Aggregation of endogenous information in large elections[1]

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Abstract

We study aggregation of information when voters can collect information of different precision, with increased precision entailing an increasing marginal cost. In order to properly understand the incentives to collect information we introduce another dimension of heterogeneity: on top of the ideological dimension we allow for different levels of intensity in preferences. Contrary to traditional models of endogenous information, in equilibrium, there are voters that use signals of different qualities.

Our strategy to show existence allows us to deal with 1) different voting rules, 2) asymmetric priors, and 3) asymmetric distribution of types. After characterizing all symmetric Bayesian equilibria in pure strategies, we show that information aggregation implies a very unique relation between the parameters of the electorate and the voting rule. In a sense, information aggregation is a knife edge result: it is not robust to small changes in the electorate. We also show that, under the same symmetric conditions in Martinelli's (2006) more specialized model, the *Condorcet Jury Theorem* holds under the same cost conditions.

Keywords: Endogenous Information, Aggregation of Information, Heterogeneity.

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1 Introduction

The *Condorcet Jury Theorem* has attracted a lot of attention from political scientists and economists. In its original form, it states that large democracies select the right candidate when voters report truthfully their information and they are, on average, correctly informed.[1] Most of the early work has been devoted to study the case where voters are exogenously informed. For example, Young (1988) and Mueller (2003) assume that "the probability that the opinion of each voter will be in conformity with the truth"(Condorcet (1976), page 47) is the same for all voters while Berend and Paroush (1998) introduce exogenous differences in the probability of selecting the right candidate among voters. The results are in line with Condorcet's: as long as an individual voter is more likely to be right than wrong, an electorate that must choose between two candidates with voters that only care about selecting the right candidate will select this candidate almost surely as the number of voters grows.

Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) show that the assumption that voters report the signal received is a rational behavior under very particular circumstances. If voters behave strategically and consider that their vote is only relevant when it is actually used to select the winner, they should incorporate more information into their reports besides the one conferred by their own signal. This is a traditional rational choice result: voters condition their behavior on the event that they are pivotal.

Moreover when voters differ on how much they suffer for making a mistake, information aggregation might fail. Indeed, Feddersen and Pesendorfer (1997) show that too much heterogeneity (uncertainty about the electorate) implies inefficient information aggregation. They point out that is not the assumption of truthful reporting what matters for information aggregation but how uncertain voters are about the environment. The *Condorcet Jury*

[1] We focus here in non costly voting models. There is a vast literature that studies voting behavior when the act of voting reduces the voter's utility. This literature discusses what is known as the "Paradox of non-voting". See Borgers (2004) for private values analysis, Krishna and Morgan (2005) for common values analysis, and Feddersen (2004) for a survey.

Theorem is only partially valid when voters are strategic. The fact that homogeneity or certainty are crucial for some results in political economy is not new. For example, Palfrey and Rosenthal (1985) show that introducing uncertainty about the other voters preferences might destroy the large and positive turnout in large elections obtained in Palfrey and Rosenthal (1983).

One relevant question that typical *Condorcet Jury Theorem* models do not address is how voters get the information they use to vote. The larger the electorate, the smaller the probability a vote will actually affect the outcome of the election; a rational voter will then have less incentives to acquire information if this information is costly and, in the limit, every voter should be rationally ignorant. Yariv (2004) uses this intuition and assume that, when the electorate grows the signal that a voter receives is less precise.[2] When voters information worsens with the size of the electorate, the speed at which the precision of the signal a voter receives decreases is crucial for information aggregation. In her model there are two effects: more voters imply more sampling which is the driving force behind early model of aggregation of information (Berend and Paroush (1998)) but each draw (a voter's signal) is not as good as it was in a smaller electorate.

Martinelli (2006) provides microfoundations for the results in Yariv (2004).[3] He allows each voter to select the quality of information they use to vote assuming an increasing marginal cost for the precision of the signal. His paper is the first one to study the rational ignorance hypothesis in a continuous quality of information set up. Unfortunately, when he introduces conflict in the electorate (voters suffer differently from mistaken decisions), his results are only valid for the simple majority rule and a particular symmetric assumption about the electorate. Indeed, if we change the voting function his existence and characterization results are no longer valid. In this set up (with conflict), he shows that information aggregation is possible iff the first three derivatives of the cost function for information

[2] She allows for abstention.

[3] He does not study the case of abstention with endogenous information. The model is significantly more complicated as Oliveros (2007) shows and aggregation results are not yet available.

acquisition are nil.[4] This result goes in line with Yariv (2004) results.

Martinelli (2006) provides sufficient conditions under very strict symmetric conditions and for the simple majority rule[5] when the electorate is not homogenous. In our paper we show that when these symmetric conditions are relaxed information aggregation requires a particular rule for each electorate. We show that no rule is robust to all electorates: small changes in the electorate lead to different rules that must be used for aggregation of information to have a chance: aggregation of endogenous information requires a particular design of rules for each electorate.

When information is endogenous the assumption about preferences is crucial and simplifications like the one presented in Martinelli (2006), Yariv (2004), Feddersen and Pesendorfer (1997) and Austen-Smith and Banks (1996) significantly weakens the predictive power of the model (see Oliveros (2007) for a discussion). We study an electorate in which each member is allowed to select the quality (or level) of information she will receive before deciding how to vote between two candidates. We model the conflict present in the electorate assuming preferences with two dimensions of heterogeneity. In one dimension, voters *differ on the ideological axis*: there is "right" and "left"; on the other dimension, voters *differ on the level of concern*: there are irresponsible and responsible members.

In behavioral terms using two dimensions of heterogeneity enriches the interpretation of voters' preferences. For example, when preferences are restricted to one dimension, a voter who suffers a high utility loss for selecting a democrat when a republican should have been elected suffers a small utility loss if a republican is elected when a democrat should have. By introducing the extra dimension (concern) we are able to generate voters that actually differ on the overall level of care for any type of mistakes. Second, for the quality of information to significantly differ across voters for any election rule and any level of asymmetry in the

[4] As it will become clear later, the first derivative of the cost function is related to the quality of information through the first order conditions of the voter's information acquisition problem, the second derivative is related to the change and the third derivative is related to speed of change in the quality of information.

[5] The simple majority rule is the optimal rule when information decreases with the size of the elctorate (see Yariv (2004)).

electorate it is not enough to use only ideological differences. Imagine a voter that suffers the same utility loss for electing the wrong candidate (say for example x) and another one that suffers y for any mistake. If $x < y$ the first voter has little incentives to acquire information, while the more concerned voter (second one) will be more willing to invest in order to receive a highly precise signal.

When voters endogenously collect information of different qualities traditional fixed point arguments require a particular restriction on the information technology.[6] This restriction rules out the aggregation conditions that Martinelli (2006) requires. Therefore, taking the easy road on the existence problem invalidates our comparison with Martinelli (2006). Since we are particularly interested in understanding how robust his results are to the introduction of more natural heterogeneity and asymmetric assumptions, we need to use a different strategy to characterize the equilibrium.

The set of equilibria presents nice geometric properties and we can use the characterization result to overcome the existence problem. We use the best response functions to construct a transformation with domain in a suitable finite dimensional space (Oliveros (2007)). Since the best response functions are embedded in this transformation, we can show that a fixed point of this transformation is an equilibrium of the game. After showing existence and characterizing the equilibrium, we study information aggregation properties of this committee under symmetric assumptions.

In equilibrium, three classes of voters emerge in the voting stage: supporters for each candidate and independents. supporters do not collect information and always vote for the same candidate: they are ideologically driven. Independents collect information and vote according to the information received. Independent voters are not homogeneously informed: in the voting stage there is a continuum of qualities of information. In contrast, all informed

[6] Each voter best response is characterized by a voting function and an investment function (C^0 almost everywhere). In order to search for a fixed point in the space of functions we require more powerful fixed point theorems like Schauder's (see Rudin (1973)). This theorem requires a strong notion of continuity in the space of functions (equicontinuity) that imposes some sort of bounded variation on the collection of candidate functions for information acquisition. In terms, this can be achieved by precluding the second derivative of the cost function for information to be nil when no information is collected.

voters in Martinelli (2006) are equally informed.

Although the model is significantly more complex, we are able to derive almost the same aggregation results that Martinelli (2006) derives when we assume the same conditions he assumes. This relates directly to our final contribution: our existence and characterization results are valid for all q-majority rules without abstention and every distribution of types.

The rest of the Paper is organized as follows. We present our model in the next section. Section 3 solves the model for arbitrary rules and a fairly arbitrary composition of the electorate. We discuss the incentives to collect information and vote separately before presenting the characterization and existence result. We then present the necessary conditions for aggregation of information in Section 4 and the aggregation result in the Appendix. Section 5 concludes.

2 The model

There are n potential voters that must decide between two options A and Q. There are two states of nature ($\omega \in \{a, q\}$) and the probability of state a is given by $\phi = \Pr(a)$ which is common knowledge. Let $\mathbf{X}= \{Q, A\}$ stand for the set of available voting actions. We refer to a generic voting rule as $\mathbf{R}$.

There are three classes of voters: **responsive, partisan for** A and **partisan for** Q. Partisan voters for A (Q) always vote for candidate A (Q), while responsive voters have contingent preferences described by $\theta = \{\theta_q, \theta_a\} \in [0, 1]^2$. Let $U(d \mid \omega)$ be the utility derived from candidate $d \in \{A, Q\}$ winning in state $\omega \in \{a, q\}$. The utility that a responsive voter with preferences θ derives for different outcomes (winning candidates) is contingent in the state and can be described by the four terms $U(A \mid a) = U(Q \mid q) = 0$, $U(A \mid q) = -\theta_q$ and $U(Q \mid a) = -\theta_a$. We refer to responsive voter i's preferences (the pair θ_q and θ_a) as her type, and to a "responsive voter type θ" simply as a "type θ".

A voter's preferences are private information. A voter is responsive with probability

$(1-\alpha)$ and partisan for A (Q) with probability $\alpha\xi_A$ $(\alpha(1-\xi_A))$, where $\alpha \in (0,1)$ and $\xi_A \in (0,1)$. If the voter is responsive, her preferences are drawn independently from a distribution with cumulative distribution function F on $[0,1]^2$ with no mass points. We assume that F, α and ξ_A are common knowledge.

Once nature selects a profile of types and preferences are assigned, a voter can invest in collecting information. Each responsive voter i can select $p \in \left[\frac{1}{2}, 1\right]$ where p is the parameter of a Bernoulli random variable S that takes values on the set $\{s_q, s_a\}$. We assume that the probability of signal $s = s_\omega$ in state $\omega \in \{a, q\}$ is equal to p:

$$\Pr(s_\omega \mid p, \omega) = p \text{ for } \omega \in \{a, q\}$$

The precision cost is given by $C : \left[\frac{1}{2}, 1\right] \rightarrow \mathcal{R}_+$ where we assume that:

Assumption 1 *The cost function C is twice continuously differentiable everywhere in $\left[\frac{1}{2}, 1\right]$ and satisfies 1) $C'(p) > 0$ and $C''(p) > 0$ for all $p > \frac{1}{2}$, 2) $C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$, 3) $C''\left(\frac{1}{2}\right) \geq 0$, and 4) $\lim\limits_{p\rightarrow 1} C'(p) \rightarrow \infty$.*

The last part of assumption (1) implies that there are no fully informed voters in equilibrium. $C'\left(\frac{1}{2}\right) = 0$ simplifies the analysis but allowing for $C'\left(\frac{1}{2}\right) > 0$ would essentially introduce a fixed cost of information acquisition.

Definition 1 *A **regular committee** of size n is a committee with mandatory voting and n members in which preferences are described by the parameters $(\alpha, \xi_A) \in (0,1)^2$ and the distribution function F over $[0,1]^2$ with no mass points, the prior probability of state a is $\phi \in (0,1)$, and the cost technology of information acquisition verifies assumption (1).*

Definition 2 *A pure strategy is an investment function $P : [0,1]^2 \rightarrow \left[\frac{1}{2}, 1\right]$ and a voting function $V : [0,1]^2 \times \{s_q, s_a\} \rightarrow \boldsymbol{X}$, such that $P(\theta)$ is the investment level of responsive voter type θ, and $V(\theta, S) = (V(\theta, s_q), V(\theta, s_a))$ is the contingent vote of responsive voter type θ*

who receives the signal $s \in \{s_q, s_a\}$.[7]

When we refer to a generic voting function, investment function or strategy, we omit the superscript indicating types. We refer to the voter's behavior (strategy) as $V(\theta, S)$ and to an arbitrary pair of votes as $(v_q, v_a) \in \mathbf{X}^2$. $V(\theta, S)$ is part of an strategy while (v_q, v_a) is basically notation. When we want to refer to a particular vote we use just v.

We will refer to a profile of strategies as $\left(\widetilde{P}, \widetilde{V}\right)$ where $\widetilde{P} = (P^1, ...P^n)$ and $\widetilde{V} = (V^1, ...V^n)$ are the profile of investment functions and voting functions for the whole committee. Analogously $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$ is the profile of strategies for all players but player i. We will say that, if $V(\theta, s) = v$ for all $s \in \{s_q, s_a\}$ type θ uses an **uninformed** voting function, and if $V(\theta, s_q) \neq V(\theta, s_a)$ type θ uses an **informed** voting function. We therefore identify strategies by the voting function they employ.

Conditional on the profile of strategies of all voters but i, $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$, we define the probability that the winner is $x \in \{Q, A\}$ in state $\omega \in \{q, a\}$, when voter i casts vote $v \in \mathbf{X}$, as

$$\Pr\left(x \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{1}$$

Since a voter selects the quality of information after observing her type but before observing the signal, while the vote is decided after observing the signal, we need to define payoffs in different stages of the game. The expected utility of player i of type $\theta \in [0,1]^2$ when she votes $v \in \mathbf{X}$, and the state is $\omega \in \{q, a\}$, is

$$u^i(v \mid \theta, \omega) \equiv -\theta_\omega \Pr\left((-\omega) \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{2}$$

where we let $(-\omega) = Q$ if $\omega = a$ and $(-\omega) = A$ if $\omega = q$. This expression is just the product of the disutility of a mistake $(-\theta_\omega)$ and the probability of a mistake in the state $\omega \in \{q, a\}$,

[7] The reader may argue that voting rules should be contingent in the level of investment performed by each voter so $V : [0,1]^2 \times \left[\frac{1}{2}, 1\right]^2 \times \{s_q, s_a\} \to \mathbf{X}$. This approach substantially complicates the model without affecting any of the results. That results are unaffected follows by the fact that between the investment decision and voting decision no other public information is revealed to the voters.

given player i's vote v. We define the expected utility of player i of type $\theta \in [0,1]^2$ and investment choice $p \in \left[\frac{1}{2}, 1\right]$, when she votes $v \in \mathbf{X}$ after receiving the signal $s \in \{s_q, s_a\}$ as

$$U^i(p, v \mid \theta, s) \equiv \sum_{\omega \in \{q,a\}} u^i(v \mid \theta, \omega) \Pr(\omega \mid s, p) \tag{3}$$

Using (3), the expected utility of player i of type $\theta \in [0,1]^2$ and investment choice $p \in \left[\frac{1}{2}, 1\right]$, for a voting function (v_q, v_a) is

$$\mathcal{U}^i(p, (v_q, v_a)) \mid \theta) \equiv \sum_{x \in \{q,a\}} U^i(p, v_x \mid \theta, s_x) \Pr(s_x \mid p) \tag{4}$$

We study symmetric Bayesian equilibria in pure strategies.

Definition 3 *A symmetric Bayesian equilibrium for the voting game in a regular committee with voting rule $\boldsymbol{R}$ and voting alternatives $\boldsymbol{X}$ is a strategy $(P^*(\theta), V^*(\theta, S))$ such that:* ***1)*** *all voters use $(V^*(\theta, S), P^*(\theta))$,* ***2)*** *for every $\theta \in [0,1]^2$, for all $s \in \{s_q, s_a\}$, and for any other feasible $v' \in \boldsymbol{X}$, the strategy $(P^*(\theta), V^*(\theta, S))$ satisfies*

$$U^i(P^*(\theta), V^*(\theta, s) \mid \theta, s) \geq U^i(P^*(\theta), v' \mid \theta, s) \tag{5}$$

and ***3)*** *for every $\theta \in [0,1]^2$, and for any other feasible (v_q, v_a) and p, the strategy $(P^*(\theta), V^*(\theta, S))$ satisfies*

$$\mathcal{U}^i(P^*(\theta), V^*(\theta, S) \mid \theta) - C(P^*(\theta)) \geq \mathcal{U}^i(p, (v_q, v_a) \mid \theta) - C(p) \tag{6}$$

From now on, we omit the strategy profile of all other players as an argument of endogenous variables. Therefore, the probability of a particular outcome of the decision $x \in \{Q, A\}$, in state ω, after player i cast a vote $v \in \mathbf{X}$, is written as $\Pr(x \mid \omega, v)$. Using the symmetry assumption, the probability that an arbitrary voter $j \neq i$ votes for $v \in \mathbf{X}$, in state ω, when all other players but i are using the strategy $(P(\theta), V(\theta, S))$ is

$$\Pr(v \mid \omega) = (1-\alpha) \int_{\theta \in [0,1]^2} \sum_{s \in \{s_q, s_a\}} I(V(\theta, s) = v) \Pr(s \mid P(\theta), \omega)\, dF(\theta) + \alpha \xi_v \tag{7}$$

The first part of the right side is just the probability that a voter is responsive multiplied by the probability that a responsive voter votes for $v \in \mathbf{X}$. The second part is the probability that a voter is partisan, multiplied by the probability that a partisan member votes for $v \in \mathbf{X}$. This expression aggregates over the two sources of private information present in the model: the type of player and the signal received after investment.[8]

Recalling the expression (1) and fixing all players' strategies but i's, we also define the change in the probability of A winning when voter i switches her vote from Q to A in state ω as,

$$\Delta \Pr(\omega, Q) \equiv \Pr(A \mid \omega, A) - \Pr(A \mid \omega, Q) \tag{8}$$

Again, we must recall that $\Delta \Pr(\omega, Q)$ for $\omega \in \{q, a\}$ are conditioned on the actual profile of strategies $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$ so they both depend on the behavior of all other players.

3 Solving the model

Let T_k stand for the total number of votes for A, when there are k voters. The voting rule is defined as a pair $\mathbf{R}= (N, r)$ with $n \geq N \geq \frac{n}{2}$[9] and $r \in [0,1]$, such that A wins if $T_n > N$ and Q wins if $T_n < N$; if $T_n = N$, A wins with probability $1-r$ and Q wins with probability r.[10]

[8] As the reader suspects $\Pr(x \mid \omega, v)$ is a combination of $\Pr(v \mid \omega)$, for $v \in \mathbf{X}$, $x \in \{Q, A\}$ and $\omega \in \{q, a\}$.

[9] The case with $N < \frac{n}{2}$ can be studied by reversing the roles of Q and A.

[10] The results are valid for all q-majority rules, such that A is the winner if the percentage of votes in favor of A is at least $N > qn$.

3.1 Voting Incentives

Responsive voters can use four possible voting functions: (Q, A), (A, Q), (A, A), and (Q, Q). It is straightforward to see that the voting functions (A, A) and (Q, Q) can not induce positive investment in information in equilibrium. Only (Q, A) and (A, Q) can induce positive investment in equilibrium. As the reader suspects, (A, Q) can not be optimal.

The next lemma provides conditions for a vote $v \in \{Q, A\}$ to satisfy the equilibrium condition (5) after receiving the signal $s \in \{s_q, s_a\}$ when the investment is p

Lemma 1 *In any regular committee, a necessary condition for a responsive voter to vote for A after receiving the signal $s \in \{s_q, s_a\}$, when she is type θ and the investment is p, is*

$$\theta_q \Delta \Pr(q, Q) \Pr(q \mid p, s) \leq \theta_a \Delta \Pr(a, Q) \Pr(a \mid p, s) \tag{9}$$

A necessary condition for a responsive voter type θ with investment p to vote for Q after receiving the signal $s \in \{s_q, s_a\}$ is obtained by reversing the sign of (9). Strict inequalities give sufficient conditions.

Proof. Using the definition of expected utility in (2) and (3), equation (5), and Bayes rule gives the result. ■

$\theta_q \Delta \Pr(q, Q) \Pr(q \mid p, s)$ is the expected cost of making a mistake (making A the winner in state q) when switching the vote from Q to A, while $\theta_a \Delta \Pr(a, Q) \Pr(a \mid p, s)$ is the expected benefit of avoiding a mistake (Q winning in state a) when switching the vote from Q to A. Therefore, (9) only states that the voter will vote in favor of A, when the expected benefit of avoiding a mistake is higher than the expected loss of making one when changing the vote from Q to A.

Responsive voters will consider how they affect the outcome of the election to decide how they vote. The next lemma shows that changing the vote always has an impact in the election so $\Delta \Pr(\omega, Q)$, $\omega \in \{q, a\}$ are strictly positive.[11]

[11] Why we use $\Delta \Pr(\omega, Q)$, $\omega \in \{A, Q\}$ instead of pivotal probabilities will be clear in the existence and

Lemma 2 *In any regular committee, for any $n \geq 2$, there is some $\zeta(\omega) > 0$ for each $\omega \in \{q, a\}$, such that $\Delta \Pr(\omega, Q) \in [\zeta(\omega), 1 - \zeta(\omega)]$. If $n = 1$, then $\Delta \Pr(\omega, Q) = 1$ for $\omega \in \{q, a\}$*

Proof. Assume that all players but i are using the strategy (P, V) and player i uses (P^i, V^i). Let $\Pr(T_m = k \mid \omega)$ be the probability that there are k votes for A out of the m voters when everybody uses the voting function V and the investment function P. Using the distribution function of a binomial random variable

$$\Pr(T_{n-1} = k \mid \omega) = \frac{(n-1)!}{(n-1-k)!k!} (\Pr(A \mid \omega))^k (1 - \Pr(A \mid \omega))^{n-1-k} \tag{10}$$

where $\Pr(A \mid \omega)$ is defined as in (7). The probability of candidate A being selected when member i votes for $x \in \{Q, A\}$ is just

$$\Pr(A \mid \omega, A) = \Pr(T_{n-1} = N - 1 \mid \omega)(1 - r) + \sum_{k=N}^{n-1} \Pr(T_{n-1} = k \mid \omega) \tag{11}$$

and

$$\Pr(A \mid \omega, Q) = \Pr(T_{n-1} = N \mid \omega)(1 - r) + \sum_{k=N+1}^{n-1} \Pr(T_{n-1} = k \mid \omega) \tag{12}$$

Therefore, using definition (8)

$$\Delta \Pr(\omega, Q) = \Pr(T_{n-1} = N - 1 \mid \omega)(1 - r) + \Pr(T_{n-1} = N \mid \omega) r \tag{13}$$

Since $\Pr(A \mid \omega) \in \left[\alpha \xi_A, 1 - \alpha \xi_Q\right]$, $\Pr(A \mid \omega, v) < 1$ for $v \in \{Q, A\}$, then $\Delta \Pr(\omega, Q) \leq 1 - \zeta^1(\omega)$ for some $\zeta^1(\omega) > 0$ small enough. On the other hand, using (10), we conclude that $\Delta \Pr(\omega, Q)$ is bigger than

$$\max\{\Pr(T_{n-1} = N - 1 \mid \omega), \Pr(T_{n-1} = N \mid \omega)\}$$

characterization section.

Again, using the fact that $\Pr(A \mid \omega) \in \left[\alpha\xi_A, 1-\alpha\xi_Q\right]$, there is some $\zeta^2(\omega) > 0$ such that $\Delta\Pr(\omega, Q) \geq \zeta^2(\omega)$. Finally, the result for $n = 1$ is straightforward. ■

Once we know that $\Delta\Pr(\omega, Q) > 0$, $\omega \in \{q, a\}$ it is easy to see that there are no equilibria in which all responsive voters vote for a particular candidate independently of their preferences. That is, there exist θ_1 and θ_2 such that $V(\theta_1, S) \neq V(\theta_2, S)$.

With Lemma (2) at hand, we can manipulate the expression (9) to show that A is optimal after signal $s \in \{s_q, s_a\}$ if

$$\frac{\theta_q}{\theta_a}\frac{\Pr(q \mid p, s)}{\Pr(a \mid p, s)} \leq \frac{\Delta\Pr(a, Q)}{\Delta\Pr(q, Q)} \tag{14}$$

Obviously, Q is optimal if the sign is reversed in expression (14). $\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}$ is determined in equilibrium, while $\frac{\theta_q}{\theta_a}\frac{\Pr(q|p,s)}{\Pr(a|p,s)}$ is the voter's private information. (14) will allow us to construct functions that separate types that prefer $v = A$ over $v = Q$ conditional on the signal and the investment.

We proceed now to determine the responsive voters' optimal voting function. There are basically two informed strategies: the strategy with the voting function (A, Q), and the strategy with the voting function (Q, A). If the signal is informative, it is not optimal for a responsive voter to vote against the information that she receives in all circumstances. A player using a strategy with the informed voting function (A, Q) is doing just that. Only uninformed voters that are indifferent between option A and Q may use (A, Q). Therefore, (A, Q) is not used in equilibrium with positive probability.

Lemma 3 *In any regular committee, the voting function (A, Q) may be used in equilibrium only by types that satisfy $\frac{\theta_a}{\theta_q} = \frac{\Delta\Pr(q,Q)}{\Delta\Pr(a,Q)}\frac{(1-\phi)}{\phi}$, and if they use it, they do not collect information; for all other types it is not optimal. The set of types who do not acquire information and use an informative strategy has measure 0 in equilibrium.*

Proof. First note that Bayes rule gives that $\frac{\Pr(q|s_a)}{\Pr(a|s_a)} = \frac{(1-p)}{p}\frac{(1-\phi)}{\phi}$ and $\frac{\Pr(q|s_q)}{\Pr(a|s_q)} = \frac{p}{(1-p)}\frac{(1-\phi)}{\phi}$.

Using (9), the informed strategy with (A, Q) must satisfy

$$\begin{aligned} \theta_q \Delta \Pr(q, Q) \frac{p}{(1-p)} \frac{(1-\phi)}{\phi} &\leq \theta_a \Delta \Pr(a, Q) \\ \theta_q \Delta \Pr(q, Q) \frac{(1-p)}{p} \frac{(1-\phi)}{\phi} &\geq \theta_a \Delta \Pr(a, Q) \end{aligned} \tag{15}$$

we must have that $\frac{\theta_q}{\theta_a} \frac{p}{(1-p)} \leq \frac{\theta_q}{\theta_a} \frac{(1-p)}{p}$. If $p > \frac{1}{2}$, we reach a contradiction.

If $p = \frac{1}{2}$, it is necessary for (A, Q) that both inequalities in (15) hold which imply the conditions stated in the hypothesis: $\frac{\theta_a}{\theta_q} = \frac{\Delta \Pr(q,Q)}{\Delta \Pr(a,Q)} \frac{(1-\phi)}{\phi}$. ■

If a responsive voter uses an uninformed strategy, this voter cannot be collecting information. If a voter type θ invests, it must be the case that she is following an informed strategy.

Now, we can separate the types in those that always vote for A, types that always vote for Q, and types that collect information and change the vote according to the signal received. We will refer to types that always vote for A (or Q) without performing any investment as **supporters** for A (or Q), and types that invest and change their vote according to the signal received as **independents**.

3.2 Information acquisition

In this section we derive the optimal investment function for independents. Using (4), the optimal investment function of players that use the informed strategy with (Q, A) is defined implicitly by[12]:

$$C'(P^*(\theta)) = \theta_q \Delta \Pr(q, Q)(1-\phi) + \theta_a \Delta \Pr(a, Q)\phi \tag{16}$$

When $C'\left(\frac{1}{2}\right) = 0$,[13] the fact that $C''(p) > 0$ for all $p > \frac{1}{2}$, and the implicit function theorem imply that $P^*(\theta)$ exists, is continuously differentiable, strictly increasing and strictly concave for all $\theta \neq (0,0)$. Recalling that $\Delta \Pr(\omega, Q) \leq 1 - \zeta(\omega)$, we conclude that

[12] Second order conditions follow directly by convexity of C.

[13] If $C'\left(\frac{1}{2}\right) > 0$, the set of types that may use the informed voting rule (Q, A) must satisfy $\min\{\theta_q, \theta_a\} > 0$.

$C'(P^*(\theta)) \leq \max_{\omega \in \{q,a\}} (1-\zeta(\omega))$, and since $\lim_{x \to 1} C'(x) \to \infty$ we know that $P^*(\theta) \leq \eta$ for some $\eta < 1$.

The informed strategy (Q, A) is used whenever its expected utility, net of investment costs, is higher than the expected utility derived from using any uninformed strategy (see condition (6)). The next lemma introduces an expression to compare the informed strategy with (Q, A) with the uninformed strategies.

Lemma 4 *In any regular committee, a necessary condition for voter type θ, to use a strategy with voting function $V(\theta, S) = (Q, A)$, and investment function $P^*(\theta) > \frac{1}{2}$ that satisfies (16), is that*

$$C'(P^*(\theta)) P^*(\theta) - C(P^*(\theta)) - \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega) \geq 0 \tag{17}$$

for all $\omega \in \{q, a\}$. A sufficient condition is obtained if (17) holds with strict inequality.

Proof. Using condition (6), the informed strategy with voting function (Q, A), is as good as an uninformed strategy $(v_q, v_a) = (X, X)$ for $X \in \{Q, A\}$, iff

$$C(P^*(\theta)) \leq \mathcal{U}^i(P^*(\theta), (Q, A) \mid \theta) - \mathcal{U}^i\left(\frac{1}{2}, (X, X) \mid \theta\right) \tag{18}$$

where $\mathcal{U}^i\left(\frac{1}{2}, (v_q, v_a) \mid \theta\right)$ is defined by (4). Using (4) and recalling $\Delta \Pr(a, Q)$ and $\Delta \Pr(q, Q)$, (18) reduces to (17), and necessity follows. Sufficiency of (17) with strict inequality is straightforward. ■

The informed strategy (Q, A) is preferred to the uninformed strategy (A, A) if we let $\omega = a$ in condition (17), and the informed strategy (Q, A) is preferred to the uninformed strategy (Q, Q) if we let $\omega = q$ in condition (17).

In order to determine which type prefers which strategy, we define implicitly the functions $g^\omega : \mathcal{R} \to \mathcal{R}$ such that the pair $(g^\omega(\theta_a), \theta_a)$ satisfies (17) with equality:

$$\begin{aligned} C'(P^*(g^a(\theta_a), \theta_a)) P^*(g^a(\theta_a), \theta_a) - C(P^*(g^a(\theta_a), \theta_a)) &= \theta_a \Delta \Pr(a, Q) \phi \\ C'(P^*(g^q(\theta_a), \theta_a)) P^*(g^q(\theta_a), \theta_a) - C(P^*(g^q(\theta_a), \theta_a)) &= g^q(\theta_a) \Delta \Pr(q, Q)(1 - \phi) \end{aligned}$$

where $P^*(g^\omega(\theta_a), \theta_a)$, $\omega \in \{q, a\}$ satisfies (16). Each function g^ω, $\omega \in \{q, a\}$ partitions the space of types $[0,1]^2$ in two regions: g^q separates types that prefer the informed strategy (Q, A) to the uninformed strategy (Q, Q), and g^a separates types that prefer the informed strategy (Q, A) to the uninformed strategy (A, A).

Let $\omega = q$ in condition (17), and note that the left side of (17) is decreasing in θ_q. Therefore, any $\theta \in [0,1]^2$ such that $\theta_q > g^q(\theta_a)$, prefers the uninformed strategy with (Q, Q) to the informed strategy with (Q, A). On the other hand, if $\theta_q < g^q(\theta_a)$ the informed strategy is preferred. If $\omega = a$ in condition (17), any pair $\theta \in [0,1]^2$ such that $\theta_q > g^a(\theta_a)$ prefers the informed strategy with (Q, A) to the uninformed strategy with (A, A).

Using the implicit function theorem, each $g^\omega(\theta_a)$ for $\omega \in \{q, a\}$ exists, is continuously differentiable and strictly increasing for all $\theta \neq (0,0)$. Moreover, $g^q(\theta_a)$ is strictly convex and $g^a(\theta_a)$ is strictly concave for all $\theta \neq (0,0)$, and $g^q(\theta_a) - g^a(\theta_a)$ is strictly increasing for all $\theta_a > 0$, with $\lim_{x \to 0} g^\omega(x) = 0$, for all $\omega \in \{q, a\}$.

It is worth noting two features of these results used later for existence. First, P^* may be defined for $\theta \notin [0,1]^2$. Indeed, as long as θ satisfies $C'(1) > \sum_{\omega \in \{a,q\}} \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega)$, $P^*(\theta)$ is well defined. Second, $g^q(\theta_a)$ and $g^a(\theta_a)$ are also properly defined for all θ that satisfy $C'(1) > \sum_{\omega \in \{a,q\}} \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega)$, even if $\theta \notin [0,1]^2$.

3.3 Characterization and existence of equilibrium

3.3.1 Characterization

The functions $g^q(\theta_a)$ and $g^a(\theta_a)$ separate the space of types in three groups that use different strategies. All responsive voters type $\theta \in [0,1]^2$ with $\theta_q < g^a(\theta_a)$ use a simple strategy described by constant functions: $P(\theta) = \frac{1}{2}$ and $V(\theta, s) = A$, for $s \in \{s_q, s_a\}$. The same can be said for responsive voters with type $\theta_q > g^q(\theta_a)$ where $V(\theta, s) = Q$, for $s \in \{s_q, s_a\}$. The interesting group is the set of responsive voters $\theta \in [0,1]^2$ that satisfy $g^a(\theta_a) \leq \theta_q \leq g^q(\theta_a)$ (independents), since both the investment function and the voting function change with the type and the signal.

The functions $g^{q}(\theta_a)$ and $g^{a}(\theta_a)$ ensure the strategies are optimal, and the backward induction process ensures that the voting function is optimal when conditional on the optimal investment level. This is formally stated in the next proposition

Proposition 1 *In any regular committee of size $n \geq 1$ in which the voting rule is $\boldsymbol{R}= (N, r)$, the strategy $(P^{*}(\theta), V^{*}(\theta, S))$ that prescribes*

1. *the investment function $P^{*}(\theta)$ as defined in (16) for every θ that satisfies $g^{a}(\theta_a) \leq \theta_q \leq g^{q}(\theta_a)$, and $P^{*}(\theta) = \frac{1}{2}$ otherwise,*
2. *the voting function $V^{*}(\theta, S) = (A, A)$ if $\theta_q < g^{a}(\theta_a)$, $V^{*}(\theta, S) = (Q, Q)$ if $\theta_q > g^{q}(\theta_a)$, and $V^{*}(\theta, S) = (Q, A)$ if $g^{a}(\theta_a) \leq \theta_q \leq g^{q}(\theta_a)$,*

is a symmetric Bayesian equilibrium.

Proof. By construction of the functions $g^{q}(\theta_a)$ and $g^{a}(\theta_a)$, all types satisfy the optimal condition (6) when using the strategies defined in the proposition. It remains to show that it is actually optimal to follow that voting function after the signal is realized (condition (5)).

For supporters for A, condition (9) is just

$$\theta_q \leq \theta_a \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \frac{\phi}{1-\phi} \tag{19}$$

for both signals. In the case of supporters for Q, we must reverse the sign of (19). In the case of the informed strategy with (Q, A) we must satisfy that

$$\begin{aligned} \theta_q &\geq \theta_a \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \frac{\phi}{1-\phi} \frac{1-P^{*}(\theta)}{P^{*}(\theta)} \\ \theta_q &\leq \theta_a \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \frac{\phi}{1-\phi} \frac{P^{*}(\theta)}{1-P^{*}(\theta)} \end{aligned} \tag{20}$$

Using the fact that $g^q(\theta_a)$ is convex and $g^a(\theta_a)$ is concave

$$\begin{aligned}
\frac{\partial g^q(0)}{\partial \theta_a}\theta_a &\leq g^q(\theta_a) \leq \frac{\partial g^q(\theta_a)}{\partial \theta_a}\theta_a \\
\frac{\partial g^a(0)}{\partial \theta_a}\theta_a &\geq g^a(\theta_a) \geq \frac{\partial g^a(\theta_a)}{\partial \theta_a}\theta_a
\end{aligned}$$

where we used that $g^\omega(0) = 0$ for $\omega \in \{a, q\}$. An application of the implicit function theorem gives that $\frac{\partial g^q(\theta_a)}{\partial \theta_a} = \frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \frac{\phi}{1-\phi} \frac{P^*(g^q(\theta_a),\theta_a)}{1-P^*(g^q(\theta_a),\theta_a)}$ and $\frac{\partial g^a(\theta_a)}{\partial \theta_a} = \frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \frac{\phi}{1-\phi} \frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}$ which implies

$$\begin{aligned}
1 &\leq \frac{g^q(\theta_a)}{\theta_a} \frac{\Delta \Pr(q,Q)}{\Delta \Pr(a,Q)} \frac{1-\phi}{\phi} \leq \frac{P^*(g^q(\theta_a),\theta_a)}{1-P^*(g^q(\theta_a),\theta_a)} \\
1 &\geq \frac{g^a(\theta_a)}{\theta_a} \frac{\Delta \Pr(q,Q)}{\Delta \Pr(a,Q)} \frac{1-\phi}{\phi} \geq \frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}
\end{aligned} \tag{21}$$

Since supporters for A satisfy $g^a(\theta_a) > \theta_q$, by the second equation in (21), condition (19) holds for these voters. Using the first equation in (21) and the fact that supporters for Q satisfy $g^q(\theta_a) < \theta_q$, condition (19) does not hold for these voters. Therefore, both uninformed strategies are consistent.

Using the right hand side of the second inequality in (21) and the fact that $\theta_q \geq g^a(\theta_a)$ for independents gives the first equation in (20). Because $g^q(\theta_a)$ is monotone it is invertible and $\theta_q \leq g^q(\theta_a)$ is equal to $\theta_a \geq (g^q)^{-1}(\theta_q)$. For any $\theta_a \geq (g^q)^{-1}(\theta_q)$ we must have that $\frac{P^*\left(\theta_q,(g^q)^{-1}(\theta_q)\right)}{1-P^*\left(\theta_q,(g^q)^{-1}(\theta_q)\right)} \leq \frac{P^*(\theta_q,\theta_a)}{1-P^*(\theta_q,\theta_a)}$; using the first equation in (21) we have that $\frac{\theta_q}{(g^q)^{-1}(\theta_q)} \frac{\Delta \Pr(q,Q)}{\Delta \Pr(a,Q)} \leq \frac{P^*(\theta_q,\theta_a)}{1-P^*(\theta_q,\theta_a)}$. using that $\theta_a \geq (g^q)^{-1}(\theta_q)$ we have that $\frac{\theta_q}{(g^q)^{-1}(\theta_q)} \geq \frac{\theta_q}{\theta_a}$ which gives the second equation in (20).

To show the characterization is complete, we need to show that no type $\theta \in [0,1]^2$ belongs to two different groups, and the union of independents and supporters covers all $[0,1]^2$. But this is obvious, since $g^a(\theta_a)$ and $g^q(\theta_a)$ cross each other only at $(0,0)$ for $\theta_a \geq 0$. ■

In Figure (1) we illustrate the equilibrium in an election where the simple majority rule is in place for n odd, $\phi = \frac{1}{2}$, F symmetric around the 45^o degree line[14], and $\xi_A = \frac{1}{2}$. In this case the equilibrium is fully symmetric around the 45^o degree line: $g^a(y) = x$ iff $g^q(x) = y$ and $P^*(\theta) = P^*(\theta')$, for every $\theta = (x, y)$ and $\theta' = (y, x)$.

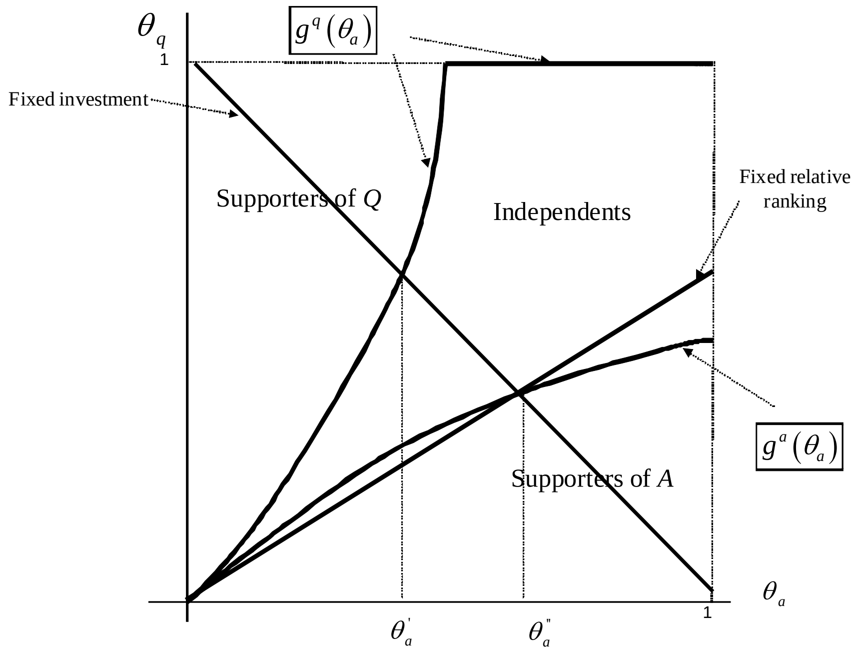

Figure 1: Equilibrium under the plurality rule and n is odd. Supporters of A (Q) always vote for A (Q) and do not collect information. Independents collect information according to equation (16), and vote according to the signal received: if $S = s_a$ then $v = A$ and if $S = s_q$ then $v = Q$.

3.3.2 Existence

The fact that the equilibrium strategy is composed of an investment function that is only C^0 almost everywhere,[15] complicates the direct use of standard fixed point theorems on the space of best responses. In order to show existence we create a transformation that uses the optimal investment function and the optimal voting strategies as arguments. In this manner,

[14]That is $F(x, y) = F(y, x)$ for all $(x, y) \in [0, 1]^2$.

[15]See the equicontinuity requirement for Schauder's Fixed Point Theorem in Rudin (1973). In turns some assumption abut the differentiability of $P^*(\theta)$ is required which can be translated into $C''\left(\frac{1}{2}\right) > 0$. This condition rules out any possible aggregation of information in the limit as shown by Martinelli (2006).

the equilibrium can be described by the functions $g^{\omega}(\theta_a)$, $\omega \in \{q, a\}$ and the investment function $P^*(\theta_a)$ that are uniquely defined by $\Delta \Pr(\omega, Q)^*$, $\omega \in \{q, a\}$.

Proposition 2 *In any regular committee of size $n \geq 1$ in which the voting rule is $\boldsymbol{R}= (N, r)$, there exists a symmetric Bayesian equilibrium. Moreover, this equilibrium is characterized by the strategy $(P^*(\theta), V^*(\theta, S))$ in Proposition (1).*

Proof. $P^*(\theta)$ changes smoothly with $\Delta \Pr(\omega, Q)$, $\omega \in \{q, a\}$ for all $\theta \neq (0, 0)$; this is a direct application of the implicit function theorem to (16). Using the definitions of $g^{\omega}(\theta_a)$, $\omega \in \{q, a\}$,

$$\begin{aligned}
\frac{\partial g^a(\theta_a)}{\partial \Delta \Pr(a, Q)} &= \frac{\theta_a}{\Delta \Pr(q, Q)} \frac{1 - P^*(g^a(\theta_a), \theta_a)}{P^*(g^a(\theta_a), \theta_a)} \frac{\phi}{(1-\phi)} \\
\frac{\partial g^a(\theta_a)}{\partial \Delta \Pr(q, Q)} &= -\frac{g^a(\theta_a)}{\Delta \Pr(q, Q)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial g^q(\theta_a)}{\partial \Delta \Pr(a, Q)} &= \frac{\theta_a}{\Delta \Pr(q, Q)} \frac{P^*(g^q(\theta_a), \theta_a)}{1 - P^*(g^q(\theta_a), \theta_a)} \frac{\phi}{(1-\phi)} \\
\frac{\partial g^q(\theta_a)}{\partial \Delta \Pr(q, Q)} &= -\frac{g^q(\theta_a)}{\Delta \Pr(q, Q)}
\end{aligned}$$

which implies that both $g^a(\theta_a)$ and $g^q(\theta_a)$ are continuous in $\Delta \Pr(\omega, Q)$, $\omega \in \{q, a\}$. Therefore $\Pr(A \mid \omega)$, $\omega \in \{q, a\}$ are continuous in $\Delta \Pr(\omega, Q)$, $\omega \in \{q, a\}$ for all θ.

Let $X = \left[\alpha \xi_A, 1 - \alpha \xi_Q\right]^2$ and $Y = [\zeta(a), 1 - \zeta(a)] \times [\zeta(q), 1 - \zeta(q)]$. Trivially $X \times Y$ is compact and convex subset of an euclidean space. Let $(x_1, x_2) \in X$ and $(y_1, y_2) \in Y$ be generic elements of these spaces.

In (16) replace $\Delta \Pr(a, Q)$ for y_1 and $\Delta \Pr(q, Q)$ for y_2 and define $P^*(\theta)$ implicitly for all $\theta \neq (0, 0)$ in terms of y_1 and y_2 as $P^*(\theta \mid y_1, y_2)$. Now define first the cut off functions $g^a(\theta_a)$ and $g^q(\theta_a)$ by replacing $\Delta \Pr(a, Q)$ for y_1 and $\Delta \Pr(q, Q)$ for y_2 in the corresponding conditions (17) and using the function $P^*(\theta \mid y_1, y_2)$: $g^a(\theta_a \mid y_1, y_2)$ and $g^q(\theta_a \mid y_1, y_2)$.

Let $K_i : Y \to \left[\alpha\xi_A, 1-\alpha\xi_Q\right]$ for $i = 1, 2$ be such that

$$\begin{aligned}
\frac{K_1(y_1, y_2) - \alpha\xi_A}{1-\alpha} &\equiv \int_0^1 \int_0^{\min\{1, g^a(\theta_a | y_1, y_2)\}} dF(\theta) + \int_0^1 \int_{\min\{1, g^a(\theta_a | y_1, y_2)\}}^{\min\{1, g^q(\theta_a | y_1, y_2)\}} P^*(\theta \mid y_1, y_2)\, dF(\theta) \\
\frac{K_2(y_1, y_2) - \alpha\xi_A}{1-\alpha} &\equiv \int_0^1 \int_0^{\min\{1, g^q(\theta_a | y_1, y_2)\}} dF(\theta) - \int_0^1 \int_{\min\{1, g^a(\theta_a | y_1, y_2)\}}^{\min\{1, g^q(\theta_a | y_1, y_2)\}} P^*(\theta \mid y_1, y_2)\, dF(\theta)
\end{aligned}$$

$K_i, i = 1, 2$ are continuous in (y_1, y_2). Here K_1 plays the role of $\Pr(A \mid a)$ and K_2 plays the role of $\Pr(A \mid q)$

Let $K_3 : \left[\alpha\xi_A, 1-\alpha\xi_Q\right] \to [\zeta(a), 1-\zeta(a)]$ and $K_4 : \left[\alpha\xi_A, 1-\alpha\xi_Q\right] \to [\zeta(q), 1-\zeta(q)]$ be defined such that

$$\begin{aligned}
K_3(x_1) &\equiv \frac{(n-1)!}{(n-1-N)!(N-1)!}(x_1)^{N-1}(1-x_1)^{n-1-N}\chi(x_1) \\
K_4(x_2) &\equiv \frac{(n-1)!}{(n-1-N)!(N-1)!}(x_2)^{N-1}(1-x_2)^{n-1-N}\chi(x_2)
\end{aligned}$$

where $\chi(x) = \left(\frac{(1-x)(1-r)}{(n-N)} + \frac{xr}{N}\right)$. Trivially, $K_i, i = 3, 4$ are continuous in x_1 and x_2 respectively. Note that $K_3(x_1)$ plays the role of $\Delta\Pr(a, Q)$ and $K_4(x_2)$ plays the role of $\Delta\Pr(q, Q)$ in (13).

Let $\Gamma : X \times Y \to X \times Y$ be defined as $\Gamma \equiv (K_1, K_2, K_3, K_4)$ which is continuous. Therefore, applying Brouwer fixed point theorem (see Border (1985)), there is some $(x_1^*, x_2^*, y_1^*, y_2^*) \in X \times Y$ such that $\Gamma(x_1^*, x_2^*, y_1^*, y_2^*) = (x_1^*, x_2^*, y_1^*, y_2^*)$.

The fact that $(x_1^*, x_2^*, y_1^*, y_2^*)$ is an equilibrium follows trivially. Let $x_1^* = \Pr(A \mid a)$, $x_2^* = \Pr(A \mid q)$, $y_1^* = \Delta\Pr(a, Q)$ and $y_2^* = \Delta\Pr(q, Q)$. Since Γ has embedded the description of the best response functions $(g^a(\theta_a), g^q(\theta_a))$ and $P^*(\theta)$, for any pair $\Delta\Pr(\omega, Q)$, $\omega \in \{q, a\}$, the transformation Γ gives the optimal probabilities of voting for A in each state $\omega \in \{q, a\}$. (x_1^*, x_2^*) only ensures that actually we have a fixed point in the probabilities of voting that are constructed using $\Delta\Pr(\omega, Q)$, $\omega \in \{q, a\}$. ■

Although we can not prove that for each set of primitives (ϕ, n, (N, r), $F(\cdot)$, α, ξ_A, $C(\cdot)$) there is a unique equilibrium (a unique set of $g^q(\theta_a)$, $g^a(\theta_a)$ and $P^*(\theta)$), we know that every symmetric Bayesian equilibrium is described by a set of cut off functions $g^q(\theta_a)$ and $g^a(\theta_a)$ and a investment function $P^*(\theta)$. Therefore, our characterization is valid for *all* symmetric Bayesian equilibria.

3.3.3 Intuitive characterization

Supporters for A are characterized by a high relative ratio of losses $\frac{\theta_a}{\theta_q}$ while supporters for Q are characterized by a low relative ratio of losses $\frac{\theta_a}{\theta_q}$. Independents present more balanced preferences and invest in order to collect information.

There are two main forces that drive a voter's behavior when information is endogenous: the relative ranking of alternatives ($\frac{\theta_a}{\theta_q}$) and the actual level of utility losses ($\min\{\theta_a, \theta_q\}$). When $\frac{\theta_a}{\theta_q}$ is high (biased towards A) a vote for Q is only possible if the evidence in favor of state q is overwhelming. When information is endogenous, this information depends on the level of losses through the function $\sum_{\omega\in\{a,q\}} \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega)$, so the information level increases as we move away from the origin. For a fixed level of $\frac{\theta_a}{\theta_q}$, the higher θ_a, the higher the precision selected if the informed strategy were used in equilibrium. For example, along the "fixed relative ranking" line in Figure (1), $\frac{\theta_a}{\theta_q}$ is fixed; when $\theta_a < \theta_a^{''}$ the information collected if (Q, A) were used is not too strong. Because of this imprecise information, the responsive voter cannot be too sure that the true state is q when the signal received is s_q; therefore, she prefers to save on the cost of information than buying reassurance that the true state is q. When $\theta_a > \theta_a^{''}$, the precision of the information collected if (Q, A) were used is high enough to induce the responsive voter to select Q when the signal is s_q.

What would happen if the information were free and its precision were exactly $P^*(\theta)$ for each type θ? For types (θ_q, θ_a) such that θ_q is much smaller than $g^a(\theta_a)$, the free signal does not alter their behavior: they would still be supporters for A. These voters decide strategically to ignore their information. But for types (θ_q, θ_a) such that θ_q is close to

$g^a(\theta_a)$, if the information were free, they would vote in favor of Q instead of A. In essence, the reason why some (θ_q, θ_a) with θ_q close but smaller than $g^a(\theta_a)$ behave as a supporter is due to information cost: a signal with the optimal precision makes her change her vote, but that signal is too costly. Saving on informational costs is preferred to be able to select the most accurate candidate according to the signal. These voters decide strategically not to collect information (rational ignorance) and free ride on the information of other voters.

Alternatively, fix the precision of the information collected by informed voters along the "fixed investment" line in Figure (1). When the type θ satisfies $\theta_a < \theta_a^{'}$, the precision of the information collected when (Q, A) is used is not high enough to make the player vote in favor of A. Then there is no reason to collect information. When θ_a is close to 0, any free information would be disregarded, and if θ_a is close to $\theta_a^{'}$, free information is welcome. When $\theta_a \in \left(\theta_a^{'}, \theta_a^{''}\right)$, preferences are balanced enough for (Q, A) to be preferred given the optimal p. In that case information is collected and the signal guides the voting function. When $\theta_a > \theta_a^{''}$, the problem is the signal in favor of Q is not strong enough, and the responsive voter becomes a supporter of A.

Besides the assume existence of partisan voters, there are supporters for A, supporters for Q and independents in any equilibrium. This is basically driven by the fact that $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)}$ is bounded by $\left[\frac{\zeta(a)}{1-\zeta(q)}, \frac{1-\zeta(q)}{\zeta(a)}\right]$ and $\frac{p}{1-p}$ is bounded by $\left[1, \frac{1-\eta}{\eta}\right]$. Using (14), there are always responsive voters with extreme types ($\frac{\theta_a}{\theta_q}$ big enough or small enough): the precision of the signal that would have been collected if (Q, A) were used could not have overturned the relative bias.

4 Information Aggregation?

Aggregation of information in general elections has been the focus of attention in political economy for quite some time. Early models derived a variation of the *Condorcet Jury Theorem* without considering strategic behavior and assuming exogenous information (see Berend and Paroush (1998)). These models are particularly appealing if there is no uncertainty (the only issue is aggregating preferences) and players do not behave strategically (they truthfully report their preferences). In these models, a voter is a random variable and larger electorates are the aggregation of more draws of these random variables; varieties of the *Law of Large Numbers* give the desired result.

Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) introduce the problem of strategic behavior assuming that information is exogenous. Feddersen and Pesendorfer (1997) show that the *Condorcet Jury Theorem* holds under some circumstances and that strategic voting may affect the aggregating properties in the presence of uncertainty about the composition of the electorate. They argue that heterogeneity might cause failure of information aggregation.

There are very few papers that actually study information aggregation when increasing the size of the electorate decreases the average level of information in the electorate. Yariv (2004) assumes that individual information decreases exogenously with the number of voters. This assumption is in line with the rational ignorance hypothesis and does not *per se* destroy the aggregation result (*Condorcet Jury Theorem*). She assumes that each voter exogenously receives the same quality of information. She shows that the "speed" at which individuals receive poorer information when the electorate grows large is relevant for elections to actually aggregate information.

Martinelli (2006) allows for endogenous quality of information in national elections. It is the first model studying both the *Condorcet Jury Theorem* and the rational ignorance hypothesis in the same set up. He gives individual rationality to the exogenous process that Yariv (2004) assumes, and he shows that the form of the cost function for information acquisition determines whether there is aggregation of information or not. Full aggregation is only possible in large electorates if $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = C'''\left(\frac{1}{2}\right) = 0$. If $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = 0$ and $C'''\left(\frac{1}{2}\right) > 0$, Martinelli (2006) obtains a specific limit (when the electorate grows) on the probability of making the right choice. This limit is decreasing in $C'''\left(\frac{1}{2}\right)$ and approaches 1 when $C'''\left(\frac{1}{2}\right) \to 0$. If $C''\left(\frac{1}{2}\right) > C'\left(\frac{1}{2}\right) = 0$ both candidates have equal chances of winning in any state of nature when the electorate is sufficiently large.[16]

Martinelli's existence and characterization results crucially depend on the fact that preferences are restricted, the simple majority rule is in place and $\phi = \frac{1}{2}$. This is because under the simple majority rule and $\phi = \frac{1}{2}$, the restriction $\theta_q + \theta_a = 1$ (which he assumes) implies that $\Delta \Pr(a, Q) = \Delta \Pr(q, Q)$ and the investment function (16) reduces to

$$C'\left(P^*\left(1-\theta_a, \theta_a\right)\right) = \frac{\Delta \Pr(\omega, Q)}{2}$$

As a consequence, in his model every informed responsive voter selects the same quality of information.

A natural question arises: how general is the aggregation result given that all informed voters collect the same quality of information? In Figure (1) Martinelli's informed voters are independents along the "fixed Investment" line ($\theta_q + \theta_a = 1$). Independents with $\theta_q + \theta_a < 1$ collect less information and independents with $\theta_q + \theta_a > 1$ collect more information. It seems that the average level of information should be higher with flexible preferences (θ_q and θ_a are not perfectly correlated) and it should be easier to aggregate information in the limit. But it is not clear that the average voter is more informed under flexible preferences than under restricted preferences. If the committee is actually making better decisions under flexible preferences it might be that the average voter is collecting poor information since his information is less valuable. Then, the requirements for aggregation of information under flexible (two dimensional) preferences might be stronger than under restricted (one dimensional) preferences.

We show in the appendix that the result in Martinelli (2006) is robust to using two dimensional preferences that allow us to unleash all the incentives to collect information. But our set up allows us to investigate how important is the symmetric assumption used for

[16] He also provides results for homogenous voters: it is sufficient for information aggregation that $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = 0$.

that result. We now proceed to show that, for a given electorate (that is α, ξ_A, ϕ and F), there is only one rule that might aggregate information.

Note that (10) is just an event of a binomial distribution with parameter p in which there are $N \leq n-1$ successes out of $n-1$ trials so in the limit its mass vanishes. This implies that $\Delta \Pr(\omega, Q) \to 0$ when $n \to \infty$ so we have that $P^*(\theta) \to \frac{1}{2}$.

Using that

$$\frac{\Pr(A \mid a) - \alpha\xi_A}{1-\alpha} \equiv \int_0^1 \int_0^{\min\{1,g^a(\theta_a)\}} dF(\theta) + \int_0^1 \int_{\min\{1,g^a(\theta_a)\}}^{\min\{1,g^q(\theta_a)\}} P^*(\theta)\, dF(\theta) \tag{22}$$

$$\frac{\Pr(A \mid q) - \alpha\xi_A}{1-\alpha} \equiv \int_0^1 \int_0^{\min\{1,g^a(\theta_a)\}} dF(\theta) + \int_0^1 \int_{\min\{1,g^a(\theta_a)\}}^{\min\{1,g^q(\theta_a)\}} (1-P^*(\theta))\, dF(\theta)$$

we have that $\Pr(A \mid a) \to \Pr(A \mid q)$ when n grows. The next result goes in line with (Austen-Smith and Banks (1996)): the conditions for information aggregation are the same conditions required for sincere voting.

Proposition 3 *For information aggregation is necessary that* $\lim\limits_{n\to\infty} \frac{N(n)}{n} = \lim\limits_{n\to\infty} \Pr(A \mid q) = \lim\limits_{n\to\infty} \Pr(A \mid a)$*, 2) the rule is proportional, and 3) the unanimity rule can not aggregate information so* $\lim\limits_{n\to\infty} \frac{N(n)}{n} < 1$.

Proof. For the first part, let $y_n(\omega) = \Pr(A \mid \omega) - \frac{1}{2} = \left(\frac{\Pr(A|\omega) - \Pr(Q|\omega)}{2}\right)$. Note that $y_n(\omega) < \frac{1}{2}$ because $\Pr(A \mid \omega) < 1$ when we consider that $\alpha \in (0,1)$.

Define the random variable $M_i^n(\omega)$ such that $M_i^n(\omega) \equiv \frac{1}{2} - y_n(\omega)$ if $v = A$ and $M_i^n(\omega) \equiv -\frac{1}{2} - (y_n(\omega))$ if $v = Q$. Note that $E(M_i^n(\omega)) = 0$, $E\left[(M_i^n(\omega))^2\right] = \left(\frac{1}{4} - (y_n(\omega))^2\right)$ and $E\left(|M_i^n(\omega)|^3\right) = 2\left(\left(\frac{1}{2}\right)^4 - (y_n(\omega))^4\right)$ for $\omega \in \{q, a\}$.

Therefore, $\overline{M}_i^n(\omega) \equiv \frac{M_i^n(\omega)}{\sqrt[2]{\left(\frac{1}{4} - (y_n(\omega))^2\right)}}$ is a random variable with zero mean, variance equal to 1 and $E\left(\left|\overline{M}_i^n(\omega)\right|^3\right)=\frac{2\left(\left(\frac{1}{2}\right)^2+(y_n(\omega))^2\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_n(\omega))^2\right)}}$. Define $M^n(\omega)\equiv\sum_{i=1}^{n}\overline{M}_i^n(\omega)$ so we have that

$$M^n(\omega)=\frac{T_n(\omega)-\left(\frac{1}{2}+y_n(\omega)\right)n}{\sqrt[2]{\left(\frac{1}{4}-(y_n(\omega))^2\right)}}$$

where we used $T_n=\sum_{i=1}^{n}I(v_i=A)$. We know that it is necessary for A to win that $T_n\geq N(n)$ and for Q to win it is necessary that $T_n\leq N(n)$; therefore a necessary condition for information aggregation is that $M^n(a)\geq\frac{N(n)-\left(\frac{1}{2}+y_n(a)\right)n}{\sqrt[2]{\left(\frac{1}{4}-(y_n(a))^2\right)}}$ and $\frac{N(n)-\left(\frac{1}{2}+y_n(q)\right)n}{\sqrt[2]{\left(\frac{1}{4}-(y_n(q))^2\right)}}\geq M^n(q)$.

Let $\mathcal{M}^n(\omega)\equiv\frac{M^n(\omega)}{\sqrt[2]{n}}$ and $\mathcal{F}^N(\omega)$ be its distribution. Note that A is winner in state a if $\mathcal{M}^n(a)\geq\sqrt[2]{n}\frac{\frac{N(n)}{n}-\left(\frac{1}{2}+y_n(a)\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_n(a))^2\right)}}$ and Q is the winner in state q if $\mathcal{M}^n(q)\leq\sqrt[2]{n}\frac{\frac{N(n)}{n}-\left(\frac{1}{2}+y_n(q)\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_n(q))^2\right)}}$; if we let $J^n(y_n(\omega))\equiv\mathcal{M}^n(\omega)$ the probability of A being the winner in state a is bounded above by $1-\mathcal{F}^n(J^n(y_n(a)))$ and the probability that Q wins in state q is bounded above by $\mathcal{F}^n(J^n(y_n(q)))$; for information aggregation we must have that $\sqrt[2]{n}\frac{\frac{N}{n}-\left(\frac{1}{2}+y_n(a)\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_n(a))^2\right)}}\to-\infty$ and $\sqrt[2]{n}\frac{\frac{N}{n}-\left(\frac{1}{2}+y_n(q)\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_n(q))^2\right)}}\to\infty$.

Let Φ be the cdf of a $(0,1)$ normal random variable, so we can apply the Berry-Esseen Theorem[17] to get that $\lim_{n\to\infty}\mathcal{F}^n(J^n(y_n(\omega)))\to\Phi(J^n(y_n(\omega)))$ if $E\left(\left|\overline{M}_i^n(\omega)\right|^3\right)$ is finite, which is the case since $y_n(\omega)<\frac{1}{2}$. Now using that Φ is continuous we must have that $\lim_{n\to\infty}\Phi(J^n(y_n(\omega)))\to\Phi\left(\lim_{n\to\infty}J^n(y_n(\omega))\right)$, which implies that $\lim_{n\to\infty}\mathcal{F}^n(J^n(y_n(\omega)))\to\Phi\left(\lim_{n\to\infty}J^n(y_n(\omega))\right)$ Let $\lim_{n\to\infty}\frac{N(n)}{n}=\lambda$ and $\lim_{n\to\infty}\Pr(A\mid\omega)=l_1$ so we must have that

$$\lim_{n\to\infty}\left(\frac{\frac{N(n)}{n}-\left(\frac{1}{2}+y_n(\omega)\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_n(\omega))^2\right)}}\right)=\frac{\lambda-l_1}{\sqrt[2]{l_1(1-l_1)}}$$

Note that if $\lambda\neq l_1$ then $\lim_{n\to\infty}\mathcal{M}^n(\omega)\notin(-\infty,\infty)$ and therefore either A wins indepen-

[17] Let $X_1,....X_n$ be *i.i.d* with mean $\mu=0$ and $\sigma^2=1$. Then, for all n

$$\sup\left|\Pr\left(\frac{\sum_{i=1}^{n}X_i}{\sqrt[2]{n}}\leq t\right)-\Phi(t)\right|\leq\frac{33}{4}\frac{E|X_1|^3}{\sqrt[2]{n}}.$$ See Bickel and Doksum (2000).

dently of the state or Q wins independently of the state. It must be then that $\lambda = l_1$. ■

Corollary 1 *For information aggregation is necessary that 1) the voting rule is proportional, and 2) the unanimity rule can not aggregate information, so 3)* $\lim_{n\to\infty} \frac{N(n)}{n} < 1$.

Proof. Assume that the rule is not proportional, then $T_n(\omega) = n - h$ where h is a fixed natural number and verifies $h < \frac{n}{2}$, and note then that we require that $M^n(a) \geq \frac{\left(\frac{1}{2} - y_n(a)\right)n - h}{\sqrt[2]{\left(\frac{1}{4} - (y_n(a))^2\right)}}$ and the probability of A winning in state a is bounded above by $1 - \mathcal{F}^n\left(\frac{\sqrt[2]{n}\left(\frac{1}{2} - y_n(a)\right) - \frac{h}{\sqrt[2]{n}}}{\sqrt[2]{\left(\frac{1}{4} - (y_n(a))^2\right)}}\right)$. Then, a necessary condition for A to win in state a is that $y_n(a) \to \frac{1}{2}$ contradicting that $y_n(\omega) < \frac{1}{2}$ for all n.

The second and third part follows immediately since the unanimity rule is just that $h = 0$ and we know that it can not aggregate information so we must have $\lambda < 1$. ■

Using (22) and the fact that $\lim_{n\to\infty} g^{\omega}(\theta_a) = \frac{\phi}{1-\phi}\theta_a$, we have that for a given electorate described by ϕ, $F(\theta)$, ξ_A and α there is only one proportional rule that might aggregate information and this rule is described by

$$\frac{N(n)}{n} \equiv \alpha\xi_A + (1-\alpha)\int_0^1 \int_0^{\min\left\{1, \frac{\phi}{1-\phi}\theta_a\right\}} dF(\theta)$$

In particular, if the electorate is symmetric (F is symmetric around the 45^o line and $\xi_A = \frac{1}{2}$) a necessary condition for information aggregation is that $\phi = \frac{1}{2}$.

Feddersen and Pesendorfer (1997) show that uncertainty about the electorate might break aggregation results otherwise robust to asymmetric priors and rules of election. They assume that information was exogenously provided to each voter. When information is endogenous the rational ignorance hypothesis creates in the limit a committee in which almost nobody is informed. If the priors are symmetric those informed end up deciding the winner although their size is arbitrarily small compared to those uninformed.

If this particular symmetry in which uninformed voters cancel each other out is destroyed, the rational ignorance hypothesis creates a committee that ends up being exogenously bias. This bias is enough for the committee to always select the same candidate. Austen-Smith and Banks (1996) show that asymmetric priors are crucial for truthful voting while Feddersen and Pesendorfer (1997) show that truthful voting is not crucial for information aggregation. In our set up, the conditions for sincere voting play a crucial role because they restore the balance between forces so that informed voters decide the election.

5 Conclusions

Allowing for endogenous information creates serious problems to well known established results.[18] For example, Stiglitz and Grossman (1980) show that efficiency and endogenous information are not as easily paired in competitive markets as Hayek (1945) suggested.[19] Even the existence of endogenous information equilibria is problematic[20]. Moreover, the non-existence problem appears in much simpler set ups as the demand for information is not well behaved.[21] Given these results and the fact that most of the literature on committees focuses on models with exogenous information we ask: are the exogenous information results in committees robust to the introduction of endogenous information?

We develop a model of voting where voters endogenously select the quality of the information they will use to vote.[22] Voters who receive reports or memos need to expend time and effort to understand the information. This decision is endogenous so there is no reason

[18]See Stiglitz (2002) for a broader survey of information economics discussing problems arising with endogenous information and incentives to collect this information.

[19]For example, in Prat (2002) allowing for endogenous information in the electorate will kill voters' incentives to collect private signals in the separating equilibrium (in Prat (2002) terms: z will not convey any information). Voters will rely solely on campaign advertisement to decide the candidates' valence and interest groups are indifferent between contributing or not to campaigns (see point 1, page 1007 in Prat (2002)).

[20]See Green (1977); see Dubey et al. (1987) for further developments departing from competitve markets.

[21]See Stiglitz and Radner (1984) for a seminal exposition in simple environments and Chade and Schlee (2002) for an extension to continuous set ups and generalizations.

[22]For any rule besides the unanimity rule, if there are no partisans, there is always an equilibrium where all voters vote for A (Q) and do not collect any information. Although we assume the existence of partisans all our results hold when we assume away these voters and let all voters be responsive and use non-weakly dominated strategies.

to expect that different voters will be equally informed. Modelling information acquisition in elections as a choice over a set of signals with different precision is more accurate than assuming a common source of information for all voters. In line with this observation, we allow voters to *select the correlation between the signal they will receive and the true state of nature.*

We assume the level of conflict among committee members to be richer than it is usually assumed in the literature. When information is exogenous, all the relevant interaction between the committee members can be represented by simple structures. Indeed, preferences modelled as a relative ranking of alternatives capture all the proper incentives to study voting decisions: ideological heterogeneity is enough. This restriction on preferences imposes correlation between the disutilities that a member derives from mistaken decisions. When information is endogenous, this restriction does not capture all the relevant strategic interaction: voters with the same ranking of alternatives may have different incentives to collect information. We assume that committee members' preferences are *flexible* and introduce *another dimension of heterogeneity:* committee members not only differ on their ideological position *but they also differ on the level of concern* about the outcome of the election.

We provide existence and characterization results for arbitrary rules of election, arbitrary distribution of types and arbitrary level of conflict. We give a natural and intuitive representation of the equilibrium behavior of committee members. This geometric representation of equilibrium is important in order to derive the existence result. In equilibrium, our model predicts that informed voters endogenously select different levels of information. Contrary to previous results in the literature, heterogenous preferences translate into heterogenous informed voters. This is directly related to the assumption about preferences and, in particular, to the second dimension of heterogeneity.

When committee members select different quality of information in equilibrium, straightforward applications of standard fixed point arguments are not possible. We solve this problem by exploiting the geometric properties of the equilibrium. Indeed, we first characterize the equilibrium completely and then apply fixed point arguments to show its existence. Our strategy is simply to transform the infinite dimensional space of players' best response functions into a more tractable object.

Aggregation results for symmetric electorates and priors are derived without imposing that every informed member must collect the same quality of information in equilibrium. Therefore, the aggregation of information derived in Martinelli (2006) does not depend on the assumed preferences and the homogeneity of information among informed voters. The "speed" at which information is lost due to the reduction in the probability of being decisive is the key ingredient in the aggregation of information (Yariv (2004)).

Unfortunately we show that necessary conditions for aggregation are very restrictive. Indeed, it is necessary that the proportional rule used to select the winner balances out voters that do not collect information. Since in the limit no voter collects information, this implies that, for a given electorate, there is only one and only one proportional rule that can actually aggregate information. We see this as a negative result: aggregation of endogenous information in democracies is a knife edge result.

References

Austen-Smith, D. and J. S. Banks (1996). Information aggregation, rationality, and the condorcet jury theorem. *The American Political Science Review 90*(1), 34–45.

Benz, M. and A. Stutzer (2004). Are voters better informed when they have a larger say in politics? *Public Choice 119*(1-2), 31–59.

Berend, D. and J. Paroush (1998). When is condorcet's jury theorem valid? *Social Choice and Welfare 15*(4), 481–88.

Bickel, P. J. and K. A. Doksum (2000). *Mathematical Statistics: Basic Ideas and Selected Topics, Vol I.* (2nd ed.). Prentice Hall.

Border, K. (1985). *Fixed Point Theorems with Applications to Economics and Game The-*

ory. Cambridge University Press.

Borgers, T. (2004). Costly voting. *The American Economic Review 94*(1), 57–66.

Chade, H. and E. Schlee (2002). Another look at radner-stiglitz nonconcavity in the value of information. *Journal of Economic Theory 107*(2), 421–452.

Condorcet, Marie Jean Antoine Nicolas Caritat, M. d. (1976). In K. M. Baker (Ed.), *Condorcet. Selected Writings*. Indianapolis: The Bobbs- Merrill Company, Inc.

Dubey, P., J. Geanakoplos, and M. Shubik (1987). The revelation of information in strategic market games: A critique of rational expectations equilibrium. *Journal of Mathematical Economics 16*(2), 105–137.

Feddersen, T. (2004). Rational choice theory and the paradox of not voting. *The Journal of Economic Perspectives 18*(1), 99–112.

Feddersen, T. and W. Pesendorfer (1997). Voting behavior and information aggregation in elections with private information. *Econometrica 65*(5), 1029–58.

Green, J. (1977). The non-existence of informational equilibria. *The Review of Economic Studies 44*(3), 451–463.

Hayek, F. A. (1945). The use of knowledge in society. *The American Economic Review 35*(4), 519–530.

Krishna, V. and J. Morgan (2005). Efficient information aggregation with costly voting. mimeo.

Martinelli, C. (2006). Would rational voters acquire costly information? *Journal of Economic Theory 129*(1), 225–251.

Mueller, D. C. (2003). *Public Choice III*. Cambridge: Cambridge University Press.

Oliveros, S. (2007). Who abstains in equilibrium? Haas School of Business, UC Berkeley, mimeo.

Palfrey, T. R. and H. Rosenthal (1983). Strategic calculus of voting. *Public Choice 41*(1), 7–53.

Palfrey, T. R. and H. Rosenthal (1985). Voter participation and strategic uncertainty. *The American Political Science Review 79*(1), 62–78.

Prat, A. (2002). Campaign advertisement and voter welfare. *The Review of Economic Studies 69*(4), 999–1017.

Rudin, W. (1973). *Functional Analysis* (2nd ed.). MacGraw- Hill Book Company. International Edition.

Stiglitz, J. and S. Grossman (1980). On the impossibility of informationally efficient markets. *The American Economic Review 70*(3), 393–408.

Stiglitz, J. and R. Radner (1984). A nonconcavity in the value of information. In M. Boyer and R. Khilstrom (Eds.), *Bayesian Models in Economic Theory*, pp. 33–52. Elsevier Science Publications.

Stiglitz, J. E. (2002). Information and the change in the paradigm in economics. *The American Economic Review 92*(3), 460–501.

Yariv, L. (2004). When majority rule yields majority ruin. Mimeo, UCLA.

Young, H. P. (1988). Condorcet's theory of voting. *The American Political Science Review 82*(4), 1231–1244.

A Information aggregation under symmetry

We study aggregation properties of general elections under a particular assumption about preferences. As it is clear in the main text our result relies on the following symmetric assumption:

Definition 4 *A symmetric committee of size n is a regular committee of size n in which both states are equally likely ($\phi = \frac{1}{2}$), partisans vote for each candidate with equal probability ($\xi_A = \frac{1}{2}$), and F is symmetric around the $45°$.*

In order to study information aggregation, first we need a characterization of the equilibrium when the majority rule is in place and the number of members is odd.

Proposition 4 *In any symmetric committee of size $n = 2N+1$ with simple majority rule, for every $N \geq 0$, there is a symmetric Bayesian equilibrium characterized by the pair $(x_N, y_N) \in (0,1)^2$, where **1)** $\Delta \Pr(\omega, Q) = x_N$ for $\omega \in \{q, a\}$, **2)** $\Pr(A \mid a) = \frac{1}{2} + y_N (1-\alpha)$, **3)** $\Pr(A \mid a) = \Pr(Q \mid q)$, and **4)** $P(\theta)$ is such that $C'(P^*(\theta)) = \frac{(\theta_q + \theta_a) x_N}{2}$.*

Proof. First we are going to prove that $\Pr(Q \mid q) = \Pr(A \mid a)$ iff $\Delta \Pr(a, Q) = \Delta \Pr(q, Q)$. Using (8) and (10),

$$\Delta \Pr(\omega, Q) = \frac{2N!}{N!N!} \Pr(Q \mid \omega)^N (1 - \Pr(Q \mid \omega))^N \tag{23}$$

If $\Pr(Q \mid q) = \Pr(A \mid a)$ it is trivial to see that $\Delta \Pr(a, Q) = \Delta \Pr(q, Q)$.

Now assume that $\Delta \Pr(a, Q) = \Delta \Pr(q, Q)$. The first order condition for investment is just $C'(P^*(\theta)) = \frac{(\theta_q + \theta_a) \Delta \Pr(a,Q)}{2}$ so $P^*(\theta_1, \theta_2) = P^*(\theta_2, \theta_1)$. By definition,

$$\begin{aligned} C(P^*(g^a(\theta_a), \theta_a)) &= \frac{((\theta_a + g^a(\theta_a)) P^*(g^a(\theta_a), \theta_a) - \theta_a) \Delta \Pr(a, Q)}{2} \\ C(P^*(g^q(\theta_a), \theta_a)) &= \frac{((\theta_a + g^q(\theta_a)) P^*(g^q(\theta_a), \theta_a) - g^q(\theta_a)) \Delta \Pr(a, Q)}{2} \end{aligned} \tag{24}$$

Let $\theta_a = \theta_1$ and $g^q(\theta_1) = \theta_2$ in the second equation of (24) to get

$$C(P^*(\theta_2, \theta_1)) = \frac{((\theta_1 + \theta_2) P^*(\theta_2, \theta_1) - \theta_2) \Delta \Pr(a, Q)}{2} \tag{25}$$

Using that $P^*(\theta_1, \theta_2) = P^*(\theta_2, \theta_1)$ on (25), it follows that

$$C(P^*(\theta_1, \theta_2)) = \frac{((\theta_1 + \theta_2) P(\theta_1, \theta_2) - \theta_2) \Delta \Pr(a, Q)}{2} \tag{26}$$

Let $\theta_2 = \theta_a$ in (26) and comparing with the first equation of (24) it follows that $g^a(\theta_2) = \theta_1$. It is easy to see the inverse also follows which implies that $g^q(\theta_a) = \theta_q \Longleftrightarrow g^a(\theta_q) = \theta_a$ or $g^a = (g^q)^{-1}$ (geometric symmetry around the 45° line).

Now we have to calculate $\Pr(Q \mid q)$ and $\Pr(A \mid a)$. This expressions are:

$$\begin{aligned}
\frac{\Pr(Q \mid q) - \frac{\alpha}{2}}{1-\alpha} &= \int_0^1 \int_{\min\{g^q(\theta_a),1\}}^1 dF(\theta) + \int_0^1 \int_{\min\{g^a(\theta_a),1\}}^{\min\{g^q(\theta_a),1\}} P^*(\theta)\, dF(\theta) \\
\frac{\Pr(A \mid a) - \frac{\alpha}{2}}{1-\alpha} &= \int_0^1 \int_0^{\min\{g^a(\theta_a),1\}} dF(\theta) + \int_0^1 \int_{\min\{g^a(\theta_a),1\}}^{\min\{g^q(\theta_a),1\}} P^*(\theta)\, dF(\theta)
\end{aligned} \tag{27}$$

Recall that $\frac{\partial g^a(\theta_a)}{\partial \theta_a} \leq \frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \leq \frac{\partial g^q(\theta_a)}{\partial \theta_a}$, which implies that there is some $\theta_a^{**} \leq 1$ such that $g^q(\theta_a^{**}) = 1$, and the previous expressions are

$$\begin{aligned}
\frac{\Pr(Q \mid q) - \frac{\alpha}{2}}{1-\alpha} &= \int_0^{\theta_a^{**}} \int_{g^q(\theta_a)}^1 dF(\theta) + \mathcal{T} \\
\frac{\Pr(A \mid a) - \frac{\alpha}{2}}{1-\alpha} &= \int_0^1 \int_0^{g^a(\theta_a)} dF(\theta) + \mathcal{T}
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
\mathcal{T} &\equiv \int_0^1 \int_{\min\{g^a(\theta_a),1\}}^{\min\{g^q(\theta_a),1\}} P^*(\theta)\, dF(\theta) \\
&\equiv \int_0^1 \int_{g^a(\theta_a)}^{\theta_a} P^*(\theta)\, dF(\theta) + \int_0^{\theta_a^{**}} \int_{\theta_a}^{g^q(\theta_a)} P^*(\theta)\, dF(\theta)
\end{aligned}$$

Reversing the order of integration, using that $g^a = (g^q)^{-1}$ and F symmetric we have

$$\begin{aligned}
\int_0^{\theta_a^{**}} \left(\int_{\theta_a}^{g^q(\theta_a)} P^*(\theta) f(\theta)\, d\theta_q \right) d\theta_a &= \int_0^1 \left(\int_{(g^q)^{-1}(\theta_q)}^{\theta_q} P^*(\theta) f(\theta)\, d\theta_a \right) d\theta_q \\
&= \int_0^1 \left(\int_{g^a(\theta_q)}^{\theta_q} P^*(\theta) f(\theta)\, d\theta_a \right) d\theta_q
\end{aligned}$$

so $\mathcal{T} = 2\int\limits_0^1 \int\limits_{g^a(\theta_a)}^{\theta_a} P^*(\theta)\, dF(\theta)$. Using the same argument we have

$$\int\limits_0^1 \int\limits_0^{g^a(\theta_a)} f(\theta)\, d\theta_a d\theta_q = \int\limits_0^{\theta_a^{**}} \int\limits_{g^q(\theta_q)}^{1} f(\theta)\, d\theta_q d\theta_a$$

which in turns implies that $\Pr(A \mid a) = \Pr(Q \mid q)$. Now we are ready to show that there is such equilibrium.

Let $X' = \{(x_1, x_2) \in X : x_1 = x_2\}$ and $Y' = \{(y_1, y_2) \in Y : y_1 = y_2\}$ where where $X = \left[\alpha\xi_A, 1 - \alpha\xi_Q\right]^2$ and $Y = [\zeta(a), 1 - \zeta(a)] \times [\zeta(q), 1 - \zeta(q)]$ as in the Proof of Proposition (2). Since X' is closed and convex in X and Y' is closed and convex in Y, the argument used in that proof with the transformation $\Gamma : X \times Y \to X \times Y$ is valid and the Brouwer fixed point theorem gives that there is $(x_1, x_2, y_1, y_2) \in X' \times Y'$ such that $\Gamma(x_1, x_2, y_1, y_2) = (x_1, x_2, y_1, y_2)$ and $x_1 = x_2$, $y_1 = y_2$.

To complete the proof we define $y_N = 2\int\limits_0^1 \int\limits_{g^a(\theta_a)}^{\theta_a} \left(P^*(\theta) - \frac{1}{2}\right) dF(\theta)$ and $x_N = \frac{2N!}{N!N!}\left(\frac{1}{4} - (y_N)^2 (1-\alpha)^2\right)^N$.

■

Along any path of equilibria indexed by N, the probability of selecting A when the state is a is equal to the probability of selecting Q when the state is q. From now on, when we refer to A being selected in state a, it should be understood that we refer to the probability that the committee selects the candidate that would have won if the true state of nature were common knowledge.

What is the effect of increasing the number of voters? Intuition suggests that the probability of being pivotal decreases when the number of voters increases. This is straightforward only if the level of information in the electorate is constant. Unfortunately, when the information collected by each voter decreases (but the number of voters remains constant) the outcome of the election becomes more random. This extra randomness translates into a higher probability of being pivotal. Nevertheless, we can prove that under the simple majority rule the effect of more voters is dominant, and investment decreases when the size of the electorate increases.

We prove this result in steps.

Lemma 5 *In any symmetric committee with the simple majority rule* $\Pr(A \mid a)$ *is decreasing in* N.

Proof. We prove this by contradiction. Let $\varphi(x) = \left(\frac{1}{2}\right)^2 - x^2 (1-\alpha)^2$. Assume that $\Pr(A \mid a) = P_N$ increases with N; therefore, we must have that $(\varphi(P_N))^N \geq (\varphi(P_{N+1}))^N$. Using that $\frac{2N!}{N!N!} >$

$\frac{1}{4}\frac{2N+2!}{N+1!N+1!}$ and $\varphi\left(P_{N+1}\right) \leq \frac{1}{4}$,

$$\frac{2N!}{N!N!}\left(\varphi\left(P_N\right)\right)^N \geq \frac{2N+2!}{N+1!N+1!}\left(\varphi\left(P_{N+1}\right)\right)^{N+1}$$

and, $\Delta \Pr(\omega, Q)$ in (23) decreases with N, which in turns imply that $P^*(\theta)$ must decrease with N as well. Therefore $\frac{1-P^*(\theta)}{P^*(\theta)}$ increases with N and the slope of the function $g^a(\theta_a)$ increases while the slope of the function $g^q(\theta_a)$ decreases. Then, the functions $g^a(\theta_a)$ and $g^q(\theta_a)$ get closer when N increases.

Recalling the expression for $\frac{\Pr(A|a)-\frac{\alpha}{2}}{1-\alpha}$ and using the symmetry of the equilibrium, and some algebra gives,

$$\frac{\Pr(A \mid a) - \frac{\alpha}{2}}{1-\alpha} = \frac{1}{2} + 2\int_0^1 \int_{g^a(\theta_a)}^{\theta_a} \left(P^*(\theta) - \frac{1}{2}\right) dF(\theta) \tag{29}$$

and because the slope of $g^a(\theta_a)$ increases and $P^*(\theta)$ decreases, we must have that $\Pr(A \mid a)$ is also decreasing with N. ■

Once we know that $\Pr(A \mid a)$ decreases with N, we must also have that investment decreases and therefore, $\Delta \Pr(\omega, Q)$ must also be decreasing in N.

Corollary 2 *In any symmetric committee with the simple majority rule, $P^*(\theta)$ and $\Delta \Pr(\omega, Q)$ decrease with N.*

Proof. Since $P^*(\theta)$ changes monotonically with $\Delta \Pr(\omega, Q)$, it must be the case that $P^*(\theta)$ changes in the same way for all types $(\theta_q, \theta_a) \in [0,1]^2$. Using equation (29), the fact that $\frac{\partial g^a(\theta_a)}{\partial \theta_a} = \frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}$ we have that if $P^*(\theta)$ increases with N, it must be the case that $\Pr(A \mid a)$ also increases. A contradiction.

Using the investment function the result on $\Delta \Pr(\omega, Q)$ follows. ■

This proves that the rational ignorance hypothesis holds in our model.[23] This does not imply that information aggregation is not possible under any circumstances. The probability that a large electorate makes the right choice depends on the speed at which information acquisition decreases in the electorate when the number of voters increases (Yariv (2004)). Indeed

Proposition 5 *In any symmetric committee of size n (odd) with the simple majority rule, if the cost function is three times differentiable with $C''' \geq 0$ and $F(\theta_q, \theta_a) = \theta_q \theta_a$,[24] then*

[23] Benz and Stutzer (2004) find empirical support for the probability of being pivotal being positively correlated with the quality of information.

[24] The uniform distribution of types and independence across parameters is a simplification: all results hold if F is symmetric arounf the 45^o degree line.

1. *if $C'\left(\frac{1}{2}\right)=0$ and $C''\left(\frac{1}{2}\right)>0$, the result of the election approaches a random variable that makes A the winner with probability $\frac{1}{2}$ in any state of nature when N grows arbitrarily large.*

2. *if $C'\left(\frac{1}{2}\right)=C''\left(\frac{1}{2}\right)=0$ and $C'''\left(\frac{1}{2}\right)>0$, when N grows arbitrarily large, the probability of making the right choice is bounded away from $\frac{1}{2}$, and the bound is decreasing on the value of $C'''\left(\frac{1}{2}\right)$.*

3. *if $C'\left(\frac{1}{2}\right)=C''\left(\frac{1}{2}\right)=C'''\left(\frac{1}{2}\right)=0$, the probability of making the right choice converges to 1 when N grows arbitrarily large.*

Proof. We follow Martinelli (2006) although we must consider a continuum of types of voters instead of an homogeneously informed voter. First we are going to construct a random variable that describes the difference between the probability of voting for one candidate and the other. Then we are going to apply Berry-Esseen Theorem (see Bickel and Doksum (2000)).

Let $y_N=2\int_0^1\int_{g^a(\theta_a)}^{\theta_a}\left(P^*(\theta)-\frac{1}{2}\right)d\theta_q d\theta_a$ on (29). Define the random variable M_i^N such that $M_i^N\equiv\frac{1}{2}-y_N(1-\alpha)$ if $v=A$ and $M_i^N\equiv-\frac{1}{2}-y_N(1-\alpha)$ if $v=Q$. It is easy to see that $E\left(M_i^N\right)=0$, $E\left(\left(M_i^N\right)^2\right)=\frac{1}{4}-\left(y_N(1-\alpha)\right)^2$ and $E\left(\left|M_i^N\right|^3\right)=2\left(\left(\frac{1}{2}\right)^4-\left(y_N(1-\alpha)\right)^4\right)$.

Therefore, $\overline{M}_i^N\equiv\frac{M_i^N}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ is a random variable with zero mean, variance equal to 1 and $E\left(\left|\overline{M}_i^N\right|^3\right)=\frac{2\left(\left(\frac{1}{2}\right)^2+(y_N(1-\alpha))^2\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$. Define $M^N\equiv\sum_{i=1}^{2N+1}\overline{M}_i^N=\frac{T_{2N+1}(A)-\frac{2N+1}{2}-(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ and recall that $T_{2N+1}\equiv\sum_{i=1}^{2N+1}I(v_i=A)$ is the number of votes for A out of $2N+1$ voters. We know that A is the winner if $T_{2N+1}>N$ and Q is the winner if $T_{2N+1}\leq N$; therefore we require that $M^N=\frac{T_{2N+1}(A)-\frac{2N+1}{2}-(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}>\frac{-\frac{1}{2}-(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ for $T_{2N+1}>N$. Let $\mathcal{M}^N\equiv\frac{M^N}{\sqrt[2]{(2N+1)}}$ and $\mathcal{F}^N$ be its distribution. The probability of A being the winner is just the probability that $\mathcal{M}^N>-\frac{\frac{\frac{1}{2}+(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}}{\sqrt[2]{(2N+1)}}$; if we let $J^N(y_N)\equiv-\frac{\frac{\frac{1}{2}+(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}}{\sqrt[2]{(2N+1)}}$ the probability of A being the winner is just $1-\mathcal{F}^N\left(J^N(y_N)\right)$. Replacing, we get

$$J^N(y_N)=-\frac{1}{2\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)(2N+1)}}-\frac{\sqrt[2]{(2N+1)}y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}.$$

Let Φ be the cdf of a $(0,1)$ normal random variable, so we can apply the Berry-Esseen Theorem[25] to get that $\lim_{N\to\infty}\mathcal{F}^N\left(J^N(y_N)\right)\to\Phi\left(J^N(y_N)\right)$ if $E\left(\left|\overline{M}_i^N\right|^3\right)$ is finite, which is the case

[25] Let $X_1,....X_n$ be *i.i.d* with mean $\mu=0$ and $\sigma^2=1$. Then, for all n

for some N big enough so y_N is close to 0 (see Lemma (5)).[26] Now using that Φ is continuous we must have that $\lim_{N\to\infty}\Phi\left(J^N(y_N)\right)\to\Phi\left(\lim_{N\to\infty}J^N(y_N)\right)$, which implies that $\lim_{N\to\infty}\mathcal{F}^N\left(J^N(y_N)\right)\to\Phi\left(\lim_{N\to\infty}J^N(y_N)\right)$.

The problem is now the limit of $J^N(y_N)$ or $\lim_{N\to\infty}\widehat{J}^N(y_N)$ where $\widehat{J}^N(y_N)=-\frac{\sqrt[2]{(2N+1)}y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ since $\frac{1}{2\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)(2N+1)}}\to 0$ as N grows. If $\sqrt[2]{(2N+1)}y_N(1-\alpha)\to\infty$, it follows that $\widehat{J}^N(y_N)\to-\infty$, and $1-\mathcal{F}^N\left(J^N(y_N)\right)\to 1$ which makes A the winner almost surely in state a.

Recall that $\Delta\Pr(a,Q)=\frac{(2N)!}{N!N!}\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)^N$ therefore

$$\begin{aligned} C'\left(P^*(x)\right) &= \frac{x}{2}\frac{(2N)!}{N!N!}\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)^N \\ y_N &= 2\int_0^1\left(\int_{\theta_a+g^a(\theta_a)}^{2\theta_a}\left(P(x)-\frac{1}{2}\right)dx\right)d\theta_a \end{aligned} \tag{30}$$

Assume that $C''\left(\frac{1}{2}\right)=l>0$; since $P^*(x)$ is concave we must have $P^*(x)\le P^*(0)+\frac{\partial P^*(x)}{\partial x}_{x=0}x$ so

$$z_N\le\frac{(1-\alpha)}{l}\frac{(2N)!}{N!N!}\frac{\sqrt[2]{N}}{2^{2N}}\left(1-\frac{(2z_N)^2}{N}\right)^N\int_0^1\left(\int_{\theta_a+g^a(\theta_a)}^{2\theta_a}xdx\right)d\theta_a \tag{31}$$

where we used that $\frac{\partial P^*(x)}{\partial x}=\frac{1}{2C''(P^*(x))}\frac{(2N)!}{N!N!}\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)^N$ and define $z_N\equiv y_N(1-\alpha)\sqrt[2]{N}$. Let $\frac{(2N)!}{N!N!}\frac{\sqrt[2]{N}}{2^{2N}}\left(1-\frac{(2z_N)^2}{N}\right)^N\equiv h(z_N,N)$ and note that $e^{-4z_N}=\lim_{N\to\infty}\left(1-\frac{(2z_N)^2}{N}\right)^N$ and $\lim_{N\to\infty}\frac{(2N)!}{N!N!}\frac{\sqrt[2]{N}}{2^{2N}}=\pi^{-1}$ so $h(z_N,N)\to e^{-4z_N}\pi^{-1}$. Therefore, since $\int_0^1\int_{\theta_a+g^a(\theta_a)}^{2\theta_a}xdx\to 0$ we must have that $\lim_{N\to\infty}z_N=0$. This proves the first part of the proposition.

$\sup\left|\Pr\left(\frac{\sum_{i=1}^n X_i}{\sqrt[2]{n}}\le t\right)-\Phi(t)\right|\le\frac{33}{4}\frac{E|X_1|^3}{\sqrt[2]{n}}$. See Bickel and Doksum (2000).

[26]Paradoxically, the fact that adding a new voter decreases the average informativeness of each vote is helpful in order to prove aggregation.

Now , for the case that $C''\left(\frac{1}{2}\right)=0$ and $C'''\left(\frac{1}{2}\right)\geq 0$ we have that

$$\begin{aligned} y_N &\geq 2\int_0^1\left(\left(P^*\left(\theta_a+g^a\left(\theta_a\right)\right)-\frac{1}{2}\right)\int_{\theta_a+g^a(\theta_a)}^{2\theta_a}dx\right)d\theta_a \\ &\geq 2\int_0^1\left(P^*\left(\theta_a+g^a\left(\theta_a\right)\right)-\frac{1}{2}\right)\left(\theta_a-g^a\left(\theta_a\right)\right)d\theta_a \end{aligned}$$

Using concavity of P^*: $P^*\left(\theta_a+g^a\left(\theta_a\right)\right)\geq P^*\left(0\right)+\frac{\partial P^*(x)}{\partial x}_{x=\theta_a+g^a(\theta_a)}\left(\theta_a+g^a\left(\theta_a\right)\right)$ it follows

$$\begin{aligned} z_N &\geq (1-\alpha)\,h\left(z_N,N\right)\int_0^1\frac{\left(\left(\theta_a\right)^2-\left(g^a\left(\theta_a\right)\right)^2\right)}{C''\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right)}d\theta_a \\ &\geq (1-\alpha)\,h\left(z_N,N\right)\int_0^1\mathcal{H}_1\left(\theta_a\right)d\theta_a \end{aligned}$$

where $\mathcal{H}_1\left(\theta_a\right)\equiv\frac{\left(\left(\theta_a\right)^2-\left(g^a\left(\theta_a\right)\right)^2\right)}{C''\left(P^*\left(2\theta_a\right)\right)}$ and we used $C'''\geq 0$. Using L'Hopital's rule we have that

$$\lim_{\Delta\Pr(a,Q)\to 0}\mathcal{H}_1\left(\theta_a\right)=\lim_{\Delta\Pr(a,Q)\to 0}\frac{-2\left(g^a\left(\theta_a\right)\right)\frac{\partial g^a(\theta_a)}{\partial\Delta\Pr(a,Q)}}{C'''\left(P^*\left(2\theta_a\right)\right)\frac{\partial P^*(2\theta_a)}{\partial\Delta\Pr(a,Q)}}$$

Using the system of equations for $P^*\left(g^a\left(\theta_a\right)+\theta_a\right)$ and $g^a\left(\theta_a\right)$

$$\begin{aligned} C\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right) &= \frac{\left(\left(\theta_a+g^a\left(\theta_a\right)\right)P^*\left(g^a\left(\theta_a\right)+\theta_a\right)-\theta_a\right)\Delta\Pr\left(a,Q\right)}{2} \\ C'\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right) &= \frac{\left(\theta_a+g^a\left(\theta_a\right)\right)\Delta\Pr\left(a,Q\right)}{2} \end{aligned}$$

we have

$$\begin{aligned} \frac{\partial g^a\left(\theta_a\right)}{\partial\Delta\Pr\left(a,Q\right)} &= -\frac{\mathcal{H}_2\left(\theta_a\right)}{P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\Delta\Pr\left(a,Q\right)} \\ \frac{\partial P^*\left(g^a\left(\theta_a\right)+\theta_a\right)}{\partial\Delta\Pr\left(a,Q\right)} &= \frac{\theta_a}{2P^*\left(g^a\left(\theta_a\right)+\theta_a\right)C''\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right)} \\ \frac{\partial P^*\left(2\theta_a\right)}{\partial\Delta\Pr\left(a,Q\right)} &= \frac{\theta_a}{C''\left(P^*\left(2\theta_a\right)\right)} \end{aligned} \tag{32}$$

where we define $\mathcal{H}_2(\theta_a) = (\theta_a + g^a(\theta_a)) P^*(g^a(\theta_a) + \theta_a) - \theta_a$. So

$$\begin{aligned}
\lim_{\Delta \Pr(a,Q)\to 0} \mathcal{H}_1(\theta_a) &= \lim_{\Delta \Pr(a,Q)\to 0} \frac{2(g^a(\theta_a)) \frac{\mathcal{H}_2(\theta_a)}{P^*(g^a(\theta_a)+\theta_a)\Delta \Pr(a,Q)}}{C'''(P^*(2\theta_a)) \frac{\theta_a}{C''(P^*(2\theta_a))}} \\
&= \lim_{\Delta \Pr(a,Q)\to 0} \frac{\frac{2}{P^*(g^a(\theta_a)+\theta_a)} \frac{(g^a(\theta_a))}{\theta_a} \frac{\mathcal{H}_2(\theta_a)}{\Delta \Pr(a,Q)}}{\frac{C'''(P^*(2\theta_a))}{C''(P^*(2\theta_a))}} \\
&= 4 \lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{C'''(P^*(2\theta_a)) \Delta \Pr(a,Q)}
\end{aligned}$$

L'Hopital again gives that $\lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)}$ is equal to

$$\lim_{\Delta \Pr(a,Q)\to 0} \left(\mathcal{H}_2(\theta_a) C'''(P^*(2\theta_a)) \frac{\partial P^*(2\theta_a)}{\partial \Delta \Pr(a,Q)} + \frac{\partial \mathcal{H}_2(\theta_a)}{\partial \Delta \Pr(a,Q)} C''(P^*(2\theta_a)) \right)$$

Using the expressions for $\frac{\partial P^*(g^a(\theta_a)+\theta_a)}{\partial \Delta \Pr(a,Q)}$ and $\frac{\partial g^a(\theta_a)}{\partial \Delta \Pr(a,Q)}$ to get $\frac{\partial \mathcal{H}_2(\theta_a)}{\partial \Delta \Pr(a,Q)}$ in (32) we have that $\lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)}$ is equal to

$$\begin{aligned}
&\lim_{\Delta \Pr(a,Q)\to 0} \frac{\theta_a (g^a(\theta_a) + \theta_a) C''(P^*(2\theta_a))}{2P^*(g^a(\theta_a) + \theta_a) C''(P^*(g^a(\theta_a) + \theta_a))} \\
&+ \lim_{\Delta \Pr(a,Q)\to 0} \left(\frac{\mathcal{H}_2(\theta_a) C'''(P^*(2\theta_a)) \theta_a}{C''(P^*(2\theta_a))} - \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)} \right)
\end{aligned} \tag{33}$$

and some algebra gives that $2 \lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)}$ is equal to

$$\begin{aligned}
&\lim_{\Delta \Pr(a,Q)\to 0} \mathcal{H}_2(\theta_a) \theta_a \frac{C'''(P^*(2\theta_a))}{C''(P^*(2\theta_a))} \\
&+ \lim_{\Delta \Pr(a,Q)\to 0} \frac{\theta_a (g^a(\theta_a) + \theta_a) C''(P^*(2\theta_a))}{2P^*(g^a(\theta_a) + \theta_a) C''(P^*(g^a(\theta_a) + \theta_a))}
\end{aligned}$$

Since $\mathcal{H}_2(\theta_a) \geq 0$ (see (15)) and $\lim_{\Delta \Pr(a,Q)\to 0} \frac{\theta_a(g^a(\theta_a)+\theta_a)}{2P^*(g^a(\theta_a)+\theta_a)} = 2(\theta_a)^2$

$$2 \lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)} \geq 2(\theta_a)^2 \lim_{\Delta \Pr(a,Q)\to 0} \frac{C''(P^*(2\theta_a))}{C''(P^*(g^a(\theta_a) + \theta_a))}$$

using that $C''' \geq 0$ and $g^a(\theta_a) < \theta_a$, it follows that

$$\lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(\theta_a + g^a(\theta_a)))}{\Delta \Pr(a,Q)} \geq (\theta_a)^2$$

so using that $C''(P^*(2\theta_a)) \geq C''(P^*(\theta_a + g^a(\theta_a)))$ we have

$$\lim_{\Delta \Pr(a,Q) \to 0} \mathcal{H}_1(\theta_a) \geq 4 \lim_{\Delta \Pr(a,Q) \to 0} \frac{(\theta_a)^2}{C'''(P^*(\theta_a + g^a(\theta_a)))}$$

and therefore if $\lim_{\Delta \Pr(a,Q) \to 0} C'''(P^*(\theta_a + g^a(\theta_a))) \to 0$ (as it is for $C'''\left(\frac{1}{2}\right) = 0$) we have $\lim_{\Delta \Pr(a,Q) \to 0} \mathcal{H}_1(\theta_a) \to \infty$, which proves that $z_N \to \infty$. If $C'''\left(\frac{1}{2}\right) = l > C''\left(\frac{1}{2}\right) = 0$, a lower bound for z_N is obtained. ■

Note that the aggregation result depends on the value of $C'''\left(\frac{1}{2}\right)$. Using that $C'(P^*(\theta)) = \frac{(\theta_q + \theta_a)\Delta \Pr(a,Q)}{2}$ we have that the direct effect of $\Delta \Pr(a,Q)$ on $P^*(\theta)$ is determined by the second derivative of the cost function while the change in this change ($\frac{\partial^2 P^*(\theta)}{\partial(\Delta \Pr(a,Q))^2}$) is affected by the third derivative of the cost function. In our model increasing the number of voters increases the chances of a vote being informative but reduces the incentives to collect information of all voters. When the speed at which the average information decreases because new voters are added is slow enough, aggregation of information is possible in the limit. The cost function for information acquisition determines whether adding another member is desirable or not.