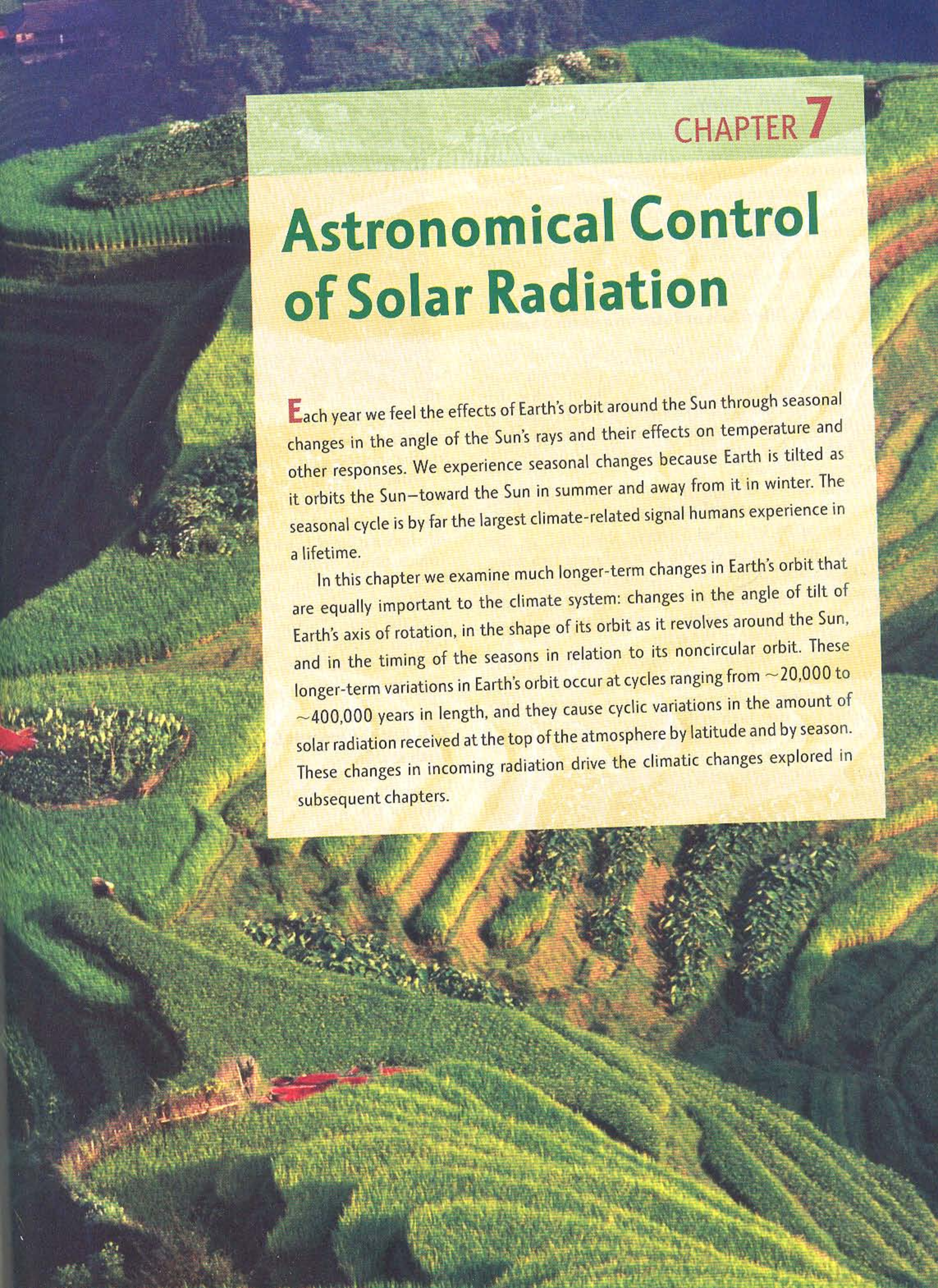


CHAPTER 7

Astronomical Control of Solar Radiation

Each year we feel the effects of Earth's orbit around the Sun through seasonal changes in the angle of the Sun's rays and their effects on temperature and other responses. We experience seasonal changes because Earth is tilted as it orbits the Sun—toward the Sun in summer and away from it in winter. The seasonal cycle is by far the largest climate-related signal humans experience in a lifetime.

In this chapter we examine much longer-term changes in Earth's orbit that are equally important to the climate system: changes in the angle of tilt of Earth's axis of rotation, in the shape of its orbit as it revolves around the Sun, and in the timing of the seasons in relation to its noncircular orbit. These longer-term variations in Earth's orbit occur at cycles ranging from ~20,000 to ~400,000 years in length, and they cause cyclic variations in the amount of solar radiation received at the top of the atmosphere by latitude and by season. These changes in incoming radiation drive the climatic changes explored in subsequent chapters.

Earth's Orbit Today

The geometry of Earth's present solar orbit is the starting point for understanding past changes in Earth-Sun geometry. Much of our knowledge of Earth's orbit dates back to investigations in the seventeenth century by the astronomer Johannes Kepler. The larger frame of reference for understanding Earth's present orbit is the plane in which it moves around the Sun, the **plane of the ecliptic** (Figure 7-1).

7-1 Earth's Tilted Axis of Rotation and the Seasons

Two fundamental motions describe today's orbit. First, Earth spins on its axis once every day. One result is the daily "rising and setting" of the Sun, but of course that description is inaccurate. Days and nights are caused by Earth's rotational spin, which carries different regions of Earth's surface into and out of the Sun's direct radiation every 24 hours.

Earth rotates around an axis (or line) that passes through its poles (see Figure 7-1). This axis is tilted at an angle of 23.5°, called Earth's "obliquity," or **tilt.** This tilt angle can be visualized in either of two ways: (1) as the angle Earth's axis of rotation makes with a line perpendicular to the plane of the ecliptic or (2) as the angle that a plane passing through Earth's equator makes with the plane of the ecliptic.

The second basic motion in Earth's present orbit is its once-a-year revolution around the Sun. This motion results in seasonal shifts between long summer days, when the Sun rises high in the sky and delivers stronger radiation, and short winter days, when the Sun stays low in the sky and delivers weaker radiation. These seasonal differences culminate at the summer and winter **solstices,** which mark the longest and shortest days of the year (June 21 and December 21 in the northern hemisphere, the reverse in the southern hemisphere).

If we move outside our Earthbound perspective, we find that the cause of the seasons, the solstices, and the changes in length of day and angle of incoming solar radiation actually lies in the changing *position* of the tilted Earth with respect to the Sun. During each yearly revolution around the Sun, Earth maintains a constant *angle* of tilt (23.5°) and a constant *direction* of this tilt in space. When the northern or southern hemisphere is tilted directly toward the Sun, it receives the more direct radiation of summer. When it tilts directly away from the Sun, it receives the less direct radiation of winter. But at both times and at all times of year it keeps the same 23.5° tilt.

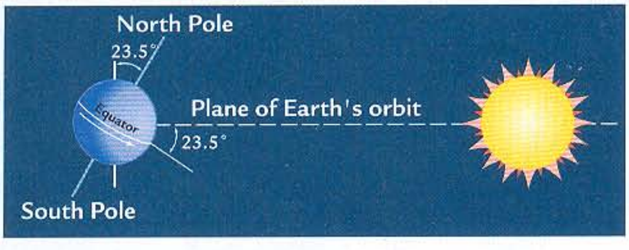


FIGURE 7-1 Earth's tilt Earth's rotational (spin) axis is currently tilted at an angle of 23.5° away from a line perpendicular to the plane of its orbit around the Sun.

If we switch back to our Earthbound perspective, we see the overhead Sun appearing to move back and forth through the year between the north tropic (Cancer) at 23.5°N and the south tropic (Capricorn) at 23.5°S. But again, this apparent movement is actually the result of Earth's revolution around the Sun with a constant 23.5° tilt. Earth's 23.5° tilt also defines the 66.5° latitude of the Arctic and Antarctic circles: 90° − 23.5° = 66.5°. Because of the 23.5° tilt away from the Sun in northern winter, no sunlight reaches latitudes poleward of 66.5° on the shortest winter day (winter solstice).

Midway between the extremes of the winter and summer solstices, during intermediate positions in Earth's revolution around the Sun, the lengths of night and day become equal in each hemisphere at the **equinoxes** (which means "equal nights"—that is, nights equal in length to days). Again, Earth's tilt angle remains at 23.5° during the equinoxes, and its direction of tilt in space stays the same. The only factor that changes is Earth's position in respect to the Sun. The two equinoxes and two solstices are handy reference points for describing distinctive features of its orbit.

7-2 Earth's Eccentric Orbit: Distance between Earth and Sun

Up to this point, everything that has been described would be true whether Earth's orbit was perfectly circular or not. But Earth's actual orbit (Figure 7-2) is not a perfect circle: it has a slightly eccentric or elliptical shape. The noncircular shape of Earth's orbit is the result of the gravitational pull of other planets on Earth as it moves through space.

Basic geometry shows that ellipses have two focal points rather than the single focus (center) of a circle. In Earth's case, the Sun lies at one of the two focal points in its elliptical orbit, as required by the physical laws of gravitation. The other focus is empty (see Figure 7-2).

Earth's distance from the Sun changes according to its position in this elliptical orbit. Not surprisingly, these changes in Earth-Sun distance affect the amount of solar radiation Earth receives, especially at two extreme positions in the orbit. The position in which Earth is closest to the Sun is called **perihelion** (the "close pass" position, from the Greek meaning "near the Sun"), while the position farthest from the Sun is called **aphelion** (the "distant pass" position, from the Greek meaning "away from the Sun"). On average, Earth lies 155.5 million

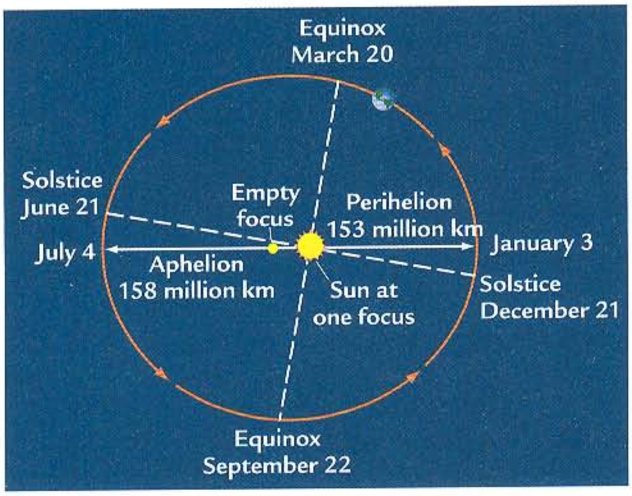


FIGURE 7-2 Earth's eccentric orbit Earth's orbit around the Sun is slightly elliptical. Earth is most distant from the sun at aphelion, on July 4, just after the June 21 solstice, and closest to the Sun at perihelion, on January 3, just after the December 21 solstice. (Modified from J. Imbrie and K. P. Imbrie, *Ice Ages: Solving the Mystery* [Short Hills, NJ: Enslow, 1979].)

kilometers from the Sun, but the distance ranges between 153 million kilometers at perihelion and 158 million at aphelion. This difference is equivalent to a total range of variation of slightly more than 3% around the mean value.

Earth is now in the perihelion position (closest to the Sun) on January 3, near the time of the December 21 winter solstice in the northern hemisphere and summer solstice in the southern hemisphere (see Figure 7-2). The fact that the close-pass position occurs in January causes winter radiation in the northern hemisphere and summer radiation in the southern hemisphere to be slightly stronger than they would be in a perfectly circular orbit.

Conversely, Earth lies farthest from the Sun on July 4, near the time of the June 21 summer solstice in the northern hemisphere and winter solstice in the southern hemisphere. The occurrence of this distant-pass position in July makes summer radiation in the northern hemisphere and winter radiation in the southern hemisphere slightly weaker than they would be in a circular orbit.

The effect of Earth's elliptical orbit on its seasons is small, enhancing or reducing the intensity of radiation received by just a few percent. Remember that the main cause of the seasons is the direction of tilt of Earth's axis in its orbit around the Sun (see Figure 7-1).

Another consequence of Earth's eccentric orbit is that the time intervals between the two equinoxes are not exactly equal: there are seven more days in the long part of the orbit, between the March 20 equinox and the September 22 equinox, than in the short part of the orbit, between September 22 and March 20. The greater length of the interval from March 20 to September 22 tends to compensate for the fact that Earth is farther from the Sun on this part of the orbit and thus is receiving less solar radiation.

Long-Term Changes in Earth's Orbit

Astronomers have known for centuries that Earth's orbit around the Sun is not fixed over long intervals of time. Instead, it varies in a regular (cyclic) way because of the mass gravitational attractions among Earth, its moon, the Sun, and the other planets and their moons. These changing gravitational attractions cause cyclic variations in Earth's angle of tilt, its eccentricity of orbit, and the relative position of the solstices and equinoxes around its elliptical orbit (Box 7–1).

7-3 Changes in Earth's Axial Tilt through Time

If we assume for simplicity that Earth has a perfectly circular orbit around the Sun, we can examine two hypothetical cases that show the most extreme differences in tilt. For both cases, we look at the summer and winter solstices, the two seasonal extremes in Earth's orbit.

For the first case, Earth's axis is not tilted at all (Figure 7-3A). Incoming solar radiation is directed straight at the equator throughout the year, and it always passes by the poles at a 90° angle. Without any tilt, no seasonal changes occur in the amount of solar

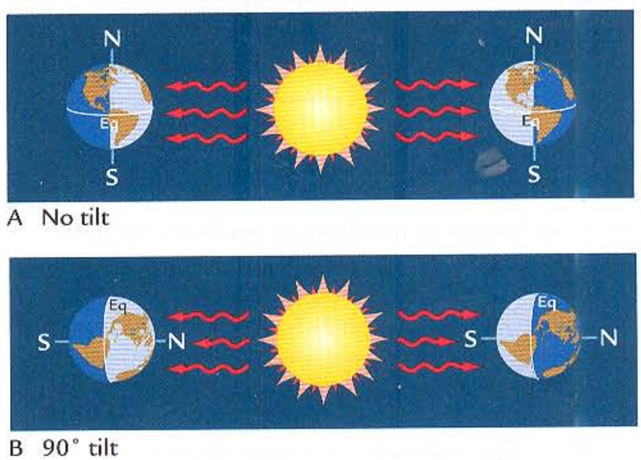


FIGURE 7-3 Extremes of tilt (A) If Earth's orbit were circular and its axis had no tilt, solar radiation would not change through the year and there would be no seasons. (B) For a 90° tilt, the poles would alternate seasonally between conditions of day-long darkness and day-long direct overhead Sun. (Adapted from J. Imbrie and K. P. Imbrie, *Ice Ages: Solving the Mystery* [Short Hills, NJ: Enslow, 1979].)

BOX 7-1 TOOLS OF CLIMATE SCIENCE

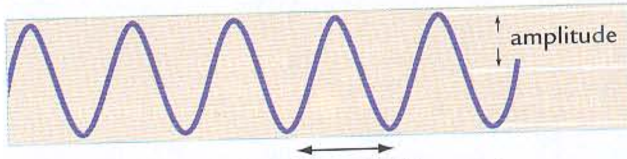
Cycles and Modulation

Slow changes in Earth's orbit around the Sun occur in a cyclic or rhythmic way, as do the changes in amount of incoming solar radiation they produce. The science of wave physics provides the terminology needed to describe these changes. The length of a cycle is referred to as its **wavelength.** Expressed in units of time, the wavelength of a cycle is called the **period,** the time span between successive pairs of peaks or valleys.

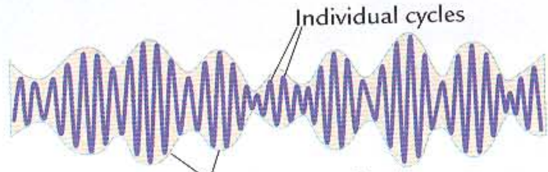
The opposite (or inverse) of the period of a cycle is its **frequency,** the number of cycles (or in this case fractions of one cycle) that occur in one year. If a cycle has a period of 10,000 years, its frequency is 0.0001 cycle per year (one cycle every 10,000 years). In this book, we will refer to cycles in terms of their periods.

Another important aspect of cycles is their **amplitude,** a measure of the amount by which they vary around their long-term average. Low-amplitude cycles barely depart from the long-term mean trend; high-amplitude cycles fluctuate more widely.

Not all cycles are perfectly regular. Commonly the sizes of peaks and valleys oscillate irregularly around the long-term mean value through time. Behavior in which the amplitude of peaks and valleys changes in a repetitive or cyclic way is called **modulation,** a concept that lies behind the principle of AM (amplitude modulation) radio. Modulation creates an envelope that encompasses the changing amplitudes that occur at a specific cycle. Note that *modulation of a cycle is not in itself a cycle;* it simply adds amplitude variations to an actual cycle.



A Sine-wave cycle



B Amplitude modulation

Description of wave behavior (A) Perfectly cyclic behavior can be represented by a sine wave with a particular period and amplitude. (B) Cycles may show regular variations in amplitude, or modulation.

If variations in a particular signal are regular in *both* period and amplitude, it is appropriate to use the term "cycle." For the case of perfect cyclicity, this behavior is described as "sinusoidal" or **sine waves.** If the variations are irregular in period, the term "cyclical" is technically incorrect; "quasi-cyclical" or "quasi-periodic" is preferable. In the case of orbital-scale changes, we will informally use the term "cyclic" or "periodic" for climatic signals that are nearly regular but vary slightly in wavelength or amplitude.

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radiation received at any latitude. As a result, solstices and equinoxes do not even exist because every day has the same length. A tilted axis is necessary for Earth to have seasons.

Next consider the opposite extreme with a maximum tilt of 90° (Figure 7-3B). Solar radiation is directed straight at the summer-season pole, while the winter-season pole lies in complete darkness. Six months later, the two poles have completely reversed position. The difference between these two extreme configurations shows that tilt is an important control on solar radiation at polar latitudes.

The angle of Earth's tilt has varied through time within a narrow range, between values as small as 22.2° and as large as almost 24.5° (Figure 7-4). The French astronomer Urbain Leverrier discovered these variations in the 1840s. Today Earth's tilt (23.5°) is near the middle of this range, and the angle is currently decreasing. Cyclic changes in tilt angle occur mainly at a period of 41,000 years, the time interval that separates successive peaks or successive valleys (see Box 7–1). The cycles are fairly regular, both in period (wavelength) and in amplitude.

Changes in tilt amplify or suppress the strength of the seasons, especially at high latitudes (Figure 7-5). Larger tilt angles turn the summer hemisphere poles more directly toward the Sun and increase the amount of solar radiation received. The increase in tilt that turns the North Pole more directly toward the Sun at its summer solstice on June 21 also turns the South Pole more directly toward the Sun at its summer solstice six months later (December 21). On the other hand, the increased angle of tilt that turns each polar region more directly toward the Sun in summer also turns each winter season pole away from the Sun.

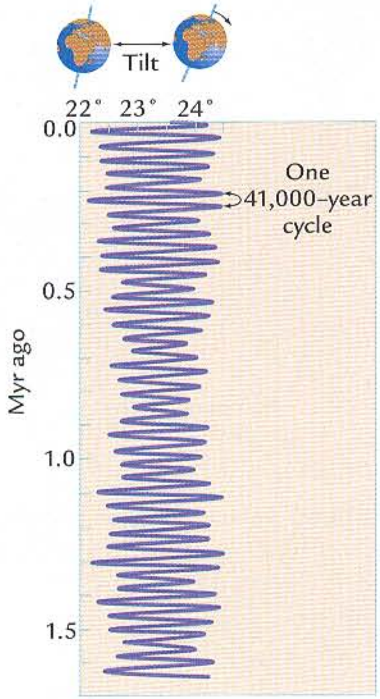


FIGURE 7-4 Long-term changes in tilt Changes in the tilt of Earth's axis have occurred at a regular 41,000-year cycle.

Decreases in tilt have the opposite effect: they diminish the amplitude of seasonal differences. Smaller tilt angles put the Earth slightly closer to the configuration shown in Figure 7-3A, which has no seasonal differences at all.

IN SUMMARY, changes in tilt mainly amplify or suppress the seasons, particularly at the poles.

7-4 Changes in Earth's Eccentric Orbit through Time

The shape of Earth's orbit around the Sun has also varied in the past, becoming at times more circular and at other times more elliptical (or "eccentric") than it is today. Leverrier discovered these variations in the 1840s. The shape of an ellipse can be described by reference to its two main axes: the "major" (or longer) axis and the "minor" (or shorter) axis (Figure 7-6). The degree of departure from a perfectly circular orbit can be described by

$$\epsilon = \frac{\sqrt{a^2 - b^2}}{a}$$

where ϵ is the **eccentricity** of the ellipse and a and b are half of the lengths of the major and minor axes (called the "semimajor" and "semiminor" axes).

The eccentricity of the elliptical orbit increases as these two axes become more unequal in length. At the extreme where the two axes become exactly equal ($a = b$), the eccentricity drops to zero because the orbit is circular ($a^2 - b^2 = 0$). Eccentricity (ϵ) has varied over time between values of 0.005 and 0.0607 (Figure 7-7). The present value (0.0167) lies toward the lower (more circular) end of the range.

Changes in orbital eccentricity are concentrated mainly at two periods. One eccentricity cycle shows up as variations at intervals near 100,000 years (see Figure 7-7). This cycle actually consists of four cycles of nearly equal strength and periods ranging between 95,000 and 131,000 years, but these cycles blend into a cycle near 100,000 years.

The second eccentricity cycle has a wavelength of 413,000 years. This longer cycle is not as obvious, but it shows up as alternations of the 100,000-year cycles between larger and smaller peak values. Larger amplitudes can be seen near 200,000, 600,000, 1,000,000, and 1,400,000 years ago (see Figure 7-7). A third eccentricity cycle also exists at a period of 2.1 Myr, but this cycle is much weaker in amplitude.

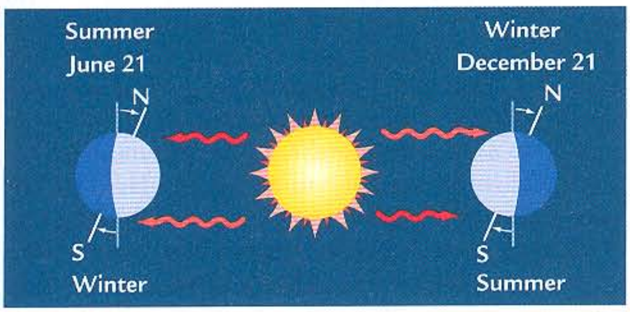
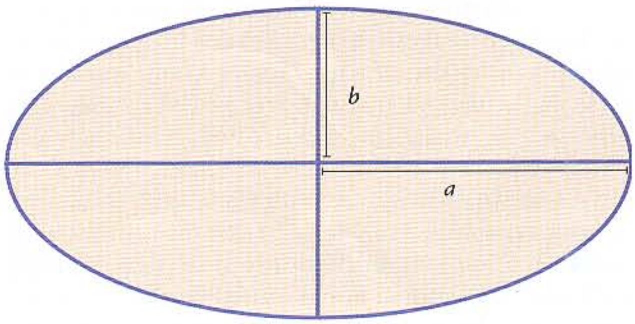


FIGURE 7-5 Effects of increased tilt on polar regions Increased tilt brings more solar radiation to the two summer season poles and less radiation to the two winter season poles.



$$\text{Eccentricity } \varepsilon = \frac{(a^2 - b^2)^{1/2}}{a}$$

FIGURE 7-6 Eccentricity of an ellipse The eccentricity of an ellipse is related to half of the lengths of its longer (major) and shorter (minor) axes.

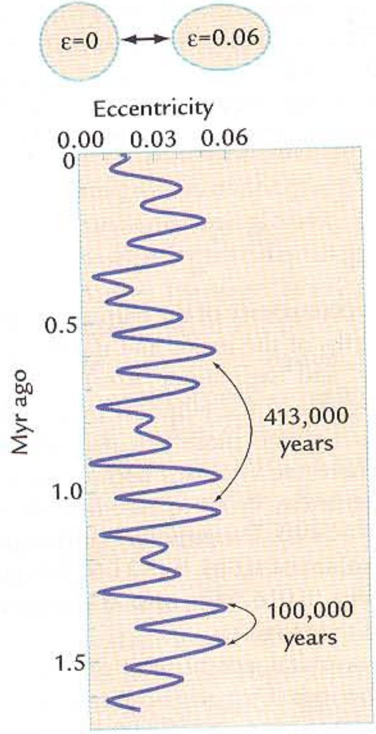


FIGURE 7-7 Long-term changes in eccentricity The eccentricity (ϵ) of Earth's orbit varies at periods of 100,000 and 413,000 years.

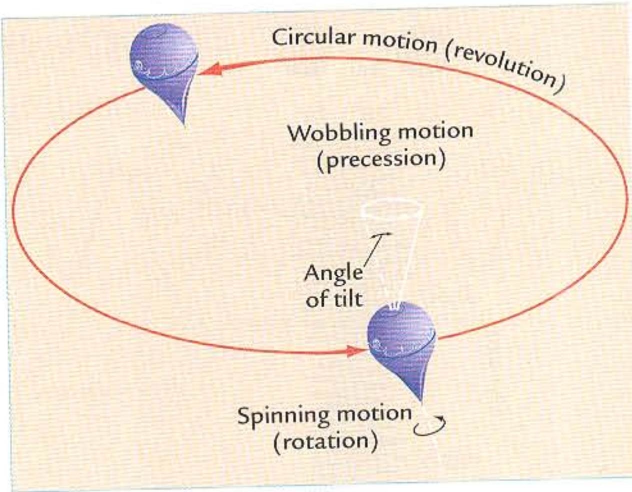


FIGURE 7-8 Earth's wobble In addition to its rapid (daily) rotational spin and its slower (yearly) revolution around the Sun, Earth wobbles slowly, like a top, with one full wobble every 25,700 years.

7-5 Precession of the Solstices and Equinoxes around Earth's Orbit

The positions of the solstices and equinoxes in relation to the eccentric orbit have not always been fixed at their present locations (see Figure 7-2). Instead, they have slowly shifted through time with respect to the eccentric orbit and the perihelion (close-pass) and aphelion (distant-pass) positions. Although Hipparchus in ancient Greece first noticed these changes, the French mathematician, scientist, and philosopher Jean Le Rond d'Alembert was the first to understand them in the eighteenth century.

The cause of these changes lies in a long-term wobbling similar to that of a top. Tops typically move with three superimposed motions (Figure 7-8). They spin very rapidly (rotate) around a tilted axis. They also revolve with a slower near-circular motion across the surface on which they spin, with many spins (rotations) for each complete revolution. Finally, tops also wobble, gradually leaning in different directions through time. This wobbling motion in not caused by changes in the *amount* by which the top leans (its angle of tilt), but rather by changes in the *direction* in which it leans.

Earth's wobbling motion, called **axial precession,** is caused by the gravitational pull of the Sun and Moon on the slight bulge in Earth's diameter at the equator. Axial precession can also be visualized as a slow turning of Earth's axis of rotation through a circular path, with one full turn every 25,700 years. Today Earth rotates around an axis that points to the North Star (Polaris), but over time the wobbling motion causes the axis of rotation to point to other celestial reference points (Figure 7-9). Earth wobbles very slowly; it revolves 25,700 times around the Sun and rotates almost 10 million times on its axis during the time it takes to complete just a single wobble.

A second kind of precessional motion is known as **precession of the ellipse.** In this case, the entire elliptically shaped orbit of the Earth rotates, with the long and short axes of the ellipse turning slowly in space (Figure 7-10). This motion is even slower than the wobbling motion of axial precession.

The combined effects of these two precessional motions (wobbling of the axis and turning of the ellipse) cause the solstices and equinoxes to move around Earth's orbit, with one full 360° orbit around the Sun completed approximately every 22,000 years (Figure 7-11). This combined movement, called the **precession of the equinoxes,** describes the absolute motion of the equinoxes and solstices in the larger reference frame of the universe. It consists of a strong cycle near 23,000 years and a weaker one near 19,000 years, with an average of one cycle every 21,700 years. For the rest of this book, we will concentrate mainly on the strong precession cycle near 23,000 years.

The precession of the equinoxes involves complicated angular motions in three-dimensional space, and these motions need to be reduced to a simple, easy-to-use mathematical form that can be plotted against time like the changes in tilt shown in Figure 7-4. To accomplish

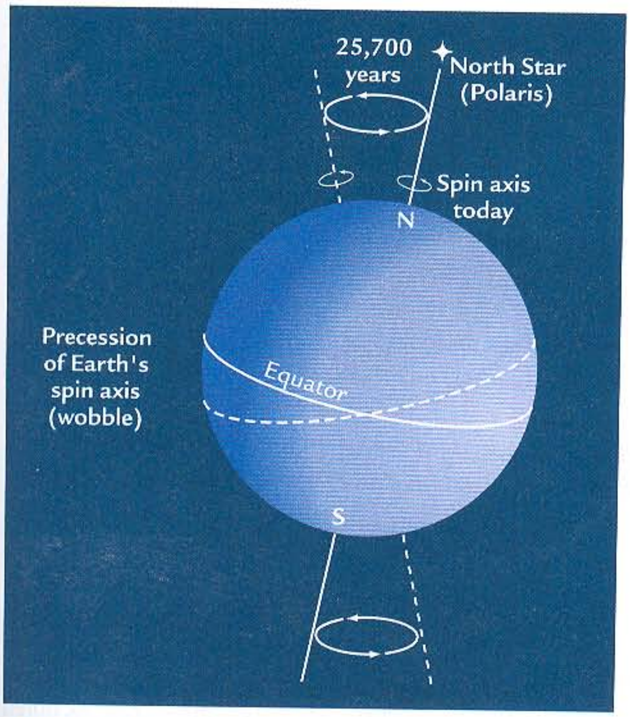


FIGURE 7-9 Precession of Earth's axis Earth's slow wobbling motion causes its rotational axis to point in different directions through time, sometimes (as today) toward the North Star, Polaris, but at other times toward other stars. (Adapted from J. Imbrie and K. P. Imbrie, *Ice Ages: Solving the Mystery* [Short Hills, NJ: Enslow, 1979].).

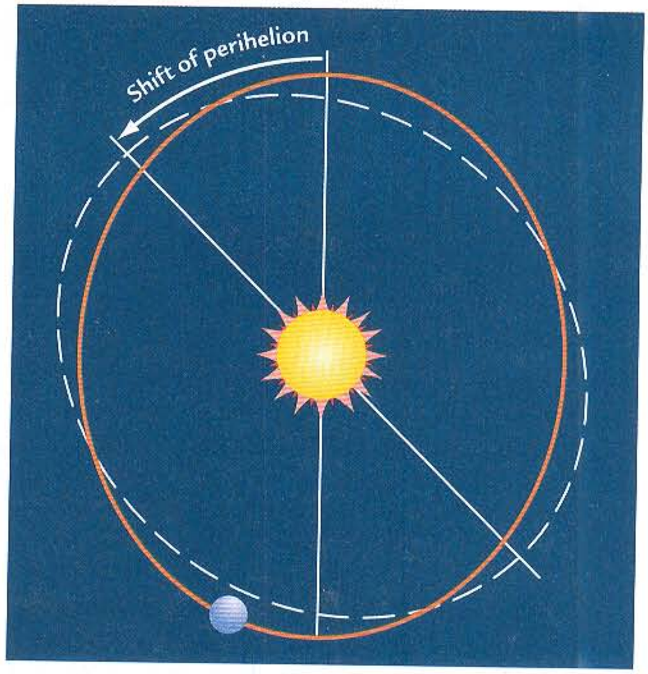


FIGURE 7-10 Precession of the ellipse The elliptical shape of Earth's orbit slowly precesses in space so that the major and minor axes of the ellipse slowly shift through time. (Adapted from N. Pisias and J. Imbrie, "Orbital Geometry, CO_2, and Pleistocene Climate," *Oceanus* 29 [1986–87]: 43–49.)

this goal, we make use of two basic geometric characteristics of precessional motion.

The first characteristic has to do with the angular form of Earth's motion with respect to the Sun. We define ω (omega) as the angle between two imaginary lines (Figure 7-12A): (1) a line connecting the Sun to Earth's position at perihelion (its closest pass to the Sun) and (2) a line connecting the Sun to Earth's position at the March 20 equinox. The first line is tied to the elliptical shape of Earth's orbit and the second to the varying positions of the seasons within the orbit. As a result, the slow change in the angle ω is a measure of Earth's wobbling motion—the very slow changes in the positions of the seasons with respect to the elliptical orbit.

The changing angle ω slowly sweeps out a 360° arc, starting at 0° (where the March 20 equinox coincides with the perihelion position), increasing to 90°, then to 180° (where the March 20 equinox occurs on the other side of the orbit, coincident with the aphelion position), later to 270°, and finally to 360°, at which point the cycle is complete and the angle returns to 0° (Figure 7-12B).

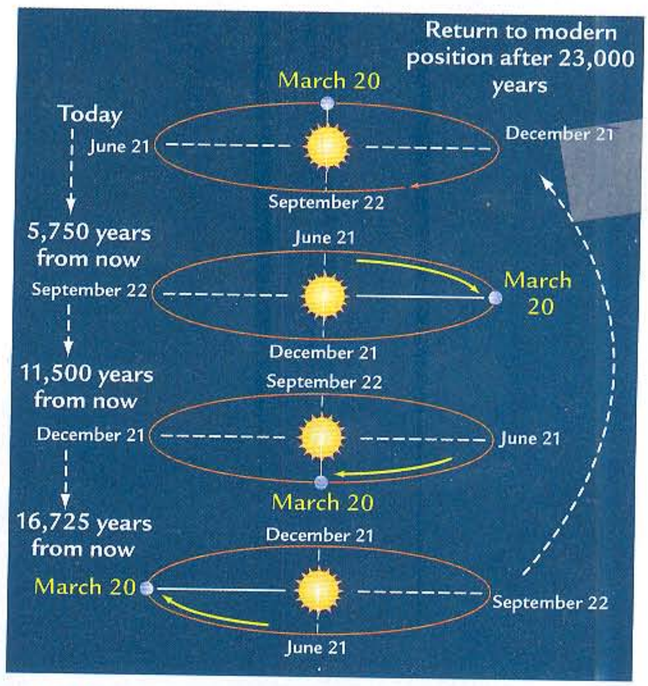


FIGURE 7-11 Precession of the equinoxes Earth's wobble and the slow turning of its elliptical orbit combine to produce the precession of the equinoxes. Both the solstices and equinoxes move slowly around the eccentric orbit in cycles of 23,000 years. (Adapted from J. Imbrie and K. P. Imbrie, *Ice Ages: Solving the Mystery* [Short Hills, NJ: Enslow, 1979].)

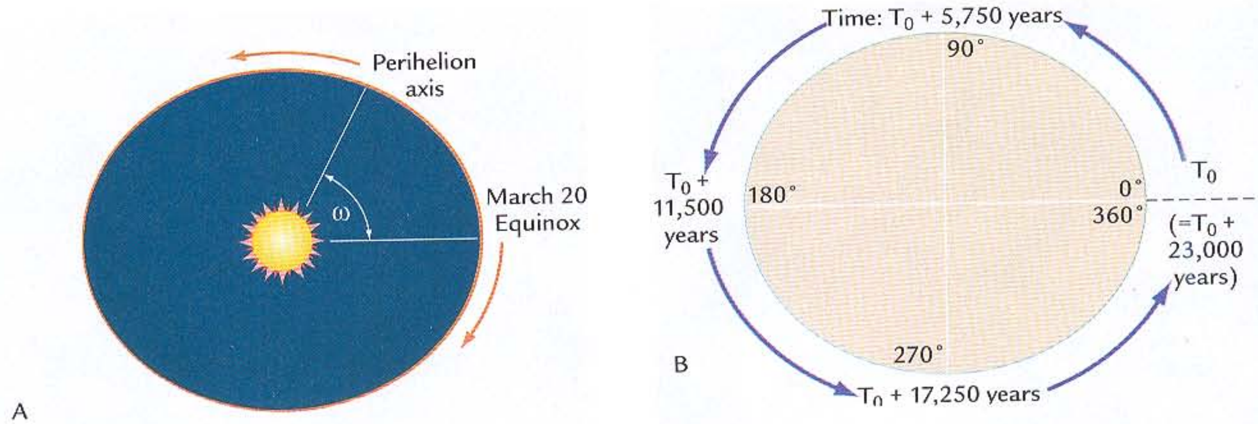


FIGURE 7-12 Precession and the angle ω (A) The angle between lines marking Earth's perihelion axis and the vernal equinox (March 20) is called ω. (B) The angle ω increases from 0° to 360° with each full 23,000-year cycle of precession.

This complicated angular motion can be represented in a simplified mathematical form by using basic geometry and trigonometry to convert the *angular* motions in Figure 7-12 to a *rectangular* coordinate system. Box 7–2 shows how the mathematical sine wave function projects the motion of a radius vector sweeping around a circle onto a vertical coordinate. This conversion allows the circular motion to be represented as an oscillating sine wave on a simple *x-y* plot. The amplitude of sinω moves from a value of +1 to −1 and back again over each 23,000-year precession cycle.

The second aspect of Earth's orbital motion that needs to be considered is its eccentricity. If Earth's orbit were perfectly circular, the slow movements of the solstices and equinoxes caused by precession would not alter the amount of sunlight received on Earth because the distance to the Sun would remain constant through time. Because the orbit is not circular, however, movements of the solstices and equinoxes (see Figure 7-11) cause long-term changes in the amount of solar radiation received on Earth.

These gradual movements of precession bring the solstices and equinoxes (and all other times of the year) into orbital positions that vary in distance from the Sun. Consider the two extreme positions of the solstices in the eccentric orbit (Figure 7-13). As noted earlier, in the present orbit, the position of the June 21 solstice (northern hemisphere summer and southern hemisphere winter) occurs very near aphelion, the most distant pass from the Sun (Figure 7-13 top). This greater Earth-Sun distance on June 21 slightly reduces the amount of solar radiation received during those seasons. Conversely, with the December 21 solstice (northern hemisphere winter and southern hemisphere summer) currently occurring near perihelion, the closest pass to the Sun, solar radiation is higher at those seasons than it would be in a perfectly circular orbit. Approximately 11,000 years ago, half of a precession cycle before now, this configuration was reversed (Figure 7-13 bottom). The June 21 solstice occurred at perihelion, and the December 21 solstice occurred at aphelion.

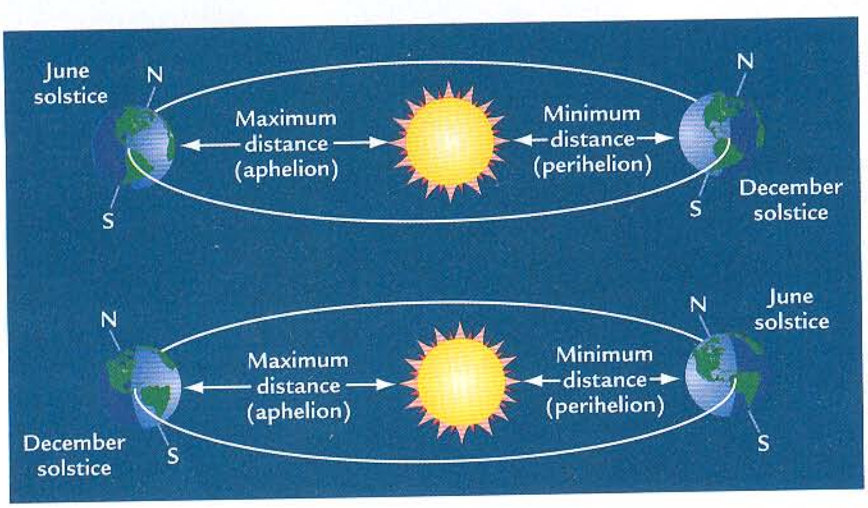


FIGURE 7-13 Extreme solstice positions Slow precessional changes in the attitude (direction) of Earth's spin axis produce changes in the distance between Earth and Sun as the summer and winter solstices move into the extreme (perihelion and aphelion) positions in Earth's eccentric orbit. (Modified from W. F. Ruddiman and A. McIntyre, "Oceanic Mechanisms for Amplification of the 23,000-Year Ice-Volume Cycle," *Science* 212 [1981]: 617–27.)

BOX 7-2 LOOKING DEEPER INTO CLIMATE SCIENCE

Earth's Precession as a Sine Wave

For a right-angle triangle (A), the sine of the angle ω is defined as the length of the opposite side over the length of the hypotenuse (the longest side). Consider a circle whose radius is a vector *r* that sweeps around in a 360° arc in an angular motion measured by the changing angle ω (B). Note that the circular motion described by the angle ω is analogous to actual changes in Earth-Sun geometry.

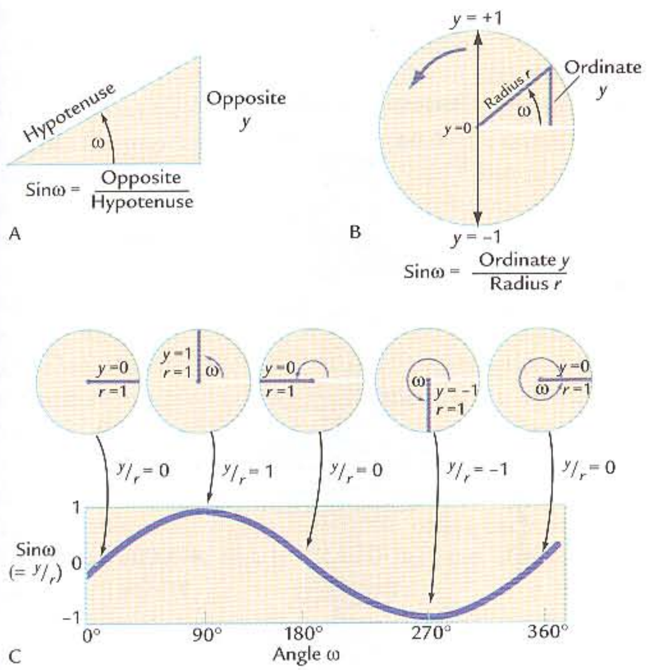
The angular motion of the radius vector *r* around the circle can be converted into changes in the dimensions of a triangle lying within the circle such that the sides of this triangle can be measured in a rectangular (horizontal and vertical) coordinate system (C). In this conversion, the hypotenuse of the triangle is also the radius vector *r* of the circle.

The sweeping motion of the radius vector *r* around the circle causes the shape of the internal triangle to change. The radius vector *r* always has a value of +1 because its length stays the same and its sign is defined within the angular coordinate system as a positive value.

But the length of the opposite side of the triangle (*y*) is defined within the rectangular coordinate system, and it can change both in amplitude and in sign (positive or negative). As the radius vector *r* sweeps around the circle, *y* increases and decreases along the vertical scale, cycling back and forth between values of +1 and −1. When *r* lies in the top half of the circle, *y* has values greater than 0. When it lies in the lower half, *y* is negative.

The angular motion of *r* can be converted to a linear mathematical form by plotting changes in sinω as the radius vector *r* sweeps out a full 360° circle, with the angle ω increasing from 0° to 90°, 180°, 270°, and back to 360° (= 0°). As before, sinω is defined as the ratio of the length of the opposite side *y* over the hypotenuse *r*.

The mathematical function sinω cycles smoothly from +1 to −1 and then back to +1 for each complete revolution of the radius vector *r*. At the starting point (ω = 0°), the length of the opposite side *y* is 0 and the radius is +1, so the value of sinω is 0/1, or 0. As the angle ω increases, the length of the opposite side of the triangle (*y*) increases in relation to the (constant) radius of the circle. When ω reaches 90°, sinω = +1 because the lengths of the opposite side and the radius (hypotenuse) are identical (1/1). At 180°, sinω has returned to 0 because the length of the opposite side (*y*) is again 0. For angles greater than 180°, the sinω values become negative because the opposite side of the triangle *y* now falls in negative rectangular coordinates (values below 0 on the vertical axis). Sinω values reach a minimum value of −1 at ω = 270° (−1/1). After that, sinω again begins to increase, returning to a value of 0 at ω = 360° (= 0°).



Converting angular motion to a sine wave (A) The sine of an angle is the length of the opposite side of a triangle over its hypotenuse. (B) This concept can be applied to a circle where the hypotenuse is the radius (amplitude = 1) and the length of the opposite side of the triangle varies from +1 to −1 along a vertical coordinate axis. (C) As the radius vector sweeps out a full circle and ω increases from 0° to 360°, the sine of ω changes from +1 to −1 and back to +1, producing a sine wave representation of circular motion and of Earth's precessional motion.

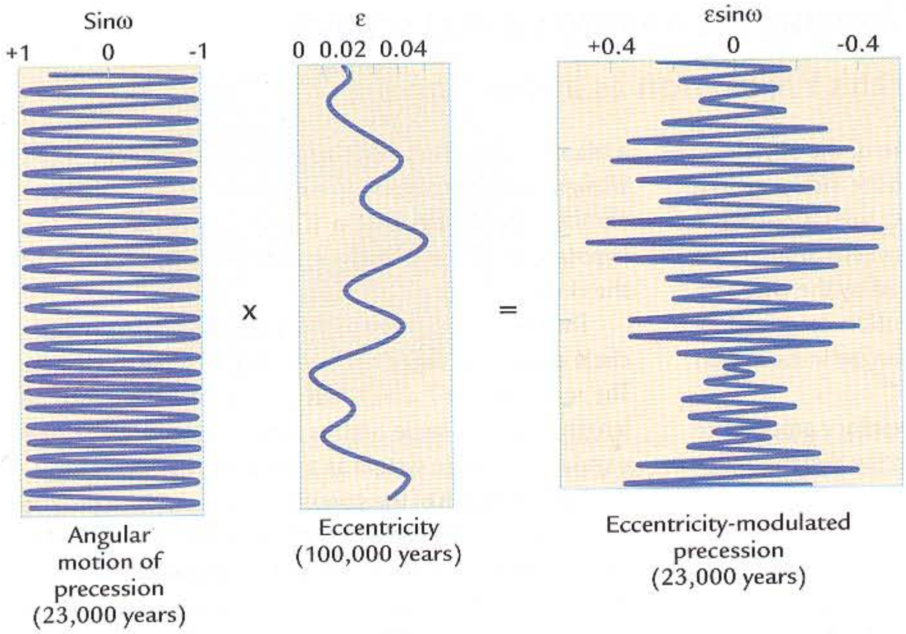


FIGURE 7-14 The precessional index The precessional index, εsinω, is the product of the sine wave function (sinω) caused by precessional motion and the eccentricity (ε) of Earth's orbit.

The two solstice positions shown in Figure 7-13 are extreme points in a continuously changing orbit. Precession also moves the solstices through orbital positions with intermediate Earth-Sun distances like those shown in Figure 7-11. In the next 11,000 years, the solstices will move from their present positions back to those shown at the bottom of Figure 7-13.

Eccentricity plays an important role in the effect of precession on the amount of solar radiation received on Earth. The full expression for this impact is εsinω, the **precessional index** (Figure 7-14). The sine part of this term is the sine wave representation of the movement of the equinoxes and solstices around the orbit (see Box 7-2). The eccentricity (ε) acts as a multiplier of the sinω term.

As noted earlier, the present value of ε is 0.0167. If this value remained constant through time, the εsinω index would cycle smoothly between values of +0.0167 and –0.0167 over each precession cycle of ~23,000 years. As shown in Section 7-4, however, the eccentricity of Earth's orbit varies through time, ranging between 0.005 and 0.06 (see Figure 7-7). These changes in ε cause the εsinω term to vary in amplitude (see Figure 7-14).

Long-term variations in the precessional index have two major characteristics (Figure 7-15). First, they occur at a cycle with a period near 23,000 years because of the regular angular motion of precession at that cycle (see Figure 7-14). Second, the individual cycles vary widely in amplitude because changes in eccentricity modulate the 23,000-year signal (see Box 7-1). At times the 23,000-year cycle swings back and forth between extreme maxima and minima; at other times the amplitude of the changes is small.

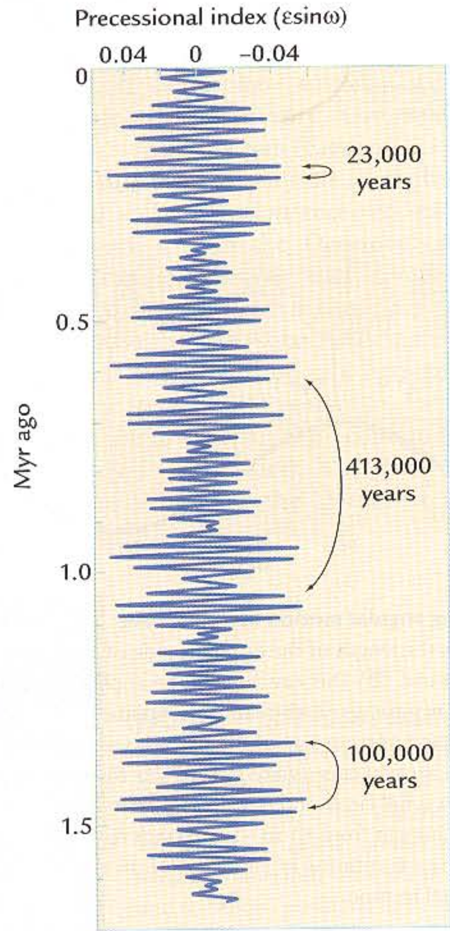


FIGURE 7-15 Long-term changes in precession The precessional index (εsinω) changes mainly at a cycle of 23,000 years. The amplitude of this cycle is modulated at the eccentricity periods of 100,000 and 413,000 years.

The changing values of $\epsilon\sin\omega$ affect the extreme perihelion and aphelion positions shown in Figure 7-13 by altering the distance between Earth and the Sun. With greater eccentricity, the differences in distance between a close pass and a distant pass are magnified. With a nearly circular orbit, differences in distance nearly vanish.

IN SUMMARY, changes in eccentricity magnify or suppress contrasts in Earth-Sun distance around the orbit at the 23,000-year precession cycle. These changes in distance to the Sun in turn alter the amount of solar radiation received on Earth (more radiation at the perihelion close-pass position, less at the distant-pass aphelion position).

The modulation of the $\epsilon\sin\omega$ signal by eccentricity is not a real cycle (see Box 7–1), even though this statement probably goes against your intuition. You have learned that eccentricity varies at cycles of 100,000 and 413,000 years (see Figures 7–7 and 7–14), and you can see that the upper and lower envelopes of the $\epsilon\sin\omega$ signal vary at these periods (see Figure 7-15). But the offsetting effects of the upper and lower envelopes cancel each other out.

For example, when the 23,000-year cycle is varying between large minima and large maxima, these adjacent minima and maxima are approximately equal in size. Over the longer (100,000-year) wavelengths of the eccentricity variations, the amplitudes of the shorter-term (23,000-year) oscillations cancel each other out, leaving a negligible amount of net variation. Similarly, short-term variations between small-amplitude maxima and minima at other times also offset each other. The importance of this point will become obvious in Chapters 9 and 11.

IN SUMMARY, the combined effects of eccentricity and precession cause the distance from the Earth to the Sun to vary by season, primarily at a cycle of 23,000 years. Times of high eccentricity produce the largest contrasts in Earth-Sun distance within the orbit, and conversely. As Earth precesses in its orbit, the changes in Earth-Sun distance are registered as seasonal changes in arriving radiation.

Changes in Insolation Received on Earth

Changes in Earth's orbit alter the amount of solar radiation received by latitude and by season. Climate scientists refer to the radiation arriving at the top of Earth's atmosphere as **insolation**. Some of this incoming insolation does not arrive at Earth's surface because clouds and other features in the climate system alter the amount that actually penetrates the atmosphere (companion Web site, pp. 2-4). Still, these calculations of insolation are the best guide to the effects of orbital changes on Earth's climate.

7-6 Insolation Changes by Month and Season

The long-term trends of tilt (see Figure 7-4) and $\epsilon\sin\omega$ (see Figure 7-15) contain all the information needed to calculate the amount of insolation arriving at any latitude and season. By convention, climate scientists usually show the amount of insolation (or the departures of insolation from a long-term average) during the solstice months of June and December in watts per square meter (W/m^2). Some studies use an alternate form, calories per square centimeter per second.

June and December insolation values over the last 300,000 years show a strong dominance of the 23,000-year precession cycle at lower and middle latitudes and also at higher latitudes during the summer season (Figure 7-16). Just like the $\epsilon\sin\omega$ precessional index, individual insolation cycles at lower latitudes occur at wavelengths near 23,000 years, but their amplitudes are modulated at periods of 100,000 and 413,000 years. The June and December monthly insolation curves at each latitude in Figure 7-16 are also opposite in sign. Both can vary by as much as 12% (40 W/m^2) around the long-term mean value for each latitude.

The 41,000-year cycle of tilt (obliquity) is not evident at lower latitudes but is visible in the low-amplitude variations of winter-season insolation at higher mid-latitudes (northern hemisphere January and southern hemisphere June at 60°). Summer season insolation changes at the tilt cycle are actually larger than those in winter, although this excess is not evident in these precession-dominated plots. One example is two precession cycles that are evident near 50,000 years ago in the June insolation signal for latitude 20°N but gradually blend and merge into a single tilt cycle at latitude 80°N (see Figure 7-16).

Changes in annual mean insolation at the 41,000-year tilt signal at high latitudes have the same sign as the summer insolation anomalies, but they are lower in amplitude. The lesser significance of winter season changes in tilt at full-polar latitudes results from the fact that no insolation at all arrives during long stretches of polar winter.

IN SUMMARY, monthly seasonal insolation changes are dominated by precession at low and middle latitudes, with the effects of tilt evident only at higher latitudes.

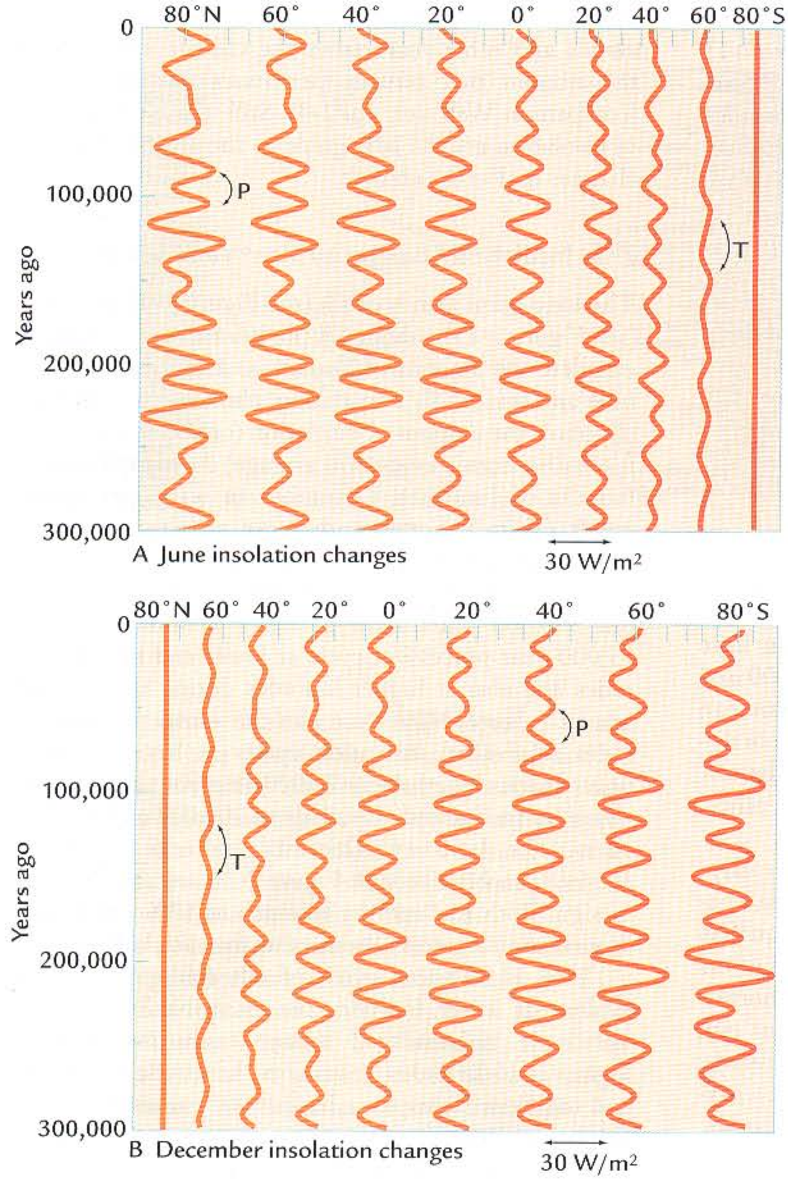
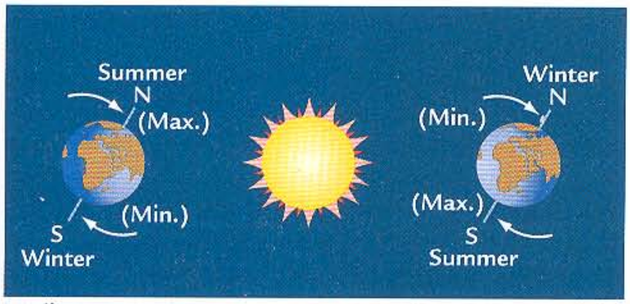


FIGURE 7-16 June and December insolation variations June and December monthly insolation values show the prevalence of precessional changes at low and middle latitudes and the presence of tilt changes at higher latitudes. Cycles of tilt and precession are indicated by T and P. The double arrows indicate variations of 30 W/m^2 for these signals.

As noted earlier, cycles of insolation change at 100,000 or 413,000 years are *not* evident in these signals because eccentricity is not a source of seasonal insolation changes. Actually, very small variations in received insolation do occur in connection with Earth's eccentric orbit around the Sun, but these appear only as changes in the total energy received by the entire Earth, not as seasonal variations. These changes are governed by the term $(1 - \epsilon^2)^{1/2}$. We have already seen that ϵ varies through time between 0.005 and 0.0607. Substituting these values for ϵ in the term above reveals that changes in total insolation received because of changes in eccentricity have varied by at most 0.002 (0.2%) around the long-term mean. Compared to changes in seasonal insolation of 10% or more at the tilt and precession cycles, these annual eccentricity changes are negligible (smaller by a factor of about 50).

The pattern of insolation changes for tilt and precession can be compared by season and by hemisphere (northern versus southern). Insolation variations at high latitudes caused by changes in tilt are *in phase* between the hemispheres from a seasonal perspective: tilt maxima in the northern winter solstice of December match tilt maxima in the southern winter solstice of June. With increased tilt (Figure 7-17A), summer (June) insolation maxima in the northern hemisphere occur at the same time in the 41,000-year cycle as summer (December) insolation maxima in the southern hemisphere on the opposite side of the orbit. Higher tilt produces more insolation at both poles in their respective summers



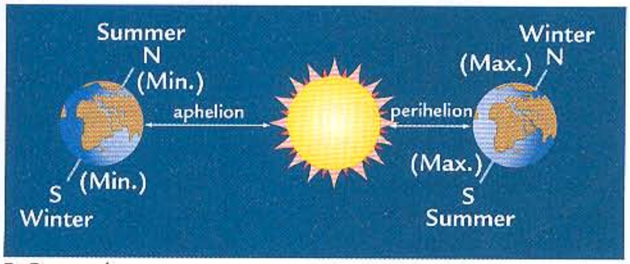


FIGURE 7-17 Phasing of insolation maxima and minima
(A) Tilt causes in-phase changes for polar regions of both hemispheres in their respective summer and winter seasons. (B) Precession causes out-of-phase changes between hemispheres for their summer and winter seasons.

because both poles are turned more directly toward the Sun. For the same reason, more pronounced insolation minima also occur at both winter poles for a higher tilt: the two winter poles are tilted away from the Sun during the same orbit.

If we compare the North Pole with the South Pole *at a particular month* in the orbit, however, the two hemispheres are exactly out of phase (see Figure 7-17A). The increased tilt angle that turns north polar regions more directly toward the Sun in northern hemisphere summer also tilts the southern polar regions farther away from the Sun at that same place in the orbit (southern hemisphere winter). As a result, tilt causes opposite insolation effects at the North and South poles for a given point in the orbit.

For precession, the relative sense of phasing between seasons and hemispheres is exactly reversed from that of tilt (Figure 7-17B). Because Earth-Sun distance is the major control on these changes in insolation, a position close to the Sun (at perihelion) produces higher insolation than normal over Earth's *entire* surface. A precessional-cycle insolation maximum occurring at June 21 (or December 21) will be simultaneous everywhere on Earth. Distant-pass positions (at aphelion) will simultaneously diminish insolation everywhere on Earth.

An important fact to remember about precession is that the seasons are reversed across the equator. As a result, an insolation maximum at June 21 is a *summer* insolation maximum in the northern hemisphere, but it is a *winter* insolation maximum in the southern hemisphere, where June 21 is the winter solstice. As a result of the seasonal reversal at the equator, insolation signals considered in terms of the season of the year are *out of phase* between the hemispheres for precession. This pattern is exactly opposite in sense to the in-phase pattern for tilt at high latitudes of both hemispheres.

Another way of looking at the relative phasing of precessional insolation is to track changes between seasons within a single hemisphere. The orbital position on the left in Figure 7-17B, which produces minimum summer (June 21) insolation in the northern hemisphere because it occurs at a distant position from the Sun (aphelion), must six months later cause maximum winter (December 21) insolation in the same hemisphere when Earth revolves around to the perihelion position (see Figure 7-17B right). As a result, precessional variations in insolation at any one location always move in opposite directions for the summer versus winter seasons.

Precessional changes in insolation have an additional characteristic not found in changes caused by tilt: an entire family of insolation curves exists for each season and month (and even day) of the year. As a matter of convention, insolation changes are typically shown only for the extreme solstice months of June and December, but in fact every season and month precesses into parts of the eccentric orbit that are alternately farther from the Sun and closer to the Sun at the same 23,000-year cycle.

As a result, each season and month experiences the same 23,000-year cycle of increasing and decreasing insolation values relative to the long-term mean, but the anomalies (departures from the mean) are offset in time from the preceding month or season. These offsets produce an entire family of monthly (and seasonal) insolation curves (Figure 7-18). Each successive month passes through perihelion (or aphelion) roughly 1916 years later than the previous month did ($1/12 \times 23{,}000 = 1916$).

7-7 Insolation Changes by Caloric Seasons

Calculations of monthly insolation are complicated by an additional factor related to the eccentricity of Earth's orbit. Earth gradually moves through a 360° arc in its orbit around the Sun, but this angular motion does not result in a constant rate of motion in space. Instead, Earth speeds up as it nears the extreme perihelion position and slows down near aphelion. As a result, as the solstices move slowly around the eccentric orbit, they gradually pass through regions of faster or slower movement in space.

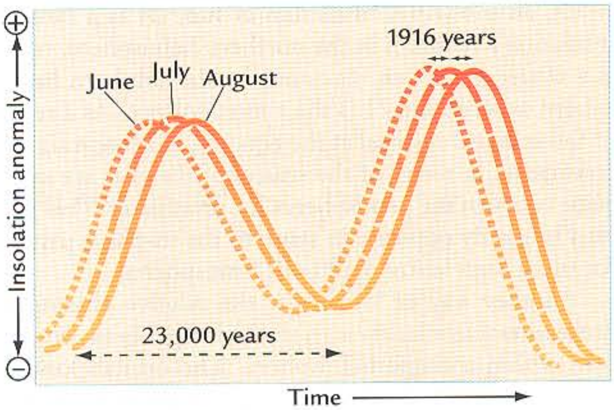


FIGURE 7-18 Family of monthly precession curves Because all seasons change position (precess) around Earth's orbit, each season (and month) has its own insolation trend through time. Monthly insolation curves are offset by slightly less than 2000 years (23,000 years divided by 12 months).

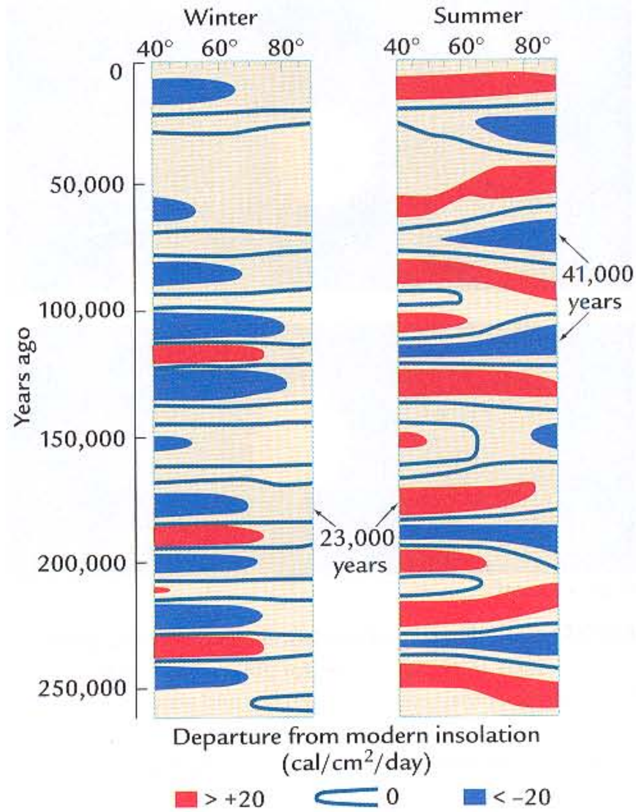


FIGURE 7-19 Caloric season insolation anomalies Plots of insolation anomalies for the summer and winter caloric half-year show a larger influence of tilt in relation to precession at higher latitudes than do the monthly anomalies. (Adapted from W. F. Ruddiman and A. McIntyre, "Oceanic Mechanisms for Amplification of the 23,000-Year Ice-Volume Cycle," *Science* 212 [1981]: 617–27.)

These changes in speed cause changes in the lengths of the months and seasons in relation to a year determined by "calendar time" (day of the year). The net effect is that changes in the amplitude of insolation variations in the monthly signals tend to be canceled by opposing changes in the lengths of the seasons. For example, times of unusually high summer insolation values at a perihelion position are also times of shorter summers. It is not obvious to scientists how to balance these two offsetting factors.

One way of minimizing these complications is to calculate the changes in insolation received on Earth within the framework of **caloric insolation seasons.** The summer caloric half-year is defined as the 182 days of the year when the incoming insolation exceeds the amount received during the other 182 days. Caloric seasons are not fixed in relation to the calendar because the insolation variations caused by orbital changes are added to or subtracted from different parts of the calendar year (see Figure 7-18). As a result, the caloric summer half-year falls during the part of the year we think of as summer, but it is not precisely centered on the June 21 summer solstice.

Changes in insolation viewed in reference to the half-year caloric seasons put a somewhat different emphasis on the relative importance of tilt and precession. Although low-latitude insolation anomalies are still dominated in both seasons by the 23,000-year precession signal, the 41,000-year tilt rhythm is much more obvious in high-latitude anomalies during the summer caloric half-year (Figure 7-19) than it is in the monthly insolation curves (see Figure 7-16). Another aspect of caloric season calculations is that the insolation values vary by a maximum of only ~5% around the mean, compared to variations as large as 12% for the monthly insolation changes.

Searching for Orbital-Scale Changes in Climatic Records

In the next four chapters we will explore abundant evidence that orbital-scale cycles are recorded in Earth's climate records. Many records contain two or even three superimposed orbital-scale cycles, and it can often be difficult to disentangle them visually.

For example, consider the three cycles shown in Figure 7-20A, with periods of 100,000 years, 41,000 years, and 23,000 years. These three cycles are equivalent to the three most prominent cycles of orbital change, but for simplicity they are shown as perfect sine waves rather than the more complex forms of the actual variations.

We can combine these three cycles by adding them together in various ways (Figure 7-20B). When the 23,000-year and 100,000-year cycles are combined, the resulting signal is obviously a simple addition of the two separate cycles. The two cycles are easy to distinguish because they differ in period by a factor of more than 4 (100,000 divided by 23,000 = 4.3).

It becomes more difficult to detect the two original signals when only the 23,000-year and 41,000-year cycles are combined. Because the periods of these two cycles are more similar, they reinforce and cancel each other in somewhat complicated ways. The task becomes even more difficult when all three cycles are combined, as in the bottom curve of Figure 7-20B. It is not at all obvious to the eye that this signal is a simple addition of three perfect sine waves.

In the case of Earth's actual climate records, the situation is even more complex because the three cycles are not only superimposed on each other but also

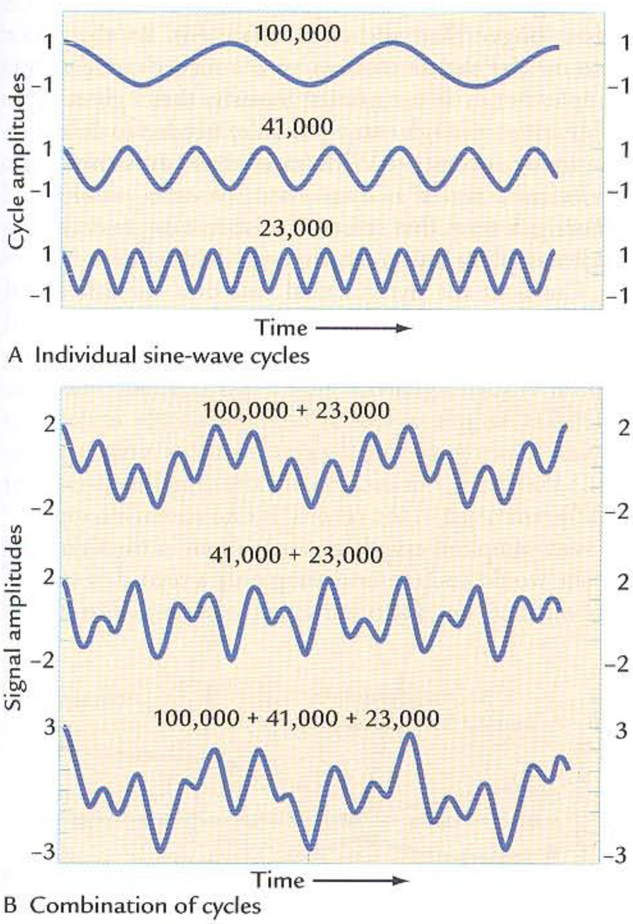


FIGURE 7-20 Complications from overlapping cycles If perfect sine wave cycles with periods of 100,000, 41,000, and 23,000 years are added together so that they are superimposed on top of one another, the original cycles are almost impossible to detect by eye in the combined signal.

change in amplitude through time (see Figures 7-4 and 7-15). Obviously, it will be impossible to disentangle all this information simply by eye.

7-8 Time Series Analysis

To simplify analyses of cyclic variations in climate changes, scientists use **time series analysis.** The term "time series" refers to records plotted against age (time). These techniques extract rhythmic cycles embedded within records of climate.

The first step in time series analysis is to convert climatic records to a time framework. After individual measurements of a climatic indicator have been made (for example, across an interval in a sediment core), all available sources of dating are used to define the ages of particular levels within the sequence. A complete time scale for the sequence is then created by interpolating the ages of all sediment depths between the dated levels. This time scale can then be used to plot the climatic record against time for further analysis.

One technique is **spectral analysis.** Modern techniques of spectral analysis are beyond the scope of this book, but we need at least a basic sense of how this technique detects cycles in records of past climate change. One way to visualize what happens in spectral analysis is to imagine taking a climate record plotted on a time axis and gradually sliding a series of sine waves of different periods across it. As this is done, the correlation between each sine wave and the full climatic signal is measured for each point in the sliding process. If the climate record that is being examined contains a strong cycle at one of the sine wave periods, the climate record will show a strong correlation with that sine wave at some point in the sliding process. The strong correlation indicates that the climatic signal contains a strong cycle at that period. As this process is repeated for different sine waves with different periods, other cycles may emerge.

Now we return to the example of the three superimposed cycles in the bottom part of Figure 7-20B. A spectral analysis run on this signal will extract the three component (orbital) cycles, which can be displayed on a plot called a **power spectrum** (Figure 7-21). The horizontal axis shows a range of periods plotted on a log scale, with the shorter periods to the right. The vertical axis represents the amplitude of the cycles (see Box 7-1), also known as their "power." The height of the lines plotted on the power spectrum is related to the square of the amplitude of the cycle at that period.

For the example shown in Figure 7-21, all three cycles detected by spectral analysis plot as narrow "line spectra," with their power concentrated entirely at the periods shown by the solid vertical lines. In this idealized example, no power occurs anywhere else in the spectrum than at these three cycles.

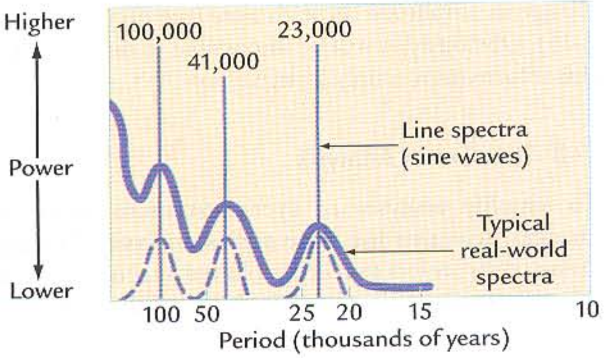


FIGURE 7-21 Spectral analysis Spectral analysis reveals the presence of cycles within complex climate signals. In this example, the original sine wave cycles from Figure 7-20A form line spectra (vertical bars) whose heights indicate their amplitudes. Actual climate records have peaks that are spread over a broader range of periods (dashed line and curving solid line).

In actual studies of climate, however, power spectra are never this simple. One reason is that even the most regular-looking orbital cycles such as the tilt changes in Figure 7-4 are not perfect sine waves but instead vary over a small range of periods. In addition, errors in dating records of climate change or in measuring their amplitude also have the effect of spreading power over a broader range of periods than would be the case for perfectly measured and dated signals. As a result of these complications, the total amount of power associated with each cycle looks like the area under the dashed curves in Figure 7-21.

Still another reason that real-world spectra are more complicated is that random noise exists in the climate system, consisting of irregular climatic responses not concentrated at orbital or other cycles. In most records, the effect of noise is spread out over a range of periods in the spectrum. In general, the amount of power tends to be larger at longer periods. As a result, spectra from real-world climatic signals tend to look like the thick curved line in Figure 7-21. The spectral peaks that rise farthest above the baseline of the trend are the most significant (believable) ones in a statistical sense.

A second useful time series analysis technique is called **filtering.** This technique extracts individual cycles at a specific period (or narrow range of periods) from the complexity of the total signal. This process is often referred to as "band-pass filtering" (filtering of a narrow band or range of the many periods present in a given signal). Filtering is analogous to using glasses with colored lenses to filter out all colors of the light spectrum except the one color (wavelength) we wish to see.

Filters are constructed directly from well-defined peaks in power spectra like those in Figure 7-21. The highest point on the spectral peak defines the central period of the filter, and the sloping sides of the spectral peak define the shape of the rest of the filter.

To understand the importance of filtering, consider again the three hypothetical sine waves in Figure 7-20. We can create filters for these three cycles based on the peaks in the power spectrum shown in Figure 7-21. If we pass these filters across the combined signal at the bottom of Figure 7-20B, the filters will extract the original form of all three individual cycles (at 23,000, 41,000, and 100,000 years). In effect, the filtering operation extracts the time-varying shapes of individual cycles embedded in the complexities of actual climate records.

7-9 Effects of Undersampling Climate Records

The technique of spectral analysis can be used only for a specific range of cycles within any climate record. Confident identification of a cycle by time series analysis requires that the cycle be repeated at least four times in the original record (the record must be at least four times longer than the cycle analyzed). At the other extreme (for the shortest cycles in a record), at least two samples per cycle are needed to verify that a given cycle is present, although many more are needed to define its amplitude accurately. With fewer than two samples per cycle, time series analysis runs into the problem of **aliasing,** a term that refers to false trends generated by undersampling the true complexity in a signal.

Consider the hypothetical case of a climatic signal that has the form of the 23,000-year cycle of orbital precession, with the wide range of amplitude variation typical of such signals (Figure 7-22). Assume that three scientists sample a record containing this underlying signal. All three sample the record at an average spacing of 23,000 years, but each begins the sampling process at a different place in the record. If one scientist happened to start sampling exactly at a maximum in the signal, he or she would end up measuring only a record of successive maxima, but if another scientist happened to start at

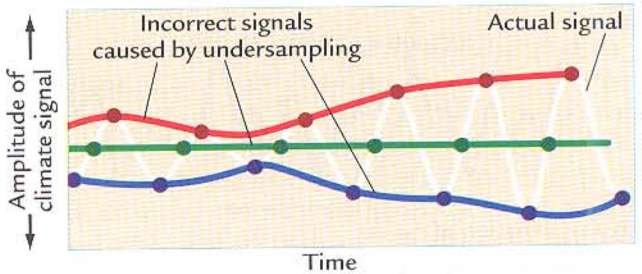


FIGURE 7-22 Aliasing (undersampling) of climate signals Undersampling of a climate signal (in this case one that is a direct response to changes in orbital precession) can produce aliased climate signals completely unlike the actual one.

a minimum, the record would show only successive minima. These sampling attempts give completely different results because they are persistently biased toward different sides of a highly modulated cycle. If the third scientist happened to start sampling exactly at a crossover point between minima and maxima, the scientist might extract a record suggesting that no signal exists at all.

These differences show the danger of aliasing. Although this example is obviously chosen to show the worst possible effect of aliasing, undersampling is a problem in most climate records.

7-10 Tectonic-Scale Changes in Earth's Orbit

Over time scales of hundreds of millions of years, some of Earth's orbital characteristics slowly evolved, as shown by evidence in ancient corals. Corals are made of banded $CaCO_3$ layers caused by changes in environmental conditions. The primary annual banding reflects seasonal changes in sunlight and water temperature (Chapter 2). A secondary banding follows the tidal cycles created by the Moon and Sun. The tidal cycles also affect water depth and other factors in the reef environment that influence coral growth.

Corals from 440 Myr ago show 11% more tidal cycles per year than modern corals do, implying that Earth spun on its rotational axis 11% more times per year than at present. As a result, each year had 11% more days. Gradually over the last 440 Myr, the spin rate and number of days decreased to their current levels. This gradual slowing in Earth's rate of rotation was caused by the frictional effect of the tides.

Other changes in Earth's orbit that can be inferred from this kind of information, such as changes in Earth-Moon distance, are thought to have affected the wavelengths of tilt and precession over tectonic-scale intervals. One estimate of the slow, long-term increases in the periods of tilt and precession toward their present values is shown in Figure 7-23.

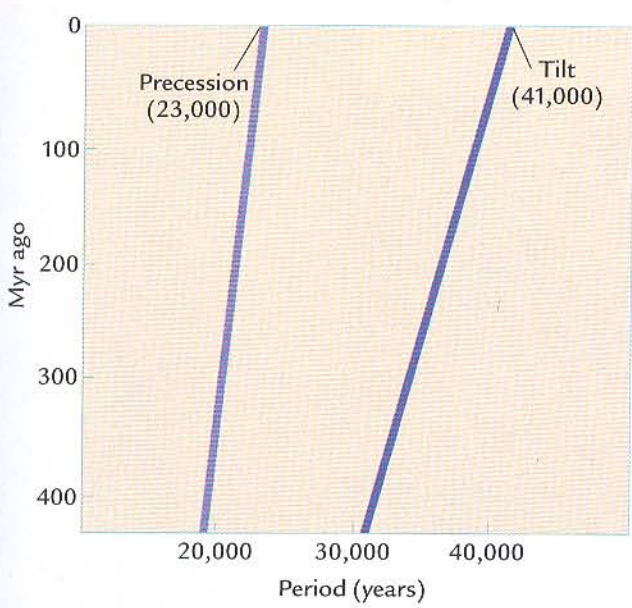


FIGURE 7-23 Tectonic-scale orbital changes Gradual changes in Earth's orbit over long tectonic time scales have caused a slow increase in the periods of the tilt and precession cycles. (Adapted from A. Berger et al., "Pre-Quaternary Milankovitch Frequencies," *Nature* 342 [1989]: 133–34.)

Key Terms

plane of the ecliptic (p. 120)
tilt (p. 120)
solstices (p. 120)
equinoxes (p. 120)
perihelion (p. 120)
aphelion (p. 120)
wavelength (p. 122)
period (p. 122)
frequency (p. 122)
amplitude (p. 122)
modulation (p. 122)
sine waves (p. 122)
eccentricity (p. 123)
axial precession (p. 124)
precession of the ellipse (p. 124)
precession of the equinoxes (p. 124)
precessional index (p. 128)
insolation (p. 129)
caloric insolation seasons (p. 132)
time series analysis (p. 133)
spectral analysis (p. 133)
power spectrum (p. 133)
filtering (p. 134)
aliasing (p. 134)

Review Questions

1. Why does Earth have seasons?
2. When is Earth closest to the Sun in its present orbit? How does this "near-pass" position affect the amount of radiation received on Earth?
3. Describe in your own words the concept of modulation of a cycle.
4. Earth's tilt is slowly decreasing today. As it does so, are the polar regions receiving more or less solar radiation in summer? In winter?
5. How is axial precession different from precession of the ellipse?
6. How does eccentricity combine with precession to control a key aspect of the amount of insolation Earth receives?
7. Do insolation changes during summer and winter have the same or opposite timing (sign) at any single location on Earth? Why or why not?

8. Do the following changes occur at the same time (same year) in Earth's orbital cycles?
   a. summer insolation maxima at both poles caused by changes in tilt
   b. summer insolation maxima in the tropics of both hemispheres caused by precession

Additional Resources

Basic Reading

Companion Web site at www.whfreeman.com/ruddiman2e, pp. 2–4.

Imbrie, J., and K. P. Imbrie, 1979. *Ice Ages: Solving the Mystery*. Short Hills, NJ: Enslow.

Ruddiman, W. F. 2005. *Plows, Plagues and Petroleum*, Chapter 3. Princeton, NJ: Princeton University Press.

www.classzone.com/book/earth_science/terc/content/visualization, Chapter 15: Earth's Orbital Motions.

Advanced Reading

Berger, A. L. 1978. "Long-Term Variations of Caloric Insolation Resulting from the Earth's Orbital Elements." *Quaternary Research* 9: 139–167.