FI2002-5 2009-2

Ejercicio 1

Solución.

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\int\frac{\sigma(\vec{r}\,')(\vec{r}-\vec{r}\,')dS'}{|\vec{r}-\vec{r}\,'|^3}=\frac{1}{4\pi\epsilon_0}\int\frac{\sigma_0 sen(\phi)(z\hat{z}-\rho\,\hat{\rho})\rho\, d\rho\, d\phi}{(z^2+\rho^{2})^{3/2}}$$

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2}\int_0^a\frac{\sigma_0 sen(\phi)(z\hat{z}-\rho\,\hat{\rho})\rho\, d\rho\, d\phi}{(z^2+\rho^{2})^{3/2}}$$

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2}\int_0^a\frac{\sigma_0 sen(\phi)(z\hat{z}-\rho\,(\cos\phi\,\hat{x}+sen\,\phi\,\hat{y}))\rho\, d\rho\, d\phi}{(z^2+\rho^{2})^{3/2}}$$

Usando que $sen\,\phi$ y $sen\,\phi\cos\phi$ son funciones impares y no así $sen^2\phi$:

$$\vec{E}=\frac{-1}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2}\int_0^a\frac{\sigma_0 sen^2\phi\,\hat{y}\,\rho\, d\rho\, d\phi}{(z^2+\rho^{2})^{3/2}}$$

,pero:

$$\int_{-\pi/٢}^{\pi/٢} sen^2(\phi)\,d\phi=\int_{-\pi/٢}^{\pi/٢}\left(\frac{١}{2}-\frac{\cos(2\phi)}{2}\right)d\phi=\frac{1}{2}(\pi/2--\pi/2)-\frac{sen(2\pi/2)}{4}-\frac{-sen(-2\pi/2)}{4}=\frac{\pi}{2}$$

$$\Rightarrow\vec{E}=\frac{\sigma_0\pi}{8\pi\epsilon_0}\int_0^a\frac{\rho^{2}\,d\rho\,\hat{y}}{(z^2+\rho^{2})^{3/2}}=\frac{\sigma_0\,\hat{y}}{8\epsilon_0}\left(\frac{-\rho}{\sqrt{\rho^{2}+z^2}}+\ln(\rho+\sqrt{\rho^{2}+z^2})\right)\Bigg|_0^a$$

$$\vec{E}=\frac{\sigma_0\,\hat{y}}{8\epsilon_0}\left(\frac{-a}{\sqrt{a^{2}+z^2}}+\ln(a+\sqrt{a^{2}+z^2})-\ln(z)\right)$$