IN71K – Auxiliar 3

"Newsvendor model & Assortment Planning"

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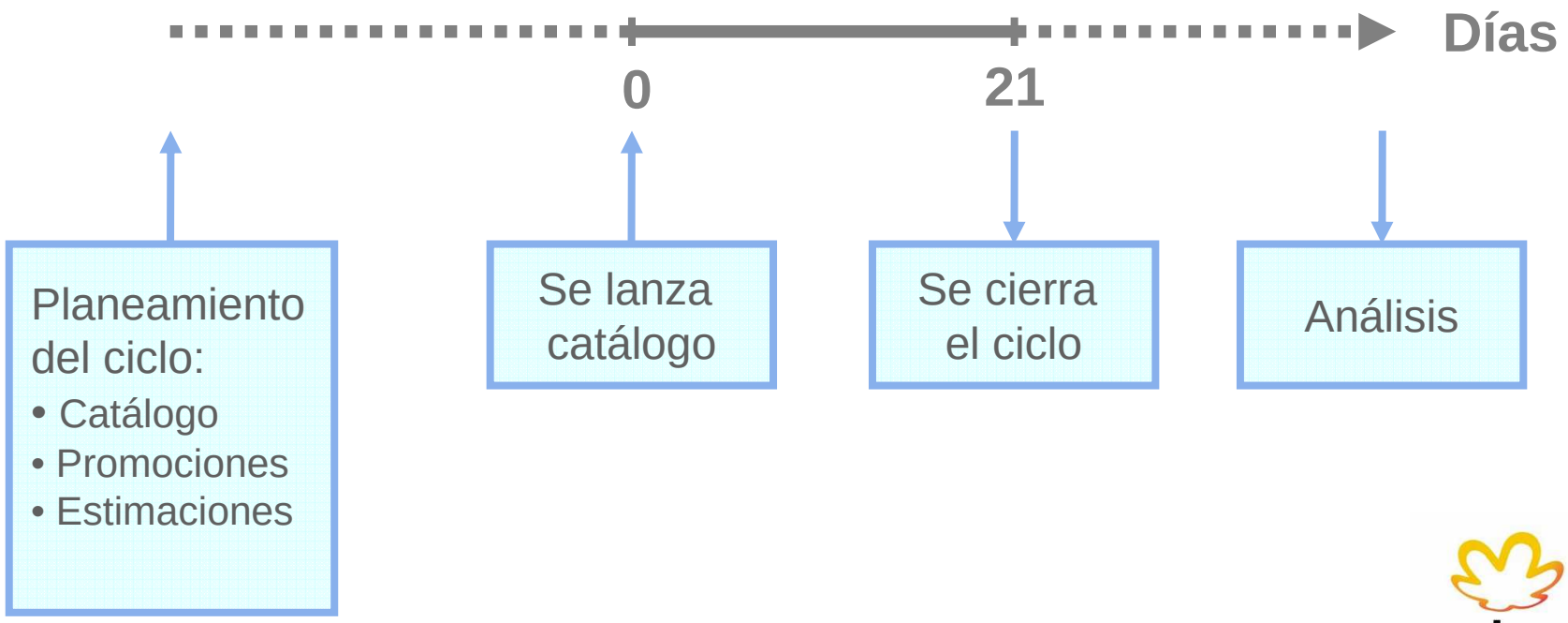
Motivación

- Existen productos con ciclo de vida corto y que se producen una sola vez
  - Ropa Fashion
  - Libros
- Necesidad de planear inventario de dichos productos

Motivación (2)

- Ejemplo: Natura



natura
bien estar bien

Modelo del Newsvendor

- Analogía: "¿Cuántos diarios debo poner para satisfacer la demanda?"
  - Si pongo menos de lo demandado => Stockouts
  - Si pongo más de lo demandado => Se pierden
- Trade-off entre overstock y understock

Modelo del Newsvendor (2)

- Definimos:
  - p = Precio
  - c = Costo unitario
  - s = Precio residual (salvage value). (s<c)

  - Costo overage (sobrante) *unitario*: O = c – s
  - Costo underage (faltante) *unitario*: U = p - c

Modelo del Newsvendor (3)

- Sea Q la cantidad a ordenar y D la demanda
  - Si **D < Q**:
    - Profit = pD – cQ + s(Q – D)
    - Overage Cost *Total* = (c – s)(Q – D).
    - Underage Cost *Total* = 0.
  - Si **D > Q**:
    - Profit = pQ – cQ
    - Overage Cost *Total* = 0
    - Underage Cost *Total* = (p – c)(D – Q).

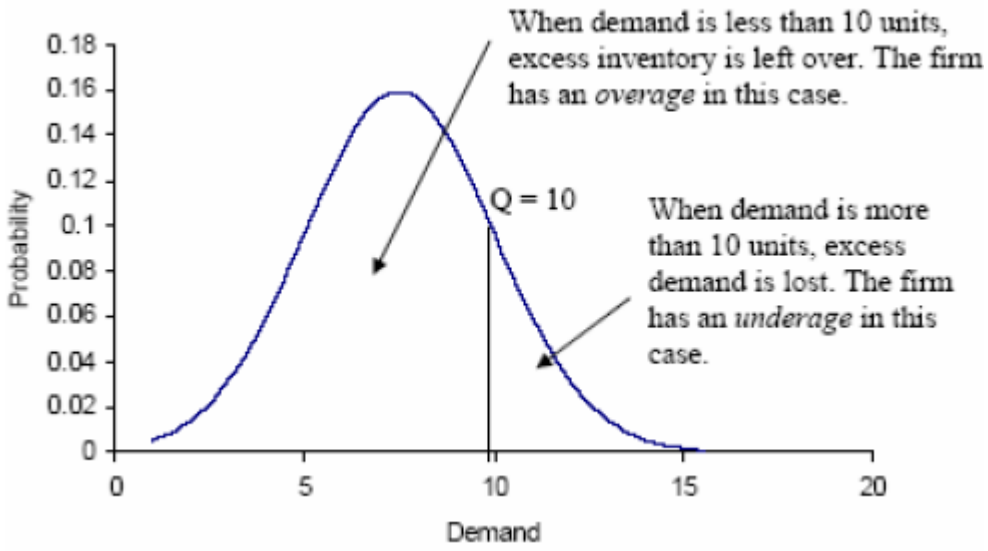
Modelo del Newsvendor (4)

- Mientras Q crece:
  - Overage cost total crece (D<Q es más probable)
  - Underage cost total decrece (D<Q es menos probable)
- Regla de optimalidad:
  - Costo esperado de obtener una unidad extra = Costo esperado de reducir orden en una unidad
  - Overage cost * Probability[Demand ≤ Q] = Underage cost * Probability[Demand ≥ Q]

$$\Pr obability\,[Demand \le Q] = \frac{Underage\ Cost}{Underage\ Cost + Overage\ Cost}$$

Modelo del Newsvendor (5)

$$\Pr obability\,[Demand \le Q] = \frac{Underage\ Cost}{Underage\ Cost + Overage\ Cost}$$



Modelo del Newsvendor (6)

- Problema 1
  - A buyer for Needless Markup, a famous “high-end” department store, must decide on the quantity of a high-priced women’s handbag to procure in Italy for the following Christmas season. The unit cost of the handbag to the store is $28.50 and the handbag will sell for $150. Handbags not sold by the end of the season are purchased by a discount firm for $20.

    A detailed analysis of historical data shows that the number of handbags sold is described by a normal distribution with mean 150 and standard deviation 20. What is the optimal number of bags to be purchased?

Modelo del Newsvendor (7)

- Calculamos U y O:
  - Underage cost = $150 – 28.50 = $121.50
  - Overage cost = $28.50 – 20.00 = $8.50
- Entonces:
  - Prob [Dda $\leq$ Z] = 121.5 / (121.5 + 8.5) = 0.9346.
  - Esto corresponde a Z = 1.51
- Por lo tanto, el número de bolsos a comprar:
  - Bolsos = 150 + 20*1.51 = 180.

Modelo del Newsvendor (8)

- Problema 2:
  - Bed, Bath and Beyond finds that in one of its stores in Manhattan, the demand for a particular brand of humidifiers during the winter season is uniformly distributed between 1 and 10 units (see table below). Each humidifier costs $25, sells for $50, and excess inventory can be returned to the supplier for a salvage value of $15.

Probability	Demand (# of units)
0.1	1
0.1	2
0.1	3
0.1	4
0.1	5
0.1	6
0.1	7
0.1	8
0.1	9
0.1	10

Modelo del Newsvendor (9)

a) Bed, Bath and Beyond decides to stock 5 units of this humidifier for the winter season. Compute the expected amount of profit that Bed, Bath and Beyond will make as a result of this decision.

b) What is the optimal inventory level that Bed, Bath and Beyond should stock? What is the expected profit at this inventory level?

Modelo del Newsvendor (10)

Parte a)

- Recordando lo visto anteriormente:
  - Si **D < Q**:
    - Profit = pD – cQ + s(Q – D)
  - Si **D > Q**:
    - Profit = pQ – cQ
- Caso 1: Si la demanda es 1
  - Me sobran 4 unidades
  - Como D<Q => Profit = pD – cQ + s(Q – D) = -15
- Caso 2: Si la demanda es 2
  - …….

Modelo del Newsvendor (11)

Probability	Demand (# of units)	Sales (# of units)	Excess Inventory (# of units)	Profit ($) = Price*Sales + Salvage*Excess Inventory – Cost*Q
0.1	1	1	4	-15
0.1	2	2	3	20
0.1	3	3	2	55
0.1	4	4	1	90
0.1	5	5	0	125
0.1	6	5	0	125
0.1	7	5	0	125
0.1	8	5	0	125
0.1	9	5	0	125
0.1	10	5	0	125

- Expected profit =

  (-15)*0.1 + 20*0.1 + 55*0.1 + 90*0.1 + 125*0.6 = $90.
- Expected Sales =

  1*0.1 + 2*0.1 + 3*0.1 + 4*0.1 + 5*0.6 = 4 units

Modelo del Newsvendor (12)

Parte b)

- Underage cost per unit, U = $25
- Overage cost per unit, O = $10
- Prob [Dda ≤ Q] = U / [U + O] = 0.714.
- Prob [Dda ≤ Q] está entre 0,7 y 0,8 (i.e. Q entre 7 y 8 unidades, dado que es una distribución uniforme). Aproximamos hacia arriba => Q=8 unidades.

Inventory	*Expected Profit*
6 units	*$97.5*
7	*$101.5*
8	*$102*
9	*$99*

Modelo del Newsvendor (13)

Problema 3:

- In the context of the Newsboy model discussed in class, consider the inventory management of a perishable product with the following features:
  - Market price = $100 per unit.
  - Purchasing cost = $50 per unit.
  - Salvage value = $10 per unit

  Based on past experience, you have collected the following information about this product demand distribution:

Modelo del Newsvendor (14)

Demand	Probability	Cumulative Probability
0	1.1%	1.1%
50	1.6%	2.7%
100	2.0%	4.7%
150	2.8%	7.5%
200	3.6%	11.1%
250	4.7%	15.8%
300	6.1%	21.9%
350	7.8%	29.7%
400	9.4%	39.1%
450	11.1%	50.2%
500	10.6%	60.8%
550	9.7%	70.5%
600	8.2%	78.7%
650	6.9%	85.6%
700	5.3%	90.9%
750	3.8%	94.7%
800	2.2%	96.9%
850	1.6%	98.5%
900	0.8%	99.3%
950	0.6%	99.9%
1000	0.1%	100.0%

Modelo del Newsvendor (15)

- a) Based on this information, what is the optimal level of inventory to procure?

SOL:

Underage Cost = $100 - $50 = $50

Overage Cost = $50 -$10 = $40

Critical Fractile = U/(U+O) = 50/90=0.555

=> Inventario óptimo entre 450 y 500 unidades

Modelo del Newsvendor (16)

- b) Suppose that instead of deciding the inventory policy using the optimal Newsboy (fractile) rule you want to pick the level of inventory so that with 95% probability you will not experience any stockout. What is the optimal inventory under this service level constraint?

SOL:

Para tener un nivel de servicio de 95%, debemos tener un inventario de ~750.

Modelo del Newsvendor (17)

- c) Your boss understands the need for offering a 95% service level but she is having a hard a time trying to justify it using a cost/benefit analysis. To help her, you point out that the solution in part a) does not take into account the goodwill cost of stocking out (which is hard to compute but must be included). What should be this stockout cost so that the solution in part b) is consistent with the Newsboy rule used in part a)? How would you go to estimate this goodwill cost in practice?

SOL:

Sea W el costo de penalización por una venta perdida

Underage Cost = $50+W

=> Critical Fractile = (50+W)/(90+W)

Para que el critical fractile sea 95% => W=$710

Assortment Planning

"Management of multi-item retail inventory systems with demand substitution"

Motivación

- Variedad del surtido como calidad de servicio
- Problemas
  - Inventarios
  - Envíos
  - Espacio en la tienda
- Nace la necesidad de decidir los niveles de stock de inventario en forma conjunta entre varios productos
- Problemas:
  - Demanda incierta
  - Efecto sustitución ante stockouts

Idea...

- Decidir política de inventarios
  - Qué items guardar
  - Nivel de inventario inicial
- Variables de decisión **deben estimarse en conjunto**
  - Ya que si hay stockouts, demanda por sustitutos aumenta

Sobre el paper…

- ¿Cómo se comporta la demanda?
  - Demanda del ciclo se origina de un número aleatorio de clientes que llega aleatoriamente a comprar una unidad de un producto específico
  - Productos de la tienda se dividen en conjuntos de categorías disjuntas (ej: café, leche, etc.)
  - Clientes eligen aleatoriamente en el set
  - Si primera opción no está disponible => sustituto

Sobre el paper… (2)

- ¿Cuál es el aporte de lo discutido en el paper?
  - ¡Considerar efecto sustitución!
  - La mayoría de los otros modelos no lo considera
- Maximizar utilidades por ciclo, sujeto a:
  - Espacio disponible
  - Presupuesto

¡Suerte!