IN759
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Porfesor: Benjamín Villena

Auxiliares: Andrés Barrera, Sergio Salgado.

P1 Money-in-the-Utility Function

a) Considere el modelo de dinero en la función de utilidad visto en clases, encuentre que la tasa marginal de sustitución entre el dinero y el consumo es $i_t/(1+i_t)$.

b) Ahora asuma que el dinero que se tiene en el periodo t da utilidad en $t+1$, es decir, la función de utilidad viene dada por $\sum \beta^i u\left(c_{t+i}, \frac{M_{t+i}}{P_{t+i}}\right)$, pero la restricción presupuestaria es $\omega_t = c_t + \frac{M_{t+1}}{P_t} + b_t + k_t$. Además la riqueza del individuo toma la forma de

$$\omega_t = f(k_{t-1}) + (1-\delta)k_{t-1} + (1+r_{t-1})b_{t-1} + m_t$$

Encuentre la nueva tasa marginal de sustitución entre el dinero y el consumo.

Sol

La función de valor es

$$V(\omega_t) = \max\{u(c_t, m_t) + \beta V(\omega_{t+1})\}$$

Sujeto a

$$\omega_{t+1} = \frac{f(k_t)}{1+n} + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)k_t + \frac{(1+i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)}$$

$$k_t = \omega_t - c_t - m_t - b_t$$

Reemplazando

$$V(\omega_t) = \max\left\{u(c_t, m_t) + \beta V\left(\frac{f(\omega_t - c_t - m_t - b_t)}{1+n} + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)(\omega_t - c_t - m_t - b_t) + \frac{(1+i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)}\right)\right\}$$

Las CPO son

Para c

$$u_c(c_t, m_t) - \beta\left[\frac{f_k(k_t) + 1 - \delta}{1+n}\right]V_\omega(\omega_{t+1}) = 0$$

Para b_t

$$\frac{1+i_t}{(1+\pi_{t+1})(1+n)} - \left[\frac{f_k(k_t) + 1 - \delta}{1+n}\right] = 0$$

Para m_t

$$u_m(c_t, m_t) - \beta\left[\frac{f_k(k_t) + 1 - \delta}{1+n}\right]V_\omega(\omega_{t+1}) + \frac{\beta V_\omega(\omega_{t+1})}{(1+\pi_{t+1})(1+n)} = 0$$

O bien

$$u_m(c_t, m_t) - u_c(c_t, m_t) + \frac{\beta V_\omega(\omega_{t+1})}{(1+\pi_{t+1})(1+n)} = 0$$

Conocido el teorema de la envolvente se tiene que

$$u_c(c_t, m_t) = V_\omega(\omega_t)$$

Utilizando esta condición para la cpo del dinero

$$u_m(c_t, m_t) + \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1+\pi_{t+1})(1+n)} = u_c(c_t, m_t)$$

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = 1 - \frac{1}{(1+\pi_{t+1})} \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1+n) u_c(c_t, m_t)}$$

Pero

$$\beta \left[\frac{f_k(k_t) + 1 - \delta}{1+n}\right] \frac{V_\omega(\omega_{t+1})}{u_c(c_t, m_t)} = 1$$

Usando la CPO de los bonos

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = 1 - \frac{1}{(1+\pi_{t+1})(1+r_t)}$$

Conociendo la regla de ficher, se sabe que $(1+\pi_{t+1})(1+r_t) \approx 1 + \pi_{t+1} + r_t = 1 + i_t$

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1+i_t}$$

b)

A maximizar queda

$$V(\omega_t) = \max\{u(c_t, m_t) + \beta u_c(c_{t+1}, m_{t+1}) + \beta V(\omega_{t+1})\}$$

$$\omega_t = c_t + \frac{M_{t+1}}{P_t} + b_t + k_t$$

$$k_t = \omega_t - c_t - m_{t+1}(1+\pi_t) - b_t$$

$$\omega_{t+1} = f(k_t) + (1-\delta)k_t + (1+r_t)b_t + m_{t+1}$$

$$V(\omega_t) = \max\{u(c_t, m_t) + \beta u_c(c_{t+1}, m_{t+1}) + \beta V(f(\omega_t - c_t - m_{t+1}(1+\pi_t) - b_t) + (1-\delta)(\omega_t - c_t - m_{t+1}(1+\pi_t) - b_t) + (1+r_t)b_t + m_{t+1})\}$$

CPO c_t

$$u_c(c_t, m_t) - \beta[f_k(k_t) + 1 - \delta]V_\omega(\omega_{t+1}) = 0$$

CPO b_T

$$1 + r_t - [f_k(k_t) + 1 - \delta] = 0$$

CPO m_{t+1}

$$\beta u_m(c_{t+1}, m_{t+1}) - \beta V_\omega(\omega_{t+1})[f_k(k_t) + (1 - \delta)](1 + \pi_t) + \beta V_\omega(\omega_{t+1}) = 0$$

$$\beta u_m(c_{t+1}, m_{t+1}) - u_c(c_t, m_t)(1 + \pi_t) + \beta V_\omega(\omega_{t+1}) = 0$$

$$\frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} = \frac{u_c(c_t, m_t)(1 + \pi_t)}{\beta u_c(c_{t+1}, m_{t+1})} - 1$$

Usando la cpo del consumo

$$\frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} = [f_k(k_t) + 1 - \delta](1 + \pi_t) - 1$$

Usando la cpo de los bonos

$$\frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} = (1 + r_t)(1 + \pi_t) - 1$$

Usando Ficher

$$\frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} = (1 + i_t) - 1$$

$$\frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} = i_t$$

P2 Shopping Time Models

Suponga que la función de producción para comprar tiene la forma $\psi = e^{x}(n^{s})^{a}m^{b}$, donde a y b son positivos y menores que 1 y x es un factor de productividad.
La utilidad de los agentes es $v(c,l) = \frac{c^{1-\Phi}}{1-\Phi} + \frac{l^{1-\eta}}{1-\eta}$,donde $l = 1 - n - n^{s}$

a) Encuentre la transaction time function $g(c,m) = n^{s}$.

b) Derive the money in the utility function specification implied by the shopping production function. How does the marginal utility of money depend on the parameters a and b? How does it depend on x?

c) Is the marginal utility of consumption increasing or decreasing in m?

Sol

a)

Se sabe que

$$c = \psi(n^{s}, m) = e^{x}(n^{s})^{a}m^{b}$$

$$(n^{s})^{a} = c\, e^{-x}m^{-b}$$

$$n^{s} = (c\, e^{-x}m^{-b})^{\frac{1}{a}}$$

b)

$$v(c,l) = \frac{c^{1-\Phi}}{1-\Phi} + \frac{l^{1-\eta}}{1-\eta}$$

Pero $l = 1 - n - n^{s} = 1 - n - g(c,m)$

Entonces

$$v(c,l) = \frac{c^{1-\Phi}}{1-\Phi} + \frac{\left(1 - n - g(c,m)\right)^{1-\eta}}{1-\eta} = \frac{c^{1-\Phi}}{1-\Phi} + \frac{\left(1 - n - (c\, e^{-x}m^{-b})^{\frac{1}{a}}\right)^{1-\eta}}{1-\eta}$$

Tanto, si x, a, b crece n^{s} decrece, aumentando el ocio, por lo que la utilidad aumentará.

c)

$$\frac{\partial v}{\partial c} = c^{-\Phi} - l^{-\eta} \cdot g_{c}$$

$$\frac{\partial^{2} v}{\partial m \partial c} = -l^{-\eta} \cdot g_{mc} - \eta\, l^{-(\eta+1)} g_{m} g_{c}$$

$$\frac{\partial^{2} v}{\partial m \partial c} = -\left(l^{-\eta} \cdot g_{mc} + \eta\, l^{-(\eta+1)} g_{m} g_{c}\right)$$

Notemos que

$$g_{c} = \frac{1}{a}\frac{g}{c}$$

Además

$$g_m = -\frac{b}{a}\frac{g}{m}$$

Así

$$\frac{\partial^2 v}{\partial m \partial c} = -\left(l^{-\eta} \cdot \left(-\frac{b}{a^2}\frac{n^s}{mc} \right) + \eta\, l^{-(\eta+1)} \left(-\frac{b}{a}\frac{n^s}{m} \right) \left(\frac{1}{a}\frac{n^s}{c} \right) \right)$$

$$\frac{\partial^2 v}{\partial m \partial c} = l^{-\eta}\frac{bn^s}{a^2 cm}[1 + \eta l^{-1} n^s] > 0$$