

THC steady state stability

$$a \dot{\Delta s} = q|q| - b|q| + c \quad (1)$$

where a is a box volume, b is $\frac{\alpha \Delta T}{K}$
and c is $\frac{FS_0 \beta}{K}$.

Consider perturbations about steady state solutions and
Take $\Delta s = \bar{\Delta s} + \Delta s'$, $q = \bar{q} + q'$

for $q > 0$

(1) becomes: $a(\dot{\bar{\Delta s}} + \dot{\Delta s}') = (\bar{q} + q')(\bar{q} + q') - b(\bar{q} + q') + c$

$$\Rightarrow a(\dot{\Delta s}') = (\bar{q}^2 - b\bar{q} + c) - b q' + 2\bar{q} q' + (q')^2$$

$$\Rightarrow a(\dot{\Delta s}') = (2\bar{q} - \frac{\alpha \Delta T}{K}) q'$$

$$\boxed{\text{for } \bar{q} > \frac{\alpha \Delta T}{2K}}$$

$$q' > 0 \Rightarrow \Delta s' > 0$$

which tends to decrease $q' \Rightarrow$ Stable

$$\boxed{\text{For } \bar{q} < \frac{\alpha \Delta T}{2K}}$$

$$q' > 0 \Rightarrow \Delta s' < 0$$

which tends to increase $q' \Rightarrow$ Unstable