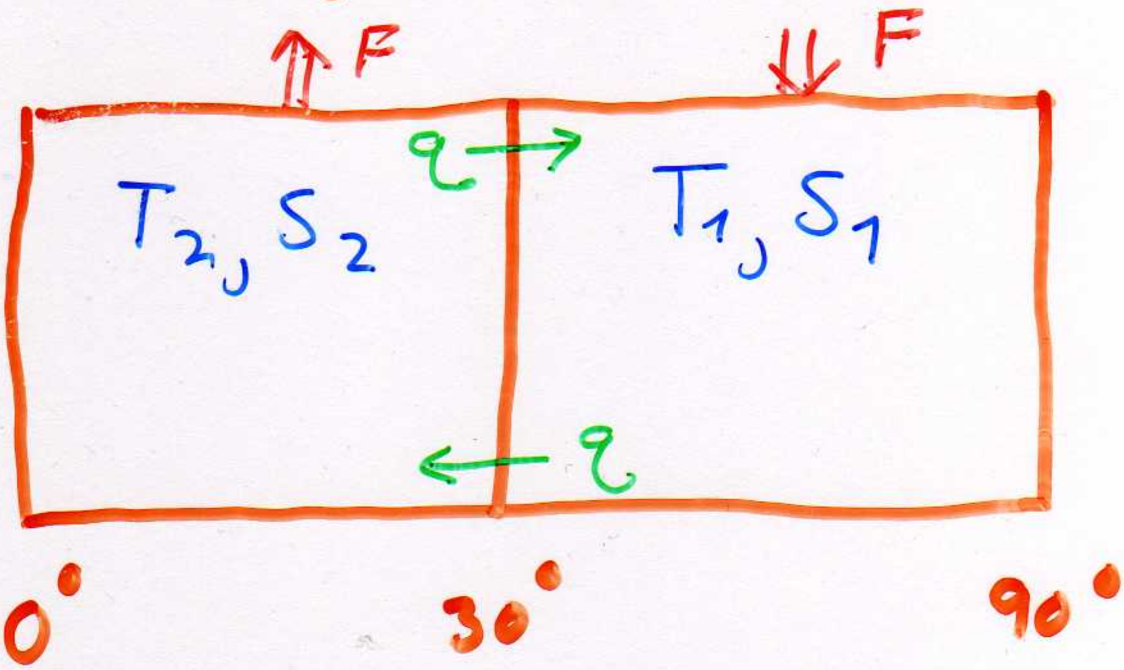
2 Box, THC Model



(1) $$\rho = \rho_0 (1 - \alpha T + \beta S)$$

(equation of state)

(2) $$q = -\kappa^{-1} \rho_0^{-1} (\rho_2 - \rho_1)$$

(flow law)

$\Rightarrow$ (3) $$q = \kappa^{-1} [\alpha (T_2 - T_1) - \beta (S_2 - S_1)]$$

Consider the case for

$T_1, T_2 \rightarrow$ constant

and for a fresh water flux from box 2 to box 1.

Steady-state, salt conservation yields

(4) $\quad -F S_0 + |q| (S_2 - S_1) = 0$

Solve for $(S_2 - S_1)$ in (3) and substitute in (4),

(5) $\quad q|q| - \frac{\alpha (T_2 - T_1)|q|}{k} + \frac{F S_0 \beta}{k} = 0$

i.e. The system is non-linear

For $q > 0$, we then have

$$q = \frac{b \pm \sqrt{b^2 - 4c}}{2}$$

with $b = \frac{\alpha \Delta T}{k}$, $c = \frac{F S_0 \beta}{K}$

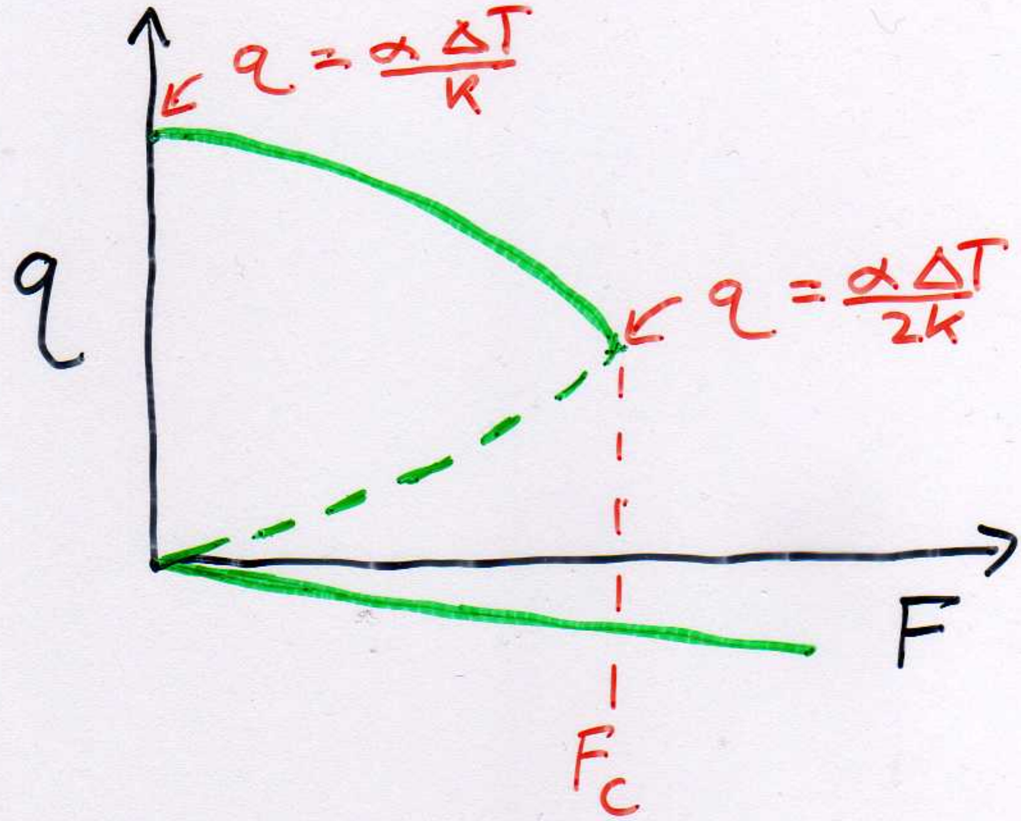
There are two solutions as long as $b^2 > 4c$

or $F \leq \frac{\alpha^2 \Delta T^2}{4 S_0 \beta K} = F_c$

For $q < 0$, we have

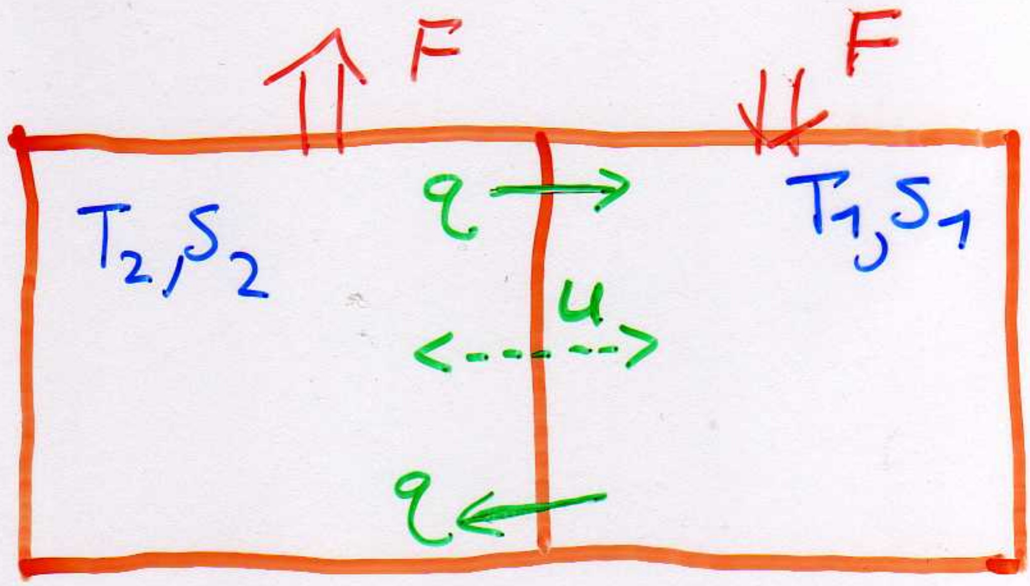
$$q = \frac{b - \sqrt{b^2 + 4c}}{2}$$

There is one solution for all values of F



For $F < F_c$, there are two stable solutions, one with large, positive q and one with small, negative q. There is also one unstable solution with small positive q.

2 Box, THC Model with "wind-driven circulation"



Now steady-state salt conservation becomes

$$(6) \quad -FS_0 + |q|(S_2 - S_1) + u(S_2 - S_1) = 0$$

and the governing equation(s) becomes

(7) $$q|q| - \frac{\alpha \Delta T}{k}|q| + uq + \frac{F S_0 \beta}{k} - \frac{u \alpha \Delta T}{k} = 0$$

where $\Delta T = T_2 - T_1$

For the case with $q > 0$

$$q = \frac{\left(\frac{\alpha \Delta T}{k} - u\right) \pm \sqrt{\left(u - \frac{\alpha \Delta T}{k}\right)^2 + 4\left(\frac{u \alpha \Delta T}{k} - \frac{F S_0 \beta}{k}\right)}}{2}$$

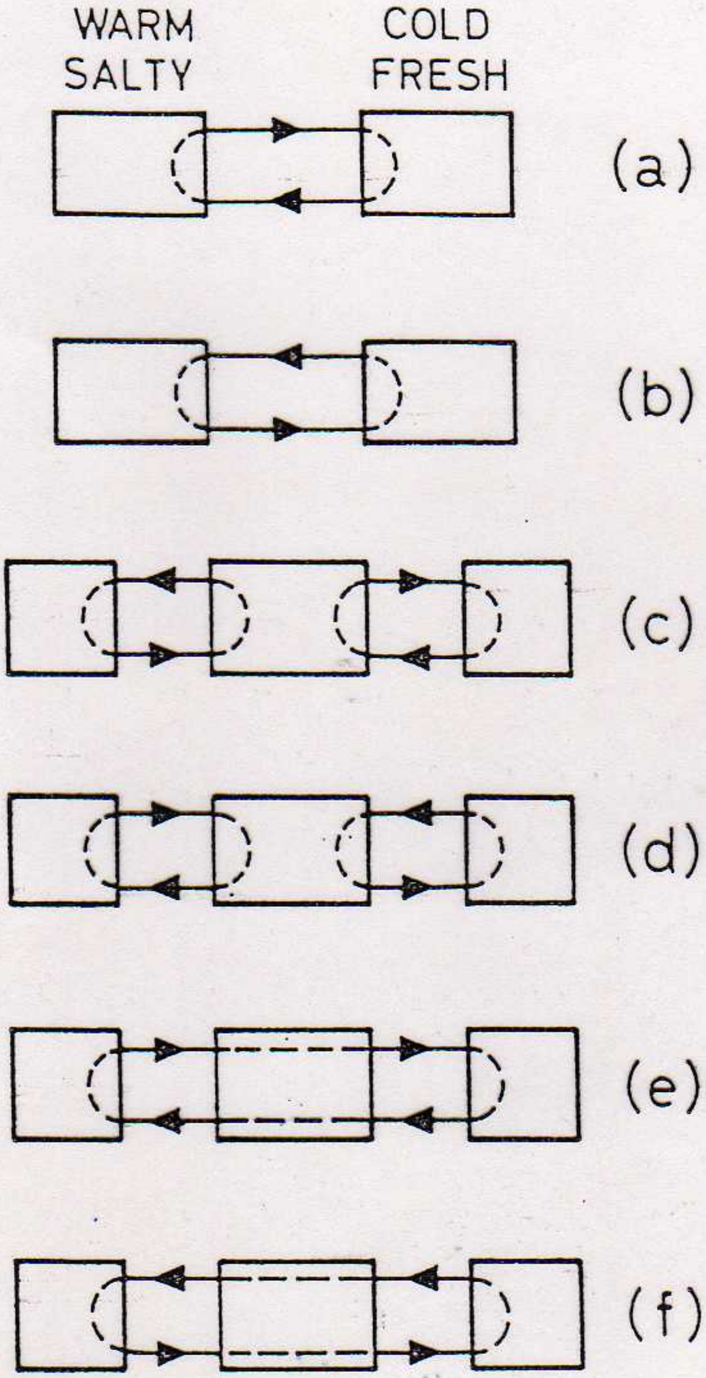


Fig. 5 (a) The stable fast-cell solution in Stommel's two-box model. The thermal torque dominates over a braking haline torque.
(b) The stable slow-cell solution in the same model. The haline torque dominates over a braking thermal torque, producing a reversed convection cell. Between these two cases exists a third, unstable solution. See the text for details.
(c), (d) Symmetric three-box solutions, corresponding to two solutions of type (a) and type (b), respectively, placed back to back.
(e), (f) Asymmetric solutions, corresponding to one solution of type (a) and one of type (b), placed back to back.