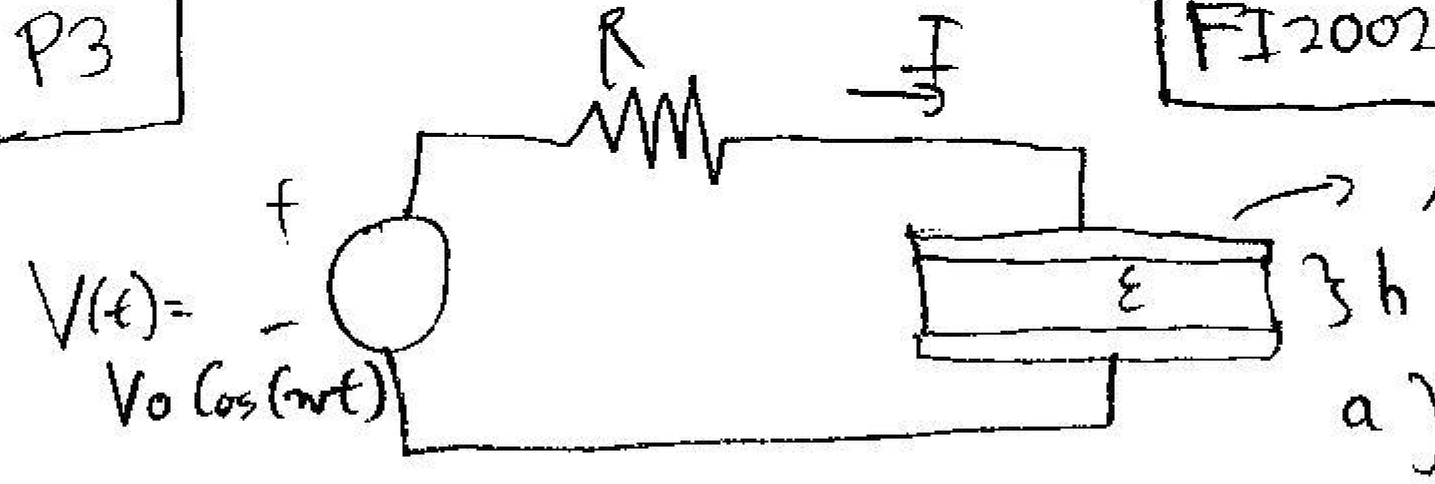
P3

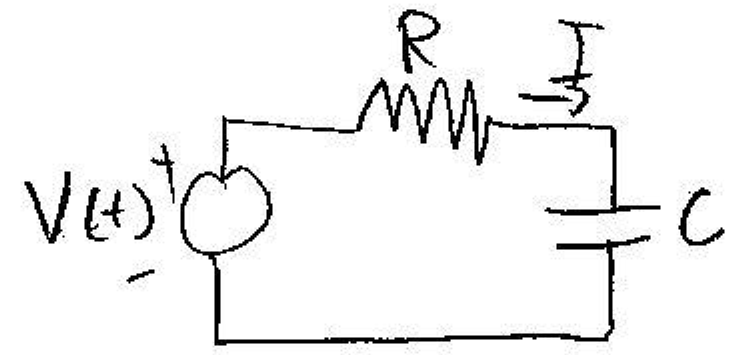
FI2002 Pauta C3



$Q(0) = Q_0$

a) ¿$I = I(t)$? b) ¿$\frac{d\vec{D}}{dt}$?

a) Calculamos la capacitancia $\boxed{C = \frac{\varepsilon \cdot A}{h}}$ (0,5 ptos) Luego circuitalmente:



LVK $\Rightarrow V(t) = R \cdot I + V_c(t)$ Pero $I = I_c = C\frac{dV_c}{dt} \Rightarrow V(t) = RC\frac{dV_c}{dt} + V_c$

$\Rightarrow (*)\ \boxed{\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V(t)}{RC}}$ (1,5 ptos) · Sol homogénea: $\boxed{V_{c_h}(t) = A \cdot e^{-t/RC}}$ (0,5 ptos)

· Sol particular: proponemos una sinusoide: $V_c(t) = B\,Cos(wt) + D\,Sen(wt)$ en $(*)$:

$$-wB\,Sen(wt) + wD\,Cos(wt) + \frac{B\,Cos(wt) + D\,Sen(wt)}{RC} = \frac{V_0 Cos(wt)}{RC}$$

· $wt \in \{0, \pi, 2\pi, \dots\} \Rightarrow wD + \frac{B}{RC} = V_0$

· $wt \in \{\pi/2, 3\pi/2, \dots\} \Rightarrow -wB + \frac{D}{RC} = 0 \Rightarrow D = wBRC \Rightarrow w^2BRC + \frac{B}{RC} = V_0$

$$\Rightarrow B = \frac{V_0}{w^2RC + \frac{1}{RC}} = \frac{RCV_0}{1+(wRC)^2} = B \Rightarrow D = \frac{wRC^2V_0}{1+(wRC)^2}$$

$$\Rightarrow \boxed{V_{c_p}(t) = \frac{V_0 RC}{1+(wRC)^2}\left[Cos(wt) + wRC\,Sen(wt)\right]} \quad (1,5 \text{ ptos})$$

$$\Rightarrow V_c(t) = Ae^{-t/RC} + \frac{V_0 RC}{1+(wRC)^2}\left[Cos(wt) + wRC\,Sen(wt)\right]$$

En $t=0$: $Q = Q_0 \Rightarrow V_0 = Q_0/C \Rightarrow V_c(t=0) = A + \frac{V_0 RC}{1+(wRC)^2} = \frac{Q_0}{C}$

$$\Rightarrow \boxed{A = \frac{Q_0}{C} - \frac{RCV_0}{1+(wRC)^2}} \ (0,5 \text{ ptos}) \Rightarrow V_c(t) = \left(\frac{Q_0}{C} - \frac{RCV_0}{1+(wRC)^2}\right)e^{-t/RC} + \frac{V_0 RC}{1+(wRC)^2}\left[Cos(wt) + wRC\,Sen(wt)\right]$$

$$\Rightarrow I = C \cdot \frac{dV_c(t)}{dt} = \boxed{-\left(\frac{Q_0}{C} - \frac{RCV_0}{1+(wRC)^2}\right)e^{-t/RC} + \frac{V_0 RCw}{1+(wRC)^2}\left[-Sen(wt) + wRC\,Cos(wt)\right] = I}$$

(0,5 ptos)

b) Ec Laplace $\Rightarrow \nabla^2 V = 0 \Rightarrow \frac{\partial^2 V}{\partial z^2} = 0 \Rightarrow V(z) = A + B$

C. de B: $V(0) = 0$, $V(h) = V_c(t) \Rightarrow V(z,t) = \frac{V_c(t) \cdot z}{h}$

$\Rightarrow \vec{E} = -\nabla \cdot V = -\frac{\partial V}{\partial z}\hat{k} = \frac{-V_c(t)}{h}\hat{k} \Rightarrow \vec{D} = \frac{-\varepsilon V_c(t)}{h}\hat{k}$

$\Rightarrow \frac{d\vec{D}}{dt} = -\frac{\varepsilon}{h} \cdot \frac{\partial(V_c(t))}{dt} = -\frac{\varepsilon}{h} \cdot (I(t)/C) \Rightarrow \boxed{\vec{J}_D = \frac{d\vec{D}}{dt} = \frac{-\varepsilon}{h \cdot C} I(t)}$

(1,5 ptos)

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