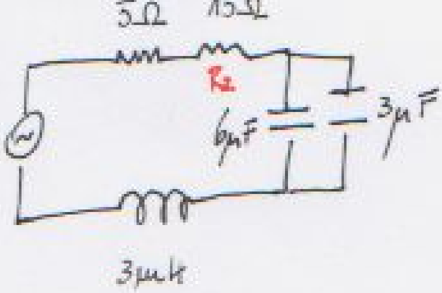
P1



$f = 1\,KHz$

$$V(t) = 20 \operatorname{sen}(\omega t + 30) = 20 \cos(\omega t + 30 - 90)$$

$$V(t) = \operatorname{Re}\left(20\, e^{i(\omega t - \pi/3)}\right)$$

a) Impedancia total

$$Z = R + i\omega L + \frac{1}{i\omega C}$$

$R = 20\ \Omega$

$C = 9\ \mu F$

1 pto por plantear bien la fórmula y calcular bien Req y Ceq.

$$Z = 20 + i\, 2\pi 10^3 \times 3\times 10^{-3} - \frac{i}{2\pi 10^3 \times 9\times 10^{-6}}$$

$$Z = 20 + i6\pi - i\,17{,}68$$

$$\boxed{Z = 20 + i\cdot 1{,}17\ \Omega}$$

0,5 pto con las unidades correctas

en $R = 15\,\Omega$

Corriente de la fuente: $I(t) = \frac{V_{in}}{Z}$

$$V_{R2}(t) = I(t) \cdot Z_{R2}$$

$$= \frac{20\, e^{i(\omega t - \pi/3)}}{(20 + i\, 1{,}17)} \cdot 15 \ [V]$$

$$V_{R2} = \underbrace{\frac{20 \cdot 15}{20 + i \cdot 1{,}17}}_{c}\, e^{i(\omega t - \pi/3)}$$

0,5 pto

$$c \in \mathbb{C} \Rightarrow c = |c|\, e^{i\delta}$$

donde $|c|$ es el Voltage (en módulo) y δ es la fase con respecto a la fuente

$$|c| = \frac{20 \cdot 15}{\sqrt{20^2 + 1{,}17^2}} \simeq 15 \ [V]$$

0,5 pto

$$c = \frac{20 \cdot 15}{20 + 1{,}17i} \cdot \frac{20 - 1{,}17i}{20 - 1{,}17i} = \frac{20 \cdot 15}{20^2 + 1{,}17^2}(20 - 1{,}17i)$$

$$c = \frac{20 \cdot 15}{\sqrt{20^2 + 1,17^2}} \frac{(20 - 1,17i)}{\sqrt{20^2 + 1,17^2}} = |c| \underbrace{\frac{(20 - 1,17i)}{\sqrt{20^2 + 1,17^2}}}_{e^{i\delta}}$$

$$e^{i\delta} = \frac{20 - 1,17i}{\sqrt{20^2 + 1,17^2}} = \cos\delta + i \operatorname{sen}\delta = \frac{1}{\sqrt{20^2 + 1,17^2}} (20 - i1,17)$$

igualando parte real y parte imaginaria se tiene

$$\left.\begin{array}{l} \cos\delta = 20 \\ \operatorname{sen}\delta = -1,17 \end{array}\right\} \quad \tan\delta = \frac{-1,17}{20} \Rightarrow \begin{array}{l} \delta = -3,3^\circ \\ \delta = -0,05 \text{ rad} \end{array}$$

0,5
pto

$$V(t) = 50 \operatorname{sen}(wt) = 50\, e^{i(wt - \pi/2)}$$

Frecuencia natural

$$w_0^2 = \frac{1}{LC}$$ Despreciando la resistencia

$$\Rightarrow \quad w_0 = 117363 \ \text{rad/s} \quad \text{o} \quad f_0 = 18679 \ \text{Hz}$$

0,3 pto

$$Z = 50 + iwL + \frac{1}{iwC}$$

$$I(t) = \frac{V_{in}}{Z} = \frac{50\, e^{i(wt-\pi/2)}}{\left(50 + iwL - \frac{i}{wC}\right)}$$

$$I(t) = \underbrace{|I(w)|\, e^{i\phi}}_{I(w)\, \in\, \mathbb{C}}\, e^{i(wt-\pi/2)}$$

$$I(w) = \frac{50}{\left(50 + iwL - \frac{i}{wC}\right)} \frac{wC}{wC} = \frac{50wC}{\left(RwC + iw^2LC - \frac{i}{\cancel{wC}}\cancel{wC}\right)} \frac{R}{R}$$

$$I(w) = \frac{50w\tau}{R} \cdot \frac{1}{w\tau + i\left(\frac{w^2}{w_0^2} - 1\right)} \quad \text{con} \quad RC = \tau$$

0,45

$$\boxed{|I(\omega)| = \omega\tau \cdot \frac{50}{R} \frac{1}{\sqrt{\omega\tau^2 + \left(\left(\frac{\omega}{\omega_0}\right)^2 - 1\right)^2}}}$$

$$\lim_{\omega \to 0} |I(\omega)| \to 0 \quad (\sim \omega)$$ 0,2 pto

$$\lim_{\omega \to \infty} |I(\omega)| \to 0 \quad (\sim 1/\omega)$$ 0,2 pto

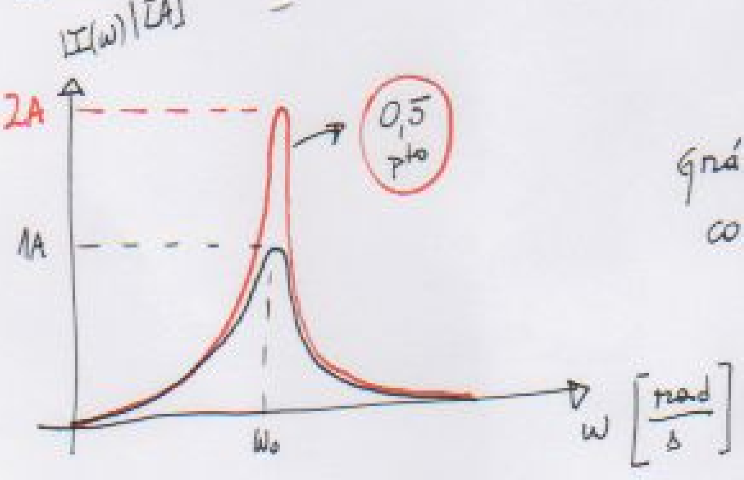


gráfico correcto 0,4 pto

Voltage en el condensador para $\omega = \omega_0$

$$V_c(t) = Z_c \cdot I(t)$$

$$V_c(t) = \frac{1}{i\omega c} |I(\omega)| e^{i\phi} e^{i(\omega t - \pi/2)}$$

$$|V_c(\omega = \omega_0)| = \frac{1 \text{ Ampere}}{\omega_0 c} \simeq 2582 \text{ V}$$ 0,75 pto