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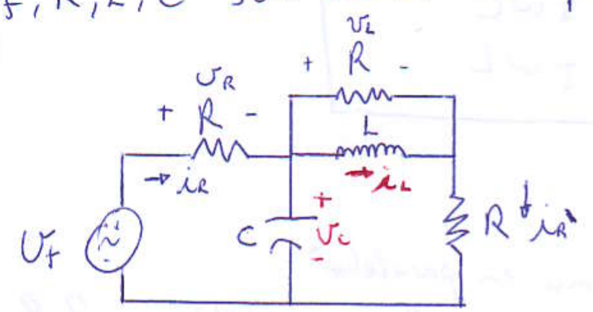
PROBLEMA 1

En el circuito de la figura, encuentre el "sistema de ecuaciones diferenciales" que permiten encontrar los valores de i_L y v_C. V_f, R, L, C son datos del problema.



Sol:

$$v_F = v_R + v_C \qquad v_R = R \cdot i_R \qquad \text{con} \quad i_R = \left(i_C + i_L + \frac{v_L}{R}\right)$$

$$\Rightarrow v_F = R\left(C \cdot \frac{dv_C}{dt} + i_L + \frac{v_L}{R}\right) + v_C$$

$$v_f = R \cdot C \cdot \frac{dv_C}{dt} + L \cdot \frac{di_L}{dt} + R i_L + v_C \qquad (*)$$

$$v_C = v_L + v_R \qquad\qquad v_R = R \cdot i_R'$$

$$i_R' = i_L + \underbrace{L \cdot \frac{di}{dt}}_{v_L} \cdot \frac{1}{R}$$

$$\Rightarrow v_C = L \frac{di_L}{dt} + R i_L + L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{v_C}{2L} - \frac{R}{2L} i_L \quad (1) \qquad \text{Reemplazando en } (*)$$

$$v_f = R \cdot C \frac{dv_C}{dt} + \frac{v_C}{2} - \frac{R}{2} i_L + R i_L + v_C$$

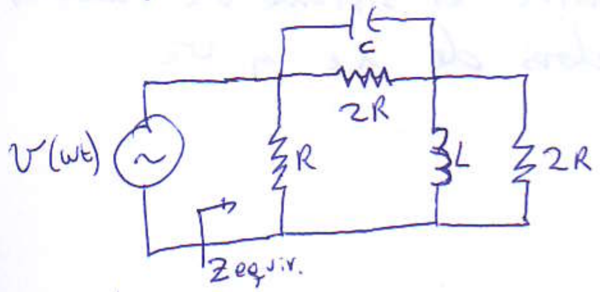
$$\Rightarrow \frac{dv_C}{dt} = -\frac{3}{2}\frac{v_C}{RC} - \frac{1}{2C} i_L + \frac{v_f}{RC} \quad (2)$$

$\therefore$ El sistema es:

$$\boxed{\begin{aligned} \frac{di_L}{dt} &= \frac{v_C}{2 \cdot L} - \frac{R}{2 \cdot L} i_L \\ \frac{dv_C}{dt} &= -\frac{3}{2}\frac{v_C}{RC} - \frac{1}{2C} i_L + \frac{v_f}{RC} \end{aligned}}$$

PROBLEMA 2

Encuentre la impedancia equivalente del sistema; si es que el voltaje de entrada, es de frecuencia "ω".



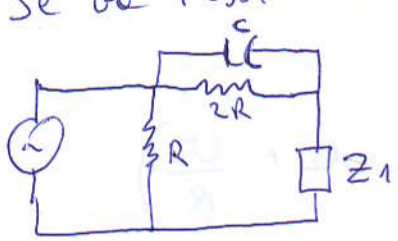
Recordar que:

$$Z_C = \frac{1}{J\omega C}$$

$$Z_L = J\omega L$$

SOL:

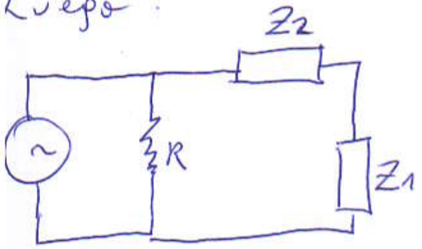
Se va resolviendo el circuito por partes:



"suma en paralelo"

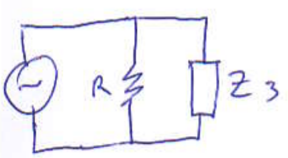
$$Z_1 = J\omega L // 2R = \left(\frac{1}{J\omega L} + \frac{1}{2R}\right)^{-1} = \frac{J\omega 2RL}{J\omega L + 2R} = \frac{2R}{1 + \frac{2R}{J\omega L}}$$

Luego:



$$Z_2 = \frac{1}{J\omega C} // 2R = \left(J\omega C + \frac{1}{2R}\right)^{-1} = \frac{\frac{2R}{J\omega C}}{\frac{1}{J\omega C} + 2R} = \frac{2R}{1 + J\omega 2RC}$$

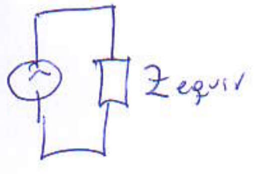
Luego



"suma en serie"

$$Z_3 = Z_1 + Z_2 = \frac{2R}{1 + \frac{2R}{J\omega L}} + \frac{2R}{1 + J\omega 2RC} = \frac{2R\left(1 + J\omega 2RC + 1 + \frac{2R}{J\omega L}\right)}{\left(1 + \frac{2R}{J\omega L}\right)\left(1 + J\omega 2RC\right)}$$

Finalmente se suman ambas en paralelo



$$Z_{equiv} = R // Z_3 = \frac{\dfrac{2R^2\left(1 + J\omega 2RC + 1 + \frac{2R}{J\omega L}\right)}{\left(1 + \frac{2R}{J\omega L}\right)\left(1 + J\omega 2RC\right)}}{R + \dfrac{2R\left(1 + J\omega 2RC + 1 + \frac{2R}{J\omega L}\right)}{\left(1 + \frac{2R}{J\omega L}\right)\left(1 + J\omega 2RC\right)}}$$

Simplificar!! //