The Condorcet Jury Theorem and Costly Information Acquisition

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1 Introduction

It is a well known fact that electors have little and uneven knowledge about policies and the backgrounds of elected officials (see, for instance, Delli Carpini and Keeter, 1996). This fact is consistent with the rational ignorance hypothesis as formulated by Schumpeter (1950) and Downs (1957): since each vote has little impact on the outcome of a large election and information acquisition is costly, individual voters will choose to acquire little information. Determining the implications of this hypothesis has important implications about the quality of democratic deliberations.

A second view suggests that aggregate opinion may be able to reflect the public interests even when most individuals are poorly informed. Condorcet (1786) argued that the larger is the population, the higher is the probability that a democracy will make the 'right' decision. According to this argument, in the process of preference aggregation, the more or less random opinion of poorly informed voters would cancel out (see Wittman 1989, 1995). The statement constitutes the so called Condorcet Jury Theorem.

In order to test this proposition we use a model where voters have to decide over two candidates. A and B is introduced. Voters have common preferences but they do not know which one of the candidates is better for them. Before voting they receive an imperfectly informative signal telling which candidate is the best one.

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As a first step we assume that voters simply follow the signal and prove that the if the quality of the signal is independent on the number of voters elections perfectly aggregate information and the probability of taking the right decision approaches one when the electorate is large. We will see that it is not necessarily the case when the quality of the signals depends on the number of electors. We will see necessary and sufficient conditions under which large elections aggregate a positive amount of information.

As a second step we will consider strategic voting. In this case, electors must condition their behavior on the event of being pivotal. We will see that strategic behavior is not necessarily an equilibrium, unless the signal is sufficiently precise or voters are ex-ante indifferent among alternatives.

The last step is to consider costly information acquisition. We assume that electors do not have free access to a reliable font of information, but they can acquire some information. As a first step we assume that the costs of acquiring information hare the same for all voters. In the last section we analyze the case of heterogeneous costs. In the latter setup we assume that the alternatives are ex-ante equal. In this case it is easier to prove the existence of equilibria with information acquisition and to derive clear results on information aggregation properties of the elections. For a more general approach the interested reader might want to read Martinelli (2006, 2007), Oliveros (2006, 2007) and Triossi (2008).

In our case the costs represent the effort of collecting and processing information. A voter who acquires information of quality x receives the correct signal with probability $\frac{1}{2} + x$ and faces a cost of $C(x)$. C is strictly convex and increasing in x. In the case where costs are heterogeneous the cost function will of the kind $C(\alpha, x)$ where x is the type of the voter.

In the case where electors have the same cost function, elections aggregate a positive amount of information, this is not, in general the case, when voters are heterogeneous.

However, the introduction of heterogeneity in information acquisition costs allows to account for three empirically relevant facts:

(i) A small fraction of the electorate is informed.

(ii) The overall quality of information electors have is limited.

(iii) The distribution of information across electors is uneven.

When costs functions are the same it is possible to account only for (i) and (ii) (Martinelli (2006). The only other paper which reflects (iii) as well is Oliveros (2006) who takes an orthogonal approach: voters have the same information acquisition costs but they differ in the losses they bear when the wrong decision is taken. He proves aggregation results similar to Martinelli (2006).

Some Literature

The first proofs of the Condorcet were entirely statistical (see Berg 1993, Berend and Paroush 1998, Ladha 1992, 1993). They assumed that each individual privately observes a signal about the right candidate and then vote sincerely according to the signal. More recently the theorem has been proved under the assumption of strategic voting (see, e.g., Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1996, 1997, 1999, and Myerson 1998). All these papers assume that the information is freely available to voters. Interestingly, Paroush (1998), in a non strategic setup, proved that elections can fail to aggregate information if the probability a voter receives the correct signal is not bounded away from one half. Yariv (2004) analyzes majority voting in common value two-option environments where voters have private information, the quality of which exogenous depends on the size of the electorate. She proves that information of low quality may lead to informational failures. In a recent work Mandler (2007) proved a similar negative result: if voters are uncertain of the quality of the initial signal elections can loose their ability to aggregate information.

The literature focusing on voting in committee has recently considered the issue of costly information acquisition. Persico (2004) and Mukkhopadhaya (2005) consider a setting in which committee members have identical and fixed costs of acquiring information. In this setup there is a maximum number of voters who can acquire information at equilibrium so that for large electorates there is no equilibrium with information acquisition. Oliveros (2007) presents a model based on Oliveros (2006) in which voters have the same information acquisition costs but they differ in the gains obtained from taking the right decisions. Voters can select whether to vote or abstain and the amount of information to acquire. He proves that there are equilibria where voters collect information of different qualities, there are informed voters that abstain, and information and abstention need not be inversely correlated for all voters.

2 The Model

There is an odd number $N = 2n + 1$ of voters, who have to elect one of two candidates, A and B by simple majority. There are two possible states of the world a and b. The prior probability of state a is $q_a = q \in (0, 1)$ and the prior probability of state b is $q_b = 1 - q$. The utility of voters depends on the candidate elected and on the state of the world. They have common preferences and agree that A is the best candidate when the state is A and B is the best candidate when the state is B. Let $U(c, \omega)$ denote the utility of any voter when candidate $c \in \{A, B\}$ is elected at state $\omega \in \{a, b\}$. We have $U(A, a) - U(B, a) = \Delta U_a > 0$ and $U(B, b) - U(A, b) = \Delta U_b > 0$.

Electors do not know the state of the world and before voting they independently receive signal $s \in \{s_a, s_b\}$. Before receiving the signal they can acquire information of quality $x \in \left[0, \frac{1}{2}\right]$.
When a voter receives a signal of quality x the likelihood of receiving the signal s_ω conditional on ω is $p(s_\omega \mid \omega, x) = \frac{1}{2} + x$. Voters have different acquisition costs. One interpretation is literal: voters bear different costs of access to information or they have access to different fonts of information. An alternative and, in my opinion more compelling, explanation is that people are exposed to some font of information which is meaningless unless they invest some effort in processing and understand it. Furthermore, the have different ability in processing information so so less skilled electors must invest more effort in order to extract the same amount of information. An elector of type α faces a cost $C(x, \alpha)$ to purchase information of quality x. Then, if candidate c has been elected and the state of the world is ω the utility of a voter of type α who has acquired information of quality x is $U(d, \omega) - C(x, \alpha)$.

The cost function is of class $C^2\left(\left[0, \frac{1}{2}\right) \times [0, 1]\right)$ and $C_x(0, 0) = 0$. Acquiring a positive amount of information has a strictly positive cost, while acquiring no information entails no costs: **NFL**(No Free Lunch) $C(0, \alpha) = 0$ and $C(x, \alpha) > 0$ for all $x > 0$ and for all α. Higher types have higher costs and acquiring information of better quality entails higher costs, formally: **MON**(Monotonicity) $C_x(x, \alpha) > 0$, $C_\alpha(x, \alpha) > 0$, for all $x \in \left(0, \frac{1}{2}\right)$, and for all $\alpha > 0$. The cost function is strictly convex in the quality of information acquired. **CONV**(Convexity) $C_{xx}(x, \alpha) > 0$, for all $x \in \left(0, \frac{1}{2}\right)$, and for all $\alpha > 0$. Finally we assume that higher types face increasingly higher costs: **SCP(**Single Crossing Property) $C_{x\alpha}(x, \alpha) > 0$ for all $x > 0$ and for all $\alpha > 0$.

Types are distributed independently across the electorate in the interval $[0, 1]$, according to a continuous density function $f : [0,1] \to \mathbb{R}_+$, with $f(0) > 0$. When there is no heterogeneity among voters, which is when the cost function is degenerate at type 0 we will write simply $C(x)$.

When there is no heterogeneity among voters, which when the cost function is degenerate we will write simply $C(x)$.

Finally, we introduce some mathematical notation that will use along the paper. Let $f, g : X \to \mathbb{R}$ where X is a metric space. Let $z \in X$. We write $f \approx g$ for $x \to z$ if $lim_{x \to z} \frac{f(x)}{g(x)} = 1$, $f = o(g)$ for $x \to z$ if $lim_{x \to z} \frac{f(x)}{g(x)} = 0$ and $f = O(g)$ for $x \to z$ if there exists $C > 0$ such that $|f(x)| \leq C |g(x)|$ in a neighborhood of z. Let $\{a_n\}_{n \in \mathbb{N}}$ and $\{b_n\}_{n \in \mathbb{N}}$ two sequences of real numbers. We write $a_n \approx b_n$ if $lim_{n \to \infty} \frac{a_n}{b_n} = 1$, $a_n = o(b_n)$ if $lim_{n \to \infty} \frac{a_n}{b_n} = 0$ and $a_n = O(b_n)$ if there exists $C > 0$ such that$|a_n| \leq C |b_n|$ for n large enough. We denote by Φ and φ the standard normal distribution and its density.

3 Naive voters.

In this part we assume that, at state ω, voters receive (for free) the correct signal with probability $\frac{1}{2} + x_n$ and follow the signal.

Proposition 1 *Let $\{n_k\}_{k \in \mathbb{N}}$ be subsequence of positive integers. Assume that*

$$l = \lim_{k \to \infty} \sqrt{n_k} x_{n_k}$$

exist for $\omega = a, b$. Assume that voters vote according to the signal. Let $P_{\omega k}$ be the probability the right decision is taken at state $\omega = a, b$, at the corresponding SBE. Then

$$\lim_{n \to \infty} P_{\omega k} \to \Phi\left(2\sqrt{2}l\right)$$

for $\omega = a, b$. In particular, the Condorcet Jury Theorem holds if and only if $l = \infty$ for $\omega = a, b$.

Proof. Without loss of generality assume that $n_k = k$ for every k. We have $x_n = x_n(p_n)$ for every n. Suppose the state is a. Given equilibrium strategies, the event of a given voter voting for A at state a is a

Bernoulli trial with probability of success $\frac{1}{2}+x_n$. For $i=1,...,2n+1$, set

$$V_i^n=\begin{cases}\frac{1}{2}-x_n \text{ if voter } i \text{ votes for} A\\ -\frac{1}{2}-x_n if voter i \text{ votes for } B\end{cases}$$

The V_i^n are i.i.d. Furthermore,$E\left(V_i^n\right)=0$, $E\left(\left(V_i^n\right)^2\right)=\frac{1}{4}-x_n^2$ and $E\left(\left|V_i^n\right|^3\right)=\frac{1}{8}-2x_n^4$.

Note that the alternative A wins the elections if and only if it gets at least $n+1$ votes which is if and only if

$$\sum_{i=1}^{2n+1}V_i^n>-\frac{1}{2}-\left(2n+1\right)x_n$$

Let W^n be the normalized sum of the V_i^n.

$$W^n=\frac{\sum_{i=1}^{2n+1}V_i^n}{\sqrt{\left(2n+1\right)E\left(\left(V_i^n\right)^2\right)}}.$$

Let F_n be the p.d.f. of W^n. The probability of choosing A at state a is $1-F_n\left(J_n\right)$ where

$$J_n=\frac{-\frac{1}{2}-\left(2n+1\right)x_n}{\sqrt{\left(\frac{1}{4}-x_n^2\right)\left(2n+1\right)}}.$$

For $n\to\infty$

$$J_n\approx\frac{-\frac{1}{2}-2\sqrt{n}\left(\sqrt{n}x_n\right)}{\sqrt{\frac{n}{2}-2\left(\sqrt{n}x_n\right)^2}}\approx-2\sqrt{2}\left(\sqrt{n}x_n\right)$$

From the Berry-Esseen Theorem (see Chow and Teicher 1997, p 322).

$$\left|F_n\left(J_n\right)-\Phi\left(J_n\right)\right|=O\left(\frac{1}{\sqrt{n}}\right)$$

uniformly in n.

$$lim_{n\to\infty}P_{an} = 1 - \Phi\left(-2\sqrt{2}l\right) = \Phi\left(2\sqrt{2}l^{I}\right)$$

whether l is finite or infinite. The proof of the case $\omega = b$ is similar. ■

The result implies that elections aggregate some information only if x_n converges to a positive real number or if it converges to 0 no faster than $\frac{1}{\sqrt{n}}$. If for instance $x_n = n^{-\alpha}$, for some $\alpha > 0$, then the probability that elections reach the best decision converges to: $\frac{1}{2}$, when $\alpha > \frac{1}{2}$, $\Phi\left(2\sqrt{2}\right)$ if $\alpha = \frac{1}{2}$ and 1 otherwise.

Corollary 1 *(Condorcet's jury theorem) If voters receive the correct signal with probability $p > \frac{1}{2}$, for every $n \in \mathbf{N}$, then large elections perfectly aggregate information.*

The result follows straightforwardly from Proposition 1 with $x_n = p - \frac{1}{2} > 0$, for every n.

4 Strategic voters

In this section we drop the assumption that voters simply follows the signal. and introduce strategic considerations. The game is as follows: before voting, electors receive a signal and then vote simultaneously for on of the two candidates.

A strategy specifies for which candidate an elector vote vote, conditional on the signal received.

Definition 1 *A strategy for voter i consists of a voting strategy $v : \{s_a, s_b\} \to \{A, B\}$.*

A strategy of player i is denoted by v_i, a strategy profile $(v_i)_{i=1,\ldots,2n+1}$ is denoted by (V) and V_{-i} is the coalitional strategy of all voter but i. Given V_{-i}, we denote by

$$U(v \mid \omega) = \sum_{d\in\{A,B\}} U(d,\omega)\Pr(d \mid \omega, v, V_{-i})$$

the expected utility from voting v at state ω, net of information acquisition costs. The term $\Pr(d \mid \omega, v, V_{-i})$ denotes the probability the outcome is d at state ω. After receiving signal $s \in \{s_a, s_b\}$, the expected utility from voting v is

$$U(v \mid s, V_{-i}) = \sum_{\omega\in\{A,B\}} U(v \mid \omega)\, Pr(\omega \mid s)$$

where $Pr\left(\omega \mid x, s\right)$ denotes the likelihood of ω given investment x and signal s.

The expected utility from a player investing x and using a voting rule from using a strategy (x, v) when other agents play $(X, V)_{-i}$ is

$$U\left(x, v \mid V_{-i}\right) = \sum_{s \in \{s_a, s_b\}} U\left(v \mid s, V_{-i}\right) p(s)$$

where $p(s)$ is the probability of receiving the signal s.

Our objective here is limited to check whether following, for a fixed signal precision x the signal is a Bayes-Nash equilibrium. If it is the case, from Proposition 1, we already know that elections will aggregate information. A more complete treatment is provided in Austen Smith and Banks (1996).

Let p_ω be the probability that a voter is pivotal at state ω, for $\omega = a, b$.

The expected utility from following the signal $U\left(x, A, B\right)$ can be written as

$$U\left(A, B\right) = \left(p_a q_a \Delta U_a + p_b q_b \Delta U_b\right)\left(\frac{1}{2} + x\right) + p_a q_a U\left(B \mid a\right) + p_b q_b U\left(A \mid b\right) + U_{-i}.$$

where U_{-i} is s term which depends only on the strategy of the other voters. In order to determine whether voting according to the signal is a Nash equilibrium we have to check that, at $U\left(A, B\right) \geq \max\left\{U\left(A, A\right), U\left(B, B\right)\right\}$. This is equivalent to check that voting A is optimal, when signal s_a has been received and voting B is optimal, when signal s_b has been received.

The utility from voting A, independently on the signal, is

$$p_a q_a U\left(A \mid a\right) + p_b q_b U\left(A \mid b\right) + U_{-i}.$$

The utility from voting B, whatever the signal, is

$$p_a q_a U\left(B \mid a\right) + p_b q_b U\left(B \mid b\right) + U_{-i}.$$

Observe that $U\left(A, A\right) \geq U\left(B, B\right)$, if and only if $p_a q_a \Delta U_a > p_b q_b \Delta U_b$.

NOTICE THAT THE OPTIMAL STRATEGY OF THE VOTER MUST DEPEND ONLY ON THE PROBABILITY OF BEING PIVOTAL.

Voting according to the signal is optimal if and only if

$$(p_a q_a \Delta U_a + p_b q_b \Delta U_b)\, x \geq \frac{p_a q_a \Delta U_a - p_b q_b \Delta U_b}{2}.$$

Observe that the p_ω depend on the strategy of the other players and that if every player but i votes according to the signal

$$p_a = p_b = p = \binom{2n}{n} \left(\frac{1}{4} - x^2\right)^n.$$

Proposition 2 *That everybody follows the signal is a Bayes-Nash equilibrium if and only if*

$$x \geq \frac{q_a \Delta U_a - q_b \Delta U_b}{2\,(q_a \Delta U_a + q_b \Delta U_b)}.$$

In particular it is an equilibrium whenever $q_a \Delta U_a - q_b \Delta U_b = 0$.

Thus naive behavior is an equilibrium either if the alternatives are ex-ante equivalent, or if the precision of the signal is high enough.

5 Costly Information acquisition:

We assume that, before receiving a signal, voters can acquire information of quality x at a cost $C(x)$, where C is of class C^2, is increasing and strictly convex.

Definition 2 *A strategy for voter i consists of a information acquisition strategy $x \in \left[0, \frac{1}{2}\right]$ and of a voting strategy $v : [0,1] \to \{A, B\}$.*

A more rigorous definition would make the voting strategy contingent on the investment $v : \left[0, \frac{1}{2}\right] \times \{s_a, s_b\} \to \{A, B\}$. However, only the signal is disclosed before the voting decision is taken so the definition presented is without loss of generality.

We use the notation introduced above. The only difference is that the signal is costly and that the precision of the signal each elector receives depends on how much information she has acquired. The equilibrium concept we employ is symmetric equilibrium.

Definition 3 *A symmetric Bayesian equilibrium (SE from now on) is given by a strategy* $(\hat{x}, \hat{v})$ *such that the profile* $\left(\hat{X}, \hat{V}\right) = (\hat{x}, \hat{v})_{i=1,\ldots 2n+1}$ *satisfies:*

1. $U\left(v\left(\hat{x}, s\right) \mid \hat{x}, s, \left(\hat{X}, \hat{V}\right)_{-i}\right) \geq U\left(v \mid \hat{x}, s, \left(\hat{X}, \hat{V}\right)_{-i}\right)$ *for all* $v \in \{A, B\}$ *and for all* $s \in \{s_a, s_b\}$.
2. $U\left(\hat{x}, \hat{v} \mid \left(\hat{X}, \hat{V}\right)_{-i}\right) - C\left(\hat{x}\right) \geq U\left(x, v \mid \left(\hat{X}, \hat{V}\right)_{-i}\right) - C\left(x\right)$ *for all voting rules* v[1].

At a symmetric equilibrium every player employs the same strategy. Individuals vote optimally conditionally on the signal received, on the investment and on the strategy of the other electors (part 1). Information acquisition and voting strategies are ex ante optimal, given the strategy of the other electors (part 2).

Note that the issue here is not the existence of a SE. An equilibrium always exists: no voter acquires information and everybody votes for the same alternative independently on the signal received. Our concern is with studying equilibria with information acquisition, if any exists, which is SE with $\hat{x} > 0$.

We will check consider now only symmetric strategic. Let $p = p_a = p_b$, the probability of being pivotal at any of the two states:

$$p_a = p_b = p_n\left(x\right) = \binom{2n}{n}\left(\frac{1}{4} - x^2\right)^n.$$

Using Stirling formula (which is $n! \approx \frac{e^{-n}n^n}{\sqrt{2\pi n}}$ for n large) and that $\left(\frac{1}{4} - x^2\right)^n \leq 2^{-2n}$ we find that $p_n\left(x\right) \to 0$ as $n \to \infty$.

The expected utility from following the signal $U\left(x, A, B\right)$ can be written as

$$U\left(A, B\right) = p\left(q_a \Delta U_a + q_b \Delta U_b\left(\frac{1}{2} + x\right) + q_a U\left(B \mid a\right) + qU\left(A \mid b\right)\right) + U_{-i} - C\left(x\right)$$

where U_{-i} is s term which depends only on the strategy of the other voters.

[1] A more rigorous definition would make the voting strategy contingent on the investment $v : [0,1] \times \left[0, \frac{1}{2}\right] \times \{s_a, s_b\} \to \{A, B\}$. However, no other information than the signal is disclose before the voting decision so the results are not affected.

Notice that $p\left(q_a \Delta U_a + q_b \Delta U_b\right)$ is the marginal benefit of acquiring information, which the expected benefit from voting for the right alternative. Thus, if a player acquire information it will use the usual "magic formula": marginal costs=marginal benefits or they'll buy nothing if the marginal costs of acquiring zero quality information are higher the its benefits.

$$p\left(q_a \Delta U_a + q_b \Delta U_b\right) = C'\left(x\right).$$

The optimal choice of information is $x\left(p\right) = 0$ if $p\left(q_a \Delta U_a + q_b \Delta U_b\right) \leq C'\left(0\right)$. And $x\left(p\right) = \frac{1}{2}$ if $p\left(q_a \Delta U_a + q_b \Delta U_b\right) \geq C'\left(\frac{1}{2}\right)$. However when n is large $p \to 0$, thus the left hand side of 1 converges to 0 so it must do the same the right-hand side. In particular it follows that:

$$p\left(q_a \Delta U_a + q_b \Delta U_b\right) = C'\left(x\right).$$

As $p \to 0$, for n large....

Proposition 3 *An equilibrium with information acquisition exists only if* $C'\left(0\right) = 0$. *If it exists the rational ignorance hypothesis holds: the quality of information acquired by voters approaches* 0 *when* n *grows large.*

Observe that if $C'\left(0\right) = 0$ a solution to the equation

$$\binom{2n}{n}\left(\frac{1}{4} - x^2\right)^n \left(q_a \Delta U_a + q_b \Delta U_b\right) = C'\left(x\right). \tag{1}$$

always exists and is unique for n large. Call it x^*. If everybody acquire information of quality x^* and votes according the signal, then it is optimal to acquire information of quality x^*. Well, we have still to check that in this case is better than ignoring the signal and buying no information. Of course, once one acquire information it is always optimal to follow the signal.

With the same calculations as the previous section we can see that buying x^* and voting according to the signal is optimal if and only if

$$p\left(q_a \Delta U_a + q_b \Delta U_b\right) x^* - C\left(x^*\right) \geq p\frac{q_a \Delta U_a - q_b \Delta U_b}{2}.$$

Proposition 4 *When n is large enough, an equilibrium with information acquisition exists if and only if $q_a \Delta U_a - q_b \Delta U_b$. The quantity of information acquired by voters solves Equation 1*

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In order to simplify computations set $r = 2q_a \Delta U_a = 2q_b \Delta U_b$

Now, in order to check whether large elections aggregate information we must study the behavior of $\sqrt{n}x^* = \sqrt{n}x_n$. Notice that, for small x_n $C'(x_n) = C'(\xi_n)\, x_n$ where $\xi_n \in (0, x_n)$. Furthermore $\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}}$. Finally $\left(\frac{1}{4} - x_n^2\right)^n = 2^{-2n}(1 - 4x_n)^n$. As $x_n \to 0$, $(1 - 4x_n)^n \approx e^{-4nx_n^2}$. So from Equation 1we can derive the following equivalence for n large

$$\frac{2^{2n}}{\sqrt{\pi n}} 2^{-2n} e^{-4nx_n^2} r \approx C'(\xi_n)\, x_n$$

and simplifying

$$re^{-4nx_n^2} r \approx C'(\xi_n) \sqrt{\pi n} x_n$$

In particular, if when $n \to \infty$ $\sqrt{n}x_n \to l$, where l solves

$$re^{-4l^2} r \approx C'(0) \sqrt{\pi} l$$

if $C'(0) \neq 0$ and $\sqrt{n}x_n \to \infty$ otherwise. We have proved.

Proposition 5 *Let $C'(0) \neq 0$. The probability of reaching the right decision in large elections approaches $\Phi\left(2\sqrt{2}l\right)$ where l solves*

$$re^{-4l^2} r \approx C'(0) \sqrt{\pi} l.$$

If $C'(0) = 0$. The probability of reaching the right decision in large elections approaches 1.

Of course this happens only if $q_a \Delta U_a - q_b \Delta U_b$ and in SE with information acquisition. Still, it's better than nothing, election are more likely to get it right than wrong when people have incentive to acquire information.

6 Costly Information acquisition: heterogeneous costs.

This part is slightly more complex technically so we assume that that $q_a \Delta U_a = q_b \Delta U_b = 1$ and that $U(B \mid a) = U(A \mid b) = 0$.

A strategy must specify how much information a voter of a given type acquires and for which candidate she votes, conditionally on the signal on the signal received.

Definition 4 *A strategy for voter i consists of a information acquisition strategy $x : [0,1] \to \left[0, \frac{1}{2}\right]$ and of a voting strategy $v : [0,1] \times \{s_a, s_b\} \to \{A, B\}$ such that x is measurable and $v(\cdot, s)$ is measurable for $s \in \{s_a, s_b\}$.*[2]

A strategy of player i is denoted by (x_i, v_i), a strategy profile $(x_i, v_i)_{i=1,\dots,2n+1}$ is denoted by (X, V) and $(X, V)_{-i}$ is the coalitional strategy of all voter but i. Given $(X, V)_{-i}$, we denote by

$$U(v \mid \omega) = \sum_{d \in \{A,B\}} U(d, \omega) \Pr\left(d \mid \omega, v, (X, V)_{-i}\right)$$

the expected utility from voting v at state ω, net of information acquisition costs. The term $\Pr\left(d \mid \omega, v, (X, V)_{-i}\right)$ denotes the probability the outcome is d at state ω. Given investment choice x and after receiving signal $s \in \{s_a, s_b\}$, the expected utility from voting v is

$$U\left(v \mid x, s, (X, V)_{-i}\right) = \sum_{\omega \in \{A,B\}} U(v \mid \omega) Pr(\omega \mid x, s)$$

where $Pr(\omega \mid x, s)$ denotes the likelihood of ω given investment x and signal s.
The expected utility from a player investing x and using a voting rule from using a strategy (x, v) when other agents play $(X, V)_{-i}$ is

$$U\left(x, v \mid (X, V)_{-i}\right) = \sum_{s \in \{s_a, s_b\}} U\left(v \mid x, s, (X, V)_{-i}\right) p(s)$$

where $p(s)$ is the probability of receiving the signal s.

[2] A more rigorous definition would make the voting strategy contingent on the investment $v : [0,1] \times \left[0, \frac{1}{2}\right] \times \{s_a, s_b\} \to \{A, B\}$. However, only the signal is disclosed before the voting decision is taken so the definition presented is without loss of generality.

The equilibrium concept we employ is symmetric Bayesian equilibrium.

Definition 5 *A symmetric Bayesian equilibrium (SBE from now on) is given by a strategy* $(\hat{x}, \hat{v})$ *such that the profile* $\left(\hat{X},\hat{V}\right) = (\hat{x}, \hat{v})_{i=1,\dots 2n+1}$ *satisfies:*

1. $U\left(v\left(\hat{x}\left(\alpha\right), s\right) \mid \hat{x}\left(\alpha\right), s, \left(\hat{X},\hat{V}\right)_{-i}\right) \geq U\left(v \mid \hat{x}\left(\alpha\right), s, \left(\hat{X},\hat{V}\right)_{-i}\right)$ *for all* $\alpha \in [0,1]$, *for all* $v \in \{A, B\}$ *and for all* $s \in \{s_a, s_b\}$.
2. $U\left(\hat{x}\left(\alpha\right), \hat{v} \mid \left(\hat{X},\hat{V}\right)_{-i}\right) - C\left(\hat{x}\left(\alpha\right), \alpha\right) \geq U\left(x, v \mid \left(\hat{X},\hat{V}\right)_{-i}\right) - C\left(x, \alpha\right)$ *for all* $\alpha \in [0,1]$ *for all voting rules* v[3].

At a symmetric Bayesian equilibrium, every player employs the same strategy. Individuals vote optimally conditionally on the signal received, on the investment and on the strategy of the other electors (part 1). Information acquisition and voting strategies are ex ante optimal, given the strategy of the other electors (part 2).

Note that the issue here is not the existence of a SBE. An equilibrium always exists: no voter acquires information and everybody votes for the same alternative independently on the type and on the signal received. The concern of the paper are equilibria with information acquisition, where a non-zero measure of of types acquire information.

Definition 6 *A* SBE $(\hat{x}, \hat{v})$ *exhibits information acquisition* $F\left(\{\alpha : x\left(\alpha\right) > 0\}\right) > 0$.

When no ambiguity is possible we omit any reference to $(X, V)_{-i}$ so, for instance, we will write $U\left(x, v\right)$ for $U\left(x, v \mid (X, V)_{-i}\right)$.

6.0.1 Optimal Strategies and Equilibria: Characterization

A voter can affect the outcome of the election only when she is pivotal, which is when there are exactly n electors voting for alternative A and n electors voting for alternative B. We denote by $p_\omega = p\left(piv \mid \omega, (X, V)_{-i}\right)$, the probability a player is pivotal at state ω. Then the utility (net of information acquisition costs) that she

[3] A more rigorous definition would make the voting strategy contingent on the investment $v : [0,1] \times \left[0, \frac{1}{2}\right] \times \{s_a, s_b\} \rightarrow \{A, B\}$. However, no other information than the signal is disclose before the voting decision so the results are not affected.

derives from a voting strategy (v_a, v_b) is the sum of a term that depends on her strategy and a term that depend only on other elector strategy:

$$\sum_{\omega\in\{a,b\}} p_\omega q_\omega \left[U\left(v_a \mid \omega\right)p\left(s_a \mid \omega\right) + U\left(v_b \mid \omega\right)p\left(s_b \mid \omega\right)\right] + U_{-i}$$

where $U_{-i} = U_{-i}\left((X,V)_{-i}\right)$ is a term which is independent only on the strategy of the other electors. Note that this formulation implies that optimal strategies only depend on pivotal probabilities/

Let us start by studying the behavior of electors who acquire information. The reader can easily check that any voter who acquires information is strictly better off by following the signal (which is by voting (A,B)) rather than voting against it (which is by voting (B,A)). A voter who ignores the signal is strictly better off by not acquiring information. Formally, $U(x,A,B) > U(x,B,A)$, $U(0,A,A) > U(x,A,A)$ and $U(0,B,B) > U(x,B,B)$ for every $x > 0$.

The benefit from acquiring x units of information and following the signal $U(x,A,B)$ can be written as

$$U(x,A,B) = (p_a + p_b)\left(\frac{1}{2} + x\right) + p_a q_a U(B \mid a) + p_b q_b U(A \mid b) - C(\alpha, x) + U_{-i}.$$

Notice that the marginal gain of acquiring information is the expected gain from voting for the best alternative at each state:

$$p_a + p_b.$$

The optimal investment of type α, $x = x(\alpha, p_a, p_b)$ solves

$$(p_a + p_b) = C_x(\alpha, x) \tag{2}$$

whenever such x exists and $x = \frac{1}{2}$ if $(p_a + p_b) \geq C_x\left(\alpha, \frac{1}{2}\right)$.

Types who acquire information equate marginal gains from acquiring information and its marginal costs (Equation 2) or the acquire the best possible information ($x = \frac{1}{2}$) if its cost is lower than marginal gains.

If the rational ignorance hypothesis holds, $(p_a + p_b) \approx 0$ when there are many voters.[4] In this case only

[4]See Proposition 7 below

Equation 2 would be relevant for our purpose. Let $\alpha = \alpha(p_a, p_b)$ be the highest type who finds optimal to acquire information:

$$(p_a + p_b) - C_x(\alpha, 0) = 0 \tag{3}$$

if any such α exists and set $\alpha = \alpha(p_a, p_b) = 1$ otherwise. For types $\alpha \geq \alpha(p_a, p_b)$ it is never optimal to acquire information because the marginal costs from acquiring any unit of information are above its marginal gains.

Let $(p_a, p_b) \neq 0$ and $0 \leq \alpha < \alpha(p_a, p_b)$. A straightforward application of the implicit function theorem yields:

$$x_\alpha(\alpha, p_a, p_b) = -\frac{C_{x\alpha}(\alpha, x)}{C_{xx}(\alpha, x)}$$

$$x_{p_\omega}(\alpha, p_a, p_b) = \frac{q_\omega}{C_{xx}(\alpha, x(\alpha, p_a, p_b))} \text{ for } \omega = a, b$$

$$x_{\Delta U_\omega}(\alpha, p_a, p_b) = \frac{p_\omega q_\omega}{C_{xx}(\alpha, x(\alpha, p_a, p_b))} \text{ for } \omega = a, b$$

and $\lim_{(p_a, p_b) \to (0,0)} x(\alpha, p_a, p_b) = 0$ for every α. Types with lower costs acquire better information and an higher probability of being decisive provides higher incentives to acquire more precise information. Finally, the larger are the stakes the better is the information acquired by electors.

The function $\alpha(p_a, p_b)$ is differentiable in the interior of the set where (p_a, p_b) satisfies 2, with partial derivatives

$$\alpha_{p_\omega}(p_a, p_b) = \frac{q_\omega}{C_{x\alpha}(\alpha(p_a, p_b), 0)} \text{ for } \omega = a, b$$

$$\alpha_{\Delta U_{\omega b}}(p_a, p_b) = \frac{p_\omega q_\omega}{C_{x\alpha}(\alpha(p_a, p_b), 0)}.$$

Furthermore, $\lim_{(p_a, p_b) \to (0,0)} \alpha(p_a, p_b) = 0$. So, for p_a and p_b small, $\alpha(p_a, p_b) \in (0, 1)$. Higher pivotal probabilities and larger stakes implies that a larger fraction of the electorate acquires information .

In order to complete the characterization of optimal strategies we have to determine the voter who prefer to acquire information to vote uninformed. The payoffs from an uninformed elector who votes for alternative A

is:

$$\frac{p_a + p_b}{2} + \frac{p_a - p_b}{2} + +U_{-i}.$$

The payoffs from an uninformed elector who votes for alternative A is:

$$\frac{p_a + p_b}{2} + \frac{p_b - p_a}{2} + U_{-i}.$$

An uninformed elector prefers to vote for alternative A if and only if the expected gain from voting for the best alternative at state a exceeds the expected gain from voting for the best alternative at state b which is if and only if

$$p_a \geq p_b.$$

We look for symmetric equilibria and conjecture that $p_a = p_b$ so uninformed voters will be always indifferent among the two alternatives. Then, an elector of type α finds optimal to acquire information if and only if they payoff from informed voting is higher than the maximal payoff from uninformed voting which is if and only if:

$$\left(p_a q_a \Delta U_a + p_b q \Delta U_b\right) x\left(\alpha, p_a, p_b\right) - C\left(\alpha, x\left(\alpha, p_a, p_b\right)\right) \geq \frac{|p_a - p_b|}{2}.$$

However we know that $\left(p_a + p_b\right) = C_x\left(\alpha, x\right)$ so the inequality can be written as

$$C_x\left(\alpha, x\left(\alpha, p_a, p_b\right)\right) x\left(\alpha, p_a, p_b\right) - C\left(\alpha, x\left(\alpha, p_a, p_b\right) \geq \frac{|p_a - p_b|}{2}\right) \tag{4}$$

Which always holds because C is strictly convex in x (check class notes) if $p_a = p_b$.

Given $\left(p_a, p_b\right)$, every type $\alpha \leq \alpha\left(p_a, p_b\right)$ finds optimal to acquire the positive amount of information determined by Equation 4 and every type $\alpha > \alpha\left(p_a, p_b\right)$ does not acquire information. Every type $\alpha \in \left[0, \alpha\left(p_a, p_b\right)\right]$ votes for the correct alternative with probability $\frac{1}{2} + x\left(\alpha, p_a, p_b\right)$.

Define $\tilde{x}\left(p_a, p_b\right)$ as the the expected amount of information acquired by a voter, when the probability of being pivotal at states a and b are p_a and p_b, respectively:

$$\tilde{x}(p_a, p_b) = \int_0^{\alpha(p_a,p_b)} x(\alpha, p_a, p_b) f(\alpha) d\alpha.$$

Let $\lambda(\alpha, p_a, p_b) \in \{0, 1\}$ be the probability a voter of type $\alpha > \alpha(p_a, p_b)$ votes for A and set $\tilde{\lambda}(p_a, p_b) = \int_{\alpha(p_a,p_b)}^1 \lambda(\alpha) f(\alpha) d\alpha = \lambda(p_a, p_b)(1 - F(\alpha(p_a, p_b)))$ for some $\lambda(p_a, p_b) \in [0, 1]$. λ is the conditional probability a voter of unknown type votes for A, given that she does not acquire information. Finally set $\mu(p_a, p_b) = \lambda(p_a, p_b) - \frac{1}{2} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. If all uninformed voters vote for alternative A we have $\lambda(p_a, p_b) = 1$ and $\mu(p_a, p_b) = \frac{1}{2}$. If all uninformed voters vote for alternative B we have $\lambda(p_a, p_b) = 0$ and $\mu(p_a, p_b) = -\frac{1}{2}$. Thus, the probability a voter of unknown type votes for alternative A at state a is:

$$\frac{F(\alpha(p_a, p_b))}{2} + \tilde{x}(p_a, p_b) + \tilde{\lambda} = \frac{1}{2} + \tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha(p_a, p_b)))$$

and the probability she votes for alternative A at state b is:

$$\frac{F(\alpha(p_a, p_b))}{2} - \tilde{x}(p_a, p_b) + \tilde{\lambda} = \frac{1}{2} - \tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha(p_a, p_b))).$$

The probability a voter is pivotal at state a is

$$P_{a\mu}(p_a, p_b) = \binom{2n}{n} \left\{ \frac{1}{4} - [\tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha(p_a, p_b)))]^2 \right\}^n. \quad (5)$$

The probability a voter is pivotal at state b is

$$P_{b\mu}(p_a, p_b) = \binom{2n}{n} \left\{ \frac{1}{4} - [\tilde{x}(p_a, p_b) - \mu(p_a, p_b)(1 - F(\alpha(p_a, p_b)))]^2 \right\}^n. \quad (6)$$

This formulation reduce the problem of finding an equilibrium, in finding pivotal probabilities and a "wedge" μ such that (p_a, p_b) are a fixed point of the map $(p_a, p_b) \mapsto (P_{a\mu}(p_a, p_b), P_{b\mu}(p_a, p_b))$.
A *SBE* equilibrium without information acquisition corresponds to the case $(p_a, p_b) = (0, 0)$ and $\mu \in \left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

Proposition 6 *A SBE with information with $p_a = p_b$ acquisition equilibrium exists if and only if there* *are* $(p_a, p_b) \in [0,1]^2 \setminus \{(0,0)\}$ *and* $\mu \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ *such that* $(P_{a\mu}(p_a,p_b), P_{b\mu}(p_a,p_b)) = (p_a,p_b)$. *Equilibrium strategies are given by* (x,v), *where:*

1. $(x,v)(\alpha) = (x(\alpha, p_a, p_b, A, B))$ *for* $\alpha \leq \alpha(p_a,p_b)$,

2. $(x,v)(\alpha) = (0, A, A)$ *if* $\alpha^*(p_a,p_b) < \alpha \leq \tilde{\alpha}(p_a,p_b)$ *and* $p_b q_b \Delta U_b - p_a q_a \Delta U_a = 0$,

3. $(x,v)(\alpha) = (0, B, B)$ *if* $\tilde{\alpha}(p_a,p_b) < \alpha \leq 1$ *and* $p_b q_b \Delta U_b - p_a q_a \Delta U_a = 0$.

Notice that we consider only pure strategy, but an alternative interpretation is that at equilibrium informed players votes according the signal and all uninformed players randomize and vote for alternative A with the same probability $\mu(p_a,p) + \frac{1}{2}$, independently their type.

Given a SBE for an election with $2n+1$ electors let (p_{an}, p_{bn}, μ_n) identifying equilibrium pivotal probabilities and equilibrium wedge. Denote by $\alpha_n = \alpha(p_{an}, p_{bn})$ the cutoff type, by $x_n(\alpha) = x(\alpha, p_{an}, p_{bn})$ the equilibrium information acquisition strategy and by $\tilde{x}_n = \tilde{x}(p_{an}, p_{bn})$ the expected amount of information acquired by a voter.

6.0.2 The rational Ignorance hypothesis

Here, we investigate whether electors have incentives in acquiring information in large electorates, which is we analyze the validity of the so called rational ignorance hypothesis.
Note that

$$P_{a\mu}(p_a,p_b) \leq \binom{2n}{n} 2^{-2n}$$

for every μ and for every $(p_a,p_b) \in [0,1]^2$. From Stirling's Formula we have

$$\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}}$$

as $n \to \infty$, so that for $\omega = a, b$

$$P_{\omega\mu}(p_a,p_b) = O\left(\frac{1}{\sqrt{\pi n}}\right),$$

uniformly in (p_a, p_b, μ).

In any sequence of SBE the probability a voter is pivotal approaches zero when the population grows large. So even if the every equilibrium might exhibits information acquisition, only vanishing fraction of electors, the ones with the smallest costs, acquire information and this information is itself of vanishing quality.

Proposition 7 *For any sequence of SBE:*

1. $lim_{n\to\infty} p_{\omega n} = 0$, *for* $\omega = a, b$.
2. $lim_{n\to\infty} \alpha_n = 0$.
3. $lim_{n\to\infty} x_n(\alpha) = 0$ *for all* $\alpha \in [0,1]$.
4. $lim_{n\to\infty} \tilde{x}_n = 0$.

Proof. The first three claim has already been proved above. Let us consider claim 4. $\tilde{x}_n = \int_0^{\alpha_n} x_n(\alpha) f(\alpha) d\alpha \leq x_n(\alpha_n) \int_0^{\alpha_n} f(\alpha) d\alpha \leq \frac{1}{2} \int_0^{\alpha_n} f(\alpha) d\alpha \to 0$ as $n \to \infty$ from 2. ■

Thanks to Proposition 7 we can prove that the condition $C_x(0,0) = 0$ is necessary for a SBE with information acquisition to exist for large electorates. By contradiction assume $C_x(0,0) > 0$ and that a SBE with information acquisition exists. For large enough n, in an equilibrium with information acquisition type $\alpha = 0$ acquires a positive amount of information which satisfies: $(p_{an} q_a \Delta U_a + p_{bn} q_b \Delta U_b) = C_x(0, x_n(0))$. From Proposition 7 the left hand side of the the equation converges to 0 as $n \to \infty$ from Proposition 7, but the right hand side converges to $C_x(0,0) > 0$, a contradiction.

Corollary 2 *A SBE information exists for arbitrarily large* n *only if* $C_x(0,0) = 0$.

From now on we will consider cost functions C that satisfy $C_x(0,0) = 0$.

6.0.3 Informative equilibria: existence and aggregation.

We circumscribe our to the generic case where $C_{xx}(0,0) > 0$ and $C_{x\alpha}(0,0) > 0$. Let's look for an equilibrium with information acquisition where $p_a = p_b = p_n$. Given the symmetry of the problem it seems natural to assume that half of the uninformed voters vote for A and half for B. The guess is right... This way we are able to prove existence. Unfortunately such equilibria with information acquisition are not so good: in large elections elections are equally likely to get it wrong than right.

Theorem 1 *For n large an equilibrium with information acquisition exists. As the number the probability of taking the right decision converges to $\frac{1}{2}$ along every sequence of equilibria with information acquisition.*

Proof. For $p \in [0,1]$, let $\alpha(p)$ satisfying $2p = C_x(\alpha, 0)$ if any such α exists and $\alpha(p) = 1$ otherwise. Let the function $x(\alpha, p)$ be defined on satisfying $2p = C_x(\alpha, x)$ for every $\alpha \in [0, \alpha(p)]$ and $x(\alpha, p) = 0$ for $\alpha \in [\alpha(p), 1]$. Define:

$$T(p) = \binom{2n}{n}\left[\frac{1}{4} - \left(\int_0^{\alpha(p)} x(\alpha, p) f(\alpha)\, d\alpha\right)^2\right]^n$$

The function $T : [0,1] \to [0,1]$ is well defined and continuous so it has a fixed point. Let p^* be a fixed point of T. Note that $p^* \neq 0$ Because $T(0) = \binom{2n}{n}2^{-2n}$. Next, define $\tilde{\alpha}$ as the conditional median of the types who do not acquire information. Formally, $F(\tilde{\alpha}) - F(\hat{\alpha}(p^*)) = \int_{\alpha(p)}^{\tilde{\alpha}} f(\alpha)\, d\alpha = \frac{1-F(\hat{\alpha}(p^*))}{2}$. Consider the strategy (x, v), where $(x, v)(\alpha) = (x(\alpha, p^*), A, B)$ for $\alpha \leq \hat{\alpha}(p^*)$, $(x, v)(\alpha) = (0, A, A)$ for $\alpha(p^*) \leq \alpha < \tilde{\alpha}$ and $(x, v)(\alpha) = (0, B, B)$ for $\tilde{\alpha} < \alpha \leq 1$. It is easily seen that $(x, v)_i = (x, v)$ for $i = 1, \ldots 2n+1$ is a SBE with (p). As $p^* \neq 0$, (x, v), is an equilibrium with information acquisition. In such equilibria the probability of being pivotal at any state:

$$p = \binom{2n}{n}\left\{\frac{1}{4} - [\tilde{x}(p, p)]^2\right\}^n$$

Then, for $\alpha < \alpha(p)$:

$$2p = C_x(\alpha, x(\alpha, p)).$$

If $\alpha(p) < 1$

$$2p = C_x(\alpha(p), 0).$$

From the implicit function theorem

$$\alpha_p(p) = \frac{2pq_a}{C_{x\alpha}(\alpha(p), 0)}$$

and for $p \to 0$

$$\alpha(p) = \frac{2}{C_{x\alpha}(0,0)} p + o(p).$$

then $\alpha(p) = O(p)$ for $p \to 0$. Similarly

$$x_\alpha(\alpha, p) = -\frac{C_{x\alpha}(\alpha, x(\alpha, p))}{C_{xx}(\alpha, x(\alpha, p))}$$

and

$$x_p(\alpha, p) = \frac{2}{C_{xx}(\alpha, x(\alpha, p))}.$$

For $\alpha < \alpha(p)$, for some $\gamma \in (\alpha, \alpha(p))$

$$x(\alpha, p) = -x_\alpha(\gamma, p)(\alpha(p) - \alpha) + x_p(\gamma, p) p = \frac{C_{x\alpha}(\gamma, p)}{C_{xx}(\gamma, p)}(\alpha(p) - \alpha) + \frac{2q_a \Delta U_a}{C_{xx}(\gamma, p)} p.$$

It follows that for $p \to 0$

$$\widetilde{x}(p) = \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)} \int_0^{\alpha(p)} (\alpha(p) - \alpha) f(\alpha) d\alpha + \frac{2}{C_{xx}(0,0)} p \int_0^{\alpha(p)} f(\alpha) d\alpha + o\left(p^2\right)$$

$$\approx \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)} f(0) \frac{\alpha^2(p)}{2} + \frac{2}{C_{xx}(0,0)} F(\alpha(p)) \approx \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)} f(0) \frac{\alpha^2(p)}{2} + \frac{2}{C_{xx}(0,0)} f(0) p\alpha(p)$$

$$\approx \frac{6^2}{C_{xx}(0,0) C_{x\alpha}(0,0)} f(0) p^2.$$

Set $C = \frac{C_{xx}(0,0) C_{x\alpha}(0,0)}{6f(0)}$. Then

$$p^2 \approx C\widetilde{x}(p) \tag{7}$$

for $p \rightarrow 0$.

At the SBE when there are $2n+1$ players the probability of being pivotal is p_n where

$$p_n = \binom{2n}{n} \left(\frac{1}{4} - \tilde{x}_n^2 \right)^n$$

and $\tilde{x}_n = \widetilde{x}(p_n)$

As$\tilde{x}_n \rightarrow 0$, $\binom{2n}{n} \left(\frac{1}{4} - \tilde{x}_n^2 \right)^n \approx \frac{1}{\sqrt{\pi n}} e^{-4\tilde{x}_n^2}$. As $p_n^2 \approx C\widetilde{x}(p_n)$, then $p_n \approx \sqrt{C\tilde{x}_n}$. So the equivalence can be written as

$$\frac{1}{\sqrt{\pi n}} e^{-4\tilde{x}_n^2} \approx \sqrt{C\tilde{x}_n}$$

and multiplying both sides of the equivalence for $\sqrt{\sqrt{n}}$ we have:

$$\frac{1}{\sqrt{\pi \sqrt{n}}} e^{-4\tilde{x}_n^2} \approx \sqrt{C\sqrt{n}\tilde{x}_n}$$

The LHS converges to zero because $e^{-4\tilde{x}_n^2}$ is always bounded above. Then $\lim_{n\rightarrow\infty} \sqrt{n}\tilde{x}_n = 0$ and the claim follows applying Proposition 1.

■

7 Conclusions

When voters can acquire information of different qualities and have different information acquisition results large election fail to aggregate information, in general. This is consistent with the most pessimistic view of the rational ignorance hypothesis. Information aggregation is possible only under quite restrictive assumptions (see Triossi (2008)).

There are aspects not reflected here could have important implications. First of all, in our model information acquisition is independent among voters. It is not clear the impact of communication or correlation among different sources of information as it would introduce new strategic considerations.[5] Furthermore, the information and its cost are exogenously provided. Competition among information providers might reduce

[5]Gerardi and Yariv (2007) study a model of pre-voting communication, without information acquisition.

its costs and improve election efficiency. Also the possibility of abstention might affect our results. If less informed voters abstain the probability an informed voter is decisive increases and so the incentive to acquire information (see also Feddersen and Pesendorfer (1999)).

For some technical references on Stirling formula look any intermediate or advanced text in probability (like Chow and Teicher (1997)) or any text about special functions.

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