# Problemas

a) $\vec{F}: \mathbb{R}^3 \to \mathbb{R}^3$, $\varphi: \mathbb{R}^3 \to \mathbb{R}$, todos $C^1$. Pdq $\operatorname{rot}(\varphi \vec{F}) = \varphi \operatorname{rot}(\vec{F}) + \nabla\varphi \times \vec{F}$

sol

$$\operatorname{rot}(\varphi \vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \varphi F_x & \varphi F_y & \varphi F_z \end{vmatrix} = \begin{pmatrix} \partial_y(\varphi F_z) - \partial_z(\varphi F_y) \\ -\left(\partial_x(\varphi F_z) - \partial_z(\varphi F_x)\right) \\ \partial_x(\varphi F_y) - \partial_y(\varphi F_x) \end{pmatrix}$$

Atención al orden del 'determinante'.

$$= \begin{pmatrix} \partial_y\varphi F_z + \varphi \partial_y F_z - \partial_z\varphi F_y - \varphi \partial_z F_y \\ -\left(\partial_x\varphi F_z + \varphi\partial_x F_z - \partial_z\varphi F_x - \varphi\partial_z F_x\right) \\ \partial_x\varphi F_y + \varphi\partial_x F_y - \partial_y\varphi F_x - \varphi\partial_y F_x \end{pmatrix}$$

$$= \varphi \operatorname{rot}(\vec{F}) + \nabla\varphi \times \vec{F} \quad //$$

$$\nabla\varphi \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x\varphi & \partial_y\varphi & \partial_z\varphi \\ F_x & F_y & F_z \end{vmatrix} = \begin{pmatrix} (\partial_y\varphi)F_z - (\partial_z\varphi)F_y \\ -\left((\partial_x\varphi)F_z - (\partial_z\varphi)F_x\right) \\ (\partial_x\varphi)F_y - (\partial_y\varphi)F_x \end{pmatrix}$$

b) $\phi, \psi$ $\mathbb{R}^3 \to \mathbb{R}$ $\in C^2$. $\Sigma$ superficie orientable y $C = \partial\Sigma$. Pdq

$$\iint_\Sigma \nabla\phi \times \nabla\psi \cdot d\vec{S} = \int_C \phi\nabla\psi \cdot d\vec{l}.$$

sol Considerar en el ejercicio (a) $\varphi = \phi$, $\vec{F} = \nabla\psi$

$$\Rightarrow \operatorname{rot}(\phi\nabla\psi) = \phi \underbrace{\operatorname{rot}(\nabla\psi)}_{=0} + \nabla\phi \times \nabla\psi = \nabla\phi \times \nabla\psi$$

$$\iint_\Sigma \nabla\phi \times \nabla\psi \, d\vec{S} = \iint_\Sigma \operatorname{rot}(\phi\nabla\psi) \, d\vec{S} \overset{\text{Teo Stokes}}{=} \oint_{\partial\Sigma} \phi\nabla\psi \, d\vec{r}$$

c) $\Omega \subseteq \mathbb{R}^3$ abto no vacío, $\vec{F}: \Omega \to \mathbb{R}^3$; $g: \Omega \to \mathbb{R}$, $C^1$.

Pdq
$$\iint_S g \operatorname{rot}(\vec{F}) = \oint_{\partial S} g\vec{F} d\vec{r} - \iint_S \nabla g \times \vec{F} d\vec{S}$$

Sol Tomando $\varphi = g$, $\vec{F}$ en la parte a),

$$\operatorname{rot}(g\vec{F}) = g \operatorname{rot}(\vec{F}) + \nabla g \times \vec{F}$$

Entonces, integrando en $S$,

$$\iint_S \operatorname{rot}(g\vec{F}) d\vec{S} = \iint_S (g \operatorname{rot}(\vec{F}) + \nabla g \times \vec{F}) d\vec{S}$$

y por teo Stokes,

$$\iint_S \operatorname{rot}(g\vec{F}) d\vec{S} = \oint_{\partial S} g\vec{F} d\vec{r}$$

$$\Rightarrow \quad \iint_S g \operatorname{rot}(\vec{F}) d\vec{S} = \oint g\vec{F} d\vec{r} - \iint_S \nabla g \times \vec{F} d\vec{S}$$

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