

PAUTA Actividad 5

Pregunta 1

Balance de masa estanque 1:

$$\rho \cdot A_1 \frac{dh_1}{dx} = \rho F - \rho \frac{h_1}{R_1}$$

$$A_1 \frac{dh_1}{dx} = F - \frac{h_1}{R_1}$$

Balance de masa estanque 2:

$$\rho \cdot A_2 \frac{dh_2}{dx} = \rho \frac{h_1}{R_1} - \rho \frac{h_2}{R_2}$$

$$A_2 \frac{dh_2}{dx} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$$

Usando variables de desviación:

$$h'_1 = h_1 - h_1^{ee}$$

$$h'_2 = h_2 - h_2^{ee}$$

$$F' = F - F^{ee}$$

Reemplazando:

$$A_1 \frac{dh'_1}{dx} = F' - \frac{h'_1}{R_1}$$

$$A_2 \frac{dh'_2}{dx} = \frac{h'_1}{R_1} - \frac{h'_2}{R_2}$$

Aplicando *LaPlace*:

$$A_1 \cdot s \cdot \overline{h'_1} = \overline{F'} - \frac{\overline{h'_1}}{R_1}$$

$$A_2 \cdot s \cdot \overline{h'_2} = \frac{\overline{h'_1}}{R_1} - \frac{\overline{h'_2}}{R_2}$$

Reordenando...

$$A_1 \cdot s \cdot \overline{h_1'} \cdot R_1 + \overline{h_1'} = \overline{F'} \cdot R_1$$

$$G_1 = \frac{\overline{h_1'}}{\overline{F'}} = \frac{R_1}{A_1 \cdot R_1 \cdot s + 1}$$

$$A_2 \cdot s \cdot \overline{h_2'} \cdot R_2 + \overline{h_2'} = \frac{R_2}{R_1} \cdot \overline{h_1'}$$

$$G_2 = \frac{\overline{h_2'}}{\overline{h_1'}} = \frac{\frac{R_2}{R_1}}{A_2 \cdot R_2 \cdot s + 1}$$

Identificando términos se tiene:

$$Kp_1 = R_1$$

$$\tau p_1 = A_1 \cdot R_1$$

$$Kp_2 = R_2/R_1$$

$$\tau p_2 = A_2 \cdot R_2$$

Por otro lado

$$\frac{\overline{h_2'}}{\overline{F'}} = G_1 \cdot G_2 = \frac{R_2}{(A_1 \cdot R_1 \cdot s + 1) \cdot (A_2 \cdot R_2 \cdot s + 1)}$$

$$\frac{\overline{h_2'}}{\overline{F'}} = \frac{R_2}{(A_1 \cdot R_1 \cdot A_2 \cdot R_2) \cdot s^2 + (A_1 \cdot R_1 + A_2 \cdot R_2) \cdot s + 1}$$

Que corresponde a un segundo orden, donde

$$Kp = R_2$$

$$\tau = \sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2}$$

$$\xi = \frac{(A_1 \cdot R_1 + A_2 \cdot R_2)}{2\sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2}}$$

Como la respuesta es críticamente amortiguada $\Rightarrow \xi = 1$

$$\frac{(A_1 \cdot R_1 + A_2 \cdot R_2)}{2\sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2}} = 1$$

$$(A_1 \cdot R_1 + A_2 \cdot R_2) = 2\sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2} / ()^2$$

$$(A_1^2 \cdot R_1^2 + 2 \cdot A_1 \cdot R_1 \cdot A_2 \cdot R_2 + A_2^2 \cdot R_2^2) = 4 \cdot A_1 \cdot R_1 \cdot A_2 \cdot R_2$$

$$(A_1^2 \cdot R_1^2 - 2 \cdot A_1 \cdot R_1 \cdot A_2 \cdot R_2 + A_2^2 \cdot R_2^2) = 0$$

$$(A_1 \cdot R_1 - A_2 \cdot R_2)^2 = 0$$

$$A_1 \cdot R_1 = A_2 \cdot R_2$$

$$\frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{1}{2} = 0,5$$

b)

$$\tau p_1 = A_1 \cdot R_1$$

$$\tau p_2 = A_2 \cdot R_2$$

$$\tau = A_1 \cdot R_1 = \tau p_1 = \tau p_2$$

Se sabe que la solución para un sistema críticamente amortiguado corresponde a:

$$y'(t) = Kp \left[1 - e^{-\frac{t}{\tau}} \left(1 + \frac{t}{\tau} \right) \right]$$

$$h_2'(t=1) = 0,5 \cdot Kp = Kp \left[1 - e^{-\frac{1}{\tau}} \left(1 + \frac{1}{\tau} \right) \right]$$

$$0,5 = e^{-\frac{1}{\tau}} \left(1 + \frac{1}{\tau} \right)$$

$$\text{Así, } \tau = 0,5958 \text{ min}$$

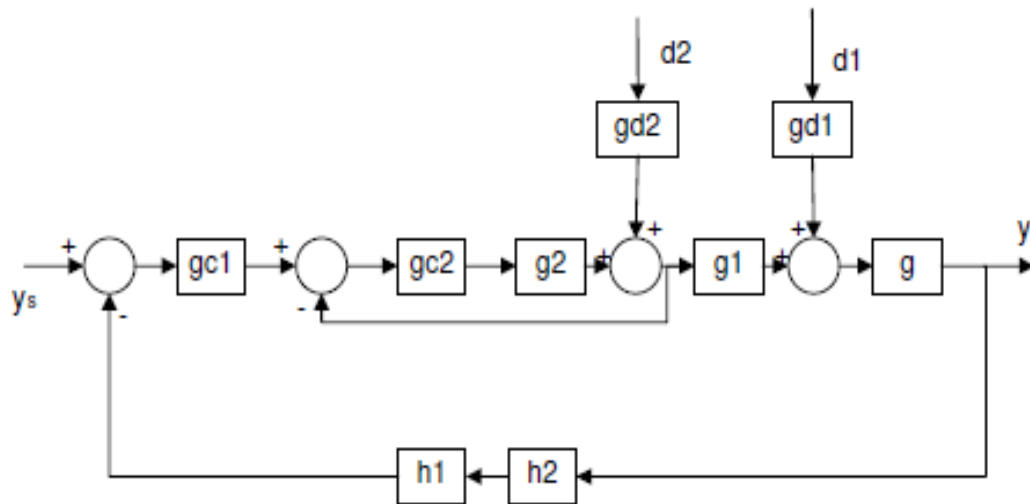
c)

$$0,9 = \left[1 - e^{-\frac{t}{0,5958}} \right]$$

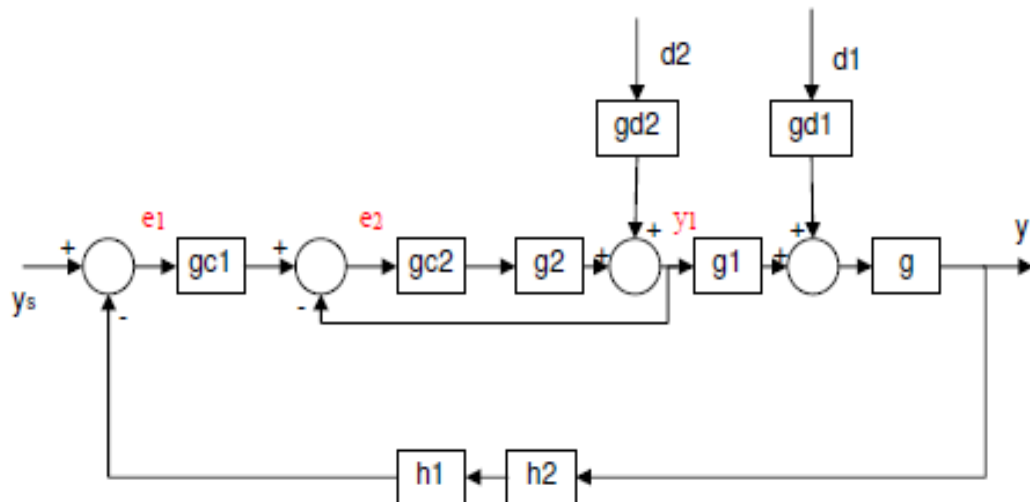
$$t = 1,3719 \text{ min}$$

Pregunta 2

a) Obtenga la función de transferencia en lazo cerrado de:



Para encontrar la función de transferencia le colocaremos nombres a algunas señales:



$$e_2 = e_1 g_{d1} - y_1$$

$$y_1 = g_{c2} g_2 e_2 + g_{d2} d_2$$

reemplazando:

$$y_1 = g_{c2}g_2g_{c1}e_1 - g_{c2}g_2y_1 + g_{d2}d_2$$

$$y_1 = \frac{g_{c2}g_2g_{c1}}{(1 + g_{c2}g_2)} e_1 + \frac{g_{d2}}{(1 + g_{c2}g_2)} d_2$$

Ahora, vemos el sistema completo:

$$e_1 = y_{sp} - h_1h_2y$$

$$y = g_{g1}y_1 + g_{g1}d_1$$

reemplazando y_1

$$y = \frac{gg_1g_{c2}g_2g_{c1}}{(1 + g_{c2}g_2)} e_1 + \frac{gg_1g_{d2}}{(1 + g_{c2}g_2)} d_2 + gg_{d1}d_1$$

reemplazando e_1

$$y = \frac{gg_1g_{c2}g_2g_{c1}}{(1 + g_{c2}g_2)} y_{sp} - \frac{h_1h_2gg_1g_{c2}g_2g_{c1}}{(1 + g_{c2}g_2)} y + \frac{gg_1g_{d2}}{(1 + g_{c2}g_2)} d_2 + gg_{d1}d_1$$

reordenando:

$$y = \frac{gg_1g_{c2}g_2g_{c1}}{(1 + g_{c2}g_2 + h_1h_2gg_1g_{c2}g_2g_{c1})} y_{sp} + \frac{gg_1g_{d2}}{(1 + g_{c2}g_2 + h_1h_2gg_1g_{c2}g_2g_{c1})} d_2 + \frac{(1 + g_{c2}g_2)gg_{d1}}{(1 + g_{c2}g_2 + h_1h_2gg_1g_{c2}g_2g_{c1})} d_1$$