

AUXILIAR # 10

P1. Calcular:

(i) $\int_0^{\infty} \frac{1}{1+x^n} dx$, para $n \geq 2$.

(ii) $\int_0^{2\pi} e^{2\cos(\theta)} d\theta$

P2. Pruebe que:

$$\int_0^{\infty} \frac{x \sin(x)}{x^2 + a^2} dx = \frac{\pi e^{-a}}{2}, \quad a < 0$$

Hint: Considere $f(z) = \frac{ze^{iz}}{z^2 + a^2}$

P3. Pruebe que:

$$\int_0^{\infty} \frac{\cosh(ax)}{\cosh(x)} dx = \frac{\pi}{2 \cos(\frac{\pi a}{2})}$$

Hint: Considere $f(z) = \frac{e^{az}}{\cosh(z)}$.

P4. Demuestre que:

$$\pi \tan(z) = \sum_{n=1}^{\infty} \frac{8z}{(2n-1)^2 - 4z^2}$$

Hint: Considere $f(z) = \frac{\pi \tan(\pi z)}{z^2 - a^2}$