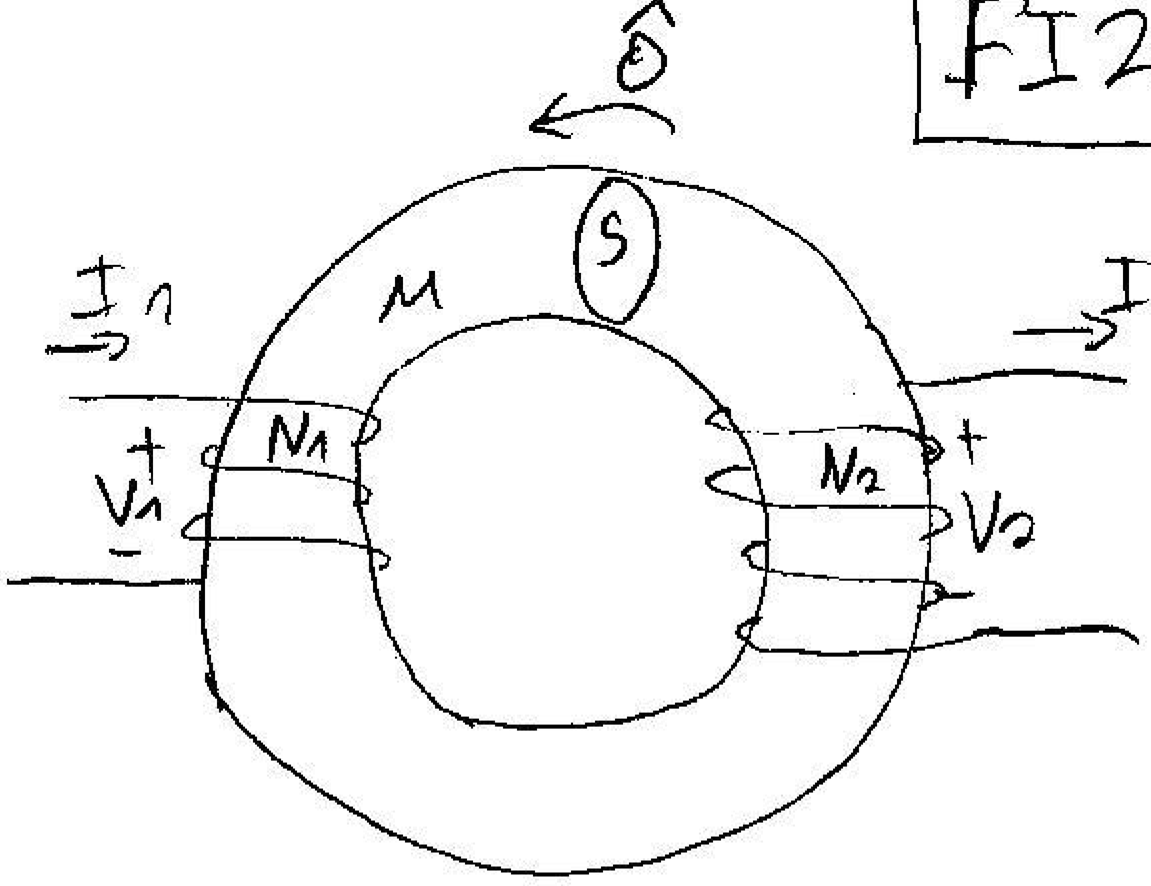
FI2002 Aux 12

P1



a) ¿V_1, V_2?

b) ¿V_1/V_2?

Sol: a) Calculamos $\vec{H}_1$, producido por espira 1, con Ley de Ampere: $\oint \vec{H}_1 \cdot d\vec{l} = I_{enc}$

Suponemos $\vec{H}_1 = {}^{+}H_1(r)\hat{\theta} \Rightarrow \int_0^{2\pi} H_1(r)\hat{\theta} \cdot r\,d\theta\hat{\theta} = -N_1 \cdot I_1 \Rightarrow \vec{H}_1 = \frac{-N_1 I_1}{2\pi r}\hat{\theta}$

$\Rightarrow \vec{B}_1 = \frac{-\mu N_1 I_1}{2\pi r}\hat{\theta}$, Análogo para 2: $\vec{B}_2 = \frac{-\mu N_2 I_2}{2\pi r}\hat{\theta}$

Calculamos los flujos inducidos: $\phi_1 = \phi_{11} + \phi_{12}$, $\phi_2 = \phi_{22} + \phi_{21}$, $\phi = \iint \vec{B} \cdot d\vec{S}$

ϕ_{11}: $\phi_{11} = N_1 \iint \vec{B}_1 \cdot d\vec{S}_1$, $d\vec{S}_1 = dS(-\hat{\theta})$ (// al $\vec{B}$)

Luego, suponiendo $\vec{B}$ = cte dentro del toroide, aproximamos por el radio medio:

$$\phi_{11} = \frac{\mu N_1^2 I_1 S}{\pi(a+b)}$$

ϕ_{22}: $\phi_{22} = N_2 \iint \vec{B}_2 \cdot d\vec{S}_2$, $d\vec{S}_2 = dS(-\hat{\theta}) \Rightarrow \phi_{22} = \frac{\mu N_2^2 I_2 S}{\pi(a+b)}$

ϕ_{12}: $\phi_{12} = N_1 \iint \vec{B}_2 \cdot d\vec{S}_1 = \frac{\mu N_1 N_2 I_2 S}{\pi(a+b)}$

ϕ_{21}: $\phi_{21} = N_2 \iint \vec{B}_1 \cdot d\vec{S}_2 = \frac{\mu N_1 N_2 I_1 \cdot S}{\pi(a+b)}$

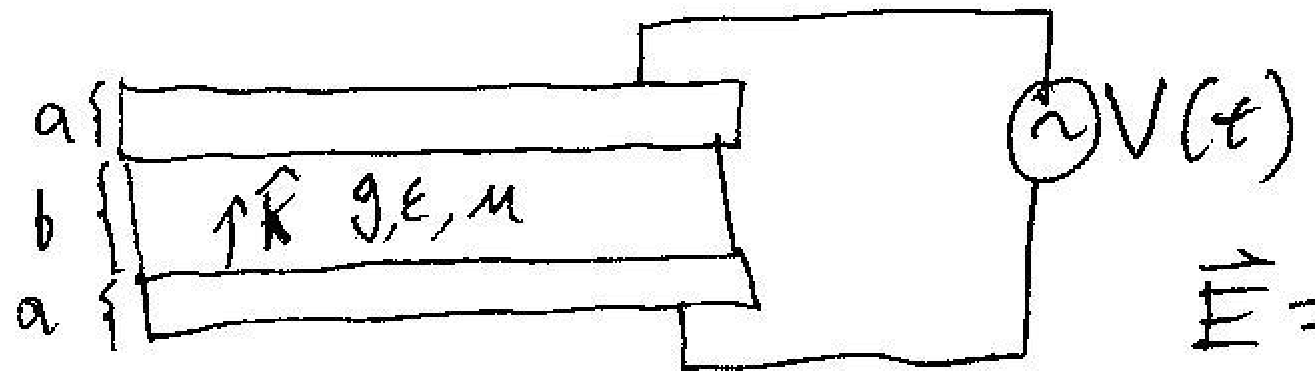
Luego por Faraday-Lenz: $\mathcal{E} = \frac{-d\phi}{dt} \Rightarrow$

$$\boxed{\begin{aligned} V_1 &= \frac{-\mu S N_1^2}{\pi(a+b)}\frac{\partial I_1}{\partial t} - \frac{\mu S N_1 N_2}{\pi(a+b)}\frac{\partial I_2}{\partial t} \\ V_2 &= \frac{-\mu S N_2^2}{\pi(a+b)} \cdot \frac{\partial I_2}{\partial t} - \frac{\mu S N_1 N_2}{\pi(a+b)} \cdot \frac{\partial I_1}{\partial t} \end{aligned}}$$

b) $V_1 = \frac{-\mu S}{\pi(a+b)} N_1\left(N_1 \cdot \frac{\partial I_1}{\partial t} + \frac{N_2 \partial I_2}{\partial t}\right)$, $V_2 = \frac{-\mu S N_2}{\pi(a+b)}\left(N_1 \frac{\partial I_1}{\partial t} + N_2 \frac{\partial I_2}{\partial t}\right)$

$\Rightarrow \boxed{V_1/V_2 = N_1/N_2}$ → Razón de conversión de voltajes de un transformador

P2 | P_1 Examen 2008:



$\vec{E} = E_0 \operatorname{Sen}(wt)\hat{k}$

i) ¿w tq $J_{máx} = J_{D\,máx}$?

ii) ¿$V(t)$?

Sol:

i) $\vec{J} = g\vec{E} = g E_0 \operatorname{Sen}(wt)\hat{k}$, $\vec{D} = \varepsilon E_0 \operatorname{Sen}(wt)\hat{k}$

$\vec{J_D} = \dfrac{\partial \vec{D}}{\partial t} = w\,\varepsilon E_0 \operatorname{Cos}(wt)\hat{k}$, los valores máximos corresponden a cuando las sinusoides valen 1, luego:

$J_{máx} = g E_0$, $J_{D\,máx} = \varepsilon E_0 w$, igualando: $\boxed{w = g/\varepsilon}$

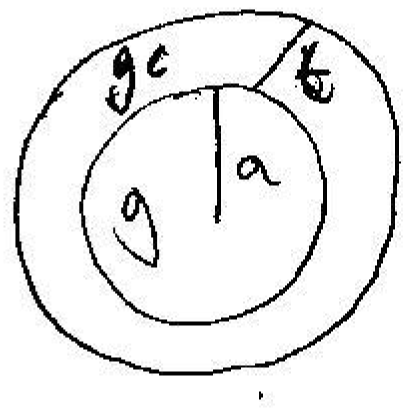
ii) $V(t) = -\int \vec{E}\cdot d\vec{l} = -E_0 \operatorname{Sen}(wt)\int_0^b \hat{k}\cdot dz\,\hat{k} = \boxed{-b E_0 \operatorname{Sen}(wt) = V(t)}$

P3 | P_2 Examen 2008:

i)

¿R? Sol: $R = \dfrac{1}{\rho}$, $\rho = \dfrac{g\cdot A}{l} = \dfrac{g\cdot\pi\cdot a^2}{h} \Rightarrow \boxed{R = \dfrac{h}{g\pi a^2}}$

ii)



¿t tq $R_{eq} = 0{,}8R$? Sol: Quedan 2 resistencias en // $\Rightarrow$

$$\frac{1}{R} + \frac{1}{R_c} = \frac{1}{0{,}8R} \Rightarrow \rho + \rho_c = \frac{\rho}{0{,}8}$$

$\Rightarrow \rho_c - \dfrac{\rho}{4} = 0$, Calculamos ρ_c como la conductancia de todo un cilindro de radio $a+t$, menos la de uno de radio a:

$$\rho_c = \frac{g_c\pi(a+t)^2}{h} - \frac{g_c\pi a^2}{h} = \frac{g_c\pi}{h}\left(t^2 + 2at\right)$$, en la ecuación:

$$\frac{g_c\cancel{\pi}}{\cancel{h}}(t^2 + 2at) - \frac{g\cancel{\pi}}{4\cancel{h}}a^2 = 0 \Rightarrow g_c t^2 + 2g_c\cdot a\cdot t - \frac{g a^2}{4} = 0$$

$\Rightarrow t = \dfrac{\pm\sqrt{4a^2 g_c^2 + g\cdot g_c\cdot a^2} - 2a\cdot g_c}{2g_c}$, descartamos la sol negativa pues no tiene sentido $\Rightarrow \boxed{t = \dfrac{a\sqrt{4g_c^2 + g\cdot g_c} - 2a\cdot g_c}{2g_c}}$

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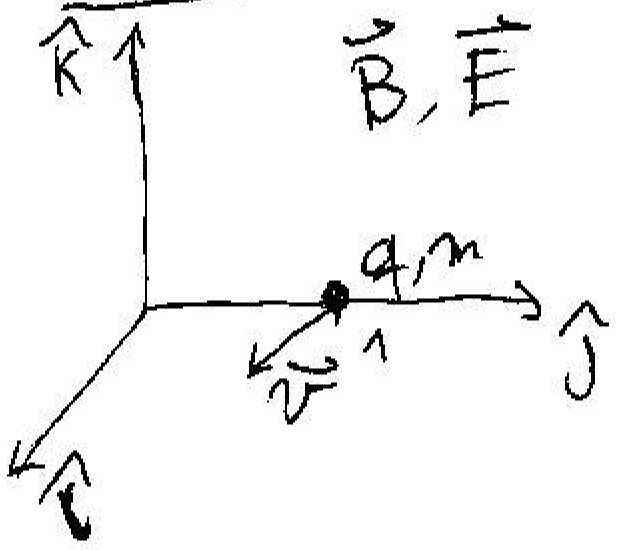
P4 | P4 Examen 2008



$\vec{B}, \vec{E}$

$\vec{B} = B_0\hat{k}$ i) ¿Ec mov?

$\vec{E} = E_0\cos(wt)\hat{x}$ ii) ¿Trayectoria?

$\vec{r}_0 = (0,1,0)$

Sol: i) $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$, En sist. cartesiano: $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$

$$\Rightarrow \vec{F} = qE_0\cos(wt)\hat{k} - qB_0\dot{x}\hat{\jmath} + qB_0\dot{y}\hat{\imath} = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k})$$

$$\boxed{\begin{array}{l} \hat{\imath})\ -qB_0\dot{y} = m\ddot{x} \\ \hat{\jmath})\ qB_0\dot{x} = m\ddot{y} \\ \hat{k})\ qE_0\cos(wt) = \ddot{z} \end{array}}$$

ii) $\hat{k}) \Rightarrow \dot{z} = \frac{qE_0}{w}\operatorname{Sen}(wt) + \dot{z}_0$, $\dot{z}_0 = 0$

$$\Rightarrow z = -\frac{qE_0}{w^2}\cos(wt) + z_0, \quad z_0 = 0$$

$$\Rightarrow \boxed{z(t) = -\frac{qE_0\cos(wt)}{w^2}}$$

Sea $a = \dot{x}$, $b = \dot{y}$:

$\hat{\imath})\ -qB_0 b = m\dot{a} \xrightarrow{d/dt} -qB_0\dot{b} = m\ddot{a} \Rightarrow \dot{b} = -\frac{m\ddot{a}}{qB_0}$ en $\hat{\jmath}$):

$\hat{\jmath})\ qB_0 a = m\dot{b}$ $\quad\hat{\jmath})$: $qB_0 a = -\frac{m^2}{qB_0}\ddot{a} \Rightarrow \ddot{a} = -\left(\frac{qB_0}{m}\right)^2 a$

$$\Rightarrow a = A\cos\left(\frac{qB_0}{m}t\right) + B\operatorname{Sen}\left(\frac{qB_0}{m}t\right) = \dot{x}$$

$\frac{d(\hat{\jmath})}{dt} \Rightarrow qB_0\dot{a} = m\ddot{b} \Rightarrow \dot{a} = \frac{-m\ddot{b}}{qB_0}$ en $\hat{\jmath}$): $\ddot{b} = -\left(\frac{qB_0}{m}\right)^2 b$

$$\Rightarrow b = C\cos\left(\frac{qB_0}{m}t\right) + D\operatorname{Sen}\left(\frac{qB_0}{m}t\right) = \dot{y}$$

En $t=0$: $\dot{x}_0 = v_0 = \dot{x}(0) = A \Rightarrow \underline{A = v_0}$, $\dot{y}_0 = 0 = y(0) = C \Rightarrow \underline{C = 0}$

$\Rightarrow \ddot{x}(t) = \frac{qB_0}{m}\left(v_0\operatorname{Sen}\left(\frac{qB_0}{m}t\right) + B\cos\left(\frac{qB_0}{m}t\right)\right)$, $\ddot{y}(t) = \frac{qB_0 D}{m}\cos\left(\frac{qB_0}{m}t\right)$

$\hat{\imath})\ -\cancel{qB_0}D\operatorname{Sen}\left(\frac{qB_0 t}{m}\right) = \cancel{m}\cdot\frac{\cancel{qB_0}}{\cancel{m}}\left(B\cos\left(\frac{qB_0 t}{m}\right) - v_0\operatorname{Sen}\left(\frac{qB_0}{t}\right)\right)$, en $t=0 \Rightarrow \underline{B = 0}$

Para $\hat{\jmath}$, en $t=0$: $\quad\dot{x}(t) = v_0\cos\left(\frac{qB_0 t}{m}\right)$

$\hat{\jmath})\ \cancel{qB_0}v_0 = \cancel{m}\cdot\frac{\cancel{qB_0}D}{\cancel{m}} \Rightarrow \underline{D = v_0} \Rightarrow \dot{y}(t) = v_0\operatorname{Sen}\left(\frac{qB_0 t}{m}\right)$

$$\therefore \boxed{x(t) = \frac{mv_0}{qB_0}\operatorname{Sen}\left(\frac{qB_0 t}{m}\right) + \cancel{x_0}^{\,0}, \quad y(t) = -\frac{mv_0}{qB_0}\cos\left(\frac{qB_0 t}{m}\right) + \underbrace{y_0}_{1}}$$