Pauta P1 C2 FI2002 2009-2.

P1/ $\mu >> b \Rightarrow$ Se desprecian efectos de borde

a) Se tiene simetría cilíndrica

$$\vec{E_i} = E_{in}(r)\,\hat{r}$$

De Laplace, $\nabla^2 V = 0 \iff \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$

$$\Rightarrow \frac{\partial V_i}{\partial r} = \frac{K_i}{r} \Rightarrow V_i(r) = K_i \ln(r) + C_i$$

Imponiendo condiciones de borde:
(Sea ① = interior del cable; ② = ext. cable)

$$V_1(a) = K_1 \ln a + C_1 = V_0 \quad (1)$$
$$V_1(b) = K_1 \ln b + C_1 = 0. \quad (2)$$

$(1)-(2)$ $V_0 = K_1 \ln\left(\frac{a}{b}\right) \Rightarrow K_1 = \frac{V_0}{\ln\left(\frac{a}{b}\right)}$

En (1), $\frac{V_0 \ln a}{\ln\left(\frac{a}{b}\right)} + C_1 = V_0.$

$$C_1 = \frac{-V_0 \ln b}{\ln\left(\frac{a}{b}\right)}$$

$$V_1(r) = \frac{V_0 \ln(r)}{\ln\left(\frac{a}{b}\right)} - \frac{V_0 \ln b}{\ln\left(\frac{a}{b}\right)} = \frac{V_0 \ln\left(\frac{r}{b}\right)}{\ln\left(\frac{a}{b}\right)}.$$

pto/ $\Rightarrow$ $\boxed{\vec{E_1}(r) = \frac{-V_0}{r\ln\left(\frac{a}{b}\right)}\hat{r} = \frac{V_0}{r\ln\left(\frac{b}{a}\right)}\hat{r}}$ (libertad en el método)

Por otro lado, fuera del cable, dado que $\lim_{r\to\infty} V(r) = 0$, necesariamente $K_2 = 0$ y entonces $\boxed{\vec{E_2}(r) = \vec{0}.}$

1 pto/

b) Sabemos $U = \iiint \vec{E}\cdot\vec{D}\, dV$; En este caso;

$$U = \varepsilon \int_0^h \int_0^{2\pi} \int_a^b \frac{V_0^2}{r^2 \ln^2\left(\frac{b}{a}\right)}\, dz\, r\, d\theta\, dr$$

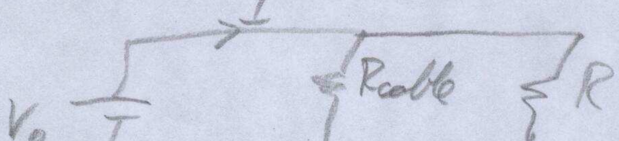
$$\boxed{U = \frac{2\pi \varepsilon V_0^2 h \ln\left(\frac{b}{a}\right)}{\ln^2\left(\frac{b}{a}\right)} = \frac{2\pi \varepsilon V_0^2 h}{\ln\left(\frac{b}{a}\right)}}\ \boxed{1\,pto}$$

c) $\vec{J} = g\vec{E} = \frac{gV_0}{r\ln\left(\frac{b}{a}\right)}\hat{r}$

$$\boxed{I_{fuga} = \int_0^{2\pi}\int_0^h \frac{gV_0}{r\ln\left(\frac{b}{a}\right)}\, r\, d\theta\, dz = \frac{2\pi h g V_0}{\ln\left(\frac{b}{a}\right)}}\ \boxed{1\,pto}$$

d) De la parte (c), $R_{cable} = \frac{V_0}{I_{fuga}} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi g h}$

Se modela la situación como




$$\Rightarrow I = \frac{V_0}{R // R_{cable}} = \left(\frac{V_0}{\frac{R\ln\left(\frac{b}{a}\right)}{2\pi g h}}\right)\left(R + \frac{\ln}{2\pi g}\right.$$

$$\boxed{I = \frac{V_0\left(2\pi R g h + \ln\left(\frac{b}{a}\right)\right)}{R\ln\left(\frac{b}{a}\right)}}\ \boxed{1\,pto}$$

e) La resistencia R disipa una potencia $\boxed{P = \frac{V_0^2}{R}}$ $\boxed{1\,pto}$