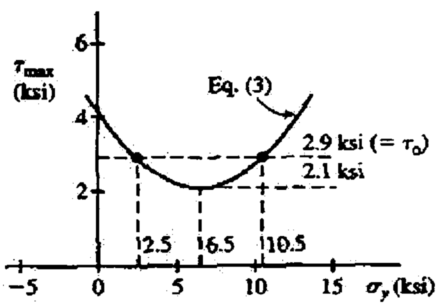
Substitute numerical values:

$$\sigma_y = 6500 \text{ psi} \pm 2\sqrt{(2900 \text{ psi})^2 - (2100 \text{ psi})^2}$$
$$= 6500 \text{ psi} \pm 4000 \text{ psi}$$

Therefore, $2500 \text{ psi} \le \sigma_y \le 10{,}500$ psi $\longleftarrow$

<u>Graph of τ_{max}</u>

From Eq. (1): $\tau_{max} = \sqrt{\left(\frac{6500 - \sigma_y}{2}\right)^2 + (2100)^2}$ Eq. (3)



7.3-18

ALLOWABLE RANGE OF VALUES

$\sigma_x = 45$ MPa $\quad \tau_{xy} = 30$ MPa $\quad \sigma_y = ?$

Find the allowable range of values for σ_y if the maximum allowable shear stress is $\tau_0 = 34$ MPa.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{Eq. (1)}$$

or

$$\tau_{max}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \text{Eq. (2)}$$

<u>Solve for σ_y</u>

$$\sigma_y = \sigma_x \pm 2\sqrt{\tau_{max}^2 - \tau_{xy}^2}$$

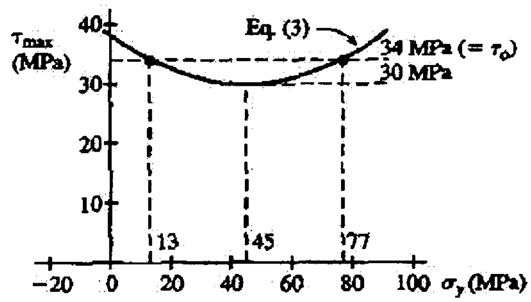
Substitute numerical values:

$$\sigma_y = 45 \text{ MPa} \pm 2\sqrt{(34 \text{ MPa})^2 - (30 \text{ MPa})^2}$$
$$= 45 \text{ MPa} \pm 32 \text{ MPa}$$

Therefore, $13 \text{ MPa} \le \sigma_y \le 77$ MPa $\longleftarrow$

<u>Graph of τ_{max}</u>

From Eq. (1): $\tau_{max} = \sqrt{\left(\frac{45 - \sigma_y}{2}\right)^2 + (30)^2}$ Eq. (3)



7.3-19

PLANE STRESS

$\sigma_x = 7620$ psi $\quad \tau_{xy} = -2910$ psi $\quad \sigma_y = ?$

One principal stress = 8400 psi (tension)

(a) Stress σ_y

Because σ_x is less than the given principal stress, we know that the given stress is the larger principal stress.

$\sigma_1 = 8400$ psi

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute numerical values and solve for σ_y:

$\sigma_y = -2457$ psi $\leftarrow$

(b) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.57755$$

$2\theta_p = -30.009°$ and $\theta_p = -15.004°$

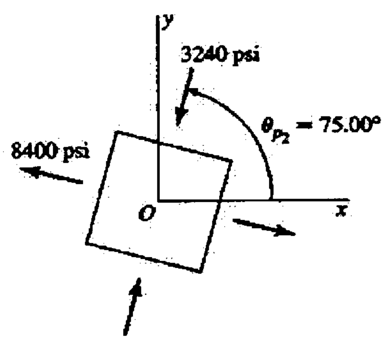
$2\theta_p = 149.991°$ and $\theta_p = 74.996°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -30.009°$: $\sigma_{x_1} = 8400$ psi

For $2\theta_p = 149.991°$: $\sigma_{x_1} = -3237$ psi

$$\left.\begin{array}{l}\text{Therefore, } \sigma_1 = 8400 \text{ psi and } \theta_{p_1} = -15.00° \\ \sigma_2 = -3237 \text{ psi and } \theta_{p_2} = 75.00°\end{array}\right\} \leftarrow$$



7.3-20

PLANE STRESS

$\sigma_x = -68.5$ MPa $\quad \tau_{xy} = 39.2$ MPa $\quad \sigma_y = ?$

One principal stress = 26.1 MPa (tension)

(a) Stress σ_y

Because σ_x is less than the given principal stress, we know that the given stress is the larger principal stress.

$\sigma_1 = 26.1$ MPa

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute numerical values and solve for σ_y:

$\sigma_y = 9.856$ MPa $\leftarrow$

(b) Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.0006$$

$2\theta_p = -45.02°$ and $\theta_p = -22.51°$

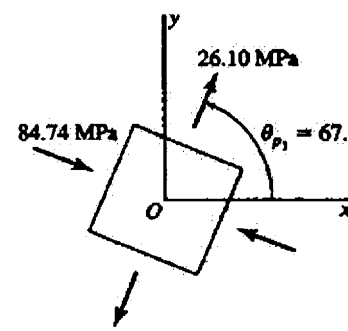
$2\theta_p = 134.98°$ and $\theta_p = 67.49°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -45.02°$: $\sigma_{x_1} = -84.74$ MPa

For $2\theta_p = 134.98°$: $\sigma_{x_1} = 26.10$ MPa

$$\left.\begin{array}{l}\text{Therefore, } \sigma_1 = 26.10 \text{ MPa and } \theta_{p_1} = 67.49° \\ \sigma_2 = -84.74 \text{ MPa and } \theta_{p_2} = -22.51°\end{array}\right\} \leftarrow$$



7.4-1

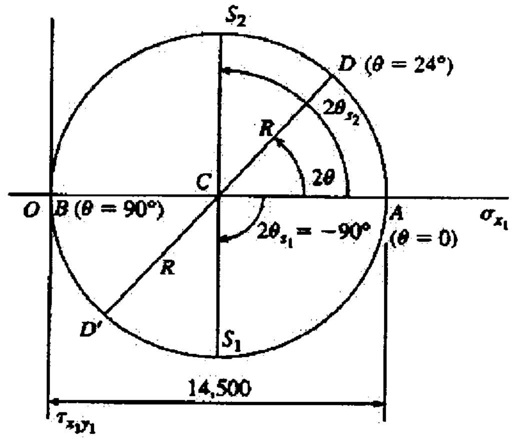
UNIAXIAL STRESS

$\sigma_x = 14{,}500$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) Element at $\theta = 24°$ (All stresses in psi)

$2\theta = 48° \quad \theta = 24° \quad R = 7250$ psi

Point C: $\sigma_{x_1} = 7250$ psi

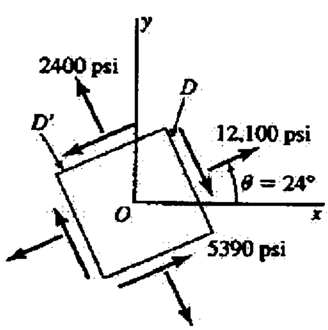


Point D: $\sigma_{x_1} = R + R\cos 2\theta = 12{,}100$ psi

$\tau_{x_1y_1} = R\sin 2\theta = -5390$ psi

Point D': $\sigma_{x_1} = R - R\cos 2\theta = 2400$ psi

$\tau_{x_1y_1} = 5390$ psi



(b) Maximum shear stresses

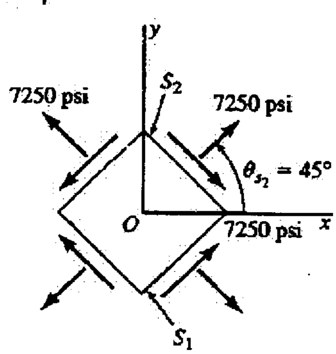
Point S_1: $2\theta_{s_1} = -90° \quad \theta_{s_1} = -45°$

$\tau_{max} = R = 7250$ psi

Point S_2: $2\theta_{s_2} = 90° \quad \theta_{s_2} = 45°$

$\tau_{min} = -R = -7250$ psi

$\sigma_{aver} = R = 7250$ psi



7.4-2

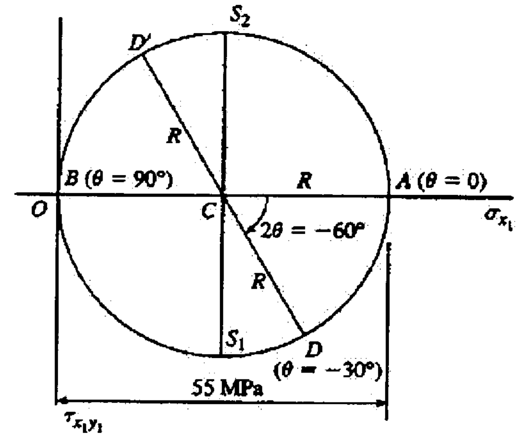
UNIAXIAL STRESS

$\sigma_x = 55$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) Element at $\theta = -30°$ (All stresses in MPa)

$2\theta = -60° \quad \theta = -30° \quad R = 27.5$ MPa

Point C: $\sigma_{x_1} = 27.5$ MPa

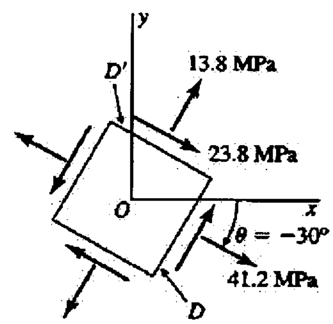


Point D: $\sigma_{x_1} = R + R\cos|2\theta| = R(1 + \cos 60°) = 41.2$ MPa

$\tau_{x_1y_1} = R\sin|2\theta| = R\sin 60° = 23.8$ MPa

Point D': $\sigma_{x_1} = R - R\cos|2\theta| = 13.8$ MPa

$\tau_{x_1y_1} = -R\sin|2\theta| = -23.8$ MPa



(b) Maximum shear stresses

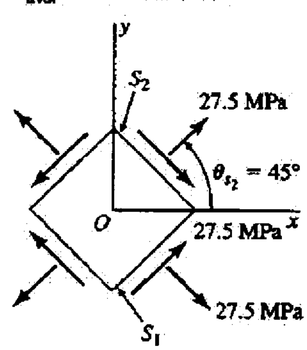
Point S_1: $2\theta_{s_1} = -90° \quad \theta_{s_1} = -45°$

$\tau_{max} = R = 27.5$ MPa

Point S_2: $2\theta_{s_2} = 90° \quad \theta_{s_2} = 45°$

$\tau_{min} = -R = -27.5$ MPa

$\sigma_{aver} = R = 27.5$ MPa



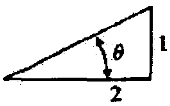
7.4-3

UNIAXIAL STRESS

$\sigma_x = -5600$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) Element at a slope of 1 on 2

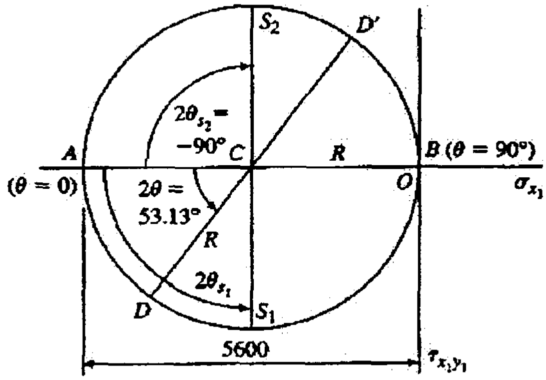
(All stresses in psi) $\theta = \arctan \frac{1}{2} = 26.565°$



$2\theta = 53.130° \qquad \theta = 26.57°$

$R = 2800$ psi

Point C: $\sigma_{x_1} = -2800$ psi

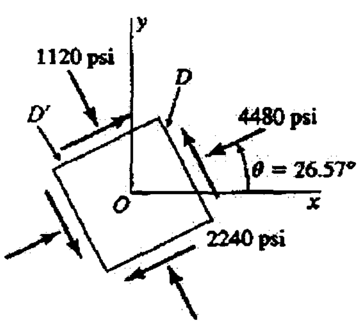


Point D: $\sigma_{x_1} = -R - R\cos 2\theta = -4480$ psi

$\tau_{x_1y_1} = R\sin 2\theta = 2240$ psi

Point D': $\sigma_{x_1} = -R + R\cos 2\theta = -1120$ psi

$\tau_{x_1y_1} = -R\sin 2\theta = -2240$ psi



(b) Maximum shear stresses

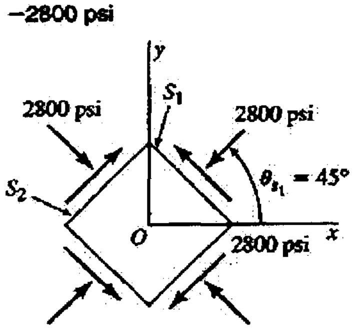
Point S_1: $2\theta_{s_1} = 90° \qquad \theta_{s_1} = 45°$

$\tau_{max} = R = 2800$ psi

Point S_2: $2\theta_{s_2} = -90° \qquad \theta_{s_2} = -45°$

$\tau_{min} = -R = -2800$ psi

$\sigma_{aver} = -R = -2800$ psi



7.4-4

BIAXIAL STRESS

$\sigma_x = -60$ MPa $\quad \sigma_y = 20$ MPa $\quad \tau_{xy} = 0$

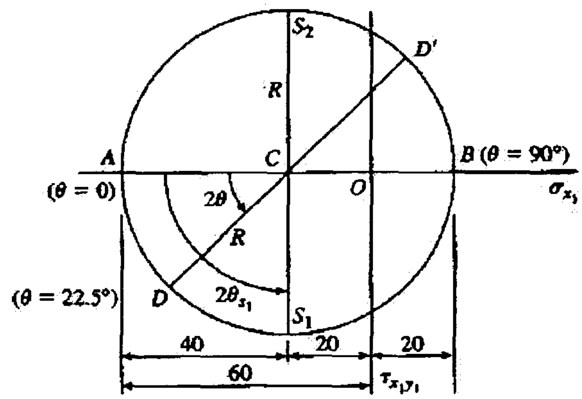
(a) Element at $\theta = 22.5°$

(All stresses in MPa)

$2\theta = 45° \qquad \theta = 22.5°$

$2R = 60 + 20 = 80$ MPa $\qquad R = 40$ MPa

Point C: $\sigma_{x_1} = -20$ MPa

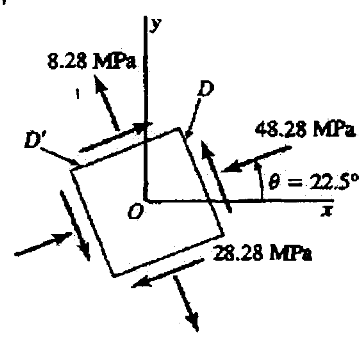


Point D: $\sigma_{x_1} = -20 - R\cos 2\theta = -48.28$ MPa

$\tau_{x_1y_1} = R\sin 2\theta = 28.28$ MPa

Point D': $\sigma_{x_1} = R\cos 2\theta - 20 = 8.28$ MPa

$\tau_{x_1y_1} = -R\sin 2\theta = -28.28$ MPa



(b) Maximum shear stresses

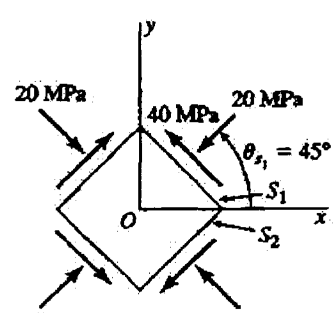
Point S_1: $2\theta_{s_1} = 90° \qquad \theta_{s_1} = 45°$

$\tau_{max} = R = 40$ MPa

Point S_2: $2\theta_{s_2} = -90° \qquad \theta_{s_2} = -45°$

$\tau_{min} = -R = -40$ MPa

$\sigma_{aver} = -20$ MPa



7.4-5

BIAXIAL STRESS

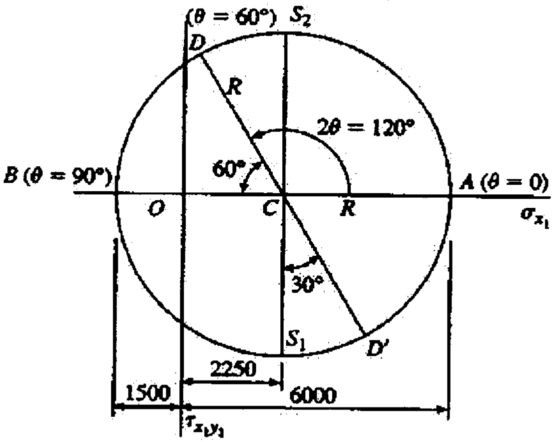
$\sigma_x = 6000$ psi $\sigma_y = -1500$ psi $\tau_{xy} = 0$

(a) Element at $\theta = 60°$

(All stresses in psi)

$2\theta = 120°$ $\theta = 60°$ $2R = 7500$ psi

$R = 3750$ psi Point C: $\sigma_{x_1} = 2250$ psi

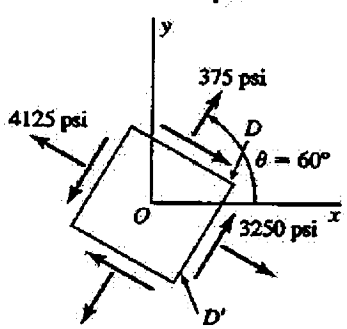


Point D: $\sigma_{x_1} = 2250 - R\cos 60° = 375$ psi

$\tau_{x_1y_1} = -R\sin 60° = -3248$ psi

Point D': $\sigma_{x_1} = 2250 + R\cos 60° = 4125$ psi

$\tau_{x_1y_1} = R\sin 60° = 3248$ psi



(b) Maximum shear stresses

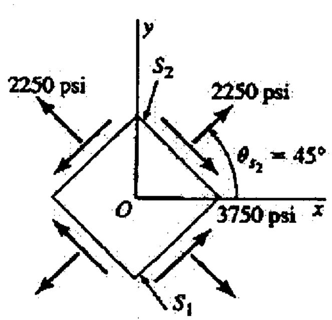
Point S_1: $2\theta_{s_1} = -90°$ $\theta_{s_1} = -45°$

$\tau_{max} = R = 3750$ psi

Point S_2: $2\theta_{s_2} = 90°$ $\theta_{s_2} = 45°$

$\tau_{min} = -R = -3750$ psi

$\sigma_{aver} = 2250$ psi



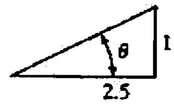
7.4-6

BIAXIAL STRESS

$\sigma_x = -24$ MPa $\sigma_y = 63$ MPa $\tau_{xy} = 0$

(a) Element at a slope of 1 on 2.5

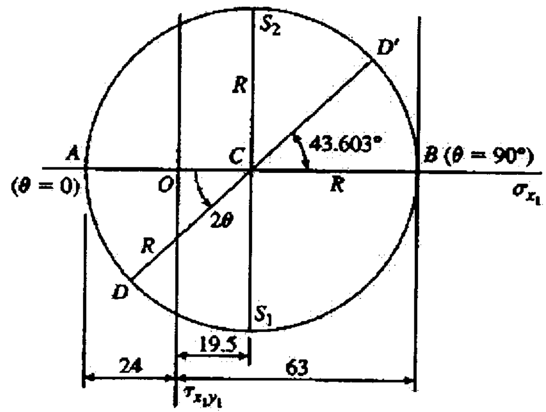
(All stresses in MPa) $\theta = \arctan\dfrac{1}{2.5} = 21.801°$



$2\theta = 43.603°$ $\theta = 21.801°$

$2R = 87$ MPa $R = 43.5$ MPa

Point C: $\sigma_{x_1} = 19.5$ MPa

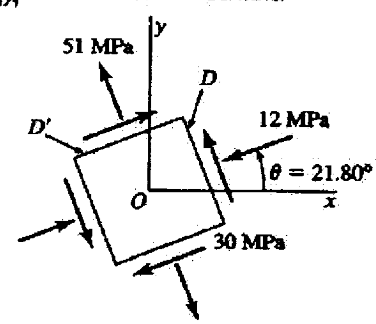


Point D: $\sigma_{x_1} = -R\cos 2\theta + 19.5 = -12$ MPa

$\tau_{x_1y_1} = R\sin 2\theta = 30$ MPa

Point D': $\sigma_{x_1} = 19.5 + R\cos 2\theta = 51$ MPa

$\tau_{x_1y_1} = -R\sin 2\theta = -30$ MPa



(b) Maximum shear stresses

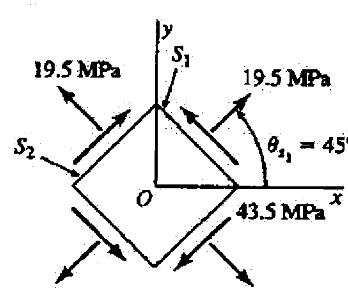
Point S_1: $2\theta_{s_1} = 90°$ $\theta_{s_1} = 45°$

$\tau_{max} = R = 43.5$ MPa

Point S_2: $2\theta_{s_2} = -90°$ $\theta_{s_2} = -45°$

$\tau_{min} = -R = -43.5$ MPa

$\sigma_{aver} = 19.5$ MPa



7.4-7

PURE SHEAR

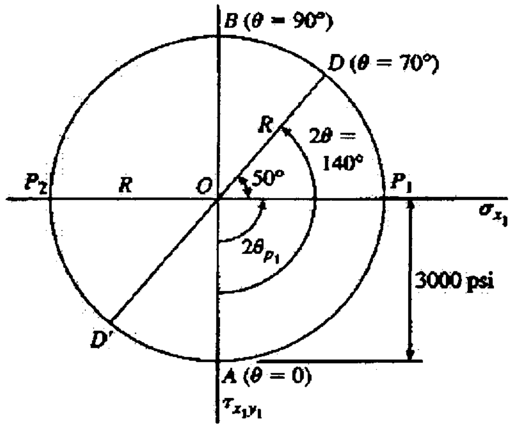
$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 3000$ psi

(a) Element at $\theta = 70°$

(All stresses in psi)

$2\theta = 140° \qquad \theta = 70° \qquad R = 3000$ psi

Origin O is at center of circle.

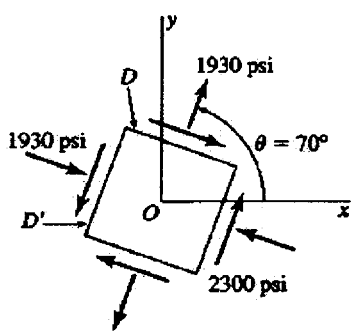


Point D: $\sigma_{x_1} = R\cos 50° = 1928$ psi

$\tau_{x_1y_1} = -R\sin 50° = -2298$ psi

Point D': $\sigma_{x_1} = -R\cos 50° = -1928$ psi

$\tau_{x_1y_1} = R\sin 50° = 2298$ psi



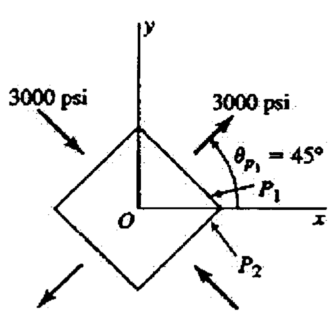
(b) Principal stresses

Point P_1: $2\theta_{p_1} = 90° \qquad \theta_{p_1} = 45°$

$\sigma_1 = R = 3000$ psi

Point P_2: $2\theta_{p_2} = -90° \qquad \theta_{p_2} = -45°$

$\sigma_2 = -R = -3000$ psi



7.4-8

PURE SHEAR

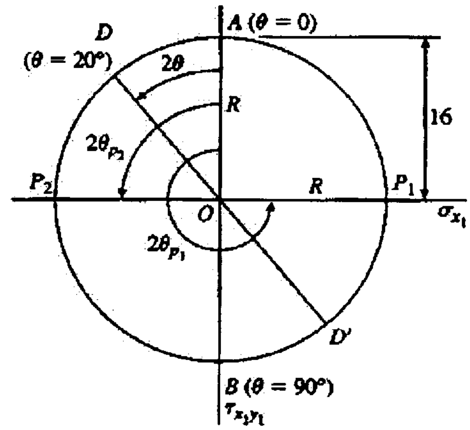
$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -16$ MPa

(a) Element at $\theta = 20°$

(All stresses in MPa)

$2\theta = 40° \qquad \theta = 20° \qquad R = 16$ MPa

Origin O is at center of circle.

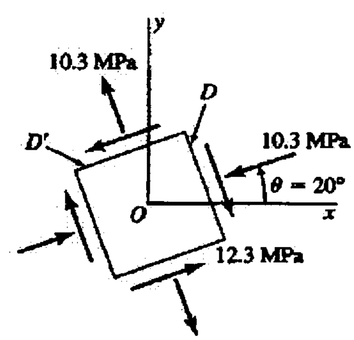


Point D: $\sigma_{x_1} = -R\sin 2\theta = -10.28$ MPa

$\tau_{x_1y_1} = -R\cos 2\theta = -12.26$ MPa

Point D': $\sigma_{x_1} = R\sin 2\theta = 10.28$ MPa

$\tau_{x_1y_1} = R\cos 2\theta = 12.26$ MPa



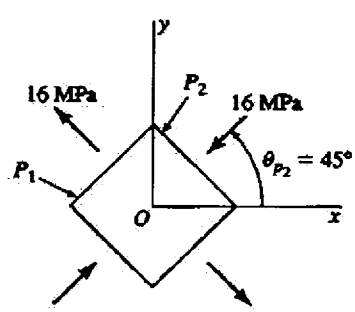
(b) Principal stresses

Point P_1: $2\theta_{p_1} = 270° \qquad \theta_{p_1} = 135°$

$\sigma_1 = R = 16$ MPa

Point P_2: $2\theta_{p_2} = 90° \qquad \theta_{p_2} = 45°$

$\sigma_2 = -R = -16$ MPa



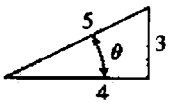
7.4-9

PURE SHEAR

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 4000 \text{ psi}$

(a) Element at a slope of 3 on 4

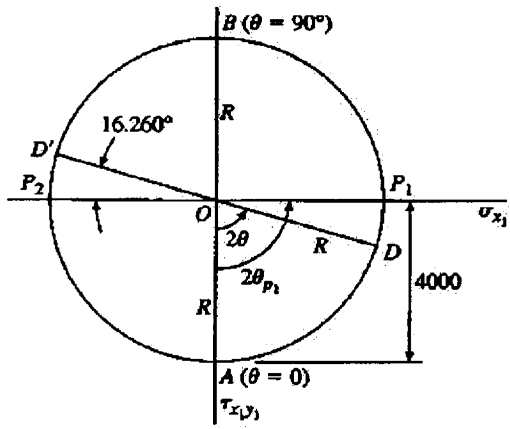
(All stresses in psi) $\quad \theta = \arctan \frac{3}{4} = 36.870°$



$2\theta = 73.740° \qquad \theta = 36.870°$

$R = 4000$ psi

Origin O is at center of circle.

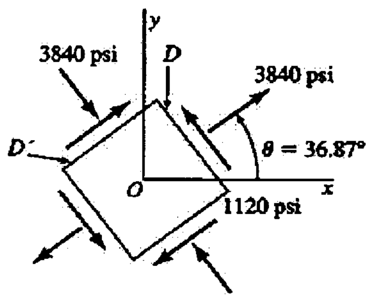


Point D: $\sigma_{x_1} = R\cos 16.260° = 3840$ psi

$\tau_{x_1y_1} = R\sin 16.260° = 1120$ psi

Point D': $\sigma_{x_1} = -R\cos 16.260° = -3840$ psi

$\tau_{x_1y_1} = -R\sin 16.260° = -1120$ psi



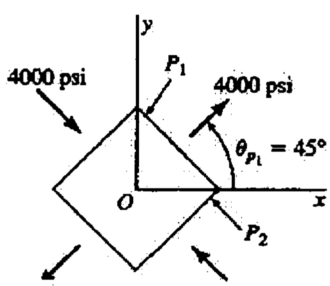
(b) Principal stresses

Point P_1: $2\theta_{p_1} = 90° \qquad \theta_{p_1} = 45°$

$\sigma_1 = R = 4000$ psi

Point P_2: $2\theta_{p_2} = -90° \qquad \theta_{p_2} = -45°$

$\sigma_2 = -R = -4000$ psi



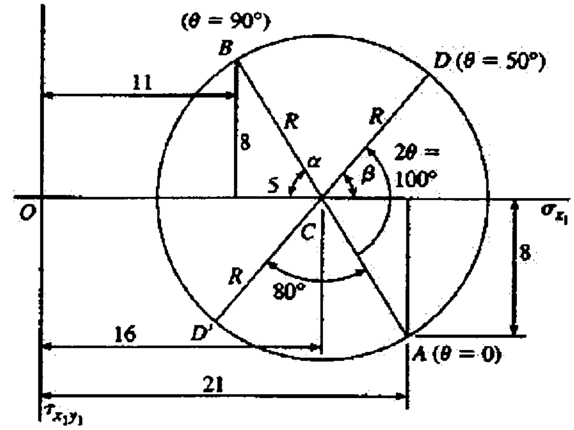
7.4-10

PLANE STRESS (ANGLE θ)

$\sigma_x = 21 \text{ MPa} \qquad \sigma_y = 11 \text{ MPa}$

$\tau_{xy} = 8 \text{ MPa} \qquad \theta = 50°$

(All stresses in MPa)



$R = \sqrt{(5)^2 + (8)^2} = 9.4340$ MPa

$\alpha = \arctan \frac{8}{5} = 57.99°$

$\beta = 2\theta - \alpha = 100° - \alpha = 42.01°$

Point D ($\theta = 50°$):

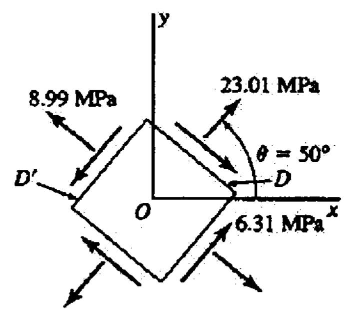
$\sigma_{x_1} = 16 + R\cos\beta = 23.01$ MPa

$\tau_{x_1y_1} = -R\sin\beta = -6.31$ MPa

Point D' ($\theta = -40°$):

$\sigma_{x_1} = 16 - R\cos\beta = 8.99$ MPa

$\tau_{x_1y_1} = R\sin\beta = 6.31$ MPa



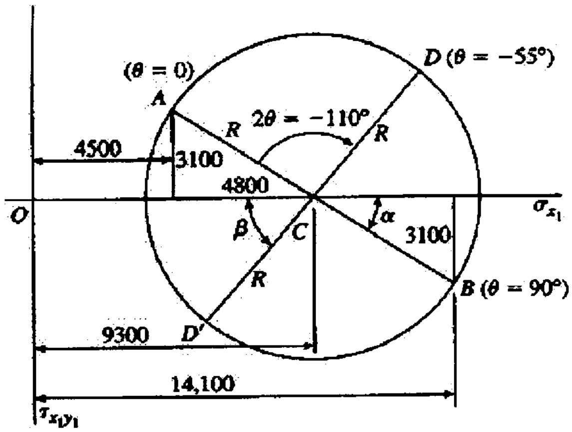
7.4-11

PLANE STRESS (ANGLE θ)

$\sigma_x = 4500$ psi $\quad \sigma_y = 14{,}100$ psi

$\tau_{xy} = -3100$ psi $\quad \theta = -55°$

(All stresses in psi)



$$R = \sqrt{(4800)^2 + (3100)^2} = 5714 \text{ psi}$$

$$\alpha = \arctan \frac{3100}{4800} = 32.86°$$

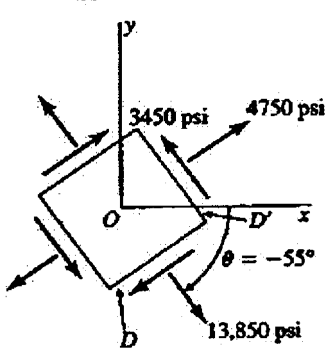
$$\beta = 180° - 110° - \alpha = 37.14°$$

Point D ($\theta = -55°$):

$$\sigma_{x_1} = 9300 + R \cos \beta = 13{,}850 \text{ psi}$$
$$\tau_{x_1y_1} = -R \sin \beta = -3450 \text{ psi}$$

Point D' ($\theta = -35°$):

$$\sigma_{x_1} = 9300 - R \cos \beta = 4750 \text{ psi}$$
$$\tau_{x_1y_1} = R \sin \beta = 3450 \text{ psi}$$



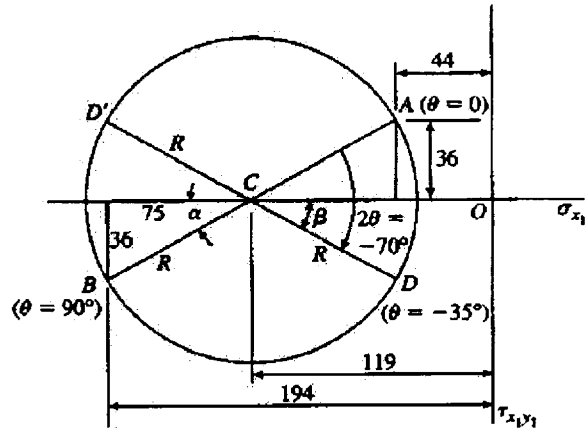
7.4-12

PLANE STRESS (ANGLE θ)

$\sigma_x = -44$ MPa $\quad \sigma_y = -194$ MPa

$\tau_{xy} = -36$ MPa $\quad \theta = -35°$

(All stresses in MPa)



$$R = \sqrt{(75)^2 + (36)^2} = 83.19 \text{ MPa}$$

$$\alpha = \arctan \frac{36}{75} = 25.64°$$

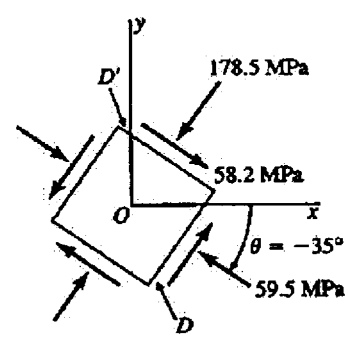
$$\beta = 70° - \alpha = 44.36°$$

Point D ($\theta = -35°$):

$$\sigma_{x_1} = -119 + R \cos \beta = -59.5 \text{ MPa}$$
$$\tau_{x_1y_1} = -R \sin \beta = 58.2 \text{ MPa}$$

Point D' ($\theta = 55°$):

$$\sigma_{x_1} = -119 - R \cos \beta = -178.5 \text{ MPa}$$
$$\tau_{x_1y_1} = -R \sin \beta = -58.2 \text{ MPa}$$



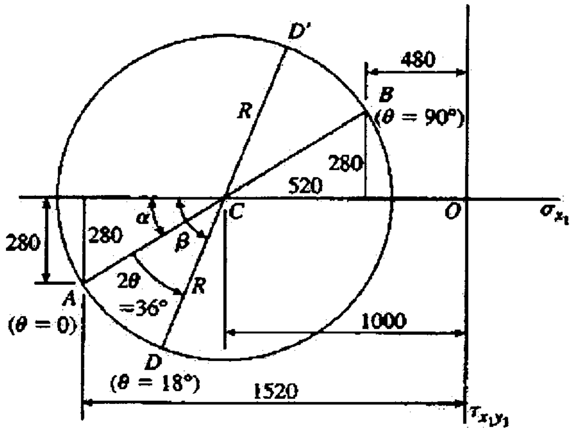
7.4-13

PLANE STRESS (ANGLE θ)

$\sigma_x = -1520$ psi $\quad \sigma_y = -480$ psi

$\tau_{xy} = 280$ psi $\quad \theta = 18°$

(All stresses in psi)



$R = \sqrt{(520)^2 + (280)^2} = 590.6$ psi

$\alpha = \arctan \dfrac{280}{520} = 28.30°$

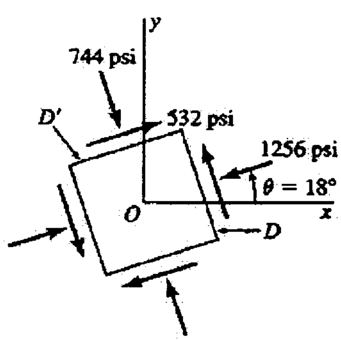
$\beta = \alpha + 36° = 64.30°$

Point D ($\theta = 18°$):

$$\sigma_{x_1} = -1000 - R\cos\beta = -1256 \text{ psi}$$
$$\tau_{x_1y_1} = R\sin\beta = 532 \text{ psi}$$

Point D'($\theta = 108°$):

$$\sigma_{x_1} = -1000 + R\cos\beta = -744 \text{ psi}$$
$$\tau_{x_1y_1} = -R\sin\beta = -532 \text{ psi}$$



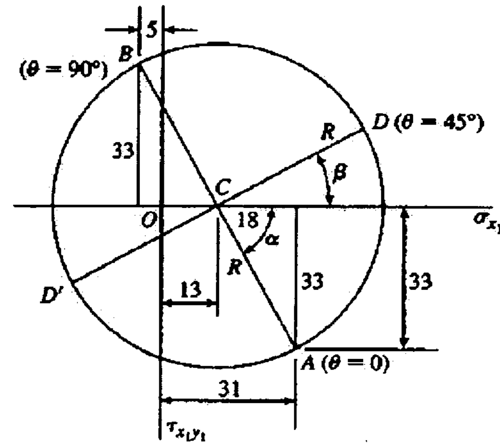
7.4-14

PLANE STRESS (ANGLE θ)

$\sigma_x = 31$ MPa $\quad \sigma_y = -5$ MPa

$\tau_{xy} = 33$ MPa $\quad \theta = 45°$

(All stresses in MPa)



$R = \sqrt{(18)^2 + (33)^2} = 37.590$ MPa

$\alpha = \arctan \dfrac{33}{18} = 61.390°$

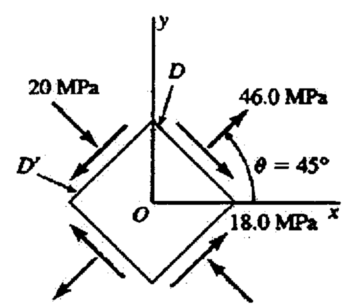
$\beta = 90° - \alpha = 28.610°$

Point D ($\theta = 45°$):

$$\sigma_{x_1} = 13 + R\cos\beta = 46.0 \text{ MPa}$$
$$\tau_{x_1y_1} = -R\sin\beta = -18.0 \text{ MPa}$$

Point D'($\theta = 135°$):

$$\sigma_{x_1} = 13 - R\cos\beta = -20.0 \text{ MPa}$$
$$\tau_{x_1y_1} = R\sin\beta = 18.0 \text{ MPa}$$



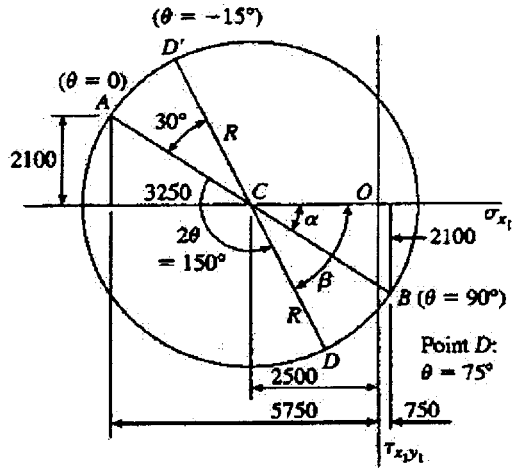
7.4-15

PLANE STRESS (ANGLE θ)

$\sigma_x = -5750$ psi $\quad \sigma_y = 750$ psi

$\tau_{xy} = -2100$ psi $\quad \theta = 75°$

(All stresses in psi)



$R = \sqrt{(3250)^2 + (2100)^2} = 3869$ psi

$\alpha = \arctan \dfrac{2100}{3250} = 32.87°$

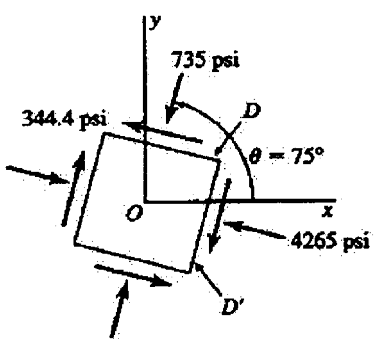
$\beta = \alpha + 30° = 62.87°$

Point D ($\theta = 75°$):

$$\sigma_{x_1} = -2500 + R\cos\beta = -735 \text{ psi}$$
$$\tau_{x_1y_1} = R\sin\beta = 3444 \text{ psi}$$

Point D' ($\theta = -15°$):

$$\sigma_{x_1} = -2500 - R\cos\beta = -4265 \text{ psi}$$
$$\tau_{x_1y_1} = -R\sin\beta = -3444 \text{ psi}$$

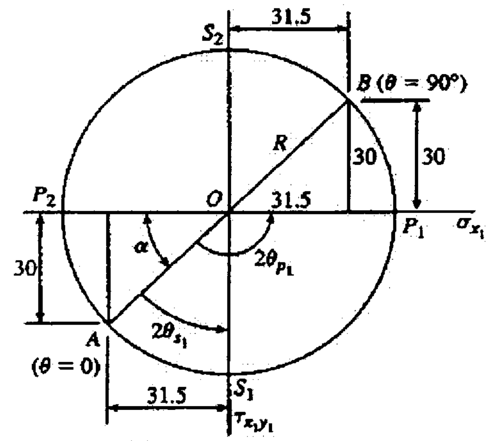


7.4-16

PRINCIPAL STRESSES

$\sigma_x = -31.5$ MPa $\quad \sigma_y = 31.5$ MPa $\quad \tau_{xy} = 30$ MPa

(All stresses in MPa)



$R = \sqrt{(31.5)^2 + (30.0)^2} = 43.5$ MPa

$\alpha = \arctan \dfrac{30}{31.5} = 43.60°$

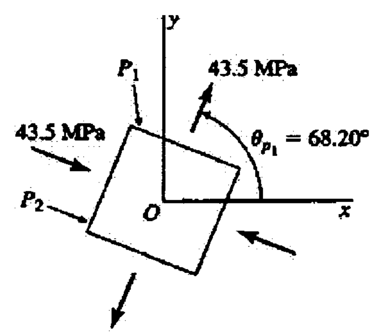
(a) Principal stresses

$2\theta_{p_1} = 180° - \alpha = 136.40° \qquad \theta_{p_1} = 68.20°$

$2\theta_{p_2} = -\alpha = -43.60° \qquad \theta_{p_2} = -21.80°$

Point P_1: $\sigma_1 = R = 43.5$ MPa

Point P_2: $\sigma_2 = -R = -43.5$ MPa



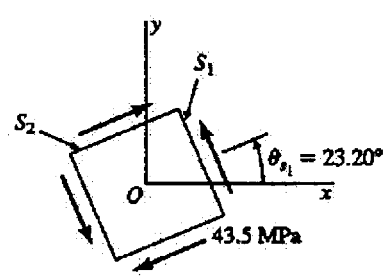
(b) Maximum shear stresses

$2\theta_{s_1} = 90° - \alpha = 46.40° \qquad \theta_{s_1} = 23.20°$

$2\theta_{s_2} = 2\theta_{s_1} + 180° = 226.40° \qquad \theta_{s_2} = 113.20°$

Point S_1: $\sigma_{aver} = 0 \qquad \tau_{max} = R = 43.5$ MPa

Point S_2: $\sigma_{aver} = 0 \qquad \tau_{min} = -43.5$ MPa

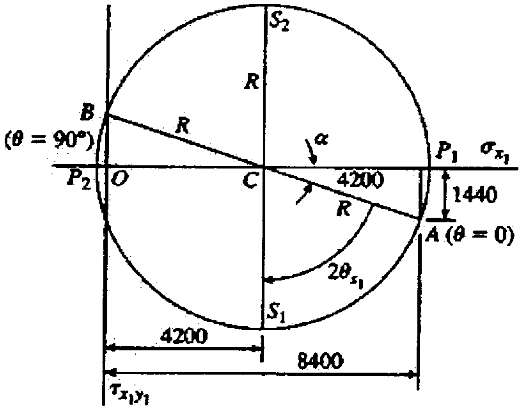


7.4-17

PRINCIPAL STRESSES

$\sigma_x = 8400$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 1440$ psi

(All stresses in psi)



$R = \sqrt{(4200)^2 + (1440)^2} = 4440$ psi

$\alpha = \arctan\dfrac{1440}{4200} = 18.92°$

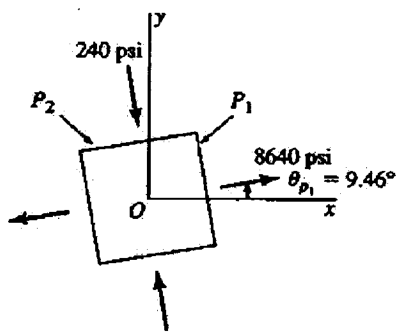
(a) Principal stresses

$2\theta_{p_1} = \alpha = 18.92° \qquad \theta_{p_1} = 9.46°$

$2\theta_{p_2} = 180° + \alpha = 198.92° \qquad \theta_{p_2} = 99.46°$

Point P_1: $\sigma_1 = 4200 + R = 8640$ psi

Point P_2: $\sigma_2 = 4200 - R = -240$ psi



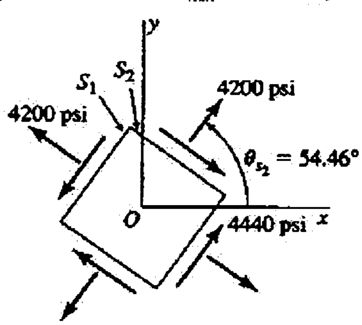
(b) Maximum shear stresses

$2\theta_{s_1} = -(90° - \alpha) = -71.08° \qquad \theta_{s_1} = -35.54°$

$2\theta_{s_2} = 90° + \alpha = 108.92° \qquad \theta_{s_2} = 54.46°$

Point S_1: $\sigma_{aver} = 4200$ psi $\qquad \tau_{max} = R = 4440$ psi

Point S_2: $\sigma_{aver} = 4200$ psi $\qquad \tau_{min} = -4440$ psi

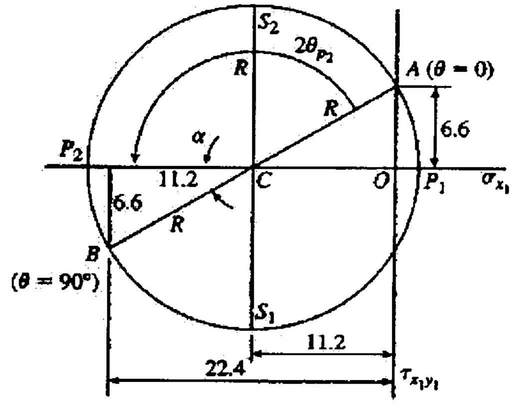


7.4-18

PRINCIPAL STRESSES

$\sigma_x = 0 \quad \sigma_y = -22.4$ MPa $\quad \tau_{xy} = -6.6$ MPa

(All stresses in MPa)



$R = \sqrt{(11.2)^2 + (6.6)^2} = 13.0$ MPa

$\alpha = \arctan\dfrac{6.6}{11.2} = 30.51°$

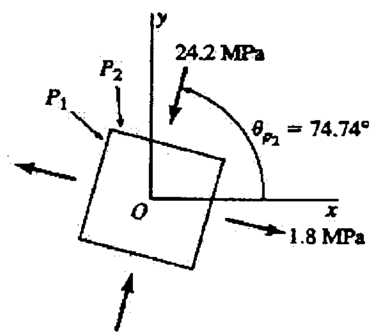
(a) Principal stresses

$2\theta_{p_1} = -\alpha = -30.51° \qquad \theta_{p_1} = -15.26°$

$2\theta_{p_2} = 180° - \alpha = 149.49° \qquad \theta_{p_2} = 74.74°$

Point P_1: $\sigma_1 = R - 11.2 = 1.8$ MPa

Point P_2: $\sigma_2 = -11.2 - R = -24.2$ MPa



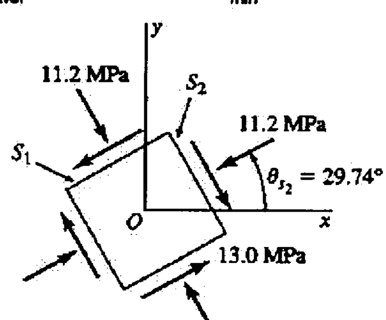
(b) Maximum shear stresses

$2\theta_{s_1} = -\alpha - 90° = -120.51° \qquad \theta_{s_1} = -60.26°$

$2\theta_{s_2} = 90° - \alpha = 59.49° \qquad \theta_{s_2} = 29.74°$

Point S_1: $\sigma_{aver} = -11.2$ MPa $\qquad \tau_{max} = R = 13.0$ MPa

Point S_2: $\sigma_{aver} = -11.2$ MPa $\qquad \tau_{min} = -13.0$ MPa

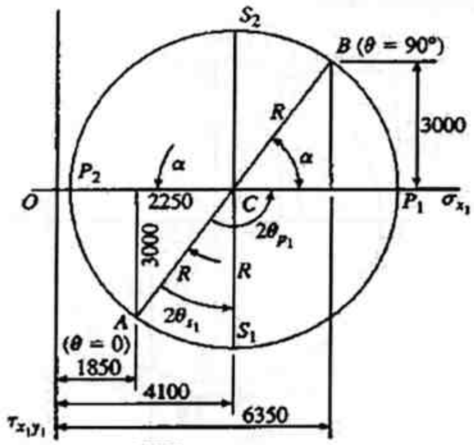


7.4-19

PRINCIPAL STRESSES

$\sigma_x = 1850$ psi $\qquad \sigma_y = 6350$ psi $\qquad \tau_{xy} = 3000$ psi

(All stresses in psi)



$R = \sqrt{(2250)^2 + (3000)^2} = 3750$ psi

$\alpha = \arctan \dfrac{3000}{2250} = 53.13°$

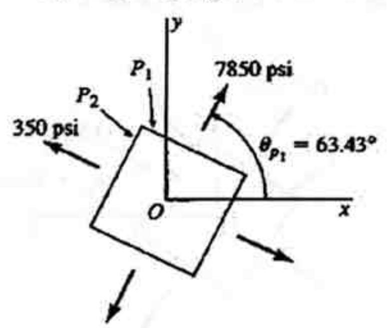
(a) Principal stresses

$2\theta_{p_1} = 180° - \alpha = 126.87° \qquad \theta_{p_1} = 63.43°$

$2\theta_{p_2} = -\alpha = -53.13° \qquad \theta_{p_2} = -26.57°$

Point P_1: $\sigma_1 = 4100 + R = 7850$ psi

Point P_2: $\sigma_2 = 4100 - R = 350$ psi



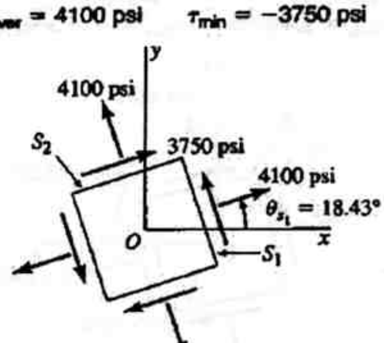
(b) Maximum shear stresses

$2\theta_{s_1} = 90° - \alpha = 36.87° \qquad \theta_{s_1} = 18.43°$

$2\theta_{s_2} = 270° - \alpha = 216.87° \qquad \theta_{s_2} = 108.43°$

Point S_1: $\sigma_{aver} = 4100$ psi $\qquad \tau_{max} = R = 3750$ psi

Point S_2: $\sigma_{aver} = 4100$ psi $\qquad \tau_{min} = -3750$ psi

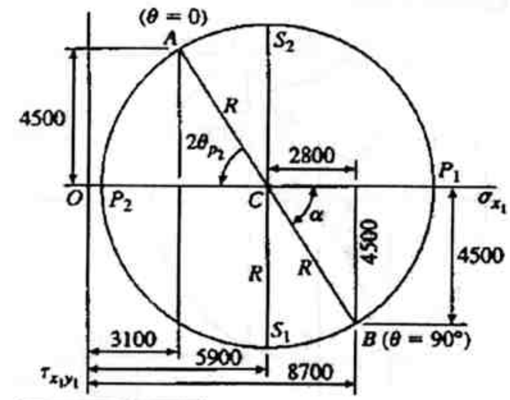


7.4-20

PRINCIPAL STRESSES

$\sigma_x = 3100$ kPa $\qquad \sigma_y = 8700$ kPa $\qquad \tau_{xy} = -4500$ kPa

(All stresses in kPa)



$R = \sqrt{(2800)^2 + (4500)^2} = 5300$ kPa

$\alpha = \arctan \dfrac{4500}{2800} = 58.11°$

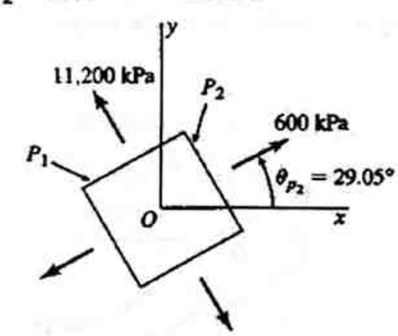
(a) Principal stresses

$2\theta_{p_1} = \alpha + 180° = 238.11° \qquad \theta_{p_1} = 119.05°$

$2\theta_{p_2} = \alpha = 58.11° \qquad \theta_{p_2} = 29.05°$

Point P_1: $\sigma_1 = 5900 + R = 11{,}200$ kPa

Point P_2: $\sigma_2 = 5900 - R = 600$ kPa



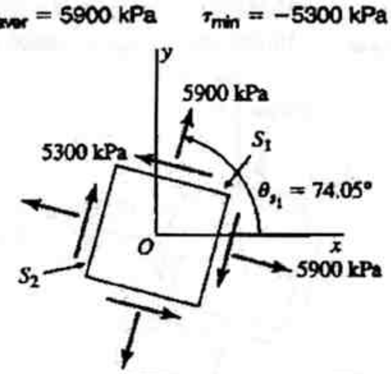
(b) Maximum shear stresses

$2\theta_{s_1} = 90° + \alpha = 148.11° \qquad \theta_{s_1} = 74.05°$

$2\theta_{s_2} = 270° + \alpha = 328.11° \qquad \theta_{s_2} = 164.05°$

Point S_1: $\sigma_{aver} = 5900$ kPa $\qquad \tau_{max} = R = 5300$ kPa

Point S_2: $\sigma_{aver} = 5900$ kPa $\qquad \tau_{min} = -5300$ kPa

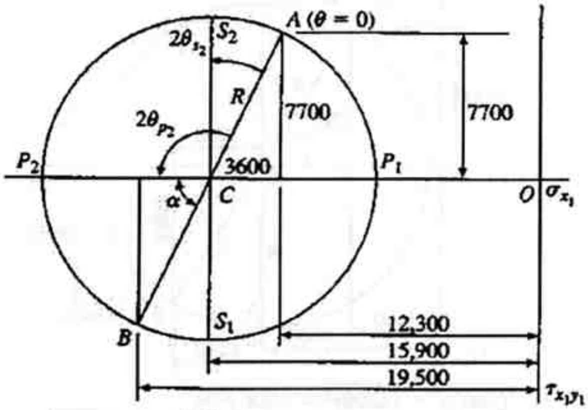


7.4-21

PRINCIPAL STRESSES

$\sigma_x = -12{,}300$ psi $\quad \sigma_y = -19{,}500$ psi $\quad \tau_{xy} = -7700$ psi

(All stresses in psi)



$R = \sqrt{(3600)^2 + (7700)^2} = 8500$ psi

$\alpha = \arctan \dfrac{7700}{3600} = 64.94°$

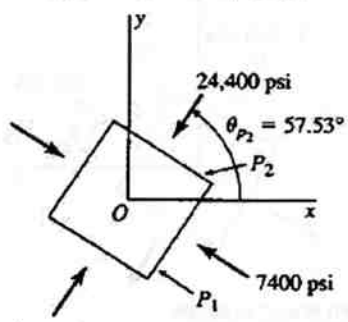
(a) Principal stresses

$2\theta_{p_1} = -\alpha = -64.94° \qquad \theta_{p_1} = -32.47°$

$2\theta_{p_2} = 180° - \alpha = 115.06° \qquad \theta_{p_2} = 57.53°$

Point P_1: $\sigma_1 = -15{,}900 + R = -7400$ psi

Point P_2: $\sigma_2 = -15{,}900 - R = -24{,}400$ psi



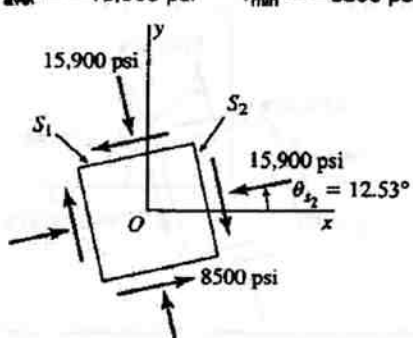
(b) Maximum shear stresses

$2\theta_{s_1} = 270° - \alpha = 205.06° \qquad \theta_{s_1} = 102.53°$

$2\theta_{s_2} = 90° - \alpha = 25.06° \qquad \theta_{s_2} = 12.53°$

Point S_1: $\sigma_{aver} = -15{,}900$ psi $\qquad \tau_{max} = R = 8500$ psi

Point S_2: $\sigma_{aver} = -15{,}900$ psi $\qquad \tau_{min} = -8500$ psi

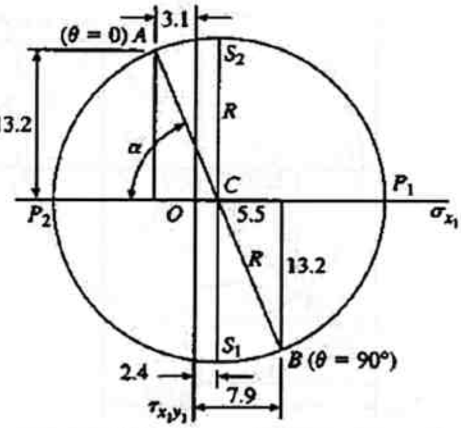


7.4-22

PRINCIPAL STRESSES

$\sigma_x = -3.1$ MPa $\qquad \sigma_y = 7.9$ MPa $\qquad \tau_{xy} = -13.2$ MPa

(All stresses in MPa)



$R = \sqrt{(5.5)^2 + (13.2)^2} = 14.3$ MPa

$\alpha = \arctan \dfrac{13.2}{5.5} = 67.38°$

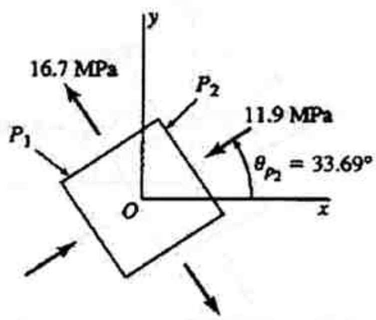
(a) Principal stresses

$2\theta_{p_1} = 180° + \alpha = 247.38° \qquad \theta_{p_1} = 123.69°$

$2\theta_{p_2} = \alpha = 67.38° \qquad \theta_{p_2} = 33.69°$

Point P_1: $\sigma_1 = 2.4 + R = 16.7$ MPa

Point P_2: $\sigma_2 = -R + 2.4 = -11.9$ MPa



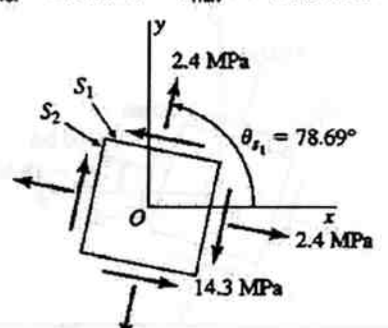
(b) Maximum shear stresses

$2\theta_{s_1} = \alpha + 90° = 157.38° \qquad \theta_{s_1} = 78.69°$

$2\theta_{s_2} = -90° + \alpha = -22.62° \qquad \theta_{s_2} = -11.31°$

Point S_1: $\sigma_{aver} = 2.4$ MPa $\qquad \tau_{max} = R = 14.3$ MPa

Point S_2: $\sigma_{aver} = 2.4$ MPa $\qquad \tau_{min} = -14.3$ MPa

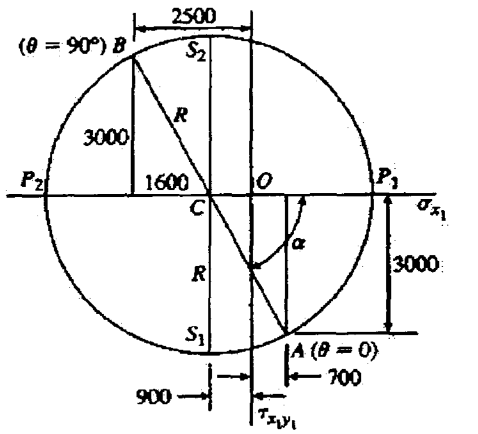


7.4-23

PRINCIPAL STRESSES

$\sigma_x = 700$ psi $\quad \sigma_y = -2500$ psi $\quad \tau_{xy} = 3000$ psi

(All stresses in psi)



$R = \sqrt{(1600)^2 + (3000)^2} = 3400$ psi

$\alpha = \arctan \dfrac{3000}{1600} = 61.93°$

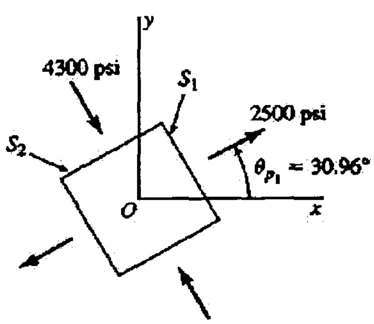
(a) Principal stresses

$2\theta_{p_1} = \alpha = 61.93° \qquad \theta_{p_1} = 30.96°$

$2\theta_{p_2} = 180° + \alpha = 241.93° \qquad \theta_{p_2} = 120.96°$

Point P_1: $\sigma_1 = -900 + R = 2500$ psi

Point P_2: $\sigma_2 = -900 - R = -4300$ psi



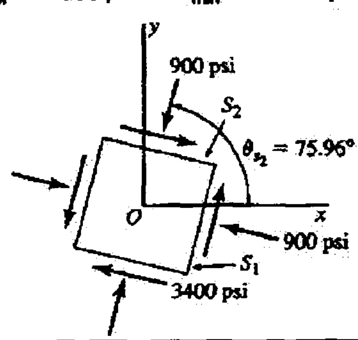
(b) Maximum shear stresses

$2\theta_{s_1} = -90° + \alpha = -28.07° \qquad \theta_{s_1} = -14.04°$

$2\theta_{s_2} = 90° + \alpha = 151.93° \qquad \theta_{s_2} = 75.96°$

Point S_1: $\sigma_{aver} = -900$ psi $\qquad \tau_{max} = R = 3400$ psi

Point S_2: $\sigma_{aver} = -900$ psi $\qquad \tau_{min} = -3400$ psi



7.4-24

RANGE OF VALUES OF SHEAR STRESS

$\sigma_x = 26$ MPa $\quad \sigma_y = -70$ MPa $\quad \tau_{allow} = 52$ MPa

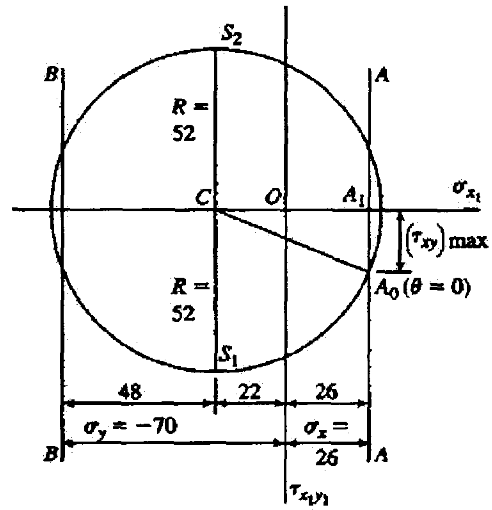
Find permissible values of τ_{xy}.

Procedure

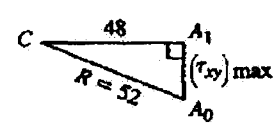
1. Construct axes σ_{x_1} and $\tau_{x_1y_1}$ with origin at point O.
2. Draw line A–A at $\sigma_x = 26$ MPa.
3. Draw line B–B at $\sigma_y = -70$ MPa.
4. Locate the center C of the circle midway between lines A–A and B–B.
5. Locate points S_1 and S_2 at $\tau_{allow} = 52$ MPa. (Therefore, radius $R = 52$ MPa.)
6. Draw Mohr's Circle with center at C and radius R.
7. Locate point A_0 at the intersection of the circle and line A–A. (Point A_0 represents the stresses on the x plane, for which $\theta = 0$.)
8. Calculate $(\tau_{xy})_{max}$ from triangle A_1CA_0.

Mohr's circle

All stresses in MPa. $R = 52$ MPa



Triangle A_1CA_0



$(\tau_{xy})_{max} = \sqrt{(52)^2 - (48)^2} = \pm 20$ MPa

If τ_{xy} is between -20 MPa and $+20$ MPa, the maximum shear stress τ_{max} does not exceed 52 MPa (see graph on next page.)

$-20 \le \tau_{xy} \le 20$ MPa $\leftarrow$

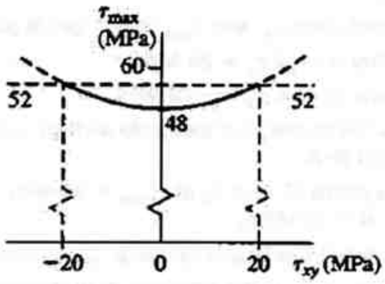
CONT.

7.4-24 (CONT.)

Graph of τ_{max} versus τ_{xy}

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

or $\tau_{max} = \sqrt{(48\text{ MPa})^2 + \tau_{xy}^2} = \sqrt{2304 + \tau_{xy}^2}$



7.4-25

RANGE OF VALUES OF SHEAR STRESS

$\sigma_x = -1800$ psi $\qquad \sigma_y = 7800$ psi $\qquad \tau_{allow} = 8000$ psi

Find permissible values of τ_{xy}.

Procedure

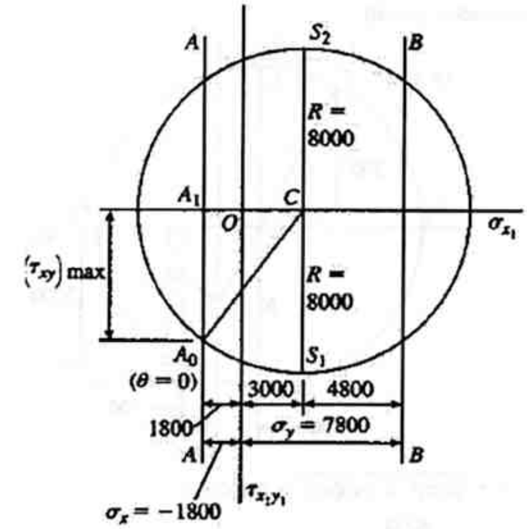
1. Construct axes σ_{x_1} and $\tau_{x_1y_1}$ with origin at point O.
2. Draw line A–A at $\sigma_x = -1800$ psi.
3. Draw line B–B at $\sigma_y = 7800$ psi.
4. Locate the center C of the circle midway between lines A–A and B–B.
5. Locate points S_1 and S_2 at $\tau_{allow} = 8000$ psi. (Therefore, radius $R = 8000$ psi.)
6. Draw Mohr's Circle with center at C and radius R.
7. Locate point A_0 at the intersection of the circle and line A–A. (Point A_0 represents the stresses on the x plane, for which $\theta = 0$.)
8. Calculate $(\tau_{xy})_{max}$ from triangle A_1CA_0.

CONT.

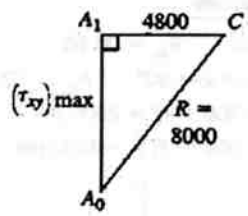
7.4-25 (CONT.)

Mohr's circle

All stresses in psi. $R = 8000$ psi



Triangle A_1CA_0



$(\tau_{xy})_{max} = \sqrt{(8000)^2 - (4800)^2} = \pm 6400$ psi

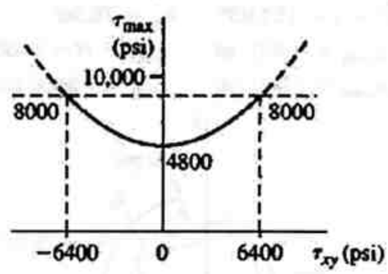
If τ_{xy} is between -6400 psi and $+6400$ psi, the maximum shear stress τ_{max} does not exceed 8000 psi (see graph below).

$-6400 \le \tau_{xy} \le 6400$ psi $\quad \leftarrow$

Graph of τ_{max} versus τ_{xy}

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

or $\tau_{max} = \sqrt{(-4800)^2 + \tau_{xy}^2} = \sqrt{23{,}040{,}000 + \tau_{xy}^2}$



7.5-1 Rectangular Plate in Biaxial Stress

$t = 0.25$ in $\quad \epsilon_x = 0.00062 \quad \epsilon_y = -0.00045$

$E = 30 \times 10^6$ psi $\quad \nu = 0.3$

Eq. (7-40a): $\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y) = 15{,}990$ psi ←

Eq. (7-40b): $\sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x) = -8{,}700$ psi ←

Eq. (7-39c): $\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -72.9 \times 10^{-6}$

$\Delta t = \epsilon_z t = -18.2 \times 10^{-6}$ in. ←

(Decrease in thickness)

7.5-2 Rectangular Plate in Biaxial Stress

$t = 10$ mm $\quad \epsilon_x = 350 \times 10^{-6} \quad \epsilon_y = 85 \times 10^{-6}$

$E = 200$ GPa $\quad \nu = 0.3$

Eq. (7-40a): $\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y) = 82.5$ MPa ←

Eq. (7-40b): $\sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x) = 41.8$ MPa ←

Eq. (7-39c): $\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -186 \times 10^{-6}$

$\Delta t = \epsilon_z t = -1860 \times 10^{-6}$ mm ←

(Decrease in thickness)

7.5-3 Plane Stress

Given: $\epsilon_x, \epsilon_y, \nu$

(a) Normal Strain ϵ_z

Eq. (7-34c): $\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$

Eq. (7-36a): $\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y)$

Eq. (7-36b): $\sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$

Substitute σ_x and σ_y into the first equation and simplify:

$$\epsilon_z = -\frac{\nu}{1-\nu}(\epsilon_x + \epsilon_y) \quad ←$$

(b) Dilatation

Eq. (7-47): $e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y)$

Substitute σ_x and σ_y into this equation and simplify:

$$e = \frac{1-2\nu}{1-\nu}(\epsilon_x + \epsilon_y) \quad ←$$

7.5-4 Biaxial Stress

$\sigma_x = 30$ MPa $\quad \sigma_y = 15$ MPa

$\epsilon_x = 550 \times 10^{-6} \quad \epsilon_y = 100 \times 10^{-6}$

Poisson's Ratio and Modulus of Elasticity

Eqs. (7-39 a and b): $\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

Substitute numerical values:

$E(550 \times 10^{-6}) = 30$ MPa $- \nu(15$ MPa$)$

$E(100 \times 10^{-6}) = 15$ MPa $- \nu(30$ MPa$)$

Solve simultaneously:

$\nu = 0.35 \quad E = 45$ GPa ←

7.5-5 Biaxial Stress

$\sigma_x = 18{,}000$ psi $\quad \sigma_y = -9000$ psi

$\epsilon_x = 700 \times 10^{-6} \quad \epsilon_y = -500 \times 10^{-6}$

Poisson's Ratio and Modulus of Elasticity

Eqs. (7-39 a and b): $\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

Substitute numerical values:

$E(700 \times 10^{-6}) = 18{,}000$ psi $- \nu(-9{,}000$ psi$)$

$E(-500 \times 10^{-6}) = -9{,}000$ psi $- \nu(18{,}000$ psi$)$

Solve simultaneously:

$\nu = 1/3 \quad E = 30 \times 10^6$ psi ←

7.5-6 Biaxial Stress

$\sigma_x = 65$ MPa $\quad \sigma_y = -20$ MPa

$E = 75$ GPa $\quad \nu = 0.33$

Dimensions of Plate: $200 \times 300 \times 15$ mm

Shear Modulus (Eq. 7-38): $G = \frac{E}{2(1+\nu)} = 28.20$ GPa

(a) Maximum in-plane Shear Strain

Principal Stresses: $\sigma_1 = 65$ MPa $\quad \sigma_2 = -20$ MPa

Eq. (7-26): $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 42.50$ MPa

Eq. (7-35): $\gamma_{max} = \frac{\tau_{max}}{G} = 1510 \times 10^{-6}$ ←

(b) Change in Thickness

Eq. (7-39c): $\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -198 \times 10^{-6}$

$\Delta t = \epsilon_z t = -2970 \times 10^{-6}$ mm ←

(Decrease in thickness)

(c) Change in Volume

From Eq. (7-47): $\Delta V = V_0\left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y)$

$V_0 = (200\text{ mm})(300\text{ mm})(15\text{ mm}) = 900 \times 10^3$ mm^3

$\left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y) = 204 \times 10^{-6}$

$\Delta V = 184$ mm^3 ←

(Increase in Volume)

7.5-7 Biaxial Stress

$\sigma_x = 9500$ psi $\quad \sigma_y = -3200$ psi

$E = 32 \times 10^6$ psi $\quad \nu = 0.29$

Dimensions of Plate: $10 \times 12 \times 1.0$ in.

Shear Modulus (Eq. 7-38): $G = \frac{E}{2(1+\nu)} = 12.40 \times 10^6$ psi

(a) Maximum in-plane Shear Strain

Principal Stresses: $\sigma_1 = 9500$ psi $\quad \sigma_2 = -3200$ psi

Eq. (7-26): $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 6350$ psi

Eq. (7-35): $\gamma_{max} = \frac{\tau_{max}}{G} = 512 \times 10^{-6}$ ←

(b) Change in Thickness

Eq. (7-39c): $\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -57.09 \times 10^{-6}$

$\Delta t = \epsilon_z t = -57.1 \times 10^{-6}$ in. ←

(Decrease in thickness)

(c) Change in Volume

From Eq. (7-47): $\Delta V = V_0\left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y)$

$V_0 = (10\text{ in.})(12\text{ in.})(1.0\text{ in.}) = 120$ in.^{3}

$\left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y) = 82.69 \times 10^{-6}$

$\Delta V = 0.00992$ in.^{3} ←

(Increase in Volume)

7.5-8 Biaxial Stress

Cube: $b = 40$ mm

$P = 120$ kN $\quad E = 100$ GPa $\quad \nu = 0.34$

$\sigma_x = \sigma_y = -\frac{P}{b^2} = -75.0$ MPa

Change in Volume

Eq. (7-47): $e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y) = -0.00048$

$V_0 = b^3 = (40\text{ mm})^3 = 64 \times 10^3$ mm^3

$\Delta V = eV_0 = -30.7$ mm^3 ←

(Decrease in Volume)

Strain Energy

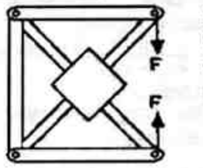
Eq. (7-50): $u = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y)$

$(\tau_{xy} = 0)$

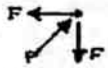
$= 0.03112$ MPa

$U = uV_0 = 2.38$ J ←

7.5-9 BIAXIAL STRESS – CONCRETE CUBE



$b = 4$ in.
$E = 3.0 \times 10^6$ psi
$\nu = 0.1$
$F = 20$ kips



$P = F\sqrt{2}$
$= 28.28$ kips

$\sigma_x = \sigma_y = -\dfrac{P}{b^2} = -1768$ psi

CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y) = -0.0009429$

$V_0 = b^3 = (4\text{ in})^3 = 64\text{ in}^3$

$\Delta V = eV_0 = -0.0603\text{ in}^3$ ←

(Decrease in Volume)

STRAIN ENERGY

Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y)$

$= 0.9377$ psi

$U = uV_0 = 60.0$ in.-lb ←

7.5-10 SQUARE PLATE IN PLANE STRESS

$b = 500$ mm $\quad t = 40$ mm
$E = 45$ GPa $\quad \nu = 0.35$

$P_x = 450$ kN $\quad \sigma_x = \dfrac{P_x}{bt} = 22.50$ MPa

$P_y = 150$ kN $\quad \sigma_y = \dfrac{P_y}{bt} = 7.50$ MPa

$V = 110$ kN $\quad \tau_{xy} = \dfrac{V}{bt} = 5.50$ MPa

CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y) = 0.0002$

$V_0 = b^2 t = 10.0 \times 10^6\text{ mm}^3$

$\Delta V = eV_0 = 2000\text{ mm}^3$ ←

(Increase in Volume)

STRAIN ENERGY

Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \dfrac{\tau_{xy}^2}{2G}$

$G = \dfrac{E}{2(1+\nu)} = 16.67$ GPa

Substitute numerical values:

$u = 5845$ Pa

$U = uV_0 = 58.5$ J ←

7.5-11 SQUARE PLATE IN PLANE STRESS

$b = 10$ in. $\quad t = 1.0$ in $\quad E = 10{,}600$ ksi $\quad \nu = 0.33$

$P_x = 120$ k $\quad \sigma_x = \dfrac{P_x}{bt} = 12{,}000$ psi

$P_y = 30$ k $\quad \sigma_y = \dfrac{P_y}{bt} = 3{,}000$ psi

$V = 20$ k $\quad \tau_{xy} = \dfrac{V}{bt} = 2{,}000$ psi

CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y) = 0.0004811$

$V_0 = b^2 t = 100\text{ in}^3$

$\Delta V = eV_0 = 0.0481\text{ in}^3$ ←

(Increase in Volume)

STRAIN ENERGY

Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \dfrac{\tau_{xy}^2}{2G}$

$G = \dfrac{E}{2(1+\nu)} = 3{,}985$ ksi

Substitute numerical values:

$u = 6.598$ psi

$U = uV_0 = 660$ in.-lb ←

7.5-12 PLATE IN BIAXIAL STRESS

$\sigma_x = 42$ MPa $\quad \sigma_y = 14$ MPa

Dimensions $400 \times 400 \times 20$ mm

Circle of diameter $d = 200$ mm

$E = 100$ GPa $\quad \nu = 0.34$

(a) CHANGE IN LENGTH OF DIAMETER IN X DIRECTION

Eq. (7-39a): $\epsilon_x = \dfrac{1}{E}(\sigma_x - \nu\sigma_y) = 0.0003724$

$\Delta ac = \epsilon_x d = 0.0745$ mm ← (increase)

(b) CHANGE IN LENGTH OF DIAMETER IN Y DIRECTION

Eq. (7-39b): $\epsilon_y = \dfrac{1}{E}(\sigma_y - \nu\sigma_x) = -0.00000280$

$\Delta bd = \epsilon_y d = -0.000560$ mm ← (decrease)

(c) CHANGE IN THICKNESS

Eq. (7-39c): $\epsilon_z = -\dfrac{\nu}{E}(\sigma_x + \sigma_y) = -0.0001904$

$\Delta t = \epsilon_z t = -0.00381$ mm ← (decrease)

(d) CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y) = 0.0001792$

$V_0 = (400)(400)(20) = 3.2 \times 10^6\text{ mm}^3$

$\Delta V = eV_0 = 573\text{ mm}^3$ ← (increase)

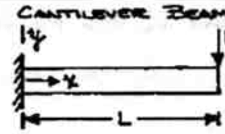
(e) STRAIN ENERGY

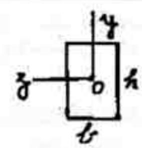
Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y)$

$= 0.007801$ MPa

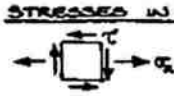
$U = uV_0 = 25.0$ J ←

7.5-13 CANTILEVER BEAM





STRESSES IN THE BEAM



$\sigma_x = \dfrac{My}{I} = \dfrac{P}{I}(L-x)y$

$\tau = \dfrac{VQ}{Ib} = \dfrac{PQ}{Ib} \qquad \sigma_y = 0$

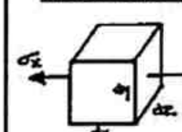
$I = \dfrac{bh^3}{12}$

The shear stress produces no change in volume.

DILATATION

Eq. (7-47): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y) = \dfrac{P(1-2\nu)}{EI}(L-x)y$

CHANGE IN VOLUME OF AN ELEMENT



dV = volume of element
$= dx\,dy\,dz$

$\Delta(dV)$ = change in volume of element

$\Delta(dV) = e(dV) = \dfrac{P(1-2\nu)}{EI}(L-x)(y)\,dx\,dy\,dz$

(a) Increase in volume of upper half of beam

$\Delta V_t = \int_{V_t} \Delta(dV) = \dfrac{P(1-2\nu)}{EI}\int_{V_t}(L-x)(y)\,dx\,dy\,dz$

To evaluate this integral over the upper half of the beam, we integrate z from $-b/2$ to $b/2$, we integrate y from 0 to $h/2$, and we integrate x from 0 to L.

CONT.

7.5-13 CONT.

$$\Delta V_t = \frac{P(1-2\nu)}{EI}\int_{-b/2}^{b/2}\int_0^{h/2}\int_0^L (L-x)(y)\,dx\,dy\,dz$$

$$= \frac{P(1-2\nu)}{EI}(z)\Big|_{-b/2}^{b/2}\left(\frac{y^2}{2}\right)\Big|_0^{h/2}\left(Lx-\frac{x^2}{2}\right)\Big|_0^L$$

$$= \frac{P(1-2\nu)}{EI}(b)\left(\frac{h^2}{8}\right)\left(\frac{L^2}{2}\right)$$

$$= \frac{3PL^2(1-2\nu)}{4Eh} \quad \text{(increase in volume)}$$

(b) Decrease in volume of lower half of beam

Because the stresses are numerically the same in both halves of the beam, ΔV_c is numerically the same as ΔV_t.

$\Delta V_c = -\Delta V_t$ (decrease in volume)

(c) Net change in volume $\Delta V = \Delta V_c + \Delta V_t = 0$

7.6-1 Triaxial Stress

$\sigma_x = 11{,}000$ psi $\quad \sigma_y = -5000$ psi $\quad \sigma_z = -1500$ psi

$a = 5$ in. $\quad b = 4$ in. $\quad c = 3$ in.

$E = 10{,}400$ ksi $\quad \nu = 0.33$

(a) Maximum Shear Stress

$\sigma_1 = 11{,}000$ psi $\quad \sigma_2 = -1500$ psi $\quad \sigma_3 = -5000$ psi

$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 8{,}000$ psi

(b) Changes in Dimensions

Eq. (7-53a): $\epsilon_x = \dfrac{\sigma_x}{E} - \dfrac{\nu}{E}(\sigma_y + \sigma_z) = 1264 \times 10^{-6}$

Eq. (7-53b): $\epsilon_y = \dfrac{\sigma_y}{E} - \dfrac{\nu}{E}(\sigma_z + \sigma_x) = -782.2 \times 10^{-6}$

Eq. (7-53c): $\epsilon_z = \dfrac{\sigma_z}{E} - \dfrac{\nu}{E}(\sigma_x + \sigma_y) = -334.6 \times 10^{-6}$

$$\left.\begin{aligned}\Delta a &= a\epsilon_x = 0.00632 \text{ in.}\\ \Delta b &= b\epsilon_y = -0.00313 \text{ in.}\\ \Delta c &= c\epsilon_z = -0.00100 \text{ in.}\end{aligned}\right\} \quad \left\{\begin{aligned}&+ = \text{increase}\\ &- = \text{decrease}\end{aligned}\right.$$

(c) Change in Volume

Eq. (7-56): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = 147.1 \times 10^{-6}$

$\Delta V = e(abc) = 0.00883$ in.^{3} (increase)

(d) Strain Energy

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z)$

$= 9.158$ psi

$U = u(abc) = 549$ in.-lb

7.6-2 Triaxial Stress

$\sigma_x = -50$ MPa $\quad \sigma_y = -40$ MPa $\quad \sigma_z = -36$ MPa

$a = 250$ mm $\quad b = 150$ mm $\quad c = 150$ mm

$E = 200$ GPa $\quad \nu = 0.30$

(a) Maximum Shear Stress

$\sigma_1 = -36$ MPa $\quad \sigma_2 = -40$ MPa $\quad \sigma_3 = -50$ MPa

$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 7.00$ MPa

(b) Changes in Dimension

Eq. (7-53a): $\epsilon_x = \dfrac{\sigma_x}{E} - \dfrac{\nu}{E}(\sigma_y + \sigma_z) = -136.0 \times 10^{-6}$

Eq. (7-53b): $\epsilon_y = \dfrac{\sigma_y}{E} - \dfrac{\nu}{E}(\sigma_z + \sigma_x) = -71.0 \times 10^{-6}$

Eq. (7-53c): $\epsilon_z = \dfrac{\sigma_z}{E} - \dfrac{\nu}{E}(\sigma_x + \sigma_y) = -45.00 \times 10^{-6}$

CONT.

7.6-2 CONT.

$$\left.\begin{aligned}\Delta a &= a\epsilon_x = -0.03400 \text{ mm}\\ \Delta b &= b\epsilon_y = -0.01065 \text{ mm}\\ \Delta c &= c\epsilon_z = -0.00675 \text{ mm}\end{aligned}\right\} \quad (- = \text{decrease})$$

(c) Change in Volume

Eq. (7-56): $e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = -252.0 \times 10^{-6}$

$\Delta V = e(abc) = -1418$ mm^3 (decrease)

(d) Strain Energy

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z) = 5630$ Pa

$U = u(abc) = 31.7$ J

7.6-3 Triaxial Stress

$\epsilon_x = -350 \times 10^{-6} \quad \epsilon_y = -65 \times 10^{-6}$

$\epsilon_z = -65 \times 10^{-6}$

$a = 3$ in. $\quad E = 14{,}000$ ksi $\quad \nu = 0.25$

(a) Normal Stresses

Eq. (7-54a): $\sigma_x = \dfrac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_x + \nu(\epsilon_y + \epsilon_z)]$

$= -6608$ psi

In a similar manner, Eqs. (7-54 b and c) give

$\sigma_y = -3416$ psi $\quad \sigma_z = -3416$ psi

(b) Maximum Shear Stress

$\sigma_1 = -3416$ psi $\quad \sigma_2 = -3416$ psi $\quad \sigma_3 = -6608$ psi

$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 1600$ psi

(c) Change in Volume

Eq. (7-55): $e = \epsilon_x + \epsilon_y + \epsilon_z = -480 \times 10^{-6}$

$\Delta V = ea^3 = -0.0130$ in.^{3} (decrease)

(d) Strain Energy

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z) = 1.378$ psi

$U = ua^3 = 37.2$ in.-lb

7.6-4 Triaxial Stress

$\epsilon_x = -620 \times 10^{-6} \quad \epsilon_y = -250 \times 10^{-6} \quad \epsilon_z = -250 \times 10^{-6}$

$a = 50$ mm $\quad E = 60$ GPa $\quad \nu = 0.25$

(a) Normal Stresses

Eq. (7-54a): $\sigma_x = \dfrac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_x + \nu(\epsilon_y + \epsilon_z)]$

$= -56.64$ MPa

In a similar manner, Eqs. (7-54 b and c) give

$\sigma_y = -38.88$ MPa $\quad \sigma_z = -38.88$ MPa

(b) Maximum Shear Stress

$\sigma_1 = -38.88$ MPa $\quad \sigma_2 = -38.88$ MPa $\quad \sigma_3 = -56.64$ MPa

$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 8.88$ MPa

(c) Change in Volume

Eq. (7-55): $e = \epsilon_x + \epsilon_y + \epsilon_z = -1120 \times 10^{-6}$

$\Delta V = ea^3 = -140$ mm^3 (decrease)

(d) Strain Energy

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z)$

$= 0.02728$ MPa

$U = ua^3 = 3.41$ J

7.6-5 Triaxial Stress

$\sigma_x = 5200$ psi $\quad \sigma_y = -4750$ psi $\quad \sigma_z = -3090$ psi

$\epsilon_x = 713.8 \times 10^{-6} \quad \epsilon_y = -502.3 \times 10^{-6}$

Find K.

Eq. (7-53a): $\epsilon_x = \dfrac{\sigma_x}{E} - \dfrac{\nu}{E}(\sigma_y + \sigma_z)$

Eq. (7-53b): $\epsilon_y = \dfrac{\sigma_y}{E} - \dfrac{\nu}{E}(\sigma_z + \sigma_x)$

CONT.

7.6-5 CONT.

Substitute numerical values and rearrange:

$$(713.8\times10^{-6})E = 5200 + 7840\,\nu \qquad (1)$$

$$(-502.3\times10^{-6})E = -4750 - 2110\,\nu \qquad (2)$$

(E has units of psi)

Solve simultaneously Eqs. (1) and (2):

$E = 10.801\times10^{6}$ psi $\quad \nu = 0.3202$

$$K = \frac{E}{3(1-2\nu)} = 10.0\times10^{6}\text{ psi} \longleftarrow$$

7.6-6 TRIAXIAL STRESS

$\sigma_x = -4.5$ MPa $\quad \sigma_y = -3.6$ MPa $\quad \sigma_z = -2.1$ MPa

$\epsilon_x = -740\times10^{-6} \quad \epsilon_y = -320\times10^{-6}$

Find K

Eq. (7-53a): $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y+\sigma_z)$

Eq. (7-53b): $\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_z+\sigma_x)$

Substitute numerical values and rearrange:

$$(-740\times10^{-6})E = -4.5 + 5.7\,\nu \qquad (1)$$

$$(-320\times10^{-6})E = -3.6 + 6.6\,\nu \qquad (2)$$

(E = MPa)

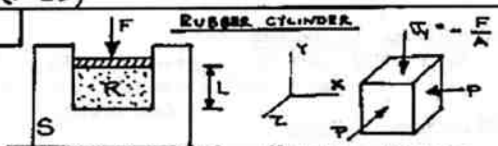
Solve simultaneously Eqs. (1) and (2):

$E = 3000$ MPa $= 3.0$ GPa $\quad \nu = 0.40$

$$K = \frac{E}{3(1-2\nu)} = 5.0\text{ GPa} \longleftarrow$$

7.6-7

RUBBER CYLINDER



$\sigma_x = -p \quad \sigma_z = -p$

$\epsilon_x = \epsilon_z = 0$

(a) Lateral Pressure

Eq. (7-53a): $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y+\sigma_z)$

or $\quad 0 = -p - \nu\left(-\frac{F}{A} - p\right)$

Solve for p: $\quad p = \frac{\nu}{1-\nu}\left(\frac{F}{A}\right) \longleftarrow$

(b) Shortening:

Eq. (7-53b): $\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_z+\sigma_x)$

$$= -\frac{F}{EA} - \frac{\nu}{E}(-2p)$$

Substitute for p and simplify:

$$\epsilon_y = \frac{F}{EA}\,\frac{(1+\nu)(-1+2\nu)}{1+\nu}$$

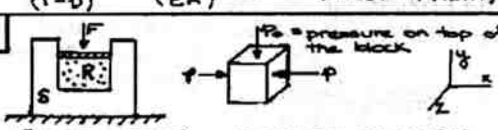
(Positive ϵ_y means increase in strain)

$\delta = -\epsilon_y L \quad$ (+ = shortening)

(Positive δ means shortening of the rubber cylinder)

$$\delta = \frac{(1+\nu)(1-2\nu)}{(1-\nu)}\left(\frac{FL}{EA}\right) \longleftarrow$$

7.6-8



$\sigma_x = -p \quad \sigma_y = -p_0 \quad \sigma_z = 0 \qquad$ R = BLOCK OF RUBBER

$\epsilon_x = 0 \quad \epsilon_y \neq 0 \quad \epsilon_z \neq 0$

(a) LATERAL PRESSURE

Eq (7-53a): $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y+\sigma_z)$

or $\quad 0 = -p - \nu(-p_0) \quad \therefore p = \nu p_0 \longleftarrow$

(b) DILATATION

Eq. (7-56): $\quad e = \frac{1-2\nu}{E}(\sigma_x+\sigma_y+\sigma_z)$

$$= \frac{1-2\nu}{E}(-p-p_0)$$

Substitute for p:

$$e = -\frac{(1+\nu)(1-2\nu)p_0}{E} \longleftarrow$$

CONT.

7.6-8 CONT.

(c) STRAIN ENERGY DENSITY

Eq. (7-57b): $\mu = \frac{1}{2E}(\sigma_x^2+\sigma_y^2+\sigma_z^2) - \frac{\nu}{E}(\sigma_x\sigma_y+\sigma_x\sigma_z+\sigma_y\sigma_z)$

Substitute for $\sigma_x, \sigma_y, \sigma_z$, and p:

$$\mu = \frac{(1-\nu^2)p_0^2}{2E} \longleftarrow$$

7.6-9 Brass Sphere $\quad E = 15\times10^{6}$ psi $\quad \nu = 0.34$

Lowered in the ocean to depth h = 10,000 ft

Diameter d = 11.0 in.

Sea water: $\gamma = 63.8$ lb/ft³

Pressure: $\sigma_0 = \gamma h = 638{,}000$ lb/ft² $= 4431$ psi

DECREASE IN DIAMETER

Eq. (7-59): $\epsilon_0 = \frac{\sigma_0}{E}(1-2\nu) = 94.53\times10^{-6}$

$\Delta d = \epsilon_0 d = 1.04\times10^{-3}$ in. $\longleftarrow$
(decrease)

DECREASE IN VOLUME

Eq. (7-60): $e = 3\epsilon_0 = 283.6\times10^{-6}$

$$V_0 = \frac{4}{3}\pi r^3 = \frac{4}{3}(\pi)\left(\frac{11.0\text{ in.}}{2}\right)^3 = 696.9\text{ in.}^3$$

$\Delta V = eV_0 = 0.198$ in.³ $\longleftarrow$
(decrease)

STRAIN ENERGY

Use Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$$\mu = \frac{3(1-2\nu)\sigma_0^2}{2E} = 0.6283\text{ psi}$$

$U = \mu V_0 = 438$ in.-lb $\longleftarrow$

7.6-10 STEEL SPHERE $\quad E = 210$ GPa $\quad \nu = 0.3$

Hydrostatic Pressure: $\Delta V = 0.004\,V_0$

$$e = \frac{\Delta V}{V_0} = 0.004$$

(a) PRESSURE

Eq. (7-60): $\quad e = \frac{3\sigma_0(1-2\nu)}{E}$

$$\sigma_0 = \frac{eE}{3(1-2\nu)} = 700\text{ MPa}$$

PRESSURE $p = 700$ MPa $\longleftarrow$

(b) VOLUME MODULUS OF ELASTICITY

Eq. (7-63): $\quad K = \frac{\sigma_0}{e} = \frac{700\text{ MPa}}{0.004}$

$= 175$ GPa $\longleftarrow$

(c) STRAIN ENERGY

$d = 150$ mm $\quad r = 75$ mm

From Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$$\mu = \frac{3(1-2\nu)\sigma_0^2}{2E} = 1.40\text{ MPa}$$

$$V_0 = \frac{4\pi r^3}{3} = 1767\times10^{-6}\text{ m}^3$$

$U = \mu V_0 = 2470$ J $\longleftarrow$

7.6-11 BRONZE SPHERE $\quad K = 14.5\times10^{6}$ psi

$\sigma_0 = 12{,}000$ psi

STRAIN IN THE SPHERE

Eq. (7-59): $\epsilon_0 = \frac{\sigma_0}{E}(1-2\nu)$

Eq. (7-61): $K = \frac{E}{3(1-2\nu)}$

Combine Equations: $\epsilon_0 = \frac{\sigma_0}{3K} = 276\times10^{-6} \longleftarrow$

CONT.

7.6-11 CONT.

UNIT VOLUME CHANGE

Eq. (7-62): $e = \frac{\sigma_0}{K} = 828 \times 10^{-6}$ ←

STRAIN ENERGY DENSITY

Eq. (7-57b): with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$$\mu = \frac{3(1-2\nu)\sigma_0^2}{2E} = \frac{\sigma_0^2}{2K}$$

$= 4.97$ psi ←

7.6-12

CUBE OF MAGNESIUM $\quad b = 100$ mm

LOWERED INTO THE OCEAN: $\Delta b = 0.018$ mm

$E = 45$ GPa $\quad \nu = 0.35 \quad \gamma = 10.0$ kN/m³

(a) DEPTH d IN THE OCEAN

STRAIN: $\epsilon_0 = \frac{0.018\text{mm}}{100\text{mm}} = 180 \times 10^{-6}$

From Eq. (7-59): $\sigma_0 = \frac{E\epsilon_0}{1-2\nu} = 27.0$ MPa

OR $p = 27.0$ MPa

$p = \gamma d \quad \therefore \quad d = \frac{p}{\gamma} = \frac{27.0 \text{ MPa}}{10.0 \text{ kN/m}^3}$

$= 2700$ m ←

(b) INCREASE IN DENSITY

Eq. (7-60): $e = 3\epsilon_0$

Decrease in Volume: $\Delta V = eV_0 = 3\epsilon_0 V_0$

Initial Volume $= V_0$

Final Volume $= V_0 - 3\epsilon_0 V_0$

$= V_0(1-3\epsilon_0)$

M = mass of sphere

Initial density: $\rho_0 = \frac{M}{V_0}$

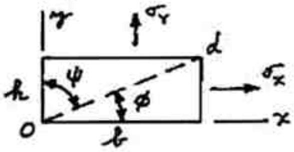
Final density: $\rho_1 = \frac{M}{V_0(1-3\epsilon_0)}$

Percent increase in density $= \left(\frac{\rho_1-\rho_0}{\rho_0}\right)100$

$= \frac{3\epsilon_0}{1-3\epsilon_0}(100) = 0.05403\%$ ←

SECTION 7.7 BEGINS ON THE NEXT PAGE →

7.7-1 PLATE IN BIAXIAL STRESS



$b = 8.0$ IN.
$h = 4.0$ IN.
$\epsilon_x = 195 \times 10^{-6}$
$\epsilon_y = -125 \times 10^{-6}$
$\gamma_{xy} = 0$

$\phi = \text{ARCTAN}\ \dfrac{h}{b} = 26.57°$

$L_d = \sqrt{b^2 + h^2} = 8.944$ IN.

(a) INCREASE IN LENGTH OF DIAGONAL

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

FOR $\theta = \phi = 26.57°$: $\epsilon_{x_1} = 130.98 \times 10^{-6}$

$\Delta d = \epsilon_{x_1} L_d = 0.00117$ IN. ←

(b) CHANGE IN ANGLE ϕ

EQ. (7-68): $\alpha = -(\epsilon_x - \epsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

FOR $\theta = \phi = 26.57°$: $\alpha = -128.0 \times 10^{-6}$ RAD

MINUS SIGN MEANS LINE od ROTATES CLOCKWISE (ANGLE ϕ DECREASES).

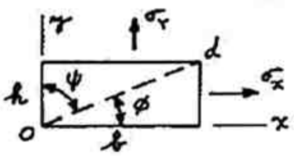
$\Delta\phi = 128 \times 10^{-6}$ RAD (DECREASE) ←

(c) CHANGE IN ANGLE ψ

ANGLE ψ INCREASES THE SAME AMOUNT THAT ϕ DECREASES.

$\Delta\psi = 128 \times 10^{-6}$ RAD (INCREASE) ←

7.7-2 PLATE IN BIAXIAL STRESS



$b = 160$ MM
$h = 60$ MM
$\epsilon_x = 410 \times 10^{-6}$
$\epsilon_y = -320 \times 10^{-6}$
$\gamma_{xy} = 0$

$\phi = \text{ARCTAN}\ \dfrac{h}{b} = 20.56°$

$L_d = \sqrt{b^2 + h^2} = 170.88$ MM

(a) INCREASE IN LENGTH OF DIAGONAL

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

FOR $\theta = \phi = 20.56°$: $\epsilon_{x_1} = 319.97 \times 10^{-6}$

$\Delta d = \epsilon_{x_1} L_d = 0.0547$ MM ←

(b) CHANGE IN ANGLE ϕ

EQ. (7-68): $\alpha = -(\epsilon_x - \epsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

FOR $\theta = \phi = 20.56°$: $\alpha = -240.0 \times 10^{-6}$ RAD

MINUS SIGN MEANS LINE od ROTATES CLOCKWISE (ANGLE ϕ DECREASES).

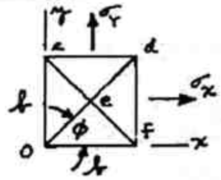
$\Delta\phi = 240 \times 10^{-6}$ RAD (DECREASE) ←

(c) CHANGE IN ANGLE ψ

ANGLE ψ INCREASES THE SAME AMOUNT THAT ϕ DECREASES.

$\Delta\psi = 240 \times 10^{-6}$ RAD (INCREASE) ←

7.7-3 SQUARE PLATE IN BIAXIAL STRESS



$b = 12.0$ IN.
$\epsilon_x = 427 \times 10^{-6}$
$\epsilon_y = 113 \times 10^{-6}$
$\phi = 45°$ $\quad \gamma_{xy} = 0$
$L_d = b\sqrt{2} = 16.97$ IN.

(a) INCREASE IN LENGTH OF DIAGONAL

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

FOR $\theta = \phi = 45°$: $\epsilon_{x_1} = 270 \times 10^{-6}$

$\Delta d = \epsilon_{x_1} L_d = 0.00458$ IN. ←

(b) CHANGE IN ANGLE ϕ

EQ. (7-68): $\alpha = -(\epsilon_x - \epsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

FOR $\theta = \phi = 45°$: $\alpha = -157 \times 10^{-6}$ RAD

MINUS SIGN MEANS LINE od ROTATES CLOCKWISE (ANGLE ϕ DECREASES).

$\Delta\phi = 157 \times 10^{-6}$ RAD (DECREASE) ←

(c) SHEAR STRAIN BETWEEN DIAGONALS

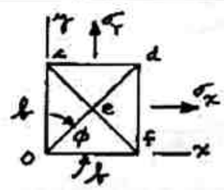
EQ. (7-71b): $\dfrac{\gamma_{x_1y_1}}{2} = -\dfrac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \dfrac{\gamma_{xy}}{2}\cos 2\theta$

FOR $\theta = \phi = 45°$: $\gamma_{x_1y_1} =$ [illegible] $\times 10^{-6}$ RAD

(NEGATIVE STRAIN MEANS ANGLE aed I[illegible]CREASES)

$\gamma = -314 \times 10^{-6}$ RAD ←

7.7-4 SQUARE PLATE IN BIAXIAL STRESS



$b = 225$ MM
$\epsilon_x = 845 \times 10^{-6}$
$\epsilon_y = 211 \times 10^{-6}$
$\phi = 45°$ $\quad \gamma_{xy} = 0$
$L_d = b\sqrt{2} = 318.2$ MM

(a) INCREASE IN LENGTH OF DIAGONAL

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

FOR $\theta = \phi = 45°$: $\epsilon_{x_1} = 528 \times 10^{-6}$

$\Delta d = \epsilon_{x_1} L_d = 0.168$ MM ←

(b) CHANGE IN ANGLE ϕ

EQ. (7-68): $\alpha = -(\epsilon_x - \epsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

FOR $\theta = \phi = 45°$: $\alpha = -317 \times 10^{-6}$ RAD

MINUS SIGN MEANS LINE od ROTATES CLOCKWISE (ANGLE ϕ DECREASES).

$\Delta\phi = 317 \times 10^{-6}$ RAD (DECREASE) ←

(c) SHEAR STRAIN BETWEEN DIAGONALS

EQ. (7-71b): $\dfrac{\gamma_{x_1y_1}}{2} = -\dfrac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \dfrac{\gamma_{xy}}{2}\cos 2\theta$

FOR $\theta = \phi = 45°$: $\gamma_{x_1y_1} = -634 \times 10^{-6}$ RAD

(NEGATIVE STRAIN MEANS ANGLE aed INCREASES)

$\gamma = -634 \times 10^{-6}$ RAD ←

7.7-5

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 220\times10^{-6} \qquad \varepsilon_y = 480\times10^{-6} \qquad \gamma_{xy} = 180\times10^{-6}$

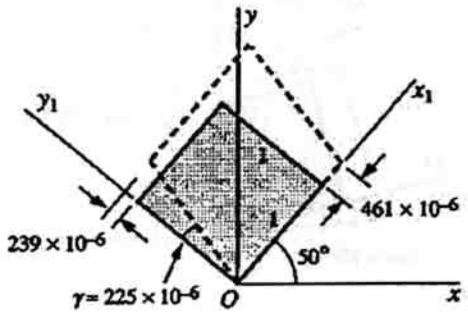
$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$$

For $\theta = 50°$:

$\varepsilon_{x_1} = 461\times10^{-6} \qquad \gamma_{x_1y_1} = 225\times10^{-6} \qquad \varepsilon_{y_1} = 239\times10^{-6}$



7.7-6

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 420\times10^{-6} \qquad \varepsilon_y = -170\times10^{-6} \qquad \gamma_{xy} = 310\times10^{-6}$

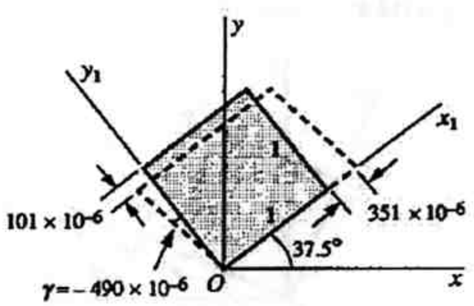
$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$$

For $\theta = 37.5°$:

$\varepsilon_{x_1} = 351\times10^{-6} \qquad \gamma_{x_1y_1} = -490\times10^{-6} \qquad \varepsilon_{y_1} = -101\times10^{-6}$



7.7-7

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 480\times10^{-6} \qquad \varepsilon_y = 140\times10^{-6} \qquad \gamma_{xy} = -350\times10^{-6}$

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 310\times10^{-6} \pm 244\times10^{-6}$$

$\varepsilon_1 = 554\times10^{-6} \qquad \varepsilon_2 = 66\times10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y} = -1.0294$$

$2\theta_p = -45.8°$ and $134.2°$

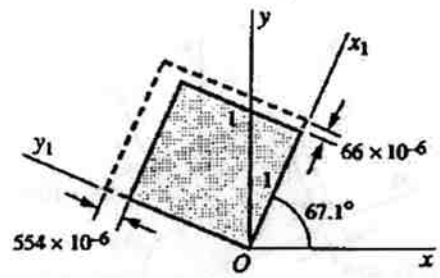
$\theta_p = -22.9°$ and $67.1°$

For $\theta_p = -22.9°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = 554\times10^{-6}$$

$\therefore \theta_{p_1} = -22.9° \qquad \varepsilon_1 = 554\times10^{-6}$ ←

$\theta_{p_2} = 67.1° \qquad \varepsilon_2 = 66\times10^{-6}$ ←



Maximum shear strains

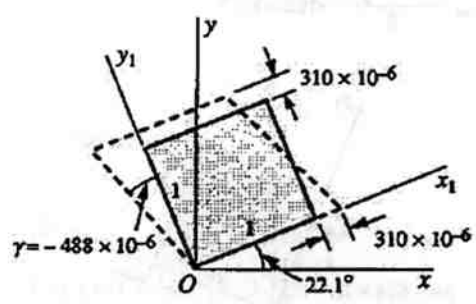
$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 244\times10^{-6}$$

$\gamma_{max} = 488\times10^{-6}$

$\theta_{s_1} = \theta_{p_1} - 45° = -67.9°$ or $112.1° \qquad \gamma_{max} = 488\times10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 22.1° \qquad \gamma_{min} = -488\times10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x+\varepsilon_y}{2} = 310\times10^{-6}$$



7.7-8

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 120\times10^{-6} \qquad \varepsilon_y = -450\times10^{-6} \qquad \gamma_{xy} = -360\times10^{-6}$

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= -165\times10^{-6} \pm 337\times10^{-6}$$

$\varepsilon_1 = 172\times10^{-6} \qquad \varepsilon_2 = -502\times10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y} = -0.6316$$

$2\theta_p = 327.7^\circ$ and 147.7°

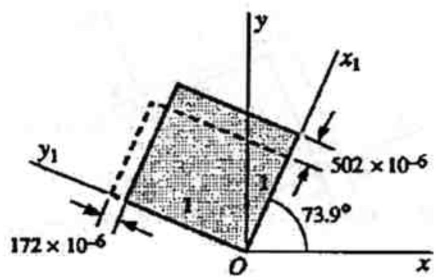
$\theta_p = 163.9^\circ$ and 73.9°

For $\theta_p = 163.9^\circ$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = 172\times10^{-6}$$

$\therefore \theta_{p_1} = 163.9^\circ \qquad \varepsilon_1 = 172\times10^{-6} \quad \leftarrow$

$\theta_{p_2} = 73.9^\circ \qquad \varepsilon_2 = -502\times10^{-6} \quad \leftarrow$



Maximum shear strains

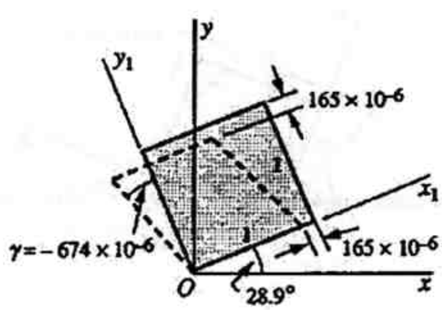
$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2} = 337\times10^{-6}$$

$\gamma_{max} = 674\times10^{-6}$

$\theta_{s_1} = \theta_{p_1} - 45^\circ = 118.9^\circ \qquad \gamma_{max} = 674\times10^{-6} \quad \leftarrow$

$\theta_{s_2} = \theta_{s_1} - 90^\circ = 28.9^\circ \qquad \gamma_{min} = -674\times10^{-6} \quad \leftarrow$

$$\varepsilon_{aver} = \frac{\varepsilon_x+\varepsilon_y}{2} = -165\times10^{-6}$$



7.7-9

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 480\times10^{-6} \qquad \varepsilon_y = 70\times10^{-6} \qquad \gamma_{xy} = 420\times10^{-6}$

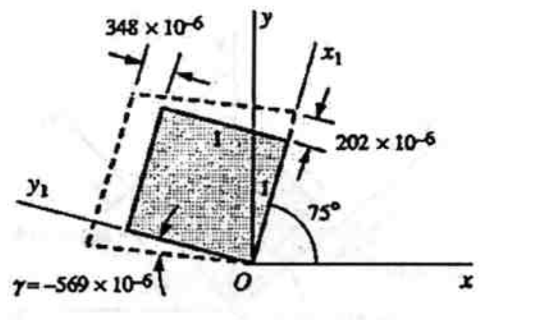
$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$

For $\theta = 75^\circ$:

$\varepsilon_{x_1} = 202\times10^{-6} \qquad \gamma_{x_1y_1} = -569\times10^{-6} \qquad \varepsilon_{y_1} = 348\times10^{-6}$



Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= 275\times10^{-6} \pm 293\times10^{-6}$$

$\varepsilon_1 = 568\times10^{-6} \qquad \varepsilon_2 = -18\times10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y} = 1.0244$$

$2\theta_p = 45.69^\circ$ and 225.69°

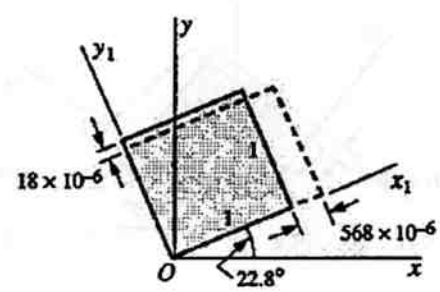
$\theta_p = 22.85^\circ$ and 112.85°

For $\theta_p = 22.85^\circ$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = 568\times10^{-6}$$

$\therefore \theta_{p_1} = 22.8^\circ \qquad \varepsilon_1 = 568\times10^{-6} \quad \leftarrow$

$\theta_{p_2} = 112.8^\circ \qquad \varepsilon_2 = -18\times10^{-6} \quad \leftarrow$



CONT.

7.7-9 CONT.

Maximum shear strains

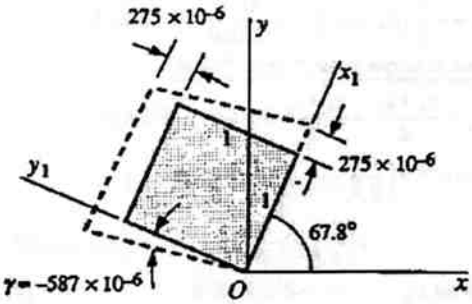
$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 293\times10^{-6}$$

$$\gamma_{max} = 587\times10^{-6}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -22.2°$ or $157.8°$ $\quad \gamma_{max} = 587\times10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 67.8°$ $\quad \gamma_{min} = -587\times10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = 275\times10^{-6}$$



7.7-10

ELEMENT IN PLANE STRAIN

$\varepsilon_x = -1120\times10^{-6}$ $\quad \varepsilon_y = -430\times10^{-6}$ $\quad \gamma_{xy} = 780\times10^{-6}$

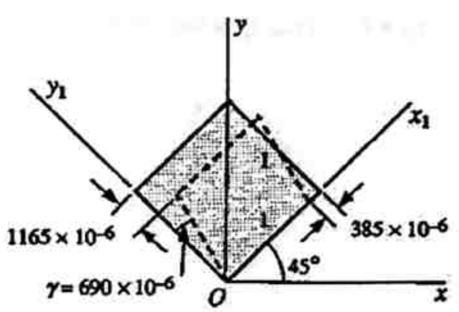
$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$$

For $\theta = 45°$:

$\varepsilon_{x_1} = -385\times10^{-6}$ $\quad \gamma_{x_1y_1} = 690\times10^{-6}$ $\quad \varepsilon_{y_1} = -1165\times10^{-6}$



7.7-10 CONT.

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= -775\times10^{-6} \pm 521\times10^{-6}$$

$\varepsilon_1 = -254\times10^{-6}$ $\quad \varepsilon_2 = -1296\times10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -1.1304$$

$2\theta_p = 131.5°$ and $311.5°$

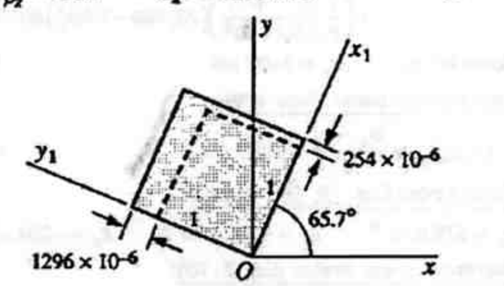
$\theta_p = 65.7°$ and $155.7°$

For $\theta_p = 65.7°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = -254\times10^{-6}$$

$\therefore \theta_{p_1} = 65.7°$ $\quad \varepsilon_1 = -254\times10^{-6}$ ←

$\theta_{p_2} = 155.7°$ $\quad \varepsilon_2 = -1296\times10^{-6}$ ←



Maximum shear strains

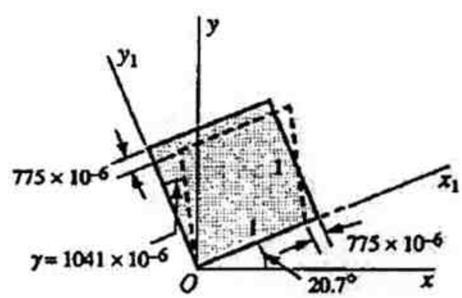
$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 521\times10^{-6}$$

$$\gamma_{max} = 1041\times10^{-6}$$

$\theta_{s_1} = \theta_{p_1} - 45° = 20.7°$ $\quad \gamma_{max} = 1041\times10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 110.7°$ $\quad \gamma_{min} = -1041\times10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = -775\times10^{-6}$$



CONT.

7.7-11

STEEL PLATE IN BIAXIAL STRESS

$\sigma_x = 18{,}000$ psi $\gamma_{xy} = 0$ $\sigma_y = ?$

$E = 30\times10^6$ psi $\nu = 0.30$

Strain gage: $\phi = 30°$ $\varepsilon = 407\times10^{-6}$

Units: All stresses in psi.

Strains in biaxial stress (Eqs. 7-39)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{30\times10^6}(18{,}000 - 0.3\sigma_y) \quad (1)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{30\times10^6}(\sigma_y - 5400) \quad (2)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{0.3}{30\times10^6}(18{,}000 + \sigma_y) \quad (3)$$

Strains at angle $\phi = 30°$ (Eq. 7-71a)

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$407\times10^{-6} = \left(\frac{1}{2}\right)\left(\frac{1}{30\times10^6}\right)(12{,}600 + 0.7\sigma_y) + \left(\frac{1}{2}\right)\left(\frac{1}{30\times10^6}\right)(23{,}400 - 1.3\sigma_y)\cos 60°$$

Solve for σ_y: $\sigma_y = 2400$ psi (4)

Maximum in-plane shear stress

$$(\tau_{max})_{xy} = \frac{\sigma_x - \sigma_y}{2} = 7800 \text{ psi} \quad \leftarrow$$

Strains from Eqs. (1), (2), and (3)

$\varepsilon_x = 576\times10^{-6}$ $\varepsilon_y = -100\times10^{-6}$ $\varepsilon_z = -204\times10^{-6}$

Maximum shear strains (Eq. 7-75)

xy plane: $$\frac{(\gamma_{max})_{xy}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = 0$ $(\gamma_{max})_{xy} = 676\times10^{-6}$ ←

xz plane: $$\frac{(\gamma_{max})_{xz}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{xz}}{2}\right)^2}$$

$\gamma_{xz} = 0$ $(\gamma_{max})_{xz} = 780\times10^{-6}$ ←

yz plane: $$\frac{(\gamma_{max})_{yz}}{2} = \sqrt{\left(\frac{\varepsilon_y - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{yz}}{2}\right)^2}$$

$\gamma_{yz} = 0$ $(\gamma_{max})_{yz} = 104\times10^{-6}$ ←

7.7-12

ALUMINUM PLATE IN BIAXIAL STRESS

$\sigma_x = 86.4$ MPa $\gamma_{xy} = 0$ $\sigma_y = ?$

$E = 72$ GPa $\nu = 1/3$

Strain gage: $\phi = 21°$ $\varepsilon = 946\times10^{-6}$

Units: All stresses in MPa.

Strains in biaxial stress (Eqs. 7-39)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{72{,}000}\left(86.4 - \frac{1}{3}\sigma_y\right) \quad (1)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{72{,}000}(\sigma_y - 28.8) \quad (2)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{1/3}{72{,}000}(86.4 + \sigma_y) \quad (3)$$

Strains at angle $\phi = 21°$ (Eq. 7-71a)

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$946\times10^{-6} = \left(\frac{1}{2}\right)\left(\frac{1}{72{,}000}\right)\left(57.6 + \frac{2}{3}\sigma_y\right) + \left(\frac{1}{2}\right)\left(\frac{1}{72{,}000}\right)\left(115.2 - \frac{4}{3}\sigma_y\right)\cos 42°$$

Solve for σ_y: $\sigma_y = 21.55$ MPa (4)

Maximum in-plane shear stress

$$(\tau_{max})_{xy} = \frac{\sigma_x - \sigma_y}{2} = 32.4 \text{ MPa} \quad \leftarrow$$

Strains from Eqs. (1), (2), and (3)

$\varepsilon_x = 1100\times10^{-6}$ $\varepsilon_y = -101\times10^{-6}$ $\varepsilon_z = -500\times10^{-6}$

Maximum shear strains (Eq. 7-75)

xy plane: $$\frac{(\gamma_{max})_{xy}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = 0$ $(\gamma_{max})_{xy} = 1200\times10^{-6}$ ←

xz plane: $$\frac{(\gamma_{max})_{xz}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{xz}}{2}\right)^2}$$

$\gamma_{xz} = 0$ $(\gamma_{max})_{xz} = 1600\times10^{-6}$ ←

yz plane: $$\frac{(\gamma_{max})_{yz}}{2} = \sqrt{\left(\frac{\varepsilon_y - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{yz}}{2}\right)^2}$$

$\gamma_{yz} = 0$ $(\gamma_{max})_{yz} = 399\times10^{-6}$ ←

7.7-13

ELEMENT IN PLANE STRESS

$\sigma_x = -8400$ psi $\quad \sigma_y = 1100$ psi $\quad \tau_{xy} = -1700$ psi

$E = 10{,}000$ ksi $\quad \nu = 0.33$

Hooke's law (Eqs. 7-34 and 7-35)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = -876.3\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 387.2\times10^{-6}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\tau_{xy}(1+\nu)}{E} = -452.2\times10^{-6}$$

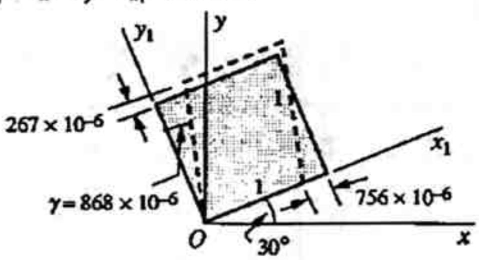
For $\theta = 30°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = -756\times10^{-6}$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta = 434\times10^{-6}$$

$$\gamma_{x_1y_1} = 868\times10^{-6}$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1} = 267\times10^{-6}$$



Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= -245\times10^{-6} \pm 671\times10^{-6}$$

$\varepsilon_1 = 426\times10^{-6} \quad \varepsilon_2 = -916\times10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y} = 0.3579$$

$2\theta_p = 19.7°$ and $199.7°$

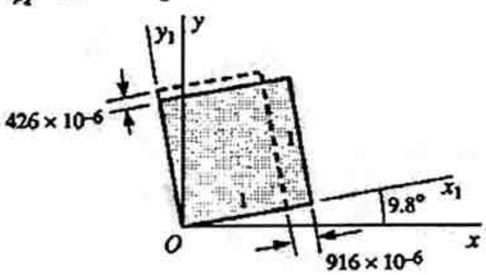
$\theta_p = 9.8°$ and $99.8°$

For $\theta_p = 9.8°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = -916\times10^{-6}$$

$\therefore \theta_{p_1} = 99.8° \quad \varepsilon_1 = 426\times10^{-6} \quad \leftarrow$

$\theta_{p_2} = 9.8° \quad \varepsilon_2 = -916\times10^{-6} \quad \leftarrow$



CONT.

7.7-13 CONT.

Maximum shear strains

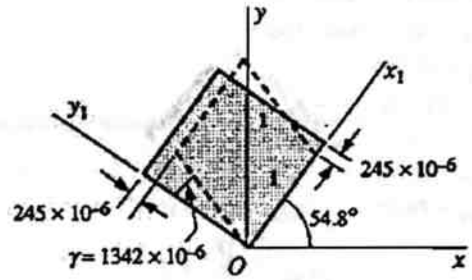
$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 671\times10^{-6}$$

$$\gamma_{max} = 1342\times10^{-6}$$

$\theta_{s_1} = \theta_{p_1} - 45° = 54.8° \quad \gamma_{max} = 1342\times10^{-6} \quad \leftarrow$

$\theta_{s_2} = \theta_{s_1} + 90° = 144.8° \quad \gamma_{min} = -1342\times10^{-6} \quad \leftarrow$

$$\varepsilon_{aver} = \frac{\varepsilon_x+\varepsilon_y}{2} = -245\times10^{-6}$$



7.7-14

ELEMENT IN PLANE STRESS

$\sigma_x = -150$ MPa $\quad \sigma_y = -210$ MPa $\quad \tau_{xy} = -16$ MPa

$E = 100$ GPa $\quad \nu = 0.34$

Hooke's law (Eqs. 7-34 and 7-35)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = -786\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = -1590\times10^{-6}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\tau_{xy}(1+\nu)}{E} = -429\times10^{-6}$$

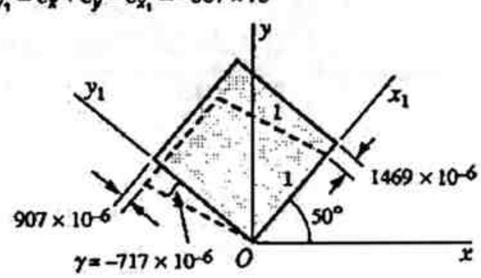
For $\theta = 50°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = -1469\times10^{-6}$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta = -358.5\times10^{-6}$$

$$\gamma_{x_1y_1} = -717\times10^{-6}$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1} = -907\times10^{-6}$$



CONT.

7.7-14 CONT.

Principal strains

$$\varepsilon_{1,2}=\frac{\varepsilon_x+\varepsilon_y}{2}\pm\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$=-1188\times10^{-6}\pm456\times10^{-6}$$

$\varepsilon_1=-732\times10^{-6}$ $\quad\varepsilon_2=-1644\times10^{-6}$

$$\tan2\theta_p=\frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y}=-0.5333$$

$2\theta_p=151.9°$ and $331.9°$

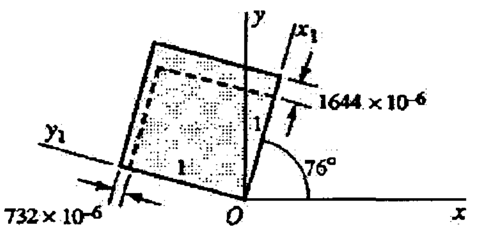
$\theta_p=76.0°$ and $166.0°$

For $\theta_p=76.0°$:

$$\varepsilon_{x_1}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos2\theta+\frac{\gamma_{xy}}{2}\sin2\theta=-1644\times10^{-6}$$

$\therefore\theta_{p_1}=166.0°$ $\quad\varepsilon_1=-732\times10^{-6}$ ←

$\theta_{p_2}=76.0°$ $\quad\varepsilon_2=-1644\times10^{-6}$ ←



Maximum shear strains

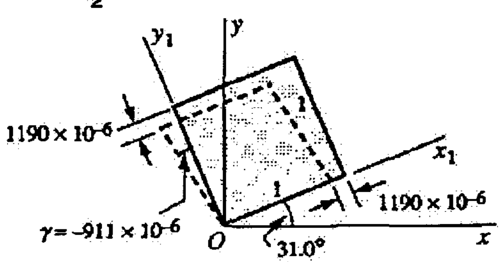
$$\frac{\gamma_{max}}{2}=\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}=456\times10^{-6}$$

$\gamma_{max}=911\times10^{-6}$

$\theta_{s_1}=\theta_{p_1}-45°=121.0°$ $\quad\gamma_{max}=911\times10^{-6}$ ←

$\theta_{s_2}=\theta_{s_1}-90°=31.0°$ $\quad\gamma_{min}=-911\times10^{-6}$ ←

$$\varepsilon_{aver}=\frac{\varepsilon_x+\varepsilon_y}{2}=-1190\times10^{-6}$$



7.7-15

45° STRAIN ROSETTE

$\varepsilon_A=520\times10^{-6}$ $\quad\varepsilon_B=360\times10^{-6}$ $\quad\varepsilon_C=-80\times10^{-6}$

From Eqs. (7-77) and (7-78) of Example 7-8:

$\varepsilon_x=\varepsilon_A=520\times10^{-6}$ $\quad\varepsilon_y=\varepsilon_C=-80\times10^{-6}$

$\gamma_{xy}=2\varepsilon_B-\varepsilon_A-\varepsilon_C=280\times10^{-6}$

Principal strains

$$\varepsilon_{1,2}=\frac{\varepsilon_x+\varepsilon_y}{2}\pm\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}=220\times10^{-6}\pm331\times10^{-6}$$

$\varepsilon_1=551\times10^{-6}$ $\quad\varepsilon_2=-111\times10^{-6}$

$$\tan2\theta_p=\frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y}=0.4667$$

$2\theta_p=25.0°$ and $205.0°$

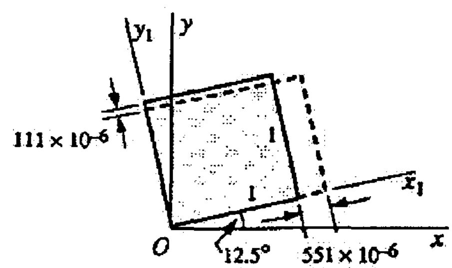
$\theta_p=12.5°$ and $102.5°$

For $\theta_p=12.5°$:

$$\varepsilon_{x_1}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos2\theta+\frac{\gamma_{xy}}{2}\sin2\theta=551\times10^{-6}$$

$\therefore\theta_{p_1}=12.5°$ $\quad\varepsilon_1=551\times10^{-6}$ ←

$\theta_{p_2}=102.5°$ $\quad\varepsilon_2=-111\times10^{-6}$ ←



Maximum shear strains

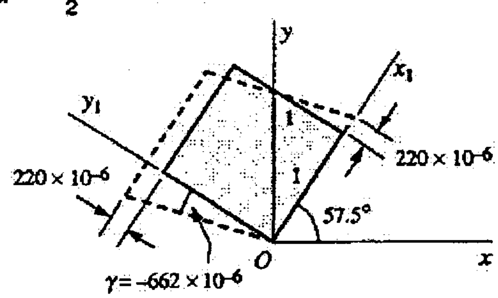
$$\frac{\gamma_{max}}{2}=\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}=331\times10^{-6}$$

$\gamma_{max}=662\times10^{-6}$

$\theta_{s_1}=\theta_{p_1}-45°=-32.5°$ or $147.5°$ $\quad\gamma_{max}=662\times10^{-6}$ ←

$\theta_{s_2}=\theta_{s_1}+90°=57.5°$ $\quad\gamma_{min}=-662\times10^{-6}$ ←

$$\varepsilon_{aver}=\frac{\varepsilon_x+\varepsilon_y}{2}=220\times10^{-6}$$



7.7-16

45° STRAIN ROSETTE

$\varepsilon_A = 310\times10^{-6}$ $\quad\varepsilon_B = 180\times10^{-6}$ $\quad\varepsilon_C = -160\times10^{-6}$

From Eqs. (7-77) and (7-78) of Example 7-8:

$\varepsilon_x = \varepsilon_A = 310\times10^{-6}$ $\quad\varepsilon_y = \varepsilon_C = -160\times10^{-6}$

$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C = 210\times10^{-6}$

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 75\times10^{-6} \pm 257\times10^{-6}$$

$\varepsilon_1 = 332\times10^{-6}$ $\quad\varepsilon_2 = -182\times10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y} = 0.4468$$

$2\theta_p = 24.1°$ and $204.1°$

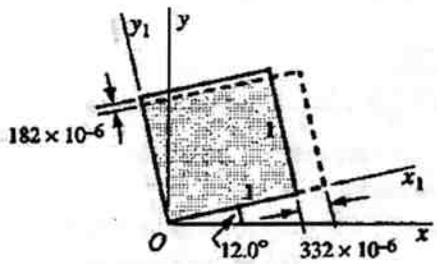
$\theta_p = 12.0°$ and $102.0°$

For $\theta_p = 12.0°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = 332\times10^{-6}$$

$\therefore \theta_{p_1} = 12.0°$ $\quad\varepsilon_1 = 332\times10^{-6}$ ←

$\theta_{p_2} = 102.0°$ $\quad\varepsilon_2 = -182\times10^{-6}$ ←



Maximum shear strains

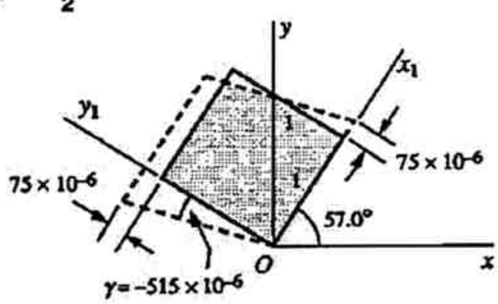
$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 257\times10^{-6}$$

$\gamma_{max} = 515\times10^{-6}$

$\theta_{s_1} = \theta_{p_1} - 45° = -33.0°$ or $147.0°$ $\quad\gamma_{max} = 515\times10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 57.0°$ $\quad\gamma_{min} = -515\times10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x+\varepsilon_y}{2} = 75\times10^{-6}$$



7.7-17

CIRCULAR BAR (PLANE STRESS)

Bar is subjected to a torque T and an axial force P.

$E = 30\times10^6$ psi $\quad\nu = 0.29$ $\quad$ Diameter d = 1.5 in.

Strain gages $\quad$ At $\theta = 0°$: $\varepsilon_A = \varepsilon_x = 100\times10^{-6}$

At $\theta = 45°$: $\varepsilon_B = -55\times10^{-6}$

Element in plane stress

$$\sigma_x = \frac{P}{A} = \frac{4P}{\pi d^2} \qquad \sigma_y = 0 \qquad \tau_{xy} = -\frac{16T}{\pi d^3}$$

$\varepsilon_x = 100\times10^{-6}$ $\quad\varepsilon_y = -\nu\varepsilon_x = -29\times10^{-6}$

Axial force P

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{4P}{\pi d^2 E} \qquad P = \frac{\pi d^2 E\varepsilon_x}{4} = 5300 \text{ lb} \quad ←$$

Shear strain

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\tau_{xy}(1+\nu)}{E} = -\frac{32T(1+\nu)}{\pi d^3 E}$$

$$= -(0.1298\times10^{-6})T \qquad (T = \text{lb-in.})$$

Strain at $\theta = 45°$

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$\varepsilon_{x_1} = \varepsilon_B = -55\times10^{-6}$ $\quad 2\theta = 90°$

Substitute numerical values into Eq. (1):

$-55\times10^{-6} = 35.5\times10^{-6} - (0.0649\times10^{-6})T$

Solve for T: $\quad T = 1390$ lb-in. ←

Maximum shear strain and maximum shear stress

$\gamma_{xy} = -(0.1298\times10^{-6})T = -180.4\times10^{-6}$ rad

Eq. (7-75): $$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 111\times10^{-6} \text{ rad}$$

$\gamma_{max} = 222\times10^{-6}$ rad ←

$\tau_{max} = G\gamma_{max} = 2580$ psi ←

7.7-18

CANTILEVER BEAM (PLANE STRESS)

Beam loaded by a force P acting at an angle α.

$E = 200$ GPa $\nu = 1/3$ $b = 25$ mm $h = 100$ mm

Axial force $F = P\sin\alpha$ Shear force $V = P\cos\alpha$

(At the neutral axis, the bending moment produces no stresses.)

<u>Strain gages</u> At $\theta = 0°$: $\varepsilon_A = \varepsilon_x = 125\times10^{-6}$

At $\theta = 60°$: $\varepsilon_B = -375\times10^{-6}$

<u>Element in plane stress</u>

$$\sigma_x = \frac{F}{A} = \frac{P\sin\alpha}{bh} \qquad \sigma_y = 0 \qquad \tau_{xy} = -\frac{3V}{2A} = -\frac{3P\cos\alpha}{2bh}$$

$$\varepsilon_x = 125\times10^{-6} \qquad \varepsilon_y = -\nu\varepsilon_x = -41.67\times10^{-6}$$

<u>Hooke's law</u>

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P\sin\alpha}{bhE} \qquad P\sin\alpha = bhE\varepsilon_x = 62{,}500 \text{ N} \qquad (1)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = -\frac{3P\cos\alpha}{2bhG} = -\frac{3(1+\nu)P\cos\alpha}{bhE}$$

$$= -(8.0\times10^{-9})P\cos\alpha \qquad (2)$$

<u>For $\theta = 60°$:</u>

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos2\theta + \frac{\gamma_{xy}}{2}\sin2\theta \qquad (3)$$

$$\varepsilon_{x_1} = \varepsilon_B = -375\times10^{-6} \qquad 2\theta = 120°$$

Substitute into Eq. (3):

$$-375\times10^{-6} = 41.67\times10^{-6} - 41.67\times10^{-6}$$
$$-(3.464\times10^{-9})P\cos\alpha$$

or $P\cos\alpha = 108{,}260$ N (4)

<u>Solve Eqs. (1) and (4):</u>

$\tan\alpha = 0.5773$ $\alpha = 30°$ ←

$P = 125$ kN ←

7.7-19

CANTILEVER BEAM (PLANE STRESS)

Beam loaded by a force P acting at an angle α.

$E = 6.0\times10^6$ psi $\nu = 0.35$ $b = 1.0$ in. $h = 3.0$ in.

Axial force $F = P\sin\alpha$ Shear force $V = P\cos\alpha$

(At the neutral axis, the bending moment produces no stresses.)

<u>Strain gages</u> At $\theta = 0°$: $\varepsilon_A = \varepsilon_x = 171\times10^{-6}$

At $\theta = 75°$: $\varepsilon_B = -266\times10^{-6}$

<u>Element in plane stress</u>

$$\sigma_x = \frac{F}{A} = \frac{P\sin\alpha}{bh} \qquad \sigma_y = 0 \qquad \tau_{xy} = -\frac{3V}{2A} = -\frac{3P\cos\alpha}{2bh}$$

$$\varepsilon_x = 171\times10^{-6} \qquad \varepsilon_y = -\nu\varepsilon_x = -59.85\times10^{-6}$$

<u>Hooke's law</u>

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P\sin\alpha}{bhE} \qquad P\sin\alpha = bhE\varepsilon_x = 3078 \text{ lb} \qquad (1)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = -\frac{3P\cos\alpha}{2bhG} = -\frac{3(1+\nu)P\cos\alpha}{bhE}$$

$$= -(225.0\times10^{-9})P\cos\alpha \qquad (2)$$

<u>For $\theta = 75°$:</u>

$$\varepsilon_{x_1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos2\theta + \frac{\gamma_{xy}}{2}\sin2\theta \qquad (3)$$

$$\varepsilon_{x_1} = \varepsilon_B = -266\times10^{-6} \qquad 2\theta = 150°$$

Substitute into Eq. (3):

$$-266\times10^{-6} = 55.575\times10^{-6} - 99.961\times10^{-6}$$
$$-(56.25\times10^{-9})P\cos\alpha$$

or $P\cos\alpha = 3939.8$ lb (4)

<u>Solve Eqs. (1) and (4):</u>

$\tan\alpha = 0.7813$ $\alpha = 38°$ ←

$P = 5000$ lb ←

7.7-20

DELTA ROSETTE (60° STRAIN ROSETTE)

Strain gages Gage A at $\theta = 0°$ Strain $= \varepsilon_A$

Gage B at $\theta = 60°$ Strain $= \varepsilon_B$

Gage C at $\theta = 120°$ Strain $= \varepsilon_C$

For $\theta = 0°$: $\varepsilon_x = \varepsilon_A$ ←

For $\theta = 60°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_B = \frac{\varepsilon_A + \varepsilon_y}{2} + \frac{\varepsilon_A - \varepsilon_y}{2}(\cos 120°) + \frac{\gamma_{xy}}{2}(\sin 120°)$$

$$\varepsilon_B = \frac{\varepsilon_A}{4} + \frac{3\varepsilon_y}{4} + \frac{\gamma_{xy}\sqrt{3}}{4} \qquad (1)$$

For $\theta = 120°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_C = \frac{\varepsilon_A + \varepsilon_y}{2} + \frac{\varepsilon_A - \varepsilon_y}{2}(\cos 240°) + \frac{\gamma_{xy}}{2}(\sin 240°)$$

$$\varepsilon_C = \frac{\varepsilon_A}{4} + \frac{3\varepsilon_y}{4} - \frac{\gamma_{xy}\sqrt{3}}{4} \qquad (2)$$

Solve Eqs. (1) and (2):

$$\varepsilon_y = \frac{1}{3}(2\varepsilon_B + 2\varepsilon_C - \varepsilon_A) \quad \leftarrow$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_B - \varepsilon_C) \quad \leftarrow$$

7.7-21

30-60-90° STRAIN ROSETTE

Magnesium alloy: $E = 6000$ ksi $\nu = 0.35$

Strain gages Gage A at $\theta = 0°$ $\varepsilon_A = 1100 \times 10^{-6}$

Gage B at $\theta = 90°$ $\varepsilon_B = 200 \times 10^{-6}$

Gage C at $\theta = 150°$ $\varepsilon_C = 200 \times 10^{-6}$

For $\theta = 0°$: $\varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

For $\theta = 90°$: $\varepsilon_y = \varepsilon_B = 200 \times 10^{-6}$

For $\theta = 150°$:

$$\varepsilon_{x_1} = \varepsilon_C = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$200 \times 10^{-6} = 650 \times 10^{-6} + 225 \times 10^{-6} - 0.43301\gamma_{xy}$$

Solve for γ_{xy}: $\gamma_{xy} = 1558.9 \times 10^{-6}$

CONT.

7.7-21 CONT.

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 650 \times 10^{-6} \pm 900 \times 10^{-6}$$

$\varepsilon_1 = 1550 \times 10^{-6}$ $\varepsilon_2 = -250 \times 10^{-6}$

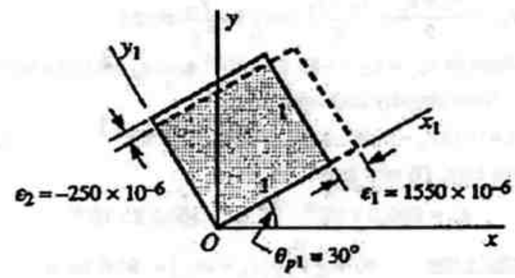
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \sqrt{3} = 1.7321 \qquad 2\theta_p = 60° \qquad \theta_p = 30°$$

For $\theta_p = 30°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = 1550 \times 10^{-6}$$

$\therefore \theta_{p_1} = 30°$ $\varepsilon_1 = 1550 \times 10^{-6}$ ←

$\theta_{p_2} = 120°$ $\varepsilon_2 = -250 \times 10^{-6}$ ←

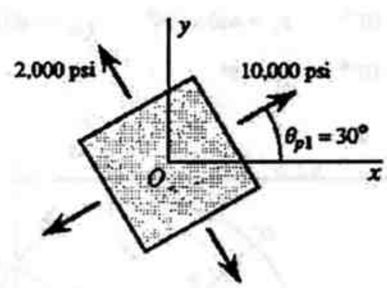


Principal stresses (see Eqs. 7-36)

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \qquad \sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

Substitute numerical values:

$\sigma_1 = 10{,}000$ psi $\sigma_2 = 2{,}000$ psi ←



7.7-22

40-40-100° STRAIN ROSETTE

Pure aluminum: $E = 70$ GPa $\quad \nu = 0.33$

Strain gages: Gage A at $\theta = 0°$ $\quad \varepsilon_A = 1100 \times 10^{-6}$

Gage B at $\theta = 40°$ $\quad \varepsilon_B = 1496 \times 10^{-6}$

Gage C at $\theta = 140°$ $\quad \varepsilon_C = -39.44 \times 10^{-6}$

For $\theta = 0°$: $\quad \varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

For $\theta = 40°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_B = 1496 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$;

then simplify and rearrange:

$$0.41318\varepsilon_y + 0.49240\gamma_{xy} = 850.49 \times 10^{-6} \qquad (1)$$

For $\theta = 140°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_C = -39.44 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$;

then simplify and rearrange:

$$0.41318\varepsilon_y - 0.49240\gamma_{xy} = -684.95 \times 10^{-6} \qquad (2)$$

Solve Eqs. (1) and (2):

$$\varepsilon_y = 200.3 \times 10^{-6} \qquad \gamma_{xy} = 1559.2 \times 10^{-6}$$

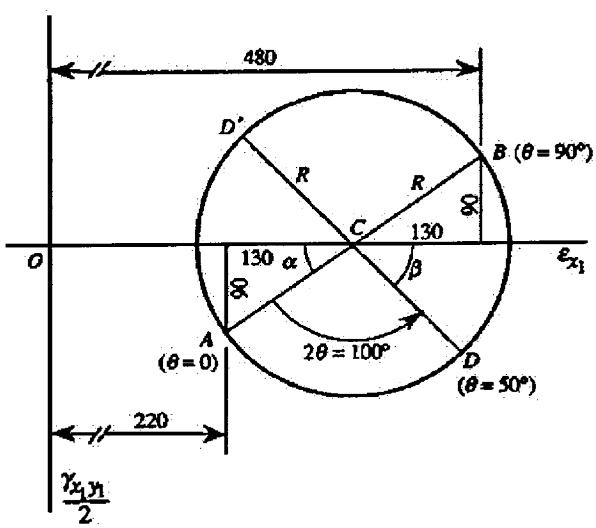
Hooke's law $\quad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = 91.6 \text{ MPa} \quad \leftarrow$

7.7-23

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 220 \times 10^{-6}$ $\quad \varepsilon_y = 480 \times 10^{-6}$ $\quad \gamma_{xy} = 180 \times 10^{-6}$

$\frac{\gamma_{xy}}{2} = 90 \times 10^{-6}$ $\quad \theta = 50°$



CONT.

7.7-23 CONT.

$$R = \sqrt{(130 \times 10^{-6})^2 + (90 \times 10^{-6})^2} = 158.11 \times 10^{-6}$$

$\alpha = \arctan\frac{90}{130} = 34.70°$ $\qquad \beta = 180° - \alpha - 2\theta = 45.30°$

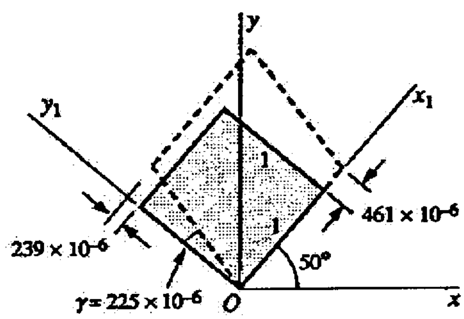
Point C: $\quad \varepsilon_{x_1} = 350 \times 10^{-6}$

Point D ($\theta = 50°$): $\quad \varepsilon_{x_1} = 350 \times 10^{-6} + R\cos\beta = 461 \times 10^{-6}$

$\frac{\gamma_{x_1y_1}}{2} = R\sin\beta = 112.4 \times 10^{-6}$ $\qquad \gamma_{x_1y_1} = 225 \times 10^{-6}$

Point D' ($\theta = 140°$): $\quad \varepsilon_{x_1} = 350 \times 10^{-6} - R\cos\beta = 239 \times 10^{-6}$

$\frac{\gamma_{x_1y_1}}{2} = -R\sin\beta = -112.4 \times 10^{-6}$ $\qquad \gamma_{x_1y_1} = -225 \times 10^{-6}$

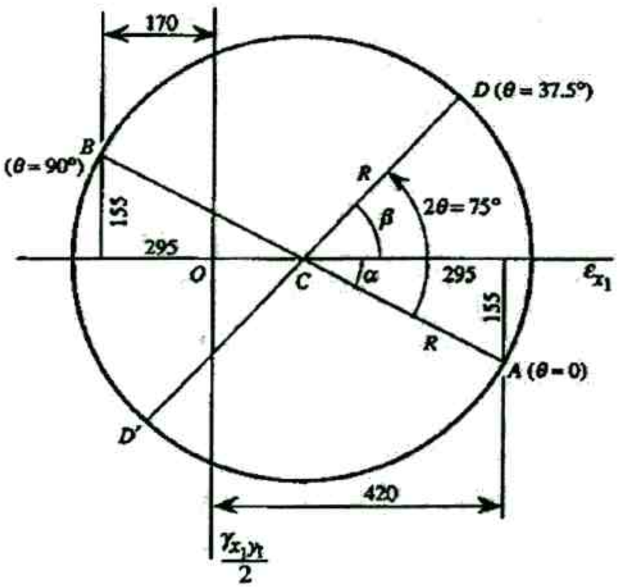


7.7-24

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 420\times10^{-6}$ $\quad\varepsilon_y = -170\times10^{-6}$ $\quad\gamma_{xy} = 310\times10^{-6}$

$\frac{\gamma_{xy}}{2} = 155\times10^{-6}$ $\quad\theta = 37.5°$



$$R = \sqrt{(295\times10^{-6})^2 + (155\times10^{-6})^2} = 333.24\times10^{-6}$$

$\alpha = \arctan\frac{155}{295} = 27.72°$ $\quad\beta = 2\theta - \alpha = 47.28°$

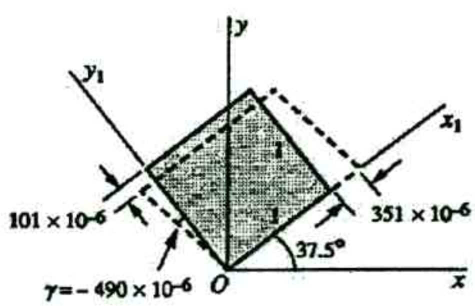
Point C: $\varepsilon_{x_1} = 125\times10^{-6}$

Point D ($\theta = 37.5°$): $\varepsilon_{x_1} = 125\times10^{-6} + R\cos\beta = 351\times10^{-6}$

$\frac{\gamma_{x_1y_1}}{2} = -R\sin\beta = -244.8\times10^{-6}$ $\quad\gamma_{x_1y_1} = -490\times10^{-6}$

Point D′ ($\theta = 127.5°$): $\varepsilon_{x_1} = 125\times10^{-6} - R\cos\beta = -101\times10^{-6}$

$\frac{\gamma_{x_1y_1}}{2} = R\sin\beta = 244.8\times10^{-6}$ $\quad\gamma_{x_1y_1} = 490\times10^{-6}$

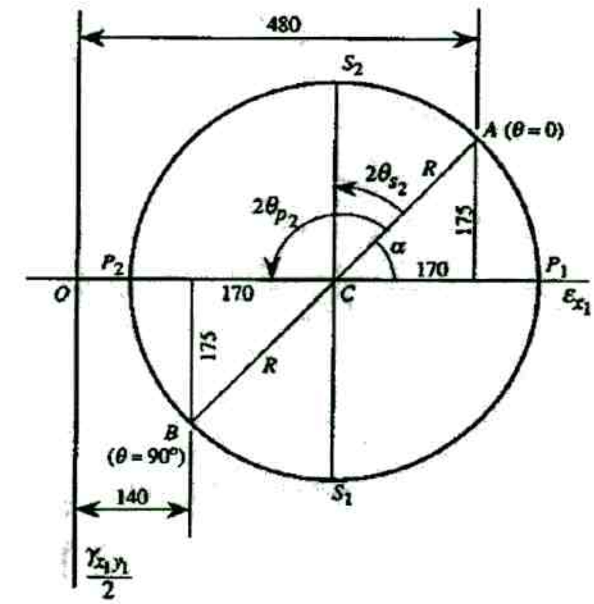


7.7-25

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 480\times10^{-6}$ $\quad\varepsilon_y = 140\times10^{-6}$ $\quad\gamma_{xy} = -350\times10^{-6}$

$\frac{\gamma_{xy}}{2} = -175\times10^{-6}$



$$R = \sqrt{(175\times10^{-6})^2 + (170\times10^{-6})^2} = 243.98\times10^{-6}$$

$\alpha = \arctan\frac{175}{170} = 45.83°$

Point C: $\varepsilon_{x_1} = 310\times10^{-6}$

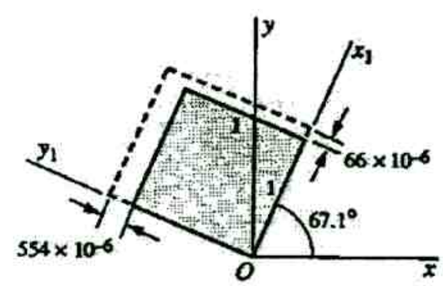
Principal strains

$2\theta_{p_2} = 180° - \alpha = 134.2°$ $\quad\theta_{p_2} = 67.1°$

$2\theta_{p_1} = 2\theta_{p_2} + 180° = 314.2°$ $\quad\theta_{p_1} = 157.1°$

Point P_1: $\varepsilon_1 = 310\times10^{-6} + R = 554\times10^{-6}$

Point P_2: $\varepsilon_2 = 310\times10^{-6} - R = 66\times10^{-6}$



CONT.

7.7-25 CONT.

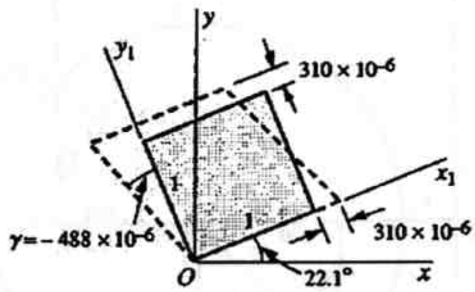
Maximum shear strains

$2\theta_{s_2} = 90° - \alpha = 44.17°$ $\quad\theta_{s_2} = 22.1°$

$2\theta_{s_1} = 2\theta_{s_2} + 180° = 224.17°$ $\quad\theta_{s_1} = 112.1°$

Point S_1: $\varepsilon_{aver} = 310\times10^{-6}$ $\quad\gamma_{max} = 2R = 488\times10^{-6}$

Point S_2: $\varepsilon_{aver} = 310\times10^{-6}$ $\quad\gamma_{min} = -488\times10^{-6}$

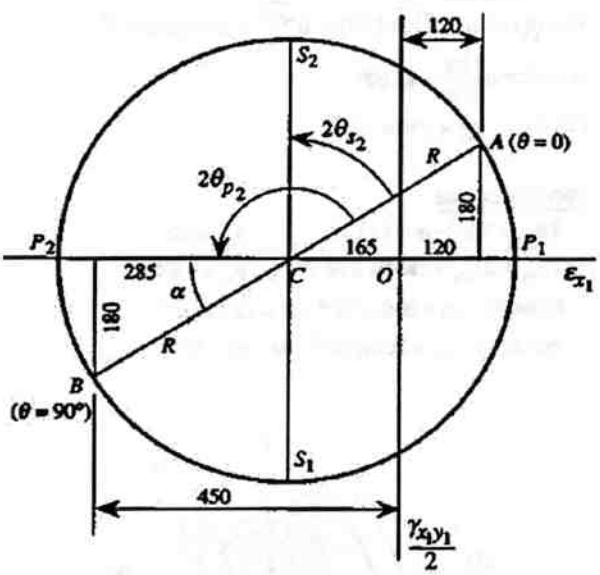


7.7-26

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 120\times10^{-6}$ $\quad\varepsilon_y = -450\times10^{-6}$ $\quad\gamma_{xy} = -360\times10^{-6}$

$\frac{\gamma_{xy}}{2} = -180\times10^{-6}$



$$R = \sqrt{(285\times10^{-6})^2 + (180\times10^{-6})^2} = 337.08\times10^{-6}$$

$\alpha = \arctan\frac{180}{285} = 32.28°$

<u>Point C:</u> $\varepsilon_{x_1} = -165\times10^{-6}$

CONT.

7.7-26 CONT.

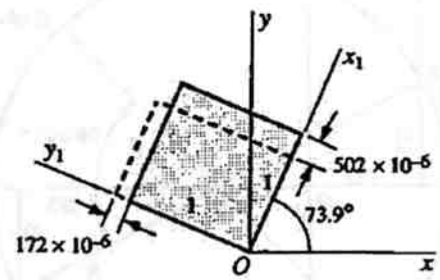
Principal strains

$2\theta_{p_2} = 180° - \alpha = 147.72°$ $\quad\theta_{p_2} = 73.9°$

$2\theta_{p_1} = 2\theta_{p_2} + 180° = 327.72°$ $\quad\theta_{p_1} = 163.9°$

Point P_1: $\varepsilon_1 = R - 165\times10^{-6} = 172\times10^{-6}$

Point P_2: $\varepsilon_2 = -165\times10^{-6} - R = -502\times10^{-6}$



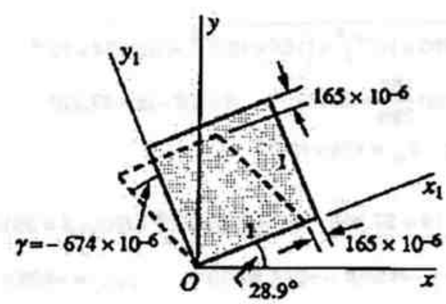
Maximum shear strains

$2\theta_{s_2} = 90° - \alpha = 57.72°$ $\quad\theta_{s_2} = 28.9°$

$2\theta_{s_1} = 2\theta_{s_2} + 180° = 237.72°$ $\quad\theta_{s_1} = 118.9°$

Point S_1: $\varepsilon_{aver} = -165\times10^{-6}$ $\quad\gamma_{max} = 2R = 674\times10^{-6}$

Point S_2: $\varepsilon_{aver} = -165\times10^{-6}$ $\quad\gamma_{min} = -674\times10^{-6}$

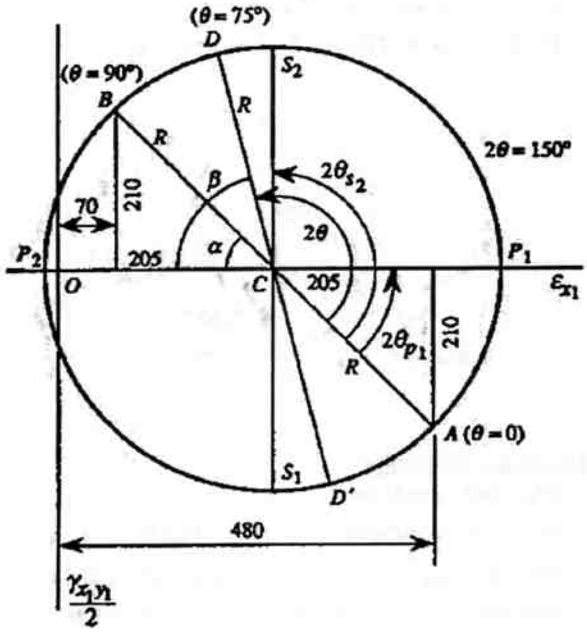


7.7-27

ELEMENT IN PLANE STRAIN

$\varepsilon_x = 480 \times 10^{-6}$ $\quad \varepsilon_y = 70 \times 10^{-6}$ $\quad \gamma_{xy} = 420 \times 10^{-6}$

$\dfrac{\gamma_{xy}}{2} = 210 \times 10^{-6}$ $\quad \theta = 75°$



$$R = \sqrt{(205 \times 10^{-6})^2 + (210 \times 10^{-6})^2} = 293.47 \times 10^{-6}$$

$\alpha = \arctan\dfrac{210}{205} = 45.69°$ $\qquad \beta = \alpha + 180° - 2\theta = 75.69°$

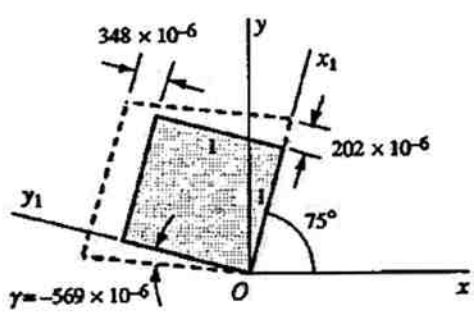
Point C: $\varepsilon_{x_1} = 275 \times 10^{-6}$

Point D ($\theta = 75°$): $\varepsilon_{x_1} = 275 \times 10^{-6} - R\cos\beta = 202 \times 10^{-6}$

$\dfrac{\gamma_{x_1y_1}}{2} = -R\sin\beta = -284.36 \times 10^{-6}$ $\qquad \gamma_{x_1y_1} = -569 \times 10^{-6}$

Point D′ ($\theta = 165°$): $\varepsilon_{x_1} = 275 \times 10^{-6} + R\cos\beta = 348 \times 10^{-6}$

$\dfrac{\gamma_{x_1y_1}}{2} = R\sin\beta = 284.36 \times 10^{-6}$ $\qquad \gamma_{x_1y_1} = 569 \times 10^{-6}$



CONT.

7.7-27 CONT.

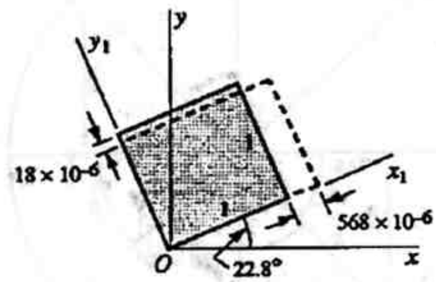
Principal strains

$2\theta_{p_1} = \alpha = 45.69°$ $\qquad \theta_{p_1} = 22.8°$

$2\theta_{p_2} = 2\theta_{p_1} + 180° = 225.69°$ $\qquad \theta_{p_2} = 112.8°$

Point P_1: $\varepsilon_1 = 275 \times 10^{-6} + R = 568 \times 10^{-6}$

Point P_2: $\varepsilon_2 = 275 \times 10^{-6} - R = -18 \times 10^{-6}$



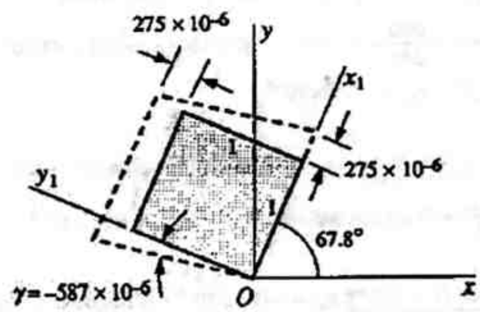
Maximum shear strains

$2\theta_{s_2} = 90° + \alpha = 135.69°$ $\qquad \theta_{s_2} = 67.8°$

$2\theta_{s_1} = 2\theta_{s_2} + 180° = 315.69°$ $\qquad \theta_{s_1} = 157.8°$

Point S_1: $\varepsilon_{aver} = 275 \times 10^{-6}$ $\qquad \gamma_{max} = 2R = 587 \times 10^{-6}$

Point S_2: $\varepsilon_{aver} = 275 \times 10^{-6}$ $\qquad \gamma_{min} = -587 \times 10^{-6}$

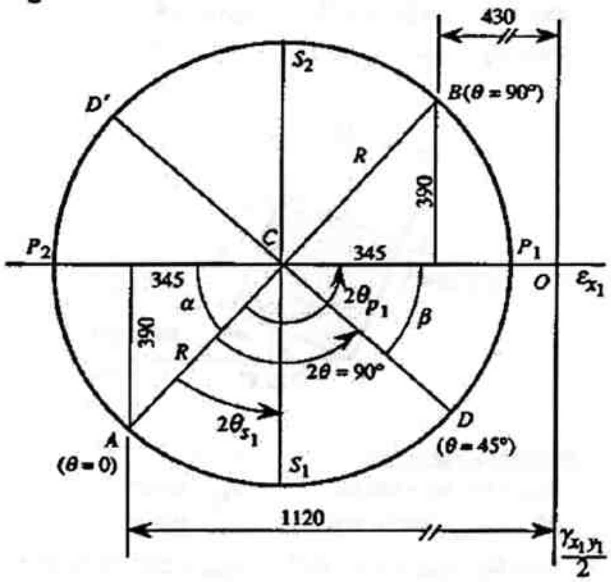


7.7-28

ELEMENT IN PLANE STRAIN

$\varepsilon_x = -1120 \times 10^{-6}$ $\quad \varepsilon_y = -430 \times 10^{-6}$ $\quad \gamma_{xy} = 780 \times 10^{-6}$

$\frac{\gamma_{xy}}{2} = 390 \times 10^{-6}$ $\quad \theta = 45°$



$$R = \sqrt{(345 \times 10^{-6})^2 + (390 \times 10^{-6})^2} = 520.70 \times 10^{-6}$$

$\alpha = \arctan\frac{390}{345} = 48.50°$ $\quad \beta = 180° - \alpha - 2\theta = 41.50°$

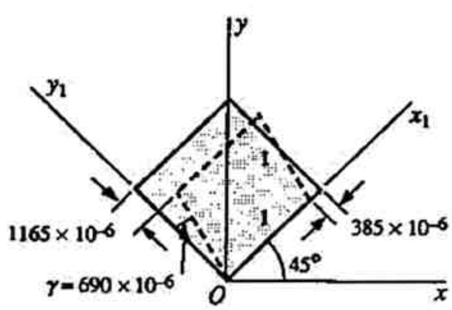
Point C: $\varepsilon_{x_1} = -775 \times 10^{-6}$

Point D ($\theta = 45°$): $\varepsilon_{x_1} = -775 \times 10^{-6} + R\cos\beta = -385 \times 10^{-6}$

$\frac{\gamma_{x_1y_1}}{2} = R\sin\beta = 345 \times 10^{-6}$ $\quad \gamma_{x_1y_1} = 690 \times 10^{-6}$

Point D' ($\theta = 135°$): $\varepsilon_{x_1} = -775 \times 10^{-6} - R\cos\beta = -1165 \times 10^{-6}$

$\frac{\gamma_{x_1y_1}}{2} = -R\sin\beta = -345 \times 10^{-6}$ $\quad \gamma_{x_1y_1} = -690 \times 10^{-6}$



CONT.

7.7-28 CONT.

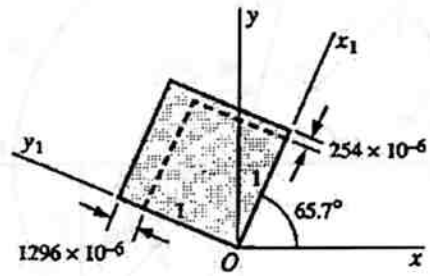
Principal strains

$2\theta_{p_1} = 180° - \alpha = 131.50°$ $\quad \theta_{p_1} = 65.7°$

$2\theta_{p_2} = 2\theta_{p_1} + 180° = 311.50°$ $\quad \theta_{p_2} = 155.7°$

Point P_1: $\varepsilon_1 = -775 \times 10^{-6} + R = -254 \times 10^{-6}$

Point P_2: $\varepsilon_2 = -775 \times 10^{-6} - R = -1296 \times 10^{-6}$



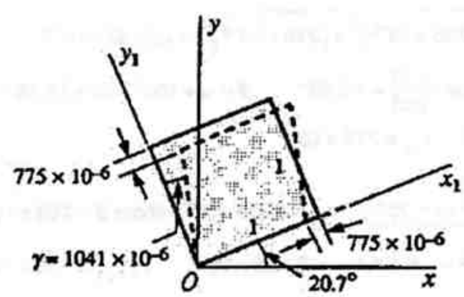
Maximum shear strains

$2\theta_{s_1} = 90° - \alpha = 41.50°$ $\quad \theta_{s_1} = 20.7°$

$2\theta_{s_2} = 2\theta_{s_1} + 180° = 221.50°$ $\quad \theta_{s_2} = 110.7°$

Point S_1: $\varepsilon_{aver} = -775 \times 10^{-6}$ $\quad \gamma_{max} = 2R = 1041 \times 10^{-6}$

Point S_2: $\varepsilon_{aver} = -775 \times 10^{-6}$ $\quad \gamma_{min} = -1041 \times 10^{-6}$



— END OF CHAPTER 7 —

8.2-1

Spherical tank

Cross-Section

Radius: $r = \frac{(60\text{ ft})(12\text{ in/ft})}{2} = 360\text{ in}$

Internal pressure: $p = 500\text{ psi}$

Yield stress: $\sigma_y = 80\text{ ksi}$ (steel)

Factor of safety: $n = 3.2$

Minimum wall thickness t_{min} (inches)

From Eq. (8-1): $\sigma_{max} = \frac{pr}{2t} \quad \therefore \quad \frac{\sigma_y}{n} = \frac{pr}{2t}$

$t = \frac{prn}{2\sigma_y} = 3.60\text{ in.}$

Use next highest ¼ inch: $t_{min} = 3.75\text{ in.}$ ←

8.2-2

Spherical tank

Cross-Section

Radius: $r = 8\text{ m}$

Internal pressure: $p = 3.0\text{ MPa}$

Yield stress: $\sigma_y = 490\text{ MPa}$

Factor of safety: $n = 2.5$

Minimum wall thickness t_{min} (meters)

From Eq. (8-1): $\sigma_{max} = \frac{pr}{2t} \quad \therefore \quad \frac{\sigma_y}{n} = \frac{pr}{2t}$

$t = \frac{prn}{2\sigma_y} = 61.2\text{ mm}$

Use next highest mm: $t_{min} = 62\text{ mm}$ ←

8.2-3

Hemispherical Viewport

Free-Body Diagram

Radius: $r = 15\text{ in.}$

Internal pressure: $p = 90\text{ psi}$

Wall thickness: $t = 1.0\text{ in.}$

18 bolts

$\Sigma F_{horiz} = T - pA = 0$ T = Total tensile force in 18 bolts

$T = pA = p(\pi r^2)$

Force F per bolt

$F = \frac{p(\pi r^2)}{18} = \frac{(90\text{ psi})(\pi)(15\text{ in})^2}{18} = 3530\text{ lb}$ ←

Tensile stress in viewport (Eq. 8-1)

$\sigma = \frac{pr}{2t} = \frac{(90\text{ psi})(15\text{ in.})}{2(1.0\text{ in.})} = 675\text{ psi}$ ←

8.2-4

Rubber Ball

Cross-Section

Radius: $r = 104\text{ mm}$

Internal pressure: $p = 80\text{ kPa}$

Wall thickness: $t = 1.2\text{ mm}$

Modulus of elasticity: $E = 3.5\text{ MPa}$ (rubber)

Poisson's ratio: $\nu = 0.45$ (rubber)

Maximum stress (Eq. 8-1)

$\sigma_{max} = \frac{pr}{2t} = \frac{(80\text{ kPa})(104\text{ mm})}{2(1.2\text{ mm})} = 3.47\text{ MPa}$ ←

Maximum strain (Eq. 8-4)

$\epsilon_{max} = \frac{pr}{2tE}(1-\nu) = \frac{(80\text{ kPa})(104\text{ mm})}{2(1.2\text{ mm})(3.5\text{ MPa})}(1-0.45) = 0.545$ ←

8.2-5

Rubber Ball

Cross-Section

Radius: $r = 4.10\text{ in.}$

Internal pressure: $p = 12\text{ psi}$

Wall thickness: $t = 0.048\text{ in.}$

Modulus of elasticity: $E = 500\text{ psi}$

Poisson's ratio: $\nu = 0.45$

Maximum stress (Eq. 8-1)

$\sigma_{max} = \frac{pr}{2t} = \frac{(12\text{ psi})(4.10\text{ in.})}{2(0.048\text{ in.})} = 513\text{ psi}$ ←

CONT.

8.2-5 CONT.

Maximum strain (Eq. 8-4)

$\epsilon_{max} = \frac{pr}{2tE}(1-\nu) = \frac{(12\text{ psi})(4.10\text{ in.})}{2(0.048\text{ in.})(500\text{ psi})}(1-0.45) = 0.564$ ←

8.2-6

Spherical vessel with brittle coating

Cross-Section

$r = 300\text{ mm}$ $E = 205\text{ GPa}$ (steel)

$t = 10\text{ mm}$ $\nu = 0.30$

Cracks occur [illegible] $\epsilon_{max} = 200\times10^{-6}$

From Eq. (8-4): $\epsilon_{max} = \frac{pr}{2tE}(1-\nu)$

$p = \frac{2tE\epsilon_{max}}{r(1-\nu)}$

$p = \frac{2(10\text{ mm})(205\text{ GPa})(200\times10^{-6})}{(300\text{ mm})(1-0.30)} = 3.90\text{ MPa}$ ←

8.2-7

Welded tank

Welded Seam

$r = 24\text{ in.}$ $E = 31\times10^6\text{ psi}$

$t = 2\text{ in.}$ $\nu = 0.29$

$p = 2500\text{ psi}$

(a) Tensile load carried by weld

T = total load   f = load per inch

$T = pA = p(\pi r^2)$ C = Circumference of tank = $2\pi r$

$f = \frac{T}{C} = \frac{p(\pi r^2)}{2\pi r} = \frac{pr}{2} = \frac{(2500\text{ psi})(24\text{ in.})}{2} = 30.0\text{ k/in.}$ ←

(b) Maximum shear stress in wall (Eq. 8-3)

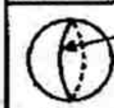
$\tau_{max} = \frac{pr}{4t} = \frac{(2500\text{ psi})(24\text{ in.})}{4(2\text{ in.})} = 7500\text{ psi}$ ←

(c) Maximum normal strain in wall (Eq. 8-4)

$\epsilon_{max} = \frac{pr}{2tE}(1-\nu) = \frac{(2500\text{ psi})(24\text{ in.})}{2(2\text{ in.})(31\times10^6\text{ psi})}(1-0.29) = 344\times10^{-6}$ ←

8.2-8

Welded tank



Welded Seam

$r = 600\text{ mm}$ $E = 210\text{ GPa}$

$t = 50\text{ mm}$ $\nu = 0.29$

$p = 20\text{ MPa}$

(a) Tensile load carried by weld

T = total load   f = load per inch

$T = pA = p(\pi r^2)$ C = Circumference of tank = $2\pi r$

$f = \frac{T}{C} = \frac{p(\pi r^2)}{2\pi r} = \frac{pr}{2} = \frac{(20\text{ MPa})(0.6\text{ m})}{2} = 6.0\text{ MN/m}$ ←

(b) Maximum shear stress in wall (Eq. 8-3)

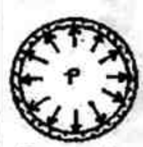
$\tau_{max} = \frac{pr}{4t} = \frac{(20\text{ MPa})(600\text{ mm})}{4(50\text{ mm})} = 60.0\text{ MPa}$ ←

(c) Maximum normal strain in wall (Eq. 8-4)

$\epsilon_{max} = \frac{pr}{2tE}(1-\nu) = \frac{(20\text{ MPa})(600\text{ mm})}{2(50\text{ mm})(210\times10^3\text{ MPa})}(1-0.29) = 406\times10^{-6}$ ←

8.2-9

Propane tank



Cross-Section

$r = 8\text{ in.}$ $E = 30\times10^6\text{ psi}$

$p = 3000\text{ psi}$ $\nu = 0.28$

$\sigma_y = 140{,}000\text{ psi}$ $n = 2.75$

$\tau_y = 65{,}000\text{ psi}$ $\epsilon_{max} = 1000\times10^{-6}$

Minimum thickness for:

(1) tension (Eq. 8-1) $\sigma_{max} = \frac{pr}{2t_1}$

$t_1 = \frac{pr}{2\sigma_{max}} = \frac{pr}{2(\sigma_y/n)} = \frac{(3000\text{ psi})(8\text{ in})}{2(140{,}000\text{ psi})/2.75} = 0.236\text{ in.}$

(2) shear (Eq. 8-3) $\tau_{max} = \frac{pr}{4t_2}$

CONT.

8.2-9 CONT.

$$t_2 = \frac{prn}{4\tau_Y} = \frac{(3000\text{ psi})(8\text{ in})(2.75)}{4(65{,}000\text{ psi})} = 0.254\text{ in.}$$

(3) Strain (from Eq. 8-4) $\quad \epsilon_{max} = \frac{pr}{2t_3 E}(1-\nu)$

$$t_3 = \frac{pr}{2\epsilon_{max}E}(1-\nu) = \frac{(3000\text{ psi})(8\text{ in})}{2(1000\times10^{-6})(30\times10^{6}\text{ psi})}(1-0.28) = 0.288\text{ in.}$$

Strain governs since $t_3 > t_1$ and $t_3 > t_2$ $\therefore$ $t_{min} = 0.29$ in. ⟵

8.2-10

Propane tank

Cross-Section

$r = 200$ mm $\qquad E = 200$ GPa
$p = 20$ MPa $\qquad \nu = 0.28$
$\sigma_Y = 950$ MPa $\qquad n = 2.4$
$\tau_Y = 450$ MPa $\qquad \epsilon_{max} = 1200\times10^{-6}$

Minimum wall thickness t

(1) tension (Eq. 8-1) $\quad \sigma_{max} = \frac{pr}{2t_1}$

$$t_1 = \frac{pr}{2\sigma_{max}} = \frac{pr}{2(\sigma_Y/n)} = \frac{(20\text{ MPa})(200\text{ mm})(2.4)}{2(950\text{ MPa})} = 5.053\text{ mm}$$

(2) Shear (Eq. 8-3) $\quad \tau_{max} = \frac{pr}{4t_2}$

$$t_2 = \frac{prn}{4\tau_Y} = \frac{(20\text{ MPa})(200\text{ mm})(2.4)}{4(450\text{ MPa})} = 5.333\text{ mm}$$

(3) Strain (Eq. 8-4) $\quad \epsilon_{max} = \frac{pr}{2t_3E}(1-\nu)$

$$t_3 = \frac{pr}{2\epsilon_{max}E}(1-\nu) = \frac{(20\text{ MPa})(200\text{ mm})}{2(1200\times10^{-6})(200\text{ GPa})}(1-0.28) = 6.000\text{ mm}$$

Strain governs since $t_3 > t_1$ and $t_3 > t_2$ $\therefore$ $t_{min} = 6.0$ mm ⟵

8.2-11

Pressurized sphere under water

CROSS-SECTIONS

① in air

$r = 6.0$ in. $\qquad p_1 = 20$ psi
$t = 0.5$ in. $\qquad \gamma$ = density of water $= 62.4$ lb/ft³

① in air: $p_1 = 20$ psi

② under water: $p_1 = 20$ psi
d = depth of water (in)

$$p_2 = \gamma d = \left(62.4\frac{\text{lb}}{\text{ft}^3}\frac{(\text{ft}^3)}{(12^3\text{ in}^3)}\right)d = 0.03611 d\ (\text{psi})$$

② under water

Compressive stress in tank wall equals 100 psi. (Note: σ is positive in tension)

$$\sigma = \frac{pr}{2t} = \frac{(p_1 - p_2)r}{2t}$$

$$-100\text{ psi} = \frac{(20\text{ psi} - 0.03611d)(6.0\text{ in.})}{2(0.5\text{ in.})}$$

Solve for d: $d = 1015$ in. $= 84.6$ ft ⟵

8.3-1

Scuba tank

Cylindrical pressure vessel
$p = 1800$ psi $\qquad n = 2.0 \qquad d = 6.0$ in.
$\sigma_Y = 42{,}000$ psi $\qquad \tau_Y = 20{,}000$ psi
$\sigma_{allow} = \frac{\sigma_Y}{n} = 21{,}000$ psi $\qquad \tau_{allow} = \frac{\tau_Y}{n} = 10{,}000$ psi

Find required wall thickness t.

(1) based on tension (Eq. 8-5) $\quad \sigma_{max} = \frac{pr}{t}$

$$t_1 = \frac{pr}{\sigma_{allow}} = \frac{(1800\text{ psi})(3.0\text{ in.})}{21{,}000\text{ psi}} = 0.257\text{ in.}$$

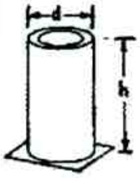
(2) based on shear (Eq. 8-10) $\quad \tau_{max} = \frac{pr}{2t}$

$$t_2 = \frac{pr}{2\tau_{allow}} = \frac{(1800\text{ psi})(3.0\text{ in.})}{2(10{,}000\text{ psi})} = 0.270\text{ in.}$$

shear governs since $t_2 > t_1$ $\therefore$ $t_{min} = 0.27$ in. ⟵

8.3-2

Vertical standpipe



$d = 2$ m $\qquad t = 5$ mm
Circumferential stress $\sigma_1 = \frac{pr}{t} = 32$ MPa (Eq. 8-5)

(a) Height h (meters)

Pressure $p = \gamma h$ $\quad \gamma$ = weight density of water $= 9.81$ kN/m³

$$\sigma_1 = \frac{\gamma h r}{t} = \frac{(9.81\text{ kN/m}^3)(h)(1\text{ m})}{5\text{ mm}} = 32\text{ MPa}$$

Solve for h: $h = 16.3$ m ⟵

(b) Axial stress due to water pressure alone
Because the top of the tank is open, there is no internal pressure at the top [illegible]. There is no axial (longitudinal) stress due to the water pressure alone. Stress equals zero. ⟵

8.3-3

Inflatable structure

Half-circular cylinder
$r = 40$ ft $\qquad p = 0.5$ psi

Longitudinal seam tears open when $T = 540$ lb/in.
T = tensile force per unit length of seam
Find factor of safety n against tearing.

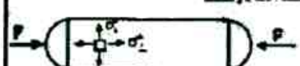
Circumferential stress $\quad \sigma_1 = \frac{pr}{t}$ (Eq. 8-5)

$$T = \sigma_1 t = pr = (0.5\text{ psi})(40\text{ ft})(12\text{ in/ft}) = 240\text{ lb/in.} \quad \left(\begin{array}{c}\text{actual}\\ \text{stress}\end{array}\right)$$

Factor of safety $\quad n = \frac{540\text{ lb/in.}}{240\text{ lb/in.}} = 2.25$ ⟵

8.3-4

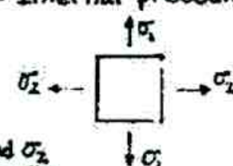
Cylindrical pressure vessel



r = Radius
p = Internal pressure

$\sigma_1 = \frac{pr}{t}$ (Eq. 8-5)

$$\sigma_2 = \frac{pr}{2t} - \frac{F}{A} = \frac{pr}{2t} - \frac{F}{2\pi r t}$$



For pure shear, the stresses σ_1 and σ_2 must be equal in magnitude and opposite in sign (See, e.g., Fig. 7-11 in Section 7.3): $\quad \sigma_1 = -\sigma_2$

$$\frac{pr}{t} = -\left(\frac{pr}{2t} - \frac{F}{2\pi r t}\right) \qquad \text{Solve for } F\text{: } F = 3\pi p r^2 \longleftarrow$$

8.3-5

Aluminum can

$\frac{r}{t} = 200 \qquad E = 10\times10^{6}$ psi $\qquad \nu = 0.33$

ϵ_0 = change in strain when pressure is released $= 170\times10^{-6}$

Find internal pressure p.

Strain in longitudinal direction (Eq. 8-11a)

$$\epsilon_2 = \frac{pr}{2tE}(1-2\nu) \quad \text{or} \quad p = \frac{2tE\epsilon_2}{r(1-2\nu)}$$

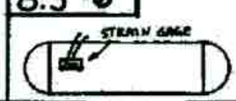
$$\epsilon_2 = \epsilon_0 \quad \therefore \quad p = \frac{2tE\epsilon_0}{r(1-2\nu)}$$

Substitute numerical values:

$$p = \frac{2(10\times10^{6}\text{ psi})(170\times10^{-6})}{(200)(1-0.66)} = 50\text{ psi} \longleftarrow$$

8.3-6

Cylindrical tank



$\tau_{max} = 82$ MPa
$E = 205$ GPa

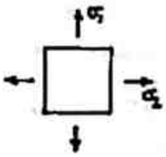
CONT.

8.3-6 CONT.

$\nu = 0.30$

$n = 2$ (factor of safety) $\quad \tau_{max} = \frac{\tau_Y}{n} = 41$ MPa

Find maximum allowable strain reading at the gage.



$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

From Eq. (8-10):

$\tau_{max} = \frac{\sigma_2}{2} = \frac{pr}{2t} \quad \therefore p_{max} = \frac{2t\tau_{max}}{r}$

From Eq. (8-11a):

$\epsilon_2 = \frac{pr}{2tE}(1-2\nu)$

$(\epsilon_2)_{max} = \frac{p_{max} r}{2tE}(1-2\nu) = \frac{\tau_{max}}{E}(1-2\nu) = \frac{41 \text{ MPa}}{205 \text{ GPa}}[1-2(0.30)]$

$\epsilon_{max} = 80 \times 10^{-6}$ ⟵

8.3-7

Cylinder with internal pressure

$d = 1.91$ in. $\quad r = 0.955$ in.

$F = 3600$ lb. $\quad \tau_{allow} = 6000$ psi

Find minimum thickness t_{min}.

Pressure in cylinder: $p = \frac{F}{A} = \frac{F}{\pi r^2}$

Maximum shear stress (Eq. 8-10): $\tau_{max} = \frac{pr}{2t} = \frac{F}{2\pi r t}$

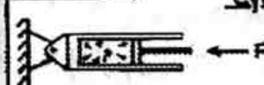
Minimum thickness: $t_{min} = \frac{F}{2\pi r \tau_{allow}}$

Substitute numerical values:

$t_{min} = \frac{3600 \text{ lb}}{2\pi(0.955 \text{ in.})(6000 \text{ psi})} = 0.10$ in. ⟵

8.3-8

Cylinder with internal pressure



$d = 90.4$ mm $\quad r = 45.2$ mm

$F = 42.6$ kN $\quad \tau_{allow} = 50$ MPa

Find minimum thickness t_{min}.

Pressure in cylinder: $p = \frac{F}{A} = \frac{F}{\pi r^2}$

Maximum shear stress (Eq. 8-10): $\tau_{max} = \frac{pr}{2t} = \frac{F}{2\pi r t}$

Minimum thickness: $t_{min} = \frac{F}{2\pi r \tau_{allow}}$

Substitute numerical values:

$t_{min} = \frac{42.6 \text{ kN}}{2\pi(45.2 \text{ mm})(50 \text{ MPa})} = 3.0$ mm ⟵

8.3-9

Vertical standpipe

$d = 12$ ft $= 144$ in. $\quad r = 72$ in.

$t = 6$ in. $\quad \gamma = 62.4$ lb/ft³ $= \frac{62.4}{1728}$ lb/in.³

σ_1 = hoop stress at bottom of stand pipe $= 130$ psi

(a) Find height h of water in the standpipe

p = pressure at bottom of standpipe $= \gamma h$

From Eq. (8-5): $\sigma_1 = \frac{pr}{t} = \frac{\gamma h r}{t}$ or $h = \frac{\sigma_1 t}{\gamma r}$

Substitute numerical values:

$h = \frac{(130 \text{ psi})(6 \text{ in.})}{\left(\frac{62.4}{1728} \frac{\text{lb}}{\text{in.}^3}\right)(72 \text{ in.})} = 300$ in. $= 25$ ft ⟵

Horizontal pipes

$d_1 = 2$ ft $= 24$ in. $\quad r_1 = 12$ in. $\quad t_1 = 1.0$ in.

(b) Find hoop stress σ in the pipes

Since the pipes are 2 ft in diameter, the depth of water to the center of the pipes is about 24 ft.

$h_1 \approx 24$ ft $= 288$ in. $\quad p_1 = \gamma h_1$

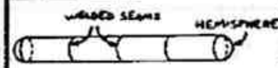
$\sigma = \frac{p_1 r_1}{t_1} = \frac{\gamma h_1 r_1}{t_1} = \frac{\left(\frac{62.4}{1728} \frac{\text{lb}}{\text{in.}^3}\right)(288 \text{ in.})(12 \text{ in.})}{1.0 \text{ in.}} = 125$ psi

Based on the average pressure in the pipes:

$\sigma = 125$ psi ⟵

8.3-10

Cylindrical tank



$d = 1.2$ m $\quad r = 0.6$ m

$t = 20$ mm $\quad p = 1800$ kPa

(a) Maximum stress in hemispheres (Eq. 8-1)

$\sigma_h = \frac{pr}{2t} = \frac{(1800 \text{ kPa})(0.6 \text{ m})}{2(20 \text{ mm})} = 27.0$ MPa ⟵

(b) Maximum stress in cylinder (Eq. 8-5)

$\sigma_c = \frac{pr}{t} = 2\sigma_h = 54.0$ MPa ⟵

(c) Maximum stress in welds (Eq. 8-6)

$\sigma_w = \frac{pr}{2t} = 27.0$ MPa ⟵

(d) Maximum shear stress in hemispheres (Eq. 8-3)

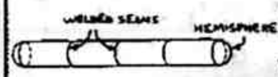
$\tau_h = \frac{pr}{4t} = \frac{\sigma_h}{2} = 13.5$ MPa ⟵

(e) Maximum shear stress in cylinder (Eq. 8-10)

$\tau_c = \frac{pr}{2t} = \frac{\sigma_c}{2} = 27.0$ MPa ⟵

8.3-11

Cylindrical tank



$d = 12$ in. $\quad r = 6$ in. $\quad p = 320$ psi

$\sigma_{allow} = 8000$ psi (tension)

$\tau_{allow} = 3200$ psi

weld: $\sigma_{allow} = 6400$ psi (tension)

(a) Find minimum thickness of cylinder

Tension: $\sigma_{max} = \frac{pr}{t}$ (Eq. 8-5)

$t_{min} = \frac{pr}{\sigma_{allow}} = \frac{(320 \text{ psi})(6 \text{ in.})}{8000 \text{ psi}} = 0.24$ in.

Shear: $\tau_{max} = \frac{pr}{2t}$ (Eq. 8-10)

$t_{min} = \frac{pr}{2\tau_{allow}} = \frac{(320 \text{ psi})(6 \text{ in.})}{(2)(3200 \text{ psi})} = 0.30$ in.

Weld: $\sigma = \frac{pr}{2t}$ (Eq. 8-6)

$t_{min} = \frac{pr}{2\sigma_{allow}} = \frac{(320 \text{ psi})(6 \text{ in.})}{2(6400 \text{ psi})} = 0.15$ in.

Shear governs. $t_{min} = 0.30$ in. ⟵

(b) Find minimum thickness of hemispheres

Tension: $\sigma_{max} = \frac{pr}{2t}$ (Eq. 8-1)

$t_{min} = \frac{pr}{2\sigma_{allow}} = \frac{(320 \text{ psi})(6 \text{ in.})}{2(8000 \text{ psi})} = 0.12$ in.

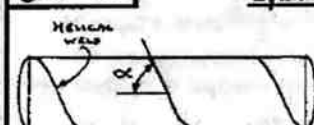
Shear: $\tau_{max} = \frac{pr}{4t}$ (Eq. 8-3)

$t_{min} = \frac{pr}{4\tau_{allow}} = \frac{(320 \text{ psi})(6 \text{ in.})}{4(3200 \text{ psi})} = 0.15$ in.

Shear governs. $t_{min} = 0.15$ in. ⟵

8.3-12

Cylindrical pressure vessel



$\alpha = 60°$ $\quad r = 0.5$ m

$t = 15$ mm $\quad p = 2.4$ MPa

$E = 200$ GPa

$\nu = 0.30$

(a) Circumferential stress $\sigma_1 = \frac{pr}{t} = \frac{(2.4 \text{ MPa})(0.5 \text{ m})}{(15 \text{ mm})} = 80$ MPa ⟵

Longitudinal stress $\sigma_2 = \frac{pr}{2t} = \frac{\sigma_1}{2} = 40$ MPa ⟵

(b) In-plane shear stress $\tau_1 = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t} = 20$ MPa ⟵

Out-of-plane shear stress $\tau_2 = \frac{\sigma_1}{2} = \frac{pr}{2t} = 40$ MPa ⟵

(c) Circumferential strain $\epsilon_1 = \frac{\sigma_2}{2E}(2-\nu)$

$\epsilon_1 = \frac{80 \text{ MPa}}{2(200 \text{ GPa})}(2-0.30) = 340 \times 10^{-6}$ ⟵

Longitudinal strain $\epsilon_2 = \frac{\sigma_2}{E}(1-2\nu)$

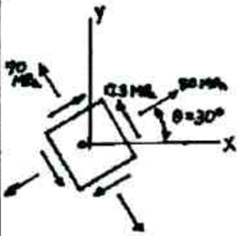
$\epsilon_2 = \frac{(40 \text{ MPa})}{(200 \text{ GPa})}[1-2(0.30)] = 80 \times 10^{-6}$ ⟵

CONT.

8.3-12 CONT.

(d) Stresses on weld

$\alpha = 60°$ $\theta = 90° - \alpha = 30°$



$\sigma_x = \sigma_2 = 40\,\text{MPa}$

$\sigma_y = \sigma_1 = 80\,\text{MPa}$

$\tau_{xy} = 0$

For $\theta = 30°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 60\,\text{MPa} - (20\,\text{MPa})(\cos 60°) + 0$$

$$= 60\,\text{MPa} - 10\,\text{MPa} = 50\,\text{MPa} \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

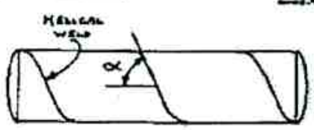
$$= +(20\,\text{MPa})(\sin 60°) + 0 = 17.3\,\text{MPa} \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 40\,\text{MPa} + 80\,\text{MPa} - 50\,\text{MPa}$$

$$= 70\,\text{MPa} \longleftarrow$$

8.3-13

Cylindrical tank



$\alpha = 65°$ $r = 18\,\text{in.}$

$t = 0.6\,\text{in.}$ $p = 200\,\text{psi}$

$E = 30 \times 10^6\,\text{psi}$

$\nu = 0.30$

(a) Circumferential stress $\sigma_1 = \frac{pr}{t} = \frac{(200\,\text{psi})(18\,\text{in.})}{0.6\,\text{in.}} = 6000\,\text{psi} \longleftarrow$

Longitudinal stress $\sigma_2 = \frac{pr}{2t} = \frac{\sigma_1}{2} = 3000\,\text{psi} \longleftarrow$

(b) In-plane shear stress $\tau_1 = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t} = 1500\,\text{psi} \longleftarrow$

Out-of-plane shear stress $\tau_2 = \frac{\sigma_1}{2} = \frac{pr}{2t} = 3000\,\text{psi} \longleftarrow$

(c) Circumferential strain $\epsilon_1 = \frac{\sigma_1}{2E}(2 - \nu)$

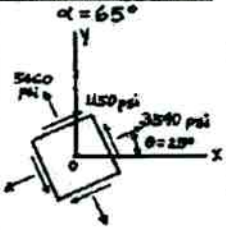
$$\epsilon_1 = \frac{6000\,\text{psi}}{2(30 \times 10^6\,\text{psi})}(2 - 0.30) = 170 \times 10^{-6} \longleftarrow$$

Longitudinal strain $\epsilon_2 = \frac{\sigma_2}{E}(1 - 2\nu)$

$$\epsilon_2 = \frac{3000\,\text{psi}}{30 \times 10^6\,\text{psi}}[1 - 2(0.30)] = 40 \times 10^{-6} \longleftarrow$$

(d) Stress on weld

$\alpha = 65°$ $\theta = 90° - \alpha = 25°$



$\sigma_x = \sigma_2 = 3000\,\text{psi}$

$\sigma_y = \sigma_1 = 6000\,\text{psi}$

$\tau_{xy} = 0$

For $\theta = 25°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 4500\,\text{psi} - (1500\,\text{psi})(\cos 50°) + 0$$

$$= 4500\,\text{psi} - 960\,\text{psi} = 3540\,\text{psi} \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

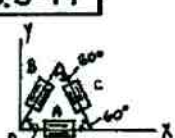
$$= +(1500\,\text{psi})(\sin 50°) + 0 = 1150\,\text{psi} \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 3000\,\text{psi} + 6000\,\text{psi} - 3540\,\text{psi}$$

$$= 5460\,\text{psi} \longleftarrow$$

8.3-14

Cylindrical tank



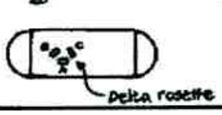
Delta rosette

$\epsilon_a = 80 \times 10^{-6}$ (Longitudinal direction)

$\epsilon_b = \epsilon_c = 275 \times 10^{-6}$

Cylindrical tank: $\frac{r}{t} = 25$

$E = 200\,\text{GPa}$



Determine the pressure p in the tank.

CONT.

8.3-14 CONT.

Strain in x direction $\epsilon_x = \epsilon_a = 80 \times 10^{-6}$

Strain in y direction

Use transformation equation of plane strain:

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Gage B $\theta = 60°$

$$\epsilon_b = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 120° + \frac{\gamma_{xy}}{2}\sin 120°$$

$$275 \times 10^{-6} = 40 \times 10^{-6} + \frac{\epsilon_y}{2} + (40 \times 10^{-6})\left(-\frac{1}{2}\right) - \frac{\epsilon_y}{2}\left(-\frac{1}{2}\right) + \gamma_{xy}\left(\frac{\sqrt{3}}{4}\right)$$

Simplify the equation: $3\epsilon_y + \sqrt{3}\,\gamma_{xy} = 1020 \times 10^{-6}$ (1)

Gage C $\theta = 120°$

$$\epsilon_c = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos(240°) + \frac{\gamma_{xy}}{2}\sin(240°)$$

$$275 \times 10^{-6} = 40 \times 10^{-6} + \frac{\epsilon_y}{2} - 20 \times 10^{-6} + \frac{\epsilon_y}{4} - \frac{\sqrt{3}}{4}\gamma_{xy}$$

Simplify the equation: $3\epsilon_y - \sqrt{3}\,\gamma_{xy} = 1020 \times 10^{-6}$ (2)

Solving Eqs. (1) and (2): $\epsilon_y = 340 \times 10^{-6}$

$\gamma_{xy} = 0$ (as expected)

Hooke's law for biaxial stress

$$\sigma_x = \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y) \qquad \sigma_y = \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x)$$

$$\sigma_x = \frac{pr}{2t} = 12.5p \qquad \sigma_y = \frac{pr}{t} = 25p$$

From σ_x: $12.5p = \frac{200\,\text{GPa}}{1-\nu^2}(80 + 340\nu)(10^{-6})$

or

$$0.625p = \frac{200\,\text{kPa}}{1-\nu^2}(4 + 17\nu) \qquad (3)$$

From σ_y: $25p = \frac{200\,\text{GPa}}{1-\nu^2}(340 + 80\nu)(10^{-6})$

or

$$1.25p = \frac{200\,\text{kPa}}{1-\nu^2}(17 + 4\nu) \qquad (4)$$

Solve simultaneously (3) and (4):

$\nu = 0.3$ $p = 3200\,\text{kPa} \longleftarrow$

Note: As an alternative solution, the following formula can be derived for the pressure p:

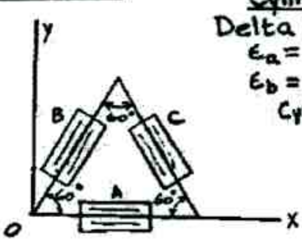
$$p = \frac{2tE}{9r}(8\epsilon_b - 5\epsilon_a)$$

Substitute numerical values:

$$p = \frac{2(200\,\text{GPa})}{9(25)}\left[8(275 \times 10^{-6}) - 5(80 \times 10^{-6})\right] = 3200\,\text{kPa} \longleftarrow$$

8.3-15

Cylindrical tank



Delta rosette

$\epsilon_a = 64 \times 10^{-6}$ (Longitudinal direction)

$\epsilon_b = \epsilon_c = 220 \times 10^{-6}$

Cylindrical tank: $\frac{r}{t} = 80$

$E = 30 \times 10^6\,\text{psi}$

Determine pressure p in the tank.

Strain in x direction

$\epsilon_x = \epsilon_a = 64 \times 10^{-6}$

Strain in y direction

Use transformation equation of plane strain:

$$\epsilon_{x_1} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Gage B $\theta = 60°$

$$\epsilon_b = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 120° + \frac{\gamma_{xy}}{2}\sin 120°$$

$$220 \times 10^{-6} = 32 \times 10^{-6} + \frac{\epsilon_y}{2} + (32 \times 10^{-6})\left(-\frac{1}{2}\right) - \frac{\epsilon_y}{2}\left(-\frac{1}{2}\right) + \gamma_{xy}\left(\frac{\sqrt{3}}{4}\right)$$

Simplify the equation: $3\epsilon_y + \sqrt{3}\,\gamma_{xy} = 816 \times 10^{-6}$ (1)

Gage C $\theta = 120°$

$$\epsilon_c = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos(240°) + \frac{\gamma_{xy}}{2}\sin(240°)$$

$$220 \times 10^{-6} = 32 \times 10^{-6} + \frac{\epsilon_y}{2} - 16 \times 10^{-6} + \frac{\epsilon_y}{4} - \frac{\sqrt{3}}{4}\gamma_{xy}$$

Simplifying the equation: $3\epsilon_y - \sqrt{3}\,\gamma_{xy} = 816 \times 10^{-6}$ (2)

Solving Eqs. (1) and (2): $\epsilon_y = 272 \times 10^{-6}$

$\gamma_{xy} = 0$ (as expected)

CONT.

8.3-15 CONT.

Hooke's Law for biaxial stress

$\sigma_x = \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y)$ $\quad \sigma_y = \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x)$

$\sigma_x = \frac{pr}{2t} = 40p$ $\quad \sigma_y = \frac{pr}{t} = 80p$

From σ_x: $40p = \frac{30\times10^6 \text{ psi}}{1-\nu^2}(64 + 272\nu)(10^{-6})$

or

$$5p = \frac{30 \text{ psi}}{1-\nu^2}(8 + 34\nu) \qquad (3)$$

From σ_y: $80p = \frac{30\times10^6 \text{ psi}}{1-\nu^2}(272 + 64\nu)(10^{-6})$

or

$$10p = \frac{30 \text{ psi}}{1-\nu^2}(34 + 8\nu) \qquad (4)$$

Solve simultaneously Eqs(3) and (4):

$\nu = 0.3$ $\quad p = 120$ psi ←

Note: As an alternative solution, the following formula can be derived for the pressure p:

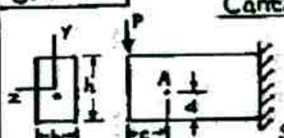
$$p = \frac{2tE}{9r}(8\epsilon_b - 5\epsilon_a)$$

Substitute numerical values:

$p = \frac{2(30\times10^6 \text{ psi})}{9(80)}\left[8(220\times10^{-6}) - 5(64\times10^{-6})\right] = 120$ psi ←

8.4-1

Cantilever beam



$P = 15$ k $\quad c = 2$ ft $= 24$ in.
$b = 4$ in. $\quad d = 3$ in.
$h = 10$ in.

Stresses at point A

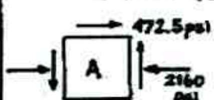
$I = \frac{bh^3}{12} = 333.3 \text{ in}^4$ $\quad M = -Pc = -360\times10^3$ lb-in.

$V = P = 15{,}000$ lb $\quad y_A = -\frac{h}{2} + d = -2.0$ in.

$\sigma_x = -\frac{My_A}{I} = -\frac{(-360\times10^3 \text{ lb-in})(-2.0 \text{ in})}{333.3 \text{ in}^4} = -2160$ psi

$\tau = \frac{VQ}{Ib}$ $\quad Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 42.0 \text{ in}^3$

$\tau = \frac{(15{,}000 \text{ lb})(42.0 \text{ in}^3)}{(333.3 \text{ in}^4)(4 \text{ in})} = 472.5$ psi



$\sigma_x = -2160$ psi
$\sigma_y = 0$
$\tau_{xy} = 472.5$ psi

Principal stresses

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.4375$

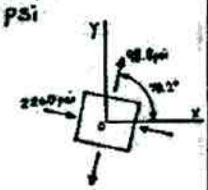
$2\theta_p = -23.63°$ and $\theta_p = -11.81°$
$2\theta_p = 156.37°$ and $\theta_p = 78.19°$

$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

For $2\theta_p = -23.63°$: $\sigma_{x_1} = -2259$ psi
For $2\theta_p = 156.37°$: $\sigma_{x_1} = 98.8$ psi

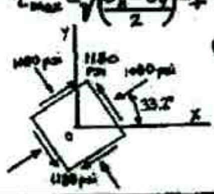
Therefore,

$\sigma_1 = 99$ psi and $\theta_{p_1} = 78.2°$
$\sigma_2 = -2260$ psi and $\theta_{p_2} = -11.8°$ ←



Maximum shear stresses

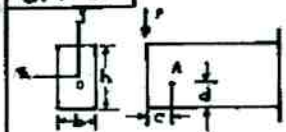
$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1179$ psi



$\theta_{s_1} = \theta_{p_1} - 45° = 33.2°$
and $\tau = 1180$ psi
$\theta_{s_2} = \theta_{s_1} + 90° = 123.2°$
and $\tau = -1180$ psi ←

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = -1080$ psi

8.4-2



Cantilever beam

$P = 120$ kN $\quad b = 100$ mm
$h = 200$ mm $\quad c = 0.5$ m
$d = 150$ mm

Stresses at point A

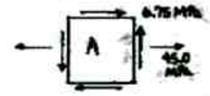
$I = \frac{bh^3}{12} = 66.67\times10^6 \text{ mm}^4$ $\quad M = -Pc = -60{,}000$ N·m

$V = P = 120$ kN $\quad y_A = -\frac{h}{2} + d = 50$ mm

$\sigma_x = -\frac{My_A}{I} = -\frac{(-60{,}000 \text{ N·m})(50 \text{ mm})}{66.67\times10^6 \text{ mm}^4} = 45.0$ MPa

$\tau = \frac{VQ}{Ib}$ $\quad Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 375{,}000 \text{ mm}^3$

$\tau = \frac{(120 \text{ kN})(375{,}000 \text{ mm}^3)}{(66.67\times10^6 \text{ mm}^4)(100 \text{ mm})} = 6.75$ MPa



$\sigma_x = 45.0$ MPa
$\sigma_y = 0$
$\tau_{xy} = 6.75$ MPa

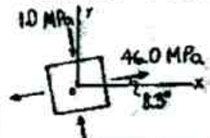
Principal stresses

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.3000$

$2\theta_p = 16.70°$ and $\theta_p = 8.35°$
$2\theta_p = 196.7°$ and $\theta_p = 98.35°$

$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

For $2\theta_p = 16.70°$: $\sigma_{x_1} = 45.99$ MPa
For $2\theta_p = 196.70°$: $\sigma_{x_1} = -0.99$ MPa

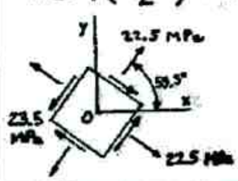


Therefore,

$\sigma_1 = 46.0$ MPa and $\theta_{p_1} = 8.3°$
$\sigma_2 = -1.0$ MPa and $\theta_{p_2} = 98.3°$ ←

Maximum shear stresses

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.5$ MPa

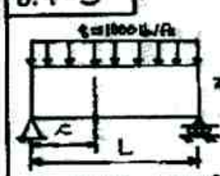


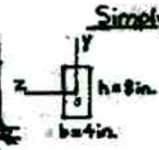
$\theta_{s_1} = \theta_{p_1} - 45° = -36.7°$
and $\tau = 23.5$ MPa
$\theta_{s_2} = \theta_{s_1} + 90° = 53.3°$
and $\tau = -23.5$ MPa ←

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 22.5$ MPa

8.4-3

Simply supported beam





$I = \frac{bh^3}{12} = 170.67 \text{ in}^4$
$A = bh = 32 \text{ in}^2$
$M = \frac{qLc}{2} - \frac{qc^2}{2}$
$= 54{,}000$ lb-in.

$c = 1.0$ ft $\quad L = 10$ ft $\quad V = \frac{qL}{2} - qc = 4{,}000$ lb

(a) Neutral axis

$\sigma_x = 0$ $\quad \sigma_y = 0$ $\quad \tau_{xy} = -\frac{3V}{2A} = -187.5$ psi

Pure shear: $\sigma_1 = 188$ psi, $\sigma_2 = -188$ psi, $\tau_{max} = 188$ psi ←

(b) 2 in. above the neutral axis

$\sigma_x = -\frac{My}{I} = -\frac{(54{,}000 \text{ lb-in})(2 \text{ in})}{170.67 \text{ in}^4} = -632.8$ psi

$\sigma_y = 0$

$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{(4000 \text{ lb})(4 \text{ in})(2 \text{ in})(3 \text{ in})}{(170.67 \text{ in}^4)(4 \text{ in})} = -140.6$ psi

From Eq.(8-22): $\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = -316.4 \pm 346.2$ psi

$\sigma_1 = 30$ psi $\quad \sigma_2 = -663$ psi ←

CONT.

8.4-3 CONT.

From Eq. (8-24): $\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = 346$ psi ⟵

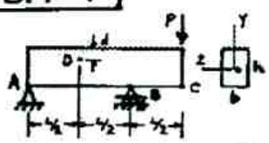
(c) Top of the beam

$\sigma_x = -\frac{Mc}{I} = -\frac{(54{,}000 \text{ lb-in.})(4 \text{ in.})}{170.67 \text{ in}^4} = -1266$ psi

$\sigma_y = 0 \qquad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 0$, $\sigma_2 = -1266$ psi, $\tau_{max} = 633$ psi ⟵

8.4-4



Overhanging beam

$L = 1.25$ m $\quad b = 25$ mm

$h = 150$ mm $\quad d = 25$ mm

Maximum principal stress at point D: $\sigma_1 = 49.0$ MPa

Determine the load P (newtons)

$M_D = -\frac{PL}{4} = -0.3125P$ (N·m) $\quad V_D = \frac{P}{2} = 0.5P$ (N)

Stresses at point D $\quad I = \frac{bh^3}{12} = 7.031 \times 10^6$ mm^4

$\sigma_x = -\frac{My}{I} = -\frac{(-0.3125P)(50 \text{ mm})}{7.031 \times 10^6 \text{ mm}^4} = 2222P$ (Pa)

$\sigma_y = 0$

$Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 39{,}062$ mm^3

$\tau_{xy} = \frac{VQ}{Ib} = \frac{(0.5P)(39{,}062 \text{ mm}^3)}{(7.031 \times 10^6 \text{ mm}^4)(25 \text{ mm})} = 111.11P$ (Pa)

Principal stress

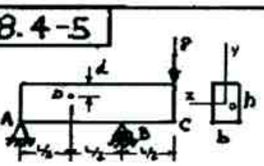
$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{2222P}{2} + \sqrt{\left(\frac{2222P}{2}\right)^2 + (111.11P)^2} = 2228P$ (Pa)

But $\sigma_1 = 49.0$ MPa $\therefore P = \frac{(49.0 \text{ MPa})}{2228 \text{ m}^{-2}} = 21{,}990$ N

$P = 22.0$ kN ⟵

8.4-5



Overhanging beam

$L = 50$ in. $\quad b = 1.0$ in.

$h = 6.0$ in. $\quad d = 1.0$ in.

Maximum principal stress at point D: $\sigma_1 = 3620$ psi

Determine the load P (pounds)

$M_D = -\frac{PL}{4} = -12.5P$ (lb-in.) $\quad V_D = \frac{P}{2} = 0.5P$ (lb)

Stresses at point D $\quad I = \frac{bh^3}{12} = 18.0$ in^4

$\sigma_x = -\frac{My}{I} = -\frac{(-12.5P)(2.0 \text{ in.})}{18.0 \text{ in}^4} = 1.3889P$ (psi)

$\sigma_y = 0 \qquad Q = bd(h/2 - d/2) = 2.5$ in^3

$\tau_{xy} = \frac{VQ}{Ib} = \frac{(0.5P)(2.5 \text{ in}^3)}{(18.0 \text{ in}^4)(1.0 \text{ in.})} = 0.06944P$ (psi)

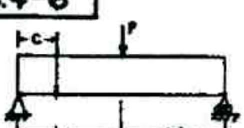
Principal stress

$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 0.6944P + \sqrt{(0.6944P)^2 + (0.06944P)^2} = 1.3923P$ (psi)

But $\sigma_1 = 3620$ psi $\therefore P = \frac{3620 \text{ psi}}{1.3923 \text{ in}^{-2}} = 2600$ lb ⟵

8.4-6



Simply supported beam

$P = 120$ kN $\quad L = 3.0$ m

$c = 1.0$ m $\quad M = \frac{Pc}{2} = 60$ kN·m

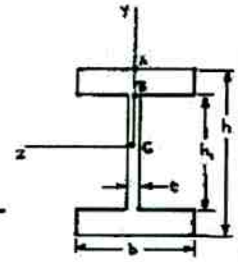
$V = \frac{P}{2} = 60$ kN

$b = 120$ mm $\quad t = 10$ mm $\quad h = 300$ mm $\quad h_1 = 260$ mm

$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = 108.89 \times 10^6$ mm^4

CONT.

8.4-6 CONT.



(a) Top of the beam (point A)

$\sigma_x = -\frac{Mc}{I} = -\frac{(60 \text{ kN·m})(150 \text{ mm})}{108.89 \times 10^6 \text{ mm}^4}$

$= -82.652$ MPa

$\sigma_y = 0 \qquad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 0$

$\sigma_2 = -82.7$ MPa $\quad \tau_{max} = 41.3$ MPa ⟵

(b) Top of the web (point B)

$\sigma_x = -\frac{My}{I} = -\frac{(60 \text{ kN·m})(130 \text{ mm})}{108.89 \times 10^6 \text{ mm}^4} = -71.63$ MPa

$\sigma_y = 0 \qquad Q = (b)\left(\frac{h - h_1}{2}\right)\left(\frac{h + h_1}{4}\right) = 336 \times 10^3$ mm^3

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(60 \text{ kN})(336 \times 10^3 \text{ mm}^3)}{(108.89 \times 10^6 \text{ mm}^4)(10 \text{ mm})} = -18.51$ MPa

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -35.82 \pm 40.32$ MPa

$\sigma_1 = 4.5$ MPa, $\sigma_2 = -76.1$ MPa ⟵

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 40.3$ MPa ⟵

(c) Neutral axis (point C) $\quad \sigma_x = 0 \quad \sigma_y = 0$

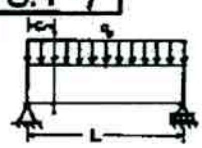
$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b-t)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right) = 420.5 \times 10^3$ mm^3

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(60 \text{ kN})(420.5 \times 10^3 \text{ mm}^3)}{(108.89 \times 10^6 \text{ mm}^4)(10 \text{ mm})} = -23.17$ MPa

Pure shear: $\sigma_1 = 23.2$ MPa, $\sigma_2 = -23.2$ MPa

$\tau_{max} = 23.2$ MPa ⟵

8.4-7



Simply supported beam

$q = 6.0$ k/ft

$L = 10$ ft $= 120$ in.

$c = 3$ ft $= 36$ in.

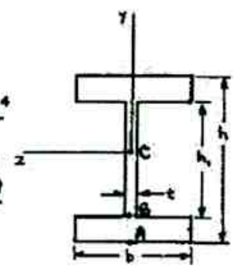
$M = \frac{qLc}{2} - \frac{qc^2}{2} = 756{,}000$ lb-in.

$V = \frac{qL}{2} - qc = 12{,}000$ lb

$b = 5.0$ in. $\quad t = 0.5$ in.

$h = 12$ in. $\quad h_1 = 10.5$ in.

$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = 285.89$ in^4



(a) Bottom of the beam (point A)

$\sigma_x = -\frac{Mc}{I} = -\frac{(756{,}000 \text{ lb-in.})(-6.0 \text{ in.})}{285.89 \text{ in}^4}$

$= 15{,}866$ psi

$\sigma_y = 0 \qquad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 15{,}870$ psi, $\sigma_2 = 0$

$\tau_{max} = 7930$ psi ⟵

(b) Bottom of the web (point B)

$\sigma_x = -\frac{My}{I} = -\frac{(756{,}000 \text{ lb-in.})(-5.25 \text{ in.})}{285.89 \text{ in}^4} = 13{,}883$ psi

$\sigma_y = 0 \qquad Q = b\left(\frac{h - h_1}{2}\right)\left(\frac{h + h_1}{4}\right) = 21.094$ in^3

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12{,}000 \text{ lb})(21.094 \text{ in}^3)}{(285.89 \text{ in}^4)(0.5 \text{ in.})} = -1771$ psi

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 6941.5 \pm 7163.9$ psi

$\sigma_1 = 14{,}100$ psi, $\sigma_2 = -220$ psi ⟵

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7160$ psi ⟵

(c) Neutral axis (point C) $\quad \sigma_x = 0 \quad \sigma_y = 0$

$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b-t)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right) = 27.984$ in.^{3}

CONT.

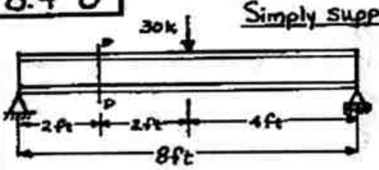
8.4-7 CONT.

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12{,}000\text{ lb})(27.984\text{ in}^3)}{(285.89\text{ in}^4)(0.5\text{ in})} = -2349\text{ psi}$$

Pure shear: $\sigma_1 = 2350$ psi, $\sigma_2 = -2350$ psi, $\tau_{max} = 2350$ psi ⟵

8.4-8

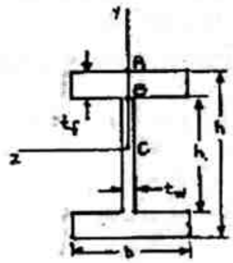
Simply supported beam



$M = 30$ k-ft $= 360$ k-in. (At section D-D)
$V = 15$ k

W 10×30

$I = 170\text{ in}^4$ $\quad b = 5.810$ in.
$t_f = 0.510$ in. $\quad t_w = 0.300$ in.
$d = 10.47$ in. $\quad h = d = 10.47$ in.
$h_1 = h - 2t_f = 9.450$ in.



(a) Top of the beam (point A)

$$\sigma_x = -\frac{Mc}{I} = -\frac{M(h/2)}{I}$$
$$= -\frac{(360\text{ k-in.})(5.235\text{ in.})}{170\text{ in.}^4} = -11{,}090\text{ psi}$$

$\sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 0$, $\sigma_2 = -11{,}090$ psi, $\tau_{max} = 5540$ psi ⟵

(b) Top of the web (point B)

$$\sigma_x = -\frac{My}{I} = -\frac{M(h_1/2)}{I} = -\frac{(360\text{ k-in.})(4.725\text{ in.})}{170\text{ in.}^4} = -10{,}006\text{ psi}$$

$\sigma_y = 0 \qquad Q = b\left(\frac{h-h_1}{2}\right)\left(\frac{h+h_1}{4}\right) = 14.76\text{ in.}^3$

$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(15\text{k})(14.76\text{ in}^3)}{(170\text{ in}^4)(0.300\text{ in.})} = -4341\text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = -5003 \pm 6624\text{ psi}$$

$\sigma_1 = 1620$ psi , $\sigma_2 = -11{,}630$ psi ⟵

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 6620\text{ psi}$$ ⟵

(c) Neutral axis (point C) $\quad \sigma_x = 0 \quad \sigma_y = 0$

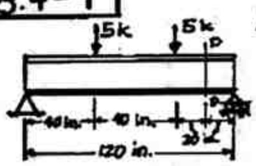
$$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b-t_w)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right) = 18.105\text{ in}^3$$

$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(15\text{ k})(18.105\text{ in}^3)}{(170\text{ in}^4)(0.300\text{ in})} = -5330\text{ psi}$$

Pure shear: $\sigma_1 = 5330$ psi, $\sigma_2 = -5330$ psi, $\tau_{max} = 5330$ psi ⟵

8.4-9

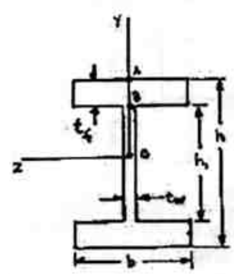
Simply supported beam



$M = 100$ k-in. (At section D-D)
$V = 5$k

W 8×28

$I = 98.0\text{ in}^4$ $\quad b = 6.535$ in.
$t_w = 0.285$ in. $\quad t_f = 0.465$ in.
$d = 8.06$ in. $\quad h = d = 8.06$ in.
$h_1 = h - 2t_f = 7.130$ in.



(a) Top of the beam (point A)

$$\sigma_x = -\frac{Mc}{I} = -\frac{M(h/2)}{I}$$
$$= -\frac{(100\text{ k-in.})(4.03\text{ in.})}{98.0\text{ in.}^4} = -4112\text{ psi}$$

$\sigma_y = 0 \qquad \tau_{xy} = 0$

Uniaxial stress : $\sigma_1 = 0$, $\sigma_2 = -4110$ psi, $\tau_{max} = 2060$ psi ⟵

CONT.

8.4-9 CONT.

(b) Top of the web (point B)

$$\sigma_x = -\frac{My}{I} = -\frac{M(h_1/2)}{I} = -\frac{(100\text{ k-in.})(3.565\text{ in.})}{98.0\text{ in.}} = -3638\text{ psi}$$

$\sigma_y = 0$

$$Q = b\left(\frac{h-h_1}{2}\right)\left(\frac{h+h_1}{4}\right) = 11.540\text{ in}^3$$

$$\tau_{xy} = \frac{VQ}{It_w} = \frac{(5\text{ k})(11.540\text{ in}^3)}{(98.0\text{ in}^4)(0.285\text{ in.})} = 2066\text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = -1819 \pm 2753\text{ psi}$$

$\sigma_1 = 930$ psi , $\sigma_2 = -4570$ psi ⟵

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 2750\text{ psi}$$ ⟵

(c) Neutral axis (point C) $\quad \sigma_x = 0 \quad \sigma_y = 0$

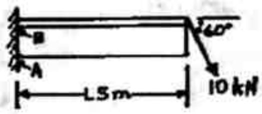
$$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b-t_w)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right) = 13.35\text{ in}^3$$

$$\tau_{xy} = \frac{VQ}{It_w} = \frac{(5\text{k})(13.35\text{ in}^3)}{(98.0\text{ in}^4)(0.285\text{ in.})} = 2390\text{ psi}$$

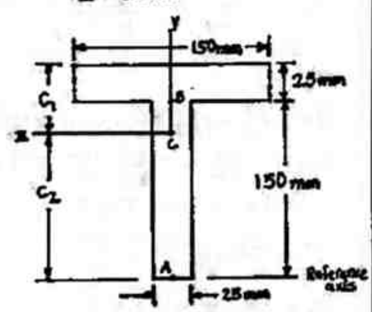
Pure shear: $\sigma_1 = 2390$ psi, $\sigma_2 = -2390$ psi, $\tau_{max} = 2390$ psi ⟵

8-4.10

Cantilever beam of T-section



$P = 10$ kN
$L = 1.5$ m



$A = 2(150\text{ mm})(25\text{ mm})$
$= 7500\text{ mm}^2$

$b = 150$ mm
$t = 25$ mm

Location of centroid C

From Eq. (12-7) in Chapter 12 :

$$c_2 = \frac{\Sigma \bar{y}_i A_i}{A}$$

For the web: $\bar{y}A = (75\text{mm})(25\text{mm})(150\text{mm}) = 281{,}250\text{ mm}^3$

For the flange: $\bar{y}A = (162.5\text{mm})(150\text{mm})(25\text{ mm}) = 609{,}375\text{ mm}^3$

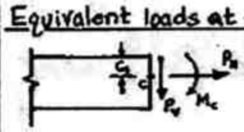
$$c_2 = \frac{281{,}250\text{ mm}^3 + 609{,}375\text{ mm}^3}{7500\text{ mm}^2} = 118.75\text{ mm}$$

$c_1 = 175\text{ mm} - c_2 = 56.25$ mm

Moment of inertia

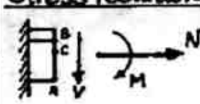
$$I_z = \frac{1}{3}tc_2^3 + \frac{1}{3}bc_1^3 - \frac{1}{3}(b-t)(c_1-t)^3 = 21.582\times10^6\text{ mm}^4$$

Equivalent loads at free end of beam



$P_H = P\cos 60° = 5.0$ kN
$P_v = P\sin 60° = 8.6603$ kN
$M_c = P_H c_1 = 281.25$ N·m

Stress resultants at fixed end of beam



$N = P_H = 5.0$ kN
$V = P_v = 8.6603$ kN
$M = M_c + P_v L = 13{,}272$ N·m

Stress at point A

$$\sigma_x = \frac{N}{A} - \frac{Mc_2}{I} = 0.67\text{ MPa} - 73.03\text{ MPa} = -72.36\text{ MPa}$$

$\sigma_y = 0 \qquad \tau_{xy} = 0$

Uniaxial stress : $\sigma_1 = 0$, $\sigma_2 = -72.4$ MPa, $\tau_{max} = 36.2$ MPa ⟵

CONT.

Stress at point B

$$\sigma_x = \frac{N}{A} + \frac{M(c_1 - t)}{I} = 0.67\text{ MPa} + 19.22\text{ MPa} = 19.89\text{ MPa}$$

$$\sigma_y = 0 \qquad Q = bt\left(c_1 - \frac{t}{2}\right) = 164.06\times10^3\text{ mm}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(8.6603\text{ kN})(164.06\times10^3\text{ mm}^3)}{(21.582\times10^6\text{ mm}^4)(25\text{ mm})} = -2.63\text{ MPa}$$

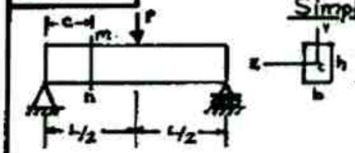
$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 9.95\text{ MPa} \pm 10.29\text{ MPa}$$

$$\sigma_1 = 20.2\text{ MPa}, \quad \sigma_2 = -0.3\text{ MPa} \longleftarrow$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 10.3\text{ MPa} \longleftarrow$$

8.4-11

Simple beam



$P = 18\text{ k}$ $\quad L = 60\text{ in.}$
$c = 20\text{ in.}$ $\quad b = 2\text{ in.}$
$h = 6\text{ in.}$

Cross section mn

$$M = \frac{Pc}{2} = 180\text{ k-in.} \qquad V = \frac{P}{2} = 9\text{ k}$$

$$\sigma_x = -\frac{My}{I} = -\frac{12My}{bh^3} = -5000y \qquad (1)$$

Units: y = in., σ_x = psi

$$\sigma_y = 0 \qquad (2)$$

$$Q = b\left(\frac{h}{2}-y\right)\left(\frac{1}{2}\right)\left(\frac{h}{2}+y\right) = \frac{b}{2}\left(\frac{h^2}{4}-y^2\right)$$

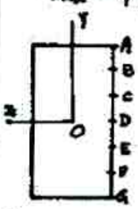
$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right) = -125(9-y^2) \qquad (3)$$

Units: y = in, τ_{xy} = psi

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \qquad (4)$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \qquad (5)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \qquad (6)$$



$$y_A = -y_G = \frac{h}{2} = 3\text{ in.}$$
$$y_B = -y_F = \frac{h}{3} = 2\text{ in.}$$
$$y_C = -y_E = \frac{h}{6} = 1\text{ in.}$$
$$y_D = 0$$

Point A (y=3 in.)

Eq. (1): $\sigma_x = -15{,}000$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$
Uniaxial stress: $\sigma_1 = 0$, $\sigma_2 = -15{,}000$ psi, $\tau_{max} = 7500$ psi

Point B (y=2 in.)

Eq. (1): $\sigma_x = -10{,}000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -625$ psi
Eqs. (4), (5), and (6): $\sigma_1 = 40$ psi, $\sigma_2 = -10{,}040$ psi, $\tau_{max} = 5040$ psi

Point C (y=1 in.)

Eq. (1): $\sigma_x = -5000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -1000$ psi
Eqs (4), (5), and (6): $\sigma_1 = 190$ psi, $\sigma_2 = -5190$ psi, $\tau_{max} = 2690$ psi

Point D (y=0)

$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\frac{3V}{2A} = -\frac{3V}{2bh} = -1125\text{ psi}$$

Pure shear: $\sigma_1 = 1125$ psi, $\sigma_2 = -1125$ psi, $\tau_{max} = 1125$ psi

Point E (y=−1 in.)

Eq. (1): $\sigma_x = 5000$ psi $\quad \sigma_y = 0$

CONT.

8.4-11 CONT.

Eq. (3): $\tau_{xy} = -1000$ psi
Eqs. (4), (5), and (6): $\sigma_1 = 5190$ psi, $\sigma_2 = -190$ psi, $\tau_{max} = 2690$ psi

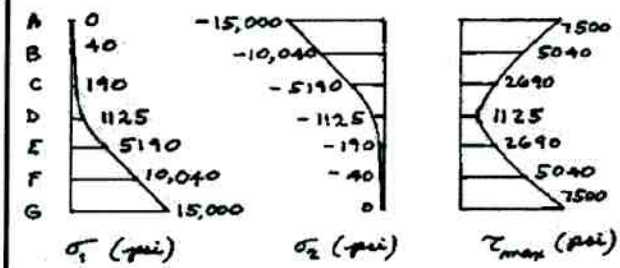
Point F (y=−2 in.)

Eq. (1): $\sigma_x = 10{,}000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -625$ psi
Eqs (4), (5), and (6): $\sigma_1 = 10{,}040$ psi, $\sigma_2 = -40$ psi, $\tau_{max} = 5040$ psi

Point G (y=−3 in.)

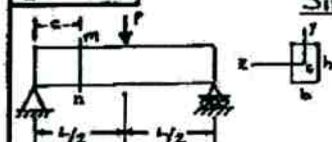
Eq. (1): $\sigma_x = 15{,}000$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$
Uniaxial stress: $\sigma_1 = 15{,}000$ psi, $\sigma_2 = 0$, $\tau_{max} = 7500$ psi

Graphs of stresses



8.4-12

Simple beam



$P = 144\text{ kN}$
$L = 0.7\text{ m}$
$c = 0.15\text{ m}$
$b = 20\text{ mm}$
$h = 120\text{ mm}$

Cross section mn

$$M = \frac{Pc}{2} = 10.8\text{ kN·m} \qquad V = \frac{P}{2} = 72\text{ kN}$$

$$\sigma_x = -\frac{My}{I} = -\frac{12My}{bh^3} = -3.75y \qquad (1)$$

Units: y = mm, σ_x = MPa

$$\sigma_y = 0 \qquad (2)$$

$$Q = b\left(\frac{h}{2}-y\right)\left(\frac{1}{2}\right)\left(\frac{h}{2}+y\right) = \frac{b}{2}\left(\frac{h^2}{4}-y^2\right)$$

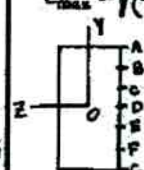
$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right) = -12.5\left(3.6-\frac{y^2}{10^3}\right) \qquad (3)$$

Units: y = mm, τ_{xy} = MPa

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \qquad (4)$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \qquad (5)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_{xy}^2} \qquad (6)$$



$$y_A = -y_G = \frac{h}{2} = 60\text{ mm}$$
$$y_B = -y_F = \frac{h}{3} = 40\text{ mm}$$
$$y_C = -y_E = \frac{h}{6} = 20\text{ mm}$$
$$y_D = 0$$

Point A (y=60 mm)

Eq. (1): $\sigma_x = -225$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$
Uniaxial stress: $\sigma_1 = 0$, $\sigma_2 = -225$ MPa, $\tau_{max} = 112$ MPa

Point B (y = 40 mm)

Eq. (1): $\sigma_x = -150$ MPa $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -25$ MPa
Eqs. (4), (5), and (6): $\sigma_1 = 4$ MPa, $\sigma_2 = -154$ MPa, $\tau_{max} = 79$ MPa

Point C (y = 20 mm)

Eq. (1): $\sigma_x = -75$ MPa $\quad \sigma_y = 0$

CONT.

8.4-12 CONT.

Eq. (3): $\tau_{xy} = -40$ MPa

Eqs. (4), (5), and (6): $\sigma_1 = 17$ MPa, $\sigma_2 = -92$ MPa, $\tau_{max} = 55$ MPa

Point D (y=0)

$\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = -\frac{3V}{2A} = -\frac{3V}{2bh} = -45$ MPa

Pure shear: $\sigma_1 = 45$ MPa, $\sigma_2 = -45$ MPa, $\tau_{max} = 45$ MPa

Point E (y = −20 mm)

Eq. (1): $\sigma_x = 75$ MPa $\quad \sigma_y = 0$

Eq. (3): $\tau_{xy} = -40$ MPa

Eqs. (4), (5), and (6): $\sigma_1 = 92$ MPa, $\sigma_2 = -17$ MPa, $\tau_{max} = 55$ MPa

Point F (y = −40 mm)

Eq. (1): $\sigma_x = 150$ MPa, $\quad \sigma_y = 0$

Eq. (3): $\tau_{xy} = -25$ MPa

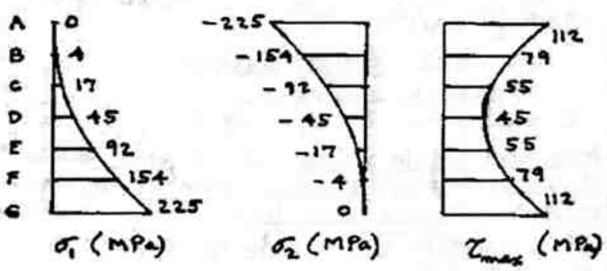
Eqs. (4), (5), and (6): $\sigma_1 = 154$ MPa, $\sigma_2 = -4$ MPa, $\tau_{max} = 79$ MPa

Point G (y = −60 mm)

Eq. (1): $\sigma_x = 225$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

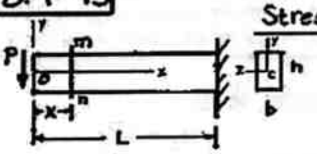
Uniaxial stress: $\sigma_1 = 225$ MPa, $\sigma_2 = 0$, $\tau_{max} = 112$ MPa

Graphs of stresses



8.4-13

Stress contours for τ_{max}



P = 200 lb
b = 1 in.
h = 3 in.
L = 18 in.

Cross section mn

$M = -Px \qquad V = P \qquad A = bh \qquad I = \frac{bh^3}{12}$

$$\sigma_x = -\frac{My}{I} = -\frac{12Pxy}{bh^3} \qquad (1)$$

$$\sigma_y = 0 \qquad (2)$$

$$Q = b\left(\frac{h}{2} - y\right)\left(\frac{1}{2}\right)\left(\frac{h}{2} + y\right) = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{6P}{bh^3}\left(\frac{h^2}{4} - y^2\right) \qquad (3)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute from Eqs. (1), (2), and (3) and simplify:

$$\tau_{max} = \frac{6P}{bh^3}\sqrt{(xy)^2 + \left(\frac{h^2}{4} - y^2\right)^2} \longleftarrow \qquad (4)$$

Eq. (4) is the equation of the contour lines for τ_{max}.

Substitute numerical values:

$$\tau_{max} = 44.44\sqrt{(xy)^2 + (2.25 - y^2)^2} \qquad (5)$$

units: x = in., y = in., τ_{max} = psi

Neutral axis (y = 0)

$$\tau_{NA} = \frac{3P}{2bh} = 100 \text{ psi} \longleftarrow \qquad (6)$$

Top and bottom of the beam ($y = \pm\frac{h}{2}$)

From Eq. (4): $\tau_{max} = \frac{3Px}{bh^2} = 66.67x$

CONT.

8.4-13 CONT.

τ_{max} (psi)	200	400	600	800	1000	1200
x (in.)	3.0	6.0	9.0	12.0	15.0	18.0

Note that the contour lines are symmetric about the neutral axis (because y is squared in Eqs. 4 and 5)

Intermediate locations

Use Eq. (5)
Select a value of τ_{max}
Select various values of x
For each value of x, solve Eq. (5) for the corresponding value of y (use a computer program or use trial-and-error).

τ_{max} = 200 psi	x	3.0	6.0	9.0	12.0	15.0	18.0
	y	1.50	0.69	0.44	0.33	0.26	0.22

τ_{max} = 400 psi	x	6.0	9.0	12.0	15.0	18.0
	y	1.5	0.99	0.74	0.59	0.49

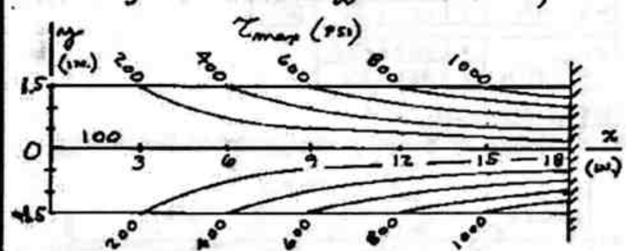
τ_{max} = 600 psi	x	9.0	12.0	15.0	18.0
	y	1.5	1.12	0.89	0.74

τ_{max} = 800 psi	x	12.0	15.0	18.0
	y	1.5	1.20	1.00

τ_{max} = 1000 psi	x	15.0	18.0
	y	1.5	1.25

Stress contours

Note: Height of beam is exaggerated for clarity.



8.4-14

Stress contours for τ_{max}

P = 1800 N
b = 30 mm
h = 90 mm
L = 540 mm

Cross section mn

$M = -Px \qquad V = P \qquad A = bh \qquad I = \frac{bh^3}{12}$

$$\sigma_x = -\frac{My}{I} = -\frac{12Pxy}{bh^3} \qquad (1)$$

$$\sigma_y = 0 \qquad (2)$$

$$Q = b\left(\frac{h}{2} - y\right)\left(\frac{1}{2}\right)\left(\frac{h}{2} + y\right) = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{6P}{bh^3}\left(\frac{h^2}{4} - y^2\right) \qquad (3)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute from Eqs. (1), (2), and (3) and simplify:

$$\tau_{max} = \frac{6P}{bh^3}\sqrt{(xy)^2 + \left(\frac{h^2}{4} - y^2\right)^2} \longleftarrow \qquad (4)$$

Eq. (4) is the equation of the contour lines for τ_{max}.

Substitute numerical values:

$$\tau_{max} = 493.83 \times 10^{-6}\sqrt{(xy)^2 + (2025 - y^2)^2} \qquad (5)$$

units: x = mm, y = mm, τ_{max} = MPa

CONT.

8.4-14 CONT.

Neutral axis (y=0)

$$\tau_{NA} = \frac{3P}{2bh} = 1.0 \text{ MPa} \leftarrow \qquad (6)$$

Top and bottom of the beam $(y = \pm\frac{h}{2})$

From Eq. (4): $\tau_{max} = \frac{3Px}{bh^2} = 0.022222\,x$

τ_{max} (MPa)	2.0	4.0	6.0	8.0	10.0
x (mm)	90	180	270	360	450

Note that the contour lines are symmetric about the neutral axis (because y is squared in Eqs. 4 and 5).

Intermediate locations

Use Eq. (5)
Select a value of τ_{max}
Select various values of x
For each value of x, solve Eq. (5) for the corresponding value of y (use a computer program or use trial-and-error).

τ_{max} = 2.0 MPa	x	90	180	270	360	450	540
	y	45	21	13	10	8	7

τ_{max} = 4.0 MPa	x	180	270	360	450	540
	y	45	30	22	18	15

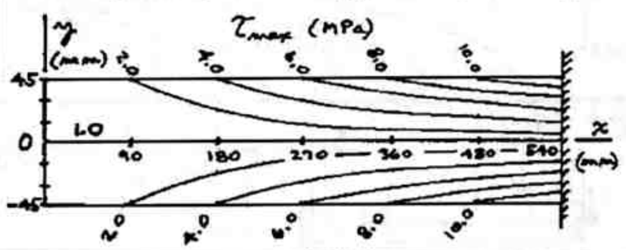
τ_{max} = 6.0 MPa	x	270	360	450	540
	y	45	34	27	22

τ_{max} = 8.0 MPa	x	360	450	540
	y	45	36	30

τ_{max} = 10.0 MPa	x	450	540
	y	45	37

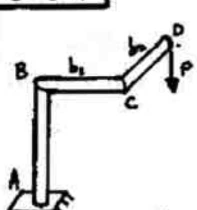
Stress contours

Note: Height of beam is exaggerated for clarity.



8-5.1

Bracket

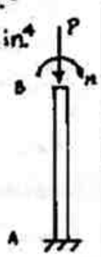


$b_1 = 4 \text{ ft} \qquad b_2 = 3 \text{ ft}$
$P = 2.0 \text{k}$
$d_2 = 8.0 \text{ in.} \qquad d_1 = 7.0 \text{ in.}$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 11.78 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 83.20 \text{ in.}^4$$

$$M = P\sqrt{b_1^2 + b_2^2} = (2.0\text{k})(5\text{ft}) = 10 \text{ k-ft} = 120{,}000 \text{ lb-in.}$$



The maximum stresses occur on opposite sides of the vertical arm.

Maximum tensile stress

$$\sigma_t = -\frac{P}{A} + \frac{M(d_2/2)}{I}$$

$$= -\frac{2000 \text{ lb}}{11.78 \text{ in.}^2} + \frac{(120{,}000 \text{ lb-in.})(4.0 \text{ in.})}{83.20 \text{ in.}^4} = -170 \text{ psi} + 5769 \text{ psi}$$

$$= 5600 \text{ psi} \leftarrow$$

CONT.

8.5-1 CONT.

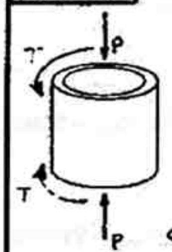
Maximum compressive stress

$$\sigma_c = -\frac{P}{A} - \frac{M(d_2/2)}{I} = -170 \text{ psi} - 5769 \text{ psi} = -5940 \text{ psi} \leftarrow$$

Maximum shear stress $\tau_{max} = \left|\frac{\sigma_c}{2}\right| = 2970 \text{ psi} \leftarrow$

8.5-2

Drill pipe



$d_1 = 120 \text{ mm} \qquad d_2 = 150 \text{ mm}$
$P = 265 \text{ kN} \qquad T = 19 \text{ kN·m}$

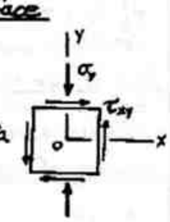
$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 6.362 \times 10^{-3} \text{ m}^2$$

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 29.34 \times 10^{-6} \text{ m}^4$$

Stresses at the surface

$$\sigma_y = -\frac{P}{A} = -\frac{265 \text{ kN}}{6.362 \times 10^{-3} \text{ m}^2} = -41.65 \text{ MPa}$$

$$\tau_{xy} = \frac{Tr}{I_P} = \frac{(19 \text{ kN·m})(75 \text{ mm})}{29.34 \times 10^{-6} \text{ m}^4} = 48.57 \text{ MPa}$$



Principal stresses

$$\sigma_{1,2} = \frac{\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\frac{41.65}{2} \pm \sqrt{\left(\frac{41.65}{2}\right)^2 + (48.57)^2}$$

$$= -20.83 \text{ MPa} \pm 52.85 \text{ MPa}$$

$\sigma_1 = 32.0 \text{ MPa} \qquad \sigma_2 = -73.7 \text{ MPa}$

Maximum tensile stress $\sigma_t = 32.0 \text{ MPa} \leftarrow$

Maximum compressive stress $\sigma_c = -73.7 \text{ MPa} \leftarrow$

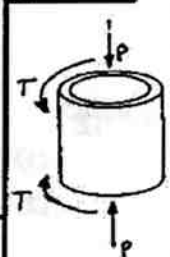
Maximum in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 52.8 \text{ MPa} \leftarrow$$

(Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.)

8.5-3

Generator shaft



$d_1 = 9 \text{ in.} \qquad d_2 = 11 \text{ in.}$
$H = 2500 \text{ hp}$
$n = 250 \text{ rpm}$
$P = 120 \text{ k}$

Eq. (3-43) from Chapter 3:

$$H = \frac{2\pi n T}{33{,}000} \qquad T = \text{lb-ft}$$

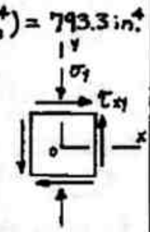
$$T = \frac{33{,}000 H}{2\pi n} = 52{,}521 \text{ lb-ft} = 630{,}300 \text{ lb-in.}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 31.42 \text{ in.}^2 \qquad I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 793.3 \text{ in.}^4$$

Stresses at the surface

$$\sigma_y = -\frac{P}{A} = -\frac{120 \text{ k}}{31.42 \text{ in.}^2} = -3819 \text{ psi}$$

$$\tau_{xy} = \frac{Tr}{I_P} = \frac{(630{,}000 \text{ lb-in.})(5.5 \text{ in.})}{793.3 \text{ in.}^4} = 4370 \text{ psi}$$



Principal stresses

$$\sigma_{1,2} = \frac{\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\frac{3819}{2} \pm \sqrt{\left(-\frac{3819}{2}\right)^2 + (4370)^2} \quad \text{(psi)}$$

$$= -1910 \text{ psi} \pm 4769 \text{ psi}$$

$\sigma_1 = 2860 \text{ psi} \qquad \sigma_2 = -6680 \text{ psi}$

Maximum tensile stress $\sigma_t = 2860 \text{ psi} \leftarrow$

Maximum compressive stress $\sigma_c = -6680 \text{ psi} \leftarrow$

CONT.

8.5-3 CONT.

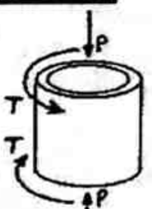
Maximum in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = 4770 \text{ psi}$$

(Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.)

8.5-4

Generator shaft



$d_1 = 160$ mm $\quad d_2 = 200$ mm

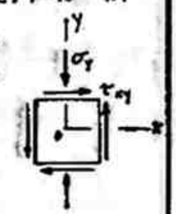
$T = 25$ kN·m

$\tau_{allow} = 45$ MPa (in-plane shear stress)

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 11.31 \times 10^{-3} \text{ m}^2$$

$$I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = 92.74 \times 10^{-6} \text{ m}^4$$

Stress at the surface



$$\sigma_y = -\frac{P}{A} = -\frac{P}{11.31 \times 10^{-3} \text{ m}^2} \quad \begin{pmatrix} P = \text{newtons} \\ \sigma_y = \text{pascals} \end{pmatrix}$$

$$\tau_{xy} = \frac{Tr}{I_p} = \frac{(25 \text{ kN·m})(100 \text{ mm})}{92.74 \times 10^{-6} \text{ m}^4} = 26.96 \text{ MPa}$$

Maximum in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Square both sides and replace τ_{max} by τ_{allow}:

$$(\tau_{allow})^2 = \left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$(45 \text{ MPa})^2 = \left[\frac{P}{2(11.31 \times 10^{-3} \text{ m}^2)(10^6)}\right]^2 + (26.96 \text{ MPa})^2$$

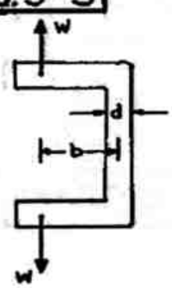
Solve for P: $\quad \frac{P^2}{511.7 \times 10^6} = 1298.2$

$P = 815{,}000$ N $= 815$ kN

(Note: The maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.)

8.5-5

Arm of gondola



$b = 7.0$ in. $\quad W = \frac{1300 \text{ lb}}{2} = 650$ lb

$\sigma_{allow} = 15{,}000$ psi

$\tau_{allow} = 7500$ psi

Find diameter d (inches)

$$A = \frac{\pi d^2}{4} \qquad S = \frac{\pi d^3}{32}$$

The maximum stresses occur on opposite sides of the arm. The tensile stress is larger than the compressive stress.

Maximum tensile stress

$$\sigma_t = \frac{W}{A} + \frac{M}{S} = \frac{650 \text{ lb}}{\pi d^2/4} + \frac{(650 \text{ lb})(7.0 \text{ in.})}{\pi d^3/32}$$

or $\quad 15{,}000 \text{ psi} = \frac{827.6}{d^2} + \frac{46{,}346}{d^3}$

$$15{,}000\, d^3 - 827.6 d - 46{,}346 = 0$$

Solve for d: $\quad d = 1.47$ in.

Maximum shear stress

τ_{max} is one-half of σ_t.

Since τ_{allow} is one-half of σ_{allow}, the required diameter is the same. $\quad d = 1.47$ in.

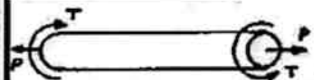
8.5-6

Circular shaft

$P = 80$ kN $\quad T = 1.1$ kN·m

CONT.

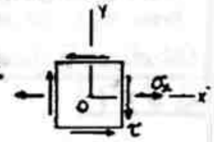
8.5-6 CONT.



$\sigma_{allow} = 60$ MPa

$\tau_{allow} = 30$ MPa

Stress at the surface



$$\sigma_x = \frac{P}{A} = \frac{80 \text{ kN}}{\pi d^2/4} = \frac{101{,}860}{d^2}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(1.1 \text{ kN·m})}{\pi d^3} = \frac{5{,}602{,}300}{d^3}$$

Units: σ_x = MPa $\quad \tau$ = MPa $\quad d$ = mm

Maximum tensile stress

$$\sigma_t = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$60 = \frac{50{,}930}{d^2} + \sqrt{\left(\frac{50{,}930}{d^2}\right)^2 + \left(\frac{5{,}602{,}300}{d^3}\right)^2}$$

Solve for d: $\quad d = 53.0$ mm

(Note: The maximum compressive stress is smaller than the maximum tensile stress.)

Maximum in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$30 = \sqrt{\left(\frac{50{,}930}{d^2}\right)^2 + \left(\frac{5{,}602{,}300}{d^3}\right)^2}$$

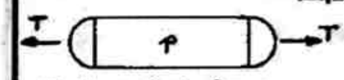
Solve for d: $\quad d = 59.7$ mm

(Note: The maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.)

Shear stress governs. $\quad d = 59.7$ mm

8.5-7

Cylindrical tank



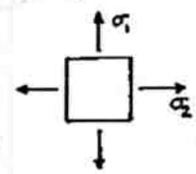
$d = 2.5$ in

$p = 600$ psi

$T = 1000$ lb (tension)

$\tau_{allow} = 3000$ psi

$T = P$ = axial force



$$\sigma_1 = \frac{pr}{t} = \frac{(600 \text{ psi})(1.25 \text{ in})}{t} = \frac{750}{t}$$

Units: σ_1 = psi $\quad t$ = in. $\quad \sigma_2$ = psi

$$\sigma_2 = \frac{pr}{2t} + \frac{T}{2\pi r t} = \frac{750}{2t} + \frac{(1000 \text{ lb})}{(2\pi)(1.25 \text{ in.})t} = \frac{502.32}{t}$$

In-plane shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\left(\frac{750}{t} - \frac{502.32}{t}\right) = \frac{123.84}{t}$$

$$\tau_{max} = 3000 \text{ psi} \qquad t = \frac{123.84}{3000} = 0.0413 \text{ in.}$$

Out-of-plane shear stress

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{375}{t} \quad \text{or} \quad \tau_{max} = \frac{\sigma_2}{2} = \frac{251.16}{t}$$

Stress σ_1 governs. $\quad t = \frac{375}{3000} = 0.125$ in.

Out-of-plane shear stress governs. $\quad t_{min} = 0.125$ in.

8.5-8

Cylindrical pressure vessel

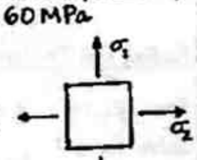
$d = 100$ mm

$t = 4$ mm

$F = 72$ kN (compression)

$\tau_{allow} = 60$ MPa

$F = P$ = axial force



$$\sigma_1 = \frac{pr}{t} = \frac{p(50 \text{ mm})}{4 \text{ mm}} = 12.5p$$

Units: σ_1 = MPa $\quad \sigma_2$ = MPa $\quad p$ = MPa

$$\sigma_2 = \frac{pr}{2t} - \frac{F}{2\pi r t} = 6.25p - 57.30 \text{ MPa}$$

In-plane shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}(12.5p - 6.25p + 57.30 \text{ MPa})$$

$$60 \text{ MPa} = 3.125p + 28.65 \text{ MPa}$$

$$p_{max} = 10.0 \text{ MPa}$$

CONT.

8-5.8 CONT.

Out-of-plane shear stress

$\tau_{max} = \frac{\sigma_1}{2}$ or $\tau_{max} = \frac{\sigma_2}{2}$

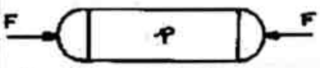
From σ_1: $60\text{ MPa} = 6.25p$ $p_{max_1} = 9.6\text{ MPa}$

From σ_2: $60\text{ MPa} = 3.125p - 28.65$ $p_{max_2} = 28.4\text{ MPa}$

Out-of-plane shear stress governs. $p_{max} = 9.6\text{ MPa}$ ←

8.5-9

Cylindrical pressure vessel



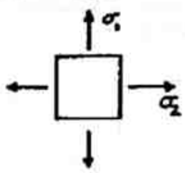
$p = 1000$ psi

$F = 12{,}000$ lb (comp.)

$\tau_{allow} = 6000$ psi

$F = P$ = axial force $10 \le \frac{r}{t} \le 20$

$A = 2\pi r t$ Units: σ_1 and σ_2 = psi r and t = in.



$$\sigma_1 = \frac{pr}{t} = 1000\left(\frac{r}{t}\right) \qquad (1)$$

$$\sigma_2 = \frac{pr}{2t} - \frac{F}{2\pi r t} = 500\left(\frac{r}{t}\right) - \frac{1909.9}{rt} \qquad (2)$$

In-plane shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 500\left(\frac{r}{t}\right) - 250\left(\frac{r}{t}\right) + \frac{954.9}{rt}$$

$$= 250\left(\frac{r}{t}\right) + \frac{954.9}{rt} \qquad (3)$$

$$6000 = 250\left(\frac{r}{t}\right) + \frac{954.9}{rt}$$

Solve for t:

$$t = \frac{r}{24} + \frac{0.15915}{r} \qquad (4)$$

Relationships between r, t, and A

Assume a value for r

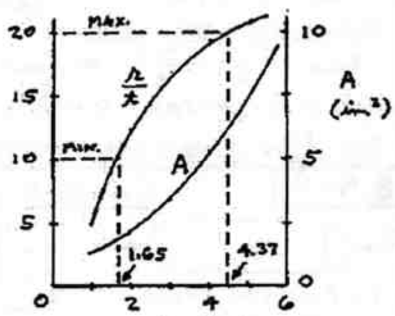
Calculate t from Eq. (4)

Calculate r/t

Calculate $A = 2\pi r t$

r (in.)	t (in.)	r/t	A (in²)
1.0	0.201	5.0	1.26
2.0	0.163	12.3	2.05
3.0	0.178	16.9	3.36
4.0	0.206	19.4	5.18
5.0	0.240	20.8	7.54
6.0	0.277	21.7	10.42

From the graph, we see that the smallest area occurs when r/t is the smallest. Therefore, use $\frac{r}{t} = 10$



Radius and Thickness

$r = 10t$

From Eq. (4): $t = \frac{10t}{24} + \frac{0.15915}{10t}$

Solve for t: $t = 0.1652$ in. $r = 10t = 1.652$ in.

Principal stresses

From Eq. (1): $\sigma_1 = 1000\left(\frac{r}{t}\right) = 10{,}000$ psi

From Eq. (2): $\sigma_2 = 500\left(\frac{r}{t}\right) - \frac{1909.9}{rt} = -1998$ psi

CONT.

8.5-9 CONT.

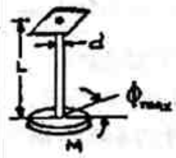
Since the principal stresses have opposite signs, the maximum out-of-plane shear stress is smaller than the maximum in-plane shear stress. Therefore, the use of Eqs. (3) and (4) is valid.

Optimum radius and thickness

$r = 1.65$ in. $t = \frac{r}{10} = 0.165$ in. ←

8.5-10

Torsional pendulum



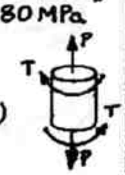
$L = 1$ m $d = 4$ mm $M = 50$ kg

$G = 80$ GPa $\sigma_{allow} = 80$ MPa

$\tau_{allow} = 50$ MPa

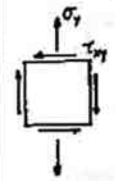
$A = \frac{\pi d^2}{4} = 12.566\text{ mm}^2$

$P = Mg = (50\text{ kg})(9.81\text{ m/s}^2)$
$= 490.5$ N



$T = \frac{GI_P \phi_{max}}{L}$ $\tau = \frac{TR}{I_P}$

$\tau = \frac{Gr\phi_{max}}{L} = 160\,\phi_{max}$ Units: ϕ_{max} = rad, τ = MPa



$\sigma_x = 0$

$\sigma_y = \frac{P}{A} = 39.03$ MPa

$\tau_{xy} = -160\,\phi_{max}$ (MPa)

Based on tension

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$80\text{MPa} = 19.52\text{ MPa} + \sqrt{380.84 + 25{,}600\phi_{max}^2}$$

Combine terms, square, and solve: $\phi_{max} = 0.358$ rad

Based on in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$50\text{ MPa} = \sqrt{380.84 + 25{,}600\,\phi_{max}^2}$$

Square and solve: $\phi_{max} = 0.288$ rad

Shear governs. $\phi_{max} = 0.288$ rad ←

Principal stresses

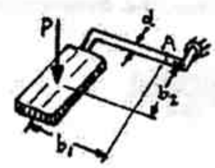
$\sigma_x = 0$ $\sigma_y = 39.03$ MPa $\tau_{xy} = -160(0.288)$
$= -46.1$ MPa

$\sigma_1 = 69.6$ MPa $\sigma_2 = -30.5$ MPa

Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-11

Pedal crank



$P = 160$ lb $d = 0.6$ in.

$b_1 = 5.0$ in. $b_2 = 2.5$ in.

Stress resultants on cross section at point A:

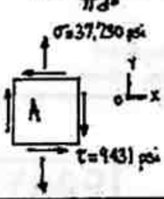
Torque $T = Pb_2 = 400$ lb-in.

Moment $M = Pb_1 = 800$ lb-in.

Shear force $V = P = 160$ lb

Stresses at point A

$\tau = \frac{16T}{\pi d^3} = 9431$ psi $\sigma = \frac{M}{S} = \frac{32M}{\pi d^3} = 37{,}730$ psi



(The shear force V produces zero shear stresses at point A)

Principal stresses and maximum shear stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

CONT.

8.5-11 CONT.

$\sigma_x = 0 \quad \sigma_y = 37{,}730\text{ psi} \quad \tau_{xy} = -9431\text{ psi}$

$\sigma_{1,2} = 18{,}860\text{ psi} \pm 21{,}090\text{ psi}$

$\sigma_1 = 39{,}950\text{ psi} \quad \sigma_2 = -2230\text{ psi}$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21{,}090\text{ psi}$$

Maximum tensile stress: $\sigma_t = 39{,}950\text{ psi}$ ←

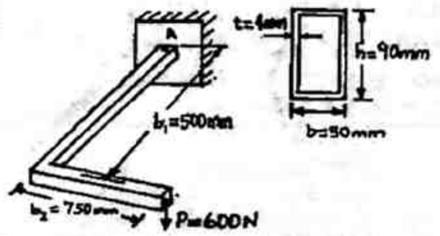
Maximum compressive stress: $\sigma_c = -2230\text{ psi}$ ←

Maximum in-plane shear stress: $\tau_{max} = 21{,}090\text{ psi}$ ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-12

L-shaped bracket



$P = 600\text{ N}$
$t = 4\text{ mm}$
$b = 50\text{ mm}$
$h = 90\text{ mm}$
$b_1 = 500\text{ mm}$
$b_2 = 750\text{ mm}$

Stress resultants at the support

Torque: $T = Pb_2 = (600\text{ N})(750\text{ mm}) = 450\text{ N·m}$

Moment: $M = Pb_1 = (600\text{N})(500\text{mm}) = 300\text{ N·m}$

Shear force: $V = P = 600\text{N}$

Stresses at point A

$\tau = \dfrac{T}{2tA_m} \qquad A_m = (b-t)(h-t) = (46\text{mm})(86\text{mm}) = 3956\text{ mm}^2$

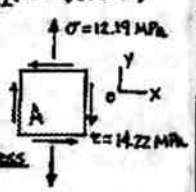
$$\tau = \frac{450\text{ N·m}}{2(4\text{mm})(3956\text{mm}^2)} = 14.22\text{ MPa}$$

$\sigma = \dfrac{Mc}{I} \qquad I = \frac{1}{12}bh^3 - \frac{1}{12}(b-2t)(h-2t)^3$

$= \frac{1}{12}(50\text{mm})(90\text{mm})^3 - \frac{1}{12}(42\text{mm})(82\text{mm})^3$

$= 1.1077 \times 10^6\text{ mm}^4$

$$\sigma = \frac{(300\text{ N·m})(45\text{mm})}{1.1077\times10^6\text{mm}^4} = 12.19\text{ MPa}$$



(The shear force V produces zero shear stress at point A)

Principal stresses and maximum shear stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = 0 \quad \sigma_y = 12.19\text{ MPa} \quad \tau_{xy} = -14.22\text{ MPa}$

$\sigma_{1,2} = 6.095\text{ MPa} \pm 15.47\text{ MPa}$

$\sigma_1 = 21.57\text{ MPa} \quad \sigma_2 = -9.38\text{ MPa}$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 15.47\text{ MPa}$$

Maximum tensile stress: $\sigma_t = 21.6\text{ MPa}$ ←

Maximum compressive stress: $\sigma_c = -9.4\text{ MPa}$ ←

Maximum shear stress: $\tau_{max} = 15.5\text{ MPa}$ ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-13

Cylindrical pressure vessel

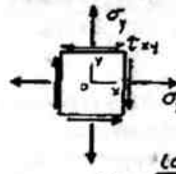
$T = 90\text{ k-ft} = 1080\text{ k-in}$

$r = 12\text{ in.} \quad t = 0.6\text{ in.}$

$p = 360\text{ psi}$

CONT.

8.5-13 CONT.



Stresses in the wall of the vessel

$\sigma_x = \dfrac{pr}{2t} = 3600\text{ psi} \quad \sigma_y = \dfrac{pr}{t} = 7200\text{ psi}$

$$\tau_{xy} = -\frac{T}{2tA_m} = -\frac{1080\text{ k-in.}}{2(0.6\text{ in.})(\pi)(12\text{ in.})^2} = -1989\text{ psi}$$

(a) Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5400 \pm 2683\text{ psi}$$

$\sigma_1 = 8080\text{ psi} \quad \sigma_2 = 2720\text{ psi} \quad \therefore \sigma_{max} = 8080\text{ psi}$ ←

Maximum in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2680\text{ psi}$$ ←

(b) Maximum allowable torque T

$\tau_{allow} = 3000\text{ psi}$ (in-plane shear stress)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (1)$$

$\sigma_x = 3600\text{ psi} \qquad \sigma_y = 7200\text{ psi}$

$\tau_{xy} = -\dfrac{T}{2tA_m} = -0.0018421\,T$

Units: τ_{xy} = psi $\quad T$ = lb-in.

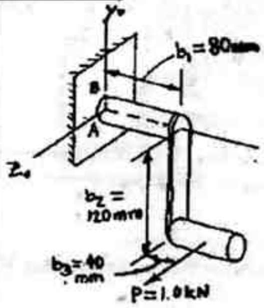
Substitute into Eq. (1):

$$3000 = \sqrt{(-1800)^2 + (-0.0018421T)^2}$$

Solve for T: $\quad T = 1.303\times10^6\text{ lb-in.} = 109\text{ k-ft}$ ←

8.5-14

Crankshaft



$P = 1.0\text{ kN} \qquad d = 20\text{mm}$

$b_1 = 80\text{mm} \qquad b_2 = 120\text{mm}$

$b_3 = 40\text{mm}$

Stress resultants at the support:

$M_y = -Pb_3 - Pb_1$

$= -40\text{N·m} - 80\text{N·m} = -120\text{N·m}$

$T_x = -Pb_2 = -120\text{N·m}$

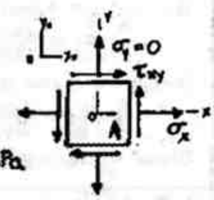
$V_z = P = 1.0\text{ kN}$

a) Stresses at point A

$$\sigma_x = \frac{M_y}{S} = \frac{32M_y}{\pi d^3} = \frac{32(-120\text{ N·m})}{\pi(20\text{mm})^3}$$

$= -152.8\text{ MPa}$ (compression)

$$\tau_{xy} = \left|\frac{16T_x}{\pi d^3}\right| = \frac{16(120\text{ N·m})}{\pi(20\text{mm})^3} = 76.39\text{ MPa}$$



(The shear force V produces zero shear stress at point A.)

Principal stresses and maximum in-plane shear stress

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$= -76.40\text{ MPa} \pm 108.04\text{ MPa}$

$\sigma_1 = 31.64\text{ MPa} \qquad \sigma_2 = -184.44\text{ MPa}$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 108.04\text{ MPa}$$

Maximum tensile stress: $\sigma_t = 32\text{ MPa}$ ←

Maximum compressive stress: $\sigma_c = -184\text{ MPa}$ ←

Maximum shear stress: $\tau_{max} = 108\text{ MPa}$ ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

(b) Stresses at point B $\qquad \sigma_x = 0 \qquad \sigma_y = 0$

$\tau_{xy} = \dfrac{16T}{\pi d^3} - \dfrac{4V}{3A} \qquad A = \dfrac{\pi d^2}{4}$

$\tau_{xy} = 76.39\text{ MPa} - 4.24\text{ MPa}$

$= 72.15\text{ MPa}$

CONT.

8.5-14 CONT.

Principal stresses and maximum in-plane shear stress

Pure shear: $\sigma_1 = 72.2$ MPa $\quad \sigma_2 = -72.2$ MPa

$\tau_{max} = 72.2$ MPa

Maximum tensile stress: $\sigma_t = 72.2$ MPa ←

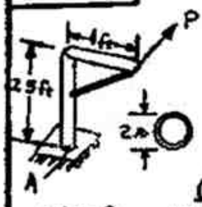
Maximum compressive stress: $\sigma_c = -72.2$ MPa ←

Maximum shear stress: $\tau_{max} = 72.2$ MPa ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-15

Post with horizontal load



$P = 360$ lb $\qquad h = 25$ ft

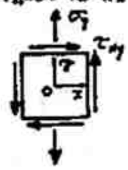
$S = 10$ in.^{3} $\qquad b = 4$ ft

$M = Ph = (360\text{ lb})(25\text{ ft})(12\text{ in/ft}) = 108{,}000$ lb-in.

$T = Pb = (360\text{ lb})(4\text{ ft})(12\text{ in/ft}) = 17{,}280$ lb-in

$V = P = 360$ lb

(a) Stresses at point A



$\sigma_x = 0 \qquad \sigma_y = \frac{M}{S} = 10{,}800$ psi

$S = \frac{I}{r} \qquad I = \frac{I_p}{2} \qquad I_p = 2I = 2Sr$

$\tau_{xy} = \frac{Tr}{I_p} = \frac{T}{2S} = 864$ psi

(The shear force V produces no stresses at point A.)

Principal stresses and maximum in-plane shear stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$= 5400$ psi ± 5469 psi

$\sigma_1 = 10{,}870$ psi $\qquad \sigma_2 = -70$ psi

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5470 \text{ psi}$$

Maximum tensile stress: $\sigma_{max} = 10{,}870$ psi ←

Maximum shear stress: $\tau_{max} = 5470$ psi ←

(b) Allowable load P_{allow}

$\sigma_{allow} = 16{,}000$ psi $\qquad \tau_{allow} = 6{,}000$ psi

The stresses at point A are proportional to the load P.

Based on tensile stress:

$\frac{P_{allow}}{360 \text{ lb}} = \frac{16{,}000 \text{ psi}}{10{,}870 \text{ psi}} \qquad P_{allow} = 530$ lb

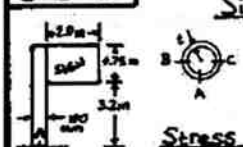
Based on shear stress:

$\frac{P_{allow}}{360 \text{ lb}} = \frac{6000 \text{ psi}}{5470 \text{ psi}} \qquad P_{allow} = 395$ lb

Shear governs. $P_{allow} = 395$ lb ←

8.5-16

Sign supported by a pipe



$d_1 = 80$ mm $\qquad d_2 = 100$ mm

$t = 10$ mm $\qquad p = 1.8$ kPa

Stress resultants at the base

Horizontal wind force on the sign = P

$P = (1.8 \text{ kPa})(2.0 \text{ m})(0.75 \text{ m}) = 2700$ N

$M = Ph = (2700 \text{ N})(3.2 \text{ m} + 0.375 \text{ m}) = 9652$ N·m

$T = Pb = (2700 \text{ N})(1.0 \text{ m} + 0.05 \text{ m}) = 2835$ N·m

$V = P = 2700$ N

Properties of the cross section

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 2.898 \times 10^6$ mm^4

$I_p = 2I = 5.796 \times 10^6$ mm^4

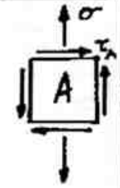
$Q = \frac{2}{3}(r_2^3 - r_1^3) = \frac{1}{12}(d_2^3 - d_1^3) = 40.67 \times 10^3$ mm^3

(From Eq. 5-43b, Chapter 5)

CONT.

8.5-16 CONT.

Stresses at point A



$\sigma = \frac{Mc}{I} = \frac{M(d_2/2)}{I} = 166.5$ MPa

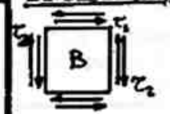
$\tau = \frac{Tr}{I_p} = \frac{T(d_2/2)}{I_p} = 24.46$ MPa

$\sigma_x = 0 \qquad \sigma_y = 166.5$ MPa $\qquad \tau_{xy} = 24.46$ MPa

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 86.8$ MPa

$\tau_A = 86.8$ MPa ←

Stresses at point B



$\tau_1 = \frac{Tr}{I_p} = 24.46$ MPa

$\tau_2 = \frac{VQ}{Ib} = \frac{VQ}{I(2t)} = \frac{(2700 \text{ N})(40.67 \times 10^3 \text{ mm}^3)}{(2.898 \times 10^6 \text{ mm}^4)(20 \text{ mm})}$

$= 1.895$ MPa

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = \tau_1 - \tau_2 = 22.57$ MPa

Pure shear. $\tau_B = 22.6$ MPa ←

Stresses at point C

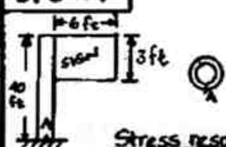
$\tau_1 = \frac{Tr}{I_p} = 24.46$ MPa $\qquad \tau_2 = \frac{VQ}{Ib} = 1.895$ MPa

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = \tau_1 + \tau_2 = 26.36$ MPa

Pure shear. $\tau_C = 26.4$ MPa ←

8.5-17

Sign on a pole



$d_1 = 8$ in. $\qquad d_2 = 10$ in.

W_1 = weight of pole = 3.8 k

W_2 = weight of sign = 400 lb

p = Wind pressure on sign = 30 lb/ft^2

Stress resultants at base of pole

Axial force: $N = W_1 + W_2 = 4.2$ k

Bending moment from wind pressure:

$M = (30 \text{ lb/ft}^2)(6 \text{ ft})(3 \text{ ft})(38.5 \text{ ft})(12 \text{ in/ft}) = 249{,}500$ lb-in

(This moment causes tension at A)

Bending moment from weight of sign:

(Omit; this moment causes zero stress at point A)

Torque from wind pressure:

$T = (30 \text{ lb/ft}^2)(6 \text{ ft})(3 \text{ ft})(41 \text{ in.}) = 22{,}140$ lb-in.

(a) Stresses at point A

$A = \pi/4(d_2^2 - d_1^2) = 28.27$ in^2

$I = \pi/64(d_2^4 - d_1^4) = 289.81$ in^4 $\qquad I_p = 2I = 579.62$ in^4

$\sigma_y = -\frac{N}{A} + \frac{Mc}{I} = -\frac{4.2 \text{ k}}{28.27 \text{ in}^2} + \frac{(249{,}500 \text{ lb-in})(5 \text{ in.})}{289.81 \text{ in}^4} = 4156$ psi ←

$\sigma_x = 0$ ←

$\tau_{xy} = \frac{Tr}{I_p} = \frac{(22{,}140 \text{ lb-in})(5 \text{ in})}{579.62 \text{ in}^4} = 191$ psi ←

(b) Principal stresses and maximum shear stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2078 \text{ psi} \pm 2087 \text{ psi}$$

$\sigma_1 = 4165$ psi $\qquad \sigma_2 = -9$ psi

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2087 \text{ psi}$$

Maximum tensile stress: $\sigma_t = 4160$ psi ←

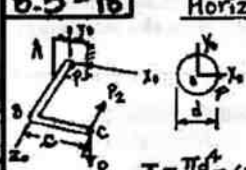
Maximum compressive stress: $\sigma_c = -10$ psi ←

Maximum shear stress: $\tau_{max} = 2090$ psi ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-18

Horizontal bracket



$d = 60$ mm $\qquad C = 0.4$ m

$P_1 = 2.02$ kN $\qquad P_2 = 3.07$ kN

Properties of the cross section

$A = \frac{\pi d^2}{4} = 2827$ mm^2

$I = \frac{\pi d^4}{64} = 636.2 \times 10^3$ mm^4

CONT.

8.5-18 CONT.

$I_p = 2I = 1272 \times 10^3\ mm^4$

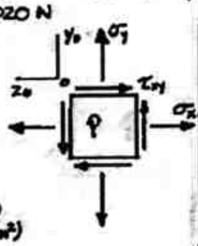
Stress resultants at support A

$M_y = P_2 c = 1228\ N\cdot m$

M_x may be omitted because it produces no stress at point p.

$T = P_1 c = 808\ N\cdot m \qquad V = P_1 = 2020\ N$

$N = P_2 = 3070\ N$ (compression)



Stresses at point p

$\sigma_x = -\frac{N}{A} - \frac{M_y c}{I} = -\frac{3070 N}{2827 mm^2} - \frac{(1228 N\cdot m)(30 mm)}{636.2 \times 10^3 mm^4}$

$= -1.09\ MPa - 57.91\ MPa$

$= -59.00\ MPa$ (compression)

$\sigma_y = 0$

$\tau_{xy} = \frac{Tr}{I_p} + \frac{4V}{3A} = \frac{(808 N\cdot m)(30 mm)}{1272 \times 10^3 mm^4} + \frac{4(2020 N)}{3(2827 mm^2)}$

$= 19.06\ MPa + 0.95\ MPa = 20.01\ MPa$

Principal stresses and maximum in-plane shear stress

$\sigma_x = -59.00\ MPa \qquad \sigma_y = 0 \qquad \tau_{xy} = 20.01\ MPa$

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -29.5\ MPa \pm 35.6\ MPa$

$\sigma_1 = 6.1\ MPa \qquad \sigma_2 = -65.1\ MPa$

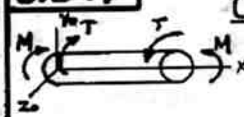
$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.6\ MPa$

Maximum stresses

$\sigma_t = 6.1\ MPa \qquad \sigma_c = -65.1\ MPa \qquad \tau_{max} = 35.6\ MPa$ ←

8.5-19

Cylindrical pressure vessel



$T = 800$ k-in. $\qquad M = 1000$ k-in.

$p = 900$ psi

$r_1 = 11.0$ in. $\qquad d_1 = 22.0$ in.

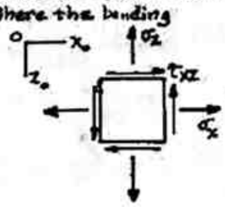
$r_2 = 12.0$ in. $\qquad d_2 = 24.0$ in.

Mean radius $= r_m = 11.5$ in $\qquad t = 1.0$ in.

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 4787\ in^4 \qquad I_p = 2I = 9574\ in^4$

Since the stresses due to T and p are the same everywhere in the cylinder, the maximum stresses occur at the top and bottom of the cylinder where the bending stresses are the largest.

Top of the cylinder



$\sigma_x = \frac{pr_m}{2t} - \frac{Mr_2}{I}$

$= \frac{(900 psi)(11.5 in.)}{2(1.0 in.)} - \frac{(1000 k\text{-}in.)(12.0 in.)}{4787 in^4}$

$= 5175\ psi - 2507\ psi = 2668\ psi$

$\sigma_z = \frac{pr_m}{t} = 2(5175\ psi) = 10{,}350\ psi$

$\tau_{xz} = -\frac{Tr_2}{I_p} = -\frac{(800 k\text{-}in.)(12.0 in.)}{9574 in^4} = -1003\ psi$

Principal stresses

$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{2668 + 10{,}350}{2} \pm \sqrt{\left(\frac{2668 - 10{,}350}{2}\right)^2 + (-1003)^2}$

$= 6509\ psi \pm 3970\ psi$

$\sigma_1 = 10{,}480\ psi \qquad \sigma_2 = 2539\ psi$

Maximum shear stresses

In-plane: $\tau = 3970\ psi$

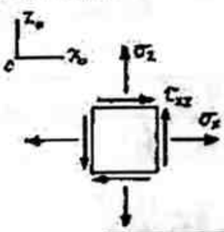
Out-of-plane: $\tau = \frac{\sigma_1}{2}$ or $\frac{\sigma_2}{2} \qquad \tau = \frac{\sigma_1}{2} = 5240\ psi$

For the top of the cylinder:

$\sigma_t = 10{,}480\ psi$

No compressive stresses

$\tau_{max} = 5240\ psi$



Bottom of the cylinder:

$\sigma_x = \frac{pr_m}{2t} + \frac{Mr_2}{I} = 5175\ psi + 2507\ psi$

$= 7682\ psi$

$\sigma_z = \frac{pr_m}{t} = 10{,}350\ psi$

$\tau_{xz} = -\frac{Tr_2}{I_p} = -1003\ psi$

CONT.

8.5-19 CONT.

Principal stresses

$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$

$= \frac{7682 + 10{,}350}{2} \pm \sqrt{\left(\frac{7682 - 10{,}350}{2}\right)^2 + (-1003)^2}$

$= 9016\ psi \pm 1669\ psi$

$\sigma_1 = 10{,}680\ psi \qquad \sigma_2 = 7350\ psi$

Maximum shear stresses

In-plane: $\tau = 1669\ psi$

Out-of-plane: $\tau = \frac{\sigma_1}{2}$ or $\frac{\sigma_2}{2} \qquad \tau_{max} = \frac{\sigma_1}{2} = 5340\ psi$

For the bottom of the cylinder: $\sigma_t = 10{,}680\ psi$

No compressive stresses $\tau_{max} = 5340\ psi$

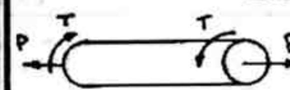
For the entire cylinder: $\sigma_t = 10{,}680\ psi$ ←

No compressive stresses ←

$\tau_{max} = 5340\ psi$ ←

8.5-20

Cylindrical tank



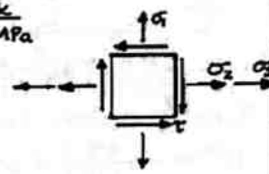
$r = 50\ mm \qquad t = 3\ mm$

$p = 3.5\ MPa \qquad T = 500\ N\cdot m$

$\sigma_{allow} = 70\ MPa$

Find allowable load P

Stresses in the wall of the tank



$\sigma_1 = \frac{pr}{t} = \frac{(3.5 MPa)(50 mm)}{3 mm} = 58.33\ MPa$

$\sigma_2 = \frac{pr}{2t} = 29.17\ MPa$

$\tau = \frac{Tr}{I_p} = \frac{Tr}{2\pi r^3 t} = \frac{T}{2\pi r^2 t}$

$= \frac{500 N\cdot m}{2\pi (50 mm)^2 (3 mm)} = 10.61\ MPa$

$\sigma_3 = \frac{P}{A} = \frac{P}{2\pi rt} = \frac{P}{2\pi (50 mm)(3 mm)} = 0.001061\ P$

Units: Stresses in megapascals (MPa)

Force in newtons (N)

$\sigma_x = \sigma_2 + \sigma_3 = 29.17 + 0.001061\ P$

$\sigma_y = \sigma_1 = 58.33 \qquad \tau_{xy} = -\tau = -10.61$

$\sigma_{tension} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$70\ MPa = 43.75 + 0.0005305P + \sqrt{(-14.58 + 0.0005305\ P)^2 + (-10.61)^2}$

or $26.25 - 0.0005305P = \sqrt{(-14.58 + 0.0005305\ P)^2 + (-10.61)^2}$

Square both sides:

$(26.25 - 0.0005305\ P)^2 = (-14.58 + 0.0005305P)^2 + 112.57$

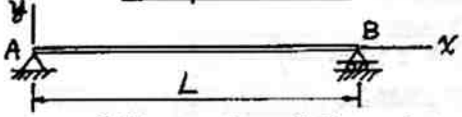
Expand and combine terms:

$0.012382\ P = 363.91$

$P = 29{,}400\ N = 29.4\ kN$ ←

- END OF CHAPTER 8 -

9.3-1 Simple beam



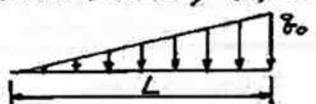
$$v = -\frac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

Take 4 consecutive derivatives and obtain:

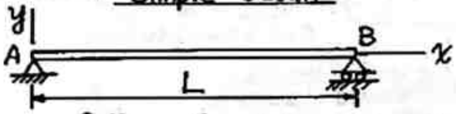
$$v'''' = -\frac{q_0 x}{LEI}$$

From Eq. (9-12c): $q = -EIv'''' = \frac{q_0 x}{L}$

The load is a triangularly distributed load of maximum intensity q_0, acting downward. ←



9.3-2 Simple beam



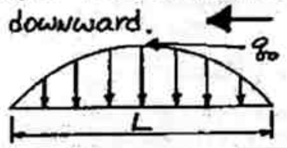
$$v = -\frac{q_0 x}{90L^2EI}(3L^5 - 5L^3x^2 + 3Lx^4 - x^5)$$

Take 4 consecutive derivatives and obtain:

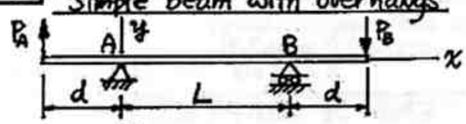
$$v'''' = -\frac{4q_0 x}{L^2EI}(L - x)$$

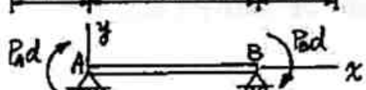
From Eq. (9-12c): $q = -EIv'''' = \frac{4q_0 x}{L^2}(L - x)$

The load is a symmetrical, parabolically distributed load of maximum intensity q_0, acting downward. ←



9.3-3 Simple beam with overhangs





$$v = -\frac{Px}{6EI}(L - x)^2 = -\frac{P}{6EI}(L^2x - 2Lx^2 + x^3)$$

$$v' = -\frac{P}{6EI}(L^2 - 4Lx + 3x^2)$$

$$v'' = \frac{P}{3EI}(2L - 3x)$$

From Eq. (9-12a): $M = EIv'' = \frac{P}{3}(2L - 3x)$

At $x = 0$: $M = \frac{2PL}{3} = P_Ad$

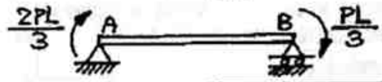
$$\therefore P_A = \frac{2PL}{3d} \quad \leftarrow$$

CONT.

9.3-3 CONT.

At $x = L$: $M = -\frac{PL}{3} = -P_Bd$

$$\therefore P_B = \frac{PL}{3d} \quad \leftarrow$$



9.3-4 Simple beam (Uniform load)

$\delta = \frac{5qL^4}{384EI}$ $\quad \theta = \frac{qL^3}{24EI}$

$$q = \frac{384EI\delta}{5L^4} = \frac{24EI\theta}{L^3} \qquad (1)$$

$$\theta = \frac{16\delta}{5L} \text{ or } L = \frac{16\delta}{5\theta} \qquad (2)$$

$\sigma = \frac{Mc}{I} = \frac{Mh}{2I}$ $\quad M = \frac{qL^2}{8}$ $\quad \sigma = \frac{qL^2h}{16I}$

Substitute for q from Eq. (1) and for L from Eq. (2):

$$\sigma = \left(\frac{24EI\theta}{L^3}\right)\left(\frac{L^2h}{16I}\right) = \frac{3E\theta h}{2L} = \frac{3E\theta h}{2(16\delta/5\theta)}$$

$$= \frac{15E\theta^2 h}{32\delta}$$

Solve for h:

$$h = \frac{32\delta\sigma}{15E\theta^2} = \frac{(32)(12\text{ mm})(75\text{ MPa})}{(15)(200\text{ GPa})(0.01\text{ rad})^2}$$

$$= 96\text{ mm} \quad \leftarrow$$

9.3-5 Simple beam (Uniform load)

W 12 × 35 $\quad L = 14$ ft $\quad I = 285$ in.^{4}

$q = 1.75$ k/ft $\quad E = 30 \times 10^6$ psi

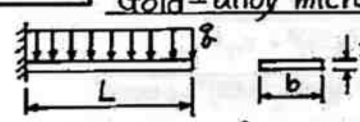
(a) $\delta_{max} = \frac{5qL^4}{384EI} = 0.177$ in. ←

$$\theta = \frac{qL^3}{24EI} = 0.003370 \text{ rad} = 0.193° \quad \leftarrow$$

(b) $\rho = \frac{EI}{M} = \frac{8EI}{qL^2}$

$$= 16{,}618 \text{ in.} = 1385 \text{ ft} \quad \leftarrow$$

9.3-6 Gold-alloy microbeam



$L = 25\ \mu m$
$b = 15\ \mu m$
$t = 0.87\ \mu m$

$qL = 44\ \mu N$ $\quad \delta_{max} = 1.3\ \mu m$

From Eq. (9-26): $\delta_{max} = \frac{qL^4}{8EI}$

$$E = \frac{qL^4}{8I\delta_{max}} = \frac{(44\ \mu N)(25\ \mu m)^3}{8(\frac{1}{12})(15\ \mu m)(0.87\ \mu m)^3(1.3\ \mu m)}$$

$$E = E_g = 80.3 \text{ GPa} \quad \leftarrow$$

9.3-7 Simple beam (Uniform load)

$M = \frac{qL^2}{8}$ $\quad c = \frac{h}{2}$ $\quad \sigma = \frac{Mc}{I} = \frac{qL^2}{8}\left(\frac{h}{2I}\right) = \frac{qL^2h}{16I}$

$$q = \frac{16I\sigma}{L^2h}$$

CONT.

9.3-7 CONT.

$$\delta = \frac{5qL^4}{384EI} = \frac{5L^4}{384EI}\left(\frac{16I\sigma}{L^2h}\right) = \frac{5\sigma L^2}{24hE}$$

$$L = \sqrt{\frac{24hE\delta}{5\sigma}} \longleftarrow$$

Substitute numerical values:

$h = 12$ in. $E = 30\times10^6$ psi $\delta = 0.1$ in.

$\sigma = 12{,}000$ psi

$$L = 120 \text{ in.} = 10 \text{ ft} \longleftarrow$$

9.3-8 Cantilever beam (Uniform load)

$$\delta = \frac{qL^4}{8EI} \qquad \frac{\delta}{L} = \frac{qL^3}{8EI}$$

$$\sigma = \frac{Mc}{I} = \frac{qL^2}{2}\left(\frac{h}{2I}\right) = \frac{qL^2h}{4I}$$

$$q = \frac{4I\sigma}{L^2h} \qquad \frac{\delta}{L} = \frac{4I\sigma}{L^2h}\left(\frac{L^3}{8EI}\right) = \frac{\sigma L}{2hE}$$

$$\frac{L}{h} = 10 \qquad \therefore \frac{\delta}{L} = \frac{5\sigma}{E} = \frac{5(130\text{ MPa})}{208\text{ GPa}} = \frac{1}{320} \longleftarrow$$

9.3-9 Simple beam (Uniform load)

(a) Maximum deflection

$$\delta = \frac{5qL^4}{384EI} \quad I = \frac{b^4}{12} \quad \delta = \frac{5qL^4}{32Eb^4} \qquad (1)$$

$$\sigma = \frac{M}{S} = \frac{6M}{b^3} \quad M = \frac{qL^2}{8} \quad \sigma = \frac{3qL^2}{4b^3}$$

$$b^3 = \frac{3qL^2}{4\sigma}$$

Substitute b into Eq.(1):

$$\delta = \frac{5}{24E}\left(\frac{4\sigma^4L^4}{3q}\right)^{1/3} \longleftarrow$$

Substitute numerical values:

$\sigma = 9000$ psi $L = 7$ ft $= 84$ in.

$q = 150$ lb/ft $= 12.5$ lb/in. $E = 10\times10^6$ psi

$$\delta_{max} = 0.680 \text{ in.} \longleftarrow$$

(b) Radius of curvature

$$\rho = \frac{EI}{M} \quad EI = \frac{5qL^4}{384\delta} \quad M = \frac{qL^2}{8}$$

$$\therefore \rho = \frac{5L^2}{48\delta} \quad \text{Also, } \delta = \frac{5}{24E}\left(\frac{4\sigma^4L^4}{3q}\right)^{1/3}$$

$$\therefore \rho = E\left(\frac{3qL^2}{32\sigma^4}\right)^{1/3} \longleftarrow$$

Substitute numerical values:

$$\rho = 1080 \text{ in.} = 90.0 \text{ ft} \longleftarrow$$

9.3-10 Simple beam (Concentrated load)

$$\delta_c = \frac{Pb(3L^2-4b^2)}{48EI} \quad (a \ge b) \quad b = L-a$$

$$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}\,LEI} \quad (a \ge b)$$

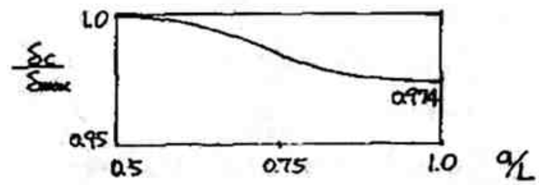
$$\frac{\delta_c}{\delta_{max}} = \frac{3\sqrt{3}\left(-1+8\frac{a}{L}-\frac{4a^2}{L^2}\right)}{16\left(\frac{2a}{L}-\frac{a^2}{L^2}\right)^{3/2}} \longleftarrow$$

CONT.

9.3-10 CONT.

If $\frac{a}{L} = \frac{1}{2}$, $\frac{\delta_c}{\delta_{max}} = 1$

If $\frac{a}{L} = 1$, $\frac{\delta_c}{\delta_{max}} = \frac{9\sqrt{3}}{16} = 0.974$



The deflection at the midpoint is almost as large as the maximum deflection. (The maximum difference is only 2.6%.)

9.3-11 Cantilever beam (Concentrated load)

$$EIv'' = M = -P(L-x)$$

$$EIv' = -PLx + \frac{Px^2}{2} + C_1$$

B.C. $v'(0) = 0$ $C_1 = 0$

$$EIv = -\frac{PLx^2}{2} + \frac{Px^3}{6} + C_2$$

B.C. $v(0) = 0$ $C_2 = 0$

$$\therefore v = -\frac{Px^2}{6EI}(3L-x) \longleftarrow$$

$$v' = -\frac{Px}{2EI}(2L-x)$$

$$\delta_B = -v(L) = \frac{PL^3}{3EI} \longleftarrow$$

$$\theta_B = -v'(L) = \frac{PL^2}{2EI} \longleftarrow$$

9.3-12 Simple beam (Couple M_0 at end)

$$EIv'' = M = M_0\left(1-\frac{x}{L}\right)$$

$$EIv' = M_0\left(x - \frac{x^2}{2L}\right) + C_1$$

$$EIv = M_0\left(\frac{x^2}{2} - \frac{x^3}{6L}\right) + C_1x + C_2$$

B.C. $v(0) = 0$ $C_2 = 0$

B.C. $v(L) = 0$ $C_1 = -\frac{M_0L}{3}$

$$v = -\frac{M_0x}{6LEI}(2L^2 - 3Lx + x^2) \longleftarrow$$

Maximum deflection

$$v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

Set $v' = 0$ and solve for x:

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \longleftarrow$$

Substitute x_1 into the equation for v:

$$\delta_{max} = -(v)_{x=x_1}$$

$$= \frac{M_0L^2}{9\sqrt{3}\,EI} \longleftarrow$$

9.3-13 Cantilever beam (Triangular load)

$EIv'' = M = -\frac{q_0}{6L}(L-x)^3$

$EIv' = \frac{q_0}{24L}(L-x)^4 + C_1$

B.C. $v'(0)=0 \qquad C_1 = -\frac{q_0L^3}{24}$

$EIv = -\frac{q_0}{120L}(L-x)^5 - \frac{q_0L^3x}{24} + C_2$

B.C. $v(0)=0 \qquad C_2 = \frac{q_0L^4}{120}$

$v = -\frac{q_0x^2}{120LEI}(10L^3 - 10L^2x + 5Lx^2 - x^3)$ ←

$v' = -\frac{q_0x}{24LEI}(4L^3 - 6L^2x + 4Lx^2 - x^3)$

$\delta_B = -v(L) = \frac{q_0L^4}{30EI}$ ←

$\theta_B = -v'(L) = \frac{q_0L^3}{24EI}$ ←

9.3-14 Cantilever beam (Distributed moment)

$EIv'' = M = -m(L-x)$

$EIv' = -m(Lx - \frac{x^2}{2}) + C_1$

B.C. $v'(0)=0 \qquad C_1=0$

$EIv = -m(\frac{Lx^2}{2} - \frac{x^3}{6}) + C_2$

B.C. $v(0)=0 \qquad C_2=0$

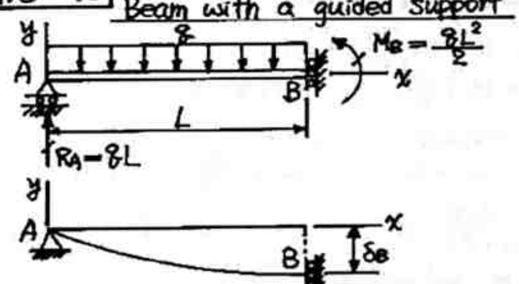
$v = -\frac{mx^2}{6EI}(3L-x)$ ←

$v' = -\frac{mx}{2EI}(2L-x)$

$\delta_B = -v(L) = \frac{mL^3}{3EI}$ ←

$\theta_B = -v'(L) = \frac{mL^2}{2EI}$ ←

9.3-15 Beam with a guided support



$EIv'' = M = qLx - \frac{qx^2}{2}$

$EIv' = \frac{qLx^2}{2} - \frac{qx^3}{6} + C_1$

B.C. $v'(L)=0 \qquad \therefore C_1 = -\frac{qL^3}{3}$

$EIv = \frac{qLx^3}{6} - \frac{qx^4}{24} - \frac{qL^3x}{3} + C_2$

B.C. $v(0)=0 \qquad \therefore C_2 = 0$

$v = -\frac{qx}{24EI}(8L^3 - 4Lx^2 + x^3)$ ←

$\delta_B = -v(L) = \frac{5qL^4}{24EI}$ ←

9.3-16 Simple beam (Couple M_0)

$EIv'' = M = \frac{M_0x}{L} \qquad (0 \le x \le a)$

$EIv' = \frac{M_0x^2}{2L} + C_1 \qquad (0 \le x \le a)$

$EIv'' = M = -\frac{M_0}{L}(L-x) \qquad (a \le x \le L)$

$EIv' = -\frac{M_0}{L}(Lx - \frac{x^2}{2}) + C_2 \qquad (a \le x \le L)$

B.C. 1: $(v')_{Left} = (v')_{Right}$ at $x=a$

$\therefore C_2 = C_1 + M_0a$

$EIv = \frac{M_0x^3}{6L} + C_1x + C_3 \qquad (0 \le x \le a)$

B.C. 2: $v(0)=0 \qquad C_3=0$

$EIv = -\frac{M_0x^2}{2} + \frac{M_0x^3}{6L} + C_1x + M_0ax + C_4 \quad (a \le x \le L)$

B.C. 3: $v(L)=0 \qquad C_4 = -M_0L(a - \frac{L}{3}) - C_1L$

B.C. 4: $(v)_{Left} = (v)_{Right}$ at $x=a$

$\therefore C_4 = -\frac{M_0a^2}{2}$

$C_1 = \frac{M_0}{6L}(2L^2 - 6aL + 3a^2)$

$v = -\frac{M_0x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \quad (0 \le x \le a)$ ←

$v = -\frac{M_0}{6LEI}(3a^2L - 3a^2x - 2L^2x + 3Lx^2 - x^3)$ ←
$(a \le x \le L)$

$\delta_0 = -v(a) = \frac{M_0a(L-a)(2a-L)}{3LEI}$

$= \frac{M_0ab(2a-L)}{3LEI}$ ←

9.3-17 Cantilever beam (Partial uniform load)

$EIv'' = M = -\frac{q}{2}(a-x)^2 = -\frac{q}{2}(a^2 - 2ax + x^2)$
$(0 \le x \le a)$

$EIv' = -\frac{q}{2}(a^2x - ax^2 + \frac{x^3}{3}) + C_1 \quad (0 \le x \le a)$

B.C. 1: $v'(0)=0 \qquad C_1=0$

$EIv'' = M = 0 \qquad (a \le x \le L)$

$EIv' = C_2 \qquad (a \le x \le L)$

B.C. 2: $(v')_{Left} = (v')_{Right}$ at $x=a$

$\therefore C_2 = -\frac{qa^3}{6}$

$EIv = -\frac{q}{2}(\frac{a^2x^2}{2} - \frac{ax^3}{3} + \frac{x^4}{12}) + C_3 \quad (0 \le x \le a)$

B.C. 3: $v(0)=0 \qquad C_3=0$

$EIv = C_2x + C_4 = -\frac{qa^3x}{6} + C_4 \quad (a \le x \le L)$

B.C. 4: $(v)_{Left} = (v)_{Right}$ at $x=a$

$\therefore C_4 = \frac{qa^4}{24}$

$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2) \quad (0 \le x \le a)$ ←

$v = -\frac{qa^3}{24EI}(4x-a) \quad (a \le x \le L)$ ←

$\delta_B = -v(L) = \frac{qa^3}{24EI}(4L-a)$ ←

9.3-18 Cantilever beam (Partial uniform load)

$EI v'' = M = -\frac{qL}{8}(3L - 4x) \qquad (0 \le x \le \frac{L}{2})$

$EI v' = -\frac{qL}{8}(3Lx - 2x^2) + C_1 \qquad (0 \le x \le \frac{L}{2})$

B.C. $v'(0) = 0 \qquad C_1 = 0$

$EI v'' = M = -\frac{q}{2}(L^2 - 2Lx + x^2) \qquad (\frac{L}{2} \le x \le L)$

$EI v' = -\frac{q}{2}(L^2x - Lx^2 + \frac{x^3}{3}) + C_2 \qquad (\frac{L}{2} \le x \le L)$

B.C. $(v')_{Left} = (v')_{Right}$ at $x = \frac{L}{2}$

$\therefore C_2 = \frac{qL^3}{48}$

$EI v = -\frac{qL}{8}(\frac{3Lx^2}{2} - \frac{2x^3}{3}) + C_3 \qquad (0 \le x \le \frac{L}{2})$

B.C. $v(0) = 0 \qquad C_3 = 0$

$EI v = -\frac{q}{2}(\frac{L^2x^2}{2} - \frac{Lx^3}{3} + \frac{x^4}{12}) + \frac{qL^3}{48}x + C_4$

$(\frac{L}{2} \le x \le L)$

B.C. $(v)_{Left} = (v)_{Right}$ at $x = \frac{L}{2}$

$\therefore C_4 = -\frac{qL^4}{384}$

$v = -\frac{qLx^2}{48EI}(9L - 4x) \qquad (0 \le x \le \frac{L}{2})$ ←

$\delta_C = -v(\frac{L}{2}) = \frac{7qL^4}{192EI}$ ←

$v = -\frac{q}{384EI}(16x^4 - 64Lx^3 + 96L^2x^2 - 8L^3x + L^4)$

$(\frac{L}{2} \le x \le L)$ ←

$\delta_B = -v(L) = \frac{41qL^4}{384EI}$ ←

9.3-19 Simple beam (Partial uniform load)

$EI v'' = M = \frac{3qLx}{8} - \frac{qx^2}{2} \qquad (0 \le x \le \frac{L}{2})$

$EI v' = \frac{3qLx^2}{16} - \frac{qx^3}{6} + C_1 \qquad (0 \le x \le \frac{L}{2})$

$EI v'' = M = \frac{qL^2}{8} - \frac{qLx}{8} \qquad (\frac{L}{2} \le x \le L)$

$EI v' = \frac{qL^2x}{8} - \frac{qLx^2}{16} + C_2 \qquad (\frac{L}{2} \le x \le L)$

B.C. $(v')_{Left} = (v')_{Right}$ at $x = \frac{L}{2}$

$\therefore C_2 = C_1 - \frac{qL^3}{48}$

$EI v = \frac{qLx^3}{16} - \frac{qx^4}{24} + C_1x + C_3 \qquad (0 \le x \le \frac{L}{2})$

B.C. $v(0) = 0 \qquad C_3 = 0$

$EI v = \frac{qL^2x^2}{16} - \frac{qLx^3}{48} + C_1x - \frac{qL^3x}{48} + C_4$

$(\frac{L}{2} \le x \le L)$

B.C. $v(L) = 0 \qquad C_4 = -C_1L - \frac{qL^4}{48}$

B.C. $(v)_{Left} = (v)_{Right}$ at $x = \frac{L}{2}$

$\therefore C_1 = -\frac{3qL^3}{128}$

$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad (0 \le x \le \frac{L}{2})$ ←

CONT.

9.3-19 CONT.

$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ ←

$(\frac{L}{2} \le x \le L)$

$\delta_C = -v(\frac{L}{2}) = \frac{5qL^4}{768EI}$ ←

9.4-1 Cantilever beam with a couple M_0

$EI v''' = V = 0$

$EI v'' = C_1$

B.C.1 $M = M_0 \qquad EI v'' = M = M_0 = C_1$

$EI v' = C_1x + C_2 = M_0x + C_2$

B.C.2 $v'(0) = 0 \qquad C_2 = 0$

$EI v = \frac{M_0x^2}{2} + C_3$

B.C.3 $v(0) = 0 \qquad C_3 = 0$

$v = \frac{M_0x^2}{2EI}$ ←

$v' = \frac{M_0x}{EI}$

$\delta_B = v(L) = \frac{M_0L^2}{2EI}$ (upward) ←

$\theta_B = v'(L) = \frac{M_0L}{EI}$ (counterclockwise) ←

9.4-2 Simple beam with a sine load

$EI v'''' = -q = -q_0 \sin\frac{\pi x}{L}$

$EI v''' = q_0(\frac{L}{\pi})\cos\frac{\pi x}{L} + C_1$

$EI v'' = q_0(\frac{L}{\pi})^2 \sin\frac{\pi x}{L} + C_1x + C_2$

B.C.1 $EI v'' = M \qquad EI v''(0) = 0 \qquad C_2 = 0$

B.C.2 $EI v''(L) = 0 \qquad C_1 = 0$

$EI v' = -q_0(\frac{L}{\pi})^3 \cos\frac{\pi x}{L} + C_3$

$EI v = -q_0(\frac{L}{\pi})^4 \sin\frac{\pi x}{L} + C_3x + C_4$

B.C.3 $v(0) = 0 \qquad C_4 = 0$

B.C.4 $v(L) = 0 \qquad C_3 = 0$

$v = -\frac{q_0L^4}{\pi^4EI}\sin\frac{\pi x}{L}$ ←

$\delta_{max} = -v(\frac{L}{2}) = \frac{q_0L^4}{\pi^4EI}$ ←

9.4-3 Simple beam with two couples

$R_A = \frac{3M_0}{L}$ downward $\qquad \therefore V = -\frac{3M_0}{L}$

$EI v''' = V = -\frac{3M_0}{L}$

$EI v'' = -\frac{3M_0x}{L} + C_1$

B.C.1 $EI v'' = M \qquad EI v''(0) = 2M_0 \qquad C_1 = 2M_0$

CONT.

9.4-3 CONT.

$$EI\,v' = -\frac{3M_0x^2}{2L} + 2M_0x + C_2$$

$$EI\,v = -\frac{M_0x^3}{2L} + M_0x^2 + C_2x + C_3$$

B.C. 2 $\quad v(0)=0 \qquad C_3 = 0$

B.C. 3 $\quad v(L)=0 \qquad C_2 = -\frac{M_0L}{2}$

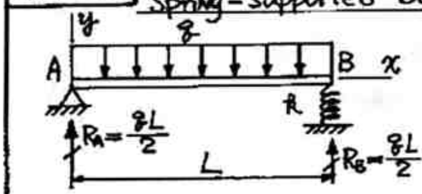
$$v = -\frac{M_0x}{2LEI}(L^2-2Lx+x^2) = -\frac{M_0x}{2LEI}(L-x)^2$$

$$v' = -\frac{M_0}{2LEI}(L-x)(L-3x)$$

Set $v'=0$: $\;x_1 = L$ and $x_1 = \frac{L}{3}$

$$\delta_{max} = -v\left(\frac{L}{3}\right) = \frac{2M_0L^2}{27EI} \quad \text{(downward)}$$

9.4-4 Spring-supported beam



$$k = \frac{48EI}{L^3}$$

$$\delta_B = \frac{R_B}{k} = \frac{qL}{2k} = \frac{qL^4}{96EI}$$

$$V = R_A - qx = \frac{q}{2}(L-2x)$$

$$EI\,v''' = V = \frac{q}{2}(L-2x)$$

$$EI\,v'' = \frac{q}{2}(Lx-x^2) + C_1$$

B.C. 1 $\quad EI\,v'' = M \qquad EI\,v''(0)=0 \qquad C_1 = 0$

$$EI\,v' = \frac{q}{2}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) + C_2$$

$$EI\,v = \frac{q}{2}\left(\frac{Lx^3}{6} - \frac{x^4}{12}\right) + C_2x + C_3$$

B.C. 2 $\quad v(0)=0 \qquad C_3 = 0$

B.C. 3 $\quad v(L) = -\delta_B = -\frac{qL^4}{96EI}$

$$\therefore C_2 = -\frac{5qL^3}{96}$$

$$v = -\frac{qx}{96EI}(5L^3 - 8Lx^2 + 4x^3)$$

$$v' = -\frac{q}{96EI}(5L^3 - 24Lx^2 + 16x^3)$$

$$\theta_A = -v'(0) = \frac{5qL^3}{96EI} \quad \text{(clockwise)}$$

9.4-5 Cantilever beam with a cosine load

$$EI\,v'''' = -q = -q_0\cos\frac{\pi x}{2L}$$

$$EI\,v''' = -q_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1$$

B.C. 1 $\quad EI\,v''' = V \qquad EI\,v'''(L)=0 \qquad C_1 = \frac{2q_0L}{\pi}$

$$EI\,v'' = q_0\left(\frac{2L}{\pi}\right)^2\cos\frac{\pi x}{2L} + \frac{2q_0Lx}{\pi} + C_2$$

B.C. 2 $\quad EI\,v'' = M \qquad EI\,v''(L)=0 \qquad C_2 = -\frac{2q_0L^2}{\pi}$

$$EI\,v' = q_0\left(\frac{2L}{\pi}\right)^3\sin\frac{\pi x}{2L} + \frac{q_0Lx^2}{\pi} - \frac{2q_0L^2x}{\pi} + C_3$$

CONT.

9.4-5 CONT.

B.C. 3 $\quad v'(0)=0 \qquad C_3 = 0$

$$EI\,v = -q_0\left(\frac{2L}{\pi}\right)^4\cos\frac{\pi x}{2L} + \frac{q_0Lx^3}{3\pi} - \frac{q_0L^2x^2}{\pi} + C_4$$

B.C. 4 $\quad v(0)=0 \qquad C_4 = \frac{16q_0L^4}{\pi^4}$

$$v = -\frac{q_0L}{3\pi^4EI}\left(48L^3\cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3Lx^2 - \pi^3x^3\right)$$

$$\delta_B = -v(L) = \frac{2q_0L^4}{3\pi^4EI}(\pi^3 - 24)$$

9.4-6 Cantilever beam with a parabolic load

$$EI\,v'''' = -q = -\frac{q_0}{L^2}(L^2-x^2)$$

$$EI\,v''' = -\frac{q_0}{L^2}\left(L^2x - \frac{x^3}{3}\right) + C_1$$

B.C. 1 $\quad EI\,v''' = V \qquad EI\,v'''(L)=0 \qquad C_1 = \frac{2q_0L}{3}$

$$EI\,v'' = -\frac{q_0}{L^2}\left(\frac{L^2x^2}{2} - \frac{x^4}{12}\right) + \frac{2q_0L}{3}x + C_2$$

B.C. 2 $\quad EI\,v'' = M \qquad EI\,v''(L)=0 \qquad C_2 = -\frac{q_0L^2}{4}$

$$EI\,v' = -\frac{q_0}{L^2}\left(\frac{L^2x^3}{6} - \frac{x^5}{60}\right) + \frac{q_0Lx^2}{3} - \frac{q_0L^2x}{4} + C_3$$

B.C. 3 $\quad v'(0)=0 \qquad C_3 = 0$

$$EI\,v = -\frac{q_0}{L^2}\left(\frac{L^2x^4}{24} - \frac{x^6}{360}\right) + \frac{q_0Lx^3}{9} - \frac{q_0L^2x^2}{8} + C_4$$

B.C. 4 $\quad v(0)=0 \qquad C_4 = 0$

$$v = -\frac{q_0x^2}{360L^2EI}(45L^4 - 40L^3x + 15L^2x^2 - x^4)$$

$$\delta_B = -v(L) = \frac{19q_0L^4}{360EI}$$

$$v' = -\frac{q_0x}{60L^2EI}(15L^4 - 20L^3x + 10L^2x^2 - x^4)$$

$$\theta_B = -v'(L) = \frac{q_0L^3}{15EI}$$

9.4-7 Simple beam with a parabolic load

$$EI\,v'''' = -q = -\frac{4q_0x}{L^2}(L-x) = -\frac{4q_0}{L^2}(Lx-x^2)$$

$$EI\,v''' = -\frac{2q_0}{3L^2}(3Lx^2 - 2x^3) + C_1$$

$$EI\,v'' = -\frac{q_0}{3L^2}(2Lx^3 - x^4) + C_1x + C_2$$

B.C. 1 $\quad EI\,v'' = M \qquad EI\,v''(0)=0 \qquad C_2 = 0$

B.C. 2 $\quad EI\,v''(L)=0 \qquad C_1 = \frac{q_0L}{3}$

$$EI\,v' = -\frac{q_0}{30L^2}(-5L^3x^2 + 5Lx^4 - 2x^5) + C_3$$

B.C. 3 (Symmetry) $\quad v'\left(\frac{L}{2}\right)=0 \qquad C_3 = -\frac{q_0L^3}{30}$

$$EI\,v = -\frac{q_0}{30L^2}\left(L^5x - \frac{5L^3x^3}{3} + Lx^5 - \frac{x^6}{3}\right) + C_4$$

B.C. 4 $\quad v(0)=0 \qquad C_4 = 0$

CONT.

9.4-7 CONT.

$$v = -\frac{q_0 x}{90L^2EI}(3L^5 - 5L^3x^2 + 3Lx^4 - x^5)$$

$$\delta_{max} = -v\left(\frac{L}{2}\right) = \frac{61 q_0 L^4}{5760EI}$$

9.4-8 Simple beam with a triangular load

$$EIv'''' = -q = -\frac{q_0 x}{L} \qquad EIv''' = -\frac{q_0x^2}{2L} + C_1$$

$$EIv'' = -\frac{q_0x^3}{6L} + C_1x + C_2$$

B.C.1 $EIv''=M$ $\quad EIv''(0)=0 \quad C_2=0$

B.C.2 $EIv''(L)=0 \quad C_1 = \frac{q_0L}{6}$

$$EIv' = -\frac{q_0x^4}{24L} + \frac{q_0Lx^2}{12} + C_3$$

$$EIv = -\frac{q_0x^5}{120L} + \frac{q_0Lx^3}{36} + C_3x + C_4$$

B.C.3 $v(0)=0 \quad C_4=0$

B.C.4 $v(L)=0 \quad C_3 = -\frac{7q_0L^3}{360}$

$$v = -\frac{q_0x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$v' = -\frac{q_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

Set $v'=0$: $\quad x_1^2 = L^2\left(1-\sqrt{\frac{8}{15}}\right) \quad x_1 = 0.5193L$

$$\delta_{max} = -v(x_1) = \frac{q_0L^4}{450EI}\left(\frac{20}{3}+\frac{8}{3}\sqrt{\frac{8}{15}}\right)^{1/2}$$

$$= 0.006522\frac{q_0L^4}{EI}$$

9.4-9 Overhanging beam

$$EIv'''' = -q = 0 \quad (0\le x\le L)$$

$$EIv''' = C_1 \quad (0\le x \le L)$$

$$EIv'' = C_1x + C_2 \quad (0\le x\le L)$$

B.C.1 $EIv''=M \quad EIv''(0)=0 \quad C_2=0$

$$EIv'''' = -q \quad \left(L\le x\le \frac{3L}{2}\right)$$

$$EIv''' = -qx + C_3 \quad \left(L\le x\le \frac{3L}{2}\right)$$

B.C.2 $EIv'''=V \quad EIv'''\left(\frac{3L}{2}\right)=0 \quad C_3=\frac{3qL}{2}$

$$EIv'' = -\frac{qx^2}{2}+\frac{3qLx}{2}+C_4 \quad \left(L\le x\le \frac{3L}{2}\right)$$

B.C.3 $EIv''=M \quad EIv''\left(\frac{3L}{2}\right)=0 \quad C_4=-\frac{9qL^2}{8}$

B.C.4 $EI(v'')_{LEFT} = EI(v'')_{RIGHT}$ at $x=L$

$$C_1L = -\frac{qL^2}{2}+\frac{3qL^2}{2}-\frac{9qL^2}{8} \qquad C_1 = -\frac{qL}{8}$$

$$EIv' = -\frac{qLx^2}{16}+C_5 \quad (0\le x\le L)$$

$$EIv' = -\frac{qx^3}{6}+\frac{3qLx^2}{4}-\frac{9qL^2x}{8}+C_6 \quad \left(L\le x\le \frac{3L}{2}\right)$$

CONT.

9.4-9 CONT.

B.C.5 $(v')_{LEFT}=(v')_{RIGHT}$ at $x=L$

$$C_6 = C_5 + \frac{23qL^3}{48} \qquad (a)$$

$$EIv = -\frac{qLx^3}{48}+C_5x+C_7 \quad (0\le x\le L)$$

B.C.6 $v(0)=0 \quad C_7=0$

B.C.7 $v(L)=0$ for $0\le x\le L \quad C_5=\frac{qL^3}{48}$

From Eq.(a): $C_6 = \frac{qL^3}{2}$

$$EIv = -\frac{qx^4}{24}+\frac{3qLx^3}{12}-\frac{9qL^2x^2}{16}+\frac{qL^3x}{2}+C_8 \quad \left(L\le x\le \frac{3L}{2}\right)$$

B.C.8 $v(L)=0$ for $L\le x\le \frac{3L}{2} \quad C_8 = -\frac{7qL^4}{48}$

$$v=\frac{qLx}{48EI}(L^2-x^2) \quad (0\le x\le L)$$

$$v = -\frac{q(L-x)}{48EI}(7L^3-17L^2x+10Lx^2-2x^3) \quad \left(L\le x\le \frac{3L}{2}\right)$$

$$\delta_C = -v\left(\frac{3L}{2}\right) = \frac{11qL^4}{384EI}$$

$$\theta_C = -v'\left(\frac{3L}{2}\right) = \frac{qL^3}{16EI}$$

9.4-10 Simple beam with a triangular load

Left-hand half (part AC): $0\le x\le \frac{L}{2}$

Right-hand half (part CB): $\frac{L}{2}\le x\le L$

Part AC $\quad q=0$

$$EIv''''=-q=0 \qquad EIv'''=C_1$$

$$EIv''=C_1x+C_2 \qquad EIv'=C_1\left(\frac{x^2}{2}\right)+C_2x+C_3$$

$$EIv = C_1\left(\frac{x^3}{6}\right)+C_2\left(\frac{x^2}{2}\right)+C_3x+C_4$$

Part CB $\quad q=\frac{q_0}{L}(2x-L)$

$$EIv''''=-q=\frac{q_0}{L}(L-2x)$$

$$EIv'''=\frac{q_0}{L}(Lx-x^2)+C_5$$

$$EIv''=\frac{q_0}{L}\left(\frac{Lx^2}{2}-\frac{x^3}{3}\right)+C_5x+C_6$$

$$EIv'=\frac{q_0}{L}\left(\frac{Lx^3}{6}-\frac{x^4}{12}\right)+C_5\left(\frac{x^2}{2}\right)+C_6x+C_7$$

$$EIv=\frac{q_0}{L}\left(\frac{Lx^4}{24}-\frac{x^5}{60}\right)+C_5\left(\frac{x^3}{6}\right)+C_6\left(\frac{x^2}{2}\right)+C_7x+C_8$$

Boundary conditions

B.C.1 $EIv'''=V \quad EI(v''')_{AC}=EI(v''')_{BC}$ at $x=\frac{L}{2}$

$$C_1-C_5=\frac{q_0L}{4} \qquad (1)$$

B.C.2 $EIv''=M \quad EIv''(0)=0$

$$C_2=0 \qquad (2)$$

B.C.3 $EIv''(L)=0 \quad C_5L+C_6=-\frac{q_0L^2}{6} \qquad (3)$

B.C.4 $(EIv'')_{AC}=(EIv'')_{CB}$ for $x=\frac{L}{2}$

$$C_1L-C_5L-2C_6=\frac{q_0L^2}{6} \qquad (4)$$

CONT.

9.4-10 CONT.

B.C. 5 $\quad (v')_{AC} = (v')_{CB} \quad$ for $x = \frac{L}{2}$

$$C_1L^2 + 8C_3 - C_5L^2 - 4C_6L - 8C_7 = \frac{q_0L^3}{8} \qquad (5)$$

B.C. 6 $\quad v(0) = 0 \qquad C_4 = 0 \qquad (6)$

B.C. 7 $\quad v(L) = 0$

$$C_5L^3 + 3C_6L^2 + 6C_7L + 6C_8 = -\frac{3q_0L^4}{20} \qquad (7)$$

B.C. 8 $\quad (v)_{AC} = (v)_{CB} \quad$ for $x = \frac{L}{2}$

$$C_1L^3 + 24C_3L - C_5L^3 - 6C_6L^2 - 24C_7L - 48C_8 = \frac{q_0L^4}{10} \qquad (8)$$

Solve Eqs. (1) through (8):

$$C_1 = \frac{q_0L}{24} \qquad C_2 = 0 \qquad C_3 = -\frac{37q_0L^3}{5760}$$

$$C_4 = 0 \qquad C_5 = -\frac{5q_0L}{24} \qquad C_6 = \frac{q_0L^2}{24}$$

$$C_7 = -\frac{67q_0L^3}{5760} \qquad C_8 = \frac{q_0L^4}{1920}$$

Substitute constants into equations for v and v'.

Deflection curve for part AC $\left(0 \le x \le \frac{L}{2}\right)$

$$v = -\frac{q_0Lx}{5760EI}(37L^2 - 40x^2) \quad \longleftarrow$$

$$v' = -\frac{q_0L}{5760EI}(37L^2 - 120x^2)$$

$$\theta_A = -v'(0) = \frac{37q_0L^3}{5760EI} \quad \longleftarrow$$

$$\delta_C = -v\left(\frac{L}{2}\right) = \frac{3q_0L^4}{1280EI} \quad \longleftarrow$$

Deflection curve for part CB $\left(\frac{L}{2} \le x \le L\right)$

$$v = -\frac{q_0}{5760LEI}\left[L^2x(37L^2 - 40x^2) + 3(2x - L)^5\right] \quad \longleftarrow$$

$$v' = -\frac{q_0}{5760LEI}\left[L^2(37L^2 - 120x^2) + 30(2x - L)^4\right]$$

$$\theta_B = v'(L) = \frac{53q_0L^3}{5760EI} \quad \longleftarrow$$

9.5-1 Cantilever beam with 3 loads

Table G-1, Cases 4 and 5

$$\theta_B = \frac{P\left(\frac{L}{3}\right)^2}{2EI} + \frac{P\left(\frac{2L}{3}\right)^2}{2EI} + \frac{PL^2}{2EI} = \frac{7PL^2}{9EI} \quad \longleftarrow$$

$$\delta_B = \frac{P\left(\frac{L}{3}\right)^2}{6EI}\left(3L - \frac{L}{3}\right) + \frac{P\left(\frac{2L}{3}\right)^2}{6EI}\left(3L - \frac{2L}{3}\right) + \frac{PL^3}{3EI}$$

$$= \frac{5PL^3}{9EI} \quad \longleftarrow$$

9.5-2 Simple beam with 5 loads

(a) Table G-2, Cases 4 and 6

$$\delta_1 = \frac{P\left(\frac{L}{6}\right)}{24EI}\left[3L^2 - 4\left(\frac{L}{6}\right)^2\right] + \frac{P\left(\frac{L}{3}\right)}{24EI}\left[3L^2 - 4\left(\frac{L}{3}\right)^2\right] + \frac{PL^3}{48EI}$$

$$= \frac{11PL^3}{144EI} \quad \longleftarrow$$

CONT.

9.5-2 CONT.

(b) Table G-2, Case 1 $\qquad qL = 5P$

$$\delta_2 = \frac{5qL^4}{384EI} = \frac{25PL^3}{384EI} \quad \longleftarrow$$

(c) $$\frac{\delta_1}{\delta_2} = \frac{11}{144}\left(\frac{384}{25}\right) = \frac{88}{75} = 1.173 \quad \longleftarrow$$

9.5-3

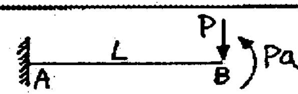


Table G-1, Cases 4 and 6

(a) $$\delta_B = \frac{PL^3}{3EI} - \frac{PaL^2}{2EI} = 0 \qquad \frac{a}{L} = \frac{2}{3} \quad \longleftarrow$$

(b) $$\theta_B = \frac{PL^2}{2EI} - \frac{PaL}{EI} = 0 \qquad \frac{a}{L} = \frac{1}{2} \quad \longleftarrow$$

9.5-4 Beam hanging from 2 springs

Table G-2, Case 4

$$\delta_C = \frac{PL^3}{48EI} + \frac{1}{2}\left(\frac{P/2}{k_1} + \frac{P/2}{k_2}\right) = \frac{PL^3}{48EI} + \frac{P}{4}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) \quad \longleftarrow$$

Substitute numerical values:

$$\delta_C = \frac{(8.0\text{ kN})(1.8\text{ m})^3}{48(216\text{ kN}\cdot\text{m}^2)} + \frac{8.0\text{ kN}}{4}\left(\frac{1}{250\text{ kN/m}} + \frac{1}{160\text{ kN/m}}\right)$$

$$= 4.5\text{ mm} + 20.5\text{ mm}$$

$$= 25\text{ mm} \quad \longleftarrow$$

9.5-5 Slightly curved beam

Let x = distance to load P

δ = downward deflection at load P

Table G-2, Case 5:

$$\delta = \frac{P(L-x)x}{6LEI}\left[L^2 - (L-x)^2 - x^2\right] = \frac{Px^2(L-x)^2}{3LEI}$$

Initial upward displacement of the beam must equal δ.

$$\therefore \quad y = \frac{Px^2(L-x)^2}{3LEI} \quad \longleftarrow$$

9.5-6

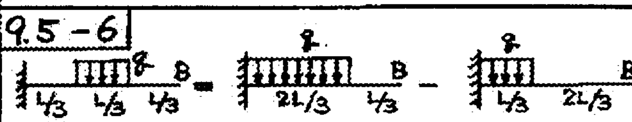


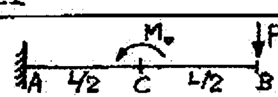
Table G-1, Case 2

$$\theta_B = \frac{q}{6EI}\left(\frac{2L}{3}\right)^3 - \frac{q}{6EI}\left(\frac{L}{3}\right)^3 = \frac{7qL^3}{162EI} \quad \longleftarrow$$

$$\delta_B = \frac{q}{24EI}\left(\frac{2L}{3}\right)^3\left(4L - \frac{2L}{3}\right) - \frac{q}{24EI}\left(\frac{L}{3}\right)^3\left(4L - \frac{L}{3}\right)$$

$$= \frac{23qL^4}{648EI} \quad \longleftarrow$$

9.5-7



$P = 3.6$ k

$M_0 = 35$ k-in.

$L = 96$ in.

$EI = 2.1 \times 10^6$ k-in.^{2}

Table G-1, Cases 4, 6, and 7

$$\delta_C = -\frac{M_0(L/2)^2}{2EI} + \frac{P(L/2)^2}{6EI}\left(3L - \frac{L}{2}\right)$$

$$= -\frac{M_0L^2}{8EI} + \frac{5PL^3}{48EI}$$

CONT.

9.5-7 CONT.

$$\delta_B = -\frac{M_0(L/2)}{2EI}\left(2L-\frac{L}{2}\right)+\frac{PL^3}{3EI} = -\frac{3M_0L^2}{8EI}+\frac{PL^3}{3EI}$$

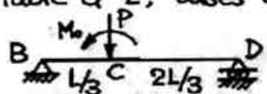
Note: + = downward, − = upward

Substitute numerical values:

$\delta_C = 0.139$ in. $\quad \delta_B = 0.448$ in. ←

9.5-8 Beam with bracket and overhang

Table G-2, Cases 5 and 9



$M_0 = Pa$

$$\theta_B = \frac{P(L/3)(2L/3)(5L/3)}{6LEI}+\frac{Pa}{6LEI}\left[6\left(\frac{L^2}{3}\right)-3\left(\frac{L^2}{9}\right)-2L^2\right] = \frac{PL}{162EI}(10L-9a)$$

(a) $\delta_A = \theta_B\left(\frac{L}{2}\right) = \frac{PL^2}{324EI}(10L-9a)$ (positive upward) ←

(b) Upward when $\frac{a}{L} < \frac{10}{9}$ and downward when $\frac{a}{L} > \frac{10}{9}$ ←

9.5-9 Beam with a bracket

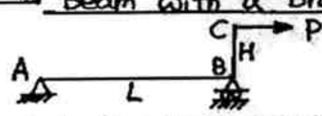
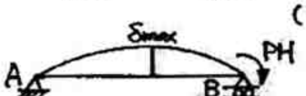


Table G-1, Case 4, and Table G-2, Case 7

(a) $\delta_C = \frac{PH^3}{3EI}+\frac{(PH)(L)(H)}{3EI} = \frac{PH^2}{3EI}(H+L)$ ←

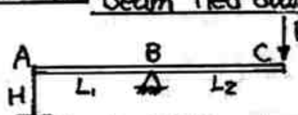
(To the right)

(b)



$\delta_{max} = \frac{PHL^2}{9\sqrt{3}EI}$ (upward) ←

9.5-10 Beam tied down by a wire



$EI = 75$ kN·m^2

$P = 480$ N

$EA = 900$ kN

$L_1 = 0.5$ m. $\quad L_2 = 0.75$ m. $\quad H = 0.5$ m

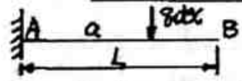
Table G-1, Case 4 and Table G-2, Case 7

$$\delta_C = \frac{PL_2^3}{3EI}+\frac{PL_2^2L_1}{3EI}+\frac{L_2}{L_1}\left(\frac{L_2}{L_1}P\right)\left(\frac{H}{EA}\right) = PL_2^2\left(\frac{L_1+L_2}{3EI}+\frac{H}{L_1^2EA}\right)$$ ←

Substitute numerical values:

$\delta_C = 2.1$ mm ←

9.5-11 Cantilever beam with a parabolic load



$q = \frac{q_0x^2}{L^2}$

Table G-1, Case 5

$$\theta_B = \int_0^L\frac{(q\,dx)(x^2)}{2EI} = \int_0^L\frac{1}{2EI}\left(q_0\frac{x^2}{L^2}\right)(x^2)dx = \frac{q_0L^3}{10EI}$$ ←

$$\delta_B = \int_0^L\frac{(q\,dx)(x^2)}{6EI}(3L-x) = \int_0^L\frac{1}{6EI}\left(q_0\frac{x^2}{L^2}\right)(x^2)(3L-x)dx = \frac{13q_0L^4}{180EI}$$ ←

9.5-12 Simple beam with a partial uniform load

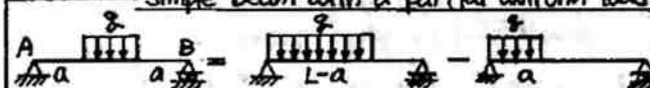


Table G-2, Case 3

$$\theta_A = \frac{q(L-a)^2}{24LEI}[2L-(L-a)]^2-\frac{qa^2}{24LEI}(2L-a)^2 = \frac{q}{24EI}(L^3-6La^2+4a^3)$$ ←

$$\delta_{max} = \frac{q(L/2)}{24LEI}\left[(L-a)^4-4L(L-a)^3+4L^2(L-a)^2+2(L-a)^2\left(\frac{L}{2}\right)^2-4L(L-a)\left(\frac{L}{2}\right)^2+L\left(\frac{L}{2}\right)^3\right]$$
$$-\frac{qa^2}{24LEI}\left[-La^2+4L^2\left(\frac{L}{2}\right)+a^2\left(\frac{L}{2}\right)-6L\left(\frac{L}{2}\right)^2+2\left(\frac{L}{2}\right)^3\right]$$

$$\delta_{max} = \frac{q}{384EI}(5L^4-24L^2a^2+16a^4)$$ ←

9.5-13 Overhanging beam

(a) Table G-2, Cases 4 and 7

$\delta_B = \frac{PL^3}{48EI}-Qa\left(\frac{L^2}{16EI}\right) = 0 \qquad \frac{P}{Q} = \frac{3a}{L}$ ←

(b) Table G-2, Case 4; Table G-1, Case 4; Table G-2, Case 7

$$\delta_D = -\frac{PL^2}{16EI}(a)+\frac{Qa^3}{3EI}+Qa\left(\frac{L}{3EI}\right)(a) = 0$$

$$\frac{P}{Q} = \frac{16a(L+a)}{3L^2}$$ ←

9.5-14 Thin metal strip

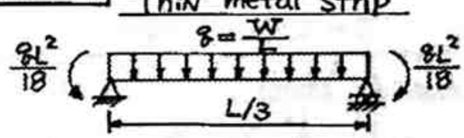


Table G-2, Cases 1 and 10

$$\delta = -\frac{5q\left(\frac{L}{3}\right)^4}{384EI}+\left(\frac{qL^2}{18}\right)\frac{\left(\frac{L}{3}\right)^2}{8EI} = \frac{19qL^4}{31{,}104EI}$$

$q = \frac{W}{L} \qquad \therefore\ \delta = \frac{19WL^3}{31{,}104EI}$ ←

9.5-15 Overhanging beam on a spring

(1) Assume that point B is on a rigid support.
Table G-2, Case 1

$\delta_C' = \frac{qL^3}{24EI}(b)$ (upward)

(2) Assume that spring shortens

$R_B = \frac{qL}{2}$ $\quad \delta_B = \frac{R_B}{k} = \frac{qL}{2k}$

$\delta_C'' = \delta_B\left(\frac{L+b}{L}\right) = \frac{q(L+b)}{2k}$ (downward)

(3) Deflection at C

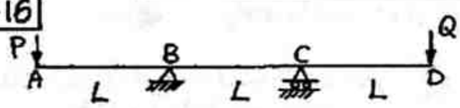
$\delta_C = \delta_C' + \delta_C'' = \frac{qL^3 b}{24EI} - \frac{q(L+b)}{2k} = 0$

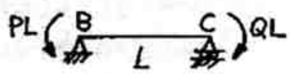
$k = \frac{12EI}{L^3}\left(1 + \frac{L}{b}\right)$ ←

Substitute numerical values:
EI = 21 k-in.² $\quad$ L = 30 in. $\quad$ b = 15 in.

k = 28 lb/in. ←

9.5-16





$\theta_B = PL\left(\frac{L}{3EI}\right) + QL\left(\frac{L}{6EI}\right) = \frac{L^2}{6EI}(2P+Q)$

$\delta_A = \theta_B L + \frac{PL^3}{3EI} = \frac{L^3}{6EI}(4P+Q)$

$4P + Q = \frac{6EI}{L^3}\delta_A$ (1)

Similarly, $\delta_D = \frac{L^3}{6EI}(4Q+P)$

$P + 4Q = \frac{6EI}{L^3}\delta_D$ (2)

Solve Eqs. (1) and (2):

$P = \frac{2EI}{5L^3}(4\delta_A - \delta_D)$ $\quad Q = \frac{2EI}{5L^3}(4\delta_D - \delta_A)$ ←

Substitute numerical values:
δ_A = 15 mm $\quad \delta_D$ = 10 mm $\quad$ L = 2.5 m
EI = 5.0×10⁶ N·m²

P = 6.4 kN $\quad$ Q = 3.2 kN ←

9.5-17 Compound beam

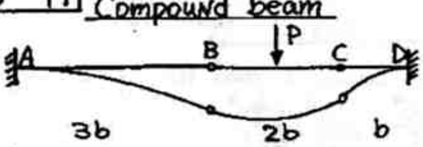


Table G-1, Case 4 and Table G-2, Case 4

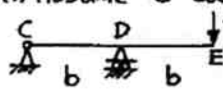
$\delta_B = \frac{\text{"}PL^3\text{"}}{3EI} = \left(\frac{P}{2}\right)(3b)^3\left(\frac{1}{3EI}\right) = \frac{9Pb^3}{2EI}$

$\delta_C = \frac{\text{"}PL^3\text{"}}{3EI} = \left(\frac{P}{2}\right)(b^3)\left(\frac{1}{3EI}\right) = \frac{Pb^3}{6EI}$

$\delta = \frac{1}{2}(\delta_B + \delta_C) + \frac{P(2b)^3}{48EI} = \frac{5Pb^3}{2EI}$ ←

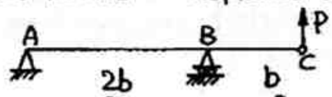
9.5-18 Compound beam

(1) Assume C does not displace



$\delta_E' = \frac{Pb^3}{3EI} + \theta_D b$
$= \frac{Pb^3}{3EI} + Pb\left(\frac{b}{3EI}\right)b = \frac{2Pb^3}{3EI}$
(Downward)

(2) Assume C displaces upward



$\delta_C = \frac{Pb^3}{3EI} + \theta_B b = \frac{Pb^3}{3EI} + Pb\left(\frac{2b}{3EI}\right)b = \frac{Pb^3}{EI}$
(Upward)

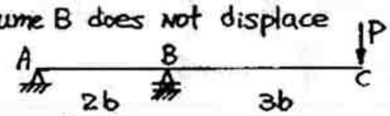
$\delta_E'' = \delta_C = \frac{Pb^3}{EI}$ (Downward)

(3) Displacement at E

$\delta_E = \delta_E' + \delta_E'' = \frac{5Pb^3}{3EI}$ ←

9.5-19 Beam supported by a wire

(1) Assume B does not displace



$\delta_C' = P(3b)^3\left(\frac{1}{3EI}\right) + \theta_B(3b)$

$\theta_B = (P)(3b)\left(\frac{2b}{3EI}\right) = \frac{2Pb^2}{EI}$

$\delta_C' = \frac{15Pb^3}{EI}$

(2) Assume B displaces downward

T = tensile force in wire $\quad T = \frac{5P}{2}$

$\delta_B = \frac{T(2b)}{EA} = \frac{5Pb}{EA}$

$\delta_C'' = \frac{5}{2}\delta_B = \frac{25Pb}{2EA}$

(3) Displacement at C

$\delta_C = \delta_C' + \delta_C'' = \frac{5Pb}{2}\left(\frac{6b^2}{EI} + \frac{5}{EA}\right)$ ←

Substitute numerical values:
P = 240 lb $\quad$ EA = 300×10³ lb
EI = 30×10⁶ lb-in² $\quad$ b = 10 in.

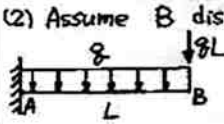
δ_C = 0.22 in. ←

9.5-20 Compound beam

(1) Assume B does not displace

$\delta_C' = \frac{5q(2L)^4}{384EI} = \frac{5qL^4}{24EI}$

(2) Assume B displaces downward



$\delta_B = \frac{qL^4}{8EI} + \frac{(qL)L^3}{3EI} = \frac{11qL^4}{24EI}$

$\delta_C'' = \frac{1}{2}\delta_B = \frac{11qL^4}{48EI}$

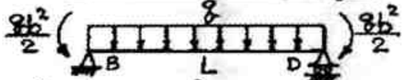
(3) Displacement at C

$\delta_C = \delta_C' + \delta_C'' = \frac{7qL^4}{16EI}$

c = clearance $\quad \frac{7qL^4}{16EI} = c$

$q = \frac{16cEI}{7L^4}$ ←

9.5-21 Beam with overhangs



$\theta_B = \frac{qb^2}{2}\left(\frac{L}{2EI}\right) - \frac{qL^3}{24EI} = \frac{qL}{24EI}(6b^2 - L^2)$

$\delta_A = \frac{qb^4}{8EI} + \theta_B b = \frac{qb}{24EI}(3b^3 + 6b^2L - L^3)$

$\delta_C = \frac{5qL^4}{384EI} - \left(\frac{qb^2}{2}\right)\left(\frac{L^2}{8EI}\right) = \frac{qL^2}{384EI}(5L^2 - 24b^2)$

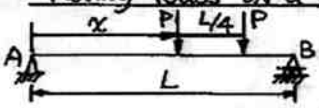
$\delta_C = \delta_A$ or $48\left(\frac{b}{L}\right)^4 + 96\left(\frac{b}{L}\right)^3 + 24\left(\frac{b}{L}\right)^2 - 16\left(\frac{b}{L}\right) - 5 = 0$

(a) Solve numerically: $\frac{b}{L} = 0.4030$ ←

(b) $\delta_C = \frac{qL^2}{384EI}[5L^2 - 24(0.40301L)^2]$

$= 0.002870\frac{qL^4}{EI}$ ←

9.5-22 Moving loads on a beam



By inspection, the loads must be near the midpoint of the beam. Thus, the range of values for x is $\frac{L}{4} \le x \le \frac{L}{2}$.

If $x = \frac{L}{4}$ or $\frac{L}{2}$, the deflection at the midpoint is $\delta_C = \frac{1}{48EI}\left(\frac{PL}{4}\right)\left[3L^2 - 4\left(\frac{L}{4}\right)^2\right] + \frac{PL^3}{48EI}$

$= \frac{9PL^3}{256EI}$ (1)

If $\frac{L}{4} < x < \frac{L}{2}$, the deflection at the midpoint is

$\delta_C = \frac{Px(3L^2 - 4x^2)}{48EI} + \frac{P\left(\frac{3L}{4} - x\right)\left[3L^2 - 4\left(\frac{3L}{4} - x\right)^2\right]}{48EI}$

$= \frac{3PL}{256EI}(L^2 + 12Lx - 16x^2)$ (2)

For the maximum deflection, $\frac{d\delta_C}{dx} = 0$

or $12L - 32x = 0$ $x = \frac{3L}{8}$ (Loads are symmetrical)

Substitute $x = \frac{3L}{8}$ into Eq.(2):

$\delta_C = \frac{39PL^3}{1024EI}$ (3)

Eq.(3) is larger than Eq.(1).

$(\delta_C)_{max} = \frac{39PL^3}{1024EI}$ ←

9.5-23 Frame

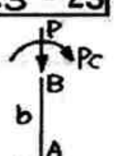


Table G-1, Case 6

$\delta_h = \frac{Pcb^2}{2EI}$ ←

$\theta_B = \frac{Pcb}{EI}$

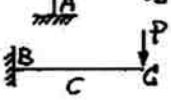


Table G-1, Case 4

$\delta_v = \frac{Pc^3}{3EI} + \theta_B c = \frac{Pc^2(c + 3b)}{3EI}$ ←

9.5-24 Frame

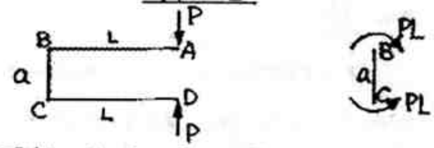


Table G-2, Case 10

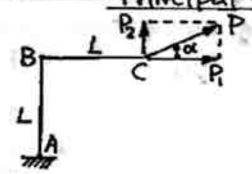
$\theta_B = \frac{PLa}{2EI}$

Table G-2, Case 4

$\delta_A = \frac{PL^3}{3EI} + \theta_B L = \frac{PL^2}{6EI}(2L + 3a)$

$\delta = 2\delta_A = \frac{PL^2}{3EI}(2L + 3a)$ ←

9.5-25 Principal directions of a frame



$P_1 = P\cos\alpha$

$P_2 = P\sin\alpha$

<u>If P_1 acts alone</u> $\delta_H' = \frac{P_1L^3}{3EI}$ (To the right)

$\delta_V' = \theta_B L = \left(\frac{P_1L^2}{2EI}\right)L = \frac{P_1L^3}{2EI}$ (Downward)

<u>If P_2 acts alone</u> $\delta_H'' = \frac{P_2L^3}{2EI}$ (To the left)

$\delta_V'' = \frac{P_2L^3}{3EI} + \theta_B L = \frac{P_2L^3}{3EI} + \left(\frac{P_2L^2}{EI}\right)L = \frac{4P_2L^3}{3EI}$ (Upward)

<u>Deflections due to load P</u>

$\delta_H = \frac{P_1L^3}{3EI} - \frac{P_2L^3}{2EI} = \frac{L^3}{6EI}(2P_1 - 3P_2)$ (To the right)

$\delta_V = -\frac{P_1L^3}{2EI} + \frac{4P_2L^3}{3EI} = \frac{L^3}{6EI}(-3P_1 + 8P_2)$ (Upward)

<u>Principal directions</u>

$\tan\alpha = \frac{P_2}{P_1} = \frac{\delta_V}{\delta_H}$

$\frac{\delta_V}{\delta_H} = \frac{-3P_1 + 8P_2}{2P_1 - 3P_2} = \frac{-3P\cos\alpha + 8P\sin\alpha}{2P\cos\alpha - 3P\sin\alpha} = \frac{-3 + 8\tan\alpha}{2 - 3\tan\alpha}$

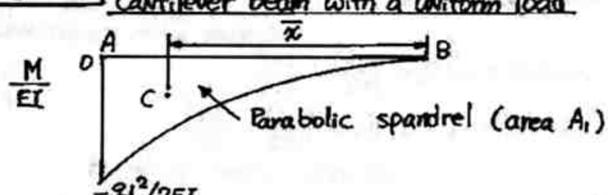
$(\tan\alpha)(2 - 3\tan\alpha) = -3 + 8\tan\alpha$

$\tan^2\alpha + 2\tan\alpha - 1 = 0$

$\tan\alpha = -1 \pm \sqrt{2}$ $\alpha = 22.5°$ and $-67.5°$ ←

(Also, $\alpha = 112.5°$ and $-157.5°$)

9.6-1 Cantilever beam with a uniform load



Use absolute values.

Appendix D, Case 18

$\bar{x} = 3L/4$, $A_1 = \frac{1}{3}(L)\left(\frac{qL^2}{2EI}\right) = \frac{qL^3}{6EI}$ CONT.

9.6-1 CONT.

$\theta_{B/A}=\theta_B-\theta_A=A_1=\frac{qL^3}{6EI}$

$\theta_A=0 \quad \theta_B=\frac{qL^3}{6EI}$ (clockwise) ←

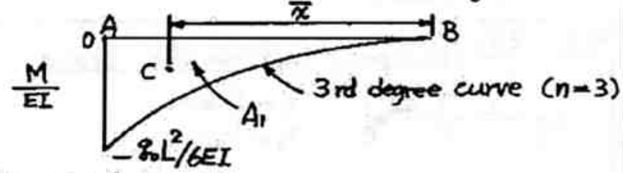
Q_1 = First moment of area A_1 with respect to B

$Q_1=A_1\bar{x}=\left(\frac{qL^3}{6EI}\right)\left(\frac{3L}{4}\right)=\frac{qL^4}{8EI}$

$\delta_B=Q_1=\frac{qL^4}{8EI}$ (Downward) ←

(These results agree with Case 1, Table G-1)

9.6-2 Cantilever beam with a triangular load



Use absolute values.

Appendix D, Case 20

$\bar{x}=\frac{b(n+1)}{n+2}=\frac{4L}{5} \qquad A_1=\frac{bh}{n+1}=\frac{L}{4}\left(\frac{q_0L^2}{6EI}\right)=\frac{q_0L^3}{24EI}$

$\theta_{B/A}=\theta_B-\theta_A=A_1=\frac{q_0L^3}{24EI}$

$\theta_A=0 \quad \theta_B=\frac{q_0L^3}{24EI}$ (clockwise) ←

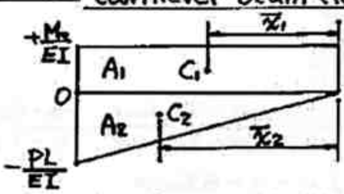
Q_1 = First moment of area A_1 with respect to B

$Q_1=A_1\bar{x}=\left(\frac{q_0L^3}{24EI}\right)\left(\frac{4L}{5}\right)=\frac{q_0L^4}{30EI}$

$\delta_B=Q_1=\frac{q_0L^4}{30EI}$ (Downward) ←

(These results agree with Case 8, Table G-1)

9.6-3 Cantilever beam (force P and couple M_0)



Use the moment-area sign conventions (Page 630)

$\bar{x}_1=\frac{L}{2} \quad A_1=\frac{M_0L}{EI} \quad \bar{x}_2=\frac{2L}{3} \quad A_2=-\frac{PL^2}{2EI}$

$A=A_1+A_2=\frac{M_0L}{EI}-\frac{PL^2}{2EI}$

$\theta_{B/A}=\theta_B-\theta_A=A \quad \theta_A=0 \quad \theta_B=\frac{M_0L}{EI}-\frac{PL^2}{2EI}$

(Positive when counterclockwise)

$Q=A_1\bar{x}_1+A_2\bar{x}_2=\frac{M_0L^2}{2EI}-\frac{PL^3}{3EI}$

$t_{B/A}=Q=\delta_B \quad \therefore \delta_B=\frac{M_0L^2}{2EI}-\frac{PL^3}{3EI}$

(Positive when upward)

Now assume that θ_B is positive when clockwise and δ_B is positive when downward. Then,

CONT.

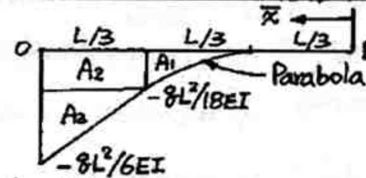
9.6-3 CONT.

$\theta_B=\frac{PL^2}{2EI}-\frac{M_0L}{EI}$ ←

$\delta_B=\frac{PL^3}{3EI}-\frac{M_0L^2}{2EI}$ ←

(These results agree with Cases 4 and 6, Table G-1).

9.6-4 Cantilever beam with partial uniform load



Use absolute values.

Appendix D, Case 1, 6, and 18

$\bar{x}_1=\frac{L}{3}+\frac{3}{4}\left(\frac{L}{3}\right)=\frac{7L}{12} \qquad A_1=\frac{1}{3}\left(\frac{L}{3}\right)\left(\frac{qL^2}{18EI}\right)=\frac{qL^3}{162EI}$

$\bar{x}_2=\frac{2L}{3}+\frac{L}{6}=\frac{5L}{6} \qquad A_2=\left(\frac{L}{3}\right)\left(\frac{qL^2}{18EI}\right)=\frac{qL^3}{54EI}$

$\bar{x}_3=\frac{2L}{3}+\frac{2}{3}\left(\frac{L}{3}\right)=\frac{8L}{9} \qquad A_3=\frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{qL^2}{9EI}\right)=\frac{qL^3}{54EI}$

$A=A_1+A_2+A_3=\frac{7qL^3}{162EI}$

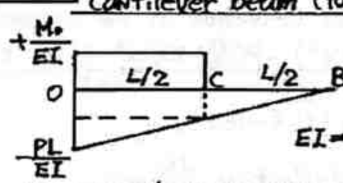
$\theta_{B/A}=\theta_B-\theta_A=A$

$\theta_A=0 \quad \theta_B=\frac{7qL^3}{162EI}$ (clockwise) ←

$Q=A_1\bar{x}_1+A_2\bar{x}_2+A_3\bar{x}_3=\frac{23qL^4}{648EI}$

$\delta_B=\frac{23qL^4}{648EI}$ (Downward) ←

9.6-5 Cantilever beam (force P and couple M_0)



$M_0 = 36$ k-in.
$P = 3.8$ k
$L = 8$ ft $= 96$ in.
$EI = 2.25\times10^7$ lb-in.^{2}

Use the moment-area sign conventions (Page 630)

δ_B = First moment of areas with respect to B

$\delta_B=\left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{4}\right)-\frac{1}{2}\left(\frac{PL}{EI}\right)(L)\left(\frac{2L}{3}\right)$

$=\frac{L^2}{24EI}(9M_0-8PL)=-0.443$ in.

(Minus means downward)

$\delta_C=\left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)-\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)-\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)$

$=\frac{L^2}{48EI}(6M_0-5PL)=-0.137$ in.

(Minus means downward)

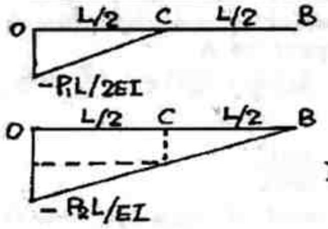
Now assume that downward deflections are positive.

$\delta_B=0.443$ in. ←

$\delta_C=0.137$ in. ←

9.6-6

Cantilever beam (forces P_1 and P_2)



$P_1 = 10$ kN
$P_2 = 5$ kN
$L = 2.6$ m
$E = 200$ GPa
$I = 20.1 \times 10^6$ mm^4

Use absolute values.

δ_B = First moment of areas with respect to B

$$\delta_B = \frac{1}{2}\left(\frac{P_1L}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{3}\right) + \frac{1}{2}\left(\frac{P_2L}{EI}\right)(L)\left(\frac{2L}{3}\right)$$

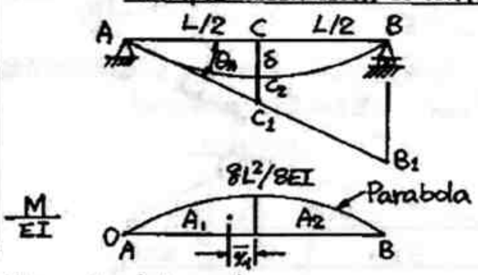
$$= \frac{5P_1L^3}{48EI} + \frac{P_2L^3}{3EI} = 11.8 \text{ mm (Downward)}$$

$$\delta_C = \frac{1}{2}\left(\frac{P_1L}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \left(\frac{P_2L}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(\frac{P_2L}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)$$

$$= \frac{P_1L^3}{24EI} + \frac{5P_2L^3}{48EI} = 4.10 \text{ mm (Downward)}$$

9.6-7

Simple beam with a uniform load



Use absolute values

Appendix D, Case 17

$\bar{x}_1 = \frac{3L}{16}$ $\quad A_1 = A_2 = \frac{qL^3}{24EI}$

BB_1 = First moment of area between A and B with respect to B

$$= (A_1 + A_2)\left(\frac{L}{2}\right) = \frac{qL^4}{24EI}$$

$$\theta_A = \frac{BB_1}{L} = \frac{qL^3}{24EI} \text{ (clockwise)}$$

$$CC_1 = \frac{BB_1}{2} = \frac{qL^4}{48EI}$$

C_2C_1 = First moment of area between A and C with respect to C

$$= A_1\bar{x}_1 = \frac{qL^4}{128EI}$$

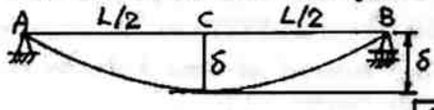
$$\delta = CC_1 - C_2C_1 = \frac{qL^4}{48EI} - \frac{qL^4}{128EI} = \frac{5qL^4}{384EI}$$

(Downward)

(These results agree with Case 1 of Table G-2)

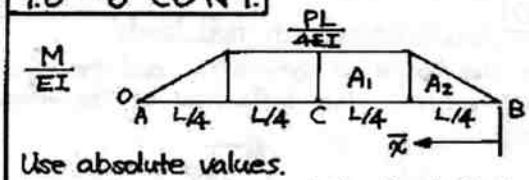
9.6-8

Simple beam with two equal loads



CONT.

9.6-8 CONT.



Use absolute values.

At point C, the tangent to the deflection curve is horizontal.

δ = First moment of area between C and B with respect to B

$$= A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= \left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{8}\right) + \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{6}\right)$$

$$= \frac{11PL^3}{384EI}$$

d = gap between support at C and the beam

$$\therefore \delta = d = \frac{11PL^3}{384EI} \qquad P = \frac{384EId}{11L^3}$$

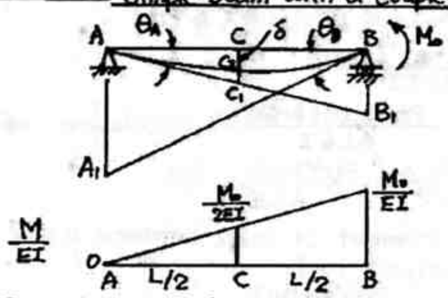
Substitute numerical values:

$d = 10$ mm $\quad L = 6$ m $\quad E = 200$ GPa

$I = 198 \times 10^6$ mm^4 $\qquad P = 64$ kN

9.6-9

Simple beam with a couple at the end



Use absolute values.

BB_1 = First moment of area between A and B with respect to B

$$= \frac{1}{2}\left(\frac{M_0}{EI}\right)(L)\left(\frac{L}{3}\right) = \frac{M_0L^2}{6EI}$$

$$\theta_A = \frac{BB_1}{L} = \frac{M_0L}{6EI} \text{ (clockwise)}$$

AA_1 = First moment of area between A and B with respect to A

$$= \frac{1}{2}\left(\frac{M_0}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{M_0L^2}{3EI}$$

$$\theta_B = \frac{AA_1}{L} = \frac{M_0L}{3EI} \text{ (Counterclockwise)}$$

$$CC_1 = \frac{BB_1}{2} = \frac{M_0L^2}{12EI}$$

C_2C_1 = First moment of area between A and C with respect to C

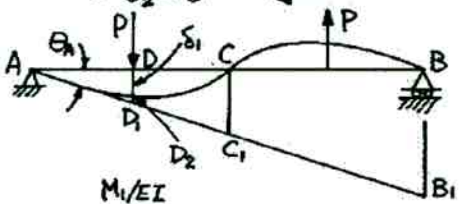
$$= \frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) = \frac{M_0L^2}{48EI}$$

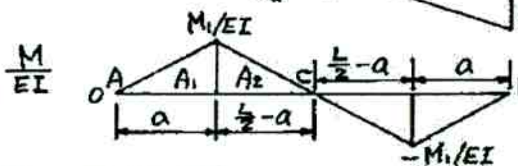
$$\delta = CC_1 - C_2C_1 = \frac{M_0L^2}{12EI} - \frac{M_0L^2}{48EI} = \frac{M_0L^2}{16EI} \text{ (Downward)}$$

(These results agree with Case 7 of Table G-2)

9.6-10 Simple beam with two loads

Because the beam is symmetric and the load is antisymmetric, the deflection at the midpoint is zero. $\therefore \delta_2 = 0$ ←





$$\frac{M_1}{EI} = \frac{Pa(L-2a)}{LEI}$$

$$A_1 = \frac{1}{2}\left(\frac{M_1}{EI}\right)(a) = \frac{Pa^2(L-2a)}{2LEI}$$

$$A_2 = \frac{1}{2}\left(\frac{M_1}{EI}\right)\left(\frac{L}{2}-a\right) = \frac{Pa(L-2a)^2}{4LEI}$$

CC_1 = First moment of area between A and C with respect to C

$$= A_1\left(\frac{L}{2}-a+\frac{a}{3}\right) + A_2\left(\frac{2}{3}\right)\left(\frac{L}{2}-a\right)$$

$$= \frac{Pa(L-a)(L-2a)}{12EI}$$

$$\theta_A = \frac{CC_1}{L/2} = \frac{Pa(L-a)(L-2a)}{6LEI} \quad \text{(clockwise)} \leftarrow$$

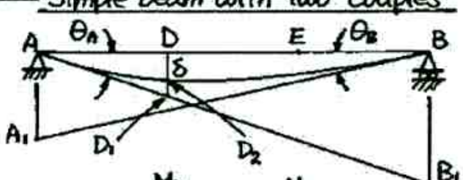
$$DD_1 = CC_1\left(\frac{a}{L/2}\right) = \frac{Pa^2(L-a)(L-2a)}{6LEI}$$

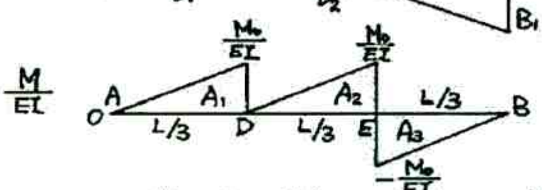
D_2D_1 = First moment of area between A and D with respect to D

$$= A_1\left(\frac{a}{3}\right) = \frac{Pa^3(L-2a)}{6LEI}$$

$$\delta_1 = DD_1 - D_2D_1 = \frac{Pa^2(L-2a)^2}{6LEI} \quad \text{(Downward)} \leftarrow$$

9.6-11 Simple beam with two couples





$$A_1 = A_2 = \frac{1}{2}\left(\frac{M_0}{EI}\right)\left(\frac{L}{3}\right) = \frac{M_0L}{6EI} \qquad A_3 = -\frac{M_0L}{6EI}$$

BB_1 = First moment of area between A and B with respect to B

$$= A_1\left(\frac{2L}{3}+\frac{L}{9}\right) + A_2\left(\frac{L}{3}+\frac{L}{9}\right) + A_3\left(\frac{2L}{9}\right) = \frac{M_0L^2}{6EI}$$

$$\theta_A = \frac{BB_1}{L} = \frac{M_0L}{6EI} \quad \text{(clockwise)} \leftarrow$$

CONT.

9.6-11 CONT.

AA_1 = First moment of area between A and B with respect to A

$$= A_1\left(\frac{2L}{9}\right) + A_2\left(\frac{L}{3}+\frac{2L}{9}\right) + A_3\left(\frac{2L}{3}+\frac{L}{9}\right) = 0$$

$$\theta_B = \frac{AA_1}{L} = 0 \leftarrow$$

$$DD_1 = BB_1\left(\frac{1}{3}\right) = \frac{M_0L^2}{18EI}$$

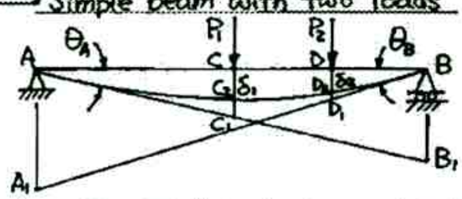
D_2D_1 = First moment of area between A and D with respect to D

$$= A_1\left(\frac{L}{9}\right) = \frac{M_0L^2}{54EI}$$

$$\delta = DD_1 - D_2D_1 = \frac{M_0L^2}{27EI} \quad \text{(Downward)} \leftarrow$$

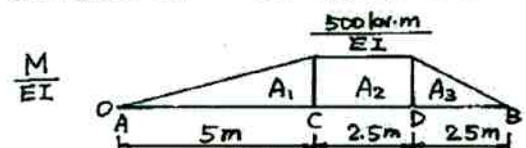
(Note: This deflection is also the maximum deflection)

9.6-12 Simple beam with two loads



$P_1 = 100$ kN $\quad P_2 = 200$ kN $\quad L = 10$ m $\quad E = 200$ GPa

$I = 1.20 \times 10^9$ mm^4 $\quad EI = 240 \times 10^3$ kN·m^2



$$A_1 = \frac{1}{2}\left(\frac{500}{EI}\right)(5) = \frac{1250 \text{ kN·m}^2}{EI}$$

$$A_2 = \left(\frac{500}{EI}\right)(2.5) = \frac{1250 \text{ kN·m}^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{500}{EI}\right)(2.5) = \frac{625 \text{ kN·m}^2}{EI}$$

BB_1 = First moment of area between A and B with respect to B

$$= A_1(6.667\text{m}) + A_2(3.75\text{m}) + A_3(1.667\text{m})$$

$$= \frac{14{,}062 \text{ kN·m}^3}{EI} = 0.058592 \text{ m}$$

$$\theta_A = \frac{BB_1}{10\text{m}} = \frac{0.058592\text{ m}}{10\text{ m}}$$

$$= 0.00586 \text{ rad} \quad \text{(clockwise)} \leftarrow$$

AA_1 = First moment of area between A and B with respect to A

$$= A_1(3.333\text{m}) + A_2(6.25\text{m}) + A_3(8.333\text{m})$$

$$= \frac{17{,}187 \text{ kN·m}^3}{EI} = 0.071613 \text{ m}$$

$$\theta_B = \frac{AA_1}{10\text{m}} = \frac{0.071613\text{ m}}{10\text{ m}}$$

$$= 0.00716 \text{ rad} \quad \text{(Counterclockwise)} \leftarrow$$

$$CC_1 = BB_1\left(\frac{1}{2}\right) = 0.029296 \text{ m}$$

C_2C_1 = First moment of area between A and C with respect to C

$$= A_1(1.667\text{m}) = \frac{2083.3 \text{ kN·m}^3}{EI} = 0.008681 \text{ m}$$

CONT.

9.6-12 CONT.

$\delta_1 = CC_1 - C_2C_1 = 20.6\ \text{mm}$ (Downward)

$DD_1 = AA_1\left(\frac{1}{4}\right) = 0.017903\ \text{m}$

D_2D_1 = First moment of area between D and B with respect to D

$= A_3(0.8333\ \text{m}) = \frac{520.833\ \text{kN}\cdot\text{m}^3}{EI} = 0.002170\ \text{m}$

$\delta_2 = DD_1 - D_2D_1 = 15.7\ \text{mm}$ (Downward)

Note: For $P_1 = P$ and $P_2 = 2P$:

$\theta_A = \frac{9PL^2}{64EI}$ $\quad \theta_B = \frac{11PL^2}{64EI}$

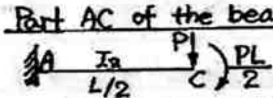
$\delta_1 = \frac{19PL^3}{384EI}$ $\quad \delta_2 = \frac{29PL^3}{768EI}$

9.7-1

(a) Part CB of the beam

$$\delta_1 = \frac{P(L/2)^3}{3EI_1} = \frac{PL^3}{24EI_1}$$

Part AC of the beam



$$\delta_C = \frac{P(L/2)^3}{3EI_2} + \frac{(PL/2)(L/2)^2}{2EI_2} = \frac{5PL^3}{48EI_2}$$

$$\theta_C = \frac{P(L/2)^2}{2EI_2} + \frac{(PL/2)(L/2)}{EI_2} = \frac{3PL^2}{8EI_2}$$

Deflection at B

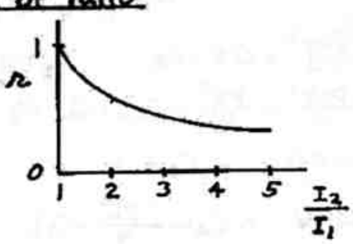
$$\delta_B = \delta_1 + \delta_C + \theta_C\left(\frac{L}{2}\right) = \frac{PL^3}{24EI_1}\left(1 + \frac{7I_1}{I_2}\right)$$

(b) Prismatic beam

$$\delta_1 = \frac{PL^3}{3EI_1}$$

Ratio $\quad r = \frac{\delta_B}{\delta_1} = \frac{1}{8}\left(1 + \frac{7I_1}{I_2}\right)$

(c) Graph of ratio

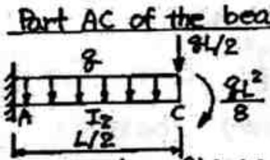


9.7-2

(a) Part CB of the beam

$$\delta_1 = \frac{q(L/2)^4}{8EI_1} = \frac{qL^4}{128EI_1}$$

Part AC of the beam



$$\delta_C = \frac{q(L/2)^4}{8EI_2} + \frac{\left(\frac{qL}{2}\right)(L/2)^3}{3EI_2} + \frac{\left(\frac{qL^2}{8}\right)\left(\frac{L}{2}\right)^2}{2EI_2} = \frac{17qL^4}{384EI_2}$$

CONT.

9.7-2 CONT.

$$\theta_C = \frac{q(L/2)^3}{6EI_2} + \frac{(qL/2)(L/2)^2}{2EI_2} + \frac{(qL^2/8)(L/2)}{EI_2} = \frac{7qL^3}{48EI_2}$$

Deflection at B

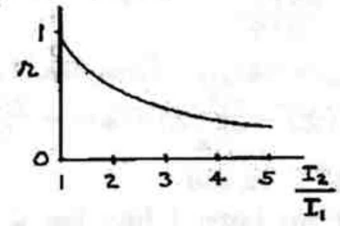
$$\delta_B = \delta_1 + \delta_C + \theta_C\left(\frac{L}{2}\right) = \frac{qL^4}{128EI_1}\left(1 + \frac{15I_1}{I_2}\right)$$

(b) Prismatic beam

$$\delta_1 = \frac{qL^4}{8EI_1}$$

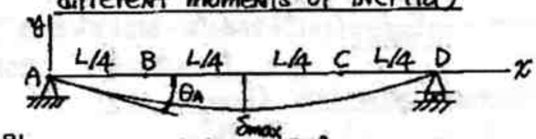
Ratio $\quad r = \frac{\delta_B}{\delta_1} = \frac{1}{16}\left(1 + \frac{15I_1}{I_2}\right)$

(c) Graph of ratio



9.7-3

Simple beam (Uniform load: two different moments of inertia)



$R_A = \frac{qL}{2} \qquad M = \frac{qLx}{2} - \frac{qx^2}{2}$

$EIv'' = M = \frac{qLx}{2} - \frac{qx^2}{2} \qquad \left(0 \le x \le \frac{L}{4}\right)$

$E(2I)v'' = M = \frac{qLx}{2} - \frac{qx^2}{2} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right)$

Integrate each equation:

$EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} + C_1 \qquad \left(0 \le x \le \frac{L}{4}\right) \qquad (1)$

$2EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} + C_2 \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right)$

B.C. 1 (Symmetry) $\quad v'\left(\frac{L}{2}\right) = 0 \qquad \therefore C_2 = -\frac{qL^3}{24}$

$2EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} - \frac{qL^3}{24} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right) \qquad (2)$

Slope at point B $\left(x = \frac{L}{4}\right)$ from Eq.(2):

$EIv'_B = -\frac{11qL^3}{768} \qquad (3)$

B.C. 2 $\quad (v'_B)_{LEFT} = (v'_B)_{RIGHT}$ (From Eqs. 1 and 3)

$\frac{qL}{4}\left(\frac{L}{4}\right)^2 - \frac{q}{6}\left(\frac{L}{4}\right)^3 + C_1 = -\frac{11qL^3}{768}$

$\therefore C_1 = -\frac{7qL^3}{256}$

Slopes of the beam

$EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} - \frac{7qL^3}{256} \qquad \left(0 \le x \le \frac{L}{4}\right) \qquad (4)$

$EIv' = \frac{qLx^2}{8} - \frac{qx^3}{12} - \frac{qL^3}{48} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right) \qquad (5)$

CONT.

9.7-3 CONT.

Angle of rotation θ_A (from Eq. 4)

$$\theta_A = -v'(0) = \frac{7qL^3}{256EI} \longleftarrow$$

Integrate Eqs. (4) and (5):

$$EIv = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{7qL^3x}{256} + C_3 \quad (0 \le x \le \tfrac{L}{4}) \qquad (6)$$

$$EIv = \frac{qLx^3}{24} - \frac{qx^4}{48} - \frac{qL^3x}{48} + C_4 \quad (\tfrac{L}{4} \le x \le \tfrac{L}{2}) \qquad (7)$$

B.C.3 $v(0) = 0$ From Eq. (6) $\therefore C_3 = 0$

Deflection at point B ($x = \frac{L}{4}$) from Eq. (6):

$$EIv_B = -\frac{35qL^4}{6144} \qquad (8)$$

B.C.4 $(v_B)_{RIGHT} = (v_B)_{LEFT}$ (From Eqs. 7 and 8)

$$\frac{qL}{24}\left(\frac{L}{4}\right)^3 - \frac{q}{48}\left(\frac{L}{4}\right)^4 - \frac{qL^3}{48}\left(\frac{L}{4}\right) + C_4 = -\frac{35qL^4}{6144}$$

$$C_4 = -\frac{13qL^4}{12{,}288}$$

Deflections of the beam (From Eqs. 6 and 7)

$$v = -\frac{qx}{768EI}(21L^3 - 64Lx^2 + 32x^3) \qquad (9) \longleftarrow$$
$$(0 \le x \le \tfrac{L}{4})$$

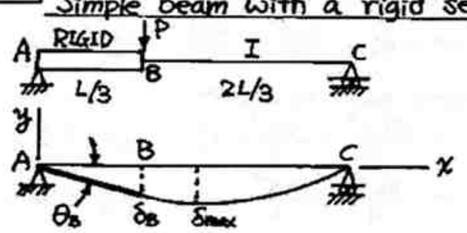
$$v = -\frac{q}{12{,}288EI}(13L^4 + 256L^3x - 512Lx^3 + 256x^4)$$
$$(\tfrac{L}{4} \le x \le \tfrac{L}{2}) \qquad (10) \longleftarrow$$

Maximum deflection (From Eq. 10)

$$\delta_{max} = -v\left(\frac{L}{2}\right) = \frac{31qL^4}{4096EI} \longleftarrow$$

9.7-4

Simple beam with a rigid segment



From A to B
$$v = -\frac{3\delta_B x}{L} \quad (0 \le x \le \tfrac{L}{3}) \qquad (1)$$
$$v' = -\frac{3\delta_B}{L} \quad (0 \le x \le \tfrac{L}{3}) \qquad (2)$$

From B to C
$$EIv'' = M = \frac{PL}{3} - \frac{Px}{3} \qquad (3)$$
$$EIv' = \frac{PLx}{3} - \frac{Px^2}{6} + C_1$$

B.C.1 At $x = L/3$, $v' = -\frac{3\delta_B}{L}$

$$\therefore C_1 = -\frac{5PL^2}{54} - \frac{3EI\delta_B}{L}$$

$$EIv' = \frac{PLx}{3} - \frac{Px^2}{6} - \frac{5PL^2}{54} - \frac{3EI\delta_B}{L} \quad (\tfrac{L}{3} \le x \le L) \qquad (4)$$

$$EIv = \frac{PLx^2}{6} - \frac{Px^3}{18} - \frac{5PL^2x}{54} - \frac{3EI\delta_B x}{L} + C_2 \quad (\tfrac{L}{3} \le x \le L)$$

B.C.2 $v(L) = 0$ $\therefore C_2 = -\frac{PL^3}{54} + 3EI\delta_B$

$$EIv = \frac{PLx^2}{6} - \frac{Px^3}{18} - \frac{5PL^2x}{54} - \frac{3EI\delta_B x}{L} - \frac{PL^2}{54} + 3EI\delta_B$$
$$(\tfrac{L}{3} \le x \le L) \qquad (5)$$

CONT.

9.7-4 CONT.

B.C.3 At $x = \frac{L}{3}$, $(v_B)_{LEFT} = (v_B)_{RIGHT}$ (Eqs. 1 and 5)

$$\therefore \delta_B = \frac{8PL^3}{729EI} \longleftarrow$$

$$\theta_A = \frac{\delta_B}{L/3} = \frac{8PL^2}{243EI} \longleftarrow$$

Substitute for δ_B in Eq. (5) and simplify:

$$v = \frac{P}{486EI}(7L^3 - 61L^2x + 81Lx^2 - 27x^3) \quad (\tfrac{L}{3} \le x \le L) \qquad (6)$$

Also,

$$v' = \frac{P}{486EI}(-61L^2 + 162Lx - 81x^2) \quad (\tfrac{L}{3} \le x \le L) \qquad (7)$$

Maximum deflection

$v' = 0$ gives $x_1 = \frac{L}{9}(9 - 2\sqrt{5}) = 0.5031L$

Substitute x_1 in Eq. (6) and simplify:

$$v_{max} = -\frac{40\sqrt{5}PL^3}{6561EI}$$

$$\delta_{max} = -v_{max} = \frac{40\sqrt{5}PL^3}{6561EI} = 0.01363\frac{PL^3}{EI} \longleftarrow$$

9.7-5

Simple beam (two different moments of inertia)

$$E\left(\frac{3I}{2}\right)v'' = M = \frac{2Px}{3} \qquad (0 \le x \le \tfrac{L}{3})$$

$$EIv'' = M = \frac{PL}{3} - \frac{Px}{3} \qquad (\tfrac{L}{3} \le x \le L)$$

Slopes
$$EIv' = \frac{4Px^2}{18} + C_1 \qquad (0 \le x \le \tfrac{L}{3})$$
$$EIv' = \frac{PLx}{3} - \frac{Px^2}{6} + C_2 \qquad (\tfrac{L}{3} \le x \le L)$$

B.C.1 Continuity condition for slopes at $x = \frac{L}{3}$

$$\frac{4P}{18}\left(\frac{L}{3}\right)^2 + C_1 = \frac{PL}{3}\left(\frac{L}{3}\right) - \frac{P}{6}\left(\frac{L}{3}\right)^2 + C_2$$

$$C_2 = C_1 - \frac{11PL^2}{162} \qquad (1)$$

Deflections
$$EIv = \frac{4Px^3}{54} + C_1x + C_3 \qquad (0 \le x \le \tfrac{L}{3})$$
$$EIv = \frac{PLx^2}{6} - \frac{Px^3}{18} + C_2x + C_4 \qquad (\tfrac{L}{3} \le x \le L)$$

B.C.2 $v(0) = 0$ $\therefore C_3 = 0$ (2)

B.C.3 $v(L) = 0$ $\therefore C_4 = -\frac{PL^3}{9} - C_2L$ (3)

B.C.4 Continuity condition for deflection at $x = \frac{L}{3}$

$$\frac{4P}{54}\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right) = \frac{PL}{6}\left(\frac{L}{3}\right)^2 - \frac{P}{18}\left(\frac{L}{3}\right)^3 + C_2\left(\frac{L}{3}\right) + C_4$$

$$C_1L = \frac{10PL^3}{243} + C_2L + 3C_4 \qquad (4)$$

Solve Eqs. (1), (2), (3), and (4):

$$C_1 = -\frac{38PL^2}{729} \qquad C_2 = -\frac{175PL^2}{1458} \qquad C_3 = 0 \qquad C_4 = \frac{13PL^3}{1458}$$

Slopes
$$v' = -\frac{2P}{729EI}(19L^2 - 81x^2) \qquad (0 \le x \le \tfrac{L}{3})$$
$$v' = -\frac{P}{1458EI}(175L^2 - 486Lx + 243x^2) \qquad (\tfrac{L}{3} \le x \le L)$$

$$\theta_A = -v'(0) = \frac{38PL^2}{729EI} \longleftarrow$$

CONT.

9.7-5 CONT.

$$\theta_C = v'(L) = \frac{34PL^2}{729EI}$$

Deflections

$$v = -\frac{2Px}{729EI}(19L^2 - 27x^2) \quad (0 \le x \le \tfrac{L}{3})$$

$$v = \frac{P}{1458EI}(13L^3 - 175L^2x + 243Lx^2 - 81x^3) \quad (\tfrac{L}{3} \le x \le L)$$

$$\delta_B = -v(\tfrac{L}{3}) = \frac{32PL^3}{2187EI}$$

9.7-6 Tapered cantilever beam

$M = -Px \quad EIv'' = -Px \quad I = \frac{I_A}{L^3}(L+x)^3$

$$v'' = -\frac{Px}{EI} = -\frac{PL^3}{EI_A}\left[\frac{x}{(L+x)^3}\right]$$

From Appendix C: $\int \frac{x\,dx}{(L+x)^3} = -\frac{L+2x}{2(L+x)^2}$

$$v' = \frac{PL^3}{EI_A}\left[\frac{L+2x}{2(L+x)^2}\right] + C_1$$

B.C.1 $\quad v'(L) = 0 \quad \therefore C_1 = -\frac{3PL^2}{8EI_A}$

$$v' = \frac{PL^3}{EI_A}\left[\frac{L+2x}{2(L+x)^2}\right] - \frac{3PL^2}{8EI_A}$$

$$v' = \frac{PL^3}{EI_A}\left[\frac{L}{2(L+x)^2}\right] + \frac{PL^3}{EI_A}\left[\frac{x}{(L+x)^2}\right] - \frac{3PL^2}{8EI_A}$$

From Appendix C:

$$\int \frac{dx}{(L+x)^2} = -\frac{1}{L+x} \qquad \int \frac{x\,dx}{(L+x)^2} = \frac{L}{L+x} + \ln(L+x)$$

$$v = \frac{PL^3}{EI_A}\left(\frac{L}{2}\right)\left(-\frac{1}{L+x}\right) + \frac{PL^3}{EI_A}\left[\frac{L}{L+x} + \ln(L+x)\right] - \frac{3PL^2}{8EI_A}x + C_2$$

$$= \frac{PL^3}{EI_A}\left[\frac{L}{2(L+x)} + \ln(L+x) - \frac{3x}{8L}\right] + C_2$$

B.C.2 $\quad v(L) = 0 \quad \therefore C_2 = \frac{PL^3}{EI_A}\left[\frac{1}{8} - \ln(2L)\right]$

$$v = \frac{PL^3}{EI_A}\left[\frac{L}{2(L+x)} - \frac{3x}{8L} + \frac{1}{8} + \ln\left(\frac{L+x}{2L}\right)\right]$$

$$\delta_A = -v(0) = \frac{PL^3}{8EI_A}(8\ln 2 - 5) = 0.06815\frac{PL^3}{EI_A}$$

9.7-7 Tapered cantilever beam

$M = -Px \quad EIv'' = -Px \quad I = \frac{I_A}{L^4}(L+x)^4$

$$v'' = -\frac{Px}{EI} = -\frac{PL^4}{EI_A}\left[\frac{x}{(L+x)^4}\right]$$

From Appendix C: $\int \frac{x\,dx}{(L+x)^4} = -\frac{L+3x}{6(L+x)^3}$

$$v' = \frac{PL^4}{EI_A}\left[\frac{L+3x}{6(L+x)^3}\right] + C_1$$

B.C.1 $\quad v'(L) = 0 \quad \therefore C_1 = -\frac{PL^2}{12EI_A}$

$$v' = \frac{PL^4}{EI_A}\left[\frac{L+3x}{6(L+x)^3}\right] - \frac{PL^2}{12EI_A}$$

$$v' = \frac{PL^4}{EI_A}\left[\frac{L}{6(L+x)^3}\right] + \frac{PL^4}{EI_A}\left[\frac{x}{2(L+x)^3}\right] - \frac{PL^2}{12EI_A}$$

CONT.

9.7-7 CONT.

From Appendix C: $\int \frac{dx}{(L+x)^3} = -\frac{1}{2(L+x)^2}$

$$\int \frac{x\,dx}{(L+x)^3} = \frac{-(L+2x)}{2(L+x)^2}$$

$$v = \frac{PL^4}{EI_A}\left(\frac{L}{6}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{L+x}\right)^2 + \frac{PL^4}{EI_A}\left(\frac{1}{2}\right)\left[-\frac{L+2x}{2(L+x)^2}\right] - \frac{PL^2}{12EI_A}x + C_2$$

$$= \frac{PL^3}{EI_A}\left[-\frac{L^2}{12(L+x)^2} - \frac{L(L+2x)}{4(L+x)^2} - \frac{x}{12L}\right] + C_2$$

B.C.2 $\quad v(L) = 0 \quad \therefore C_2 = \frac{PL^3}{EI_A}\left(\frac{7}{24}\right)$

$$v = \frac{PL^3}{24EI_A}\left[7 - \frac{4L(2L+3x)}{(L+x)^2} - \frac{2x}{L}\right]$$

$$\delta_A = -v(0) = \frac{PL^3}{24EI_A}$$

9.7-8 Tapered cantilever beam

$M = -Px \quad EIv'' = -Px \quad I = \frac{I_A}{8L^3}(2L+x)^3$

$$v'' = -\frac{Px}{EI} = -\frac{8PL^3}{EI_A}\left[\frac{x}{(2L+x)^3}\right]$$

From Appendix C: $\int \frac{x\,dx}{(2L+x)^3} = -\frac{2L+2x}{2(2L+x)^2}$

$$v' = \frac{8PL^3}{EI_A}\left[\frac{L+x}{(2L+x)^2}\right] + C_1$$

B.C.1 $\quad v'(L) = 0 \quad \therefore C_1 = -\frac{16PL^2}{9EI_A}$

$$v' = \frac{8PL^3}{EI_A}\left[\frac{L+x}{(2L+x)^2}\right] - \frac{16PL^2}{9EI_A}$$

$$v' = \frac{8PL^3}{EI_A}\left[\frac{L}{(2L+x)^2}\right] + \frac{8PL^3}{EI_A}\left[\frac{x}{(2L+x)^2}\right] - \frac{16PL^2}{9EI_A}$$

From Appendix C: $\int \frac{dx}{(2L+x)^2} = -\frac{1}{2L+x}$

$$\int \frac{x\,dx}{(2L+x)^2} = \frac{2L}{2L+x} + \ln(2L+x)$$

$$v = \frac{8PL^3}{EI_A}\left(-\frac{L}{2L+x}\right) + \frac{8PL^3}{EI_A}\left[\frac{2L}{2L+x} + \ln(2L+x)\right] - \frac{16PL^2}{9EI_A}x + C_2$$

$$= \frac{PL^3}{EI_A}\left[\frac{8L}{2L+x} + 8\ln(2L+x) - \frac{16x}{9L}\right] + C_2$$

B.C.2 $\quad v(L) = 0 \quad \therefore C_2 = -\frac{8PL^3}{EI_A}\left[\frac{1}{9} + \ln(3L)\right]$

$$v = \frac{8PL^3}{EI_A}\left[\frac{L}{2L+x} - \frac{2x}{9L} - \frac{1}{9} + \ln\left(\frac{2L+x}{3L}\right)\right]$$

$$\delta_A = -v(0) = \frac{8PL^3}{EI_A}\left[\ln\left(\frac{3}{2}\right) - \frac{7}{18}\right]$$

9.7-9 Simple beam (tapered) with a uniform load

Total length $= 2L \quad I = \frac{I_A}{L^4}(L+x)^4 \quad (0 \le x \le L)$

$$EIv'' = M = qLx - \frac{qx^2}{2}$$

$$v'' = \frac{qL^5x}{EI_A(L+x)^4} - \frac{qL^4x^2}{2EI_A(L+x)^4}$$

From Appendix C: $\int \frac{x\,dx}{(L+x)^4} = -\frac{L+3x}{6(L+x)^3}$

$$\int \frac{x^2\,dx}{(L+x)^4} = -\frac{L^2+3Lx+3x^2}{3(L+x)^3}$$

CONT.

9.7-9 CONT.

$$v' = \frac{qL^5}{EI_A}\left[-\frac{L+3x}{6(L+x)^3}\right] - \frac{qL^4}{2EI_A}\left[-\frac{L^2+3Lx+3x^2}{3(L+x)^3}\right] + C_1$$

$$= \frac{qL^4x^2}{2EI_A(L+x)^3} + C_1$$

B.C.1 (Symmetry) $v'(L)=0 \quad \therefore C_1 = -\frac{qL^3}{16EI_A}$

$$v' = \frac{qL^4x^2}{2EI_A(L+x)^3} - \frac{qL^3}{16EI_A}$$

$$= -\frac{qL^3}{16EI_A}\left[1 - \frac{8Lx^2}{(L+x)^3}\right] \quad (0 \le x \le L) \quad \leftarrow$$

$$\theta_A = -v'(0) = \frac{qL^3}{16EI_A} \quad \leftarrow$$

From Appendix C: $\int \frac{x^2dx}{(L+x)^3} = \frac{L(3L+4x)}{2(L+x)^2} + \ln(L+x)$

$$v = -\frac{qL^3}{16EI_A}\left[x - \frac{8L^2(3L+4x)}{2(L+x)^2} - 8L\ln(L+x)\right] + C_2$$

B.C.2 $v(0)=0 \quad \therefore C_2 = -\frac{qL^4}{2EI_A}\left(\frac{3}{2} + \ln L\right)$

Substitute for C_2 and simplify (lengthy):

$$v = -\frac{qL^4}{2EI_A}\left[\frac{(9L^2+14Lx+x^2)x}{8L(L+x)^2} - \ln\left(1+\frac{x}{L}\right)\right] \quad \leftarrow$$

$(0 \le x \le L)$

$$\delta_C = -v(L) = \frac{qL^4}{8EI_A}(3 - 4\ln 2) \quad \leftarrow$$

9.8-1 Simple beam with concentrated load

(a) $M = \frac{Px}{2} \quad \left(0 \le x \le \frac{L}{2}\right)$

$$U = 2\int_0^{L/2}\frac{M^2dx}{2EI} = \frac{2}{EI}\int_0^{L/2}\left(\frac{Px}{2}\right)^2dx = \frac{P^2L^3}{96EI} \quad \leftarrow$$

(b) From Table G-2, Case 4:

$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \quad \left(0 \le x \le \frac{L}{2}\right)$$

$$\frac{dv}{dx} = -\frac{P}{16EI}(L^2 - 4x^2) \qquad \frac{d^2v}{dx^2} = \frac{Px}{2EI}$$

$$U = 2\int_0^{L/2}\frac{EI}{2}\left(\frac{d^2v}{dx^2}\right)^2dx = EI\int_0^{L/2}\left(\frac{Px}{2EI}\right)^2dx$$

$$= \frac{P^2L^3}{96EI} \quad \leftarrow$$

(c) $\delta = \frac{2U}{P} = \frac{PL^3}{48EI} \quad \leftarrow$

9.8-2 Simple beam with uniform load

GIVEN: L, b, h, σ_{max} FIND: U

$$M = \frac{qLx}{2} - \frac{qx^2}{2} \qquad U = \int_0^L \frac{M^2dx}{2EI} = \frac{q^2L^5}{240EI} \qquad (1)$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{M_{max}h}{2I} \qquad M_{max} = \frac{qL^2}{8}$$

$$\sigma_{max} = \frac{qL^2h}{16I} \qquad q = \frac{16I\sigma_{max}}{L^2h}$$

Substitute q into Eq.(1): $U = \frac{16I\sigma_{max}^2L}{15h^2E}$

Substitute $I = \frac{bh^3}{12}$: $\quad U = \frac{4bhL\sigma_{max}^2}{45E} \quad \leftarrow$

9.8-3 Cantilever beam with a uniform load

(a) Measure x from the right-hand end of the beam

$$M = -\frac{qx^2}{2} \qquad U = \int_0^L \frac{M^2dx}{2EI} = \frac{1}{2EI}\int_0^L\left(-\frac{qx^2}{2}\right)^2dx$$

$$= \frac{q^2L^5}{40EI} \quad \leftarrow$$

(b) From Table G-1, Case 1.

Measure x from the left-hand end of the beam

$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2)$$

$$\frac{dv}{dx} = -\frac{q}{6EI}(3L^2x - 3Lx^2 + x^3)$$

$$\frac{d^2v}{dx^2} = -\frac{q}{2EI}(L^2 - 2Lx + x^2)$$

$$U = \int_0^L \frac{EI}{2}\left(\frac{d^2v}{dx^2}\right)^2dx = \frac{EI}{2}\int_0^L\left(-\frac{q}{2EI}\right)^2(L^2 - 2Lx + x^2)^2dx$$

$$= \frac{q^2L^5}{40EI} \quad \leftarrow$$

(c) W 8×15 $\quad L = 6\text{ ft} = 72$ in.

$\sigma_{max} = 16{,}000$ psi $\quad E = 30\times10^6$ psi

$I = 48.0$ in.^{4} $\quad c = \frac{d}{2} = \frac{8.11}{2} = 4.055$ in.

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{qL^2c}{2I} \qquad q = \frac{2I\sigma_{max}}{L^2c}$$

$$U = \left(\frac{2I\sigma_{max}}{L^2c}\right)^2\left(\frac{L^5}{40EI}\right) = \frac{IL\sigma_{max}^2}{10Ec^2}$$

$$U = \frac{(48.0\text{ in.}^4)(72\text{ in.})(16{,}000\text{ psi})^2}{10(30\times10^6\text{ psi})(4.055\text{ in.})^2} = 179\text{ in.-lb} \quad \leftarrow$$

9.8-4 Simple beam

GIVEN: L, EI, δ (at midpoint)

FIND: U

(a) Deflection curve is a parabola

$$v = \frac{4\delta x}{L^2}(L - x) \qquad \frac{dv}{dx} = \frac{4\delta}{L^2}(L - 2x)$$

$$\frac{d^2v}{dx^2} = -\frac{8\delta}{L^2}$$

$$U = \int_0^L \frac{EI}{2}\left(\frac{d^2v}{dx^2}\right)^2dx = \frac{EI}{2}\int_0^L\left(-\frac{8\delta}{L^2}\right)^2dx = \frac{32EI\delta^2}{L^3} \quad \leftarrow$$

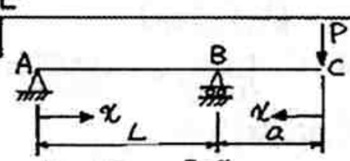
(b) Sine curve

$$v = \delta\sin\frac{\pi x}{L} \qquad \frac{dv}{dx} = \frac{\pi\delta}{L}\cos\frac{\pi x}{L} \qquad \frac{d^2v}{dx^2} = -\frac{\pi^2\delta}{L^2}\sin\frac{\pi x}{L}$$

$$U = \int_0^L \frac{EI}{2}\left(\frac{d^2v}{dx^2}\right)^2dx = \frac{EI}{2}\int_0^L\left(-\frac{\pi^2\delta}{L^2}\right)^2\sin^2\frac{\pi x}{L}\,dx$$

$$= \frac{\pi^4EI\delta^2}{4L^3} \quad \leftarrow$$

9.8-5



(a) From A to B: $M = -\frac{Pax}{L}$

$$U_{AB} = \int\frac{M^2dx}{2EI} = \int_0^L\frac{1}{2EI}\left(-\frac{Pax}{L}\right)^2dx = \frac{P^2a^2L}{6EI}$$

From B to C: $M = -Px$

$$U_{BC} = \int_0^a \frac{1}{2EI}(-Px)^2dx = \frac{P^2a^3}{6EI}$$

$$U = U_{AB} + U_{BC} = \frac{P^2a^2}{6EI}(L + a) \quad \leftarrow$$

CONT.

9.8-5 CONT.

(b) $\delta_C = \dfrac{2U}{P} = \dfrac{Pa^2}{3EI}(L+a)$

(c) $L = 8\text{ ft} = 96\text{ in.}$ $\quad a = 3\text{ ft} = 36\text{ in.}$

W10×12 $\quad E = 29\times10^6$ psi $\quad \sigma_{max} = 12{,}000$ psi

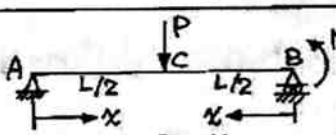
$I = 53.8\text{ in.}^4 \quad c = \dfrac{d}{2} = \dfrac{9.87}{2} = 4.935\text{ in.}$

$\sigma_{max} = \dfrac{Mc}{I} = \dfrac{Pac}{I} \qquad P = \dfrac{\sigma_{max} I}{ac}$

$U = \dfrac{P^2a^2(L+a)}{6EI} = \dfrac{\sigma_{max}^2 I(L+a)}{6c^2E} = 241\text{ in.-lb}$

$\delta_C = \dfrac{Pa^2(L+a)}{3EI} = \dfrac{\sigma_{max} a(L+a)}{3cE} = 0.133\text{ in.}$

9.8-6



From A to C $\quad M = \left(\dfrac{P}{2} + \dfrac{M_0}{L}\right)x$

$U_{AC} = \int\dfrac{M^2dx}{2EI} = \dfrac{1}{2EI}\int_0^{L/2}\left(\dfrac{P}{2}+\dfrac{M_0}{L}\right)^2x^2dx$

$= \dfrac{L}{192EI}(P^2L^2 + 4PLM_0 + 4M_0^2)$

From C to B $\quad M = \left(\dfrac{P}{2} - \dfrac{M_0}{L}\right)x + M_0$

$U_{CB} = \int\dfrac{M^2dx}{2EI} = \dfrac{1}{2EI}\int_0^{L/2}\left[\left(\dfrac{P}{2}-\dfrac{M_0}{L}\right)x + M_0\right]^2dx$

$= \dfrac{L}{192EI}(P^2L^2 + 8PLM_0 + 28M_0^2)$

Entire beam

$U = U_{AC} + U_{CB} = \dfrac{L}{96EI}(P^2L^2 + 6PLM_0 + 16M_0^2)$

$= \dfrac{P^2L^3}{96EI} + \dfrac{PM_0L^2}{16EI} + \dfrac{M_0^2L}{6EI}$

9.8-7 Frame with beam ACB and strut CD

Part CB of the beam: $M = Px$

$U_{CB} = \int\dfrac{M^2dx}{2EI} = \dfrac{1}{2EI}\int_0^L(Px)^2dx = \dfrac{P^2L^3}{6EI}$

Entire beam: $U_{ACB} = 2U_{CB} = \dfrac{P^2L^3}{3EI}$

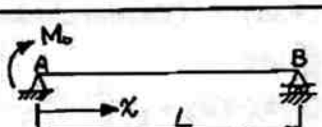
Strut CD: Axial force $F = 2\sqrt{2}P \qquad L_{CD} = L\sqrt{2}$

$U_{CD} = \dfrac{F^2L_{CD}}{2EA} = \dfrac{1}{2EA}(2\sqrt{2}P)^2(L\sqrt{2}) = \dfrac{4\sqrt{2}P^2L}{EA}$

Frame: $U = U_{ACB} + U_{CD} = \dfrac{P^2L^3}{3EI} + \dfrac{4\sqrt{2}P^2L}{EA}$

$\delta_B = \dfrac{2U}{P} = \dfrac{2PL^3}{3EI} + \dfrac{8\sqrt{2}PL}{EA}$

9.9-1

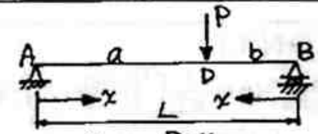


$M = -\dfrac{M_0x}{L}$

$U = \int\dfrac{M^2dx}{2EI} = \dfrac{1}{2EI}\int_0^L\left(-\dfrac{M_0x}{L}\right)^2dx = \dfrac{M_0^2L}{6EI}$

$\theta_A = \dfrac{dU}{dM_0} = \dfrac{M_0L}{3EI}$ (clockwise)

9.9-2



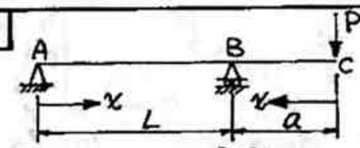
$M_{AD} = \dfrac{Pbx}{L} \qquad M_{DB} = \dfrac{Pax}{L}$

$U = \int\dfrac{M^2dx}{2EI} = \dfrac{1}{2EI}\int_0^a\left(\dfrac{Pbx}{L}\right)^2dx + \dfrac{1}{2EI}\int_0^b\left(\dfrac{Pax}{L}\right)^2dx$

$= \dfrac{P^2a^2b^2}{6LEI}$

$\delta_D = \dfrac{dU}{dP} = \dfrac{Pa^2b^2}{3LEI}$

9.9-3



Segment AB: $\quad M = -\dfrac{Pax}{L}$

$U_{AB} = \int\dfrac{M^2dx}{2EI} = \int_0^L\dfrac{1}{2EI}\left(-\dfrac{Pax}{L}\right)^2dx = \dfrac{P^2a^2L}{6EI}$

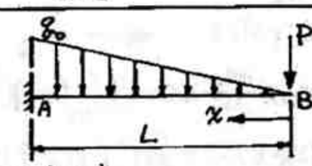
Segment BC: $\quad M = -Px$

$U_{BC} = \int_0^a\dfrac{1}{2EI}(-Px)^2dx = \dfrac{P^2a^3}{6EI}$

$U = U_{AB} + U_{BC} = \dfrac{P^2a^2}{6EI}(L+a)$

$\delta_C = \dfrac{dU}{dP} = \dfrac{Pa^2(L+a)}{3EI}$

9.9-4



P = fictitious load

$M = -Px - \dfrac{q_0x^3}{6L}$

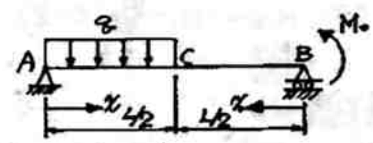
$U = \int\dfrac{M^2dx}{2EI} = \dfrac{1}{2EI}\int_0^L\left(-Px - \dfrac{q_0x^3}{6L}\right)^2dx$

$= \dfrac{P^2L^3}{6EI} + \dfrac{Pq_0L^4}{30EI} + \dfrac{q_0^2L^5}{42EI}$

$\delta_B = \dfrac{\partial U}{\partial P} = \dfrac{PL^3}{3EI} + \dfrac{q_0L^4}{30EI}$

Set $P = 0$: $\quad \delta_B = \dfrac{q_0L^4}{30EI}$

9.9-5



M_0 = fictitious moment

$R_A = \dfrac{3qL}{8} + \dfrac{M_0}{L} \qquad R_B = \dfrac{qL}{8} - \dfrac{M_0}{L}$

Segment AC:

$M_{AC} = R_Ax - \dfrac{qx^2}{2} = \left(\dfrac{3qL}{8} + \dfrac{M_0}{L}\right)x - \dfrac{qx^2}{2} \quad (0 \le x \le \frac{L}{2})$

$\dfrac{\partial M_{AC}}{\partial M_0} = \dfrac{x}{L}$

Segment CB: $\quad M_{CB} = R_Bx + M_0 = \left(\dfrac{qL}{8} - \dfrac{M_0}{L}\right)x + M_0$
$(0 \le x \le L/2)$

$\dfrac{\partial M_{CB}}{\partial M_0} = -\dfrac{x}{L} + 1$

CONT.

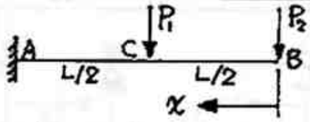
9.9-5 CONT.

$$\theta_B = \int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial M_0}\right)dx = \frac{1}{EI}\int_0^{L/2}\left[\left(\frac{3qL}{8}+\frac{M_0}{L}\right)x-\frac{qx^2}{2}\right]\left[\frac{x}{L}\right]dx$$
$$+\frac{1}{EI}\int_0^{L/2}\left[\left(\frac{qL}{8}-\frac{M_0}{L}\right)x+M_0\right]\left[1-\frac{x}{L}\right]dx$$

Set $M_0=0$:

$$\theta_B = \frac{1}{EI}\int_0^{L/2}\left(\frac{3qLx}{8}-\frac{qx^2}{2}\right)\left(\frac{x}{L}\right)dx+\frac{1}{EI}\int_0^{L/2}\left(\frac{qLx}{8}\right)\left(1-\frac{x}{L}\right)dx$$
$$=\frac{qL^3}{128EI}+\frac{qL^3}{96EI}=\frac{7qL^3}{384EI}$$ (Counterclockwise)

9.9-6



Segment CB: $M_{CB}=-P_2x \quad (0\le x\le \frac{L}{2})$

$$\frac{\partial M_{CB}}{\partial P_1}=0 \qquad \frac{\partial M_{CB}}{\partial P_2}=-x$$

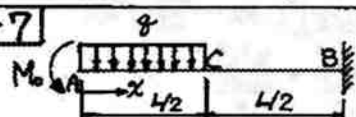
Segment AC: $M_{AC}=-P_1(x-\frac{L}{2})-P_2x \quad (\frac{L}{2}\le x\le L)$

$$\frac{\partial M_{AC}}{\partial P_1}=\frac{L}{2}-x \qquad \frac{\partial M_{AC}}{\partial P_2}=-x$$

$$\delta_C=\frac{1}{EI}\int_0^{L/2}(M_{CB})\left(\frac{\partial M_{CB}}{\partial P_1}\right)dx+\frac{1}{EI}\int_{L/2}^{L}(M_{AC})\left(\frac{\partial M_{AC}}{\partial P_1}\right)dx$$
$$=0+\frac{1}{EI}\int_{L/2}^{L}\left[-P_1\left(x-\frac{L}{2}\right)-P_2x\right]\left(\frac{L}{2}-x\right)dx$$
$$=\frac{L^3}{48EI}(2P_1+5P_2)$$

$$\delta_B=\frac{1}{EI}\int_0^{L/2}(M_{CB})\left(\frac{\partial M_{CB}}{\partial P_2}\right)dx+\frac{1}{EI}\int_{L/2}^{L}(M_{AC})\left(\frac{\partial M_{AC}}{\partial P_2}\right)dx$$
$$=\frac{1}{EI}\int_0^{L/2}(-P_2x)(-x)dx+\frac{1}{EI}\int_{L/2}^{L}\left[-P_1\left(x-\frac{L}{2}\right)-P_2x\right](-x)dx$$
$$=\frac{P_2L^3}{24EI}+\frac{L^3}{48EI}(5P_1+14P_2)=\frac{L^3}{48EI}(5P_1+16P_2)$$

9.9-7



M_0 = fictitious moment

Segment AC: $M_{AC}=-M_0-\frac{qx^2}{2} \quad (0\le x\le\frac{L}{2})$

$$\frac{\partial M_{AC}}{\partial M_0}=-1$$

Segment CB: $M_{CB}=-M_0-\frac{qL}{2}\left(x-\frac{L}{4}\right) \quad (\frac{L}{2}\le x\le L)$

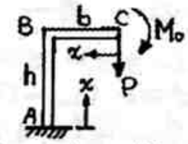
$$\frac{\partial M_{CB}}{\partial M_0}=-1$$

$$\theta_A=\int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial M_0}\right)dx$$
$$=\frac{1}{EI}\int_0^{L/2}\left(-M_0-\frac{qx^2}{2}\right)(-1)dx+\frac{1}{EI}\int_{L/2}^{L}\left[-M_0-\frac{qL}{2}\left(x-\frac{L}{4}\right)\right](-1)dx$$

Set $M_0=0$:

$$\theta_A=\frac{1}{EI}\int_0^{L/2}\frac{qx^2}{2}dx+\frac{1}{EI}\int_{L/2}^{L}\left(\frac{qL}{2}\right)\left(x-\frac{L}{4}\right)dx$$
$$=\frac{qL^3}{48EI}+\frac{qL^3}{8EI}$$
$$=\frac{7qL^3}{48EI}$$ (Counterclockwise)

9.9-8



M_0 = fictitious moment

Member AB: $M_{AB}=Pb+M_0 \quad (0\le x\le h)$

$$\frac{\partial M_{AB}}{\partial P}=b \qquad \frac{\partial M_{AB}}{\partial M_0}=1$$

Member BC: $M_{BC}=Px+M_0 \quad (0\le x\le b)$

$$\frac{\partial M_{BC}}{\partial P}=x \qquad \frac{\partial M_{BC}}{\partial M_0}=1$$

$$\delta_C=\int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial P}\right)dx$$
$$=\frac{1}{EI}\int_0^h(Pb+M_0)(b)dx+\frac{1}{EI}\int_0^b(Px+M_0)(x)dx$$

Set $M_0=0$:

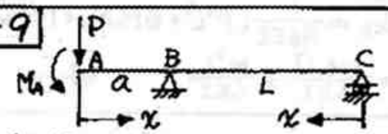
$$\delta_C=\frac{1}{EI}\int_0^h Pb^2dx+\frac{1}{EI}\int_0^b Px^2dx$$
$$=\frac{Pb^2}{3EI}(3h+b)$$ (Downward)

$$\theta_C=\int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial M_0}\right)dx$$
$$=\frac{1}{EI}\int_0^h(Pb+M_0)(1)dx+\frac{1}{EI}\int_0^b(Px+M_0)(1)dx$$

Set $M_0=0$:

$$\theta_C=\frac{1}{EI}\int_0^h Pb\,dx+\frac{1}{EI}\int_0^b Px\,dx$$
$$=\frac{Pb}{2EI}(2h+b)$$ (Clockwise)

9.9-9



P = fictitious load

Segment AB: $M_{AB}=-M_A-Px \quad (0\le x\le a)$

$$\frac{\partial M_{AB}}{\partial M_A}=-1 \qquad \frac{\partial M_{AB}}{\partial P}=-x$$

Segment BC: $M_{BC}=-\frac{M_Ax}{L}-\frac{Pax}{L} \quad (0\le x\le L)$

$$\frac{\partial M_{BC}}{\partial M_A}=-\frac{x}{L} \qquad \frac{\partial M_{BC}}{\partial P}=-\frac{ax}{L}$$

$$\theta_A=\int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial M_A}\right)dx$$
$$=\frac{1}{EI}\int_0^a(-M_A-Px)(-1)dx+\frac{1}{EI}\int_0^L\left(-\frac{M_Ax}{L}-\frac{Pax}{L}\right)\left(-\frac{x}{L}\right)dx$$

Set $P=0$:

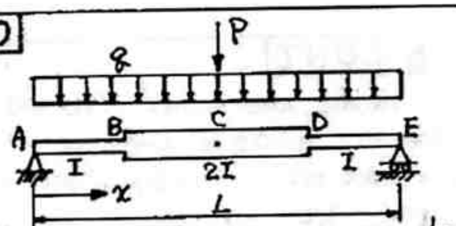
$$\theta_A=\frac{1}{EI}\int_0^a M_Adx+\frac{1}{EI}\int_0^L\left(\frac{M_Ax}{L}\right)\left(\frac{x}{L}\right)dx$$
$$=\frac{M_A}{3EI}(L+3a)$$ (Counterclockwise)

$$\delta_A=\int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial P}\right)dx$$
$$=\frac{1}{EI}\int_0^a(-M_A-Px)(-x)dx+\frac{1}{EI}\int_0^L\left(-\frac{M_Ax}{L}-\frac{Pax}{L}\right)\left(-\frac{ax}{L}\right)dx$$

Set $P=0$:

$$\delta_A=\frac{1}{EI}\int_0^a M_Ax\,dx+\frac{1}{EI}\int_0^L\left(\frac{M_Ax}{L}\right)\left(\frac{ax}{L}\right)dx$$
$$=\frac{M_Aa}{6EI}(2L+3a)$$ (Downward)

9.9-10



P = fictitious load $\quad AB = BC = CD = DE = \frac{L}{4}$

$$M = \frac{qLx}{2} - \frac{qx^2}{2} + \frac{Px}{2} \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\frac{\partial M}{\partial P} = \frac{x}{2}$$

Integrate from A to C and multiply by 2.

Set $P = 0$

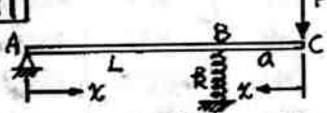
$$\delta_C = \int \left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial P}\right) dx$$

$$= 2\left(\frac{1}{EI}\right)\int_0^{L/4}\left(\frac{qLx}{2} - \frac{qx^2}{2}\right)\left(\frac{x}{2}\right)dx + 2\left(\frac{1}{2EI}\right)\int_{L/4}^{L/2}\left(\frac{qLx}{2} - \frac{qx^2}{2}\right)\left(\frac{x}{2}\right)dx$$

$$= \frac{13qL^4}{6{,}144EI} + \frac{67qL^4}{12{,}288EI}$$

$$\delta_C = \frac{31qL^4}{4096EI} \quad \text{(Downward)} \quad \longleftarrow$$

9.9-11



$$R_A = -\frac{Pa}{L}$$

$$R_B = \frac{P}{L}(L+a)$$

Segment AB: $M_{AB} = -\frac{Pax}{L} \qquad \frac{dM_{AB}}{dP} = -\frac{ax}{L} \qquad (0 \le x \le L)$

Segment BC: $M_{BC} = -Px \qquad \frac{dM_{BC}}{dP} = -x \qquad (0 \le x \le a)$

Strain energy of the spring:

$$U_S = \frac{R_B^2}{2k} = \frac{P^2(L+a)^2}{2kL^2} \qquad \text{(Eq. 2-32a)}$$

Strain energy of the beam:

$$U_B = \int \frac{M^2 dx}{2EI} \qquad \text{(Eq. 9-80a)}$$

$$\delta_C = \frac{dU}{dP} = \frac{d}{dP}\int \frac{M^2 dx}{2EI} + \frac{d}{dP}\left[\frac{P^2(L+a)^2}{2kL^2}\right]$$

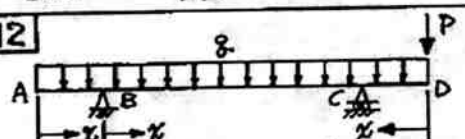
$$= \int \left(\frac{M}{EI}\right)\left(\frac{dM}{dP}\right)dx + \frac{P(L+a)^2}{kL^2}$$

$$= \frac{1}{EI}\int_0^L \left(-\frac{Pax}{L}\right)\left(-\frac{ax}{L}\right)dx + \frac{1}{EI}\int_0^a (-Px)(-x)dx + \frac{P(L+a)^2}{kL^2}$$

$$= \frac{Pa^2L}{3EI} + \frac{Pa^3}{3EI} + \frac{P(L+a)^2}{kL^2}$$

$$\delta_C = \frac{Pa^2(L+a)}{3EI} + \frac{P(L+a)^2}{kL^2} \quad \longleftarrow$$

9.9-12



P = fictitious load $\quad AB = CD = \frac{L}{4} \quad BC = L$

$$R_B = \frac{3qL}{4} - \frac{P}{4}$$

Segment AB: $M_{AB} = -\frac{qx^2}{2} \qquad \frac{\partial M_{AB}}{\partial P} = 0 \qquad \left(0 \le x \le \frac{L}{4}\right)$

Segment BC: $M_{BC} = -\frac{q}{2}\left(x + \frac{L}{4}\right)^2 + \left(\frac{3qL}{4} - \frac{P}{4}\right)x$

$$\frac{\partial M_{BC}}{\partial P} = -\frac{x}{4} \qquad (0 \le x \le L)$$

CONT.

9.9-12 CONT.

Segment CD: $M_{CD} = -\frac{qx^2}{2} - Px$

$$\frac{\partial M_{CD}}{\partial P} = -x \qquad \left(0 \le x \le \frac{L}{4}\right)$$

Set $P = 0$

$$\delta_D = \int \left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial P}\right)dx$$

$$= \frac{1}{EI}\int_0^{L/4}\left(-\frac{qx^2}{2}\right)(0)dx$$

$$+ \frac{1}{EI}\int_0^L \left[-\frac{q}{2}\left(x + \frac{L}{4}\right)^2 + \frac{3qL}{4}x\right]\left[-\frac{x}{4}\right]dx$$

$$+ \frac{1}{EI}\int_0^{L/4}\left(-\frac{qx^2}{2}\right)(-x)dx$$

$$= 0 - \frac{5qL^4}{768EI} + \frac{qL^4}{2048EI} = -\frac{37qL^4}{6144EI}$$

$$\therefore \delta_D = \frac{37qL^4}{6144EI} \quad \text{(Upward)} \quad \longleftarrow$$

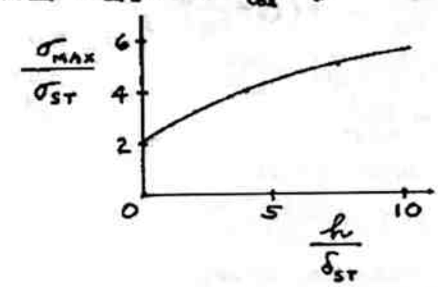
9.10-1 Weight W dropping onto a simple beam

Eq. (9-94): $\delta_{max} = \delta_{st} + (\delta_{st}^2 + 2h\,\delta_{st})^{1/2}$

For a linearly elastic beam, the stress σ is proportional to the deflection δ.

$$\therefore \frac{\sigma_{max}}{\sigma_{st}} = \frac{\delta_{max}}{\delta_{st}} = 1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}$$

$$\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right] \quad \longleftarrow$$



9.10-2 Weight W dropping onto a simple beam

Height h is very large.

Eq. (9-95): $\delta_{max} = \sqrt{2h\,\delta_{st}}$

For a linearly elastic beam, the stress σ is proportional to the deflection δ.

$$\therefore \frac{\sigma_{max}}{\sigma_{st}} = \frac{\delta_{max}}{\delta_{st}} = \sqrt{\frac{2h}{\delta_{st}}}$$

$$\sigma_{max} = \sqrt{\frac{2h\sigma_{st}^2}{\delta_{st}}} \qquad (1)$$

$$\sigma_{st} = \frac{M}{S} = \frac{WL}{4S} \qquad \delta_{st} = \frac{WL^3}{48EI}$$

$$\frac{\sigma_{st}^2}{\delta_{st}} = \frac{3WEI}{S^2L} \qquad (2)$$

For a rectangular beam, $\frac{I}{S^2} = \frac{3}{A} \qquad (3)$

Substitute (2) and (3) into (1):

$$\sigma_{max} = \sqrt{\frac{18WEh}{AL}} \quad \longleftarrow$$

9.10-3 Cantilever beam

$L = 6 \text{ ft} = 72 \text{ in.}$ $W = 1500 \text{ lb}$ $h = 0.25 \text{ in.}$

$E = 30 \times 10^6 \text{ psi}$ W8×21 $I = 75.3 \text{ in.}^4$ $S = 18.2 \text{ in.}^3$

$\delta_{st} = \frac{WL^3}{3EI} = 0.08261 \text{ in.}$

Eq. (9-94):

$\delta_{max} = \delta_{st} + (\delta_{st}^2 + 2h\delta_{st})^{1/2} = 0.302 \text{ in.}$ ←

Relationships between σ_{max} and δ_{max}:

$\sigma_{max} = \frac{M_{max}}{S} = \frac{PL}{S}$ $\delta_{max} = \frac{PL^3}{3EI}$

$\therefore \sigma_{max} = \frac{3EI\delta_{max}}{SL^2} = 21{,}700 \text{ psi}$ ←

Note: Eq. (9-94) is valid for any type of linearly elastic structure, including a cantilever beam

9.10-4 Simple beam with square cross section (d×d)

$I = \frac{d^4}{12}$ $S = \frac{d^3}{6}$

$\delta_{max} = \delta_{st}[1 + (1 + \frac{2h}{\delta_{st}})^{1/2}]$ (From Eq. 9-94)

For a linearly elastic beam, the stress σ is proportional to the deflection δ.

$\therefore \frac{\sigma_{max}}{\sigma_{st}} = \frac{\delta_{max}}{\delta_{st}} = 1 + (1 + \frac{2h}{\delta_{st}})^{1/2}$ (1)

$\sigma_{st} = \frac{M}{S} = (\frac{WL}{4})(\frac{6}{d^3}) = \frac{3WL}{2d^3}$

$\delta_{st} = \frac{WL^3}{48EI} = \frac{WL^3}{4Ed^4}$

Substitute into Eq. (1):

$\frac{2\sigma_{max} d^3}{3WL} = 1 + (1 + \frac{8hEd^4}{WL^3})^{1/2}$

Substitute numerical values:

$\sigma_{max} = \sigma_{allow} = 10 \text{ MPa}$ $W = 20 \text{ kN}$ $L = 3 \text{ m}$

$h = 1.0 \text{ mm}$ $E = 12 \text{ GPa}$

$\frac{1000}{9} d^3 = 1 + (1 + \frac{1600}{9} d^4)^{1/2}$

After rearranging and squaring:

$2500 d^3 - 36d - 45 = 0$ (d = meters)

Solve numerically: $d = 0.2804 \text{ m} = 280.4 \text{ mm}$

For minimum value, round upward.

$\therefore d = 281 \text{ mm}$ ←

9.10-5 Simple beam of wide-flange shape

Eq. (9-94): $\delta_{max} = \delta_{st} + (\delta_{st}^2 + 2h\delta_{st})^{1/2}$

or $\frac{\delta_{max}}{\delta_{st}} = 1 + (1 + \frac{2h}{\delta_{st}})^{1/2}$

For a linearly elastic beam, the stress σ is proportional to the deflection δ.

$\therefore \frac{\sigma_{max}}{\sigma_{st}} = 1 + (1 + \frac{2h}{\delta_{st}})^{1/2}$ (1)

CONT.

9.10-5 CONT.

Units: Pounds and inches (lb and in.)

$W = 4000 \text{ lb}$ $h = 0.5 \text{ in.}$ $L = 120 \text{ in.}$

$\sigma_{allow} = 18{,}000 \text{ psi}$ $E = 30 \times 10^6 \text{ psi}$

$\sigma_{st} = \frac{M_{st}}{S} = \frac{WL}{4S} = \frac{120{,}000}{S}$ ($S = \text{in.}^3$)

$\delta_{st} = \frac{WL^3}{48EI} = \frac{24}{5I}$ ($I = \text{in.}^4$)

$\frac{\sigma_{max}}{\sigma_{st}} = \frac{\sigma_{allow}}{\sigma_{st}} = \frac{18{,}000S}{120{,}000} = \frac{3S}{20}$ (2)

$\frac{2h}{\delta_{st}} = \frac{2(0.5)(5I)}{24} = \frac{5I}{24}$ (3)

Substitute (2) and (3) into (1):

$\frac{3S}{20} = 1 + (1 + \frac{5I}{24})^{1/2}$

$\therefore$ Required $S = \frac{20}{3}[1 + (1 + \frac{5I}{24})^{1/2}]$

Trial beam	Actual I	Actual S	Required S
W8×35	127	31.2	416 (NG)
W10×45	248	49.1	550 (NG)
W10×60	341	66.7	633 (OK)
W12×50	394	64.7	674 (NG)
W14×53	541	77.8	778 (OK)
W16×31	375	47.2	660 (NG)

Lightest beam is W14×53 ←

9.10-6 Overhanging beam

From Prob. 9.8-5 or Prob. 9.9-3:

$\delta_C = \frac{Pa^2(L+a)}{3EI}$ $L = L_{AB}$ $a = L_{BC} = 2L_{AB}$

$\therefore \delta_C = \frac{4PL_{AB}^3}{EI}$ $\delta_{st} = \frac{4WL_{AB}^3}{EI}$ (1)

Eq. (9-94) is valid for any type of linearly elastic structure, including an overhanging beam:

$\delta_{max} = \delta_{st} + (\delta_{st}^2 + 2h\delta_{st})^{1/2}$

or

$\frac{\delta_{max}}{\delta_{st}} = 1 + (1 + \frac{2h}{\delta_{st}})^{1/2}$

Stress σ is proportional to deflection δ.

$\therefore \frac{\sigma_{max}}{\sigma_{st}} = 1 + (1 + \frac{2h}{\delta_{st}})^{1/2}$ (2)

$\sigma_{st} = \frac{M_{st}}{S} = \frac{WL_{BC}}{S} = \frac{2WL_{AB}}{S}$ (3)

Solve Eq. (2) for h: $h = \frac{\delta_{st}}{2}(\frac{\sigma_{max}}{\sigma_{st}})(\frac{\sigma_{max}}{\sigma_{st}} - 2)$

Substitute δ_{st} from Eq. (1), σ_{st} from Eq. (3), and $\sigma_{max} = \sigma_{allow}$:

$h = \frac{2WL_{AB}^3}{EI}(\frac{\sigma_{allow} S}{2WL_{AB}})(\frac{\sigma_{allow} S}{2WL_{AB}} - 2)$ (4)

Substitute numerical values into Eq. (4):

$W = 750 \text{ N}$ $L_{AB} = 1.2 \text{ m}$ $E = 12 \text{ GPa}$

CONT.

9.10-6 CONT.

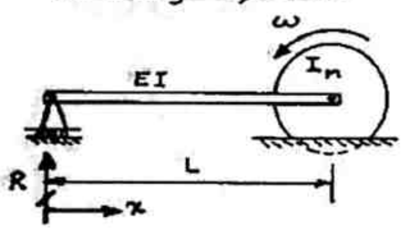
$I = \frac{1}{12}(500\ mm)(40\ mm)^3 = 2.667 \times 10^6\ mm^4$
$= 2.667 \times 10^{-6}\ m^4$

$\sigma_{allow} = 45\ MPa$

$S = \frac{1}{6}(500\ mm)(40\ mm)^2 = 133.3 \times 10^3\ mm^3$
$= 0.1333 \times 10^{-3}\ m^3$

$h = (0.08100\ m)(3.333)(3.333-2) = 0.360\ m$
$= 360\ mm$ ←

9.10-7 Rotating flywheel



Disregard the mass of the beam and all energy losses due to the sudden stopping of the rotating flywheel.

Kinetic energy of flywheel:

$$KE = \frac{1}{2} I_m \omega^2$$

Strain energy of beam:

$$U = \int \frac{M^2 dx}{2EI}$$

$M = Rx \qquad U = \int_0^L \frac{(Rx)^2 dx}{2EI} = \frac{R^2 L^3}{6EI}$

Conservation of energy:

$KE = U \qquad \frac{I_m \omega^2}{2} = \frac{R^2 L^3}{6EI}$

$$R = \sqrt{\frac{3EI\, I_m \omega^2}{L^3}} \quad \leftarrow$$

9.11-1 Simple beam with temperature differential

Eq. (9-100): $v'' = \frac{d^2v}{dx^2} = \frac{\alpha(T_2 - T_1)}{h}$

$v' = \frac{dv}{dx} = \frac{\alpha(T_2-T_1)x}{h} + C_1$

B.C.1 (Symmetry) $v'(\frac{L}{2}) = 0 \quad \therefore C_1 = -\frac{\alpha L(T_2-T_1)}{2h}$

$v = \frac{\alpha(T_2-T_1)x^2}{2h} - \frac{\alpha L(T_2-T_1)x}{2h} + C_2$

B.C.2 $v(0) = 0 \quad \therefore C_2 = 0$

$v = -\frac{\alpha(T_2-T_1)(x)(L-x)}{2h}$ (Positive v is upward) ←

$v' = -\frac{\alpha(T_2-T_1)(L-2x)}{2h}$

$\theta_A = -v'(0) = \frac{\alpha L(T_2-T_1)}{2h}$ ←

(Positive clockwise)

CONT.

9.11-1 CONT.

$\delta_{max} = -v(\frac{L}{2}) = \frac{\alpha L^2(T_2-T_1)}{8h}$ ←

(Positive downward)

9.11-2 Cantilever beam with temperature differential

Eq. (9-100): $v'' = \frac{d^2v}{dx^2} = \frac{\alpha(T_2-T_1)}{h}$

$v' = \frac{dv}{dx} = \frac{\alpha(T_2-T_1)}{h} x + C_1$

B.C.1 $v'(0) = 0 \quad \therefore C_1 = 0$

$v' = \frac{\alpha(T_2-T_1)}{h} x$

$v = \frac{\alpha(T_2-T_1)}{h}(\frac{x^2}{2}) + C_2$

B.C.2 $v(0) = 0 \quad \therefore C_2 = 0$

$v = \frac{\alpha(T_2-T_1)x^2}{2h}$ ←

(Positive v is upward)

$\theta_B = v'(L) = \frac{\alpha L(T_2-T_1)}{h}$ ←

(Positive counterclockwise)

$\delta_B = v(L) = \frac{\alpha L^2(T_2-T_1)}{2h}$ (Positive upward) ←

9.11-3 Overhanging beam with temperature differential

Eq. (9-100): $v'' = \frac{d^2v}{dx^2} = \frac{\alpha(T_2-T_1)}{h}$

(This equation is valid for the entire length of the beam.)

$v' = \frac{\alpha(T_2-T_1)x}{h} + C_1$

$v = \frac{\alpha(T_2-T_1)x^2}{2h} + C_1 x + C_2$

B.C.1 $v(0) = 0 \quad \therefore C_2 = 0$

B.C.2 $v(L) = 0 \quad \therefore C_1 = -\frac{\alpha(T_2-T_1)L}{2h}$

$v = \frac{\alpha(T_2-T_1)}{2h}(x^2 - Lx)$ (Positive v is upward) ←

$v' = \frac{\alpha(T_2-T_1)}{2h}(2x - L)$

$\theta_C = v'(L+a) = \frac{\alpha(T_2-T_1)}{2h}(L+2a)$ ←

(Positive counterclockwise)

$\delta_C = v(L+a) = \frac{\alpha(T_2-T_1)(L+a)(a)}{2h}$ ←

(Positive upward)

9.11-4 Simple beam with temperature differential proportional to distance x

$T_2 - T_1 = T_0 x$

Eq. (9-100): $v'' = \frac{d^2v}{dx^2} = \frac{\alpha(T_2-T_1)}{h} = \frac{\alpha T_0 x}{h}$

CONT.

9.11-4 CONT.

$$v' = \frac{dv}{dx} = \frac{\alpha T_0 x^2}{2h} + C_1$$

$$v = \frac{\alpha T_0 x^3}{6h} + C_1 x + C_2$$

B.C.1 $\quad v(0) = 0 \qquad \therefore C_2 = 0$

B.C.2 $\quad v(L) = 0 \qquad \therefore C_1 = -\frac{\alpha T_0 L^2}{6h}$

$$v = -\frac{\alpha T_0 x}{6h}(L^2 - x^2) \quad \text{(Positive } v \text{ is upward)}$$

$$v' = -\frac{\alpha T_0}{6h}(L^2 - 3x^2)$$

Maximum deflection

$v' = 0 \qquad L^2 - 3x_1^2 = 0 \qquad x_1 = \frac{L}{\sqrt{3}}$

$$v_{max} = v(x_1) = -\frac{\alpha T_0 L^3}{9\sqrt{3}\,h}$$

$$\delta_{max} = -v_{max} = \frac{\alpha T_0 L^3}{9\sqrt{3}\,h} \quad \text{(Positive downward)} \longleftarrow$$

— END OF CHAPTER 9 —

10.3-1 PROPPED CANTILEVER BEAM
M_0 = APPLIED LOAD

SELECT M_A AS THE REDUNDANT REACTION

REACTIONS (FROM EQUILIBRIUM)

$$R_A = \frac{M_A}{L} + \frac{M_0}{L} \quad (1) \qquad R_B = -R_A \quad (2)$$

BENDING MOMENT (FROM EQUILIBRIUM)

$$M = R_A x - M_A = \frac{M_A}{L}(x-L) + \frac{M_0 x}{L} \quad (3)$$

DIFFERENTIAL EQUATIONS

$$EIv'' = M = \frac{M_A}{L}(x-L) + \frac{M_0 x}{L}$$

$$EIv' = \frac{M_A}{L}\left(\frac{x^2}{2} - Lx\right) + \frac{M_0 x^2}{2L} + C_1 \quad (4)$$

B.C. 1 $\quad v'(0) = 0 \quad \therefore C_1 = 0$

$$EIv = \frac{M_A}{L}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) + \frac{M_0 x^3}{6L} + C_2 \quad (5)$$

B.C. 2 $\quad v(0) = 0 \quad \therefore C_2 = 0$

B.C. 3 $\quad v(L) = 0 \quad \therefore M_A = \frac{M_0}{2}$

REACTIONS (SEE EQS. 1 AND 2)

$$M_A = \frac{M_0}{2} \qquad R_A = \frac{3M_0}{2L} \qquad R_B = -\frac{3M_0}{2L} \quad \longleftarrow$$

SHEAR FORCE (FROM EQUILIBRIUM)

$$V = R_A = \frac{3M_0}{2L} \quad \longleftarrow$$

BENDING MOMENT (FROM EQ. 3)

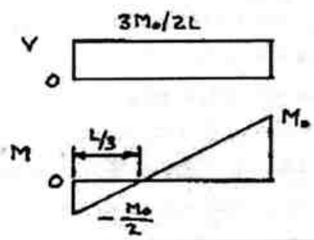
$$M = \frac{M_0}{2L}(3x - L) \quad \longleftarrow$$

SLOPE (FROM EQ. 4)

$$v' = -\frac{M_0 x}{4LEI}(2L - 3x) \quad \longleftarrow$$

DEFLECTION (FROM EQ. 5)

$$v = -\frac{M_0 x^2}{4LEI}(L - x) \quad \longleftarrow$$



10.3-2 FIXED-END BEAM (UNIFORM LOAD)

SELECT M_A AS THE REDUNDANT REACTION

REACTIONS (FROM SYMMETRY AND EQUILIBRIUM)

$$R_A = R_B = \frac{qL}{2} \qquad M_B = M_A$$

BENDING MOMENT (FROM EQUILIBRIUM)

$$M = R_A x - M_A - \frac{qx^2}{2} = -M_A + \frac{q}{2}(Lx - x^2) \quad (1)$$

DIFFERENTIAL EQUATIONS

$$EIv'' = M = -M_A + \frac{q}{2}(Lx - x^2)$$

$$EIv' = -M_A x + \frac{q}{2}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) + C_1 \quad (2)$$

B.C. 1 $\quad v'(0) = 0 \quad \therefore C_1 = 0$

$$EIv = -\frac{M_A x^2}{2} + \frac{q}{2}\left(\frac{Lx^3}{6} - \frac{x^4}{12}\right) + C_2 \quad (3)$$

B.C. 2 $\quad v(0) = 0 \quad \therefore C_2 = 0$

B.C. 3 $\quad v(L) = 0 \quad \therefore M_A = \frac{qL^2}{12}$

CONT.

10.3-2 CONT.

REACTIONS

$$R_A = R_B = \frac{qL}{2} \qquad M_A = M_B = \frac{qL^2}{12} \quad \longleftarrow$$

SHEAR FORCE (FROM EQUILIBRIUM)

$$V = R_A - qx = \frac{q}{2}(L - 2x) \quad \longleftarrow$$

BENDING MOMENT (FROM EQ. 1)

$$M = -\frac{q}{12}(L^2 - 6Lx + 6x^2) \quad \longleftarrow$$

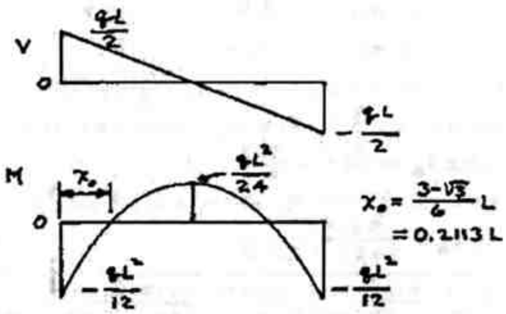
SLOPE (FROM EQ. 2)

$$v' = -\frac{qx}{12EI}(L^2 - 3Lx + 2x^2) \quad \longleftarrow$$

DEFLECTION (FROM EQ. 3)

$$v = -\frac{qx^2}{24EI}(L - x)^2 \quad \longleftarrow$$

$$\delta_{max} = -v\left(\frac{L}{2}\right) = \frac{qL^4}{384EI}$$



10.3-3 CANTILEVER BEAM WITH IMPOSED DISPLACEMENT δ_B

REACTIONS (FROM EQUILIBRIUM)

$$R_A = R_B \quad (1) \qquad M_A = R_B L \quad (2)$$

DIFFERENTIAL EQUATIONS

$$EIv'''' = -q = 0 \quad (3)$$

$$EIv''' = V = C_1 \quad (4)$$

$$EIv'' = M = C_1 x + C_2 \quad (5)$$

$$EIv' = C_1 x^2/2 + C_2 x + C_3 \quad (6)$$

$$EIv = C_1 x^3/6 + C_2 x^2/2 + C_3 x + C_4 \quad (7)$$

B.C. 1 $\quad v(0) = 0 \quad \therefore C_4 = 0$

B.C. 2 $\quad v'(0) = 0 \quad \therefore C_3 = 0$

B.C. 3 $\quad v''(L) = 0 \quad \therefore C_1 L + C_2 = 0 \quad (8)$

B.C. 4 $\quad v(L) = -\delta_B \quad \therefore C_1 L + 3C_2 = -6EI\delta_B/L^2 \quad (9)$

SOLVE EQUATIONS (8) AND (9):

$$C_1 = \frac{3EI\delta_B}{L^3} \qquad C_2 = -\frac{3EI\delta_B}{L^2}$$

SHEAR FORCE (EQ. 4)

$$V = \frac{3EI\delta_B}{L^3} \qquad R_A = V(0) = \frac{3EI\delta_B}{L^3}$$

REACTIONS (EQS. 1 AND 2)

$$R_A = R_B = \frac{3EI\delta_B}{L^3} \qquad M_A = R_B L = \frac{3EI\delta_B}{L^2} \quad \longleftarrow$$

DEFLECTION (FROM EQ. 7):

$$v = -\frac{\delta_B x^2}{2L^3}(3L - x) \quad \longleftarrow$$

SLOPE (FROM EQ. 6):

$$v' = -\frac{3\delta_B x}{2L^3}(2L - x)$$

10.3-4 BEAM WITH SPRING SUPPORT

q = INTENSITY OF UNIFORM LOAD

EQUILIBRIUM
$$R_A = qL - R_B \qquad (1)$$
$$M_A = \frac{qL^2}{2} - R_B L \qquad (2)$$

SPRING
$$R_B = k\delta_B \qquad (3)$$

δ_B = DOWNWARD DISPLACEMENT OF POINT B

BENDING MOMENT (FROM EQUILIBRIUM)
$$M = R_A x - M_A - \frac{qx^2}{2}$$

DIFFERENTIAL EQUATIONS
$$EIv'' = M = R_A x - M_A - \frac{qx^2}{2}$$
$$EIv' = R_A\frac{x^2}{2} - M_A x - \frac{qx^3}{6} + C_1$$
$$EIv = R_A\frac{x^3}{6} - M_A\frac{x^2}{2} - \frac{qx^4}{24} + C_1 x + C_2$$

B.C. 1 $v'(0) = 0 \quad \therefore C_1 = 0$

B.C. 2 $v(0) = 0 \quad \therefore C_2 = 0$

B.C. 3 $v(L) = -\delta_B \quad \therefore -EI\delta_B = \frac{R_A L^3}{6} - \frac{M_A L^2}{2} - \frac{qL^4}{24}$

SUBSTITUTE R_A AND M_A FROM EQS. (1) AND (2):
$$-EI\delta_B = \frac{R_B L^3}{3} - \frac{qL^4}{8}$$

SUBSTITUTE FOR R_B FROM EQ. (3) AND SOLVE:
$$\delta_B = \frac{3qL^4}{24EI + 8kL^3} \longleftarrow$$

10.3-5 PROPPED CANTILEVER BEAM

TRIANGULAR LOAD $q = q_0(L-x)/L$

DIFFERENTIAL EQUATIONS
$$EIv'''' = -q = -\frac{q_0}{L}(L-x) \qquad (1)$$
$$EIv''' = V = -q_0 x + \frac{q_0 x^2}{2L} + C_1 \qquad (2)$$
$$EIv'' = M = -\frac{q_0 x^2}{2} + \frac{q_0 x^3}{6L} + C_1 x + C_2 \qquad (3)$$
$$EIv' = -\frac{q_0 x^3}{6} + \frac{q_0 x^4}{24L} + C_1\frac{x^2}{2} + C_2 x + C_3 \qquad (4)$$
$$EIv = -\frac{q_0 x^4}{24} + \frac{q_0 x^5}{120L} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \qquad (5)$$

B.C. 1 $v''(L) = 0 \quad \therefore C_1 L + C_2 = \frac{q_0 L^2}{3} \qquad (6)$

B.C. 2 $v'(0) = 0 \quad \therefore C_3 = 0$

B.C. 3 $v(0) = 0 \quad \therefore C_4 = 0$

B.C. 4 $v(L) = 0 \quad \therefore C_1 L + 3C_2 = \frac{q_0 L^2}{5} \qquad (7)$

SOLVE EQS. (6) AND (7):
$$C_1 = \frac{2q_0 L}{5} \qquad C_2 = -\frac{q_0 L^2}{15}$$

SHEAR FORCE (EQ. 2)
$$V = \frac{q_0}{10L}(4L^2 - 10Lx + 5x^2)$$

REACTIONS
$$R_A = V(0) = \frac{2q_0 L}{5} \longleftarrow$$
$$R_B = -V(L) = \frac{q_0 L}{10} \longleftarrow$$

FROM EQUILIBRIUM:
$$M_A = \frac{q_0 L^2}{6} - R_B L = \frac{q_0 L^2}{15} \longleftarrow$$

DEFLECTION CURVE (FROM EQ. 5)
$$EIv = -\frac{q_0 x^4}{24} + \frac{q_0 x^5}{120L} + \frac{2q_0 L}{5}\left(\frac{x^3}{6}\right) - \frac{q_0 L^2}{15}\left(\frac{x^2}{2}\right)$$

OR
$$v = -\frac{q_0 x^2}{120LEI}(4L^3 - 8L^2 x + 5Lx^2 - x^3) \longleftarrow$$

10.3-6 PROPPED CANTILEVER BEAM

PARABOLIC LOAD $q = q_0(1 - x^2/L^2)$

DIFFERENTIAL EQUATIONS
$$EIv'''' = -q = -q_0(1 - x^2/L^2) \qquad (1)$$
$$EIv''' = V = -q_0(x - x^3/3L^2) + C_1 \qquad (2)$$
$$EIv'' = M = -q_0\left(\frac{x^2}{2} - \frac{x^4}{12L^2}\right) + C_1 x + C_2 \qquad (3)$$
$$EIv' = -q_0\left(\frac{x^3}{6} - \frac{x^5}{60L^2}\right) + C_1\frac{x^2}{2} + C_2 x + C_3 \qquad (4)$$
$$EIv = -q_0\left(\frac{x^4}{24} - \frac{x^6}{360L^2}\right) + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \qquad (5)$$

B.C. 1 $v''(L) = 0 \quad \therefore C_1 L + C_2 = 5q_0 L^2/12 \qquad (6)$

B.C. 2 $v'(0) = 0 \quad \therefore C_3 = 0$

B.C. 3 $v(0) = 0 \quad \therefore C_4 = 0$

B.C. 4 $v(L) = 0 \quad \therefore C_1 L + 3C_2 = 7q_0 L^2/30 \qquad (7)$

SOLVE EQS. (6) AND (7):
$$C_1 = 61q_0 L/120 \qquad C_2 = -11q_0 L^2/120$$

SHEAR FORCE (EQ. 2)
$$V = \frac{q_0}{120L^2}(61L^3 - 120L^2 x + 40x^3)$$

REACTIONS
$$R_A = V(0) = 61q_0 L/120 \longleftarrow$$
$$R_B = -V(L) = 19q_0 L/120 \longleftarrow$$

FROM EQUILIBRIUM:
$$M_A = \frac{2}{3}(q_0)(L)\left(\frac{3L}{8}\right) - R_B L = \frac{11q_0 L^2}{120} \longleftarrow$$

DEFLECTION CURVE (FROM EQ. 5)
$$v = -\frac{q_0 x^2}{720L^2 EI}(33L^4 - 61L^3 x + 30L^2 x^2 - 2x^4)$$
$$= -\frac{q_0 x^2(L-x)}{720L^2 EI}(33L^3 - 28L^2 x + 2Lx^2 + 2x^3) \longleftarrow$$

10.3-7 FIXED-END BEAM (SINE LOAD)

$q = q_0 \sin \pi x/L$

FROM SYMMETRY: $R_A = R_B \quad M_A = M_B$

DIFFERENTIAL EQUATIONS
$$EIv'''' = -q = -q_0 \sin \pi x/L \qquad (1)$$
$$EIv''' = V = \frac{q_0 L}{\pi}\cos\frac{\pi x}{L} + C_1 \qquad (2)$$
$$EIv'' = M = \frac{q_0 L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2 \qquad (3)$$
$$EIv' = -\frac{q_0 L^3}{\pi^3}\cos\frac{\pi x}{L} + C_1\frac{x^2}{2} + C_2 x + C_3 \qquad (4)$$
$$EIv = -\frac{q_0 L^4}{\pi^4}\sin\frac{\pi x}{L} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \qquad (5)$$

B.C. 1 FROM SYMMETRY, $V(\frac{L}{2}) = 0 \quad \therefore C_1 = 0$

B.C. 2 $v'(0) = 0 \quad \therefore C_3 = q_0 L^3/\pi^3$

B.C. 3 $v'(L) = 0 \quad \therefore C_2 = -2q_0 L^2/\pi^3$

B.C. 4 $v(0) = 0 \quad \therefore C_4 = 0$

SHEAR FORCE (EQ. 2)
$$V = \frac{q_0 L}{\pi}\cos\frac{\pi x}{L} \qquad R_A = V(0) = \frac{q_0 L}{\pi} \longleftarrow$$
$$R_B = R_A = \frac{q_0 L}{\pi} \longleftarrow$$

BENDING MOMENT (EQ. 3)
$$M = \frac{q_0 L^2}{\pi^3}\left(\pi \sin\frac{\pi x}{L} - 2\right)$$
$$M_A = -M(0) = \frac{2q_0 L^2}{\pi^3} \qquad M_B = M_A = \frac{2q_0 L^2}{\pi^3} \longleftarrow$$

DEFLECTION CURVE (FROM EQ. 5)
$$EIv = -\frac{q_0 L^4}{\pi^4}\sin\frac{\pi x}{L} - \frac{q_0 L^2 x^2}{\pi^3} + \frac{q_0 L^3 x}{\pi^3}$$

OR
$$v = -\frac{q_0 L^2}{\pi^4 EI}\left(L^2 \sin\frac{\pi x}{L} + \pi x^2 - \pi L x\right) \longleftarrow$$

10.3-8 FIXED-END BEAM (TRIANGULAR LOAD)

$q = q_0(1 - x/L)$

DIFFERENTIAL EQUATIONS

$$EI\,v'''' = -q = -q_0\left(1 - \frac{x}{L}\right) \qquad (1)$$

$$EI\,v''' = V = -q_0\left(x - \frac{x^2}{2L}\right) + C_1 \qquad (2)$$

$$EI\,v'' = M = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L}\right) + C_1 x + C_2 \qquad (3)$$

$$EI\,v' = -q_0\left(\frac{x^3}{6} - \frac{x^4}{24L}\right) + C_1\frac{x^2}{2} + C_2 x + C_3 \qquad (4)$$

$$EI\,v = -q_0\left(\frac{x^4}{24} - \frac{x^5}{120L}\right) + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \qquad (5)$$

B.C. 1 $\quad v'(0) = 0 \quad \therefore C_3 = 0$

B.C. 2 $\quad v'(L) = 0 \quad \therefore C_1 L + 2C_2 = \frac{q_0 L^2}{4} \qquad (6)$

B.C. 3 $\quad v(0) = 0 \quad \therefore C_4 = 0$

B.C. 4 $\quad v(L) = 0 \quad \therefore C_1 L + 3C_2 = \frac{q_0 L^2}{5} \qquad (7)$

SOLVE EQS. (6) AND (7):

$$C_1 = \frac{7q_0 L}{20} \qquad C_2 = -\frac{q_0 L^2}{20}$$

SHEAR FORCE (EQ. 2)

$$V = \frac{q_0}{20L}\left(7L^2 - 20Lx + 10x^2\right)$$

REACTIONS $\quad R_A = V(0) = \frac{7q_0 L}{20} \longleftarrow$

$$R_B = -V(L) = \frac{3q_0 L}{20} \longleftarrow$$

BENDING MOMENT (EQ. 3)

$$M = -\frac{q_0}{60L}\left(3L^3 - 21L^2 x + 30Lx^2 - 10x^3\right)$$

REACTIONS $\quad M_A = -M(0) = \frac{q_0 L^2}{20} \longleftarrow$

$$M_B = -M(L) = \frac{q_0 L^2}{30} \longleftarrow$$

DEFLECTION CURVE (EQ. 5)

$$v = -\frac{q_0 x^2}{120LEI}\left(3L^3 - 7L^2 x + 5Lx^2 - x^3\right)$$

OR $\quad v = -\frac{q_0 x^2}{120LEI}(L-x)^2(3L-x) \longleftarrow$

10.3-9 FIXED-END BEAM (M_0 = APPLIED LOAD)

BEAM IS SYMMETRIC; LOAD IS ANTISYMMETRIC.
THEREFORE, $R_A = -R_B \quad M_A = -M_B \quad \delta_C = 0$

DIFFERENTIAL EQUATIONS $(0 \le x \le L/2)$

$$EI\,v'' = M = R_A x - M_A \qquad (1)$$

$$EI\,v' = R_A\frac{x^2}{2} - M_A x + C_1 \qquad (2)$$

$$EI\,v = R_A\frac{x^3}{6} - M_A\frac{x^2}{2} + C_1 x + C_2 \qquad (3)$$

B.C. 1 $\quad v'(0) = 0 \quad \therefore C_1 = 0$

B.C. 2 $\quad v(0) = 0 \quad \therefore C_2 = 0$

B.C. 3 $\quad v\left(\frac{L}{2}\right) = 0 \quad \therefore M_A = \frac{R_A L}{6} \quad$ ALSO, $M_B = \frac{-R_A L}{6}$

EQUILIBRIUM (OF ENTIRE BEAM)

$$\Sigma M_B = 0 \quad M_A + M_0 - M_B - R_A L = 0$$

OR, $\quad \frac{R_A L}{6} + M_0 + \frac{R_A L}{6} - R_A L = 0$

$$\therefore R_A = -R_B = \frac{3M_0}{2L} \longleftarrow$$

$M_A = \frac{R_A L}{6} \qquad \therefore M_A = -M_B = \frac{M_0}{4} \longleftarrow$

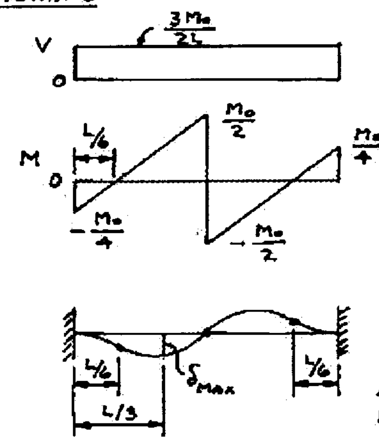
CONT.

10.3-9 CONT.

DEFLECTION CURVE (EQ. 3)

$$v = -\frac{M_0 x^2}{8LEI}(L - 2x) \qquad \left(0 \le x \le \frac{L}{2}\right) \longleftarrow$$

DIAGRAMS



$\delta_{MAX} = \frac{M_0 L^2}{216EI}$

AT POINT OF INFLECTION: $\delta = \delta_{MAX}/2$

10.3-10 PROPPED CANTILEVER BEAM

P = APPLIED LOAD AT $x = L/2$

SELECT R_B AS REDUNDANT REACTION

REACTIONS (FROM EQUILIBRIUM)

$$R_A = P - R_B \quad (1) \qquad M_A = \frac{PL}{2} - R_B L \quad (2)$$

BENDING MOMENTS (FROM EQUILIBRIUM)

$$M = R_A x - M_A = (P - R_B)x - \left(\frac{PL}{2} - R_B L\right) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$M = R_B(L - x) \qquad \left(\frac{L}{2} \le x \le L\right)$$

DIFFERENTIAL EQUATIONS $(0 \le x \le L/2)$

$$EI\,v'' = M = (P - R_B)x - \left(\frac{PL}{2} - R_B L\right) \qquad (3)$$

$$EI\,v' = (P - R_B)\frac{x^2}{2} - \left(\frac{PL}{2} - R_B L\right)x + C_1 \qquad (4)$$

$$EI\,v = (P - R_B)\frac{x^3}{6} - \left(\frac{PL}{2} - R_B L\right)\frac{x^2}{2} + C_1 x + C_2 \qquad (5)$$

B.C. 1 $\quad v'(0) = 0 \quad \therefore C_1 = 0$

B.C. 2 $\quad v(0) = 0 \quad \therefore C_2 = 0$

DIFFERENTIAL EQUATIONS $(L/2 \le x \le L)$

$$EI\,v'' = M = R_B(L - x) \qquad (6)$$

$$EI\,v' = R_B L x - R_B\frac{x^2}{2} + C_3 \qquad (7)$$

$$EI\,v = R_B L\frac{x^2}{2} - R_B\frac{x^3}{6} + C_3 x + C_4 \qquad (8)$$

B.C. 3 $\quad v(L) = 0 \quad \therefore C_3 L + C_4 = -\frac{R_B L^3}{3} \qquad (9)$

B.C. 4 $\quad$ CONTINUITY CONDITION AT POINT C

AT $x = \frac{L}{2}$: $\quad (v')_{LEFT} = (v')_{RIGHT}$

$$(P - R_B)\left(\frac{L^2}{8}\right) - \left(\frac{PL}{2} - R_B L\right)\left(\frac{L}{2}\right) = R_B L\left(\frac{L}{2}\right) - R_B\left(\frac{L^2}{8}\right) + C_3$$

OR $\quad C_3 = -\frac{PL^2}{8} \qquad (10)$

FROM EQ. (9): $\quad C_4 = -\frac{R_B L^3}{3} + \frac{PL^3}{8} \qquad (11)$

CONT.

10.3-10 CONT.

B.C. 5 CONTINUITY CONDITION AT POINT C

AT $x = \frac{L}{2}$: $(v)_{LEFT} = (v)_{RIGHT}$

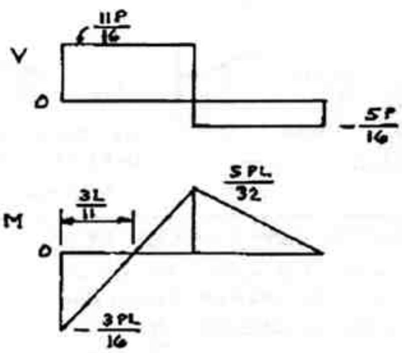
$$(P - R_B)\frac{L^3}{48} - \left(\frac{PL}{2} - R_B L\right)\frac{L^2}{8} =$$

$$R_B L\left(\frac{L^2}{8}\right) - R_B\left(\frac{L^3}{48}\right) - \frac{PL^2}{8}\left(\frac{L}{2}\right) - \frac{R_B L^3}{3} + \frac{PL^3}{8}$$

OR $R_B = \frac{5P}{16}$ ←

FROM EQ. (1): $R_A = P - R_B = \frac{11P}{16}$ ←

FROM EQ. (2): $M_A = \frac{PL}{2} - R_B L = \frac{3PL}{16}$ ←



DEFLECTION CURVE FOR $0 \le x \le L/2$ (FROM EQ. 5)

$$v = -\frac{Px^2}{96EI}(9L - 11x)$$ ←

DEFLECTION CURVE FOR $L/2 \le x \le L$ (FROM EQ. 8)

$$v = -\frac{P}{96EI}(-2L^3 + 12L^2x - 15Lx^2 + 5x^3)$$

$$= -\frac{P}{96EI}(L - x)(-2L^2 + 10Lx - 5x^2)$$ ←

SLOPE IN RIGHT-HAND PART OF THE BEAM

FROM EQ. (7): $v' = -\frac{P}{32EI}(4L^2 - 10Lx + 5x^2)$

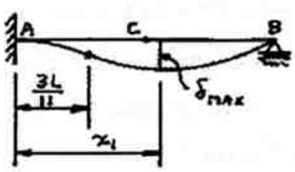
POINT OF ZERO SLOPE:

$5x_1^2 - 10Lx_1 + 4L^2 = 0 \qquad x_1 = \frac{L}{5}(5 - \sqrt{5})$

$= 0.5528\,L$

MAXIMUM DEFLECTION

$$\delta_{max} = -(v)_{x = x_1} = 0.009317\,\frac{PL^3}{EI}$$



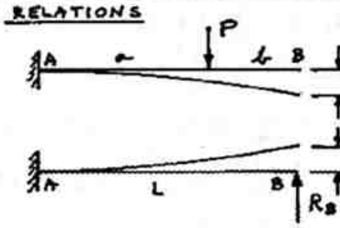
10.4-1 PROPPED CANTILEVER BEAM

SELECT R_B AS REDUNDANT

EQUILIBRIUM

$R_A = P - R_B \qquad M_A = Pa - R_B L$

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$(\delta_B)_1 = \frac{Pa^2}{6EI}(3L - a)$

$(\delta_B)_2 = \frac{R_B L^3}{3EI}$

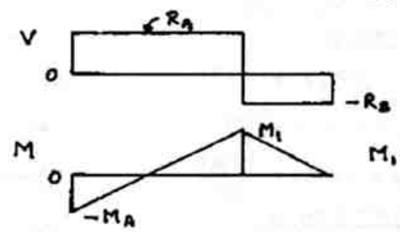
COMPATIBILITY $\quad \delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) - \frac{R_B L^3}{3EI} = 0$$

$$R_B = \frac{Pa^2}{2L^3}(3L - a)$$ ←

OTHER REACTIONS (FROM EQUILIBRIUM)

$$R_A = \frac{Pb}{2L^3}(3L^2 - b^2) \qquad M_A = \frac{Pab}{2L^2}(L + b)$$ ←



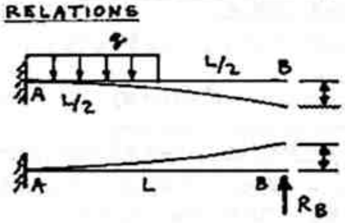
$M_1 = R_B b$

10.4-2 PROPPED CANTILEVER BEAM

SELECT R_B AS REDUNDANT

EQUILIBRIUM $\quad R_A = \frac{qL}{2} - R_B \qquad M_A = \frac{qL^2}{8} - R_B L$

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$(\delta_B)_1 = \frac{7qL^4}{384EI}$

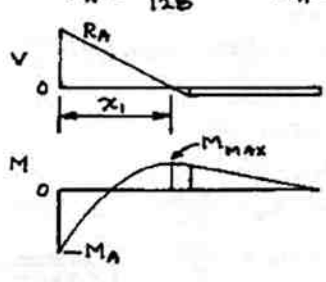
$(\delta_B)_2 = \frac{R_B L^3}{3EI}$

COMPATIBILITY $\quad \delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

SUBSTITUTE FOR $(\delta_B)_1$ AND $(\delta_B)_2$ AND SOLVE FOR R_B: $\quad R_B = \frac{7qL}{128}$ ←

OTHER REACTIONS (FROM EQUILIBRIUM)

$$R_A = \frac{57qL}{128} \qquad M_A = \frac{9qL^2}{128}$$ ←



$x_1 = \frac{57L}{128}$

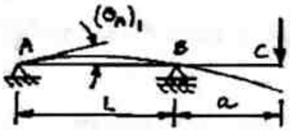
$M_{MAX} = \frac{9457qL^2}{32{,}768}$

10.4-3 BEAM WITH AN OVERHANG

SELECT M_A AS REDUNDANT

EQUILIBRIUM $R_A = \frac{1}{L}(M_A + Pa)$ $R_B = \frac{1}{L}(M_A + PL + Pa)$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$(\theta_A)_1 = \frac{PaL}{6EI}$

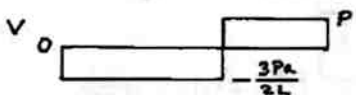
$(\theta_A)_2 = \frac{M_A L}{3EI}$

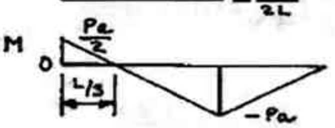
COMPATIBILITY $\theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$

SUBSTITUTE FOR $(\theta_A)_1$ AND $(\theta_A)_2$ AND SOLVE FOR M_A: $M_A = \frac{Pa}{2}$ ←

OTHER REACTIONS (FROM EQUILIBRIUM)

$R_A = \frac{3Pa}{2L}$ $R_B = \frac{P}{2L}(2L + 3a)$ ←





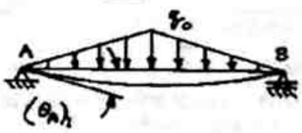
10.4-4 FIXED-END BEAM (TRIANGULAR LOAD)

SELECT M_A AND M_B AS REDUNTANTS

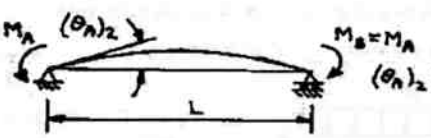
SYMMETRY $M_A = M_B$ $R_A = R_B$

EQUILIBRIUM $R_A = R_B = q_0 L/4$ ←

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$(\theta_A)_1 = \frac{5q_0 L^2}{192EI}$

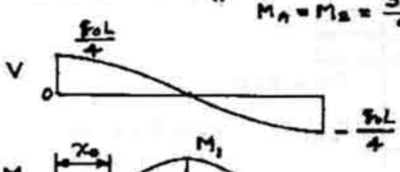


$(\theta_A)_2 = \frac{M_A L}{2EI}$

COMPATIBILITY $\theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$

SUBSTITUTE FOR $(\theta_A)_1$ AND $(\theta_A)_2$ AND SOLVE FOR M_A:

$M_A = M_B = \frac{5q_0 L^2}{96}$ ←



$M_1 = \frac{q_0 L^2}{32}$

$x_0 = 0.2231\,L$

10.4-5 TWO BEAMS SUPPORTING A LOAD P

FOR ALL FOUR REACTIONS TO BE THE SAME, EACH BEAM MUST SUPPORT ONE-HALF OF THE LOAD P.

DEFLECTIONS

$\delta_{AB} = \frac{(P/2)L_{AB}^3}{48EI_{AB}}$ $\delta_{CD} = \frac{(P/2)L_{CD}^3}{48EI_{CD}}$

COMPATIBILITY

$\delta_{AB} = \delta_{CD}$ OR $\frac{L_{AB}^3}{I_{AB}} = \frac{L_{CD}^3}{I_{CD}}$

MOMENT OF INERTIA

$I_{AB} = \frac{1}{12} b t_{AB}^3$ $I_{CD} = \frac{1}{12} b t_{CD}^3$

$\therefore \frac{L_{AB}^3}{t_{AB}^3} = \frac{L_{CD}^3}{t_{CD}^3}$ $\frac{t_{AB}}{t_{CD}} = \frac{L_{AB}}{L_{CD}}$ ←

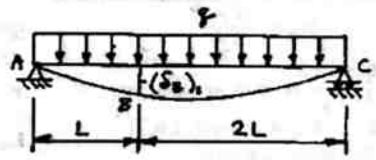
10.4-6 CONTINUOUS BEAM WITH TWO SPANS

SELECT R_B AS REDUNDANT

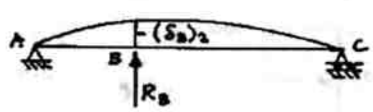
EQUILIBRIUM

$R_A = \frac{3qL}{2} - \frac{2}{3}R_B$ $R_C = \frac{3qL}{2} - \frac{1}{3}R_B$

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$(\delta_B)_1 = \frac{11qL^4}{12EI}$



$(\delta_B)_2 = \frac{4R_B L^3}{9EI}$

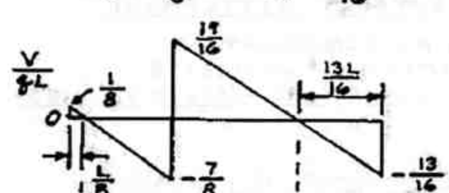
COMPATIBILITY

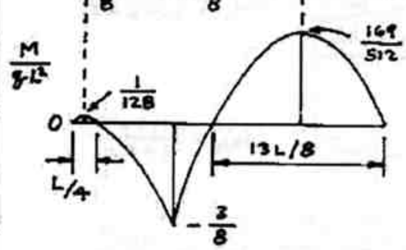
$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

$\frac{11qL^4}{12EI} - \frac{4R_B L^3}{9EI} = 0$ $R_B = \frac{33qL}{16}$ ←

OTHER REACTIONS (FROM EQUILIBRIUM)

$R_A = \frac{qL}{8}$ $R_C = \frac{13qL}{16}$ ←





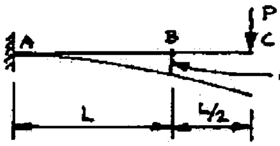
10.4-7 BEAM WITH SPRING SUPPORT

SELECT R_B AS REDUNDANT

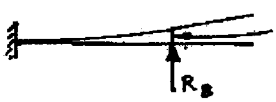
EQUILIBRIUM

$R_A = R_B - P \qquad M_A = R_B L - 3PL/2$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$(\delta_B)_1 = \dfrac{7PL^3}{12EI}$



$(\delta_B)_2 = \dfrac{R_B L^3}{3EI}$

COMPATIBILITY $\quad \delta_B = (\delta_B)_1 - (\delta_B)_2 = \dfrac{R_B}{k}$

BEAM DE: $k = \dfrac{48EI}{L^3}$

$\dfrac{7PL^3}{12EI} - \dfrac{R_B L^3}{3EI} = \dfrac{R_B L^3}{48EI} \qquad R_B = \dfrac{28P}{17}$ ←

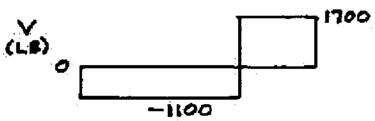
OTHER REACTIONS (FROM EQUILIBRIUM)

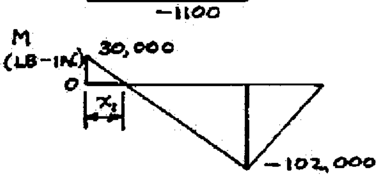
$R_A = \dfrac{11P}{17} \qquad M_A = \dfrac{5PL}{34}$ ←

NUMERICAL VALUES

$P = 1700$ LB $\quad L = 10$ FT $= 120$ IN.

$\left.\begin{array}{l} R_A = 1100 \text{ LB} \quad R_B = 2800 \text{ LB} \\ M_A = 30{,}000 \text{ LB-IN.} \end{array}\right\}$ ←





$x_1 = \dfrac{300}{11}$ IN.

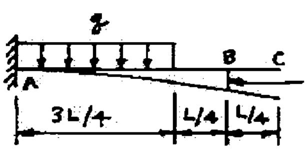
$= 27.27$ IN.

10.4-8 OVERHANGING BEAM WITH SUPPORT SETTLEMENT

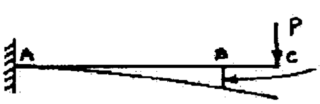
SELECT R_B AS REDUNDANT

Δ = SETTLEMENT OF SUPPORT B

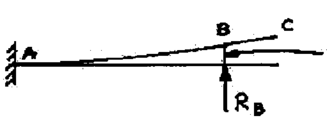
RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$(\delta_B)_1 = \dfrac{117qL^4}{2048EI}$



$(\delta_B)_2 = \dfrac{11PL^3}{24EI}$



$(\delta_B)_3 = \dfrac{R_B L^3}{3EI}$

CONT.

10.4-8 CONT.

COMPATIBILITY $\quad \delta_B = (\delta_B)_1 + (\delta_B)_2 - (\delta_B)_3 = \Delta$

SUBSTITUTE FOR $(\delta_B)_1$, $(\delta_B)_2$, AND $(\delta_B)_3$ AND SOLVE FOR R_B:

$R_B = \dfrac{1}{2048}\left(351qL + 2816P - 6144\dfrac{EI\Delta}{L^3}\right)$ ←

NUMERICAL VALUES

$q = 6$ kN/m $\quad P = 3$ kN $\quad \Delta = 6.0$ mm

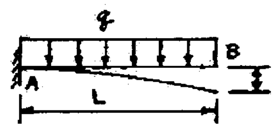
$L = 4.0$ m $\quad EI = 4.0$ MN·m²

$R_B = 7.11$ kN ←

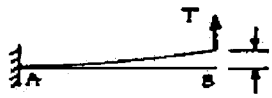
10.4-9 BEAM SUPPORTED BY A TIE ROD

SELECT THE FORCE T IN THE TIE ROD AS REDUNDANT

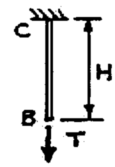
RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$(\delta_B)_1 = \dfrac{qL^4}{8EI}$



$(\delta_B)_2 = \dfrac{TL^3}{3EI}$



$(\delta_B)_3 = \dfrac{TH}{EA}$

COMPATIBILITY $\quad (\delta_B)_1 - (\delta_B)_2 = (\delta_B)_3$

OR $\quad \dfrac{qL^4}{8EI} - \dfrac{TL^3}{3EI} = \dfrac{TH}{EA}$

$T = \dfrac{3qAL^4}{8AL^3 + 24IH}$ ←

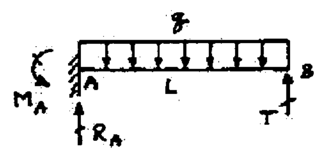
NUMERICAL VALUES

$q = 200$ LB/FT $\quad L = 6$ FT $\quad H = 3$ FT $\quad E = 30 \times 10^6$ PSI

BEAM: S 6 × 12.5 $\quad I = 22.1$ IN.⁴

TIE ROD: $d = 0.25$ IN. $\quad A = 0.04909$ IN.²

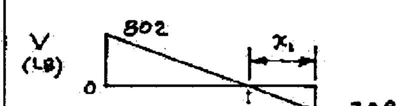
SUBSTITUTE: $\quad T = 398$ LB ←

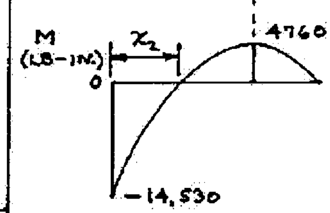


$R_A = qL - T = 802$ LB

$M_A = \dfrac{qL^2}{2} - TL$

$= 14{,}530$ LB-IN.





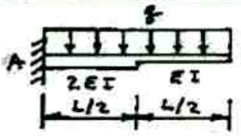
$x_1 = 23.9$ IN.

$x_2 = 24.2$ IN.

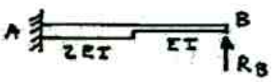
10.4-10 NONPRISMATIC BEAM

SELECT R_B AS REDUNDANT

RELEASED STRUCTURE



$(\delta_B)_1$ = DOWNWARD DEFLECTION OF END B DUE TO q



$(\delta_B)_2$ = UPWARD DEFLECTION DUE TO R_B

FORCE-DISPLACEMENT RELATIONS

FROM PROB. 9.7-2: $\delta_B = \frac{qL^4}{128EI_1}\left(1 + 15\frac{I_1}{I_2}\right)$

$I_1 \rightarrow I \quad I_2 \rightarrow 2I \quad \therefore (\delta_B)_1 = \frac{17qL^4}{256EI}$

FROM PROB. 9.7-1:

$\delta_B = \frac{PL^3}{24EI_1}\left(1 + 7\frac{I_1}{I_2}\right) \quad \therefore (\delta_B)_2 = \frac{3R_BL^3}{16EI}$

COMPATIBILITY $\quad \delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

OR $\frac{17qL^4}{256EI} - \frac{3R_BL^3}{16EI} = 0 \quad R_B = \frac{17qL}{48}$

EQUILIBRIUM

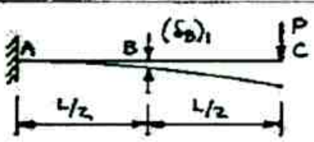
$R_A = qL - R_B = \frac{31qL}{48} \quad M_A = \frac{qL^2}{2} - R_BL = \frac{7qL^2}{48}$

10.4-11 BEAM SUPPORTED BY A BEAM

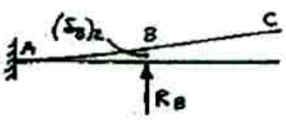
LET R_B = INTERACTION FORCE BETWEEN BEAMS

SELECT R_B AS REDUNDANT

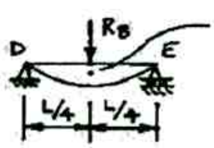
RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$(\delta_B)_1 = \frac{5PL^3}{48EI}$



$(\delta_B)_2 = \frac{R_BL^3}{24EI}$



$(\delta_B)_3 = \frac{R_BL^3}{384EI}$

COMPATIBILITY $\quad (\delta_B)_1 - (\delta_B)_2 = (\delta_B)_3$

SUBSTITUTE AND SOLVE: $\quad R_B = \frac{40P}{17}$

SYMMETRY AND EQUILIBRIUM

$R_D = R_E = \frac{R_B}{2} = \frac{20P}{17}$

$R_A = P - R_D - R_E = -\frac{23P}{17}$

(MINUS MEANS DOWNWARD)

$M_A = R_B\left(\frac{L}{2}\right) - PL = \frac{3PL}{17}$

BEAM ABC: $\quad M_{MAX} = M_B = -\frac{PL}{2}$

BEAM DE: $\quad M_{MAX} = M_E = \frac{5PL}{17}$

$|M_{MAX}| = \frac{PL}{2}$

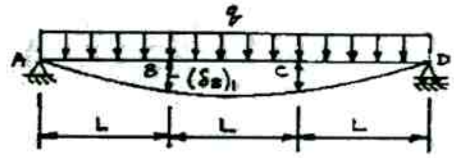
10.4-12 THREE-SPAN CONTINUOUS BEAM

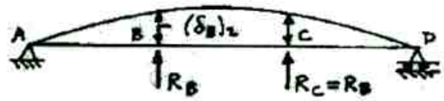
SELECT R_B AND R_C AS REDUNDANTS

SYMMETRY AND EQUILIBRIUM

$R_C = R_B \quad R_A = R_D = \frac{3qL}{2} - R_B$

RELEASED STRUCTURE





FORCE-DISPLACEMENT RELATIONS

$(\delta_B)_1 = \frac{11qL^4}{12EI} \qquad (\delta_B)_2 = \frac{5R_BL^3}{6EI}$

COMPATIBILITY

$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0 \quad \therefore R_B = \frac{11qL}{10}$

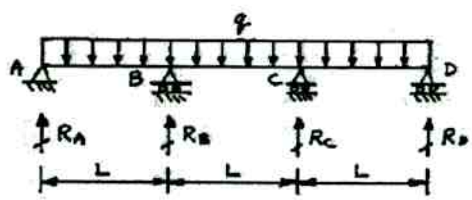
OTHER REACTIONS

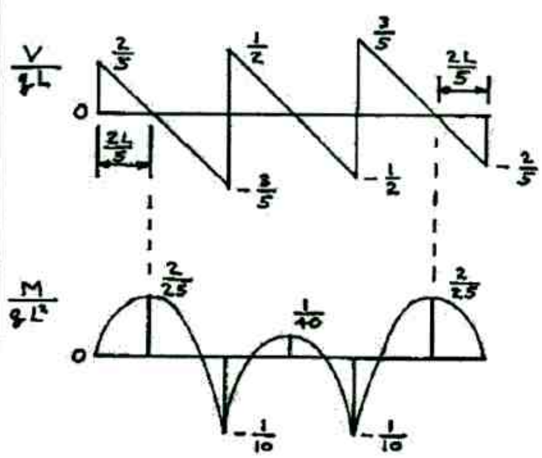
FROM SYMMETRY AND EQUILIBRIUM:

$R_C = R_B = \frac{11qL}{10}$

$R_A = R_D = \frac{2qL}{5}$

LOADING, SHEAR-FORCE, AND BENDING-MOMENT DIAGRAMS





$M_B = M_C = -\frac{qL^2}{10}$

$M_{MAX} = \frac{2qL^2}{25}$

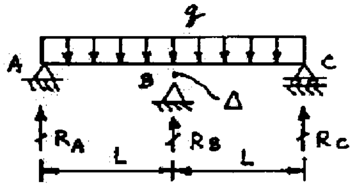
10.4-13 BEAM ON A SUPPORT WITH A GAP

q_0 = LOAD REQUIRED TO CLOSE THE GAP

Δ = MAGNITUDE OF GAP

$(M_B)_0$ = BENDING MOMENT WHEN $q = q_0$

CASE 1 $q < q_0$



$$\delta_B = \frac{5qL^4}{24EI} \qquad M_B = \frac{qL^2}{2}$$

$$R_A = R_C = qL$$

CASE 2 $q = q_0$

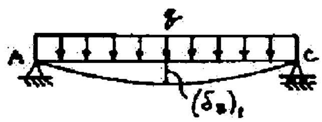
$$\delta_B = \Delta = \frac{5q_0L^4}{24EI} \qquad q_0 = \frac{24EI\Delta}{5L^4} \qquad (1)$$

$$(M_B)_0 = \frac{q_0L^2}{2} = \frac{12EI\Delta}{5L^2} \qquad (2)$$

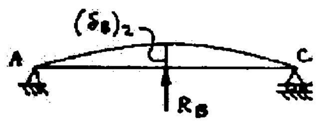
CASE 3 $q > q_0$ (STATICALLY INDETERMINATE)

SELECT R_B AS REDUNDANT

RELEASED STRUCTURE



$$(\delta_B)_1 = \frac{5qL^4}{24EI}$$



$$(\delta_B)_2 = \frac{R_BL^3}{6EI}$$

COMPATIBILITY $\delta_B = (\delta_B)_1 - (\delta_B)_2 = \Delta$

OR $\dfrac{5qL^4}{24EI} - \dfrac{R_BL^3}{6EI} = \Delta \qquad R_B = \dfrac{5qL}{4} - \dfrac{6EI\Delta}{L^3}$

EQUILIBRIUM

$R_A = R_C \qquad 2R_A - 2qL + R_B = 0$

$$R_A = R_C = \frac{3qL}{8} + \frac{3EI\Delta}{L^3}$$

$$M_B = R_AL - \frac{qL^2}{2} = \frac{3EI\Delta}{L^2} - \frac{qL^2}{8}$$

NUMERICAL VALUES

$\Delta = 0.4$ IN. $\quad L = 40$ IN. $\quad EI = 0.4 \times 10^9$ LB-IN.^{2}

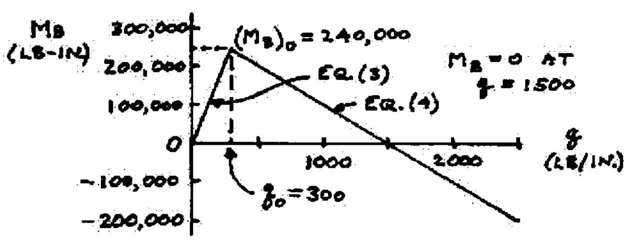
UNITS: LB, IN.

FROM EQS. (1) AND (2): $\quad q_0 = 300$ LB/IN.

$(M_B)_0 = 240{,}000$ LB-IN.

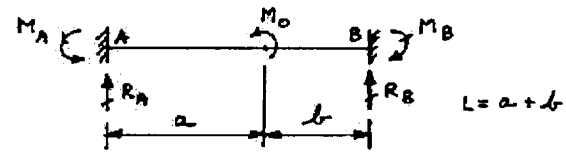
FOR $q < q_0$: $\quad M_B = 800q \qquad (3)$

FOR $q > q_0$: $\quad M_B = 300{,}000 - 200q \qquad (4)$



10.4-14 FIXED-END BEAM (M_0 = APPLIED LOAD)

SELECT R_B AND M_B AS REDUNDANTS

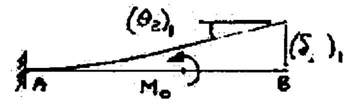


$L = a + b$

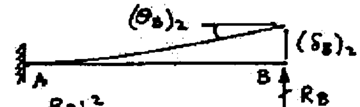
EQUILIBRIUM

$R_A = -R_B \qquad M_A = M_B - R_BL - M_0$

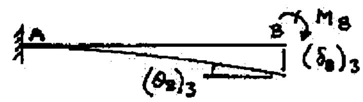
RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_B)_1 = \frac{M_0a}{EI} \qquad (\delta_B)_1 = \frac{M_0a}{2EI}(a+2b)$$



$$(\theta_B)_2 = \frac{R_BL^2}{2EI} \qquad (\delta_B)_2 = \frac{R_BL^3}{3EI}$$



$$(\theta_B)_3 = \frac{M_BL}{EI} \qquad (\delta_B)_3 = \frac{M_BL^2}{2EI}$$

COMPATIBILITY

$\delta_B = -(\delta_B)_1 - (\delta_B)_2 + (\delta_B)_3 = 0$

OR $\quad 2R_BL^3 - 3M_BL^2 = -3M_0a(a+2b) \qquad (1)$

$\theta_B = (\theta_B)_1 + (\theta_B)_2 - (\theta_B)_3 = 0$

OR $\quad R_BL^2 - 2M_BL = -2M_0a \qquad (2)$

SOLVE EQS. (1) AND (2):

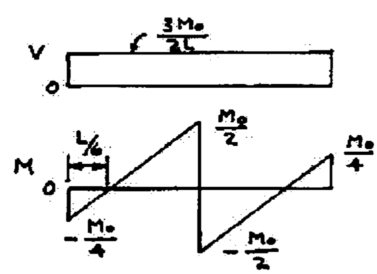
$$R_B = -\frac{6M_0ab}{L^3} \qquad M_B = -\frac{M_0a}{L^2}(3b - L) \quad \longleftarrow$$

FROM EQUILIBRIUM:

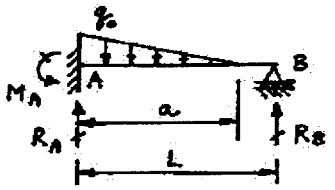
$$R_A = \frac{6M_0ab}{L^3} \qquad M_A = \frac{M_0b}{L^2}(3a - L) \quad \longleftarrow$$

SPECIAL CASE $a = b = L/2$

$$R_A = -R_B = \frac{3M_0}{2L} \qquad M_A = -M_B = \frac{M_0}{4}$$



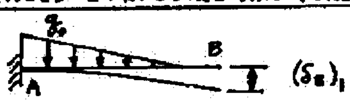
10.4-15 SIDE WALL OF A WOOD FLUME



SELECT R_B AS REDUNDANT.

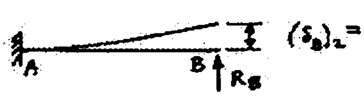
EQUILIBRIUM: $M_A = \frac{q_0 a^2}{6} - R_B L$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



FROM TABLE G-1, CASE 8:

$$(\delta_B)_1 = \frac{q_0 a^4}{30EI} + \frac{q_0 a^3}{24EI}(L-a) = \frac{q_0 a^3}{120EI}(5L-a)$$



$$(\delta_B)_2 = \frac{R_B L^3}{3EI}$$

COMPATIBILITY

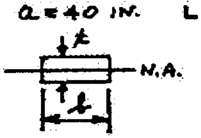
$$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0 \quad \therefore R_B = \frac{q_0 a^3(5L-a)}{40L^3}$$

MAXIMUM BENDING MOMENT

$$M_{MAX} = M_A = \frac{1}{6} q_0 a^2 - R_B L = \frac{q_0 a^2}{120L^2}(20L^2 - 15aL + 3a^2)$$

NUMERICAL VALUES

$a = 40$ IN. $\quad L = 50$ IN. $\quad t = 1.5$ IN.



b = WIDTH OF BEAM

$S = \frac{bt^2}{6} \quad \sigma = \frac{M_{MAX}}{S}$

$\gamma = 62.4$ LB/FT^3 = 0.03611 LB/IN.^{3}

PRESSURE $p = \gamma a \quad q_0 = pb = \gamma a b$

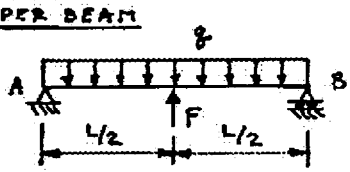
$$M_{MAX} = \frac{\gamma a^3 b}{120L^2}(20L^2 - 15aL + 3a^2) = 191.05\,b$$

$$S = \frac{bt^2}{6} = 0.3750\,b \quad \sigma = \frac{M_{MAX}}{S} = 509 \text{ PSI} \leftarrow$$

10.4-16 TWO BEAMS THAT CROSS

F = INTERACTION FORCE BETWEEN THE BEAMS

UPPER BEAM



$(\delta_B)_1$ = DOWNWARD DEFLECTION DUE TO q

$$= \frac{5qL^4}{384EI}$$

$(\delta_B)_2$ = UPWARD DEFLECTION DUE TO F

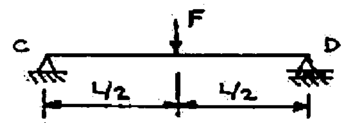
$$= \frac{FL^3}{48EI}$$

$$\delta_{AB} = (\delta_B)_1 - (\delta_B)_2 = \frac{5qL^4}{384EI} - \frac{FL^3}{48EI}$$

CONT.

10.4-16 CONT.

LOWER BEAM

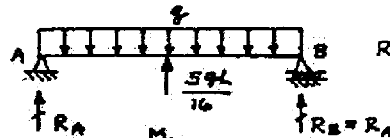


$$\delta_{CD} = \frac{FL^3}{48EI}$$

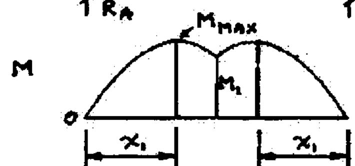
COMPATIBILITY $\quad \delta_{AB} = \delta_{CD}$

$$\frac{5qL^4}{384EI} - \frac{FL^3}{48EI} = \frac{FL^3}{48EI} \quad \therefore F = \frac{5qL}{16}$$

UPPER BEAM



$$R_A = \frac{11qL}{32}$$



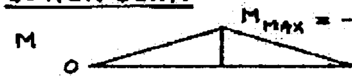
$$x_1 = \frac{11L}{32}$$

$$M_{MAX} = \frac{121qL^2}{2048}$$

$$M_1 = \frac{3qL^2}{64}$$

$$(M_{AB})_{MAX} = \frac{121qL^2}{2048} \leftarrow$$

LOWER BEAM



$$M_{MAX} = \frac{FL}{4} = \frac{5qL^2}{64}$$

$$(M_{CD})_{MAX} = \frac{5qL^2}{64} \leftarrow$$

NUMERICAL VALUES

$q = 6.4$ kN/m $\quad (M_{AB})_{MAX} = 6.05$ kN·m $\leftarrow$

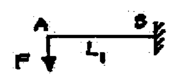
$L = 4$ m $\quad (M_{CD})_{MAX} = 8.0$ kN·m $\leftarrow$

10.4-17 BEAMS JOINED BY A HANGER

F = TENSILE FORCE IN HANGER

SELECT F AS REDUNDANT

① CANTILEVER BEAM AB



S 6 × 12.5 $\quad I_1 = 22.1$ IN.^{4}

$L_1 = 6$ FT = 72 IN.

$E_1 = 30 \times 10^6$ PSI

$$(\delta_A)_1 = \frac{FL_1^3}{3E_1 I_1} = 187.66 \times 10^{-6} F \quad \begin{cases} F = \text{LB} \\ \delta = \text{IN.} \end{cases}$$

② HANGER AC

$d = 0.25$ IN. $\quad L_2 = 10$ FT = 120 IN.

$E_2 = 30 \times 10^6$ PSI

$$A_2 = \frac{\pi d^2}{4} = 0.049087 \text{ IN.}^2$$

Δ = ELONGATION OF AC

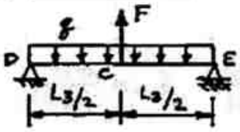
$$\Delta = \frac{FL_2}{E_2 A_2} = 81.488 \times 10^{-6} F$$

(F = LB, Δ = IN.)

CONT.

10.4-17 CONT.

③ BEAM DCE



$L_3 = 20$ FT $= 240$ IN.

$q = 400$ LB/FT $= 33.333$ LB/IN.

$E_3 = 1.5 \times 10^6$ PSI

4 IN. X 12 IN. (NOMINAL)

$I_3 = 415.28$ IN.^{4}

$$(\delta_C)_3 = \frac{5qL_3^4}{384E_3I_3} - \frac{FL_3^3}{48E_3I_3} = 2.3117 \text{ IN.} - 462.34\times10^{-6}F \quad \begin{cases}F = \text{LB}\\ \delta = \text{IN.}\end{cases}$$

COMPATIBILITY

$$(\delta_A)_1 + \Delta = (\delta_C)_3$$

$$18266\times10^{-6}F + 81.488\times10^{-6}F = 2.3117 - 462.34\times10^{-6}F$$

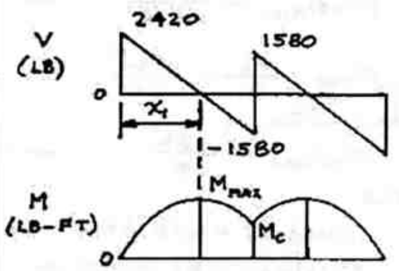
$$F = 3160 \text{ LB} \quad \longleftarrow$$

① MAX. MOMENT IN AB

$$M_{AB} = FL_1 = (3160 \text{ LB})(6 \text{ FT}) = 18{,}960 \text{ LB-FT} \quad \longleftarrow$$

③ MAX. MOMENT IN DCE

$$R_D = \frac{qL_3}{2} - \frac{F}{2} = 2420 \text{ LB}$$



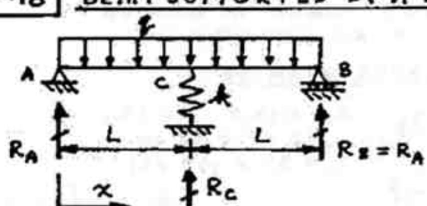
$x_1 = 6.050$ FT

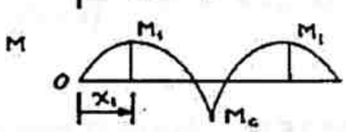
$M_{MAX} = 7320$ LB-FT

$M_C = 4200$ LB-FT

$$M_{DE} = 7320 \text{ LB-FT} \quad \longleftarrow$$

10.4-18 BEAM SUPPORTED BY A SPRING





BENDING MOMENT $\quad M = R_A x - \dfrac{qx^2}{2}$

LOCATION OF MAXIMUM POSITIVE MOMENT

$$\frac{dM}{dx} = 0 \quad R_A - qx = 0 \quad x_1 = \frac{R_A}{q}$$

MAXIMUM POSITIVE MOMENT

$$M_1 = (M)_{x=x_1} = \frac{R_A^2}{2q}$$

MAXIMUM NEGATIVE MOMENT

$$M_C = (M)_{x=L} = R_A L - \frac{qL^2}{2}$$

CONT.

10.4-18 CONT.

FOR THE SMALLEST MAXIMUM MOMENT:

$$|M_1| = |M_C| \quad \text{OR} \quad M_1 = -M_C$$

$$\frac{R_A^2}{2q} = -R_A L + \frac{qL^2}{2}$$

SOLVE FOR R_A: $\quad R_A = qL(\sqrt{2} - 1)$

EQUILIBRIUM

$$\Sigma F_{VERT} = 0 \quad 2R_A + R_C - 2qL = 0$$

$$R_C = 2qL(2 - \sqrt{2})$$

DOWNWARD DEFLECTION OF BEAM

$$(\delta_C)_1 = \frac{5qL^4}{24EI} - \frac{R_C L^3}{6EI} = \frac{qL^4}{24EI}(8\sqrt{2} - 11)$$

DOWNWARD DISPLACEMENT OF SPRING

$$(\delta_C)_2 = \frac{R_C}{k} = \frac{2qL}{k}(2 - \sqrt{2})$$

COMPATIBILITY $\quad (\delta_C)_1 = (\delta_C)_2$

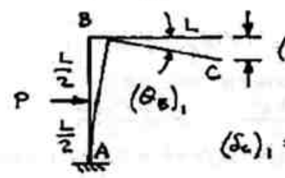
SOLVE FOR k: $\quad k = \dfrac{48EI}{7L^3}(6 + 5\sqrt{2}) = 89.63\dfrac{EI}{L^3} \quad \longleftarrow$

10.4-19 FRAME ABC WITH FIXED SUPPORT

SELECT V_C AS REDUNDANT

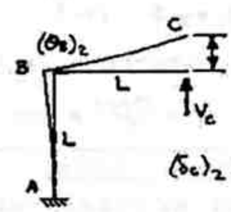
EQUILIBRIUM $\quad V_A = V_C \quad H_A = P \quad M_A = PL/2 - V_C L$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_B)_1 = \frac{PL^2}{8EI}$$

$$(\delta_C)_1 = (\theta_B)_1 L = \frac{PL^3}{8EI}$$



$$(\theta_B)_2 = \frac{V_C L^2}{EI}$$

$$(\delta_C)_2 = (\theta_B)_2 L + \frac{V_C L^3}{3EI} = \frac{4V_C L^3}{3EI}$$

COMPATIBILITY $\quad (\delta_C)_1 = (\delta_C)_2$

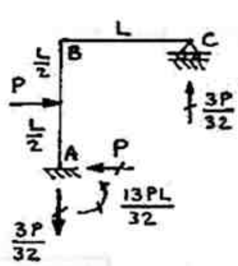
SUBSTITUTE FOR $(\delta_C)_1$ AND $(\delta_C)_2$ AND SOLVE:

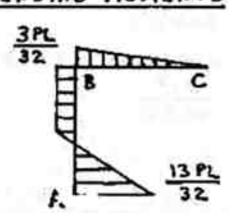
$$V_C = \frac{3P}{32} \quad \longleftarrow$$

FROM EQUILIBRIUM:

$$V_A = \frac{3P}{32} \quad H_A = P \quad M_A = \frac{13PL}{32} \quad \longleftarrow$$

REACTIONS AND BENDING MOMENTS





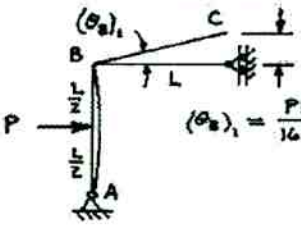
$$M_{MAX} = \frac{13PL}{32} \quad \longleftarrow$$

10.4-20 FRAME ABC WITH PINNED SUPPORTS

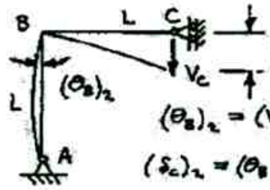
SELECT V_C AS REDUNDANT

EQUILIBRIUM $V_A = V_C \quad H_A = \frac{P}{2} - V_C \quad H_C = \frac{P}{2} + V_C$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_B)_1 = \frac{PL^2}{16EI} \qquad (\delta_C)_1 = (\theta_B)_1 L = \frac{PL^3}{16EI}$$



$$(\theta_B)_2 = (V_C L)\frac{L}{3EI} = \frac{V_C L^2}{3EI}$$

$$(\delta_C)_2 = (\theta_B)_2 L + \frac{V_C L^3}{3EI} = \frac{2V_C L^3}{3EI}$$

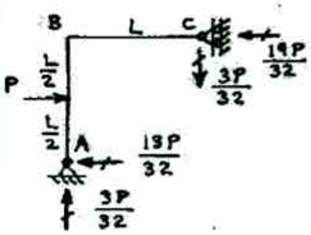
COMPATIBILITY

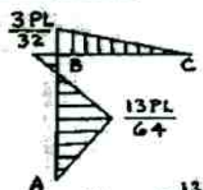
$$(\delta_C)_1 = (\delta_C)_2 \qquad \frac{PL^3}{16EI} = \frac{2V_C L^3}{3EI} \qquad V_C = \frac{3P}{32} \quad \longleftarrow$$

FROM EQUILIBRIUM:

$$V_A = \frac{3P}{32} \qquad H_A = \frac{13P}{32} \qquad H_C = \frac{19P}{32} \quad \longleftarrow$$

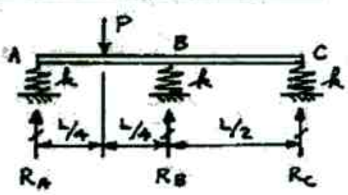
REACTIONS AND BENDING MOMENTS





$$M_{MAX} = \frac{13PL}{64} \quad \longleftarrow$$

10.4-21 / 10.4-22 BEAM ON THREE SPRINGS

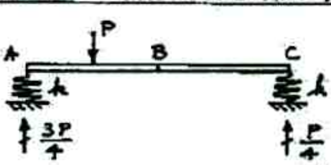


SELECT R_B AS REDUNDANT

EQUILIBRIUM

$$R_A = \frac{3P}{4} - \frac{R_B}{2} \qquad R_C = \frac{P}{4} - \frac{R_B}{2}$$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\delta_A)_1 = \frac{3P}{4k} \qquad (\delta_C)_1 = \frac{P}{4k}$$

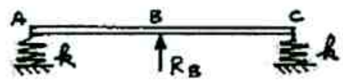
$$(\delta_B)_1 = \frac{1}{2}\left[(\delta_A)_1 + (\delta_C)_1\right] + \frac{P\left(\frac{L}{4}\right)\left[3L^2 - 4\left(\frac{L}{4}\right)^2\right]}{48EI}$$

(CASE 5, TABLE G-2)

$$(\delta_B)_1 = \frac{P}{2k} + \frac{11PL^3}{768EI} \quad \text{(DOWNWARD)}$$

CONT.

10.4-21 CONT.
10.4-22 CONT.



$$(\delta_A)_2 = \frac{R_B}{2k} \qquad (\delta_C)_2 = \frac{R_B}{2k}$$

$$(\delta_B)_2 = \frac{1}{2}\left[(\delta_A)_2 + (\delta_C)_2\right] + \frac{R_B L^3}{48EI} = \frac{R_B}{2k} + \frac{R_B L^3}{48EI} \quad \text{(UPWARD)}$$

COMPATIBILITY $\quad (\delta_B)_1 - (\delta_B)_2 = \frac{R_B}{k}$

SUBSTITUTE AND SOLVE:

$$R_B = P\left(\frac{384EI + 11kL^3}{1152EI + 16kL^3}\right)$$

$$\text{LET } k^* = \frac{kL^3}{EI} \quad \text{(NONDIMENSIONAL)} \quad \longleftarrow$$

$$R_B = \frac{P}{16}\left(\frac{384 + 11k^*}{72 + k^*}\right) \quad \longleftarrow$$

FROM EQUILIBRIUM:

$$R_A = \frac{P}{32}\left(\frac{1344 + 13k^*}{72 + k^*}\right) \quad \longleftarrow$$

$$R_C = \frac{3P}{32}\left(\frac{64 - k^*}{72 + k^*}\right) \quad \longleftarrow$$

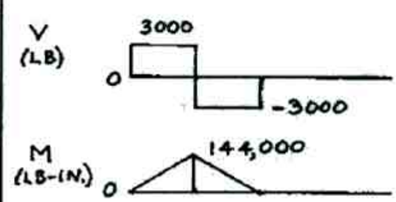
10.4-21 NUMERICAL VALUES

$EI = 6912 \times 10^6$ LB-IN.^{2} $\quad k = 62{,}500$ LB/IN.

$L = 16$ FT $= 192$ IN. $\quad P = 6000$ LB

$$k^* = \frac{kL^3}{EI} = 64 \qquad R_B = 3000 \text{ LB} \quad \longleftarrow$$

$$R_A = 3000 \text{ LB} \qquad R_C = 0 \quad \longleftarrow$$



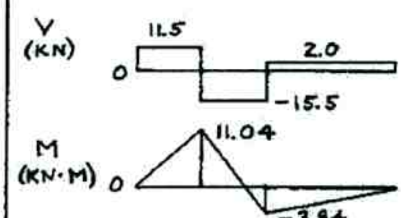
10.4-22 NUMERICAL VALUES

$EI = 9.216 \times 10^6$ N·M^2 $\quad k = 93{,}750 \times 10^3$ N/M

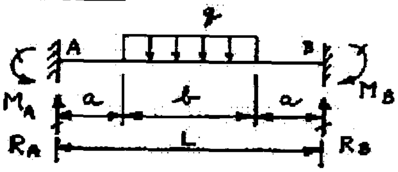
$L = 3.84$ M $\quad P = 27$ KN

$$k^* = \frac{kL^3}{EI} = 576 \qquad R_B = 17.5 \text{ KN} \quad \longleftarrow$$

$$R_A = 11.5 \text{ KN} \qquad R_C = -2.0 \text{ KN} \quad \longleftarrow$$



10.4-23 FIXED-END BEAM

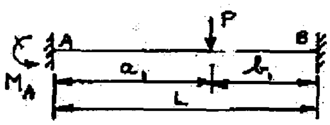


$M_A = M_B$

$R_A = R_B = \dfrac{qb}{2}$

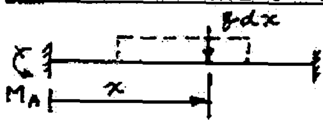
$a = \dfrac{L-b}{2}$

FROM EXAMPLE 10-4, EQ. (10-25a):



$M_A = \dfrac{P a_1 b_1^2}{L^2}$

FOR THE PARTIAL UNIFORM LOAD



$dM_A = \dfrac{(q\,dx)(x)(L-x)^2}{L^2}$

$$M_A = \int_a^{a+b} dM_A = \int_{(L-b)/2}^{(L+b)/2} dM_A$$

$$= \frac{q}{L^2}\int_{(L-b)/2}^{(L+b)/2} x(L-x)^2\,dx$$

$$= \frac{q}{L^2}\int_{(L-b)/2}^{(L+b)/2} (L^2x - 2Lx^2 + x^3)\,dx$$

$$= \frac{q}{L^2}\left[\frac{L^2x^2}{2} - \frac{2Lx^3}{3} + \frac{x^4}{4}\right]_{(L-b)/2}^{(L+b)/2}$$

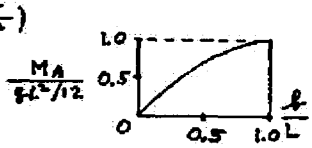
...(LENGTHY SUBSTITUTION)...

$$= \frac{qb}{24L}(3L^2 - b^2)$$

(a) $M_A = M_B = \dfrac{qb}{24L}(3L^2 - b^2)$ ←

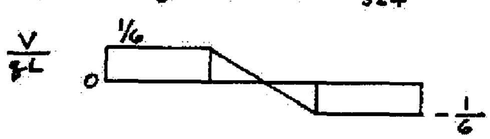
(b) GRAPH OF FIXED-END MOMENT

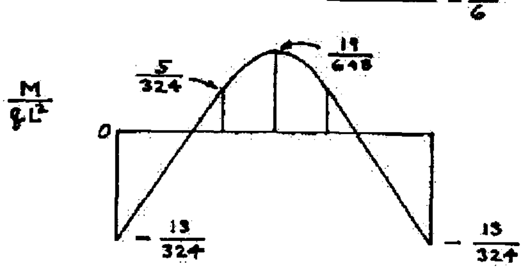
$$\frac{M_A}{qL^2/12} = \frac{b}{2L}\left(3 - \frac{b^2}{L^2}\right)$$



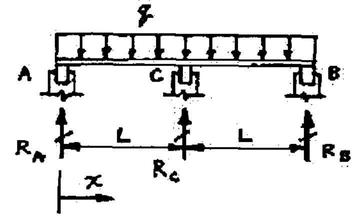
(c) SPECIAL CASE $a = b = L/3$

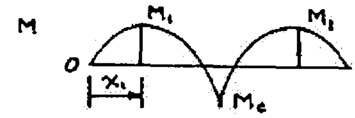
$R_A = R_B = \dfrac{qL}{6}$ $\quad M_A = M_B = \dfrac{13qL^2}{324}$





10.4-24 BEAM SUPPORTED BY PISTONS





BENDING MOMENT $\quad M = R_A x - \dfrac{qx^2}{2}$

LOCATION OF MAXIMUM POSITIVE MOMENT

$\dfrac{dM}{dx} = 0 \quad R_A - qx = 0 \quad x_1 = \dfrac{R_A}{q}$

MAXIMUM POSITIVE MOMENT

$M_1 = (M)_{x=x_1} = \dfrac{R_A^2}{2q}$

MAXIMUM NEGATIVE MOMENT

$M_c = (M)_{x=L} = R_A L - \dfrac{qL^2}{2}$

FOR THE SMALLEST MAXIMUM MOMENT:

$|M_1| = |M_c| \quad$ OR $\quad M_1 = -M_c$

$\dfrac{R_A^2}{2q} = -R_A L + \dfrac{qL^2}{2}$

SOLVE FOR R_A: $\quad R_A = qL(\sqrt{2} - 1)$

EQUILIBRIUM

$\Sigma F_{VERT} = 0 \quad 2R_A + R_c - 2qL = 0$

$R_c = 2qL(2 - \sqrt{2})$

REACTIONS BASED UPON PRESSURE

$R_A = R_B = p\left(\dfrac{\pi d_1^2}{4}\right) \quad R_c = p\left(\dfrac{\pi d_2^2}{4}\right)$

(a) $\therefore \dfrac{d_2}{d_1} = \sqrt{\dfrac{R_c}{R_A}} = \sqrt{\dfrac{2(2-\sqrt{2})}{\sqrt{2}-1}} = \sqrt[4]{8}$

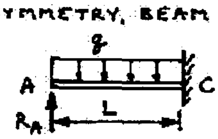
$= 1.682$ ←

(b) $M_{MAX} = M_1 = \dfrac{R_A^2}{2q} = \dfrac{qL^2}{2}(3 - 2\sqrt{2})$

$= 0.08579\,qL^2$ ←

(c) DIFFERENCE IN ELEVATION

BY SYMMETRY, BEAM HAS ZERO SLOPE AT C.



$R_A = qL(\sqrt{2} - 1)$

δ_A = DIFFERENCE IN ELEV

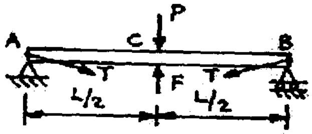
$\delta_A = \dfrac{R_A L^3}{3EI} - \dfrac{qL^4}{8EI} = \dfrac{qL^4}{24EI}(8\sqrt{2} - 11)$

$= 0.01307\,qL^4/EI$ ←

POINT C IS BELOW POINTS A AND B BY THE AMOUNT $0.01307\,qL^4/EI$.

10.4-25 TRUSSED BEAM

BEAM



$P = 20$ K
W 18 X 119
$I = 2190$ IN.^{4}
$L = 36$ FT
$= 432$ IN.

F = COMPRESSIVE FORCE IN STRUT
SELECT F AS REDUNDANT

T = TENSILE FORCE IN TWO TIE RODS

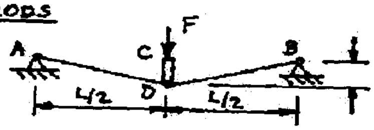
$(\delta_c)_1$ = DOWNWARD DEFLECTION OF POINT C

$$(\delta_c)_1 = (P-F)\frac{L^3}{48EI}$$

SUBSTITUTE NUMERICAL VALUES:
UNITS: LB, IN.

$$E(\delta_c)_1 = 766.95(P-F) = 15.339 \times 10^6 - 766.95F$$

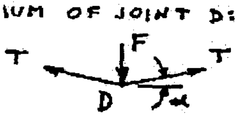
TIE RODS



$H = 3$ FT
$= 36$ IN.

DISREGARD SHORTENING OF STRUT.
$(\delta_c)_2$ = DOWNWARD DISPLACEMENT OF C

EQUILIBRIUM OF JOINT D:



$$\tan\alpha = \frac{H}{L/2} = \frac{1}{6}$$

$$\sin\alpha = \frac{1}{\sqrt{37}}$$

$2T\sin\alpha = F$

$$T = \frac{F}{2\sin\alpha} = \frac{F\sqrt{37}}{2} = 3.0414F$$

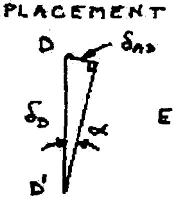
δ_{AD} = ELONGATION OF TIE ROD

$$\delta_{AD} = \frac{TL_{AD}}{EA} \qquad E\delta_{AD} = \frac{1}{A}(3.0414F)\left(\frac{H}{\sin\alpha}\right)$$

$$= \frac{1}{4.0}(3.0414F)(36\sqrt{37})$$

$$= 166.50F$$

DISPLACEMENT DIAGRAM:



$$\delta_D = \overline{DD'} = (\delta_c)_2 = \frac{\delta_{AD}}{\sin\alpha}$$

$$E(\delta_c)_2 = (166.50F)\sqrt{37}$$

$$= 1012.8F$$

COMPATIBILITY $(\delta_c)_1 = (\delta_c)_2$

$$15.339 \times 10^6 - 766.95F = 1012.8F$$

(a) $F = 8619$ LB SAY, $F = 8620$ LB ←

(b) MAXIMUM BENDING STRESS IN BEAM

W 18 X 119 $S = 231$ IN.^{3} $\sigma_c = M_{max}/S$

$$M_{max} = \frac{(P-F)L}{4} = 1.229 \times 10^6 \text{ LB-IN.}$$

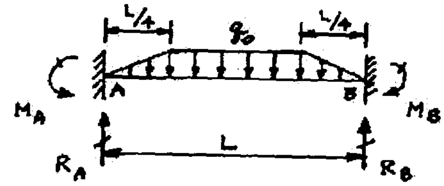
$$\sigma_c = 5320 \text{ PSI} \leftarrow$$

TENSILE STRESS IN TIE RODS

$T = 3.0414F = 26{,}214$ LB $A = 4.0$ IN.^{2}

$$\sigma_t = \frac{T}{A} = 6550 \text{ PSI} \leftarrow$$

10.4-26 FIXED-END BEAM (TRAPEZOIDAL LOAD)



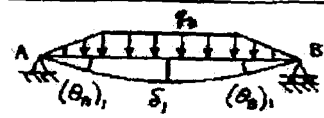
$$c = \frac{L}{4}$$

FROM SYMMETRY AND EQUILIBRIUM

$$M_A = M_B \qquad R_A = R_B = \frac{3q_0L}{8}$$

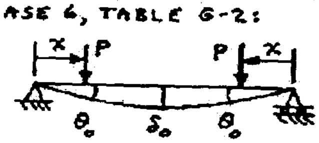
SELECT M_A AND M_B AS REDUNDANTS

RELEASED STRUCTURE WITH APPLIED LOAD



$$(\theta_A)_1 = (\theta_B)_1$$

CONSIDER THE FOLLOWING BEAM FROM CASE 6, TABLE G-2:



$$\theta_0 = \frac{Px(L-x)}{2EI} \qquad \delta_0 = \frac{Px}{24EI}(3L^2 - 4x^2)$$

CONSIDER THE LOAD P AS AN ELEMENT OF THE DISTRIBUTED LOAD.
REPLACE P BY $q\,dx$, WHERE

$q = \dfrac{4q_0x}{L}$ x FROM 0 TO L/4

$q = q_0$ x FROM L/4 TO L/2

$$(\theta_A)_1 = \frac{1}{2EI}\int_0^{L/4}\left(\frac{4q_0x}{L}\right)(x)(L-x)\,dx$$

$$+ \frac{1}{2EI}\int_{L/4}^{L/2} q_0x(L-x)\,dx$$

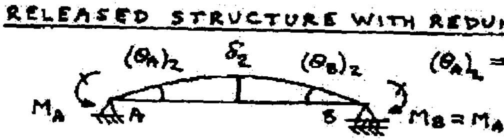
$$= \frac{13q_0L^3}{1536EI} + \frac{11q_0L^3}{384EI} = \frac{19q_0L^3}{512EI}$$

$$\delta_1 = \frac{1}{24EI}\int_0^{L/4}\left(\frac{4q_0x}{L}\right)(x)(3L^2 - 4x^2)\,dx$$

$$+ \frac{1}{24EI}\int_{L/4}^{L/2} q_0x(3L^2 - 4x^2)\,dx$$

$$= \frac{19q_0L^4}{7680EI} + \frac{19q_0L^4}{2048EI} = \frac{361q_0L^4}{30{,}720EI}$$

RELEASED STRUCTURE WITH REDUNDANTS



$$(\theta_A)_2 = (\theta_B)_2$$

FROM CASE 10, TABLE G-2:

$$(\theta_A)_2 = \frac{M_AL}{2EI} \qquad \delta_2 = \frac{M_AL^2}{8EI}$$

COMPATIBILITY

$$\theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$$

$$\frac{19q_0L^3}{512EI} - \frac{M_AL}{2EI} = 0 \qquad M_A = \frac{19q_0L^2}{256} \leftarrow$$

CONT.

10.4-26 CONT.

DEFLECTION AT THE MIDPOINT

$$\delta_{MAX} = \delta_1 - \delta_2 = \frac{361 q_0 L^4}{30{,}720 EI} - \frac{M_A L^2}{8EI}$$
$$= \frac{361 q_0 L^4}{30{,}720 EI} - \left(\frac{19 q_0 L^2}{256}\right)\left(\frac{L^2}{8EI}\right)$$
$$= \frac{19 q_0 L^4}{7680 EI} \longleftarrow$$

BENDING MOMENT AT THE MIDPOINT

$$M_C = R_A\left(\frac{L}{2}\right) - M_A - \frac{q_0 L^2}{24} - \frac{q_0 L^2}{32}$$
$$= \frac{3 q_0 L}{8}\left(\frac{L}{2}\right) - \frac{19 q_0 L^2}{256} - \frac{7 q_0 L^2}{96} = \frac{31 q_0 L^2}{768} \longleftarrow$$

MAXIMUM BENDING MOMENT

$$M_A > M_C \quad \therefore M_{max} = M_A = \frac{19 q_0 L^2}{256} \longleftarrow$$

NUMERICAL VALUES

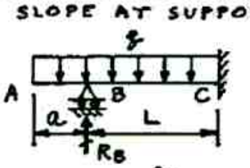
$q_0 = 18$ kN/m $\quad L = 200$ mm

$b = 60$ mm $\quad h = 20$ mm $\quad E = 200$ GPa

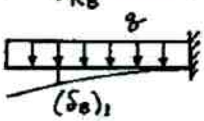
$$S = \frac{bh^2}{6} = 4.0 \times 10^{-6}\ \text{m}^3$$
$$I = \frac{bh^3}{12} = 40 \times 10^{-9}\ \text{m}^4$$
$$M_{MAX} = \frac{19 q_0 L^2}{256} = 53.44\ \text{N·m}$$
$$\sigma_{MAX} = \frac{M_{MAX}}{S} = 13.4\ \text{MPa} \longleftarrow$$
$$\delta_{MAX} = \frac{19 q_0 L^4}{7680 EI} = 0.00891\ \text{mm} \longleftarrow$$

10.4-27 BEAM WITH OVERHANGS

BECAUSE THE BEAM IS SYMMETRIC, THE SLOPE AT SUPPORT C IS ZERO.

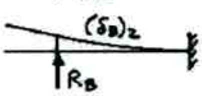


SELECT R_B AS REDUNDANT



RELEASED STRUCTURE WITH APPLIED LOAD

$$(\delta_B)_1 = \frac{qL^2}{24EI}(3L^2 + 8aL + 6a^2)$$



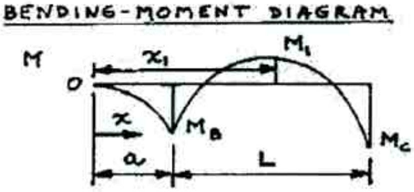
RELEASED STRUCTURE WITH REDUNDANT

$$(\delta_B)_2 = \frac{R_B L^3}{3EI}$$

COMPATIBILITY $\quad \delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

SUBSTITUTE AND SOLVE FOR R_B: $\quad R_B = \dfrac{q}{8L}(3L^2 + 8aL + 6a^2)$

BENDING-MOMENT DIAGRAM



CONT.

10.4-27 CONT.

BENDING MOMENTS

$$M_B = -\frac{qa^2}{2}$$
$$M_C = -\frac{q(L+a)^2}{2} + R_B L = -\frac{q}{8}(L^2 - 2a^2)$$

POSITIVE MOMENT M_1 $\quad (a \le x \le L+a)$

$V = R_B - qx$

SET $V = 0$ AND SOLVE FOR x_1:

$$x_1 = \frac{R_B}{q} = \frac{1}{8L}(3L^2 + 8aL + 6a^2)$$
$$x_1 - a = \frac{1}{8L}(3L^2 + 6a^2) = \frac{3}{8L}(L^2 + 2a^2)$$
$$M_1 = R_B(x_1 - a) - \frac{qx_1^2}{2}$$

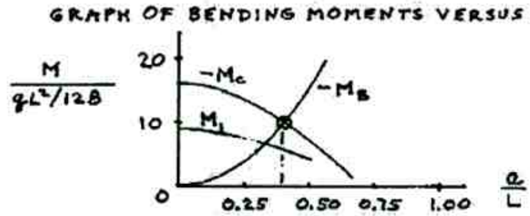
SUBSTITUTE FOR R_B AND x_1 AND SIMPLIFY:

$$M_1 = \frac{q}{128L^2}(9L^4 - 28a^2L^2 + 36a^4)$$

BENDING MOMENTS (NONDIMENSIONAL FORM)

$$\frac{M_B}{qL^2/128} = -64\left(\frac{a}{L}\right)^2$$
$$\frac{M_C}{qL^2/128} = -16\left(1 - \frac{2a^2}{L^2}\right)$$
$$\frac{M_1}{qL^2/128} = 9 - \frac{28a^2}{L^2} + \frac{36a^4}{L^4}$$

GRAPH OF BENDING MOMENTS VERSUS a/L:



THE BEAM HAS THE NUMERICALLY SMALLEST BENDING MOMENT WHEN $-M_B = -M_C$.

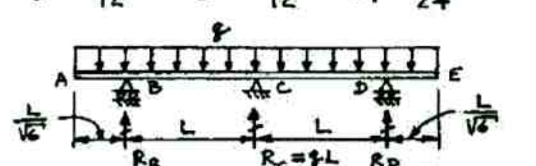
THEREFORE, $\quad 64\left(\dfrac{a}{L}\right)^2 = 16\left(1 - \dfrac{2a^2}{L^2}\right)$

SOLVE FOR a/L:

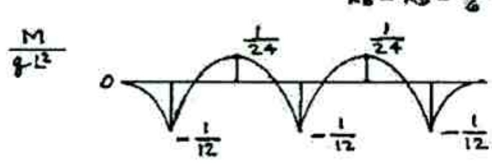
$$\frac{a}{L} = \frac{1}{\sqrt{6}} \qquad a = \frac{L}{\sqrt{6}} = 0.4082L \longleftarrow$$

OPTIMUM CONDITION

$$\frac{a}{L} = \frac{1}{\sqrt{6}} \qquad x_1 = \frac{L}{2} + a \qquad x_1 - a = \frac{L}{2}$$
$$M_B = -\frac{qL^2}{12} \qquad M_C = -\frac{qL^2}{12} \qquad M_1 = \frac{qL^2}{24}$$



$$R_B = R_D = \frac{qL}{6}(3 + \sqrt{6})$$

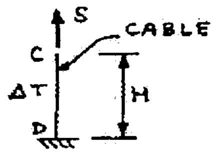


10.5-1 UNIFORM TEMPERATURE CHANGE

ΔT = DECREASE IN TEMPERATURE
USE METHOD OF SUPERPOSITION.
SELECT TENSILE FORCE S IN THE CABLE AS REDUNDANT.

RELEASED STRUCTURE

I = MOMENT OF INERTIA



A = CROSS-SECTIONAL AREA

BEAM $(\delta_C)_1 = \dfrac{SL^3}{48EI}$ (DOWNWARD)

CABLE $(\delta_C)_2 = \alpha H(\Delta T) - \dfrac{SH}{EA}$ (DOWNWARD)

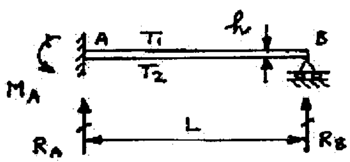
COMPATIBILITY $(\delta_C)_1 = (\delta_C)_2$

$$\frac{SL^3}{48EI} = \alpha H(\Delta T) - \frac{SH}{EA}$$

SOLVE FOR S:

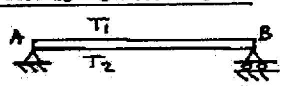
$$S = \frac{48EIAH\alpha(\Delta T)}{AL^3 + 48IH} \longleftarrow$$

10.5-2 BEAM WITH TEMPERATURE DIFFERENTIAL

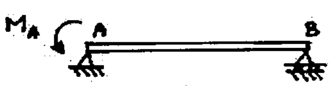


USE THE METHOD OF SUPERPOSITION.
SELECT M_A AS REDUNDANT.

RELEASED STRUCTURE



$(\theta_A)_1 = \dfrac{\alpha L(T_2 - T_1)}{2h}$ (CLOCKWISE) (FROM THE ANSWER TO PROB. 9.11-1)



$(\theta_A)_2 = \dfrac{M_A L}{3EI}$ (COUNTERCLOCKWISE)

COMPATIBILITY $(\theta_A)_1 = (\theta_A)_2$

$$\frac{\alpha L(T_2 - T_1)}{2h} = \frac{M_A L}{3EI} \qquad M_A = \frac{3\alpha EI(T_2 - T_1)}{2h} \longleftarrow$$

EQUILIBRIUM

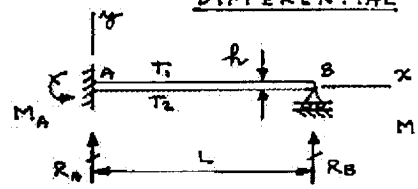
$\Sigma M_B = 0 \quad M_A - R_A L = 0$

$$R_A = \frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

$\Sigma F_{VERT} = 0 \quad R_B = -R_A$

$$R_B = -\frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

10.5-3 BEAM WITH TEMPERATURE DIFFERENTIAL



$M = R_B(L - x)$

DIFFERENTIAL EQUATION (EQ. 10-39 b)

$$EIv'' = M + \frac{\alpha EI(T_2 - T_1)}{h}$$

OR $EIv'' = R_B(L - x) + \dfrac{\alpha EI(T_2 - T_1)}{h}$

$$EIv' = R_B L x - R_B\left(\frac{x^2}{2}\right) + \frac{\alpha EI(T_2 - T_1)}{h}x + C_1$$

B.C. 1 $v'(0) = 0 \quad \therefore C_1 = 0$

$$EIv = R_B L\left(\frac{x^2}{2}\right) - R_B\left(\frac{x^3}{6}\right) + \frac{\alpha EI(T_2 - T_1)}{2h}x^2 + C_2$$

B.C. 2 $v(0) = 0 \quad \therefore C_2 = 0$

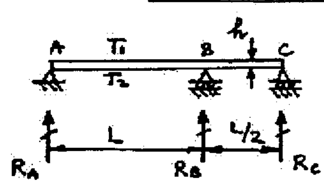
B.C. 3 $v(L) = 0 \quad \therefore R_B = -\dfrac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$

FROM EQUILIBRIUM:

$$R_A = -R_B = \frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

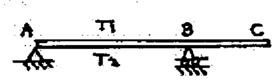
$M_A = R_A L \qquad M_A = \dfrac{3\alpha EI(T_2 - T_1)}{2h} \longleftarrow$

10.5-4 BEAM WITH TEMPERATURE DIFFERENTIAL

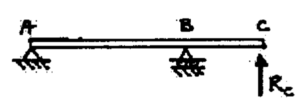


USE THE METHOD OF SUPERPOSITION.
SELECT R_C AS REDUNDANT.

RELEASED STRUCTURE



FROM PROB. 9.11-3:
$(\delta_C)_1 = \dfrac{3\alpha L^2(T_2 - T_1)}{8h}$ (UPWARD)



FROM PROB. 9.8-5:
$(\delta_C)_2 = \dfrac{R_C L^3}{8EI}$ (UPWARD)

COMPATIBILITY $(\delta_C)_1 + (\delta_C)_2 = 0$

$$\frac{3\alpha L^2(T_2 - T_1)}{8h} = -\frac{R_C L^3}{8EI}$$

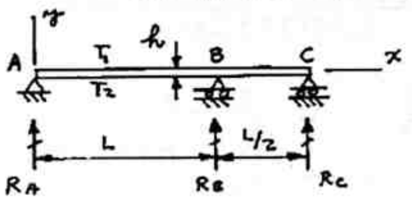
$$R_C = -\frac{3\alpha EI(T_2 - T_1)}{hL} \longleftarrow$$

FROM EQUILIBRIUM:

$R_A = \dfrac{R_C}{2} \qquad R_A = -\dfrac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$

$R_B = -\dfrac{3R_C}{2} \qquad R_B = \dfrac{9\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$

10.5-5 BEAM WITH TEMPERATURE DIFFERENTIAL



DIFFERENTIAL EQUATION (EQ. 10-27b)

$$EIv'' = M + \frac{\alpha EI(T_2-T_1)}{h}$$

FOR CONVENIENCE, LET $\beta = \frac{\alpha EI(T_2-T_1)}{h}$ (1)

$EIv'' = M + \beta$ (2)

PART AB OF THE BEAM $(0 \le x \le L)$

$M = R_A x \qquad EIv'' = R_A x + \beta$

$EIv' = R_A x^2/2 + \beta x + C_1$ (3)

$EIv = R_A x^3/6 + \beta x^2/2 + C_1 x + C_2$ (4)

B.C. 1 $\quad v(0) = 0 \qquad \therefore C_2 = 0$

B.C. 2 $\quad v(L) = 0 \qquad \therefore R_A L^2 + 6C_1 = -3\beta L$ (5)

PART BC OF THE BEAM $(L \le x \le 3L/2)$

$M = R_A x + R_B(x-L)$

FROM EQUILIBRIUM, $R_B = -3R_A$ (6)

$\therefore M = -2R_A x + 3R_A L$

$EIv'' = M + \beta = -2R_A x + 3R_A L + \beta$

$EIv' = -R_A x^2 + 3R_A L x + \beta x + C_3$ (7)

$EIv = -R_A x^3/3 + 3R_A L x^2/2 + \beta x^2/2 + C_3 x + C_4$ (8)

B.C. 3 $\quad v(L) = 0$

$\therefore 7R_A L^3 + 6C_3 L + 6C_4 = -3\beta L^2$ (9)

B.C. 4 $\quad v(3L/2) = 0$

$\therefore 18R_A L^3 + 12C_3 L + 8C_4 = -9\beta L^2$ (10)

CONTINUITY CONDITION AT B

$(EIv')_{AB} = (EIv')_{BC}$ AT $x = L$

FROM EQS. (3) AND (7):

$R_A(L^2/2) + \beta L + C_1 = -R_A L^2 + 3R_A L^2 + \beta L + C_3$

OR $\quad 3R_A L^2 - 2C_1 + 2C_3 = 0$ (11)

SOLVE EQS. (5), (9), (10), AND (11) FOR R_A:

$$R_A = -\frac{3\beta}{2L} = -\frac{3\alpha EI(T_2-T_1)}{2hL} \quad \longleftarrow$$

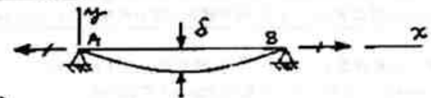
ALSO: $C_1 = -\beta L/4 \quad C_2 = 0 \quad C_3 = 2\beta L \quad C_4 = -3\beta L^2/4$

FROM EQ. (6): $\quad R_B = \frac{9\alpha EI(T_2-T_1)}{2hL} \quad \longleftarrow$

FROM EQUILIBRIUM:

$$R_C = 2R_A = -\frac{3\alpha EI(T_2-T_1)}{hL} \quad \longleftarrow$$

10.6-1 BEAM WITH IMMOVABLE SUPPORTS



(a)

$v = -\delta \sin\frac{\pi x}{L} \qquad \frac{dv}{dx} = -\frac{\pi\delta}{L}\cos\frac{\pi x}{L}$

EQ. (10-42): $\lambda = \frac{1}{2}\int_0^L \left(\frac{dv}{dx}\right)^2 dx = \frac{\pi^2\delta^2}{4L}$

EQ. (10-45): $H = \frac{EA\lambda}{L} = \frac{\pi^2 EA\delta^2}{4L^2} \quad \longleftarrow$

EQ. (10-46): $\sigma_t = \frac{H}{A} = \frac{\pi^2 E\delta^2}{4L^2} \quad \longleftarrow$

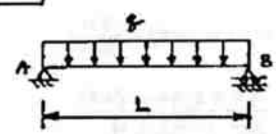
(b) ALUMINUM ALLOY

$E = 10 \times 10^6$ PSI $\qquad \sigma_t = 24.67 \times 10^6 \left(\frac{\delta}{L}\right)^2$ (PSI)

$\frac{\delta}{L}$	$\frac{1}{200}$	$\frac{1}{400}$	$\frac{1}{600}$
σ_t (PSI)	617	154	69

NOTE: THE AXIAL STRESS INCREASES AS THE DEFLECTION INCREASES.

10.6-2 BEAM WITH UNIFORM LOAD



h = HEIGHT OF BEAM

(a) CURVATURE SHORTENING

FROM CASE 1, TABLE G-2:

$\frac{dv}{dx} = -\frac{q}{24EI}(L^3 - 6Lx^2 - 4x^3)$

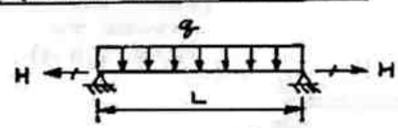
EQ. (10-42): $\lambda = \frac{1}{2}\int_0^L \left(\frac{dv}{dx}\right)^2 dx = \frac{17q^2L^7}{40{,}320E^2I^2} \quad \longleftarrow$

(b) BENDING STRESS

$M_{max} = \frac{qL^2}{8} \qquad c = \frac{h}{2}$

$\sigma_b = \frac{Mc}{I} = \frac{qhL^2}{16I} \quad \longleftarrow$

(c) IMMOVABLE SUPPORTS



EQ. (10-45): $H = \frac{EA\lambda}{L}$

EQ. (10-46): $\sigma_t = \frac{H}{A} = \frac{E\lambda}{L} = \frac{17q^2L^6}{40{,}320EI^2} \quad \longleftarrow$

(d) NUMERICAL VALUES $\qquad q = 25$ kN/m

$L = 3$ m $\quad h = 300$ mm $\quad E = 200$ GPa $\quad I = 36 \times 10^6$ mm^4

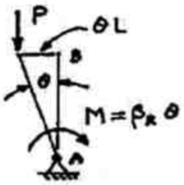
$\lambda = 0.01112$ mm $\qquad \sigma_b = 117.2$ MPa $\longleftarrow$

$\sigma_t = 0.7411$ MPa $\longleftarrow$

THE BENDING STRESS IS MUCH LARGER THAN THE AXIAL TENSILE STRESS DUE TO CURVATURE SHORTENING.

– END OF CHAPTER 10 –

11.2-1

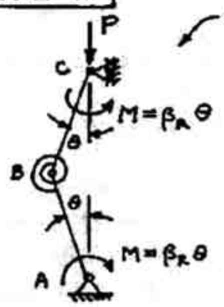


$\Sigma M_A = 0$

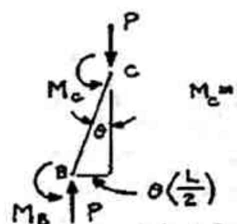
$P(\theta L) - \beta_R \theta = 0$

$P_{cr} = \dfrac{\beta_R}{L}$ ←

11.2-2



$\Sigma M_A = 0$ SHOWS THAT THERE ARE NO HORIZONTAL REACTIONS AT THE SUPPORTS.



$M_C = \beta_R \theta$

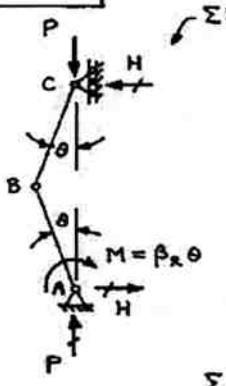
$M_B = \beta_R (2\theta)$

$\Sigma M_B = 0 \quad M_B + M_C - P\theta\left(\frac{L}{2}\right) = 0$

$\beta_R(2\theta) + \beta_R \theta = \dfrac{PL\theta}{2}$

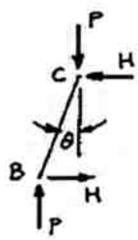
$P_{cr} = \dfrac{6\beta_R}{L}$ ←

11.2-3



$\Sigma M_A = 0 \quad HL - \beta_R \theta = 0$

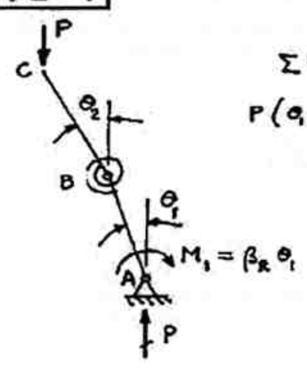
$H = \dfrac{\beta_R \theta}{L}$



$\Sigma M_B = 0 \quad H\left(\frac{L}{2}\right) - P\left(\frac{\theta L}{2}\right) = 0$

$P_{cr} = \dfrac{H}{\theta} = \dfrac{\beta_R}{L}$ ←

11.2-4



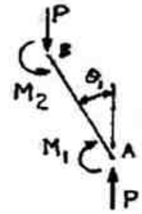
$\Sigma M_A = 0$

$P\left(\theta_1 \frac{L}{2} + \theta_2 \frac{L}{2}\right) - \beta_R \theta_1 = 0$

$\theta_2 = \left(\dfrac{2\beta_R}{PL} - 1\right)\theta_1 \qquad (1)$

CONT.

11.2-4 CONT.



$M_1 = \beta_R \theta_1 \qquad M_2 = \beta_R(\theta_2 - \theta_1)$

$\Sigma M_A = 0 \quad M_2 - M_1 + P\left(\frac{\theta_1 L}{2}\right) = 0$

$\beta_R(\theta_2 - \theta_1) - \beta_R \theta_1 + \frac{PL}{2}\theta_1 = 0$

SOLVE FOR θ_2:

$\theta_2 = \left(2 - \dfrac{PL}{2\beta_R}\right)\theta_1 \qquad (2)$

COMBINE EQS. (1) AND (2) AND REARRANGE:

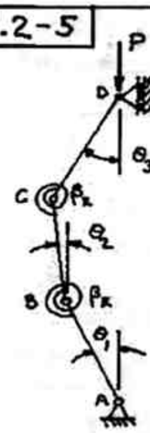
$(PL)^2 - 6\beta_R(PL) + 4\beta_R^2 = 0$

SOLVE FOR THE LOWER VALUE OF PL:

$PL = (3 - \sqrt{5})\beta_R \qquad P_{cr} = (3 - \sqrt{5})\dfrac{\beta_R}{L}$

$= 0.764\,\dfrac{\beta_R}{L}$ ←

11.2-5



$\Sigma M_A = 0$ SHOWS THAT THERE ARE NO HORIZONTAL REACTIONS AT THE SUPPORTS.

$\theta_1\left(\frac{L}{3}\right) + \theta_2\left(\frac{L}{3}\right) = \theta_3\left(\frac{L}{3}\right)$

$\therefore\ \theta_2 = \theta_3 - \theta_1$

SPRING AT B:

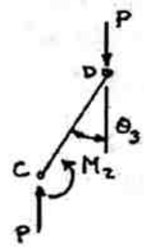
$M_1 = \beta_R(\theta_1 - \theta_2)$

$= \beta_R(2\theta_1 - \theta_3)$

SPRING AT C:

$M_2 = \beta_R(\theta_3 + \theta_2)$

$= \beta_R(2\theta_3 - \theta_1)$

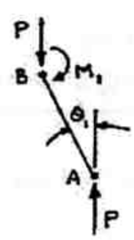


$\Sigma M_C = 0 \quad M_2 - P\left(\frac{\theta_3 L}{3}\right) = 0$

$\beta_R(2\theta_3 - \theta_1) - \frac{PL}{3}\theta_3 = 0$

SOLVE FOR θ_3:

$\theta_3 = \left(\dfrac{3\beta_R}{6\beta_R - PL}\right)\theta_1 \qquad (1)$



$\Sigma M_A = 0 \quad -M_1 + P\left(\frac{\theta_1 L}{3}\right) = 0$

$-\beta_R(2\theta_1 - \theta_3) + \frac{PL}{3}\theta_1 = 0$

SOLVE FOR θ_2:

$\theta_3 = \left(\dfrac{6\beta_R - PL}{3\beta_R}\right)\theta_1 \qquad (2)$

COMBINE EQS. (1) AND (2) AND REARRANGE:

$(PL)^2 - 12\beta_R(PL) + 27\beta_R^2 = 0$

SOLVE FOR THE LOWER VALUE OF PL:

$PL = 3\beta_R \qquad P_{cr} = \dfrac{3\beta_R}{L}$ ←

NOTE: FOR THIS VALUE OF P, WE OBTAIN $\theta_3 = \theta_1$, $\theta_2 = 0$.

11.3-1

W 8 x 35 COLUMN WITH PINNED SUPPORTS

$L = 24 \text{ FT} = 288 \text{ IN.} \quad E = 30 \times 10^6 \text{ PSI}$

$I_1 = 127 \text{ IN.}^4 \quad I_2 = 42.6 \text{ IN.}^4 \quad A = 10.3 \text{ IN.}^2$

(a) BUCKLING ABOUT STRONG AXIS

$$P_{CR} = \frac{\pi^2 E I_1}{L^2} = 453 \text{ K} \longleftarrow$$

(b) BUCKLING ABOUT WEAK AXIS

$$P_{CR} = \frac{\pi^2 E I_2}{L^2} = 152 \text{ K} \longleftarrow$$

NOTE: $\sigma_{CR} = \frac{P_{CR}}{A} = \frac{453 \text{ K}}{10.3 \text{ in}^2} = 44 \text{ KSI}$

$\therefore$ SOLUTION IS SATISFACTORY IF $\sigma_{PL} \geq 44$ KSI

11.3-2

W 10 x 60 COLUMN WITH PINNED SUPPORTS

$L = 30 \text{ FT} = 360 \text{ IN.} \quad E = 30 \times 10^6 \text{ PSI}$

$I_1 = 341 \text{ IN.}^4 \quad I_2 = 116 \text{ IN.}^4 \quad A = 17.6 \text{ IN.}^2$

(a) BUCKLING ABOUT STRONG AXIS

$$P_{CR} = \frac{\pi^2 E I_1}{L^2} = 779 \text{ K} \longleftarrow$$

(b) BUCKLING ABOUT WEAK AXIS

$$P_{CR} = \frac{\pi^2 E I_2}{L^2} = 265 \text{ K} \longleftarrow$$

NOTE: $\sigma_{CR} = \frac{P_{CR}}{A} = \frac{779 \text{ K}}{17.6 \text{ IN.}^2} = 44 \text{ KSI}$

$\therefore$ SOLUTION IS SATISFACTORY IF $\sigma_{PL} \geq 44$ KSI

11.3-3

W 10 X 45 COLUMN WITH PINNED SUPPORTS

$L = 28 \text{ FT} = 336 \text{ IN.} \quad E = 30 \times 10^6 \text{ PSI}$

$I_1 = 248 \text{ IN.}^4 \quad I_2 = 53.4 \text{ IN.}^4 \quad A = 13.3 \text{ IN.}^2$

(a) BUCKLING ABOUT STRONG AXIS

$$P_{CR} = \frac{\pi^2 E I_1}{L^2} = 650 \text{ K} \longleftarrow$$

(b) BUCKLING ABOUT WEAK AXIS

$$P_{CR} = \frac{\pi^2 E I_2}{L^2} = 140 \text{ K} \longleftarrow$$

NOTE: $\sigma_{CR} = \frac{P_{CR}}{A} = \frac{650 \text{ K}}{13.3 \text{ IN.}^2} = 49 \text{ KSI}$

$\therefore$ SOLUTION IS SATISFACTORY IF $\sigma_{PL} \geq 49$ KSI

11.3-4

FOR COLUMN CD:

$E = 200 \text{ GPa} \quad L = 1.8 \text{ m} \quad b = 50 \text{ mm} \quad n = 2.0$

$I = \frac{b^4}{12} = 520.8 \times 10^3 \text{ mm}^4$

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 E I}{n L^2} = 158.6 \text{ kN}$$

FOR BEAM ACB: $Q_{ALLOW} = \frac{P_{ALLOW}}{3}$

$$= 52.9 \text{ kN} \longleftarrow$$

11.3-5

FOR COLUMN CD:

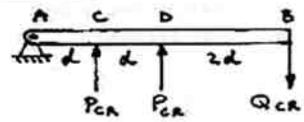
$E = 10 \times 10^6 \text{ PSI} \quad L = 25 \text{ IN.} \quad b = 1.5 \text{ IN.} \quad n = 1.8$

$I = \frac{b^4}{12} = 0.4219 \text{ IN.}^4$

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 E I}{n L^2} = 37.01 \text{ K}$$

FOR BEAM ACB: $Q_{ALLOW} = \frac{P_{ALLOW}}{3} = 12.3 \text{ K} \longleftarrow$

11.3-6



COLLAPSE OCCURS WHEN BOTH COLUMNS REACH THE CRITICAL LOAD.

$P_{CR} = \frac{\pi^2 E I}{L^2}$

$$\Sigma M_A = 0 \quad Q_{CR} = \frac{3}{4} P_{CR} = \frac{3\pi^2 E I}{4L^2} \longleftarrow$$

11.3-7

(TEMPERATURE INCREASE = ΔT)

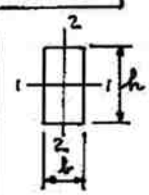
P = AXIAL COMPRESSIVE FORCE IN THE BAR

EQ. (2-17) OF CHAPTER 2: $P = EA\alpha(\Delta T)$

FOR BUCKLING: $P = P_{CR} = \frac{\pi^2 E I}{L^2}$

$$\therefore \Delta T = \frac{\pi^2 I}{\alpha A L^2} \longleftarrow$$

11.3-8



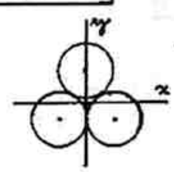
$P_1 = \frac{\pi^2 E I_1}{L^2} \qquad P_2 = \frac{\pi^2 E I_2}{(L/2)^2} = \frac{4\pi^2 E I_2}{L^2}$

$P_1 = P_2 \quad \therefore I_1 = 4 I_2$

$I_1 = \frac{bh^3}{12} \quad I_2 = \frac{hb^3}{12}$

$$bh^3 = 4hb^3 \qquad \frac{h}{b} = 2 \longleftarrow$$

11.3-9



r = RADIUS $\quad L$ = LENGTH

(a) RODS ACT INDEPENDENTLY

$P_{CR} = \frac{\pi^2 E I}{L^2}(3) \qquad I = \frac{\pi r^4}{4}$

$$P_{CR} = \frac{3\pi^3 E r^4}{4L^2} \longleftarrow$$

(b) RODS ARE BONDED TOGETHER

THE x AND y AXES HAVE THEIR ORIGIN AT THE CENTROID OF THE CROSS SECTION. BECAUSE THERE ARE THREE DIFFERENT CENTROIDAL AXES OF SYMMETRY, ALL CENTROIDAL AXES ARE PRINCIPAL AXES AND ALL CENTROIDAL MOMENTS OF INERTIA ARE EQUAL. (SEE SECTION 12.9)

FROM CASE 9, APPENDIX D:

$$I = I_y = \frac{\pi r^4}{4} + 2\left(\frac{5\pi r^4}{4}\right) = \frac{11\pi r^4}{4}$$

$$P_{CR} = \frac{\pi^2 E I}{L^2} = \frac{11\pi^3 E r^4}{4L^2} \longleftarrow$$

NOTE: JOINING THE RODS INCREASES THE CRITICAL LOAD BY A FACTOR OF 3.67.

11.3-10

E, L, AND A ARE THE SAME.

$P_{CR} = \frac{\pi^2 EI}{L^2}$ $\quad \therefore P_1 : P_2 : P_3 = I_1 : I_2 : I_3$

(1) CIRCLE CASE 9, APPENDIX D

$I = \frac{\pi d^4}{64}$ $\quad A = \frac{\pi d^2}{4}$ $\quad \therefore I_1 = \frac{A^2}{4\pi}$

(2) SQUARE CASE 1, APPENDIX D

$I = \frac{b^4}{12}$ $\quad A = b^2$ $\quad \therefore I_2 = \frac{A^2}{12}$

(3) EQUILATERAL TRIANGLE CASE 5

$I = \frac{b^4\sqrt{3}}{96}$ $\quad A = \frac{b^2\sqrt{3}}{4}$ $\quad \therefore I_3 = \frac{A^2\sqrt{3}}{18}$

$P_1 : P_2 : P_3 = 1 : \frac{\pi}{3} : \frac{2\pi\sqrt{3}}{9} = 1.000 : 1.047 : 1.209$ ⟵

NOTE: FOR EACH OF THE ABOVE CROSS SECTIONS, EVERY CENTROIDAL AXIS HAS THE SAME MOMENT OF INERTIA.

11.3-11

W 10 × 45 $\quad E = 30 \times 10^6$ PSI

$I_1 = 248$ IN.^{4} $\quad I_2 = 53.4$ IN.^{4} $\quad L = 18$ FT

BUCKLING ABOUT AXIS 1-1

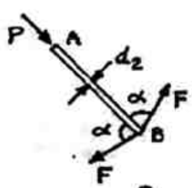
$$P_{CR} = \frac{\pi^2 EI_1}{(2L)^2} = 393.5 \text{ K}$$

BUCKLING ABOUT AXIS 2-2

$$P_{CR} = \frac{\pi^2 EI_2}{L^2} = 338.9 \text{ K}$$

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{338.9 \text{ K}}{2.5} = 136 \text{ K} \longleftarrow$$

11.3-12



$\alpha = 75°$ $\quad E = 72$ GPa

$d_2 = 50$ MM $\quad d_1 = 40$ MM

$L = 1.0$ M $\quad n = 2.25$

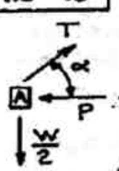
$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 181.13 \times 10^3 \text{ MM}^4$$

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 EI}{nL^2} = 57.21 \text{ KN}$$

JOINT B: $P = 2F \cos 75°$

$$\therefore F_{ALLOW} = \frac{P_{ALLOW}}{2\cos 75°} = 110 \text{ KN} \longleftarrow$$

11.3-13



$\tan\alpha = \frac{7}{10}$ $\quad L = 8.5$ FT $= 102$ IN.

$d_2 = 2.75$ IN. $\quad d_1 = 2.25$ IN.

$E = 29 \times 10^6$ PSI $\quad n = 2.0$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.549 \text{ IN.}^4$$

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 EI}{nL^2} = 21.31 \text{ K}$$

JOINT A: $\left.\begin{aligned} \Sigma F_{HORIZ} &= 0 \quad -P + T\cos\alpha = 0 \\ \Sigma F_{VERT} &= 0 \quad T\sin\alpha - \frac{W}{2} = 0 \end{aligned}\right\}$

SOLVE FOR W: $W = 2P\tan\alpha$

$$\therefore W_{MAX} = 2P_{ALLOW}\tan\alpha = 29.8 \text{ K} \longleftarrow$$

11.3-14

HOLLOW TUBE

$E = 72$ GPa $\quad L = 1.8$ M $\quad d = 50$ MM (OUTSIDE)

$P = 18$ KN $\quad n = 2.0$ $\quad$ FIND THICKNESS t

$$P_{CR} = nP_{ALLOW} = 2.0(18 \text{ KN}) = 36 \text{ KN}$$

$P_{CR} = \frac{\pi^2 EI}{L^2}$ $\quad P_{CR}L^2 = \pi^2 EI$

UNITS: NEWTONS AND METERS

$$(36{,}000 \text{ N})(1.8 \text{ M})^2 = \pi^2(72 \times 10^9 \text{ N/M}^2)I$$

$$I = 164.14 \times 10^{-9} \text{ M}^4$$

$$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right]$$

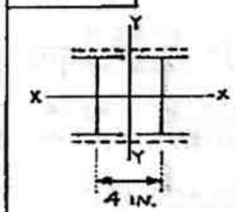
$$d^4 - (d-2t)^4 = 3.3438 \times 10^{-6} \text{ M}^4$$

$$(d-2t)^4 = (0.050 \text{ M})^4 - 3.3438 \times 10^{-6} \text{ M}^4 = 2.9062 \times 10^{-6} \text{ M}^4$$

$$d - 2t = 0.04129 \text{ M} = 41.29 \text{ MM}$$

$$2t = 8.71 \text{ MM} \quad t_{MIN} = 4.36 \text{ MM} \longleftarrow$$

11.3-15



S 6 × 17.25 $\quad E = 30 \times 10^6$ PSI

$I_1 = 26.3$ IN.^{4} $\quad I_2 = 2.31$ IN.^{4}

$L = 27.5$ FT $= 330$ IN.

$I_X = 2I_1 = 52.6$ IN.^{4}

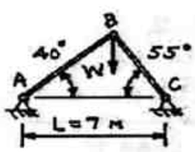
$I_Y = 2(I_2 + Ad^2)$

$A = 5.07$ IN.^{2} $\quad d = 2.0$ IN.

$I_Y = 45.18$ IN.^{4}

$$P_{CR} = \frac{\pi^2 EI_Y}{L^2} = 123 \text{ K} \longleftarrow$$

11.3-16



$E = 200$ GPa $\quad L = 7$ M

$d_2 = 100$ MM $\quad t = 6$ MM

$d_1 = 88$ MM

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.965 \times 10^6 \text{ MM}^4$$

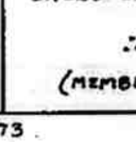
FROM THE LAW OF SINES (APP. C):

$L_{AB} = 5.756$ M $\quad L_{BC} = 4.517$ M

BUCKLING OCCURS WHEN EITHER MEMBER REACHES ITS CRITICAL LOAD.

$$(P_{CR})_{AB} = \frac{\pi^2 EI}{L_{AB}^2} = 117.1 \text{ KN}$$

$$(P_{CR})_{BC} = \frac{\pi^2 EI}{L_{BC}^2} = 190.1 \text{ KN}$$



AT JOINT B:

$\Sigma F_{HORIZ} = 0 \quad F_{AB}\sin 50° - F_{BC}\sin 35° = 0$

$\Sigma F_{VERT} = 0 \quad F_{AB}\cos 50° + F_{BC}\cos 35° - W = 0$

SOLVE THE TWO EQUATIONS:

$W = 1.7368\, F_{AB}$ $\quad W = 1.3004\, F_{BC}$

BASED ON AB: $W_{CR} = 1.7368(P_{CR})_{AB} = 203$ KN

BASED ON BC: $W_{CR} = 1.3004(P_{CR})_{BC} = 247$ KN

$$\therefore W_{CR} = 203 \text{ KN} \longleftarrow$$

(MEMBER AB BUCKLES)

11.3-17

FRAMEWORK ABCD WITH STRUT AB

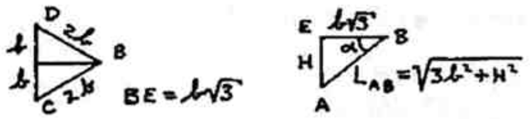
$W = 2000$ LB $\quad b = 45$ IN.

STRUT AB $\quad E = 30 \times 10^6$ PSI $\quad n = 2$

$d_2 = 2.0$ IN. $\quad d_1 = 1.8$ IN.

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 0.27010 \text{ IN.}^4$$

TRIANGLE BCD $\qquad$ TRIANGLE BEA

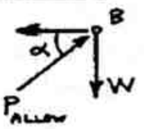


CRITICAL LOAD OF STRUT AB

$$P_{CR} = \frac{\pi^2 EI}{L_{AB}^2} = \frac{\pi^2 EI}{3b^2 + H^2}$$

ALLOWABLE LOAD $\quad P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 EI}{2(3b^2 + H^2)}$

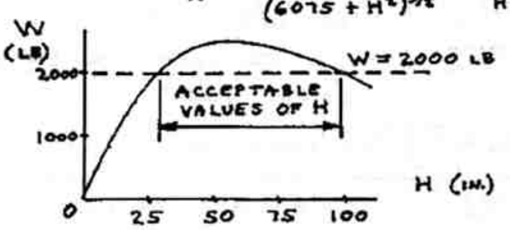
JOINT B



$$\frac{W}{P_{ALLOW}} = \frac{H}{L_{AB}} \quad \text{(SEE TRIANGLE BEA)}$$

$$W = P_{ALLOW}\left(\frac{H}{L_{AB}}\right) = \frac{\pi^2 EIH}{2(3b^2 + H^2)^{3/2}}$$

$$W = \frac{39.487 \times 10^6 H}{(6075 + H^2)^{3/2}} \quad \begin{matrix} W = \text{LB} \\ H = \text{IN.} \end{matrix}$$



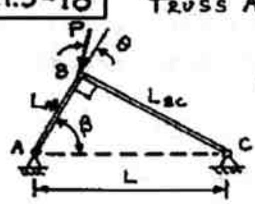
SMALLEST ALLOWABLE VALUE OF H

SET $W = 2000$ LB AND SOLVE FOR H.

$H_{MAX} = 97.8$ IN. $\quad H_{MIN} = 28.6$ IN. ⟵

11.3-18

TRUSS ABC WITH LOAD P



EQUILIBRIUM AT JOINT B:

$F_{AB} = P\cos\theta \quad F_{BC} = P\sin\theta$

FROM TRIANGLE ABC:

$L_{AB} = L\cos\beta$

$L_{BC} = L\sin\beta$

CRITICAL LOADS

$$(F_{AB})_{CR} = \frac{\pi^2 EI}{L_{AB}^2} = \frac{\pi^2 EI}{L^2\cos^2\beta}$$

$$(F_{BC})_{CR} = \frac{\pi^2 EI}{L_{BC}^2} = \frac{\pi^2 EI}{L^2\sin^2\beta}$$

MAXIMUM LOAD P BASED UPON BAR AB

$$P_{AB} = \frac{(F_{AB})_{CR}}{\cos\theta} = \frac{\pi^2 EI}{L^2\cos^2\beta\cos\theta} \quad \text{Eq. (1)}$$

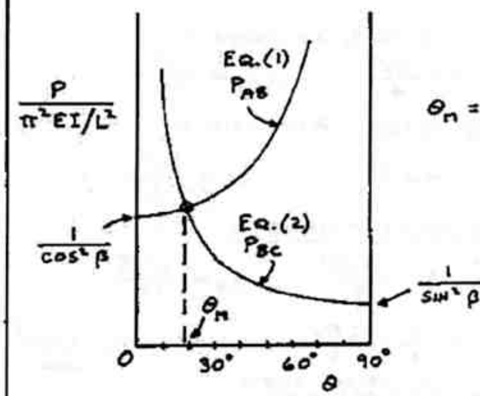
MAXIMUM LOAD P BASED UPON BAR BC

$$P_{BC} = \frac{(F_{BC})_{CR}}{\sin\theta} = \frac{\pi^2 EI}{L^2\sin^2\beta\sin\theta} \quad \text{Eq. (2)}$$

CONT.

11.3-18 CONT.

GRAPH OF LOAD P VERSUS ANGLE θ



θ_m = ANGLE FOR MAXIMUM LOAD P

(a) P HAS ITS LARGEST VALUE WHEN

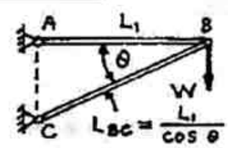
$P_{AB} = P_{BC}$

OR $\cos^2\beta\cos\theta = \sin^2\beta\sin\theta$

$\tan\theta = \cot^2\beta \qquad \theta_m = \arctan(\cot^2\beta)$ ⟵

(b) $\beta = 60°\qquad \theta_m = 18.43°$ ⟵

11.3-19



EQUILIBRIUM AT JOINT B:

$$F_{BC} = \frac{W}{\sin\theta}$$

SOLID CIRCULAR BAR:

$$I = \frac{\pi d^4}{64} \quad A = \frac{\pi d^2}{4}$$

$$I = \frac{A^2}{4\pi}$$

$$P_{CR} = \frac{\pi^2 EI}{L_{BC}^2} = \frac{\pi EA^2\cos^2\theta}{4L_1^2}$$

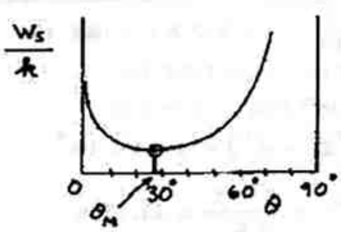
$$F_{BC} = P_{CR} \quad \frac{W}{\sin\theta} = \frac{\pi EA^2\cos^2\theta}{4L_1^2}$$

SOLVE FOR AREA A:

$$A = \left(\frac{4WL_1^2}{\pi E\cos^2\theta\sin\theta}\right)^{1/2} \qquad \gamma = \text{WEIGHT DENSITY}$$

$$\text{WEIGHT OF STRUT} = W_S = \gamma A L_{BC} = \gamma A\left(\frac{L_1}{\cos\theta}\right)$$

$$W_S = \frac{2\gamma L_1^2}{\cos^2\theta}\left(\frac{W}{\pi E\sin\theta}\right)^{1/2} = \frac{k}{\cos^2\theta\sqrt{\sin\theta}}$$



θ_m = ANGLE FOR MINIMUM WEIGHT

FOR MINIMUM WEIGHT, $\cos^2\theta\sqrt{\sin\theta}$ IS A MAXIMUM.

$$\frac{d}{d\theta}\left(\cos^2\theta\sqrt{\sin\theta}\right) = 0 \qquad 4\sin^2\theta = \cos^2\theta$$

$\tan\theta = \frac{1}{2} \qquad \theta_m = \arctan\frac{1}{2} = 26.57°$ ⟵

11.4-1 ALUMINUM PIPE COLUMN

$d_2 = 6.0$ IN. $d_1 = 5.0$ IN. $E = 10{,}400$ KSI

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 32.94 \text{ IN.}^4$ $L = 10.5$ FT $= 126$ IN.

(1) PINNED-PINNED

$$P_{CR} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(10{,}400 \text{ KSI})(32.94 \text{ IN.}^4)}{(126 \text{ IN.})^2}$$

$= 213$ K ←

(2) FIXED-FREE $P_{CR} = \frac{\pi^2 EI}{4L^2} = 53.2$ K ←

(3) FIXED-PINNED $P_{CR} = \frac{2.046\pi^2 EI}{L^2} = 436$ K ←

(4) FIXED-FIXED $P_{CR} = \frac{4\pi^2 EI}{L^2} = 852$ K ←

11.4-2 STEEL PIPE COLUMN

$d_2 = 40$ MM $d_1 = 36$ MM $E = 210$ GPa

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 43.22 \times 10^3 \text{ MM}^4$ $L = 1.5$ M

(1) PINNED-PINNED $P_{CR} = \frac{\pi^2 EI}{L^2} = 39.8$ KN ←

(2) FIXED-FREE $P_{CR} = \frac{\pi^2 EI}{4L^2} = 10.0$ KN ←

(3) FIXED-PINNED $P_{CR} = \frac{2.046\pi^2 EI}{L^2} = 81.5$ KN ←

(4) FIXED-FIXED $P_{CR} = \frac{4\pi^2 EI}{L^2} = 159$ KN ←

11.4-3 W 12 × 87 WIDE-FLANGE COLUMN

$E = 30 \times 10^6$ PSI $L = 30$ FT $= 360$ IN.

$n = 2.25$ $I_2 = 241 \text{ IN.}^4$

(1) PINNED-PINNED $P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 EI_2}{nL^2}$

$= 245$ K ←

(2) FIXED-FREE $P_{ALLOW} = \frac{\pi^2 EI_2}{4nL^2} = 61.2$ K ←

(3) FIXED-PINNED $P_{ALLOW} = \frac{2.046\pi^2 EI_2}{nL^2} = 501$ K ←

(4) FIXED-FIXED $P_{ALLOW} = \frac{4\pi^2 EI_2}{nL^2} = 979$ K ←

11.4-4 W 10 × 60 WIDE-FLANGE COLUMN

$E = 30 \times 10^6$ PSI $L = 25$ FT $= 300$ IN.

$n = 2.25$ $I_2 = 116 \text{ IN.}^4$

(1) PINNED-PINNED $P_{ALLOW} = \frac{P_{CR}}{n} = \frac{\pi^2 EI_2}{nL^2}$

$= 170$ K ←

(2) FIXED-FREE $P_{ALLOW} = \frac{\pi^2 EI_2}{4nL^2} = 42.4$ K ←

(3) FIXED-PINNED $P_{ALLOW} = \frac{2.046\pi^2 EI_2}{nL^2} = 347$ K ←

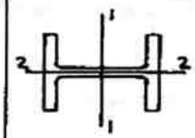
(4) FIXED-FIXED $P_{ALLOW} = \frac{4\pi^2 EI_2}{nL^2} = 678$ K ←

11.4-5

W 8 × 21 $L = 14$ FT $= 168$ IN.

$E = 30 \times 10^3$ KSI

$I_1 = 75.3 \text{ IN.}^4$ $I_2 = 9.77 \text{ IN.}^4$



AXIS 1-1 (FIXED-FREE)

$$P_{CR} = \frac{\pi^2 EI_1}{4L^2} = 197 \text{ K}$$

AXIS 2-2 (FIXED-PINNED)

$$P_{CR} = \frac{2.046\pi^2 EI_2}{L^2} = 210 \text{ K}$$

BUCKLING ABOUT AXIS 1-1 GOVERNS.

$P_{CR} = 197$ K ←

11.4-6 STEEL TUBE $E = 200$ GPa

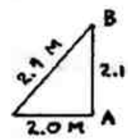
$d_2 = 40$ MM $d_1 = 30$ MM $L = 2.1$ M

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 85{,}903 \text{ MM}^4$ $n = 3.0$

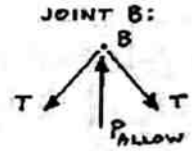
BUCKLING IN THE PLANE OF THE FIGURE MEANS FIXED-PINNED END CONDITIONS.

$$P_{CR} = \frac{2.046\pi^2 EI}{L^2} = 78.67 \text{ KN}$$

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{78.67 \text{ KN}}{3.0} = 26.22 \text{ KN}$$



JOINT B:



T = TENSILE FORCE IN A CABLE

$$\Sigma F_{VERT} = P_{ALLOW} - 2T\left(\frac{2.1 \text{ M}}{2.9 \text{ M}}\right) = 0$$

$$\therefore T_{ALLOW} = P_{ALLOW}\left(\frac{2.9}{4.2}\right) = 18.1 \text{ KN} \leftarrow$$

11.4-7 BEAM SUPPORTED BY TWO COLUMNS

COLUMN BD $E = 30 \times 10^6$ PSI $L = 35$ IN.

$b = 0.625$ IN. $I = \frac{b^4}{12} = 0.012716 \text{ IN.}^4$

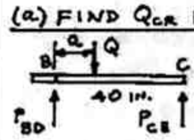
$$P_{CR} = \frac{2.046\pi^2 EI}{L^2} = 6288 \text{ LB}$$

COLUMN CE $E = 30 \times 10^6$ PSI $L = 45$ IN.

$b = 0.625$ IN. $I = \frac{b^4}{12} = 0.012716 \text{ IN.}^4$

$$P_{CR} = \frac{\pi^2 EI}{L^2} = 1859 \text{ LB}$$

(a) FIND Q_{CR} IF $a = 10$ IN.



$P_{BD} = \frac{3Q}{4}$ $P_{CE} = \frac{Q}{4}$

$Q = \frac{4}{3}P_{BD}$ $Q = 4P_{CE}$

COLUMN BD BUCKLES: $Q = \frac{4}{3}(6288 \text{ LB}) = 8380$ LB

COLUMN CE BUCKLES: $Q = 4(1859 \text{ LB}) = 7440$ LB

$\therefore Q_{CR} = 7440$ LB ←

(b) MAXIMUM VALUE OF Q_{CR}

BOTH COLUMNS BUCKLE SIMULTANEOUSLY.

$P_{BD} = 6288$ LB $P_{CE} = 1859$ LB

$\Sigma F_{VERT} = 0$ $Q_{CR} = P_{BD} + P_{CE} = 8150$ LB ←

$\Sigma M_B = 0$ $Q_{CR}(a) = P_{CE}(40 \text{ IN.})$

$$a = \frac{(1859 \text{ LB})(40 \text{ IN.})}{6288 \text{ LB} + 1859 \text{ LB}} = 9.13 \text{ IN.} \leftarrow$$

11.4-8 PIPE COLUMN

$E = 210$ GPa
$L = 4.9$ m

$d_2 = 100$ mm
$d_1 = 90$ mm

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1688 \times 10^3 \text{ mm}^4$$

(1) UPPER END IS PINNED

$$P_{CR} = \frac{2.046\pi^2 EI}{L^2} = 298 \text{ kN} \longleftarrow$$

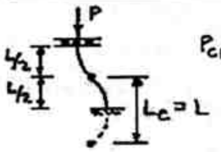
(2) UPPER END IS FIXED

$$P_{CR} = \frac{4\pi^2 EI}{L^2} = 583 \text{ kN} \longleftarrow$$

(3) UPPER END IS FREE

$$P_{CR} = \frac{\pi^2 EI}{4L^2} = 36 \text{ kN} \longleftarrow$$

(4) UPPER END IS GUIDED



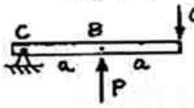
$$P_{CR} = \frac{\pi^2 EI}{L^2} = 146 \text{ kN} \longleftarrow$$

11.4-9 ALUMINUM TUBE (FIXED-PINNED)

$E = 10{,}500$ ksi $\quad L = 10$ ft $= 120$ in. $\quad n = 2.5$

$d_2 = 4.5$ in. $\quad d_1 = 4.5$ in. $- 2t$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = \frac{\pi}{64}\left[(4.5 \text{ in.})^4 - (4.5 \text{ in.} - 2t)^4\right]$$



$\Sigma M_C = 0 \quad Q = \frac{P}{2}$

$Q = 45$ k $\quad P = 90$ k

$$P_{ALLOW} = \frac{P_{CR}}{n} = \frac{2.046\pi^2 EI}{nL^2}$$

$$P = P_{ALLOW} \qquad 90 \text{ k} = \frac{2.046\pi^2(10{,}500 \text{ ksi})I}{(2.5)(120 \text{ in.})^2}$$

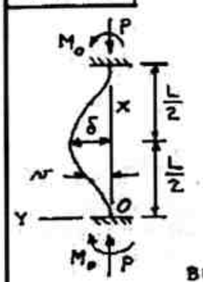
SOLVE FOR I: $\quad I = 15.281$ in.^{4}

$$\frac{\pi}{64}\left[(4.5)^4 - (4.5 - 2t)^4\right] = 15.281$$

$(4.5 - 2t)^4 = 98.761 \qquad 4.5 - 2t = 3.152$

$$t_{min} = 0.674 \text{ in.} \longleftarrow$$

11.4-10



$EIv'' = M = M_0 - Pv \qquad k^2 = \frac{P}{EI}$

$v'' + k^2 v = \frac{M_0}{EI}$

$v = C_1 \sin kx + C_2 \cos kx + \frac{M_0}{P}$

B.C. $v(0) = 0 \quad C_2 = -\frac{M_0}{P}$

B.C. $v'(0) = 0 \quad C_1 = 0$

$v = \frac{M_0}{P}(1 - \cos kx)$

B.C. $v(L) = 0 \quad 1 - \cos kL = 0$

BUCKLING EQUATION: $\cos kL = 1$

$$kL = 2\pi \qquad k^2 = \frac{4\pi^2}{L^2} \qquad P_{CR} = \frac{4\pi^2 EI}{L^2} \longleftarrow$$

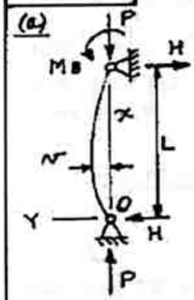
BUCKLED MODE SHAPE:
AT $x = L/2$, $v = \delta = \frac{M_0}{P}\left(1 - \cos\frac{kL}{2}\right)$

$$\delta = \frac{M_0}{P}(1 - \cos\pi) = \frac{2M_0}{P} \qquad \frac{M_0}{P} = \frac{\delta}{2}$$

$$v = \frac{\delta}{2}\left(1 - \cos\frac{2\pi x}{L}\right) \longleftarrow$$

11.4-11 COLUMN WITH ELASTIC SUPPORT

(a)



θ_B = ANGLE OF ROTATION AT UPPER END

$M_B = \beta_R \theta_B$

$\Sigma M_0 = 0 \quad H = \frac{M_B}{L} = \frac{\beta_R \theta_B}{L}$

$EIv'' = M = Hx - Pv \qquad k^2 = \frac{P}{EI}$

$v'' + k^2 v = \frac{\beta_R \theta_B}{LEI}x$

$v = C_1 \sin kx + C_2 \cos kx + \frac{\beta_R \theta_B}{PL}x$

B.C. $v(0) = 0 \quad \therefore C_2 = 0$

B.C. $v(L) = 0 \quad \therefore C_1 = -\frac{\beta_R \theta_B}{P \sin kL}$

$v' = C_1 k \cos kx + \frac{\beta_R \theta_B}{PL}$

B.C. $v'(L) = -\theta_B$

$$-\theta_B = -\frac{\beta_R \theta_B}{P \sin kL} k \cos kL + \frac{\beta_R \theta_B}{PL}$$

OR, AFTER REARRANGING, WE OBTAIN THE BUCKLING EQUATION:

$$\frac{\beta_R L}{EI}(kL \cot kL - 1) - k^2L^2 = 0 \longleftarrow$$

(b) CRITICAL LOAD FOR $\beta_R = 3EI/L$

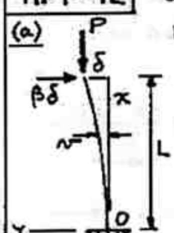
$$3(kL \cot kL - 1) - (kL)^2 = 0$$

SOLVE NUMERICALLY: $\quad kL = 3.7264$

$$P_{CR} = k^2 EI = (kL)^2 \frac{EI}{L^2} = 13.89 \frac{EI}{L^2} \longleftarrow$$

11.4-12 COLUMN WITH ELASTIC SUPPORT

(a)



$EIv'' = M = P(\delta - v) - \beta\delta(L - x)$

$k^2 = P/EI$

$v'' + k^2 v = \frac{\delta}{EI}(P - \beta L + \beta x)$

$v = C_1 \sin kx + C_2 \cos kx + \frac{\delta}{P}(P - \beta L + \beta x)$

$v' = C_1 k \cos kx - C_2 k \sin kx + \frac{\beta\delta}{P}$

B.C. $v(0) = 0 \quad \therefore C_2 + \delta\left(1 - \frac{\beta L}{P}\right) = 0 \qquad (1)$

B.C. $v'(0) = 0 \quad \therefore C_1 + \delta\left(\frac{\beta}{Pk}\right) = 0 \qquad (2)$

B.C. $v(L) = \delta \quad \therefore C_1 \tan kL + C_2 = 0 \qquad (3)$

FROM EQ. (3): $\quad C_2 = -C_1 \tan kL$

SUBSTITUTE C_2 IN EQ. (1): $\quad C_1 = \delta\left(1 - \frac{\beta L}{P}\right)\cot kL$

SUBSTITUTE C_1 IN EQ. (2) AND REARRANGE TO OBTAIN THE BUCKLING EQUATION:

$$\frac{\beta L^3}{EI}(\tan kL - kL) + (kL)^3 = 0 \longleftarrow$$

(b) CRITICAL LOAD FOR $\beta = 2EI/L^3$

$$2(\tan kL - kL) + (kL)^3 = 0$$

SOLVE NUMERICALLY: $\quad kL = 2.0174$

$$P_{CR} = k^2 EI = (kL)^2 \frac{EI}{L^2} = 4.070 \frac{EI}{L^2} \longleftarrow$$

11.5-1 COLUMN WITH ECCENTRIC LOADS (SEE FIG. 11-21)

EQ. (11-49): $v = -e\left(\tan\frac{kL}{2}\sin kx + \cos kx - 1\right)$

$M = Pe - Pv = Pe\left(\tan\frac{kL}{2}\sin kx + \cos kx\right)$ ←

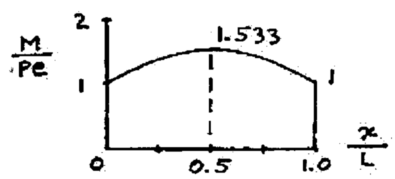
FOR $P = 0.3\,P_{CR}$:

EQ. (11-52): $kL = \pi\sqrt{\frac{P}{P_{CR}}} = \pi\sqrt{0.3} = 1.7207$

$\frac{M}{Pe} = 1.162\left(\sin 1.721\frac{x}{L}\right) + \cos 1.721\frac{x}{L}$ ←

BENDING-MOMENT DIAGRAM

$P = 0.3\,P_{CR}$



11.5-2 LOAD-DEFLECTION DIAGRAM

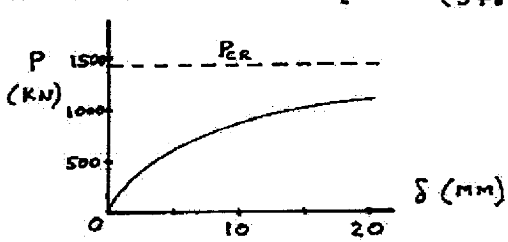
EQ. (11-54): $\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{CR}}}\right) - 1\right]$

$e = 5$ MM $L = 3.6$ M $E = 210$ GPa $I = 9.0 \times 10^6$ MM^4

$P_{CR} = \frac{\pi^2 EI}{L^2} = 1439$ KN

$\delta = 5(\sec 0.04140\sqrt{P} - 1)$ P = KN, δ = MM

SOLVE FOR P: $P = 583.4\left[\arccos\left(\frac{5}{5+\delta}\right)\right]^2$



11.5-3 LOAD-DEFLECTION DIAGRAM

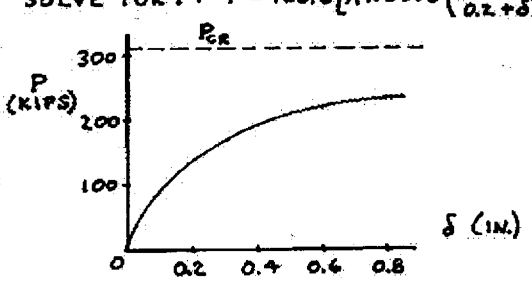
EQ. (11-54): $\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{CR}}}\right) - 1\right]$

$e = 0.20$ IN. $L = 144$ IN. $E = 30 \times 10^6$ PSI $I = 21.7$ IN.^{4}

$P_{CR} = \frac{\pi^2 EI}{L^2} = 309.9$ K

$\delta = 0.20(\sec 0.08924\sqrt{P} - 1)$ P = KIPS, δ = INCHES

SOLVE FOR P: $P = 125.6\left[\arccos\left(\frac{0.2}{0.2+\delta}\right)\right]^2$



11.5-4 BAR WITH SQUARE CROSS SECTION

$b = 50$ MM $L = 2$ M $P = 60$ KN $e = 25$ MM

$E = 210$ GPa $I = \frac{b^4}{12} = 520.8 \times 10^3$ MM^4

$kL = L\sqrt{\frac{P}{EI}} = 1.481$

EQ. (11-51): $\delta = e\left(\sec\frac{kL}{2} - 1\right) = 8.87$ MM ←

EQ. (11-56): $M_{MAX} = Pe\sec\frac{kL}{2} = 2.03$ KN·M ←

11.5-5 BAR WITH RECTANGULAR CROSS SECTION

$b = 2.0$ IN. $h = 1.0$ IN. $L = 30$ IN.

$P = 2800$ LB $e = 0.5$ IN. $E = 10 \times 10^6$ PSI

$I = \frac{bh^3}{12} = 0.1667$ IN.^{4} $kL = L\sqrt{\frac{P}{EI}} = 1.230$

EQ. (11-51): $\delta = e\left(\sec\frac{kL}{2} - 1\right) = 0.112$ IN. ←

EQ. (11-56): $M_{MAX} = Pe\sec\frac{kL}{2} = 1710$ LB-IN. ←

11.5-6 W 8 × 15 E = 29,000 KSI

$L = 20$ FT $= 240$ IN. $\delta_{MAX} = 0.25$ IN.

$I = 48.0$ IN.^{4} $e = \frac{8.11\text{ IN.}}{2} = 4.055$ IN.

$P_{CR} = \frac{\pi^2 EI}{L^2} = 238{,}500$ LB

EQ. (11-54): $\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{CR}}}\right) - 1\right]$

$0.25\text{ IN.} = 4.055\text{ IN.}\left[\sec(0.003216\sqrt{P}) - 1\right]$

REARRANGE AND SIMPLIFY:

$\cos(0.003216\sqrt{P}) = 0.9419$

$0.003216\sqrt{P} = 0.3426$ $P_{ALLOW} = 11{,}300$ LB ←

11.5-7 W 10 × 30 E = 29,000 KSI

$I = 16.7$ IN.^{4} $e = \frac{5.810\text{ IN.}}{2} = 2.905$ IN.

$\delta = \frac{L}{400}$ $P = 20$ K $k = \sqrt{\frac{P}{EI}} = 0.006426$ IN.$^{-1}$

EQ. (11-51): $\delta = e\left(\sec\frac{kL}{2} - 1\right)$

$\frac{L}{400} = (2.905\text{ IN.})\left[\sec(0.003213\,L) - 1\right]$

REARRANGE AND SIMPLIFY:

$\sec(0.003213\,L) - 1 - \frac{L}{1162} = 0$

SOLVE NUMERICALLY:

$L_{MAX} = 150.5$ IN. $= 12.5$ FT ←

11.5-8 W 10 × 30 E = 29,000 KSI

$I = 16.7$ IN.^{4} $e = \frac{5.810\text{ IN.}}{2} = 2.905$ IN.

$\delta = \frac{L}{400}$ $P = 25$ K $k = \sqrt{\frac{P}{EI}} = 0.007185$ IN.$^{-1}$

EQ. (11-51): $\delta = e\left(\sec\frac{kL}{2} - 1\right)$

$\frac{L}{400} = (2.905\text{ IN.})\left[\sec(0.003592\,L) - 1\right]$

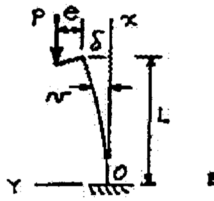
REARRANGE AND SIMPLIFY:

$\sec(0.003592\,L) - 1 - \frac{L}{1162} = 0$

SOLVE NUMERICALLY:

$L_{MAX} = 122.6$ IN. $= 10.2$ FT ←

11.5-9 FIXED-FREE COLUMN



$EIv'' = M = P(e + \delta - v)$

$v'' + k^2 v = k^2(e+\delta) \qquad k^2 = \frac{P}{EI}$

$v = C_1 \sin kx + C_2 \cos kx + e + \delta$

$v' = C_1 k \cos kx - C_2 k \sin kx$

B.C. $v(0) = 0 \qquad C_2 = -e - \delta$

B.C. $v'(0) = 0 \qquad C_1 = 0$

$v = (e+\delta)(1 - \cos kx)$

B.C. $v(L) = \delta \quad \therefore \ \delta = e(\sec kL - 1)$ ←

$M_{max} = P(e+\delta) = Pe \sec kL$ ←

NOTE: $v = e(\sec kL)(1 - \cos kx)$

11.5-10 FIXED-FREE COLUMN

δ = DEFLECTION AT THE TOP

USE EQ. (11-51) WITH L REPLACED BY THE EQUIVALENT LENGTH 2L:

$\delta = e(\sec kL - 1)$

SOLVE FOR L: $L = \frac{1}{k} \arccos \frac{e}{e+\delta}$

$k = \sqrt{\frac{P}{EI}} \qquad L = \sqrt{\frac{EI}{P}} \arccos \frac{e}{e+\delta} \qquad (1)$

SUBSTITUTE NUMERICAL VALUES:

$E = 73$ GPa $\quad b = 100$ mm $\quad t = 8$ mm

$P = 50$ kN $\quad \delta = 30$ mm $\quad e = 50$ mm

$I = \frac{1}{12}b^4 - \frac{1}{12}(b-2t)^4 = 4.184 \times 10^6$ mm^4

FROM EQ. (1): $L_{max} = 2.21$ m ←

11.5-11 FIXED-FREE COLUMN

δ = DEFLECTION AT THE TOP

USE EQ. (11-51) WITH L REPLACED BY THE EQUIVALENT LENGTH 2L:

$\delta = e(\sec kL - 1)$

SOLVE FOR L: $L = \frac{1}{k} \arccos \frac{e}{e+\delta}$

$k = \sqrt{\frac{P}{EI}} \qquad L = \sqrt{\frac{EI}{P}} \arccos \frac{e}{e+\delta} \qquad (1)$

SUBSTITUTE NUMERICAL VALUES:

$E = 10.6 \times 10^3$ ksi $\quad b = 6.0$ in. $\quad t = 0.5$ in.

$P = 30$ k $\quad \delta = 2.0$ in. $\quad e = 3.0$ in.

$I = \frac{1}{12}b^4 - \frac{1}{12}(b-2t)^4 = 55.917$ in.^{4}

FROM EQ. (1): $L_{max} = 130.3$ in. $= 10.9$ ft ←

11.5-12 FIXED-FREE COLUMN

δ = DEFLECTION AT THE TOP

USE EQ. (11-51) WITH L REPLACED BY THE EQUIVALENT LENGTH 2L:

$\delta = e(\sec kL - 1) \qquad k = \sqrt{\frac{P}{EI}}$

$\delta = e\left(\sec\sqrt{\frac{PL^2}{EI}} - 1\right)$

SOLVE FOR P:

$\sqrt{\frac{PL^2}{EI}} = \arccos \frac{e}{e+\delta}$

$P = \frac{EI}{L^2}\left(\arccos \frac{e}{e+\delta}\right)^2 \qquad (1)$

CONT.

11.5-12 CONT.

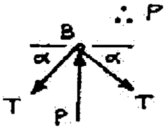
SUBSTITUTE NUMERICAL VALUES:

$E = 205$ GPa $\quad L = 4.0$ m $\quad e = 100$ mm

$\delta = 20$ mm $\quad d_2 = 110$ mm $\quad d_1 = 96$ mm

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.018 \times 10^6$ mm^4

$\therefore P = 13.26$ kN



$\alpha = 53.13°$

$\Sigma F_{VERT} = 0$

$P - 2T \sin\alpha = 0$

$T = \frac{P}{2\sin\alpha} = \frac{5}{8}P$

$T_{max} = \frac{5}{8}(13.26 \text{ kN}) = 8.29$ kN ←

11.5-13 FIXED-FREE COLUMN

USE EQ. (11-56) WITH L REPLACED BY $L_e = 2L$:

$M_{max} = Pe \sec kL \qquad k = \sqrt{\frac{P}{EI}}$

$M_{max} = Pe \sec\sqrt{\frac{PL^2}{EI}} \qquad E = 30 \times 10^3$ ksi $\quad M_{max} = 40$ k-in.

S 8 × 23:

$I_2 = 4.31$ in.$^4 \quad b = 4.171$ in. $\quad e = \frac{b}{2} = 2.086$ in.

$L = 14$ ft $= 168$ in.

SUBSTITUTE NUMERICAL VALUES:

40 k-in. $= P(2.086 \text{ in.}) \sec(0.4672\sqrt{P})$

SOLVE NUMERICALLY: $P = 6.73$ k ←

11.5-14 FIXED-FREE COLUMN

USE EQ. (11-56) WITH L REPLACED BY $L_e = 2L$:

$M_{max} = Pe \sec kL \qquad k = \sqrt{\frac{P}{EI}}$

$M_{max} = Pe \sec\sqrt{\frac{PL^2}{EI}} \qquad E = 30 \times 10^3$ ksi $\quad M_{max} = 70$ k-in.

S 10 × 35:

$I_2 = 8.36$ in.$^4 \quad b = 4.944$ in. $\quad e = \frac{b}{2} = 2.472$ in.

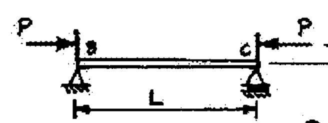
$L = 16$ ft $= 192$ in.

SUBSTITUTE NUMERICAL VALUES:

70 k-in. $= P(2.472 \text{ in.}) \sec(0.3834\sqrt{P})$

SOLVE NUMERICALLY: $P = 9.97$ k ←

11.5-15



$e = \frac{h}{3}$

$P = \frac{q_0 h}{2}$

W 8 × 21 $\quad E = 30 \times 10^6$ psi

$I_2 = 9.77$ in.$^4 \quad L = 20$ ft $= 240$ in.

$h = 4$ ft $= 48$ in. $\quad e = 16$ in.

(a) USE EQ. (11-56)

$M_{max} = Pe \sec\frac{kL}{2} \qquad k = \sqrt{\frac{P}{EI}} \qquad (1)$

$M_{max} = Pe \sec\sqrt{\frac{PL^2}{4EI}} \qquad M_{max} = 80$ k-in.

SUBSTITUTE NUMERICAL VALUES:

80,000 lb-in. $= P(16 \text{ in.}) \sec(0.007009\sqrt{P})$

SOLVE NUMERICALLY:

$P = 4462$ lb $\qquad q_0 = \frac{2P}{h} = 186$ lb/in. $= 2230$ lb/ft ←

(b) ONE-HALF THE LOAD

$P = 2231$ lb

SUBSTITUTE IN EQ. (1): $M_{max} = 37.7$ k-in.

RATIO $= \frac{37.7}{80.0} = 0.47$ ←

11.6-1 SQUARE CROSS SECTION PINNED ENDS

$b = 2.0$ in. $\quad L = 3$ ft $= 36$ in. $\quad P = 20$ k
$e = 0.75$ in. $\quad E = 29{,}000$ ksi

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$\frac{P}{A} = \frac{P}{b^2} = 5.0$ ksi $\quad c = \frac{b}{2} = 1.0$ in.

$I = \frac{b^4}{12} = 1.333$ in.^{4} $\quad r^2 = \frac{I}{A} = 0.3333$ in.^{2}

$\frac{ec}{r^2} = 2.25 \quad \frac{L}{r} = 62.354 \quad \frac{P}{EA} = 0.00017241$

SUBSTITUTE INTO EQ. (1):

$\sigma_{max} = 17.3$ ksi ←

(b) MAXIMUM LENGTH ($\sigma_{allow} = 18$ ksi)

SOLVE EQ. (1) FOR THE LENGTH L:

$$L = 2\sqrt{\frac{EI}{P}}\arccos\left[\frac{P(ec/r^2)}{\sigma_{max}A - P}\right] \quad (2)$$

SUBSTITUTE NUMERICAL VALUES:

$L_{max} = 46.2$ in. ←

11.6-2 SQUARE CROSS SECTION PINNED ENDS

$b = 30$ mm $\quad L = 0.6$ m $\quad \sigma_{allow} = 150$ MPa
$e = 10$ mm $\quad E = 100$ GPa

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$\sigma_{max} = 150$ MPa $\quad P = $ newtons $\quad A = b^2 = 900$ mm^2

$c = \frac{b}{2} = 15$ mm $\quad r^2 = \frac{I}{A} = \frac{b^2}{12} = 75$ mm^2

$\frac{ec}{r^2} = 2.0 \quad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.003651\sqrt{P}$

SUBSTITUTE INTO EQ. (1):

$$150 = \frac{P}{900}\left[1 + 2\sec\left(0.003651\sqrt{P}\right)\right]$$

SOLVE NUMERICALLY:

$P_{allow} = 37{,}200$ N ←

11.6-3 SQUARE CROSS SECTION PINNED ENDS

$P = 25$ k $\quad e = 2$ in. $\quad L = 54$ in.
$E = 10{,}600$ ksi $\quad \sigma_{max} = 6$ ksi

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$A = b^2 \quad c = \frac{b}{2} \quad r^2 = \frac{I}{A} = \frac{b^2}{12}$

$\frac{ec}{r^2} = \frac{12}{b} \quad \frac{L}{2r}\sqrt{\frac{P}{EA}} = \frac{4.5423}{b^2}$

SUBSTITUTE INTO EQ. (1):

$$6 = \frac{25}{b^2}\left[1 + \frac{12}{b}\sec\left(\frac{4.5423}{b^2}\right)\right]$$

SOLVE NUMERICALLY:

$b_{min} = 4.10$ in. ←

11.6-4 PIPE COLUMN WITH PINNED ENDS

$L = 2.1$ m $\quad E = 210$ GPa $\quad P = 10$ kN $\quad e = 30$ mm
$d_2 = 68$ mm $\quad d_1 = 60$ mm

$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 804.25$ mm^2

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 413.38 \times 10^3$ mm^4

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$\frac{P}{A} = 12.434$ MPa $\quad c = \frac{d_2}{2} = 34$ mm

$r^2 = \frac{I}{A} = 513.99$ mm^2 $\quad \frac{ec}{r^2} = 1.9845$

$r = 22.671$ mm $\quad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.35638$

SUBSTITUTE INTO EQ. (1):

$\sigma_{max} = 38.8$ MPa ←

(b) MAXIMUM LENGTH ($\sigma_{allow} = 50$ MPa)

SOLVE EQ. (1) FOR THE LENGTH L:

$$L = 2\sqrt{\frac{EI}{P}}\arccos\left[\frac{P(ec/r^2)}{\sigma_{max}A - P}\right] \quad (2)$$

SUBSTITUTE NUMERICAL VALUES:

$L_{max} = 5.03$ m ←

11.6-5 PIPE COLUMN WITH PINNED ENDS

$L = 5.2$ ft $= 62.4$ in. $\quad E = 30 \times 10^3$ ksi
$P = 2.0$ k $\quad e = 1.0$ in.

$d_2 = 2.2$ in. $\quad d_1 = 2.0$ in.

$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 0.65973$ in.^{2}

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 0.36450$ in.^{4}

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$\frac{P}{A} = 3.0315$ ksi $\quad c = \frac{d_2}{2} = 1.1$ in.

$r^2 = \frac{I}{A} = 0.55250$ in.^{2} $\quad \frac{ec}{r^2} = 1.9910$

$r = 0.74330$ in. $\quad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.42195$

SUBSTITUTE INTO EQ. (1):

$\sigma_{max} = 9.65$ ksi ←

(b) ALLOWABLE LOAD

$\sigma_Y = 42$ ksi $\quad n = 2 \quad$ FIND P_{allow}

SUBSTITUTE INTO EQ. (1):

$$42 = \frac{P}{0.65973}\left[1 + 1.9910\sec\left(0.29836\sqrt{P}\right)\right]$$

SOLVE NUMERICALLY: $P = P_Y = 7.184$ k

$P_{allow} = \frac{P_Y}{n} = 3.59$ k ←

11.6-6 ALUMINUM TUBE WITH PINNED ENDS

$P = 18\text{ kN}$ $e = 50\text{ mm}$ $L = 3.5\text{ m}$
$E = 73\text{ GPa}$ $\sigma_{max} = 20\text{ MPa}$ $d_1/d_2 = 0.9$

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}\left[d_2^2 - (0.9d_2)^2\right] = 0.14923\,d_2^2$$

$$\frac{P}{A} = \frac{120{,}620}{d_2^2} \qquad \left(\frac{P}{A} = \text{MPa};\ d_2 = \text{mm}\right)$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 0.016881\,d_2^4 \qquad c = \frac{d_2}{2}$$

$$r^2 = \frac{I}{A} = 0.11312\,d_2^2 \qquad r = 0.33633\,d_2$$

$$\frac{ec}{r^2} = \frac{221.00}{d_2} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = \frac{6688.4}{d_2^2}$$

SUBSTITUTE INTO EQ. (1):

$$20 = \frac{120{,}620}{d_2^2}\left[1 + \frac{221.00}{d_2}\sec\left(\frac{6688.4}{d_2^2}\right)\right]$$

SOLVE NUMERICALLY:

$$d_2 = 131\text{ mm}$$

11.6-7 COPPER TUBE WITH PINNED SUPPORTS

$L = 30\text{ in.}$ $d_2 = 2.0\text{ in.}$ $t = 0.1\text{ in.}$ $d_1 = 1.8\text{ in.}$
$E = 18{,}000\text{ ksi}$ $c = d_2/2 = 1.0\text{ in.}$

UNITS: KIPS, INCHES

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 0.59690\text{ in.}^2$$
$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 0.27010\text{ in.}^4$$
$$r^2 = \frac{I}{A} = 0.45250\text{ in.}^2$$

$r = 0.67268\text{ in.}$ $\frac{L}{r} = 44.598$ $EA = 10{,}744\text{ k}$

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

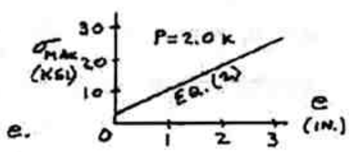
(a) COMPRESSIVE STRESS VERSUS ECCENTRICITY e

$P = 2.0\text{ k}$ $\frac{P}{A} = 3.3506\text{ ksi}$ $\frac{ec}{r^2} = 2.2099\,e$

$\frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.30424$ SUBSTITUTE INTO EQ. (1):

$$\sigma_{max} = (3.3506\text{ ksi})\left[1 + 2.2099\,e\sec(0.30424)\right]$$
$$= 3.3506(1 + 2.3163\,e) \quad (2)$$

e (IN.)	σ_{max} (KSI)
0	3.35
1	11.11
2	18.87
3	26.63



σ_{max} IS LINEAR WITH e.

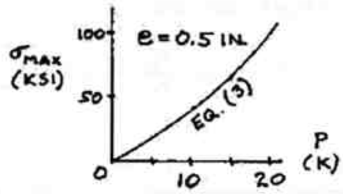
(b) COMPRESSIVE STRESS VERSUS FORCE P

$e = 0.5\text{ in.}$ $\frac{P}{A} = 1.6753\,P$ $\frac{ec}{r^2} = 1.1050$

$\frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.21513\sqrt{P}$ SUBSTITUTE INTO EQ. (1):

$$\sigma_{max} = 1.6753\,P\left[1 + 1.1050\sec(0.21513\sqrt{P})\right] \quad (3)$$

P (K)	σ_{max} (KSI)
0	0
5	18.8
10	40.6
15	66.4
20	98.3



11.6-8 W 10 × 60 COLUMN WITH PINNED ENDS

$L = 24\text{ ft} = 288\text{ in.}$ $E = 30 \times 10^3\text{ ksi}$ $e = 2.0\text{ in.}$
$A = 17.6\text{ in.}^2$ $I = 341\text{ in.}^4$ $d = 10.22\text{ in.}$

$r^2 = \frac{I}{A} = 19.38\text{ in.}^2$ $r = 4.402\text{ in.}$ $c = \frac{d}{2} = 5.11\text{ in.}$

$\frac{L}{r} = 65.42$ $\frac{ec}{r^2} = 0.5273$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$P = 120\text{ k}$ $\frac{P}{A} = 6.818\text{ ksi}$ $\frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.4931$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 10.9\text{ ksi}$

(b) ALLOWABLE LOAD

$\sigma_Y = 42\text{ ksi}$ $n = 2.5$ FIND P_{allow}

SUBSTITUTE INTO EQ. (1):

$$42 = \frac{P}{17.6}\left[1 + 0.5273\sec(0.04502\sqrt{P})\right]$$

SOLVE NUMERICALLY: $P = P_Y = 399.9\text{ k}$

$P_{allow} = P_Y/n = 160\text{ k}$

11.6-9 STEEL COLUMN WITH PINNED ENDS

W 16 × 57 $A = 16.8\text{ in.}^2$ $I = I_2 = 43.1\text{ in.}^4$
$b = 7.120\text{ in.}$
$c = b/2 = 3.560\text{ in.}$

$e = 1.5\text{ in.}$ $r^2 = \frac{I}{A} = 2.565\text{ in.}^2$

$\frac{ec}{r^2} = 2.082$ $r = 1.602\text{ in.}$

$P = 75\text{ k}$ $E = 30 \times 10^3\text{ ksi}$ $\frac{P}{EA} = 148.8 \times 10^{-6}$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$L = 10\text{ ft} = 120\text{ in.}$

$\frac{P}{A} = 4.464\text{ ksi}$ $\frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.4569$

SUBSTITUTE INTO EQ. (1):

$$\sigma_{max} = 4.464\left[1 + 2.082\sec(0.4569)\right]$$
$$= 14.8\text{ ksi}$$

(b) MAXIMUM LENGTH

SOLVE EQ. (1) FOR THE LENGTH L:

$$L = 2\sqrt{\frac{EI}{P}}\arccos\left[\frac{P(ec/r^2)}{\sigma_{max}A - P}\right] \quad (2)$$

$\sigma_Y = 36\text{ ksi}$ $n = 2.0$ $P_Y = nP = 150\text{ k}$

SUBSTITUTE P_Y FOR P AND σ_Y FOR σ_{max}:

$$L_{max} = 2\sqrt{\frac{EI}{P_Y}}\arccos\left[\frac{P_Y(ec/r^2)}{\sigma_Y A - P_Y}\right]$$
$$= 151.1\text{ in.}$$

$$L_{max} = 12.6\text{ ft}$$

11.6-10 W 8 × 35 COLUMN (FIXED-FREE)

$L_e = 2L = 2(9.0\text{ FT}) = 18\text{ FT} = 216\text{ IN.}$

$E = 30 \times 10^3\text{ KSI} \quad e = 1.25\text{ IN.}$

$A = 10.3\text{ IN.}^2 \quad I = I_2 = 42.6\text{ IN.}^4 \quad b = 8.020\text{ IN.}$

$r^2 = \dfrac{I}{A} = 4.136\text{ IN.}^2 \quad r = 2.034\text{ IN.} \quad c = \dfrac{b}{2} = 4.010\text{ IN.}$

$\dfrac{L_e}{r} = 106.2 \qquad \dfrac{ec}{r^2} = 1.212$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$P = 40\text{ K} \quad \dfrac{P}{A} = 3.883\text{ KSI} \quad \dfrac{L_e}{2r}\sqrt{\dfrac{P}{EA}} = 0.6042$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 9.60\text{ KSI}$ ←

(b) ALLOWABLE LOAD

$\sigma_Y = 36\text{ KSI} \quad n = 2.1 \quad$ FIND P_{allow}

SUBSTITUTE INTO EQ. (1):

$$36 = \frac{P}{10.3}\left[1 + 1.212\sec\left(0.09552\sqrt{P}\right)\right]$$

SOLVE NUMERICALLY: $P = P_Y = 112.6\text{ K}$

$P_{allow} = P_Y/n = 53.6\text{ K}$ ←

11.6-11 W 12 × 50 COLUMN (FIXED-FREE)

$L_e = 2L = 2(12.5\text{ FT}) = 25\text{ FT} = 300\text{ IN.}$

$A = 14.7\text{ IN.}^2 \quad I = I_2 = 56.3\text{ IN.}^4$

$r^2 = \dfrac{I}{A} = 3.830\text{ IN.}^2 \qquad r = 1.957\text{ IN.}$

$\dfrac{ec}{r^2} = 0.25 \quad E = 30 \times 10^3\text{ KSI}$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$P = 70\text{ K} \quad \dfrac{P}{A} = 4.762\text{ KSI} \quad \dfrac{L_e}{2r}\sqrt{\dfrac{P}{EA}} = 0.9657$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 6.85\text{ KSI}$ ←

(b) FACTOR OF SAFETY

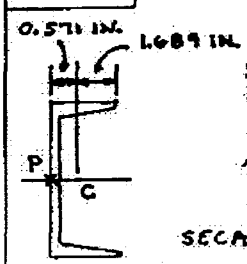
$\sigma_Y = 42\text{ KSI} \quad$ SUBSTITUTE INTO EQ. (1):

$$42 = \frac{P}{14.7}\left[1 + 0.25\sec\left(0.1154\sqrt{P}\right)\right]$$

SOLVE NUMERICALLY: $P = P_Y = 164.5\text{ K}$

$P = 70\text{ K} \quad n = \dfrac{P_Y}{P} = 2.35$ ←

11.6-12 C 8 × 11.5 POST (FIXED-FREE)



$L = 5\text{ FT} = 60\text{ IN.} \quad L_e = 2L = 120\text{ IN.}$

$E = 30 \times 10^3\text{ KSI} \quad P = 4.0\text{ K}$

$A = 3.38\text{ IN.}^2 \quad I = I_2 = 1.32\text{ IN.}^4$

$r^2 = \dfrac{I}{A} = 0.3905\text{ IN.}^2$

$r = 0.6249\text{ IN.} \quad e = 0.571\text{ IN.}$

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

MAXIMUM COMPRESSIVE STRESS

$c = 0.571\text{ IN.} \qquad ec/r^2 = 0.8349$

SUBSTITUTE INTO EQ. (1): $\sigma_c = 2.38\text{ KSI}$ ←

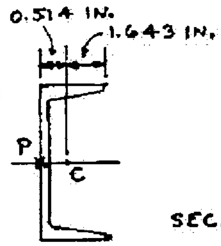
MAXIMUM TENSILE STRESS

$c = -1.689\text{ IN.} \qquad ec/r^2 = -2.470$

SUBSTITUTE INTO EQ. (1):

$\sigma_{max} = -2.37\text{ KSI} \quad (+ = \text{COMP.}) \quad \sigma_t = 2.37\text{ KSI}$ ←

11.6-13 C 6 × 13 POST (FIXED-FREE)



$L = 4.0\text{ FT} = 48\text{ IN.} \quad L_e = 2L = 96\text{ IN.}$

$E = 29 \times 10^6\text{ KSI} \quad P = 6.0\text{ K}$

$A = 3.83\text{ IN.}^2 \quad I = I_2 = 1.05\text{ IN.}^4$

$r^2 = \dfrac{I}{A} = 0.2742\text{ IN.}^2$

$r = 0.5236\text{ IN.} \quad e = 0.514\text{ IN.}$

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

MAXIMUM COMPRESSIVE STRESS

$c = 0.514\text{ IN.} \qquad ec/r^2 = 0.9635$

SUBSTITUTE INTO EQ. (1): $\sigma_c = 3.50\text{ KSI}$ ←

MAXIMUM TENSILE STRESS

$c = -1.643\text{ IN.} \qquad ec/r^2 = -3.080$

SUBSTITUTE INTO EQ. (1):

$\sigma_{max} = -4.61\text{ KSI} \quad (+ = \text{COMP.}) \quad \sigma_t = 4.61\text{ KSI}$ ←

11.6-14 W 12 × 87 COLUMN (PINNED ENDS)

$L = 18\text{ FT} = 216\text{ IN.}$

$P_1 = 180\text{ K} \qquad P_2 = 75\text{ K} \qquad b = 5.0\text{ IN.}$

$E = 29{,}000\text{ KSI} \quad \sigma_Y = 36\text{ KSI}$

$P = P_1 + P_2 = 255\text{ K} \qquad e = \dfrac{P_2 b}{P} = 1.471\text{ IN.}$

$A = 25.6\text{ IN.}^2 \quad I = I_1 = 740\text{ IN.}^4 \quad d = 12.53\text{ IN.}$

$r^2 = \dfrac{I}{A} = 28.91\text{ IN.}^2 \quad r = 5.376\text{ IN.}$

$c = \dfrac{d}{2} = 6.265\text{ IN.} \qquad \dfrac{ec}{r^2} = 0.3188$

$\dfrac{P}{A} = 9.961\text{ KSI} \qquad \dfrac{L}{2r}\sqrt{\dfrac{P}{EA}} = 0.3723$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 13.4\text{ KSI}$ ←

(b) FACTOR OF SAFETY

$\sigma_{max} = \sigma_Y = 36\text{ KSI} \qquad P = P_Y$

SUBSTITUTE INTO EQ. (1):

$$36 = \frac{P_Y}{25.6}\left[1 + 0.3188\sec\left(0.02332\sqrt{P_Y}\right)\right]$$

SOLVE NUMERICALLY:

$P_Y = 664.7\text{ K} \qquad n = \dfrac{P_Y}{P} = 2.61$ ←

11.6-15 W10 X 45 COLUMN (PINNED ENDS)

$L = 13.5 \text{ FT} = 162 \text{ IN.}$

$P_1 = 100 \text{ K} \quad P_2 = 60 \text{ K} \quad \Delta = 4.0 \text{ IN.}$

$E = 29{,}000 \text{ KSI} \quad \sigma_Y = 42 \text{ KSI}$

$P = P_1 + P_2 = 160 \text{ K} \quad e = \frac{P_2 \Delta}{P} = 1.50 \text{ IN.}$

$A = 13.3 \text{ IN.}^2 \quad I = I_1 = 248 \text{ IN.}^4 \quad d = 10.10 \text{ IN.}$

$r^2 = \frac{I}{A} = 18.65 \text{ IN.}^2 \quad r = 4.318 \text{ IN.}$

$c = \frac{d}{2} = 5.05 \text{ IN.} \quad \frac{ec}{r^2} = 0.4062$

$\frac{P}{A} = 12.03 \text{ KSI} \quad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.3821$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 17.3 \text{ KSI}$ ←

(b) LARGEST VALUE OF LOAD P_2 IF $n = 2.0$

UNITS: K, IN. $\quad P = P_1 + P_2 = 100 + P_2$

$e = \frac{P_2 \Delta}{P} = \frac{P_2(4.0)}{100 + P_2} \quad \frac{ec}{r^2} = \frac{1.0831 P_2}{100 + P_2}$

$\sigma_{max} = \sigma_Y = 42 \text{ KSI} \quad P_Y = nP = 2.0(100 + P_2)$

SUBSTITUTE INTO EQ. (1):

$$\sigma_Y = \frac{P_Y}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{L}{2r}\sqrt{\frac{P_Y}{EA}}\right)\right]$$

$$42 = \frac{2.0(100 + P_2)}{13.3}\left[1 + \frac{1.0831 P_2}{100 + P_2} \sec\left(0.04272\sqrt{100 + P_2}\right)\right]$$

SOLVE NUMERICALLY: $P_2 = 78.4 \text{ K}$ ←

11.6-16 W14 X 53 COLUMN (FIXED-FREE)

$L = 15 \text{ FT} = 180 \text{ IN.} \quad L_e = 2L = 360 \text{ IN.}$

$P_1 = 120 \text{ K} \quad P_2 = 40 \text{ K} \quad \Delta = 12 \text{ IN.}$

$E = 29{,}000 \text{ KSI} \quad \sigma_Y = 36 \text{ KSI}$

$P = P_1 + P_2 = 160 \text{ K} \quad e = \frac{P_2 \Delta}{P} = 3.0 \text{ IN.}$

$A = 15.6 \text{ IN.}^2 \quad I = I_1 = 541 \text{ IN.}^4 \quad d = 13.92 \text{ IN.}$

$r^2 = \frac{I}{A} = 34.68 \text{ IN.}^2 \quad r = 5.889 \text{ IN.}$

$c = \frac{d}{2} = 6.960 \text{ IN.} \quad \frac{ec}{r^2} = 0.6021$

$\frac{P}{A} = 10.26 \text{ KSI} \quad \frac{L_e}{2r}\sqrt{\frac{P}{EA}} = 0.5748$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 17.6 \text{ KSI}$ ←

(b) FACTOR OF SAFETY

$\sigma_{max} = \sigma_Y = 36 \text{ KSI} \quad P = P_Y$

SUBSTITUTE INTO EQ. (1):

$$36 = \frac{P_Y}{15.6}\left[1 + 0.6021 \sec\left(0.04544\sqrt{P_Y}\right)\right]$$

SOLVE NUMERICALLY:

$P_Y = 302.6 \text{ K} \quad n = \frac{P_Y}{P} = \frac{302.6}{160} = 1.89$ ←

11.6-17 W12 X 35 COLUMN (FIXED-FREE)

$L = 16 \text{ FT} = 192 \text{ IN.} \quad L_e = 2L = 384 \text{ IN.}$

$P_1 = 75 \text{ K} \quad P_2 = 25 \text{ K} \quad \Delta = 10.0 \text{ IN.}$

$E = 29{,}000 \text{ KSI} \quad \sigma_Y = 42 \text{ KSI}$

$P = P_1 + P_2 = 100 \text{ K} \quad e = \frac{P_2 \Delta}{P} = 2.5 \text{ IN.}$

$A = 10.3 \text{ IN.}^2 \quad I = I_1 = 285 \text{ IN.}^4 \quad d = 12.50 \text{ IN.}$

$r^2 = \frac{I}{A} = 27.67 \text{ IN.}^2 \quad r = 5.260 \text{ IN.}$

$c = \frac{d}{2} = 6.25 \text{ IN.} \quad \frac{ec}{r^2} = 0.5647$

$\frac{P}{A} = 9.709 \text{ KSI} \quad \frac{L_e}{2r}\sqrt{\frac{P}{EA}} = 0.6679$

(a) MAXIMUM COMPRESSIVE STRESS

SECANT FORMULA (EQ. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

SUBSTITUTE INTO EQ. (1): $\sigma_{max} = 16.7 \text{ KSI}$ ←

(b) LARGEST VALUE OF LOAD P_2 IF $n = 1.8$

UNITS: K, IN. $\quad P = P_1 + P_2 = 75 + P_2$

$e = \frac{P_2 \Delta}{P} = \frac{P_2(10.0)}{75 + P_2} \quad \frac{ec}{r^2} = \frac{2.259 P_2}{75 + P_2}$

$\sigma_{max} = \sigma_Y = 42 \text{ KSI} \quad P_Y = nP = 1.8(75 + P_2)$

SUBSTITUTE INTO EQ. (1):

$$\sigma_Y = \frac{P_Y}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{L_e}{2r}\sqrt{\frac{P_Y}{EA}}\right)\right]$$

$$42 = \frac{1.8(75 + P_2)}{10.3}\left[1 + \frac{2.259 P_2}{75 + P_2} \sec\left(0.08961\sqrt{75 + P_2}\right)\right]$$

SOLVE NUMERICALLY: $P_2 = 34.3 \text{ K}$ ←

11.9-1 W 8 X 35 COLUMN (PINNED ENDS; K=1)

$A = 10.3 \text{ IN.}^2 \quad r_2 = 2.03 \text{ IN.} \quad E = 29{,}000 \text{ KSI}$

$\sigma_Y = 36 \text{ KSI} \quad \text{EQ. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\, r = 256.0 \text{ IN.} = 21.3 \text{ FT}$

L	16 FT	20 FT	24 FT	28 FT
L/r	94.58	118.2	141.9	165.5
n_1 (EQ. 11-79)	1.895	1.915	—	—
n_2 (EQ. 11-80)	—	—	1.917	1.917
σ_{allow}/σ_Y (EQ. 11-81)	0.3793	0.2931	—	—
σ_{allow}/σ_Y (EQ. 11-82)	—	—	0.2060	0.1514
σ_{allow} (KSI)	13.65	10.55	7.42	5.45
$P_{allow} = A\sigma_{allow}$	141 K	109 K	76 K	56 K

11.9-2 W10 × 30 COLUMN (PINNED ENDS; K=1)

$A = 8.84 \text{ IN.}^2$ $r_2 = 1.37$ IN. $E = 29{,}000$ KSI

$\sigma_Y = 36$ KSI EQ. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\, r = 172.8$ IN. $= 14.4$ FT

L	12 FT	14 FT	16 FT	18 FT
L/r	105.1	122.6	140.1	157.7
n_1 (EQ. 11-79)	1.907	1.916	—	—
n_2 (EQ. 11-80)	—	—	1.917	1.917
σ_{allow}/σ_Y (EQ. 11-81)	0.3422	0.2751	—	—
σ_{allow}/σ_Y (EQ. 11-82)	—	—	0.2113	0.1668
σ_{allow} (KSI)	12.32	9.905	7.607	6.004
$P_{allow} = A\sigma_{allow}$	109 K	88 K	67 K	53 K

11.9-3 W12 × 50 COLUMN (PINNED ENDS; K=1)

$A = 14.7 \text{ IN.}^2$ $r_2 = 1.96$ IN. $E = 29{,}000$ KSI

$\sigma_Y = 50$ KSI EQ. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 107.0$

$L_c = 107.0\, r = 209.7$ IN. $= 17.5$ FT

L	12 FT	15 FT	18 FT	21 FT
L/r	73.47	91.84	110.2	128.6
n_1 (EQ. 11-79)	1.884	1.909	—	—
n_2 (EQ. 11-80)	—	—	1.917	1.917
σ_{allow}/σ_Y (EQ. 11-81)	0.4057	0.3308	—	—
σ_{allow}/σ_Y (EQ. 11-82)	—	—	0.2459	0.1806
σ_{allow} (KSI)	20.29	16.54	12.29	9.028
$P_{allow} = A\sigma_{allow}$	298 K	243 K	181 K	133 K

11.9-4 SELECT A COLUMN OF W 10 SHAPE

$P = 180$ K $L = 14$ FT $= 168$ IN. $K = 1$ $\sigma_Y = 36$ KSI

$E = 29{,}000$ KSI EQ. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

(1) TRIAL VALUE OF σ_{allow}

UPPER LIMIT: USE EQ. (11-81) WITH $L/r = 0$

MAX. $\sigma_{allow} = \frac{\sigma_Y}{n_1} = \frac{\sigma_Y}{5/3} = 21.6$ KSI

TRY $\sigma_{allow} = 16$ KSI

(2) TRIAL VALUE OF AREA

$A = \frac{P}{\sigma_{allow}} = \frac{180 \text{ K}}{16 \text{ KSI}} = 11.25 \text{ IN.}^2$

(3) TRIAL COLUMN W 10 × 45 $A = 13.3 \text{ IN.}^2$ $r = 2.01$ IN.

(4) ALLOWABLE STRESS FOR TRIAL COLUMN

$\frac{L}{r} = \frac{168 \text{ IN.}}{2.01 \text{ IN.}} = 83.58$ $\frac{L}{r} < \left(\frac{L}{r}\right)_c$

EQS. (11-79) AND (11-81): $n_1 = 1.879$

$\frac{\sigma_{allow}}{\sigma_Y} = 0.4153$ $\sigma_{allow} = 14.95$ KSI

(5) ALLOWABLE LOAD FOR TRIAL COLUMN

$P_{allow} = \sigma_{allow} A = 199 \text{ K} > 180 \text{ K}$ (OK)

CONT.

11.9-4 CONT.

NEXT SMALLER SIZE IS W 10 × 30, FOR WHICH WE FIND $P_{allow} = 88 \text{ K} < 180 \text{ K}$ (N.G.)

SELECT W 10 × 45 ←

11.9-5 SELECT A COLUMN OF W 12 SHAPE

$P = 175$ K $L = 35$ FT $= 420$ IN. $K = 1$ $\sigma_Y = 36$ KSI

$E = 29{,}000$ KSI EQ. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

(1) TRIAL VALUE OF σ_{allow}

UPPER LIMIT: USE EQ. (11-81) WITH $L/r = 0$

MAX. $\sigma_{allow} = \frac{\sigma_Y}{n_1} = \frac{\sigma_Y}{5/3} = 21.6$ KSI

TRY $\sigma_{allow} = 8$ KSI (BECAUSE COLUMN IS VERY LONG)

(2) TRIAL VALUE OF AREA

$A = \frac{P}{\sigma_{allow}} = \frac{175 \text{ K}}{8 \text{ KSI}} = 22 \text{ IN.}^2$

(3) TRIAL COLUMN W 12 × 87 $A = 25.6 \text{ IN.}^2$ $r = 3.07$ IN.

(4) ALLOWABLE STRESS FOR TRIAL COLUMN

$\frac{L}{r} = \frac{420 \text{ IN.}}{3.07 \text{ IN.}} = 136.8$ $\frac{L}{r} > \left(\frac{L}{r}\right)_c$

EQS. (11-80) AND (11-82): $n_2 = 1.917$

$\frac{\sigma_{allow}}{\sigma_Y} = 0.2216$ $\sigma_{allow} = 7.978$ KSI

(5) ALLOWABLE LOAD FOR TRIAL COLUMN

$P_{allow} = \sigma_{allow} A = 204 \text{ K} > 175 \text{ K}$ (OK)

NEXT SMALLER SIZE IS W 12 × 50, FOR WHICH WE FIND $L/r = 214 > 200$ (N.G.)

SELECT W 12 × 87 ←

11.9-6 SELECT A COLUMN OF W14 SHAPE

$P = 250$ K $L = 20$ FT $= 240$ IN. $K = 1$ $\sigma_Y = 50$ KSI

$E = 29{,}000$ KSI EQ. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 107$

(1) TRIAL VALUE OF σ_{allow}

UPPER LIMIT: USE EQ. (11-81) WITH $L/r = 0$

MAX. $\sigma_{allow} = \frac{\sigma_Y}{n_1} = \frac{\sigma_Y}{5/3} = 30$ KSI

TRY $\sigma_{allow} = 12$ KSI

(2) TRIAL VALUE OF AREA

$A = \frac{P}{\sigma_{allow}} = \frac{250 \text{K}}{12 \text{ KSI}} = 21 \text{ IN.}^2$

(3) TRIAL COLUMN W 14 × 82 $A = 24.1 \text{ IN.}^2$ $r = 2.48$ IN.

(4) ALLOWABLE STRESS FOR TRIAL COLUMN

$\frac{L}{r} = \frac{240 \text{ IN.}}{2.48 \text{ IN.}} = 96.77$ $\frac{L}{r} < \left(\frac{L}{r}\right)_c$

EQS. (11-79) AND (11-81): $n_1 = 1.913$

$\frac{\sigma_{allow}}{\sigma_Y} = 0.3090$ $\sigma_{allow} = 15.45$ KSI

(5) ALLOWABLE LOAD FOR TRIAL COLUMN

$P_{allow} = \sigma_{allow} A = 372 \text{ K} > 250 \text{ K}$ (OK)

NEXT SMALLER SIZE IS W 14 × 53, FOR WHICH WE FIND $P_{allow} = 149 \text{ K} < 250 \text{ K}$ (N.G.)

SELECT W 14 × 82 ←

11.9-7 STEEL PIPE COLUMN (FIXED-FREE; K=2)

$d = 6.625$ IN. $t = 0.280$ IN. $E = 29{,}000$ KSI

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 5.581 \text{ IN.}^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 28.14 \text{ IN.}^4 \quad r = \sqrt{\frac{I}{A}} = 2.245 \text{ IN.}$

$\sigma_Y = 36$ KSI EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = (126.1)\frac{r}{K} = 141.5$ IN. $= 11.8$ FT

L	6 FT	9 FT	12 FT	15 FT
KL/r	64.14	96.21	128.3	160.4
n_1 (EQ. 11-79)	1.841	1.897	—	—
n_2 (EQ. 11-80)	—	—	1.917	1.917
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.4729	0.3737	—	—
σ_{ALLOW}/σ_Y (EQ. 11-82)	—	—	0.2520	0.1612
σ_{ALLOW} (KSI)	17.03	13.45	9.07	5.80
$P_{ALLOW} = A\sigma_{ALLOW}$	95 K	75 K	51 K	32 K

11.9-8 STEEL PIPE COLUMN (FIXED-FREE; K=2)

$d = 140$ MM $t = 7$ MM $E = 200$ GPa

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 2925 \text{ MM}^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 6485 \times 10^3 \text{ MM}^4 \quad r = \sqrt{\frac{I}{A}} = 47.09 \text{ MM}$

$\sigma_Y = 250$ MPa EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$

$L_c = (125.7)\frac{r}{K} = 2960$ MM $= 2.960$ M

L	2.6 M	2.8 M	3.0 M	3.2 M
KL/r	110.4	118.9	127.4	135.9
n_1 (EQ. 11-79)	1.911	1.916	—	—
n_2 (EQ. 11-80)	—	—	1.917	1.917
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.3215	0.2884	—	—
σ_{ALLOW}/σ_Y (EQ. 11-82)	—	—	0.2539	0.2231
σ_{ALLOW} (MPa)	80.37	72.11	63.48	55.79
$P_{ALLOW} = A\sigma_{ALLOW}$	235 kN	211 kN	186 kN	163 kN

11.9-9 STEEL PIPE COLUMN (FIXED-FREE; K=2)

$d = 4.50$ IN. $t = 0.237$ IN. $E = 29{,}000$ KSI

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 3.174 \text{ IN.}^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 7.233 \text{ IN.}^4 \quad r = \sqrt{\frac{I}{A}} = 1.510 \text{ IN.}$

$\sigma_Y = 36$ KSI EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = (126.1)\frac{r}{K} = 95.21$ IN. $= 7.93$ FT

L	4 FT	6 FT	8 FT	10 FT
KL/r	63.58	95.36	127.2	158.9
n_1 (EQ. 11-79)	1.840	1.896	—	—
n_2 (EQ. 11-80)	—	—	1.917	1.917
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.4745	0.3766	—	—
σ_{ALLOW}/σ_Y (EQ. 11-82)	—	—	0.2563	0.1643
σ_{ALLOW} (KSI)	17.08	13.56	9.227	5.913
$P_{ALLOW} = A\sigma_{ALLOW}$	54 K	43 K	29 K	19 K

11.9-10 STEEL PIPE COLUMN (FIXED-FREE; K=2)

$d = 200$ MM $t = 10$ MM $E = 200$ GPa

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 5969 \text{ MM}^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 27.01 \times 10^6 \text{ MM}^4 \quad r = \sqrt{\frac{I}{A}} = 67.27 \text{ MM}$

$\sigma_Y = 250$ MPa EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$

$L_c = (125.7)\frac{r}{K} = 4227$ MM $= 4.227$ M

$P = 500$ kN

SELECT VARIOUS VALUES OF L UNTIL WE OBTAIN $P_{ALLOW} = P$ (BY INTERPOLATION).

IF $L \leq L_c$, USE EQS. (11-79) AND (11-81).
IF $L \geq L_c$, USE EQS. (11-80) AND (11-82).

L	3.50 M	3.55 M	3.60 M	3.59 M
KL/r	104.1	105.5	107.0	106.7
n_1 (EQ. 11-79)	1.906	1.908	1.909	1.909
n_2 (EQ. 11-80)	—	—	—	—
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.3449	0.3396	0.3339	0.3351
σ_{ALLOW}/σ_Y (EQ. 11-82)	—	—	—	—
σ_{ALLOW} (MPa)	86.22	84.90	83.48	83.78
$P_{ALLOW} = A\sigma_{ALLOW}$	514 kN	506 kN	498 kN	500 kN

FOR $P = 500$ kN, $L_{max} = 3.59$ M ←

11.9-11 STEEL PIPE COLUMN (FIXED-FREE; K=2)

$d = 4.0$ IN. $t = 0.226$ IN. $E = 29{,}000$ KSI

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 2.680 \text{ IN.}^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 4.788 \text{ IN.}^4 \quad r = \sqrt{\frac{I}{A}} = 1.337 \text{ IN.}$

$\sigma_Y = 42$ KSI EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$

$L_c = (116.7)\frac{r}{K} = 78.04$ IN. $= 6.50$ FT

$P = 40$ K

SELECT VARIOUS VALUES OF L UNTIL WE OBTAIN $P_{ALLOW} = P$ (BY INTERPOLATION).

IF $L \leq L_c$, USE EQS. (11-79) AND (11-81).
IF $L \geq L_c$, USE EQS. (11-80) AND (11-82).

L	5.20 FT	5.25 FT	5.30 FT	5.23 FT
KL/r	93.34	94.24	95.14	93.88
n_1 (EQ. 11-79)	1.903	1.904	1.905	1.903
n_2 (EQ. 11-80)	—	—	—	—
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.3574	0.3540	0.3505	0.3555
σ_{ALLOW}/σ_Y (EQ. 11-82)	—	—	—	—
σ_{ALLOW} (KSI)	15.01	14.87	14.72	14.93
$P_{ALLOW} = A\sigma_{ALLOW}$	40.2 K	39.8 K	39.5 K	40.0 K

FOR $P = 40$ K, $L_{max} = 5.23$ FT ←

11.9-12 STEEL PIPE COLUMN (FIXED-FREE; K=2)

$d = 160$ MM $\quad t = 8$ MM $\quad E = 210$ GPa

$A = \frac{\pi}{4}\left[d^2-(d-2t)^2\right] = 3820 \text{ MM}^2$

$I = \frac{\pi}{64}\left[d^4-(d-2t)^4\right] = 11.063 \times 10^6 \text{ MM}^4 \quad r = \sqrt{\frac{I}{A}} = 53.82$ MM

$\sigma_Y = 290$ MPa EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 119.6$

$L_c = (119.6)\frac{r}{K} = 3220 \text{ MM} = 3.22$ M

$P = 400$ KN

SELECT VARIOUS VALUES OF L UNTIL WE OBTAIN $P_{ALLOW} = P$ (BY INTERPOLATION).

IF $L \leq L_c$, USE EQS. (11-79) AND (11-81).
IF $L \geq L_c$, USE EQS. (11-80) AND (11-82).

L	2.50 M	2.55 M	2.60 M
KL/r	92.90	94.76	96.62
n_1 (EQ. 11-79)	1.899	1.902	1.904
n_2 (EQ. 11-80)	—	—	—
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.3677	0.3607	0.3538
σ_{ALLOW}/σ_Y (EQ. 11-82)	—	—	—
σ_{ALLOW} (MPa)	106.6	104.6	102.6
$P_{ALLOW} = A\sigma_{ALLOW}$	407 KN	400 KN	392 KN

FOR $P = 400$ KN, $L_{max} = 2.55$ M ⟵

11.9-13 COLUMN BC (FIXED-PINNED; K=0.699). FRAME ABC WITH HORIZ. FORCE H.

$H = P \cot 60°$ $\quad P$ = COMP. FORCE IN BC

$d_2 = 3.5$ IN. $\quad d_1 = 3.0$ IN. $\quad E = 29{,}000$ KSI

$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.553 \text{ IN.}^2$

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.390 \text{ IN.}^4 \quad r = \sqrt{\frac{I}{A}} = 1.152$ IN.

$\sigma_Y = 50$ KSI EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 107.0$

$L_c = (107.0)\frac{r}{K} = 176.3 \text{ IN.} = 14.7$ FT

(a) L = 12 FT; FIND H_{MAX}

$\frac{KL}{r} = 87.38$ $\quad$ EQ. (11-79): $n_1 = 1.905$

EQ. (11-81): $\sigma_{ALLOW}/\sigma_Y = 0.3499$ $\quad \sigma_{ALLOW} = 17.49$ KSI

$P_{ALLOW} = \sigma_{ALLOW} A = 44.7$ K

$H_{MAX} = P_{ALLOW} \cot 60° = 25.8$ K ⟵

(b) H = 16 K; FIND L_{MAX}

$P = H \tan 60° = 27.71$ K $\quad$ ASSUME $L_{MAX} > L_c$ (14.7 FT)

$\therefore n_2 = \frac{23}{12}$ (EQ. 11-80)

EQ. (11-82): $\frac{\sigma_{ALLOW}}{\sigma_Y} = \frac{(KL/r)_c^2}{2n_2(KL/r)^2}$ $\quad$ (1)

$\sigma_{ALLOW} = \frac{P}{A} = \frac{27.71 \text{ K}}{2.553 \text{ IN.}^2} = 10.85$ KSI

SUBSTITUTE INTO EQ. (1):

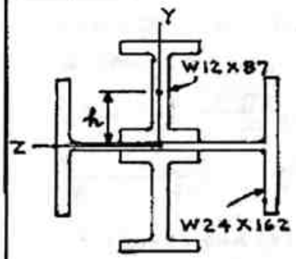
$\frac{10.85 \text{ KSI}}{50 \text{ KSI}} = \frac{(107.0)^2(1.152)^2}{2(23/12)(0.699)^2(L^2)}$

SOLVE FOR L:

$L^2 = 37{,}380 \text{ IN.}^2$ $\quad L_{MAX} = 193.3 \text{ IN.} = 16.1$ FT ⟵

$L_{MAX} > L_c$ $\therefore$ ASSUMPTION IS VALID.

11.9-14 PINNED-END COLUMN (K=1)



$L = 55 \text{ FT} = 660$ IN.
$E = 29{,}000$ KSI
$\sigma_Y = 36$ KSI

W12X87

$A = 25.6 \text{ IN.}^2$ $\quad d = 12.53$ IN.
$I_1 = 740 \text{ IN.}^4$ $\quad I_2 = 241 \text{ IN.}^4$

W24X162

$A = 47.7 \text{ IN.}^2$ $\quad t_w = 0.705$ IN.
$I_1 = 5170 \text{ IN.}^4$ $\quad I_2 = 443 \text{ IN.}^4$

FOR THE ENTIRE CROSS SECTION

$A = 2(25.6) + 47.7 = 98.9 \text{ IN.}^2$

$I_Y = 2(241) + 5170 = 5652 \text{ IN.}^4$

$h = d/2 + t_w/2 = 6.6175$ IN.

$I_Z = 443 + 2\left[740 + (25.6)(6.6175)^2\right] = 4165 \text{ IN.}^4$

MIN. $r = \sqrt{\frac{I_Z}{A}} = \sqrt{\frac{4165}{98.9}} = 6.489$ IN.

EQ. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = (126.1)r = (126.1)(6.489 \text{ IN.}) = 818.3$ IN.

$L < L_c$ $\therefore$ USE EQS. (11-79) AND (11-81)

$\frac{L}{r} = \frac{660 \text{ IN.}}{6.489 \text{ IN.}} = 101.7$

FROM EQ. (11-79): $n_1 = 1.904$

FROM EQ. (11-81): $\sigma_{ALLOW}/\sigma_Y = 0.3544$

$\sigma_{ALLOW} = 0.3544\,\sigma_Y = 12.76$ KSI

$P_{ALLOW} = \sigma_{ALLOW} A = (12.76 \text{ KSI})(98.9 \text{ IN.}^2)$
$= 1260$ K ⟵

11.9-15 W 8 X 28 COLUMN (PINNED ENDS; K=1)

$A = 8.25 \text{ IN.}^2$ $\quad I = 21.7 \text{ IN.}^4$ $\quad \sigma_Y = 36$ KSI

$r = \sqrt{I/A} = 1.622$ IN. $\quad E = 29{,}000$ KSI

EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

LET $\beta = \frac{L/r}{(L/r)_c} = \frac{L/r}{126.1}$ $\quad \frac{\sigma_{ALLOW}}{\sigma_Y} = \frac{P}{A\sigma_Y}$

IF $\beta \leq 1$, USE EQS. (11-79) AND (11-81):

$$\frac{P}{A\sigma_Y} = \frac{1-\frac{1}{2}\beta^2}{\frac{5}{3}+\frac{3}{8}\beta-\frac{1}{8}\beta^3} \quad (\beta \leq 1) \quad (1)$$

IF $\beta \geq 1$, USE EQS. (11-80) AND (11-82):

$$\frac{P}{A\sigma_Y} = \frac{1}{2\left(\frac{23}{12}\right)\beta^2} \quad (\beta \geq 1) \quad (2)$$

(a) P = 60 K $\quad$ ASSUME $\beta \geq 1$ (EQ. 2)

$\frac{60 \text{ K}}{(8.25)(36)} = \frac{6}{23\beta^2}$ $\quad \beta = 1.136 > 1$

$\frac{L}{r} = (126.1)\beta = 143.3$ $\quad L_{MAX} = (143.3)r$
$= 232.4 \text{ IN.} = 19.4$ FT ⟵

(b) P = 120 K $\quad$ ASSUME $\beta \leq 1$ (EQ. 1)

$\frac{120 \text{ K}}{(8.25)(36)} = \frac{1-\frac{1}{2}\beta^2}{\frac{5}{3}+\frac{3}{8}\beta-\frac{1}{8}\beta^3}$ $\quad \beta = 0.6908$

$\frac{L}{r} = (126.1)\beta = 87.11$ $\quad L_{MAX} = (87.11)r$
$= 141.3 \text{ IN.} = 11.8$ FT ⟵

ALTERNATE METHOD: USE TRIAL AND ERROR.

11.9-16 W10X45 COLUMN (PINNED ENDS; K=1)

$A = 13.3 \text{ IN.}^2$ $I = 53.4 \text{ IN.}^4$ $\sigma_Y = 42$ KSI

$r = \sqrt{I/A} = 2.004$ IN. $E = 29{,}000$ KSI

EQ. (11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$

LET $\beta = \frac{L/r}{(L/r)_c} = \frac{L/r}{116.7}$ $\frac{\sigma_{ALLOW}}{\sigma_Y} = \frac{P}{A\sigma_Y}$

IF $\beta \le 1$, USE EQS. (11-79) AND (11-81):

$$\frac{P}{A\sigma_Y} = \frac{1 - \frac{1}{2}\beta^2}{\frac{5}{3} + \frac{3}{8}\beta - \frac{1}{8}\beta^3} \quad (\beta \le 1) \qquad (1)$$

IF $\beta \ge 1$, USE EQS. (11-80) AND (11-82):

$$\frac{P}{A\sigma_Y} = \frac{1}{2\left(\frac{23}{12}\right)\beta^2} \quad (\beta \ge 1) \qquad (2)$$

(a) $P = 125$ K ASSUME $\beta \ge 1$ (EQ. 2)

$$\frac{125 \text{ K}}{(13.3)(42)} = \frac{6}{23\beta^2} \qquad \beta = 1.080 > 1$$

$\frac{L}{r} = (116.7)\beta = 126.0$ $L_{MAX} = (126.0)\,r$

$= 252.5$ IN. $= 21.0$ FT ←

(b) $P = 200$ K ASSUME $\beta \le 1$ (EQ. 1)

$$\frac{200 \text{ K}}{(13.3)(42)} = \frac{1 - \frac{1}{2}\beta^2}{\frac{5}{3} + \frac{3}{8}\beta - \frac{1}{8}\beta^3} \qquad \beta = 0.7986$$

$\frac{L}{r} = (116.7)\beta = 93.19$ $L_{MAX} = (93.19)\,r$

$= 186.8$ IN. $= 15.6$ FT ←

11.9-17 PIPE COLUMN (PINNED ENDS; K=1)

$L = 20$ FT $= 240$ IN. $P = 25$ K $t = d/20$

$E = 29{,}000$ KSI $\sigma_Y = 36$ KSI

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 0.14923\,d^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 0.016881\,d^4$ $r = \sqrt{\frac{I}{A}}$

$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$ $L_c = (126.1)\,r$

SELECT VARIOUS VALUES OF DIAMETER d UNTIL WE OBTAIN $P_{ALLOW} = P$.

IF $L \le L_c$, USE EQS. (11-79) AND (11-81).
IF $L \ge L_c$, USE EQS. (11-80) AND (11-82).

d (IN.)	4.80	4.90	5.00
A (IN.^{2})	3.438	3.583	3.731
I (IN.^{4})	8.961	9.732	10.551
r (IN.)	1.614	1.648	1.682
L_c (IN.)	204	208	212
L/r	148.7	145.6	142.7
n_2 (EQ. 11-80)	23/12	23/12	23/12
σ_{ALLOW}/σ_Y (EQ. 11-82)	0.1876	0.1957	0.2037
σ_{ALLOW} (KSI)	6.754	7.044	7.333
$P_{ALLOW} = A\sigma_{ALLOW}$	23.2 K	25.2 K	27.4 K

FOR $P = 25$ K, $d = 4.89$ IN. ←

11.9-18 PIPE COLUMN (PINNED ENDS; K=1)

$L = 3.5$ m $P = 130$ kN $t = d/20$

$E = 200$ GPa $\sigma_Y = 275$ MPa

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] = 0.14923\,d^2$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right] = 0.016881\,d^4$ $r = \sqrt{\frac{I}{A}}$

$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 119.8$ $L_c = (119.8)\,r$

SELECT VARIOUS VALUES OF DIAMETER d UNTIL WE OBTAIN $P_{ALLOW} = P$.

IF $L \le L_c$, USE EQS. (11-79) AND (11-81).
IF $L \ge L_c$, USE EQS. (11-80) AND (11-82).

d (MM)	98	99	100
A (MM^2)	1433	1463	1492
I (MM^4)	1557×10^3	1622×10^3	1688×10^3
r (MM)	32.96	33.30	33.64
L_c (MM)	3950	3989	4030
L/r	106.2	105.1	104.0
n_1 (EQ. 11-79)	1.912	1.911	1.910
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.3175	0.3219	0.3263
σ_{ALLOW} (MPa)	87.32	88.53	89.73
$P_{ALLOW} = A\sigma_{ALLOW}$	125.1 KN	129.5 KN	133.9 KN

FOR $P = 130$ KN, $d = 99$ MM ←

11.9-19 PIPE COLUMN (PINNED ENDS; K=1)

$L = 11.5$ FT $= 138$ IN. $P = 80$ K $t = 0.30$ IN.

$E = 29{,}000$ KSI $\sigma_Y = 42$ KSI

$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right]$ $r = \sqrt{\frac{I}{A}}$

$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right]$

$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$ $L_c = (116.7)\,r$

SELECT VARIOUS VALUES OF DIAMETER d UNTIL WE OBTAIN $P_{ALLOW} = P$.

IF $L \le L_c$, USE EQS. (11-79) AND (11-81).
IF $L \ge L_c$, USE EQS. (11-80) AND (11-82).

d (IN.)	5.20	5.25	5.30
A (IN.^{2})	4.618	4.665	4.712
I (IN.^{4})	13.91	14.34	14.78
r (IN.)	1.736	1.753	1.771
L_c (IN.)	203	205	207
L/r	79.49	78.72	77.92
n_1 (EQ. 11-79)	1.883	1.881	1.880
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.4079	0.4107	0.4133
σ_{ALLOW} (KSI)	17.13	17.25	17.36
$P_{ALLOW} = A\sigma_{ALLOW}$	79.1 K	80.5 K	81.8 K

FOR $P = 80$ K, $d = 5.23$ IN. ←

11.9-20

PIPE COLUMN (PINNED ENDS; K=1)

L = 3.0 M P = 800 KN t = 9 MM

E = 200 GPa σ_Y = 300 MPa

$$A = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] \qquad r = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right]$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 114.7 \qquad L_c = (114.7)r$$

SELECT VARIOUS VALUES OF DIAMETER d UNTIL WE OBTAIN P_{ALLOW} = P.

IF L ≤ L_c, USE EQS. (11-79) AND (11-81).
IF L ≥ L_c, USE EQS. (11-80) AND (11-82).

d (MM)	193	194	195
A (MM²)	5202	5231	5259
I (MM⁴)	22.08×10⁶	22.43×10⁶	22.80×10⁶
r (MM)	65.13	65.48	65.84
L_c (MM)	7470	7510	7550
L/r	46.06	45.82	45.57
n_1 (EQ. 11-79)	1.809	1.809	1.808
σ_{ALLOW}/σ_Y (EQ. 11-81)	0.5082	0.5087	0.5094
σ_{ALLOW} (MPa)	152.5	152.6	152.8
$P_{ALLOW} = A\sigma_{ALLOW}$	793.1 KN	798.3 KN	803.8 KN

FOR P = 800 KN, d = 194 MM ←

11.9-21

ALUMINUM PIPE COLUMN (2014-T6)

PINNED ENDS d_2 = 5.60 IN.
(K=1) d_1 = 4.80 IN.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 6.535 \text{ IN.}^2 \qquad r = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 22.22 \text{ IN.}^4 \qquad r = 1.844 \text{ IN.}$$

USE EQS. (11-84a, b, c):

σ_{ALLOW} = 28 KSI 0 ≤ L/r ≤ 12
σ_{ALLOW} = 30.7 − 0.23(L/r) KSI 12 ≤ L/r ≤ 55
σ_{ALLOW} = 54,000/(L/r)² KSI 55 ≤ L/r

L (FT)	6 FT	8 FT	10 FT	12 FT
L/r	39.05	52.06	65.08	78.09
σ_{ALLOW} (KSI)	21.72	18.73	12.75	8.86
$P_{ALLOW} = \sigma_{ALLOW} A$	142 K	122 K	83 K	58 K

11.9-22

ALUMINUM PIPE COLUMN (2014-T6)

PINNED ENDS d_2 = 120 MM = 4.7244 IN.
(K=1) d_1 = 110 MM = 4.3307 IN.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.800 \text{ IN.}^2 \qquad r = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 7.188 \text{ IN.}^4 \qquad r = 1.602 \text{ IN.}$$

USE EQS. (11-84a, b, c):

σ_{ALLOW} = 28 KSI 0 ≤ L/r ≤ 12
σ_{ALLOW} = 30.7 − 0.23(L/r) KSI 12 ≤ L/r ≤ 55
σ_{ALLOW} = 54,000/(L/r)² KSI 55 ≤ L/r

L (M)	1 M	2 M	3 M	4 M
L (IN.)	39.37	78.74	118.1	157.5
L/r	24.58	49.15	73.73	98.30
σ_{ALLOW} (KSI)	25.05	19.40	9.934	5.588
$P_{ALLOW} = \sigma_{ALLOW} A$	70.14 K	54.31 K	27.81 K	15.65 K
P_{ALLOW} (KN)	312 KN	242 KN	124 KN	70 KN

11.9-23

ALUMINUM PIPE COLUMN (6061-T6)

FIXED-FREE d_2 = 3.25 IN.
(K=2) d_1 = 3.00 IN.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.227 \text{ IN.}^2 \qquad r = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.500 \text{ IN.}^4 \qquad r = 1.106 \text{ IN.}$$

USE EQS. (11-85a, b, c):

σ_{ALLOW} = 19 KSI 0 ≤ KL/r ≤ 9.5
σ_{ALLOW} = 20.2 − 0.126(KL/r) KSI 9.5 ≤ KL/r ≤ 66
σ_{ALLOW} = 51,000/(KL/r)² KSI 66 ≤ KL/r

L (FT)	2 FT	3 FT	4 FT	5 FT
KL/r	43.40	65.10	86.80	108.5
σ_{ALLOW} (KSI)	14.73	12.00	6.77	4.33
$P_{ALLOW} = \sigma_{ALLOW} A$	18.1 K	14.7 K	8.3 K	5.3 K

11.9-24

ALUMINUM PIPE COLUMN (6061-T6)

FIXED-FREE d_2 = 80 MM = 3.1496 IN.
(K=2) d_1 = 72 MM = 2.8346 IN.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.480 \text{ IN.}^2 \qquad r = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.661 \text{ IN.}^4 \qquad r = 1.059$$

USE EQS. (11-85a, b, c):

σ_{ALLOW} = 19 KSI 0 ≤ KL/r ≤ 9.5
σ_{ALLOW} = 20.2 − 0.126(KL/r) KSI 9.5 ≤ KL/r ≤ 66
σ_{ALLOW} = 51,000/(KL/r)² KSI 66 ≤ KL/r

L (M)	0.6 M	0.8 M	1.0 M	1.2 M
KL (IN.)	47.24	62.99	78.74	94.49
KL/r	44.61	59.48	74.35	89.23
σ_{ALLOW} (KSI)	14.58	12.71	9.226	6.405
$P_{ALLOW} = \sigma_{ALLOW} A$	21.58 K	18.81 K	13.65 K	9.48 K
P_{ALLOW} (KN)	96 KN	84 KN	61 KN	42 KN

11.9-25 ALUMINUM BAR (2014-T6)

PIN SUPPORTS (K=1) P = 60 K

(a) FIND L_{max} IF $d = 2.0$ IN.

$A = \frac{\pi d^2}{4} = 3.142 \text{ IN.}^2 \qquad I = \frac{\pi d^4}{64}$

$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.5 \text{ IN.}$

$\sigma_{allow} = \frac{P}{A} = \frac{60 \text{ K}}{3.142 \text{ IN.}^2} = 19.10 \text{ KSI}$

ASSUME L/r IS BETWEEN 12 AND 55:

EQ. (11-84b): $\sigma_{allow} = 30.7 - 0.23(L/r)$ KSI

OR $19.10 = 30.7 - 0.23(L/r)$

SOLVE FOR L/r: $\frac{L}{r} = 50.43 \quad (12 \le \frac{L}{r} \le 55)$

$L_{max} = (50.43)r = 25.2$ IN. ←

(b) FIND d_{min} IF L = 30 IN.

$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{30 \text{ IN.}}{d/4} = \frac{120}{d}$

$\sigma_{allow} = \frac{P}{A} = \frac{60 \text{ K}}{\pi d^2/4} = \frac{76.39}{d^2}$ (KSI)

ASSUME L/r IS GREATER THAN 55:

EQ. (11-84c): $\sigma_{allow} = \frac{54{,}000 \text{ KSI}}{(L/r)^2}$

OR $\frac{76.39}{d^2} = \frac{54{,}000}{(120/d)^2}$

$d^4 = 20.37 \text{ IN}^4 \qquad d_{min} = 2.12$ IN. ←

$L/r = 120/d = 56.6 > 55$ (OK)

11.9-27 ALUMINUM BAR (6061-T6)

PIN SUPPORTS (K=1) P = 10 K

(a) FIND L_{max} IF $d = 1.0$ IN.

$A = \frac{\pi d^2}{4} = 0.7854 \text{ IN.}^2 \qquad I = \frac{\pi d^4}{64}$

$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2500 \text{ IN.}$

$\sigma_{allow} = \frac{P}{A} = \frac{10 \text{ K}}{0.7854 \text{ IN.}^2} = 12.73 \text{ KSI}$

ASSUME L/r IS BETWEEN 9.5 AND 66:

EQ. (11-85b): $\sigma_{allow} = 20.2 - 0.126(L/r)$ KSI

OR $12.73 = 20.2 - 0.126(L/r)$

SOLVE FOR L/r: $\frac{L}{r} = 59.29 \quad (9.5 \le \frac{L}{r} \le 66)$

$L_{max} = (59.29)r = 14.8$ IN. ←

(b) FIND d_{min} IF L = 20 IN.

$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{20 \text{ IN.}}{d/4} = \frac{80}{d}$

$\sigma_{allow} = \frac{P}{A} = \frac{10 \text{ K}}{\pi d^2/4} = \frac{12.73}{d^2}$ (KSI)

ASSUME L/r IS GREATER THAN 66:

EQ. (11-85c): $\sigma_{allow} = \frac{51{,}000 \text{ KSI}}{(L/r)^2}$

OR $\frac{12.73}{d^2} = \frac{51{,}000}{(80/d)^2}$

$d^4 = 1.597 \text{ IN}^4 \qquad d_{min} = 1.12$ IN. ←

$L/r = 80/d = 71 > 66$ (OK)

11.9-26 ALUMINUM BAR (2014-T6)

PIN SUPPORTS (K=1) P = 175 kN = 39.34 K

(a) FIND L_{max} IF $d = 40$ MM = 1.575 IN.

$A = \frac{\pi d^2}{4} = 1.948 \text{ IN}^2 \qquad I = \frac{\pi d^4}{64}$

$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.3938 \text{ IN.}$

$\sigma_{allow} = \frac{P}{A} = \frac{39.34 \text{ K}}{1.948 \text{ IN.}^2} = 20.20 \text{ KSI}$

ASSUME L/r IS BETWEEN 12 AND 55:

EQ. (11-84b): $\sigma_{allow} = 30.7 - 0.23(L/r)$ KSI

OR $20.20 = 30.7 - 0.23(L/r)$

SOLVE FOR L/r: $\frac{L}{r} = 45.65 \quad (12 \le \frac{L}{r} \le 55)$

$L_{max} = (45.65)r = 18.0$ IN. = 457 MM ←

(b) FIND d_{min} IF L = 0.6 M = 23.62 IN.

$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{23.62 \text{ IN.}}{d/4} = \frac{94.48}{d}$

$\sigma_{allow} = \frac{P}{A} = \frac{39.34 \text{ K}}{\pi d^2/4} = \frac{50.09}{d^2}$ (KSI)

ASSUME L/r IS GREATER THAN 55:

EQ. (11-84c): $\sigma_{allow} = \frac{54{,}000 \text{ KSI}}{(L/r)^2}$

OR $\frac{50.09}{d^2} = \frac{54{,}000}{(94.48/d)^2}$

$d^4 = 8.280 \text{ IN}^4 \qquad d_{min} = 1.696$ IN. = 43.1 MM ←

$L/r = 94.48/1.696 = 55.7 > 55$ (OK)

11.9-28 ALUMINUM BAR (6061-T6)

PIN SUPPORTS (K=1) P = 60 kN = 13.49 K

(a) FIND L_{max} IF $d = 30$ MM = 1.181 IN.

$A = \frac{\pi d^2}{4} = 1.095 \text{ IN.}^2 \qquad I = \frac{\pi d^4}{64}$

$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2953 \text{ IN.}$

$\sigma_{allow} = \frac{P}{A} = \frac{13.49 \text{ K}}{1.095 \text{ IN.}^2} = 12.32 \text{ KSI}$

ASSUME L/r IS BETWEEN 9.5 AND 66:

EQ. (11-85b): $\sigma_{allow} = 20.2 - 0.126(L/r)$ KSI

OR $12.32 = 20.2 - 0.126(L/r)$

SOLVE FOR L/r: $\frac{L}{r} = 62.54 \quad (9.5 \le \frac{L}{r} \le 66)$

$L_{max} = (62.54)r = 18.47$ IN. = 469 MM ←

(b) FIND d_{min} IF L = 0.6 M = 23.62 IN.

$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{23.62 \text{ IN.}}{d/4} = \frac{94.48}{d}$

$\sigma_{allow} = \frac{P}{A} = \frac{13.49 \text{ K}}{\pi d^2/4} = \frac{17.18}{d^2}$ (KSI)

ASSUME L/r IS GREATER THAN 66:

EQ. (11-85c): $\sigma_{allow} = \frac{51{,}000 \text{ KSI}}{(L/r)^2}$

OR $\frac{17.18}{d^2} = \frac{51{,}000}{(94.48/d)^2}$

$d^4 = 3.007 \text{ IN}^4 \qquad d_{min} = 1.317$ IN. = 33.4 MM ←

$L/r = 94.48/1.317 = 72 > 66$ (OK)

11.9-29 WOOD POST (PINNED ENDS)

$b = 3.5$ IN. $h = 5.5$ IN. $A = bh = 19.25$ IN.^{2}

$F_c = 2000$ PSI $E = 1{,}800{,}000$ PSI $d = b = 3.5$ IN.

$K_c = \sqrt{0.45E/F_c} = 20.12$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

L (FT)	2.5 FT	5.0 FT	7.5 FT	10.0 FT
d (IN.)	3.5	3.5	3.5	3.5
L/d	8.57	17.14	25.71	34.29
σ_{ALLOW} (PSI)	2000	1649	816.9	459.3
$P_{ALLOW} = \sigma_{ALLOW} A$	38.5 K	31.7 K	15.7 K	8.8 K

11.9-30 WOOD POST (PINNED ENDS)

$b = 100$ MM $h = 150$ MM $A = bh = 15{,}000$ MM^2

$F_c = 14$ MPa $E = 12$ GPa $d = b$

$K_c = \sqrt{0.45E/F_c} = 19.64$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

L (M)	1.0 M	1.5 M	2.0 M	2.5 M
d (MM)	100	100	100	100
L/d	10	15	20	25
σ_{ALLOW} (MPa)	14	12.41	9.00	5.76
$P_{ALLOW} = \sigma_{ALLOW} A$	210 KN	186 KN	135 KN	86 KN

11.9-31 WOOD POST (PINNED ENDS)

$b = 3.5$ IN. $h = 7.25$ IN. $A = bh = 25.38$ IN.^{2}

$F_c = 1000$ PSI $E = 1{,}300{,}000$ PSI $d = b$

$K_c = \sqrt{0.45E/F_c} = 24.19$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

L (FT)	4 FT	6 FT	8 FT	10 FT
d (IN.)	3.5	3.5	3.5	3.5
L/d	13.71	20.57	27.43	34.29
σ_{ALLOW} (PSI)	965.6	825.7	518.3	331.7
$P_{ALLOW} = \sigma_{ALLOW} A$	24.5 K	21.0 K	13.2 K	8.4 K

11.9-32 WOOD POST (PINNED ENDS)

$b = 140$ MM $h = 210$ MM $A = bh = 29{,}400$ MM^2

$F_c = 12$ MPa $E = 10$ GPa $d = b$

$K_c = \sqrt{0.45E/F_c} = 19.36$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

L (M)	1.5 M	2.5 M	3.5 M	4.5 M
d (MM)	140	140	140	140
L/d	10.71	17.86	25.00	32.14
σ_{ALLOW} (MPa)	12.00	9.103	4.800	2.904
$P_{ALLOW} = \sigma_{ALLOW} A$	353 KN	268 KN	141 KN	85 KN

11.9-33

WOOD COLUMN (PINNED ENDS)

$F_c = 1700$ PSI $E = 1{,}400{,}000$ PSI $P = 40$ K

$K_c = \sqrt{0.45E/F_c} = 19.25 \qquad A = b^2$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

(a) FIND L_{MAX} IF $b = d = 5.5$ IN.

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{40\text{ K}}{(5.5\text{ IN.})^2} = 1322$ PSI

ASSUME L/d IS BETWEEN 11 AND K_c:

$1322\text{ PSI} = (1700\text{ PSI})\left[1 - \frac{1}{3}\left(\frac{L/d}{19.25}\right)^4\right]$

SOLVE FOR L/d: $\frac{L}{d} = 17.40 \qquad \left(11 \le \frac{L}{d} \le K_c\right)$

$L_{MAX} = (17.40)d = 95.68$ IN. $= 7.97$ FT ←

(b) FIND b ($=d$) IF $L = 11$ FT $= 132$ IN.

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{40{,}000}{d^2}$ (PSI)

ASSUME L/d IS BETWEEN K_c AND 50:

$$\frac{40{,}000}{d^2} = \frac{0.3(1{,}400{,}000)}{(132/d)^2}$$

SOLVE FOR d: $d = b = 6.38$ IN. $b_{min} = 6.38$ IN. ←

L/d = 20.7 (OK)

11.9-34 WOOD COLUMN (PINNED ENDS)

$F_c = 10.5$ MPa $\quad E = 12$ GPa $\quad P = 200$ kN

$K_c = \sqrt{0.45E/F_c} = 22.68 \quad A = b^2$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

(a) FIND L_{MAX} IF $b = d = 150$ MM

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{200 \text{ kN}}{(150 \text{ mm})^2} = 8.889$ MPa

ASSUME L/d IS BETWEEN 11 AND K_c:

$8.889 \text{ MPa} = (10.5 \text{ MPa})\left[1 - \frac{1}{3}\left(\frac{L/d}{22.68}\right)^4\right]$

SOLVE FOR L/d: $\frac{L}{d} = 18.68 \qquad \left(11 \le \frac{L}{d} \le K_c\right)$

$L_{MAX} = (18.68)d = 2800$ MM $= 2.80$ M ←

(b) FIND b (= d) IF L = 4.0 M

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{200 \text{ kN}}{d^2}$

ASSUME L/d IS BETWEEN K_c AND 50:

$\frac{200{,}000 \text{ N}}{d^2} = \frac{0.3(12{,}000 \text{ MPa})}{(4000/d)^2}$

SOLVE FOR d: $d = b = 173$ mm $\quad b_{min} = 173$ mm ←

L/d = 23.1 (OK)

11.9-35 WOOD COLUMN (PINNED ENDS)

$F_c = 900$ PSI $\quad E = 1{,}500{,}000$ PSI $\quad P = 8.0$ K

$K_c = \sqrt{0.45E/F_c} = 27.39 \quad A = b^2$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

(a) FIND L_{MAX} IF $b = d = 3.5$ IN.

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{8.0 \text{ K}}{(3.5 \text{ in.})^2} = 653.1$ PSI

ASSUME L/d IS BETWEEN 11 AND K_c:

$653.1 \text{ PSI} = (900 \text{ PSI})\left[1 - \frac{1}{3}\left(\frac{L/d}{27.39}\right)^4\right]$

SOLVE FOR L/d: $\frac{L}{d} = 26.09 \qquad \left(11 \le \frac{L}{d} \le K_c\right)$

$L_{MAX} = (26.09)d = 91.31$ IN. $= 7.61$ FT ←

(b) FIND b (= d) IF L = 10 FT = 120 IN.

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{8000 \text{ LB}}{d^2}$

ASSUME L/d IS BETWEEN K_c AND 50:

$\frac{8000}{d^2} = \frac{0.3(1{,}500{,}000)}{(120/d)^2}$

SOLVE FOR d: $d = b = 4.00$ IN. $\quad b_{min} = 4.00$ IN. ←

L/d = 30 (OK)

11.9-36 WOOD COLUMN (PINNED ENDS)

$F_c = 8.0$ MPa $\quad E = 8.5$ GPa $\quad P = 100$ kN

$K_c = \sqrt{0.45E/F_c} = 21.87 \quad A = b^2$

USE EQS. (11-86 a, b, c):

$\sigma_{ALLOW} = F_c \qquad 0 \le \frac{L}{d} \le 11$

$\sigma_{ALLOW} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K_c}\right)^4\right] \qquad 11 \le \frac{L}{d} \le K_c$

$\sigma_{ALLOW} = \frac{0.3E}{(L/d)^2} \qquad K_c \le \frac{L}{d} \le 50$

(a) FIND L_{MAX} IF $b = d = 120$ MM

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{100 \text{ kN}}{(120 \text{ mm})^2} = 6.944$ MPa

ASSUME L/d IS BETWEEN 11 AND K_c:

$6.944 \text{ MPa} = (8.0 \text{ MPa})\left[1 - \frac{1}{3}\left(\frac{L/d}{21.87}\right)^4\right]$

SOLVE FOR L/d: $\frac{L}{d} = 17.35 \qquad \left(11 \le \frac{L}{d} \le K_c\right)$

$L_{MAX} = (17.35)d = 2080$ mm $= 2.08$ m ←

(b) FIND b (= d) IF L = 4.0 M

$\sigma_{ALLOW} = \frac{P}{A} = \frac{P}{b^2} = \frac{100 \text{ kN}}{d^2}$

ASSUME L/d IS BETWEEN K_c AND 50:

$\frac{100{,}000 \text{ N}}{d^2} = \frac{0.3(8500 \text{ MPa})}{(4000/d)^2}$

SOLVE FOR d: $d = b = 158$ mm $\quad b_{min} = 158$ mm ←

L/d = 25.3 (OK)

– END OF CHAPTER 11 –

12.2-1

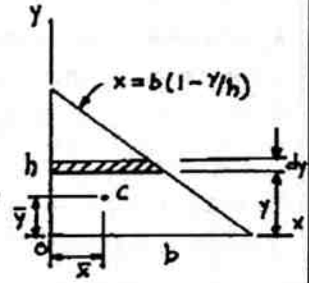
$dA = x\,dy = b(1 - y/h)\,dy$

$A = \int dA = \int_0^h b(1 - y/h)\,dy = \dfrac{bh}{2}$

$Q_x = \int y\,dA = \int_0^h y\,b(1 - y/h)\,dy = \dfrac{bh^2}{6}$

$\bar{y} = \dfrac{Q_x}{A} = \dfrac{h}{3} \longleftarrow$

Similarly, $\bar{x} = \dfrac{b}{3} \longleftarrow$

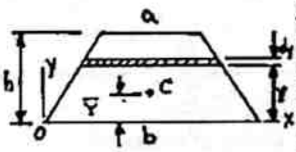


12.2-2

Width of element $= b + (a-b)y/h$

$dA = [b + (a-b)y/h]\,dy$

$A = \int dA = \int_0^h [b + (a-b)y/h]\,dy = \dfrac{h(a+b)}{2}$



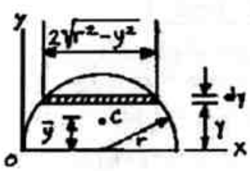
$Q_x = \int y\,dA = \int_0^h y[b + (a-b)y/h]\,dy = \dfrac{h^2}{6}(b + 2a)$

$\bar{y} = \dfrac{Q_x}{A} = \dfrac{h(2a+b)}{3(a+b)} \longleftarrow$

12.2-3

$dA = 2\sqrt{r^2 - y^2}\,dy$

$A = \int dA = \int_0^r 2\sqrt{r^2 - y^2}\,dy = \dfrac{\pi r^2}{2}$



$Q_x = \int y\,dA = \int_0^r 2y\sqrt{r^2 - y^2}\,dy = \dfrac{2r^3}{3}$

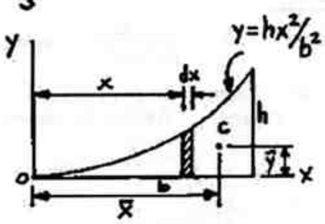
$\bar{y} = \dfrac{Q_x}{A} = \dfrac{4r}{3\pi} \longleftarrow$

12.2-4

$dA = y\,dx = \dfrac{hx^2\,dx}{b^2}$

$A = \int dA = \int_0^b \dfrac{hx^2}{b^2}\,dx = \dfrac{bh}{3}$

$Q_y = \int x\,dA = \int_0^b \dfrac{hx^3}{b^2}\,dx = \dfrac{b^2h}{4}$



$\bar{x} = \dfrac{Q_y}{A} = \dfrac{3b}{4} \longleftarrow$

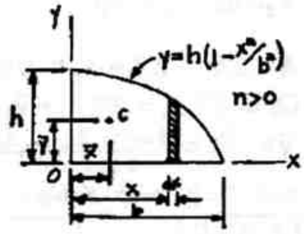
$Q_x = \int y/2\,dA = \int_0^b \dfrac{1}{2}\left(\dfrac{hx^2}{b^2}\right)\left(\dfrac{hx^2}{b^2}\right)dx = \dfrac{bh^2}{10}$

$\bar{y} = \dfrac{Q_x}{A} = \dfrac{3h}{10} \longleftarrow$

12.2-5

$dA = y\,dx = h\left(1 - \dfrac{x^n}{b^n}\right)dx$

$A = \int dA = \int_0^b h\left(1 - \dfrac{x^n}{b^n}\right)dx = bh\left(\dfrac{n}{n+1}\right)$



$Q_y = \int x\,dA = \int_0^b x\,h\left(1 - \dfrac{x^n}{b^n}\right)dx = \dfrac{hb^2}{2}\left(\dfrac{n}{n+2}\right)$

$\bar{x} = \dfrac{Q_y}{A} = \dfrac{b(n+1)}{2(n+2)} \longleftarrow$

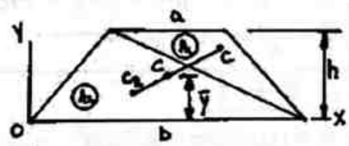
$Q_x = \int \dfrac{y}{2}\,dA = \int_0^b \dfrac{1}{2} h\left(1 - \dfrac{x^n}{b^n}\right)(h)\left(1 - \dfrac{x^n}{b^n}\right)dx = bh^2\left[\dfrac{n^2}{(n+1)(2n+1)}\right]$

$\bar{y} = \dfrac{Q_x}{A} = \dfrac{hn}{2n+1} \longleftarrow$

12.3-1

$A_1 = \dfrac{ah}{2} \qquad \bar{y}_1 = \dfrac{2h}{3}$

$A_2 = \dfrac{bh}{2} \qquad \bar{y}_2 = \dfrac{h}{3}$



$A = \Sigma A_i = \dfrac{ah}{2} + \dfrac{bh}{2} = \dfrac{h}{2}(a+b)$

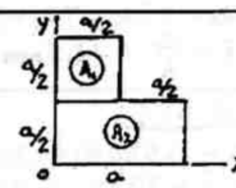
$Q_x = \Sigma \bar{y}_i A_i = \dfrac{2h}{3}\left(\dfrac{ah}{2}\right) + \dfrac{h}{3}\left(\dfrac{bh}{2}\right) = \dfrac{h^2}{6}(2a+b)$

$\bar{y} = \dfrac{Q_x}{A} = \dfrac{h(2a+b)}{3(a+b)} \longleftarrow$

12.3-2

$A_1 = \dfrac{a^2}{4} \qquad \bar{y}_1 = \dfrac{3a}{4}$

$A_2 = \dfrac{a^2}{2} \qquad \bar{y}_2 = \dfrac{a}{4}$



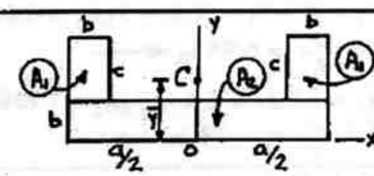
$A = \Sigma A_i = \dfrac{3a^2}{4}$

$Q_x = \Sigma \bar{y}_i A_i = \dfrac{3a}{4}\left(\dfrac{a^2}{4}\right) + \dfrac{a}{4}\left(\dfrac{a^2}{2}\right) = \dfrac{5a^3}{16}$

$\bar{x} = \bar{y} = \dfrac{Q_x}{A} = \dfrac{5a}{12} \longleftarrow$

12.3-3

$a = 6$ in.
$b = 1$ in.
$c = 2$ in.



$A_1 = bc = 2\text{ in.}^2 \qquad \bar{y}_1 = 2\text{ in.}$

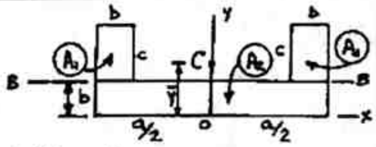
$A_2 = ab = 6\text{ in.}^2 \qquad \bar{y}_2 = 0.5\text{ in.}$

$A = \Sigma A_i = 2A_1 + A_2 = 10\text{ in.}^2$

$Q_x = \Sigma \bar{y}_i A_i = 2\bar{y}_1 A_1 + \bar{y}_2 A_2 = 11.0\text{ in.}^3$

$\bar{y} = \dfrac{Q_x}{A} = 1.1\text{ in.} \longleftarrow$

12.3-4



$A_1 = bc \quad \bar{y}_1 = b + c/2$

$A_2 = ab \quad \bar{y}_2 = b/2$

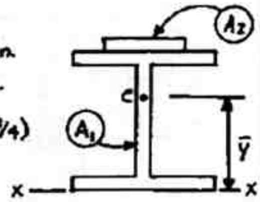
$A = \Sigma A_i = 2A_1 + A_2 = b(2c + a)$

$Q_x = \Sigma \bar{y}_i A_i = 2\bar{y}_1 A_1 + \bar{y}_2 A_2 = \frac{b}{2}(4bc + 2c^2 + ab)$

$\bar{y} = \frac{Q_x}{A} = \frac{4bc + 2c^2 + ab}{2(2c + a)}$

Set $\bar{y} = b$ and solve: $2c^2 = ab$ ←

12.3-5



W 24 × 162 Plate: 8 in. × 0.75 in.

$A_1 = 47.7 \text{ in.}^2$ $\quad \bar{y}_1 = d/2 = 12.5 \text{ in.}$

$d = 25.00$ in.

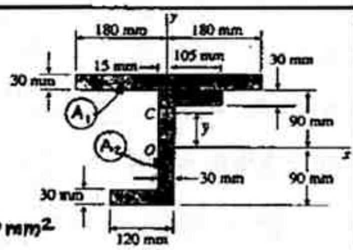
$A_2 = (8)(3/4) = 6 \text{ in.}^2 \quad \bar{y}_2 = 25.00 + \frac{1}{2}(3/4) = 25.375$ in.

$A = \Sigma A_i = A_1 + A_2 = 53.70 \text{ in.}^2$

$Q_x = \Sigma \bar{y}_i A_i = y_1 A_1 + y_2 A_2 = 748.5 \text{ in.}^3$

$\bar{y} = \frac{Q_x}{A} = 13.9$ in. ←

12.3-6



$A_1 = (360)(30) = 10{,}800 \text{ mm}^2$

$\bar{y}_1 = 105$ mm

$A_2 = 2(120)(30) + (120)(30)$
$= 10{,}800 \text{ mm}^2$

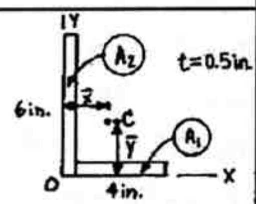
$\bar{y}_2 = 0$

$A = \Sigma A_i = A_1 + A_2 = 21{,}600 \text{ mm}^2$

$Q_x = \Sigma \bar{y}_i A_i = \bar{y}_1 A_1 + \bar{y}_2 A_2 = 1.134 \times 10^6 \text{ mm}^3$

$\bar{y} = \frac{Q_x}{A} = 52.5$ mm ←

12.3-7



$A_1 = (3.5)(0.5) = 1.75 \text{ in.}^2$

$\bar{y}_1 = 0.25$ in. $\quad \bar{x}_1 = 2.25$ in.

$A_2 = (6)(0.5) = 3.0 \text{ in.}^2$

$\bar{y}_2 = 3.0$ in. $\quad \bar{x}_2 = 0.25$ in.

$A = \Sigma A_i = A_1 + A_2 = 4.75 \text{ in.}^2$

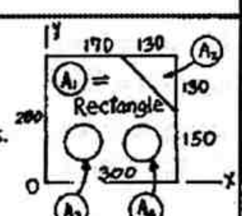
$Q_y = \Sigma \bar{x}_i A_i = \bar{x}_1 A_1 + \bar{x}_2 A_2 = 4.688 \text{ in.}^3$

$\bar{x} = \frac{Q_y}{A} = 0.99$ in. ←

$Q_x = \Sigma \bar{y}_i A_i = \bar{y}_1 A_1 + \bar{y}_2 A_2 = 9.438 \text{ in.}^3$

$\bar{y} = \frac{Q_x}{A} = 1.99$ in. ←

12.3-8



All dimensions are in millimeters.
Diameter of holes = 50 mm
Centers of holes are 80 mm from edges.

$A_1 = (280)(300) = 84{,}000 \text{ mm}^2$

$\bar{x}_1 = 150 \text{ mm} \quad \bar{y}_1 = 140$ mm

$A_2 = \frac{1}{2}(130)^2 = 8450 \text{ mm}^2$

$\bar{x}_2 = 300 - 130/3 = 256.7$ mm

$\bar{y}_2 = 280 - 130/3 = 236.7$ mm

$A_3 = \frac{\pi d^2}{4} = \frac{\pi}{4}(50)^2 = 1963 \text{ mm}^2$

$\bar{x}_3 = 80 \text{ mm} \quad \bar{y}_3 = 80$ mm

CONT.

12.3-8 CONT.

$A_4 = 1963 \text{ mm}^2 \quad \bar{x}_4 = 220 \text{ mm} \quad \bar{y}_4 = 80$ mm

$A = \Sigma A_i = A_1 - A_2 - A_3 - A_4 = 71{,}620 \text{ mm}^2$

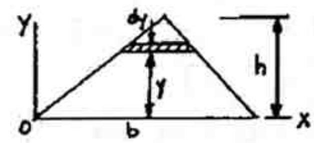
$Q_y = \Sigma \bar{x}_i A_i = \bar{x}_1 A_1 - \bar{x}_2 A_2 - \bar{x}_3 A_3 - \bar{x}_4 A_4 = 9.842 \times 10^6 \text{ mm}^3$

$\bar{x} = \frac{Q_y}{A} = \frac{9.842 \times 10^6}{71{,}620} = 137$ mm ←

$Q_x = \Sigma \bar{y}_i A_i = \bar{y}_1 A_1 - \bar{y}_2 A_2 - \bar{y}_3 A_3 - \bar{y}_4 A_4 = 9.446 \times 10^6 \text{ mm}^3$

$\bar{y} = \frac{Q_x}{A} = \frac{9.446 \times 10^6}{71{,}620} = 132$ mm ←

12.4-1



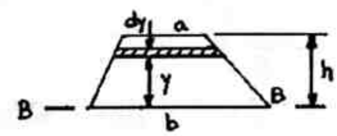
Width of element
$= b\left(\frac{h - y}{h}\right)$

$dA = \frac{b(h - y)}{h}\, dy$

$I_x = \int y^2 dA = \int_0^h y^2 b\left(\frac{h - y}{h}\right) dy$

$= \frac{bh^3}{12}$ ←

12.4-2

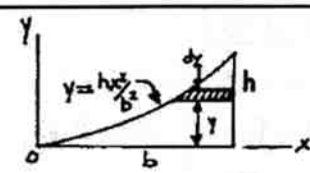


Width of element
$= a + (b - a)\left(\frac{h - y}{h}\right)$

$dA = \left[a + (b - a)\left(\frac{h - y}{h}\right)\right] dy$

$I_{BB} = \int y^2 dA = \int_0^h y^2 \left[a + (b - a)\left(\frac{h - y}{h}\right)\right] dy = \frac{h^3(3a + b)}{12}$ ←

12.4-3



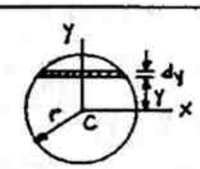
Width of element
$= b - x = b - b\sqrt{\frac{y}{h}}$

$= b(1 - \sqrt{y/h})$

$dA = b(1 - \sqrt{y/h})\, dy$

$I_x = \int y^2 dA = \int_0^h y^2 b(1 - \sqrt{y/h})\, dy = \frac{bh^3}{21}$ ←

12.4-4



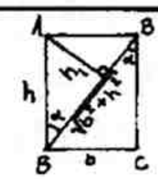
Width of element $= 2\sqrt{r^2 - y^2}$

$dA = 2\sqrt{r^2 - y^2}\, dy$

$I_x = \int y^2 dA = \int_{-r}^{r} y^2 \left(2\sqrt{r^2 - y^2}\right) dy$

$= \frac{\pi r^4}{4}$ ←

12.4-5



$\sin \alpha = \frac{h_1}{h} = \frac{b}{\sqrt{b^2 + h^2}} \qquad h_1 = \frac{bh}{\sqrt{b^2 + h^2}}$

I_1 = Moment of inertia of triangle ABB with respect to BB

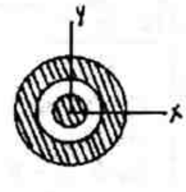
$I_1 = \frac{1}{12}\left(\sqrt{b^2 + h^2}\right) h_1^3$

$= \frac{b^3 h^3}{12(b^2 + h^2)}$

$I_{BB} = 2I_1 = \frac{b^3 h^3}{6(b^2 + h^2)}$ ←

12.4-6

Diameters = 50, 100, and 150 mm

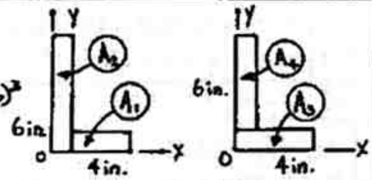


$I_x = \frac{\pi d^4}{64}$ (for a circle)

$I_x = \frac{\pi}{64}\left[(150)^4 - (100)^4 + (50)^4\right]$

$I_x = 20.2 \times 10^6\ mm^4$ ⟵

12.4-7

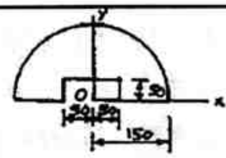


$I_x = I_1 + I_2$
$= \frac{1}{3}(3.5)(0.5)^3 + \frac{1}{3}(0.5)(6)^3$
$= 36.1\ in.^4$ ⟵

$I_y = I_3 + I_4$
$= \frac{1}{3}(0.5)(4)^3 + \frac{1}{3}(5.5)(0.5)^3$
$= 10.9\ in.^4$ ⟵

12.4-8

All dimensions in millimeters



$r = 150\ mm$
$b = 100\ mm$
$h = 50\ mm$

$I_x = (I_x)_{semicircle} - (I_x)_{rectangle} = \frac{\pi r^4}{8} - \frac{bh^3}{3}$
$= 195 \times 10^6\ mm^4$ ⟵

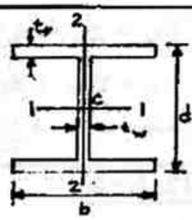
$I_y = I_x$ ⟵

$A = \frac{\pi r^2}{2} - bh = 30.34 \times 10^3\ mm^2$

$r_x = \sqrt{I_x/A} = 80.1\ mm$ ⟵

$r_y = r_x$ ⟵

12.4-9



W16×100 $d = 16.97$ in.
$t_w = t_{web} = 0.585$ in.
$b = 10.425$ in.
$t_F = t_{Flange} = 0.985$ in.
All dimensions in inches.

$I_1 = \frac{1}{12}bd^3 - \frac{1}{12}(b - t_w)(d - 2t_F)^3$
$= \frac{1}{12}(10.425)(16.97)^3 - \frac{1}{12}(9.840)(15.00)^3$
$= 1478\ in.^4$ say, $I_1 = 1480\ in.^4$ ⟵

$I_2 = 2\left(\frac{1}{12}\right)t_F b^3 + \frac{1}{12}(d - 2t_F)t_w^3$
$= \frac{1}{6}(0.985)(10.425)^3 + \frac{1}{12}(15.00)(0.585)^3$
$= 186.2\ in.^4$ say, $I_2 = 186\ in.^4$ ⟵

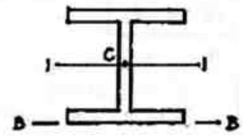
$A = 2(bt_F) + (d - 2t_F)t_w$
$= 2(10.425)(0.985) + (15.00)(0.585)$
$= 29.31\ in.^2$

$r_1 = \sqrt{I_1/A} = 7.10$ in. ⟵

$r_2 = \sqrt{I_2/A} = 2.52$ in. ⟵

Note that these results are in close agreement with the tabulated values.

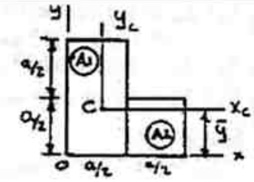
12.5-1



W 12×50
$I_1 = 394\ in.^4$ $A = 14.7\ in.^2$
$d = 12.19$ in.

$I_b = I_1 + A\left(\frac{d}{2}\right)^2$
$= 394 + 14.7\,(6.095)^2 = 940\ in.^4$ ⟵

12.5-2



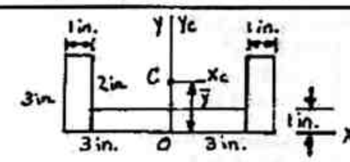
From Prob. 12.3-2:
$A = 3a^2/4$
$\bar{y} = 5a/12$

$I_x = \frac{1}{3}\left(\frac{a}{2}\right)(a^3) + \frac{1}{3}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)^3 = \frac{3a^4}{16}$

$I_x = I_{x_c} + A\bar{y}^2$

$I_c = I_{x_c} = I_x - A\bar{y}^2 = \frac{3a^4}{16} - \frac{3a^2}{4}\left(\frac{5a}{12}\right)^2 = \frac{11a^4}{192}$ ⟵

12.5-3



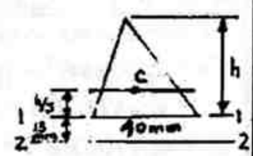
From Prob. 12.3-3:
$A = 10.0\ in.^2$
$\bar{y} = 1.10$ in.

$I_x = \frac{1}{3}(4)(1)^3 + 2\left(\frac{1}{3}\right)(1)(3)^3 = 19.33\ in.^4$

$I_x = I_{x_c} + A\bar{y}^2$

$I_{xc} = I_x - A\bar{y}^2 = 19.33 - (10.0)(1.10)^2 = 7.23\ in.^4$ ⟵

12.5-4



$b = 40\ mm$ $I_1 = 90 \times 10^3\ mm^4$

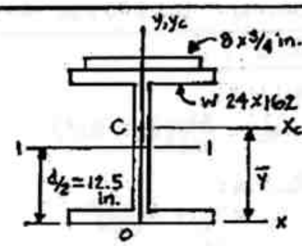
$I_1 = \frac{1}{12}bh^3$

$h = \sqrt[3]{\frac{12 I_1}{b}} = 30\ mm$

$I_c = \frac{1}{36}bh^3 = 30 \times 10^3\ mm^4$

$I_2 = I_c + Ad^2 = 30 \times 10^3 + \frac{1}{2}(40)(30)(25)^2$
$= 405 \times 10^3\ mm^4$ ⟵

12.5-5



From Prob. 12.3-5:
$\bar{y} = 13.9$ in.

W24 × 162
$d = 25.00$ in.
$I_1 = 5170\ in.^4$
$A = 47.7\ in.^2$
$I_2 = 443\ in.^4$

$I'_{x_c} = I_1 + A(\bar{y} - d/2)^2 = 5170 + (47.7)(1.40)^2 = 5260\ in.^4$

$I'_{y_c} = I_2 = 443\ in.^4$

Plate

$I''_{x_c} = \frac{1}{12}(8)(3/4)^3 + (8)(3/4)(d + 3/8 - \bar{y})^2$
$= 0.281 + 6(25.00 + 0.375 - 13.9)^2$
$= 0.281 + 6(11.48)^2 = 790\ in.^4$

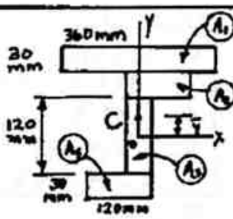
$I''_{y_c} = \frac{1}{12}(3/4)(8)^3 = 32.0\ in.^4$

Entire cross section

$I_{x_c} = I'_{x_c} + I''_{x_c} = 5260 + 790 = 6050\ in.^4$ ⟵

$I_{y_c} = I'_{y_c} + I''_{y_c} = 443 + 32 = 475\ in.^4$ ⟵

12.5-6



From Prob. 12.3-6:
$\bar{y} = 52.50$ mm
$A = 21,600$ mm^2

A_1: $I_x = 1/12(360)(30)^3 + (360)(30)(105)^2 = 119.9 \times 10^6$ mm^4

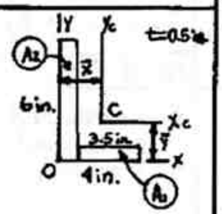
A_2: $I_x = 1/12(120)(30)^3 + (120)(30)(75)^2 = 20.52 \times 10^6$ mm^4

A_3: $I_x = 1/12(30)(120)^3 = 4.32 \times 10^6$ mm^4

A_4: $I_x = 20.52 \times 10^6$ mm^4

Entire area:
$I_x = 165.2 \times 10^6$ mm^4
$I_{xc} = I_x - A\bar{y}^2 = 165.2 \times 10^6 - (21,600)(52.50)^2$
$= 106 \times 10^6$ mm^4 ⟵

12.5-7



From Prob. 12.3-7:
$A = 4.75$ in.^{2}
$\bar{y} = 1.987$ in.
$\bar{x} = 0.9869$ in.

From Problem 12.4-7:
$I_x = 36.15$ in.^{4}
$I_y = 10.90$ in.^{4}

$I_{xc} = I_x - A\bar{y}^2 = 36.15 - (4.75)(1.987)^2 = 17.40$ in.^{4} ⟵

$I_{yc} = I_y - A\bar{x}^2 = 10.90 - (4.75)(0.9869)^2 = 6.27$ in.^{4} ⟵

12.5-8 Wide-Flange Beam

All dimensions in millimeters.

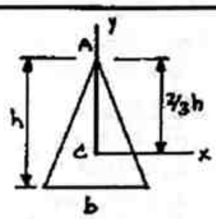
$I_x = \frac{1}{12}(b)(250)^3 - \frac{1}{12}(b-15)(220)^3$
$= 0.4147 \times 10^6 b + 13.31 \times 10^6$ (mm^4)

$I_y = 2\left(\frac{1}{12}\right)(15)(b^3) + \frac{1}{12}(220)(15)^3$
$= 2.5b^3 + 61,880$ (mm^4)

Equate I_x to $3I_y$ and rearrange:
$7.5b^3 - 0.4147 \times 10^6 b - 13.12 \times 10^6 = 0$

Solving: $b = 250$ mm ⟵

12.6-1

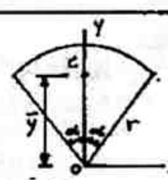


Point C:
$(I_p)_c = \frac{bh}{144}(4h^2 + 3b^2)$

Point A:
$I_p = I_{pc} + A\left(\frac{2h}{3}\right)^2$
$= \frac{bh}{144}(4h^2+3b^2) + \frac{bh}{2}\left(\frac{2h}{3}\right)^2$

$I_p = \frac{bh}{48}(b^2 + 12h^2)$ ⟵

12.6-2



$(I_p)_o = I_x + I_y = \frac{\alpha r^4}{2}$

$A = \alpha r^2$

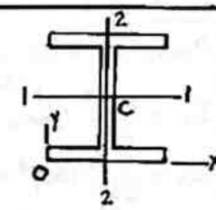
$\bar{y} = \frac{2r\sin\alpha}{3\alpha}$

$(I_p)_c = (I_p)_o - A\bar{y}^2 = \frac{\alpha r^4}{2} - \alpha r^2\left(\frac{2r\sin\alpha}{3\alpha}\right)^2$

$= \frac{r^4}{18\alpha}(9\alpha^2 - 8\sin^2\alpha)$ ⟵

(α = radians)

12.6-3



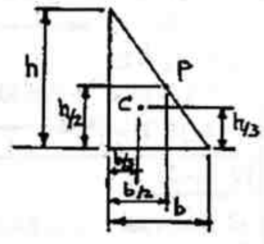
W 8×21
$I_1 = 75.3$ in.^{4} $\quad I_2 = 9.77$ in.^{4}
$A = 6.16$ in.^{2}
Depth $d = 8.28$ in.
Width $b = 5.27$ in.

$I_x = I_1 + A(d/2)^2 = 75.3 + 6.16(4.14)^2 = 180.9$ in.^{4}

$I_y = I_2 + A(b/2)^2 = 9.77 + 6.16(2.635)^2 = 52.5$ in.^{4}

$I_P = I_x + I_y = 233$ in.^{4} ⟵

12.6-4



$I_P = (I_P)_c + Ad^2$

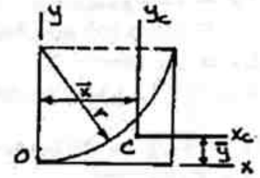
$(I_P)_c = \frac{bh}{36}(h^2 + b^2)$

$A = \frac{bh}{2}$

$d^2 = \left(\frac{b}{2} - \frac{b}{3}\right)^2 + \left(\frac{h}{2} - \frac{h}{3}\right)^2$
$= \frac{b^2}{36} + \frac{h^2}{36}$

$I_P = \frac{bh}{36}(h^2+b^2) + \frac{bh}{2}\left(\frac{b^2}{36} + \frac{h^2}{36}\right) = \frac{bh}{24}(b^2+h^2)$ ⟵

12.6-5



$A = (1 - \pi/4)r^2$

$I_x = (1 - 5\pi/16)r^4$

$I_y = (1/3 - \pi/16)r^4$

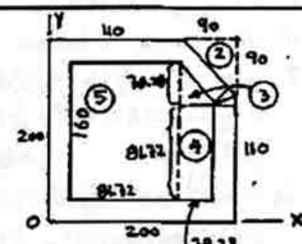
$\bar{x} = \frac{2r}{3(4-\pi)} \qquad \bar{y} = \frac{(10-3\pi)r}{3(4-\pi)}$

$I_{xc} = I_x - A\bar{y}^2 = \frac{r^4}{144}\left(\frac{176 - 84\pi + 9\pi^2}{4-\pi}\right)$

$I_{yc} = I_{xc}$ (By symmetry)

$(I_P)_c = 2I_{xc} = \frac{r^4}{72}\left(\frac{176 - 84\pi + 9\pi^2}{4-\pi}\right)$ ⟵

12.6-6



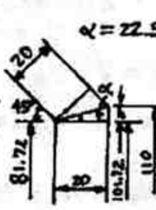
All dimensions in millimeters.

Area No. 1 (Outer square)
$A_1 = (200)(200) = 40 \times 10^3$ mm^2
$\bar{y}_1 = 100$ mm
$I_{x_1} = 1/3(200)(200)^3 = 533.33 \times 10^6$ mm^4

Area No. 2 (Triangle)
$A_2 = 1/2(90)(90) = 4.050 \times 10^3$ mm^2
$\bar{y}_2 = 110 + 2/3(90) = 170$ mm
$I_{x_2} = 1/36(90)(90)^3 + (4.05 \times 10^3)(170)^2 = 118.87 \times 10^6$ mm^4

Area No. 3 (Triangle)
$A_3 = 1/2(78.28)(78.28) = 3.064 \times 10^3$ mm^2
$\bar{y}_3 = 101.72 + 1/3(78.28) = 127.8$ mm
$I_{x_3} = 1/36(78.28)(78.28)^3 + (3.064 \times 10^3)(127.8)^2 = 51.09 \times 10^6$ mm^4

Figure for obtaining the dimensions of areas 3 and 4



$h = 20(\tan\alpha) = 8.28$

Area No. 4 (Rectangle)
$A_4 = (78.28)(81.72) = 6.397 \times 10^3$ mm^2
$\bar{y}_4 = 20 + 1/2(81.72) = 60.86$ mm

CONT.

12.6-6 CONT.

$I_{x4} = 1/12(78.28)(81.72)^3 + (6.397\times10^3)(60.86)^2 = 27.25\times10^6\ mm^4$

Area No. ⑤ (Rectangle)

$A_5 = (81.72)(160) = 13.075\times10^3\ mm^2$

$\bar{y}_5 = 20 + 1/2(160) = 100\ mm$

$I_{x5} = 1/12(81.72)(160)^3 + (13.075\times10^3)(100)^2 = 158.64\times10^6\ mm^4$

Composite Area

$A = \Sigma A_i = A_1 - A_2 - A_3 - A_4 - A_5 = 13.41\times10^3\ mm^2$

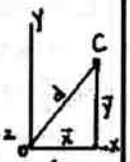
$Q_x = \Sigma \bar{y}_i A_i = \bar{y}_1 A_1 - \bar{y}_2 A_2 - \bar{y}_3 A_3 - \bar{y}_4 A_4 - \bar{y}_5 A_5$

$= 1.223\times10^6\ mm^3$

$\bar{y} = \frac{Q_x}{A} = 91.20\ mm \qquad \bar{x} = \bar{y}$

$I_x = \Sigma I_{xi} = I_{x1} - I_{x2} - I_{x3} - I_{x4} - I_{x5}$

$= 177.5\times10^6\ mm^4$

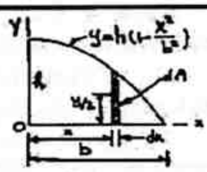


Origin O: $(I_P)_O = 2I_x = 355.0\times10^6\ mm^4$

Centroid C: $(I_P)_C = (I_P)_O - Ad^2 \qquad d^2 = \bar{x}^2 + \bar{y}^2 = 2\bar{y}^2$

$(I_P)_C = 355.0\times10^6 - (13.41\times10^3)(2)(91.20)^2 = 132\times10^6\ mm^4$ ←

12.7-1



Product of inertia of element dA with respect to axes through its own centroid equals zero.

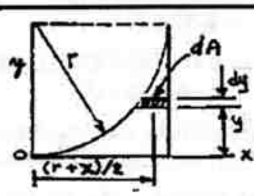
Parallel-axis theorem applied to element dA:

$dI_{xy} = 0 + (dA)(d_1 d_2) = (y\,dx)(x)(y/2)$

$= \frac{h^2 x}{2}\left(1 - \frac{x^2}{b^2}\right)^2 dx$

$I_{xy} = \int dI_{xy} = \frac{h^2}{2}\int_0^b x\left(1 - \frac{x^2}{b^2}\right)^2 dx = \frac{b^2h^2}{12}$ ←

12.7-2



$x^2 + (y-r)^2 = r^2$

$r^2 - x^2 = (y-r)^2$

Product of inertia of element dA with respect to axes through its own centroid equals zero.

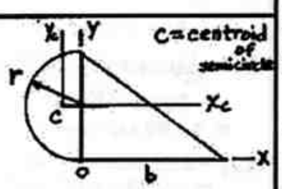
Parallel-axis theorem applied to element dA:

$dI_{xy} = 0 + (dA)(d_1 d_2) = (r-x)(dy)\left(\frac{r+x}{2}\right)(y)$

$= \frac{1}{2}(r^2 - x^2)y\,dy = \frac{1}{2}(y-r)^2 y\,dy$

$I_{xy} = 1/2\int_0^r y(y-r)^2 dy = \frac{r^4}{24}$ ←

12.7-3



Triangle

$I_{xy} = \frac{b^2h^2}{24} = \frac{b^2(2r)^2}{24} = \frac{b^2r^2}{6}$

Semicircle

$I_{xy} = I_{x_c y_c} + A d_1 d_2$

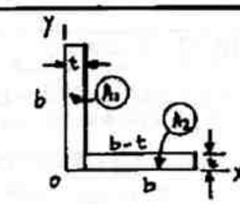
$I_{x_c y_c} = 0 \qquad A = \frac{\pi r^2}{2} \qquad d_1 = r \qquad d_2 = -\frac{4r}{3\pi}$

$I_{xy} = 0 + \frac{\pi r^2}{2}(r)\left(-\frac{4r}{3\pi}\right) = -\frac{2r^4}{3}$

Composite area

$I_{xy} = \frac{b^2r^2}{6} - \frac{2r^4}{3} = 0 \qquad b = 2r$ ←

12.7-4



$(I_{xy})_1 = \frac{t^2b^2}{4}$

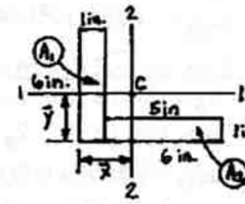
$(I_{xy})_2 = I_{x_c y_c} + A_2 d_1 d_2$

$= 0 + (b-t)(t)(t/2)\left(\frac{b+t}{2}\right)$

$= \frac{t^2}{4}(b^2 - t^2)$

$I_{xy} = (I_{xy})_1 + (I_{xy})_2 = \frac{t^2}{4}(2b^2 - t^2)$ ←

12.7-5



$A = (6)(1) + (5)(1) = 11.0\ in.^2$

$\bar{y} = \frac{(6)(1)(3) + (5)(1)(0.5)}{11.0}$

$= 1.864\ in. \qquad \bar{x} = \bar{y}$

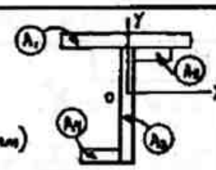
$I_{12} = A_1(3.0 - \bar{y})(-\bar{x} + 0.5) +$

$A_2(-\bar{y} + 0.5)(3.5 - \bar{x})$

$= (6)(1)(1.136)(-1.364) + (5)(1)(-1.364)(1.636)$

$I_{12} = -20.5\ in.^4$ ←

12.7-6



$(I_{xy})_1 = (I_{xy})_3 = 0$

$(I_{xy})_2 = I_{x_c y_c} + A_2 d_1 d_2$

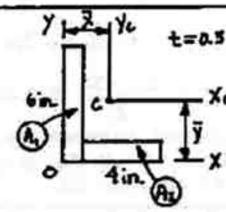
$= 0 + (90\,mm)(30\,mm)(75\,mm)(60\,mm)$

$= 12.15\times10^6\ mm^4$

$(I_{xy})_4 = (I_{xy})_2$

$I_{xy} = 2(12.15\times10^6) = 24.3\times10^6\ mm^4$ ←

12.7-7



t = 0.5 in.

$A_1 = (6)(0.5) = 3\ in.^2$

$A_2 = (3.5)(0.5) = 1.75\ in.^2$

$\bar{x} = 0.9868\ in.$

$\bar{y} = 1.9868\ in.$

$(I_{xy})_1 = \frac{(0.5)^2(6)^2}{4} = 2.25\ in.^4$

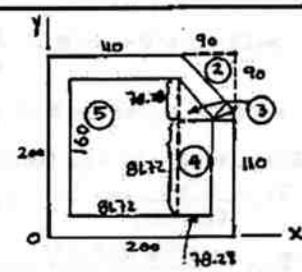
$(I_{xy})_2 = I_c + A_2 d_1 d_2 = 0 + (1.75\,in.^2)\left(\frac{0.5\,in.}{2}\right)(2.25\,in.) = 0.9844\ in.^4$

$I_{xy} = 2.25 + 0.9844 = 3.234\ in.^4$

$I_{x_c y_c} = I_{xy} - A\bar{x}\bar{y} = 3.234 - (4.75)(0.9868)(1.9868)$

$= -6.079\ in.^4$ ←

12.7-8



All dimensions in millimeters.

See solution to Prob. 12.6-6 for dimensions, areas, and centroidal distances.

$I_{xy} = I_{x_c y_c} + A\bar{x}\bar{y}$

Area No. ① (outer square)

$A_1 = 40\times10^3\ mm^2$

$\bar{y}_1 = 100\ mm \qquad \bar{x}_1 = 100\ mm$

$I_{xy_1} = 1/4(200)^2(200)^2 = 400\times10^6\ mm^4$

Area No. ② (Triangle)

$A_2 = 4.050\times10^3\ mm^2$

$\bar{y}_2 = 170\ mm \qquad \bar{x}_2 = 170\ mm$

$I_{xy_2} = -1/72(90)^2(90)^2 + (4.050\times10^3)(170)(170) = 116.13\times10^6\ mm^4$

CONT.

12.7-8 CONT.

Area No. ③ (Triangle)

$A_3 = 3.064 \times 10^3\ mm^2 \quad \bar{y}_3 = 127.8\ mm \quad \bar{x}_3 = 127.8\ mm$

$I_{xy_3} = -\frac{1}{72}(78.28)^2(78.28)^2 + (3.064\times10^3)(127.8)(127.8)$

$= 49.52 \times 10^6\ mm^4$

Area No. ④ (Rectangle)

$A_4 = 6.397 \times 10^3\ mm^2$

$\bar{y} = 60.86\ mm \qquad \bar{x}_4 = 20 + 81.72 + \frac{1}{2}(78.28) = 140.86\ mm$

$I_{xy_4} = (6.397\times10^3)(60.86)(140.86) = 54.84 \times 10^6\ mm^4$

Area No. ⑤ (Rectangle)

$A_5 = 13.075 \times 10^3\ mm^2$

$\bar{y}_5 = 100\ mm \qquad \bar{x}_5 = 20 + \frac{1}{2}(81.72) = 60.86\ mm$

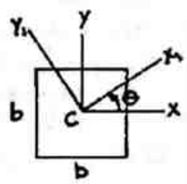
$I_{xy_5} = (13.075\times10^3)(100)(60.86) = 79.57 \times 10^6\ mm^4$

Composite area

$I_{xy} = I_{xy_1} - I_{xy_2} - I_{xy_3} - I_{xy_4} - I_{xy_5}$

$= 99.9 \times 10^6\ mm^4 \longleftarrow$

12.8-1



$I_x = I_y = \frac{b^4}{12} \qquad I_{xy} = 0$

$I_{x_1} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

$I_{x_1} = \frac{I_x + I_y}{2} = \frac{b^4}{12} \longleftarrow$

$I_{x_1} + I_{y_1} = I_x + I_y \qquad I_{y_1} = \frac{b^4}{12} \longleftarrow$

$I_{x_1y_1} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = 0 \longleftarrow$

Note that all moments of inertia are the same and the product of inertia is always zero (with respect to axes through C).

12.8-2

Rectangle

$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = 0$

$\cos\theta = \frac{b}{\sqrt{b^2+h^2}} \qquad \sin\theta = \frac{h}{\sqrt{b^2+h^2}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{b^2 - h^2}{b^2 + h^2}$

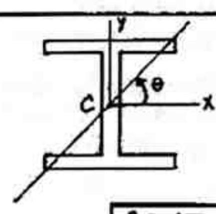
$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2bh}{b^2+h^2}$

Substitute into Eqs. (12-25), (12-28), (12-27):

$I_{x_1} = \frac{b^3h^3}{6(b^2+h^2)} \longleftarrow \qquad I_{y_1} = \frac{bh(b^4+h^4)}{12(b^2+h^2)} \longleftarrow$

$I_{x_1y_1} = \frac{b^2h^2(h^2-b^2)}{12(b^2+h^2)} \longleftarrow$

12.8-3



W 12×50 $\quad I_x = 394\ in.^4$

$I_y = 56.3\ in.^4 \qquad I_{xy} = 0$

Depth $d = 12.19$ in.

Width $b = 8.080$ in.

$\tan\theta = \frac{d}{b} = \frac{12.19}{8.080} = 1.509$

CONT.

12.8-3 CONT.

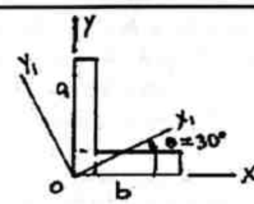
$\theta = 56.46° \qquad 2\theta = 112.9°$

$I_d = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

$= \frac{394 + 56.3}{2} + \frac{394 - 56.3}{2}\cos(112.9°) - 0$

$= 159\ in.^4 \longleftarrow$

12.8-4



$a = 150\ mm \qquad b = 100\ mm$

$t = 15\ mm \qquad \theta = 30°$

$I_x = \frac{1}{3}(15)(150)^3 + \frac{1}{3}(85)(15)^3$

$= 16.97 \times 10^6\ mm^4$

$I_y = \frac{1}{3}(135)(15)^3 + \frac{1}{3}(15)(100)^3 = 5.152 \times 10^6\ mm^4$

$I_{xy} = \frac{1}{4}(15)^2(150)^2 + (85)(15)\left(\frac{15}{2}\right)(57.5) = 1.815 \times 10^6\ mm^4$

Substitute into Eq. (12-26) with $\theta = 30°$:

$I_{x_1} = 12.4 \times 10^6\ mm^4 \longleftarrow$

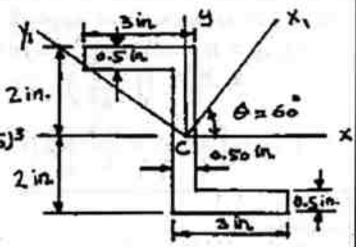
Substitute into Eq. (12-25) with $\theta = 120°$:

$I_{y_1} = 9.68 \times 10^6\ mm^4 \longleftarrow$

Substitute into Eq. (12-27) with $\theta = 30°$:

$I_{x_1y_1} = 6.02 \times 10^6\ mm^4 \longleftarrow$

12.8-5



$I_x = \frac{1}{12}(3)(4)^3 - \frac{1}{12}(2.5)(3)^3$

$= 10.38\ in.^4$

$I_y = \frac{1}{12}(4)(0.5)^3 + 2\left(\frac{1}{12}\right)(0.5)(2.5)^3 + 2(2.5)(0.5)(1.5)^2$

$= 6.97\ in.^4$

$I_{xy} = 2(2.5)(0.5)(1.75)(-1.5)$

$= -6.562\ in.^4$

Substitute into Eq. (12-25) with $\theta = 60°$:

$I_{x_1} = 13.5\ in.^4 \longleftarrow$

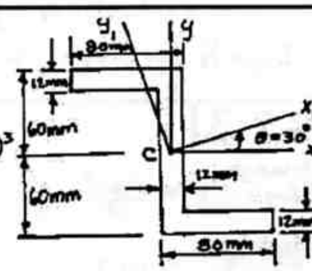
Substitute into Eq. (12-25) with $\theta = 150°$:

$I_{y_1} = 3.84\ in.^4 \longleftarrow$

Substitute into Eq. (12-27) with $\theta = 60°$:

$I_{x_1y_1} = 4.76\ in.^4 \longleftarrow$

12.8-6



$I_x = \frac{1}{12}(80)(120)^3 - \frac{1}{12}(68)(96)^3$

$= 6.506 \times 10^6\ mm^4$

$I_y = \frac{1}{12}(120)(12)^3 + (2)\left(\frac{1}{12}\right)(12)(68)^3 + 2(12)(68)(40)^2$

$= 3.257 \times 10^6\ mm^4$

$I_{xy} = 2(68)(12)(54)(-40)$

$= -3.525 \times 10^6\ mm^4$

Substitute into Eq. (12-25) with $\theta = 30°$:

$I_{x_1} = 8.75 \times 10^6\ mm^4 \longleftarrow$

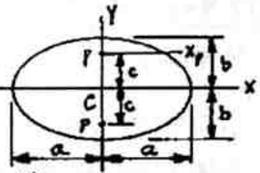
Substitute into Eq. (12-25) with $\theta = 120°$:

$I_{y_1} = 1.02 \times 10^6\ mm^4 \longleftarrow$

Substitute into Eq. (12-27) with $\theta = 30°$:

$I_{x_1y_1} = -0.356 \times 10^6\ mm^4 \longleftarrow$

12.9-1



(a) At a principal point, all moments of inertia are equal.

$\therefore I_{x_p} = I_y \qquad (1)$

$I_y = \dfrac{\pi b a^3}{4}$ (Appendix D, Case 16)

Parallel-axis theorem:

$I_{x_p} = I_x + Ac^2 \qquad I_x = \dfrac{\pi a b^3}{4} \qquad A = \pi ab$

$I_{x_p} = \dfrac{\pi a b^3}{4} + \pi a b c^2$

From Eq. (1): $\dfrac{\pi a b^3}{4} + \pi a b c^2 = \dfrac{\pi b a^3}{4}$

$\therefore c = \frac{1}{2}\sqrt{a^2 - b^2} \longleftarrow$

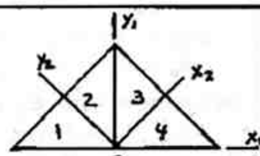
(b) $c = b \qquad b = \frac{1}{2}\sqrt{a^2 - b^2}$

Solving, $\dfrac{a}{b} = \sqrt{5} \longleftarrow$

(c) If $\dfrac{a}{b} = 1$, $c = 0$

$\therefore 1 \leq \dfrac{a}{b} < \sqrt{5} \longleftarrow$

12.9-2



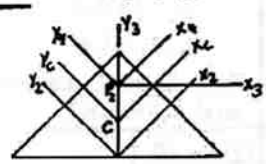
Consider Point P_1:

$I_{x_1 y_1} = 0$ because y_1 is an axis of symmetry.

$I_{x_2 y_2} = 0$ because areas 1 and 2 are symmetrical about the y_2 axis and areas 3 and 4 are symmetrical about the x_2 axis.

Two different sets of principal axes exist at point P_1.

$\therefore P_1$ is a principal point $\longleftarrow$



Consider Point P_2:

$I_{x_3 y_3} = 0$ because y_3 is an axis of symmetry.

$I_{x_2 y_2} = 0$ (see above).

Parallel-axis theorem:

$I_{x_2 y_2} = I_{x_c y_c} + A d_1 d_2 \qquad A = \dfrac{b^2}{4} \qquad d_1 = d_2 = \dfrac{b}{6\sqrt{2}}$

$I_{x_c y_c} = -\left(\dfrac{b^2}{4}\right)\left(\dfrac{b}{6\sqrt{2}}\right)^2 = -\dfrac{b^4}{288}$

Parallel-axis theorem:

$I_{x_4 y_4} = I_{x_c y_c} + A d_1 d_2 \qquad d_1 = d_2 = \dfrac{-b}{6\sqrt{2}}$

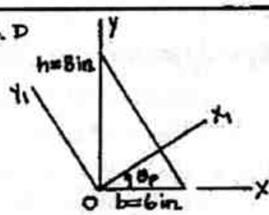
$I_{x_4 y_4} = \dfrac{-b^4}{288} + \dfrac{b^2}{4}\left(-\dfrac{b}{6\sqrt{2}}\right)^2 = 0$

Two different sets of principal axes ($x_3 y_3$ and $x_4 y_4$) exist at point P_2.

$\therefore P_2$ is a principal point $\longleftarrow$

12.9-3

See Case 7, App. D



$I_x = \dfrac{bh^3}{12} = 256 \text{ in.}^4$

$I_y = \dfrac{hb^3}{12} = 144 \text{ in.}^4$

$I_{xy} = \dfrac{b^2 h^2}{24} = 96 \text{ in.}^4$

$\tan 2\theta_p = -\dfrac{2I_{xy}}{I_x - I_y} = -1.7143$

$2\theta_p = -59.743°$ and $120.26°$

$\theta_p = -29.87°$ and $60.13°$

CONT.

12.9-3 CONT.

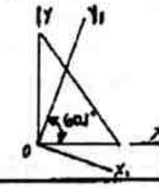
Substitute into Eq. (12-33a): $I_1 = 311.1 \text{ in.}^4$

Substitute into Eq. (12-33b): $I_2 = 88.9 \text{ in.}^4$

Substitute into Eq. (12-25) with $\theta = -29.87°$:

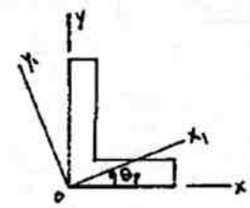
$I_{x_1} = 311.1 \text{ in.}^4$

$\therefore \theta_{p_1} = -29.87°$ and $\theta_{p_2} = 60.13°$



$\begin{cases} I_1 = 311.1 \text{ in.}^4 & \theta_{p_1} = -29.9° \longleftarrow \\ I_2 = 88.9 \text{ in.}^4 & \theta_{p_2} = 60.1° \longleftarrow \end{cases}$

12.9-4



From Prob. 12.8-4:

$I_x = 16.97 \times 10^6 \text{ mm}^4$

$I_y = 5.152 \times 10^6 \text{ mm}^4$

$I_{xy} = 1.815 \times 10^6 \text{ mm}^4$

$\tan 2\theta_p = -\dfrac{2I_{xy}}{I_x - I_y} = -0.3072$

$2\theta_p = -17.07°$ and $162.9°$

$\theta_p = -8.54°$ and $81.46°$

Substitute into Eq. (12-25) with $\theta = -8.54°$:

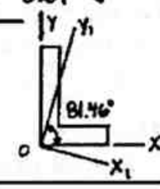
$I_{x_1} = 17.24 \times 10^6 \text{ mm}^4$

Substitute into Eq. (12-25) with $\theta = 81.46°$:

$I_{x_1} = 4.88 \times 10^6 \text{ mm}^4$

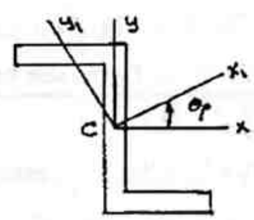
Therefore, $I_1 = 17.24 \times 10^6 \text{ mm}^4 \quad \theta_{p_1} = -8.54° \longleftarrow$

$I_2 = 4.88 \times 10^6 \text{ mm}^4 \quad \theta_{p_2} = 81.46° \longleftarrow$



Note: I_1 and I_2 can also be found from Eqs. (12-33a) and (12-33b).

12.9-5



From Prob. 12.8-5:

$I_x = 10.38 \text{ in.}^4$

$I_y = 6.97 \text{ in.}^4$

$I_{xy} = -6.562 \text{ in.}^4$

$\tan 2\theta_p = -\dfrac{2I_{xy}}{I_x - I_y} = 3.848$

$2\theta_p = 75.43°$ and $255.43°$

$\theta_p = 37.72°$ and $127.7°$

Substitute into Eq. (12-25) with $\theta = 37.72°$:

$I_{x_1} = 15.45 \text{ in.}^4$

Substitute into Eq. (12-25) with $\theta = 127.7°$:

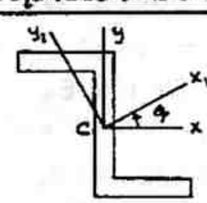
$I_{x_1} = 1.90 \text{ in.}^4$

Therefore, $I_1 = 15.5 \text{ in.}^4 \quad \theta_{p_1} = 37.7° \longleftarrow$

$I_2 = 1.9 \text{ in.}^4 \quad \theta_{p_2} = 127.7° \longleftarrow$

Note: I_1 and I_2 can also be found from Eqs. (12-33a) and (12-33b).

12.9-6



From Prob. 12.8-6:

$I_x = 6.506 \times 10^6 \text{ mm}^4$

$I_y = 3.257 \times 10^6 \text{ mm}^4$

$I_{xy} = -3.525 \times 10^6 \text{ mm}^4$

$\tan 2\theta_p = -\dfrac{2I_{xy}}{I_x - I_y} = 2.170$

$2\theta_p = 65.26°$ and $245.26°$

CONT.

12.9-6 CONT.

$\theta_p = 32.63^\circ$ and 122.63°

Substitute into Eq. (12-25) with $\theta = 32.63^\circ$:

$I_{x_1} = 8.76 \times 10^6\ mm^4$

Substitute into Eq. (12-25) with $\theta = 122.63^\circ$:

$I_{x_1} = 1.00 \times 10^6\ mm^4$

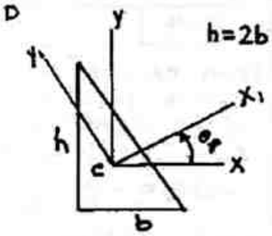
Therefore, $I_1 = 8.76 \times 10^6\ mm^4 \qquad \theta_{p_1} = 32.6^\circ \longleftarrow$

$I_2 = 1.00 \times 10^6\ mm^4 \qquad \theta_{p_2} = 122.6^\circ \longleftarrow$

Note: I_1 and I_2 can also be found from Eqs. (12-33a) and (12-33b).

12.9-7

See Case 6, App. D



$$I_x = \frac{bh^3}{36} = \frac{2b^4}{9}$$

$$I_y = \frac{hb^3}{36} = \frac{b^4}{18}$$

$$I_{xy} = -\frac{b^2h^2}{72} = -\frac{b^4}{18}$$

$$\tan 2\theta_p = -\frac{2I_{xy}}{I_x - I_y} = 2/3$$

$2\theta_p = 33.690^\circ$ and 213.690°, $\theta_p = 16.85^\circ$ and 106.85°

For $2\theta_p = 33.690^\circ$, $\sin 2\theta_p = \frac{2}{\sqrt{13}}$ and $\cos 2\theta_p = \frac{3}{\sqrt{13}}$

Substitute into Eq. (12-25):

$$I_{x_1} = \frac{b^4}{36}(5 + \sqrt{13}) = 0.23090\, b^4$$

For $2\theta_p = 213.690^\circ$, $\sin 2\theta_p = \frac{-2}{\sqrt{13}}$ and $\cos 2\theta_p = -\frac{3}{\sqrt{13}}$

Substitute into Eq. (12-25):

$$I = \frac{b^4}{36}(5 - \sqrt{13}) = 0.03873\, b^4$$

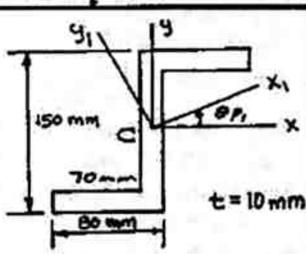
Therefore,

$I_1 = (5 + \sqrt{13})\frac{b^4}{36} = 0.2390\, b^4 \qquad \theta_{p_1} = 16.8^\circ \longleftarrow$

$I_2 = (5 - \sqrt{13})\frac{b^4}{36} = 0.03873\, b^4 \qquad \theta_{p_2} = 106.8^\circ \longleftarrow$

Note: I_1 and I_2 can also be found from Eqs. (12-33a) and (12-33b).

12.9-8



$$I_x = \frac{1}{12}(80)(150)^3 - \frac{1}{12}(70)(130)^3$$
$$= 9.684 \times 10^6\ mm^4$$

$$I_y = \frac{1}{12}(150)(10^3) + 2\left(\frac{1}{12}\right)(10)(70)^3 + 2(70)(10)(40)^2$$
$$= 2.824 \times 10^6\ mm^4$$

$$I_{xy} = 2(70)(10)(40)(70) = 3.920 \times 10^6\ mm^4$$

$$\tan 2\theta_p = -\frac{2I_{xy}}{I_x - I_y} = -1.1429$$

$2\theta_p = -48.81^\circ$ and 131.19°

$\theta_p = -24.41^\circ$ and 65.59°

Substitute into Eq. (12-25) with $\theta = -24.41^\circ$

$I_{x_1} = 11.46 \times 10^6\ mm^4$

Substitute into Eq. (12-25) with $\theta = 65.59^\circ$

$I_{x_1} = 1.05 \times 10^6\ mm^4$

CONT.

12.9-8 CONT.

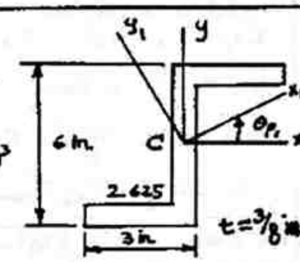
Therefore,

$I_1 = 11.46 \times 10^6\ mm^4 \qquad \theta_{p_1} = -24.4^\circ \longleftarrow$

$I_2 = 1.05 \times 10^6\ mm^4 \qquad \theta_{p_2} = 65.6^\circ \longleftarrow$

Note: I_1 and I_2 can also be found from Eqs. (12-33a) and (12-33b).

12.9-9



$$I_x = \frac{1}{12}(3)(6)^3 - \frac{1}{12}(2.625)(5.25)^3$$
$$= 22.35\ in.^4$$

$$I_y = \frac{1}{12}(6)\left(\frac{3}{8}\right)^3 + 2\left(\frac{1}{12}\right)(0.375)(2.625)^3 + 2(2.625)(0.375)(1.5)^2$$
$$= 5.587\ in.^4$$

$$I_{xy} = 2(2.625)\left(\frac{3}{8}\right)(2.8125)(1.5)$$
$$= 8.306\ in.^4$$

$$\tan 2\theta_p = -\frac{2I_{xy}}{I_x - I_y} = -0.9910$$

$2\theta_p = -44.74^\circ$ and 135.26°

$\theta_p = -22.37^\circ$ and 67.63°

Substitute into Eq. (12-25) with $\theta = -22.37^\circ$:

$I_{x_1} = 25.77\ in.^4$

Substitute into Eq. (12-25) with $\theta = 67.63^\circ$:

$I_{x_1} = 2.17\ in.^4$

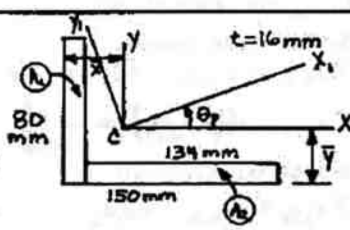
Therefore,

$I_1 = 25.8\ in.^4 \qquad \theta_{p_1} = -22.4^\circ \longleftarrow$

$I_2 = 2.2\ in.^4 \qquad \theta_{p_2} = 67.6^\circ \longleftarrow$

Note: I_1 and I_2 can also be found from Eqs. (12-33a) and (12-33b).

12.9-10



$$A = A_1 + A_2$$
$$= 3.424 \times 10^3\ mm^2$$

$$\bar{x} = \frac{(8)(80)(16) + (83)(134)(16)}{3.424 \times 10^3}$$
$$= 54.96\ mm$$

$$\bar{y} = \frac{(40)(80)(16) + (8)(134)(16)}{3.424 \times 10^3} = 19.96\ mm$$

$$I_x = \frac{1}{12}(16)(80)^3 + (80)(16)(40 - 19.96)^2 + \frac{1}{12}(134)(16)^3 + (134)(16)(19.96 - 8)^2$$
$$= 1.549 \times 10^6\ mm^4$$

$$I_y = \frac{1}{12}(80)(16)^3 + (80)(16)(54.96 - 8.00)^2 + \frac{1}{12}(16)(134)^3 + (134)(16)(83 - 54.96)^2$$
$$= 7.744 \times 10^6\ mm^4$$

$$I_{xy} = (80)(16)(40 - 19.96)(-54.96 + 8) + (134)(16)(83 - 54.96)(-19.96 + 8)$$
$$= -1.924 \times 10^6\ mm^4$$

$$\tan 2\theta_p = -\frac{2I_{xy}}{I_x - I_y} = -0.6211$$

$2\theta_p = -31.85^\circ$ and 148.15°

CONT.

12.9-10 CONT.

$\theta_p = -15.92°$ and $74.08°$

Substitute into Eq. (12-25) with $\theta = -15.92°$:

$I_{x_1} = 1.00 \times 10^6 \text{ mm}^4$

Substitute into Eq. (12-25) with $\theta = 74.08°$:

$I_{x_1} = 8.29 \times 10^6 \text{ mm}^4$

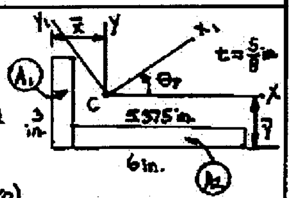
Therefore,

$I_1 = 8.29 \times 10^6 \text{ mm}^4 \qquad \theta_{p_1} = 74.1°$ ←

$I_2 = 1.00 \times 10^6 \text{ mm}^4 \qquad \theta_{p_2} = -15.9°$ ←

Note: I_1 and I_2 can also be obtained from Eqs. (12-33a) and (12-33b)

12.9-11



$A = A_1 + A_2 = 5.2344 \text{ in.}^2$

$\bar{x} = \dfrac{(5/16)(3)(5/8) + (3.3125)(5.375)(5/8)}{5.2344}$

$= 2.238$ in.

$\bar{y} = \dfrac{(1.5)(3)(5/8) + (5/16)(5.375)(5/8)}{5.2344} = 0.7379$ in.

$I_x = \frac{1}{12}\left(\frac{5}{8}\right)(3)^3 + (3)\left(\frac{5}{8}\right)(1.5 - 0.7379)^2 + \frac{1}{12}(5.375)\left(\frac{5}{8}\right)^3$
$+ (5.375)\left(\frac{5}{8}\right)\left(0.7379 - \frac{5}{16}\right)^2 = 3.213 \text{ in.}^4$

$I_y = \frac{1}{12}(3)\left(\frac{5}{8}\right)^3 + (3)\left(\frac{5}{8}\right)\left(2.238 - \frac{5}{16}\right)^2 + \frac{1}{12}\left(\frac{5}{8}\right)(5.375)^3$
$+ (5.375)\left(\frac{5}{8}\right)(3.3125 - 2.238)^2 = 18.979 \text{ in.}^4$

$I_{xy} = (3)\left(\frac{5}{8}\right)(1.5 - 0.7379)\left(\frac{5}{16} - 2.238\right)$
$+ (5.375)\left(\frac{5}{8}\right)(3.3125 - 2.238)\left(\frac{5}{16} - 0.7379\right)$
$= -4.287 \text{ in.}^4$

$\tan 2\theta_p = -\dfrac{2I_{xy}}{I_x - I_y} = -0.5438$

$2\theta_p = -28.54°$ and $151.46°$

$\theta_p = -14.27°$ and $75.73°$

Substitute into Eq. (12-25) with $\theta = -14.27°$:

$I_{x_1} = 2.12 \text{ in.}^4$

Substitute into Eq. (12-25) with $\theta = 75.73°$:

$I_{x_1} = 20.1 \text{ in.}^4$

Therefore,

$I_1 = 20.1 \text{ in.}^4 \qquad \theta_{p_1} = 75.7°$ ←

$I_2 = 2.12 \text{ in.}^4 \qquad \theta_{p_2} = -14.3°$ ←

Note: I_1 and I_2 can also be obtained from Eq. (12-33a) and (12-33b).

- END OF SOLUTIONS -