CHAPTER 58

MODELING PARTY COMPETITION IN GENERAL ELECTIONS

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1 INTRODUCTION

POPULAR elections are the central political act of democracies,¹ and citizens in all advanced democracies organize their political competition through parties that compete in general elections. While political historians have studied parties for many years, it is remarkable that only in the last decade or so have there been serious attempts at abstract conceptualizations—that is, formal models—of inter- and intraparty competition in a democracy. In this chapter, I will report on the attempts to model political equilibrium among parties and its applications. Indeed, it appears that a satisfactory model of interparty competition can only be constructed by paying careful attention to *intra*party competition between conflicting interests or factions.

In the advanced democracies, between 27 and 50 per cent of the gross national product is collected through taxation and disbursed by the state, and state policies are decided, ultimately, by popular elections. We no longer view the state as a benevolent social planner, which maximizes some social welfare function whose arguments are the utilities of its citizens; rather, in the new political economy, the state is pictured as

1 After William Riker 1982.

implementing the favored policies of whichever coalition of citizens manages to win control of it. (In one extreme view, that coalition could be the bureaucrats who run the state.) Thus, the theory of political competition should be, and is becoming in fact, a sub-field of public economics.

Furthermore, the issues with which the state deals are myriad, involving law, religion, language, and ethnic and racial conflict, as well as traditional economic issues of taxation and the provision of public goods. This means that any realistic theory of political competition must represent parties as taking positions in a *multidimensional policy space*.

Yet the most commonly used theory of political competition, of Harold Hotelling (1929), later elaborated by Anthony Downs (1957), with its principal result, the socalled median voter theorem, posits unidimensional political competition. Moreover, many believe that the Arrow impossibility theorem tells us that there can be no theory of multidimensional political competition—that there is no satisfactory procedure whereby citizens can aggregate their preferences to decide upon which multidimensional policy will be implemented. Our aim in this chapter is to rectify these Downsian and Arrovian pessimisms.

We will begin by introducing some notation, and then proceed to a review of the two main theories of political competition when it is assumed that the policy space is unidimensional. We will then note the problems involved in generalizing these theories to the multidimensional context, and propose a resolution to these problems, a theory of multidimensional political competition. Finally we will discuss some applications of this theory, and pose some open questions.

2 THE POLITICAL ENVIRONMENT

We model a polity as follows. There is a *policy space* T, a subset of some n-dimensional real space. There is a set of *voter types*, denoted H, which is a sample space endowed with a probability measure **F**. A voter of type h has preferences over the policy space represented by a utility function $v(\cdot; h)$, on T.

In the simplest economic application, we might think of h as describing a citizen's income or wealth and her preference for public goods, and T as a set of vectors each of which specifies some tax policy and supply of public goods. Given any tax policy t, the voter's after-tax income will be determined, as will be the supply of public goods, engendering a utility level for this citizen. v(t; h) is the utility citizen h enjoys at policy t; the function v is thus an indirect utility function, derived from the citizen's direct utility function over consumption of private and public goods.

Suppose the voters face two policies, t^1 and t^2 . The set of voter types who prefer the first policy to the second is denoted:

$$W(t^1, t^2) = \{h | v(t^1, h) > v(t^2, h)\}.$$

If everyone votes, then the fraction voting for t^1 should be $F(W(t^1, t^2))$, and if these are the only two policies in the election, then t^1 wins exactly when $F(W(t^1, t^2)) >$ 0.5.² In reality, however, the outcomes of elections are uncertain, because not everyone votes, not everyone is rational, random shocks may occur, and so on. We wish to capture this uncertainty in a simple way. We suppose that the fraction who will, in the event, vote for policy t^1 is $F(W(t^1, t^2)) + X$, where X is a random variable that is uniformly distributed on some interval $[-\delta, \delta]$, where δ is a (fairly small) positive number. Think of δ as the error term that newspapers report, when they say "We estimate that 53% will vote Democratic, but our forecast is subject to a 4% margin of error." Translation: $F(W(t^1, t^2)) = 0.53$ and $\delta = .04$. We can now compute the probability that t^1 will win the election; it is:

$$prob\left[F(W(t^{1}, t^{2}) + X > \frac{1}{2}\right] = prob\left[X > \frac{1}{2} - F(W(t^{1}, t^{2})\right] = \begin{cases} 0 \text{ if } F(W(t^{1}, t^{2})) \le \frac{1}{2} - \delta \\ \frac{\delta + F(W(t^{1}, t^{2})) - \frac{1}{2}}{2\delta}, \text{ if } \frac{1}{2} - \delta \le F(W(t^{1}, t^{2})) \le \frac{1}{2} + \delta \\ 1 \text{ if } F(W(t^{1}, t^{2})) \ge \frac{1}{2} + \delta \end{cases}$$
(1)

This formula is derived as follows. The fraction of the vote for policy t^1 can fall anywhere between $F(W(t^1, t^2)) - \delta$ and $F(W(t^1, t^2)) + \delta$, and it is uniformly distributed on this interval, by hypothesis. We simply compute the fraction of this interval that lies above 0.5; this produces formula (1). We denote the above probability by $\pi(t^1, t^2)$.

If we apply formula (1) to the newspaper report quoted above, then we see that the probability of Democratic victory is 0.875. When the fraction of voters voting for t^1 ranges from 49 to 57 per cent, 7/8 or 87.5 per cent of the time the fraction will be larger than 50 per cent.

Although I suggested that δ is a small number, note that it is really appropriate to measure uncertainty, from the parties' viewpoints, at the time that they announce their policies. The party manifestos, or the party conventions, typically take place months before the elections, when uncertainty may be substantial. Consequently, the appropriate δ could be fairly large; there could be at that time substantial uncertainty concerning the election outcome.

Because we wish to model large polities, where no type is of noticeable size in the entire population, the default assumption is that H is a continuum of types, and F is a continuous probability measure. Note that, even with a continuum of types, uncertainty in the outcome of voting does not disappear in our model. We assume that the random variable X applies, as defined above. The interpretation must be that the "misbehavior" of voters is correlated; it is not i.i.d. across voters. This may be because a scandal occurs in a campaign, which will cause some unpredictable fraction of voters to vote "against" their supposed preferences, or because one candidate is

² F is a probability measure, not a distribution function. Thus F(A) is the fraction of the polity whose type is in the set A.

more telegenic than another. In sum, it is reasonable that uncertainty concerning the outcome of the elections is produced by shocks that correlate deviations by voters from "rational" behavior in the same direction. So even when there is a very large number of voter types, and large numbers of voters in each type, uncertainty does not disappear. Because of uncertainty, it will sometimes be appropriate to assume that $v(\cdot; h)$ is a von Neumann–Morgenstern (vNM) utility function on the policy space.

3 UNIDIMENSIONAL POLITICAL COMPETITION

We now specialize to the case that *T* is an interval of real numbers: a unidimensional policy space. For example, *T* might be the interval [0, 1], and $t \in [0, 1]$ could be proportional income tax rate.

Suppose that the functions $\{v(\cdot; h)|h \in H\}$ are all *single peaked* on *T*: that is, each function has a unique local maximum on *T*, which is also its global maximum. Suppose there are two political candidates: each wishes to propose the policy that will maximize his probability of victory, given what the other candidate is proposing. In other words, if Candidate 2 proposes t^2 , then Candidate 1 will choose

t to maximize $\pi(t, t^2)$

and if Candidate 1 chooses t^1 then Candidate 2 will choose

t to maximize $1 - \pi(t^1, t)$

A Nash equilibrium in this game is a pair of policies (t^1, t^2) such that:

 t^1 solves $\max_t \pi(t, t^2)$ t^2 solves $\max_t (1 - \pi(t^1, t))$

If the functions $\{v(\cdot; h)|h \in H\}$ are single peaked, then Hotelling (1929) showed there is a unique such equilibrium: both candidates must play the policy that is the median in the set of ideal policies of all voters: that is, $t^1 = t^2 = t^*$, where t^* has the property that exactly one-half of the set of types has an ideal policy at least large as t^* and exactly one-half of the set of types has an ideal policy no larger than t^* .

Neither Hotelling nor Downs had uncertainty in the model, as we do, but the extension of the "median voter theorem" to our environment, with uncertainty, is immediate. Writing before Nash, Hotelling of course did not speak of Nash equilibrium. In fact, the Hotelling equilibrium is a dominant strategy equilibrium, a simpler concept than Nash equilibrium. However, when we introduce uncertainty, we must resort to the full power of Nash equilibrium to deduce the "median voter theorem."

There are two central problems with Hotelling–Downs equilibrium as a conceptualization of political competition: the first is its realism, the second is mathematical. The reality problem is that political parties, the soul of democracy, have not been modeled. In fact, as Downs tells the story, the two candidates are completely opportunist: they have no interest in policies per se, and use them only as vehicles for winning the election. To be precise, Downs *does* speak of parties, but his parties are evidently controlled completely by venal opportunistic politicians who have no accountability to constituents. He writes:

[Party members] act solely in order to obtain the income, prestige, and power which comes from being in office. Thus politicians in our model never seek office as a means of carrying out particular policies; their only goal is to reap the rewards of holding office per se... Upon this reasoning rests the fundamental hypothesis of our model: parties formulate policies in order to win elections, rather than win elections to formulate policies. (Downs 1957, 28)

Historically, however, parties are associated with particular ideologies—presumably the views, or preferences, of the coalition of citizens whom they, in some way, represent. So the Downsian model is missing something important—perhaps the essence—of democratic competition.

Indeed, it is interesting—and puzzling—to compare the development of general equilibrium theory and formal political equilibrium theory, with respect to the issue of agency. In the Arrow–Debreu model, no agency problem is mentioned: it is assumed that firms maximize profits, without any friction between owners/shareholders and managers. Not until the early 1970s did the principal–agent problem enter into *formal* economic theory—although, of course, Berle and Means (1932) had discussed the problem of ownership vs. control much earlier. In contrast, the first formal model of political competition, the Downs model, assumes that political parties are completely in control of the *agents*, the political entrepreneurs, who, somehow, completely escape supervision by their collective principal, the parties' constituents.

In Downsian equilibrium, both candidates play a *Condorcet winner* in the policy space, a policy that defeats or ties all other policies. Each candidate wins with probability one-half, if we assume that every voter casts her vote randomly, and the policies of both candidates are identical.

The mathematical problem I alluded to above is that Downsian equilibrium does not generalize to the case of a multidimensional policy space. If T is a subset of \mathbb{R}^2 or some higher-dimensional space, there is in general no Nash equilibrium (in pure strategies) of the game in which each politician has, as her payoff function, her probability-of-victory function. Only in a singular case, first observed by Plott (1967), will an interior Nash equilibrium in this game exist. (There may be a Nash equilibrium on the boundary of the policy space, if it is compact. See Roemer 2001, ch. 6 for details.)

Although historians and political scientists had (informally) studied parties with ideological commitments for many years, it appears that the first formal model of ideological parties was proposed by Donald Wittman (1973). In that model, each party has a (von Neumann–Morgenstern) utility function on policies, and seeks to maximize its expected utility, given the policy played by the opposition party. Given parties called A and B, with utility functions $v^A : T \rightarrow \mathbf{R}$, $v^B : T \rightarrow \mathbf{R}$, a Wittman

equilibrium is a pair of policies (t^A, t^B) such that

$$t^{A}$$
 solves $\max_{t} \pi(t, t^{B})v^{A}(t) + (1 - \pi(t, t^{B}))v^{A}(t^{B})$, and t^{B} solves $\max \pi(t^{A}, t)v^{B}(t^{A}) + (1 - \pi(t^{A}, t))v^{B}(t)$.

In other words, it is a Nash equilibrium of the game played by expected-utilitymaximizing parties, where utility depends on policy outcomes.

Perhaps the central weakness in Wittman's concept is that the parties' utility functions are exogenous, so the model is incomplete. To put it politically, parties do not *represent* citizens in the Wittman model. Ortuño-Ortin and Roemer (1998) remedied this as follows. For any partition of the set of types, $A \cup B = H$, $A \cap B = \emptyset$, define the utility functions

$$V^{A}(t) = \int_{h \in A} v^{h}(t) d\mathbf{F}(h), \quad V^{B}(t) = \int_{h \in B} v^{h}(t) d\mathbf{F}(h);$$

these are utility functions of two parties, should coalitions A and B form parties. We say that a partition (A, B) and a pair of policies (t^A, t^B) comprise an *endogenous*-party Wittman equilibrium (EPW) if

1.
$$(t^A, t^B)$$
 is a Wittman equilibrium for the utility functions (V^A, V^B) , and
 $h \in A \Rightarrow v^h(t^A) \ge v^h(t^B)$,

$$h \in B \Rightarrow v^h(t^B) \ge v^h(t^A)$$

Condition (1) says that each party maximizes the expected utility of an "average constituent" of the party, facing the policy of the other party. Condition (2) states that each citizen (weakly) prefers the policy of her own party to the policy of the other party. This condition means that each citizen will vote (modulo the uncertainty element) for the party that, by hypothesis, accepts him as a constituent.³ In other words, at an EPW equilibrium, the set of *voters* for a party comprise exactly its *constituency*, and the party represents its constituency in the sense of maximizing their average expected utility.

There is (to date) no simple proof of equilibrium existence for EPW equilibrium, as there is for Downs equilibrium. (There are some difficult proofs that are not completely general: e.g. see Roemer 2001, ch. 3.) The difficulty comes from the fact that even with the kind of simple specification of the probability function that we have given, the conditional payoff functions of the parties are not quasi-concave, and so the premisses of the usual fixed-point theorems do not hold. Still, in practice, it seems that EPW equilibria exist whenever one has a specific environment to work with.

The EPW equilibrium is a self-contained concept: given only the political environment defined in Section 2, equilibrium can be calculated. In this sense, the concept has the same informational standing as Downs equilibrium. Unlike Downs equilibrium,

³ Readers will note that for the integration of member utility functions, in constructing the party utility function, to be meaningful, member utility functions should be cardinally unit comparable. There are other ways of aggregating member preferences into party preferences which avoid this, but I will not discuss them here (see Roemer 2001, sec. 5.3).

parties play different policies (generically) in EPW equilibrium, and so the concept provides an escape from the tyranny of the median voter. It is also the case that, generically, parties do not win with probability one-half in EPW equilibrium: this, too, provides a realistic contrast to the Downsian prediction.⁴

Naturally, the EPW equilibrium concept is harder to work with than Downsian equilibrium: for applications that arise from particular economic environments, such as the determination of tax rates to finance public goods, it is usually easy to compute the EPW equilibrium (on a computer), but the comparative statics are often difficult to deduce analytically: one must resort to simulation. Political economists are in the habit of constructing politico-economic models that are quite complex on the economic side, and simplistic (that is, Downsian) on the political side. To replace the political module of these models with EPW equilibrium will often complicate the analysis substantially. I believe, however, that the extra effort is worth taking, because the EPW concept is the simplest model of *party competition* that we have. Of course, it formulates an ideal view of representation—every citizen "belongs" to, or is represented by, a party, and each citizen's influence on his party's utility function is equal. It is, however, a far better approximation to democratic reality than the Downs model.

I summarize one application, taken from Lee and Roemer (2005), to show the payoff of using EPW equilibrium in political economy. The polity consists of workers and capital owners. A worker's type is her real wage or skill level; the distribution of real wages is given. There is a trade union that represents all workers. Two political parties form endogenously, which jointly represent all citizens. In the equilibrium to be described, one party (the "left") represents all workers whose real wage is less than some endogenously determined value, and the other party ("right") represents all more skilled workers and all capital owners. A game will be played between the two parties and the union. The union's strategy is a mark-up on the Walrasian equilibrium real wage will be *sw.*) The mark-up determines the degree of unemployment, since firms choose their labor demand to maximize profits. The income tax rate, set by political competition, determines the size of government revenues, which are used to finance an unemployment benefit for those who cannot find work at the non-Walrasian wages.

An endogenous party Wittman equilibrium is, in this case:

- (a) a skill level s^* , defining two parties, L, consisting of all workers whose skill level is $s \leq s^*$, and R, consisting of all other workers and all capital owners;
- (b) payoff functions for the two parties and the union, defined on vectors (t^L, t^R, λ) , where t^J is the tax rate proposed by party J = L, R, and λ is the rate of unemployment, which can be viewed as the union's strategy choice. A party's payoff function is the average expected utility of its members, and the union's payoff function is the average expected utility of its members.

⁴ One reason that I have introduced uncertainty is that, under certainty, EPW equilibrium also consists in both parties proposing the same policy. So to escape the unrealistic prediction of the Downsian model, one must introduce *both* parties that care about policies, *and* uncertainty.

- (c) a Nash equilibrium $(t^{L^*}, t^{R^*}, \lambda^*)$ in the game played among the two parties and the union;⁵
- (d) each party member (weakly) prefers her party's policy to the opposition's, given the equilibrium unemployment rate and mark-up.

We compare the welfare of citizens, in this equilibrium, to their welfare in a fullemployment Walrasian equilibrium. This allows us to say something about why some societies have a highly unionized labor market, and some (such as the USA) one with much less union strength. We view the choice of "labor market regime" as made by citizens. If the majority of citizens fare better in the Walrasian equilibrium, we expect to have a quite unregulated labor market, whereas if the majority fare better in the union equilibrium described above, we expect to have highly regulated labor markets. The main result is with regard to a comparative static that alters the *degree of skill inequality* among workers: when that inequality coefficient is low or high, the majority of citizens prefer the unionized regime; when it has an intermediate value, the majority prefer the Walrasian equilibrium. Thus, the mapping from degree of *skill inequality* to choice of labor market regime is U shaped. We test for this result econometrically, and find support for it.

We also study the relationship between inequality and tax rates. A number of authors have studied this question, using the Downsian model (Alesina and Rodrik 1994; Persson and Tabellini 1994). In those models, increasing inequality of skill engenders increasing tax rates. There is, however, an extensive empirical literature arguing that this does not hold in reality (for example, Alvarez, Garrett, and Lange 1991). In our model, the result is more nuanced: we find that as inequality of skill increases among workers, the left party proposes higher tax rates, while the right party proposes *lower* tax rates. Not only do the two parties propose different tax rates (unlike the Downsian model), but their proposals move in different directions as inequality changes. We test this result econometrically, and find support for it.

Thus the feature of Wittman equilibrium, that parties generically propose different policies as long as there is some uncertainty, becomes important in explaining a "puzzle" in the empirical literature.

The Downsian model, in other words, mis-specifies the problem. We claim that an understanding of the relationship between taxation and inequality requires specifying, as well, whether the left or the right party holds power.

4 Multidimensional Generalizations

As I said earlier, multidimensional political competition is ubiquitous. And even if one is interested only in, say, tax policy, it would mis-specify the model to work with a

⁵ There are two forms of uncertainty represented in the payoff function: first, the uncertainty associated with a citizen's being unemployed or not, and second the uncertainty concerning the size of the tax rate and the unemployment benefit, deriving from electoral uncertainty.

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unidimensional policy space, because the positions of voters on *other* issues will affect the equilibrium in tax policy. As we will see, the preferences of voters on the religious issue or the race issue will significantly affect the equilibrium policies that emerge on economic issues. So a proper specification of political competition requires a theory where parties compete on multidimensional policy spaces.

Unfortunately, neither the Downs nor the Wittman model generalizes in what I think is a satisfactory way to multidimensional policy spaces.⁶ Wittman equilibrium, or EPW equilibrium, sometimes exists on multidimensional policy spaces but existence is undependable. Interested readers are referred to Roemer (2001, sec. 8.5) for the details. Besides crafting the "probabilistic voting" models referred to in the previous footnote, political scientists responded to the non-existence of Downsian equilibrium in the multidimensional environment in the following ways:

- mixed strategy equilibrium;
- sequential games;
- institutions;
- the uncovered set;
- cycling.

A quick summary: often a game without pure strategy equilibria possesses mixed strategy equilibria. But mixed strategy equilibrium is best justified by assuming that players do not know the types of other players. In our case, the players are political parties, which are public institutions. It is, I submit, not reasonable to say that parties do not know each other's preferences. In the sequential game approach, one party moves first, and the other second, giving a Stackelberg equilibrium. These often exist in multidimensional policy spaces. But I submit that it is more appropriate to model the game as one of simultaneous moves, and so I have not found the sequential-game approach to public elections convincing. Shepsle (1979) is associated with the view that political equilibrium exists, in multidimensional contexts, because institutions restrict the moves that players can make. Actually, Shepsle's model is one of legislative equilibrium, not general elections. In the legislative context, his approach is credible. But concerning general elections, one still faces the fact that parties seem to be playing a fairly straightforward game with two players and simultaneous moves. The uncovered set is a "cooperative" kind of solution concept; its logical foundations are suspect, and it is not strategic. The uncovered set always contains the Condorcet winner if one exists, so it is, mathematically, a generalization of Downsian equilibrium. (For critique, see Roemer 2001, sec. 8.1.) Finally, many political scientists took the nonexistence of multidimensional equilibrium in the known models to mean that in reality there was no equilibrium in the party competition game, and hence one should observe *cycling*: each party plays its best response to the previous move of the other party, and this generates a sequence of moves which end only with the election.

An equilibrium theorist, however, does not conclude that if her model fails to produce equilibrium, there *is* no equilibrium in the real world; this would be a last resort. Instead, she looks for another model. The failure of the Downs and Wittman models does not necessarily tell us something about the world, but rather, something about the models. For we *do* seem to observe equilibrium in real-world party competition.

In the last decade, two models have been offered that do produce political equilibrium with multidimensional policy spaces, in which parties propose different policies: the party-faction model of Roemer (1998, 1999, 2001), and the citizen-candidate model of Osborne and Slivinski (1996) and Besley and Coate (1997). I will spend most of the remaining space discussing the party-faction model, because it appears to be more realistic, easier to work with, and has more applications at present than the citizen-candidate model. I will discuss the citizen-candidate model only briefly.

The party-faction model is a generalization of both the Downsian model and the EPW model: it contains both of them as special cases. We assume, now, that the decision-makers in parties form factions. Each faction possesses its own payoff function in the game of party competition. Thus, as in the EPW model, let (A, B) be a partition of the space of types: $A \cup B = H$. $A \cap B = \emptyset$. As before, we define the average utility functions of these two coalitions:

$$V^{A}(t) = \int_{h \in A} v(t;h) d\mathbf{F}(h), \quad V^{B}(t) = \int_{h \in B} v(t;h) d\mathbf{F}(h).$$

The first faction in party A are the Opportunists; as in the Downsian model, they wish only to maximize the probability of their party's victory against party B. Thus, the payoff function of the Opportunists in A is:

$$^{Opp}\Pi^{A}(t^{A}, t^{B}) = \pi(t^{A}, t^{B}).$$
(2)

The second faction in *A* are the Reformists: they are the characters of the Wittman model, who wish to maximize the expected utility of the average party member. Thus, their payoff function is:

$${}^{Ref}\Pi^{A}(t^{A}, t^{B}) = \pi(t^{A}, t^{B})V^{A}(t^{A}) + (1 - \pi(t^{A}, t^{B}))V^{A}(t^{B}).$$
(3)

The third faction in *A* are the Militants (or the Guardians): they are concerned with ideology only, and want to play a policy as close as possible to the ideal policy of the "average" party member. Their payoff function is:

$$^{Mil}\Pi^A(t^A, t^B) = V^A(t^A).$$

$$\tag{4}$$

In like manner, party *B* has the analogous three factions.

Party factions are not to be associated with particular voter types. The factions are formed by professional party activists, and are small relative to the size of the population.

⁶ Two very similar models of multidimensional Downs equilibrium were indeed proposed by Lindbeck and Weibull 1987 and Enelow and Hinich 1989. Coughlin 1992 also proposed a model of this type. Existence is secured by having voters behave probabilistically, in a way which "convexifies" the conditional payoff functions of the Downsian parties. Uncertainty exists about electoral outcomes, but only when the set of voters is finite. Moreover, the equilibria have both parties playing the same policy, an unrealistic prediction that we wish to avoid.

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The idea is that, while parties *compete* with each other strategically, factions within parties *bargain* with each other over policy. I state the equilibrium concept and then explain it:

A partition of types (A, B) and a pair of policies (t^A, t^B) comprise a partyunanimity Nash equilibrium (PUNE) if:

- 1. Given the policy t^B , there is no policy t that all three factions of party A would prefer to play, instead of t^A ;
- 2. Given the policy t^A there is no policy t that all three factions of party Bwould prefer to play, instead of t^B ;
- 3. Every member of each party (weakly) prefers the policy of his party to the policy of the other party.

The phrase "that all three factions would prefer to play" is shorthand for: "that all three factions would weakly prefer to play and at least one would prefer to play."

Requirement (1) means that, given policy t^{B} , policy t^{A} is Pareto optimal for the three factions in A: there is no policy choice that would increase all their payoffs. We can thus think of t^{A} as the outcome of *efficient bargaining* among the factions of A. when facing t^{B} . In like manner, (2) means that policy t^{B} is the outcome of efficient bargaining among the factions of B, when facing t^{A} .

There is much historical evidence to justify the choice of these factions. One could quibble, and define other factions. These three, however, seem fairly canonical. It is the Militants who seem the most surprising. Yet there are many examples of Militants in history. The Militants' strategy seems to be to use the elections as a platform for advertising the party's preferences—perhaps with an eye to changing the preferences of voters for future elections.

The interesting fact is that the Reformists are expendable (or gratuitous) in this equilibrium concept: that is to say, *we get exactly the same set of equilibria* if only the Opportunist and Militant factions are active in the parties. The Reformists are, in an appropriate mathematical sense, just a convex combination of the Opportunists and the Militants.

Although there is no satisfactory *general* existence theorem (as in the case with endogenous-party Wittman equilibrium), in all applications that I have studied on multidimensional policy spaces, PUNEs exist. Moreover, there is a two-dimensional manifold (set) of equilibria. We can understand this as follows.

It turns out (see Roemer 2001, sec. 8.3) that the bargaining that takes place in the intraparty faction struggle can be represented as generalized Nash bargaining, when appropriate convexity properties hold. Take the threat point of the intraparty bargaining game in our party to be the bad situation that the opposition party wins for sure, because our party does not succeed in solving its bargaining problem and defaults. In generalized Nash bargaining, the bargainers maximize the product, raised to some power, of their utility gains from the threat point. Thus, for the Militants and Opportunists in *A*, this means:

$$\max_{t \in T} [\pi(t, t^B) - 0]^{\alpha} [V^A(t) - V^A(t^B)]^{1-\alpha}.$$
(5)

Party *B*'s factions do the same thing. So I am claiming that a PUNE can be expressed as a pair of policies (t^A, t^B) such that

$$t^{A} = \arg\max_{t \in T} [\pi(t, t^{B})]^{a} [V^{A}(t) - V^{A}(t^{B})]^{1-a},$$

$$t^{B} = \arg\max_{t \in T} [1 - \pi(t^{A}, t)]^{\beta} [V^{B}(t) - V^{B}(t^{A})]^{1-\beta}$$

for *some* numbers α . β in [0,1].

In words, recall that, if party A fails to propose a policy, then its probability of victory is zero, and the utility of its average constituent will be $V^A(t^B)$ since party B will win for sure. Thus expression (5) states that bargaining maximizes the weighted product of the "utility" gains from the threat point of the Opportunist and Militant factions. This, as I said earlier, is the upshot of the Nash bargaining game.

We call $a(\beta)$ the *relative strength* of the Opportunists in party A (resp., B). Now if such a pair of policies exists for a particular pair of numbers (a, β) then the implicit function theorem tells us (generically) that there will exist solutions for all values of the relative strengths in a small neighborhood of (a, β) . This describes the twodimensional manifold of PUNEs: each equilibrium is indexed by a pair of relative strengths of the factions in the intraparty bargaining game.

In other words, if we wanted to specify a particular pair of relative strengths of the factions in the two parties as a datum of the problem, we would have a unique equilibrium. The problem is that, *we cannot be guaranteed* that an equilibrium will exist with any *pre-specified* pair of bargaining strengths.

In fact, it is easy to deduce that an endogenous party Wittman equilibrium is a PUNE where $a = \beta = \frac{1}{2}$, in other words, a PUNE where the Opportunists and Militants *have equal strengths*. This is a nice characterization of Wittman equilibrium—indeed, one that applies as well in the unidimensional model. Unfortunately, there is no guarantee that a PUNE with *this* pair of relative strengths exists when the policy space is multidimensional. For some environments it does, and for others it does not.

Hence, PUNE is a generalization of Wittman equilibrium. It is also a generalization of a Downs equilibrium: set $a = \beta = 1$ for Downs equilibrium. We know, however, that this equilibrium rarely exists.

Here is a second story that gives rise to exactly the same equilibrium concept.⁷ Each party has two factions, the Opportunists and the Guardians. The Opportunists are as above; the Guardians insist that, whatever the Opportunists do, they (Guardians) will not accept a policy that would give their party's constituents, on average, too low a utility. Thus, we can express the bargaining problem in party *A* as follows:

$$\max_{\substack{t \in T \\ s \in L}} \pi(t, t^{B}) = k^{A}$$
(6)

The bigger the number k^A , the tougher are the Guardians. In like manner, party *B*'s bargaining problem is characterized by a number k^B . It is easy to see that there is a

⁷ It was remarked by Gérard Debreu that formal models often, virtuously, support several interpretations of reality. Here is a case in point.

2-manifold of equilibria of this game, indexed by pairs of numbers (k^A, k^B) , and that this manifold is identical to the PUNE manifold.⁸

Therefore we have the freedom to conceptualize the "tough" guys in party bargaining as either Militants (who use the party as a platform to advertise) or Guardians (who hold the fort in the interest of constituents). Perhaps the Guardian story is more appealing.

As I said, I have no suitably general existence theorem for PUNE: all I can say is that in many applications that I have studied, PUNEs exist. The intuition for existence is that it is much harder to find a successful deviation to a proposal in the PUNE game than in the Wittman or Downs game. To deviate, two payoff functions must be satisfied—and the Militants and Opportunists have sufficiently "orthogonal" preferences that that is often hard to do. So many pairs of policies survive the deviation test necessary to qualify as a Nash equilibrium.

I now briefly describe citizen-candidate equilibrium, which is the second equilibrium concept that survives the generalization to multidimensional policy spaces and produces differentiated policies at equilibrium. We begin with the same data (H, F, v, T), which define the environment. Each citizen now considers whether or not to stand for election. If a citizen enters the contest, she pays a cost, and if she wins, she enjoys a benefit from holding office, as well as deriving utility from implementing the policy upon which she ran. It is assumed that, if a candidate stands for election, she must announce her ideal policy; to do otherwise would not be credible in this one-shot game. An equilibrium consists of a set of citizens each of whom enters the race, and each announces her ideal policy. Once the policies have been announced, we can compute the coalitions of citizens that will vote for each candidate, absent uncertainty. We can then append an element of uncertainty as we have above. The equilibrium is a Nash equilibrium; to be so, it must satisfy two tests. First, each candidate must not have higher expected utility, should he decide not to run. (Under that deviation, he does not have to pay the cost of running, but forfeits the expected gain from winning.) Secondly, each non-candidate must not have higher utility should she throw her hat into the ring. The model generally possesses pure-strategy equilibria with a small set of candidates, even when the policy space is multidimensional. Thus, both limitations of the Downs model are overcome, because candidates are explicitly "ideological," as well as caring about the spoils of office, and equilibria exist.

I see two problems with the model. First, it is not a model of party competition, and so ignores the central institutions of democratic political competition. Second, the element of compromise in political competition is ignored, in the sense that each candidate proposes her ideal policy. The justification of this move is that the game is one-shot, and candidates cannot commit themselves to do otherwise. This strikes me as unrealistic, even if it is logically consistent within the framework of a one-shot game. In the PUNE model, parties do compromise, although we ignore the credibility of their proposing non-ideal policies in a one-shot game. (There is, indeed, a (locally) unique equilibrium in the PUNE model where both parties play the ideal point of their average member, but I consider this to be an uninteresting equilibrium.)

5 APPLICATIONS

Two of the virtues of the PUNE model are that it is often possible to derive interesting analytical results in specific applications, and it is possible to estimate the model econometrically, which enables one to conduct policy experiments for specific polities. In this section I present four applications of PUNE.

5.1 Progressive Taxation

We observe that, in all advanced democracies, income taxation is progressive, in the sense that marginal tax rates rise with income. Why is this so? A standard answer has been that progressive taxation seems fair. Many, however, would consider this explanation not to be parsimonious: it would be better to have a completely "political" explanation, one that did not presuppose any assumption that citizens are motivated by a sense of justice or fairness. Thus, one can ask, will the income tax proposals that survive in cut-throat democratic competition be progressive ones? An early discussion of this problem is due to Kramer and Snyder (1988), which takes a Downsian approach, and places an ad hoc assumption on the nature of the policy space in order to produce equilibria. The unidimensional Downsian and Wittman models are ill equipped to answer this question. The standard unidimensional policy space of tax regimes consists of the set of affine income tax functions, characterized by a constant marginal tax rate (in the interval [0, 1]) and a lump-sum transfer to all, financed by that tax rate. None of these tax regimes have increasing marginal tax rates. Now one could work with a unidimensional policy space constructed to possess both convex and concave tax functions, but the unidimensional restriction is really too constraining. The ideal model is one that poses a space of tax policies that is genuinely multidimensional, and contains both progressive and regressive tax functions.

In Roemer (1999), the tax-function space contains all quadratic income tax functions, constrained to require that no citizen pay a tax greater than her income, and that after-tax income be non-decreasing in pre-tax income (an incentive compatibility constraint). This is a two-dimensional policy space. We posit a distribution of income-earning capacities (wages). Citizens desire only to maximize their after-tax income—they have no desire for leisure—and so everyone works at his full capacity. The income tax is purely redistributive (no public goods). We study the two-party PUNEs of this model. It is shown that, if the median income is less than mean income,

⁸ One can check the claim that the two stories engender the same equilibria by noting that the first-order conditions for the solution of (5) and (6) are equivalent. Of course, the same holds for the corresponding FOCs for the *B* party.

then in *every* PUNE,⁹ the probability that a progressive tax scheme wins the election is unity. (In other words, either both parties propose progressive schemes, or if not, the one proposing a regressive scheme wins with probability zero.) Here, then, is a completely "positive" explanation of the ubiquity of progressive taxation.¹⁰

5.2 The Effect of Non-economic Issues on Taxation

In the introduction, I wrote that a central reason to model political competition as multidimensional is that apparently non-economic issues can affect political outcomes on economic issues. Suppose that the electorate is concerned with two issues, taxation and religion. (Religion is a place-holder for many other issues, of course.) Thus, voters have preferences over the tax policy and the religious policy of the state, and parties compete on this policy space. To be specific, let us suppose that a voter's type is a pair (w, ρ) , a policy is a pair (t, r), mean income is μ , and the voter's utility function is:

$$v(t, r; w, \rho) = (1 - t)w + t\mu - a(r - \rho)^2.$$
⁽⁷⁾

Thus, w is this voter's income, ρ is the voter's religious position, t is an affine income tax which distributes the lump-sum $t\mu$ to all citizens, and the voter's preferences over the religious issue are Euclidean (she suffers a quadratic loss as the state's policy becomes further away from her religious view). We call a the *salience* of the religious issue, which is here assumed to be the same for all citizens.

Here the space of types is two dimensional, as is the policy space. Given a distribution of types F, the environment is complete, and we can study the two-party PUNEs. The question is: when do citizens' views on the religious issue affect the equilibrium tax rates proposed by the parties in PUNEs?

If a = 0, this model reduces to a unidimensional model on tax policies, and it is not hard to show that in the endogenous-party Wittman equilibrium, the two parties consist of the "poor" and the "rich," and they propose tax rates of one and zero, respectively. This is the benchmark. We can ask: is it ever the case that, when *a* is positive, *both* parties propose a tax rate of zero (or a tax rate of one)? That would show that religious views can have an extreme effect on economic policy.

The answer is there is such a case. Suppose the following condition on the distribution F holds:

Condition A. The mean income of the cohort of voters who hold the median religious view is greater than mean income in the population as a whole.

Then it can be shown (see Roemer 1998, 2001) that if α is sufficiently large, and if uncertainty is sufficiently small, then in *all* PUNEs, both parties propose a tax rate of

zero! Correspondingly, if we change "greater" to "less" in the statement of Condition A, then the conclusion is that, in all PUNEs, both parties propose a tax rate of one.

An intuition behind this result is as follows. As a gets large, the model approaches one where political competition is unidimensional, and the only policy is the religious issue. If uncertainty is small, then in such competition, both parties will propose policies close to the ideal policy of the voter(s) with the median religious view. But if this cohort of voters has income greater than the mean, on average, then they want zero taxation. Conversely, if this cohort has mean income less than the mean, they want a tax rate of one.

The substantial result is that convergence of both parties to proposing a tax rate of zero, if Condition A holds, happens at finite α and with a positive degree of uncertainty.

More generally, the comparative static is that as α increases, the tax rates proposed in PUNEs fall. In other words, we should see economic policy moving to the right (left), as the salience of the religious increases, if Condition A (resp., its negation) holds.

The applications of this result seem myriad. The religious issue could be nationalism, racism, language policy, civil rights, etc. In Section 5.4 below, I discuss an application where the second issue is "racial policy" in the USA.

5.3 The Flypaper Effect

It has been noted by many authors that an increase in the wealth of a community by one unit engenders a smaller increase in the level of locally financed public goods than an increase by one unit of a federal grant to the community engenders: Hines and Thaler (1995) find that a federal grant increases the financing of public goods by about \$637 per thousand dollars of the grant, a substantially greater increment than occurs with an increase in the community's average wealth by an equivalent amount. This has been dubbed the flypaper effect. Many authors have viewed it as an anomaly, because if the community is assumed to be composed of homogeneous citizens, the increase in the supply of the public good should be identical in the two cases.

However, if the community is heterogeneous in income, but homogeneous in preferences over income and the public good, the flypaper effect is predicted theoretically.

How should one model the political problem here? There are two things to be decided: the tax policy and the value of the public good. This can be done on a unidimensional policy space, if one restricts taxation to be proportional to income, and finances the public good from the tax revenues. So a Downsian formulation is possible. Indeed, with a Downsian formulation, we do predict the flypaper effect, with heterogeneous incomes.

But proportional taxation is unusual. More realistically, tax policy is affine—a constant marginal tax rate and a transfer payment to all citizens. Thus, here we have, naturally, a two-dimensional policy space: three variables must be chosen—the income tax rate, the lump sum transfer payment to all citizens, and the value of

⁹ There is, as usual, a 2-manifold of PUNEs.

¹⁰ An extension for future research would be to study this problem on a small-dimensional space of piecewise linear tax functions, which are prevalent in reality. The problem of characterizing PUNEs on such a space is much harder than on the space of quadratic functions.

the public good. The budget constraint states that tax revenues must equal the sum of transfers and the public good, so the policy space is two dimensional.

Roemer and Silvestre (2002) model the problem using PUNE. We parameterize the model to the US income distribution, and choose some reasonable values for the parameters of the utility function, which determine the relative preference of citizens for private income and the public good. We compute PUNEs for three economies:

- E1. An economy at date zero, with a given distribution of income;
- E2. An economy at date one, with the same distribution of income and a external subsidy of \$1,000 per capita;
- E3. An economy at date one, with the distribution of income whose mean is \$1,000 more than in E1, and no external subsidy.

Each PUNE consists of two policy proposals (by the two parties) and the probability of left victory. We take the expected expenditure on public goods as the value to examine. There is a 2-manifold of PUNEs: we take the average of the expected expenditures on public goods over this manifold. We find that the political equilibria in E2 have expected expenditures on public goods that are \$635 higher than the political equilibria in E1: this is almost exactly the average found in the Hines and Thaler (1995) studies. It is substantially more than the increase in expected expenditures in the move from E1 to E3, which is \$157.

5.4 The Effect of Racism on Redistribution in the USA

In Lee and Roemer (2006), we take the "religious" issue of Section 5.2 above to be the race issue in the United States. We fit a model of citizen preferences to the US polity, and attempt to compute the effect of racism in the electorate on the degree of redistribution that takes place through income tax policy, where the policy space is two dimensional, representing income taxation and the position of the party on the race question. We fit the model to the data for every presidential election in the period 1976–92, achieving an excellent fit. We then conduct counterfactual experiments, asking what the equilibrium would be on the tax rate dimension, if the degree of voter racism should decline. (The distribution of voter racism is estimated from the American National Election Studies.) The punchline is that (we predict) the marginal tax rate would increase by at least ten points, were American voters not racist, making the US fiscal system much closer in size to that of the northern European democracies.

6 CONCLUSION

We have argued that in modern democracies, an understanding of the apparatus of political competition, whereby citizens with divergent interests organize to battle for control of state policy, is of the highest importance. In this chapter, we have discussed only one of the several arenas of political competition: general popular elections. Indeed, contemporary practice lags reality: the vast majority of scholarly papers in political economy model political competition using the Hotelling–Downs apparatus, one which predicts that, in two-party competition, both parties propose the same policy. Were this indeed the case, it is hard to understand how parties would finance themselves: what motivation would the rational citizen have to contribute to one party over another in such a situation? Moreover, the Hotelling–Downs model is incapable of describing political competition which is complex, in the sense of taking place over several issues. All general elections are concerned with a multitude of issues.

We argued that a variation on Wittman's model provides a superior description of reality to Hotelling–Downs in the unidimensional context. The basic data of a political environment—preferences of citizens and the policy space—determine a partition of citizens into two parties, an equilibrium pair of policy proposals, and a probability that each party wins the election. The model can be estimated and its predictions tested.

Neither the Hotelling-Downs model nor the endogenous-party Wittman model generally possess equilibria, however, when the policy space is multidimensional. We proposed that the way to solve this problem is not to complexify the concept of Nash equilibrium (to a stage game, for instance) but rather to articulate further the conception of what a party is. Parties are, in reality, complex institutions, and they are the soul of modern democracy: hence, good modeling impels us to think carefully about what parties are. We proposed to think of the decision-makers in parties as forming factions, with different concerns: Opportunists, Reformists, and Militants or Guardians. Interparty competition is strategic, in the sense of Nash equilibrium; intraparty competition is "cooperative" in the sense of Nash bargaining among factions. (Thus our PUNE can be thought of as a "Nash-Nash" equilibrium.) Formally, the PUNE is a generalization of both Hotelling-Downs and endogenousparty Wittman equilibrium, but unlike those two special cases, PUNEs exist with multidimensional policy spaces. We argued that interesting analytical results can be derived about PUNE in specific applications, and moreover, the model can be fit to data, in order to study policy and comparative statics for actual political economies.

The models described here have all been ones of *perfectly representative democracy*. In the PUNE, every citizen is a member (constituent) of one party, and each party aggregates the preferences of its constituent types according to their population sizes. This is an ideal type of party behavior. In the USA, where private financing of parties is the norm, one might expect that parties would represent their *contributors* according to their *contributions*, rather than their constituents according to their numbers. The models of this chapter can be generalized to study that kind of imperfectly representative democracy (see Roemer 2006).

Moreover, we have stayed with the assumption of two parties. The citizencandidate model allows the number of candidates to be endogenous: however, there are many equilibria so that it can hardly be said to have *determined* the number of candidates. PUNE can be generalized to deal with more than two parties. But it must be said that models with more than two parties are inherently more complex, because the natural political game then has two stages: first, an election, and second, the formation of a government among a set of parties that comprise a majority coalition.¹¹ That coalition-formation process must be modeled, and then the citizen-voter must take into account the nature of that process when she votes. There is no conceptual problem in using the PUNE concept to study multidimensional political competition with several parties: the main conceptual issue, about which disagreement among political scientists persists, is the nature of the coalition formation process in the second stage.

Many open questions are posed by the factional approach to party competition. What are the microfoundations of the formation of the particular factions I have presumed to exist? Do voters form factions? How do candidates emerge from factional bargaining? Can we formulate a theory of how the results of primary elections influence the bargaining powers of factions? More generally, how can one endogenize the relative bargaining powers of the factions? In a federal system, one might conceive of factions in national parties as representing different regional interests. This, too, would suffice to provide existence of equilibria in multidimensional competition, as long as the regional interests were suitably different.

Finally, to return to a point alluded to much earlier, how does the Arrow impossibility theorem fit into all this? To see, we must first formulate the political environments described here as Arrovian environments. Thus, let the set of *social alternatives* be lotteries whose elements are policy pairs taken from the given policy space. A *profile* is a function $\{v(\cdot; h)|h \in H\}$ where h is distributed according to F. A *social choice function* maps a profile into orderings of social alternatives. We could take the ordering of lotteries associated with a given profile to be as follows: all lotteries that are engendered by PUNEs are socially indifferent, and all other lotteries are socially indifferent, and inferior to the ones generated by PUNEs. This social choice function violates the Arrow postulates as follows:

- it is not defined on *preferences* but on *utility functions*, which must be cardinally unit comparable (or else adding up [integrating] members' utilities to form the party's utility function makes no sense);
- it is not Pareto efficient (in fact, each party proposes, in a PUNE, a policy that *is* a Pareto efficient social alternative, but the *lottery* between these policies, engendered because of uncertainty, might not be Pareto efficient, because of risk aversion);
- the axiom of binary independence of alternatives fails.

In the *modified* Arrovian framework, where utility functions are cardinally unit comparable, the unique social choice function to satisfy the Arrovian axioms is utilitarianism:¹² but certainly the PUNE is not the utilitarian rule.

¹¹ Reality is still more complex. There are times when governments are formed by coalitions that together won less than one-half of the votes.

Does this mean that political equilibrium, as we have described it in this chapter, is not a legitimate way for a society to aggregate its members' preferences? Hardly; it means the Arrovian framework is not the right abstraction to capture the nature of political competition. (Let me simply note that if Nash equilibrium is involved in political competition, we cannot expect outcomes to be Pareto efficient, immediately violating an Arrovian axiom.) Although it is *desirable* to have Pareto efficient outcomes, that might not be compatible with democratic competition.

To put the same point somewhat differently, defining the set of feasible allocations for a society in the classical way is an apolitical approach. Why should some allocations be "feasible" if there are no political institutions that could bring them about? The same point has been made with regard to asymmetric information: Why should an allocation be regarded as "feasible"¹⁵ if asymmetric information makes it impossible for it ever to be brought about? The constraint of asymmetric information is just as real as a technological constraint; similarly, a complex society must have politics, and it is therefore myopic to conceive of feasibility apolitically.

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¹³ For instance, one achieved through certain kinds of lump-sum taxation.

¹² See d'Aspremont and Gevers 1977, theorem 3; also Roemer 1996, theorem 1.4.

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