



FI 2002

ELECTROMAGNETISMO

Clase 18

Magnetostática II

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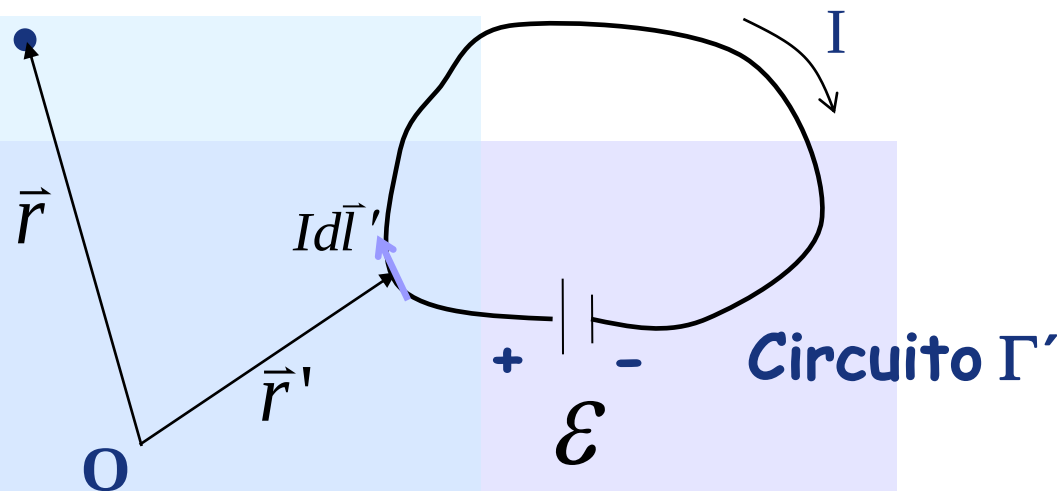
- Campo magnético
- Ejemplo
- Ley de Biot y Savart
- Torque magnético
- Motor Elemental

Paul Klee, "Rose Garden", 1920





Campo magnético



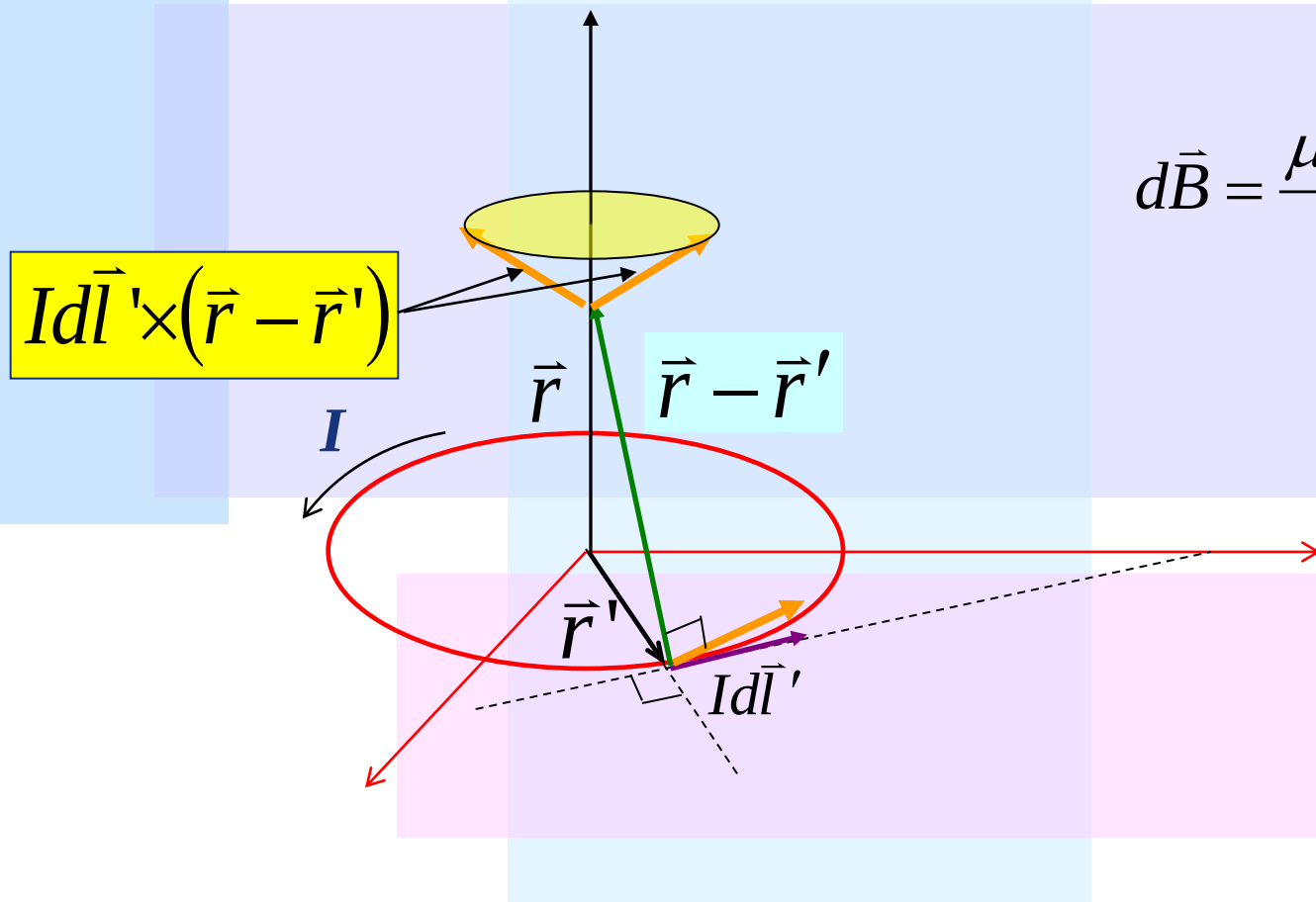
Campo producido por
circuito Γ'

$$\vec{B} = \oint_{\Gamma'} \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



Regla de la mano derecha

Dirección de campo está dado por el producto $I d\vec{l}' \times (\vec{r} - \vec{r}')$

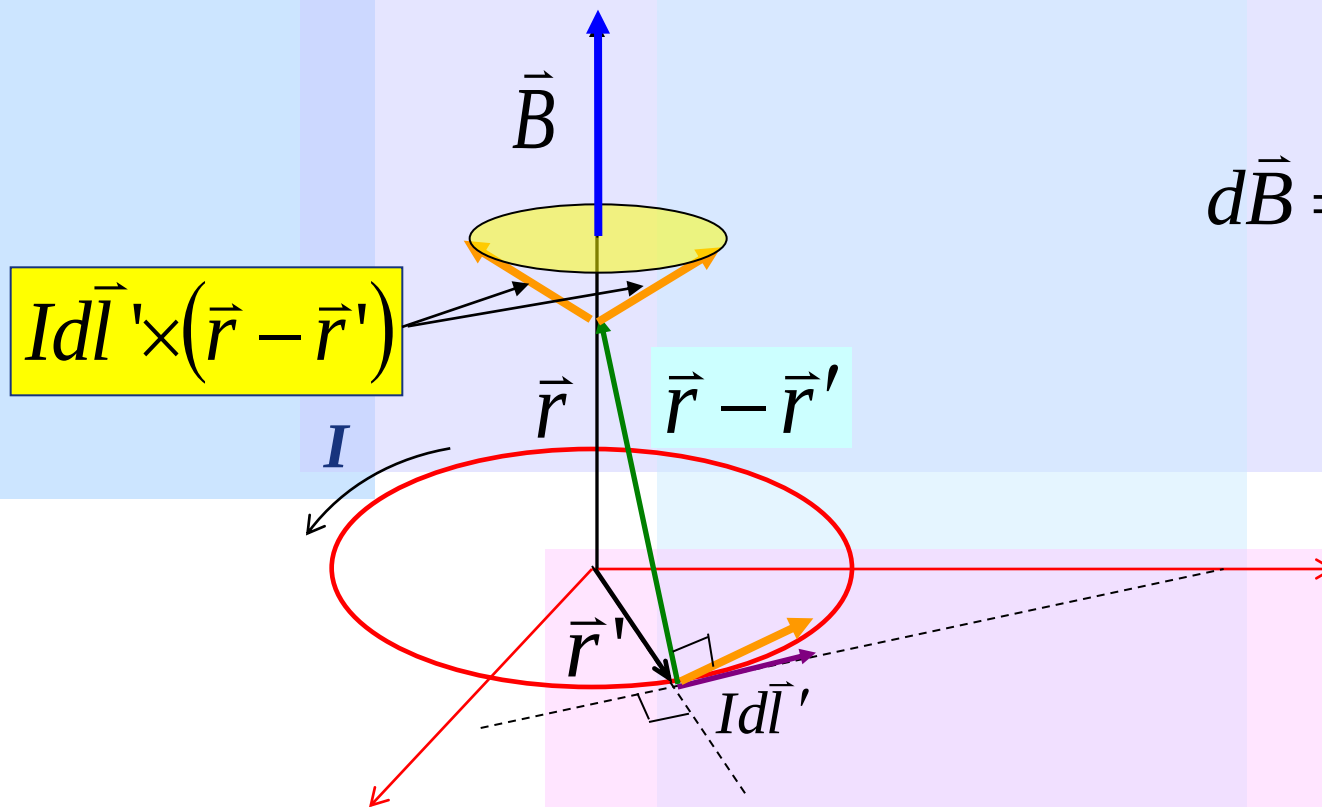


$$d\vec{B} = \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



Regla de la mano derecha

Campo magnético resultante sólo tiene dirección según eje z.



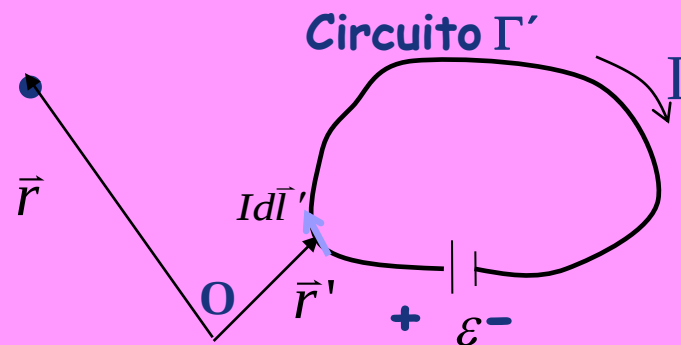
$$d\vec{B} = \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



Campo magnético

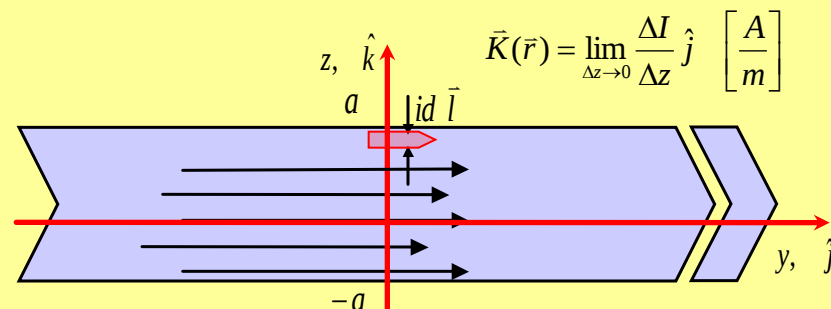
Campo producido por circuito Γ'

$$\vec{B} = \oint_{\Gamma'} \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



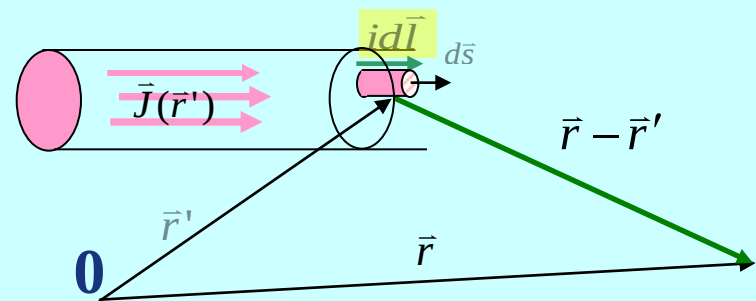
Campo producido por densidad de corriente superficial $\vec{K}(\vec{r})$

$$\vec{B} = \frac{\mu_0}{4\pi} \iint_S \frac{\vec{K}(\vec{r}') \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3} ds'$$



Campo producido por densidad de corriente en volumen $\vec{J}(\vec{r})$

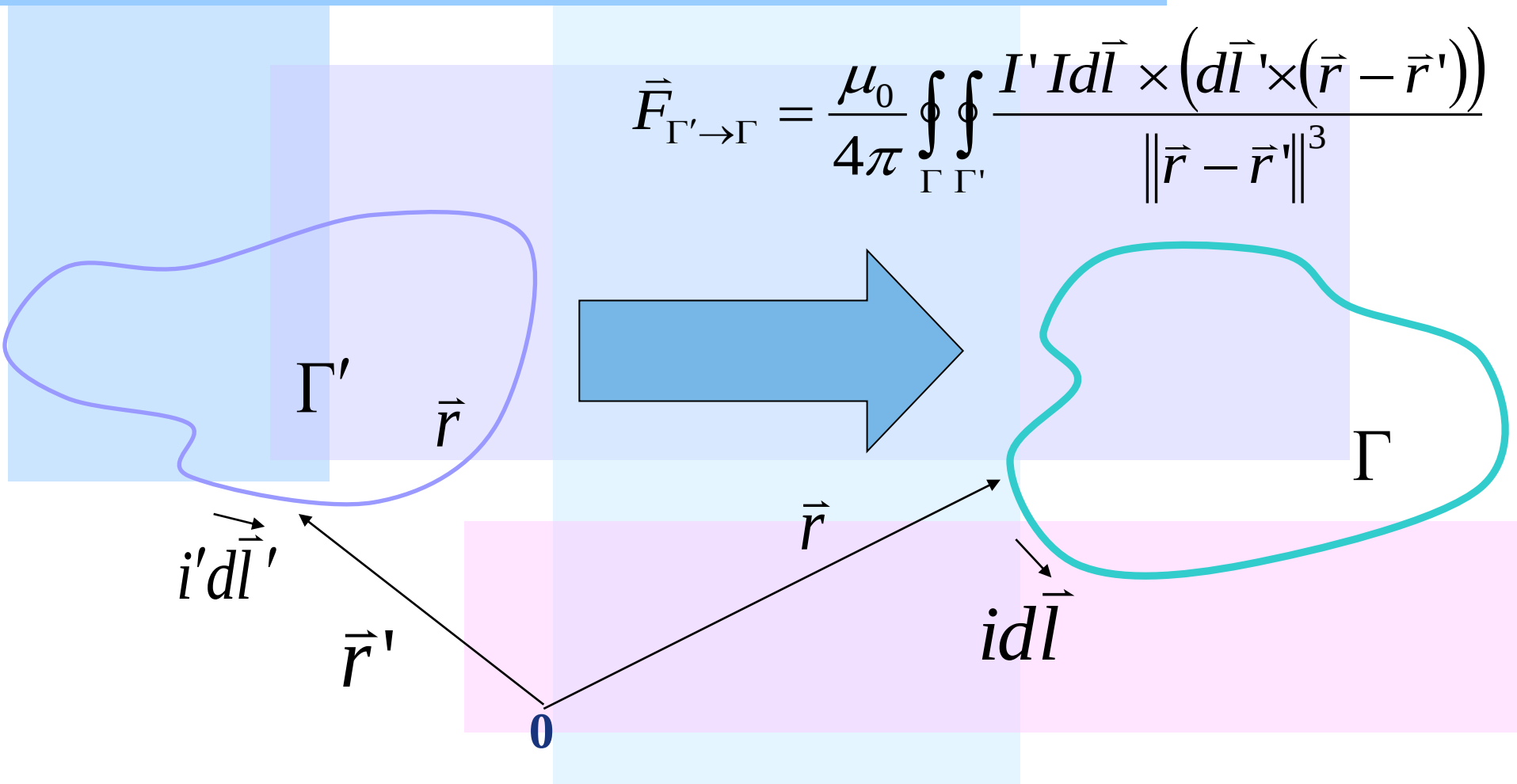
$$\vec{B} = \frac{\mu_0}{4\pi} \iiint_{\Omega} \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3} dv'$$





Ley de Biot y Savart

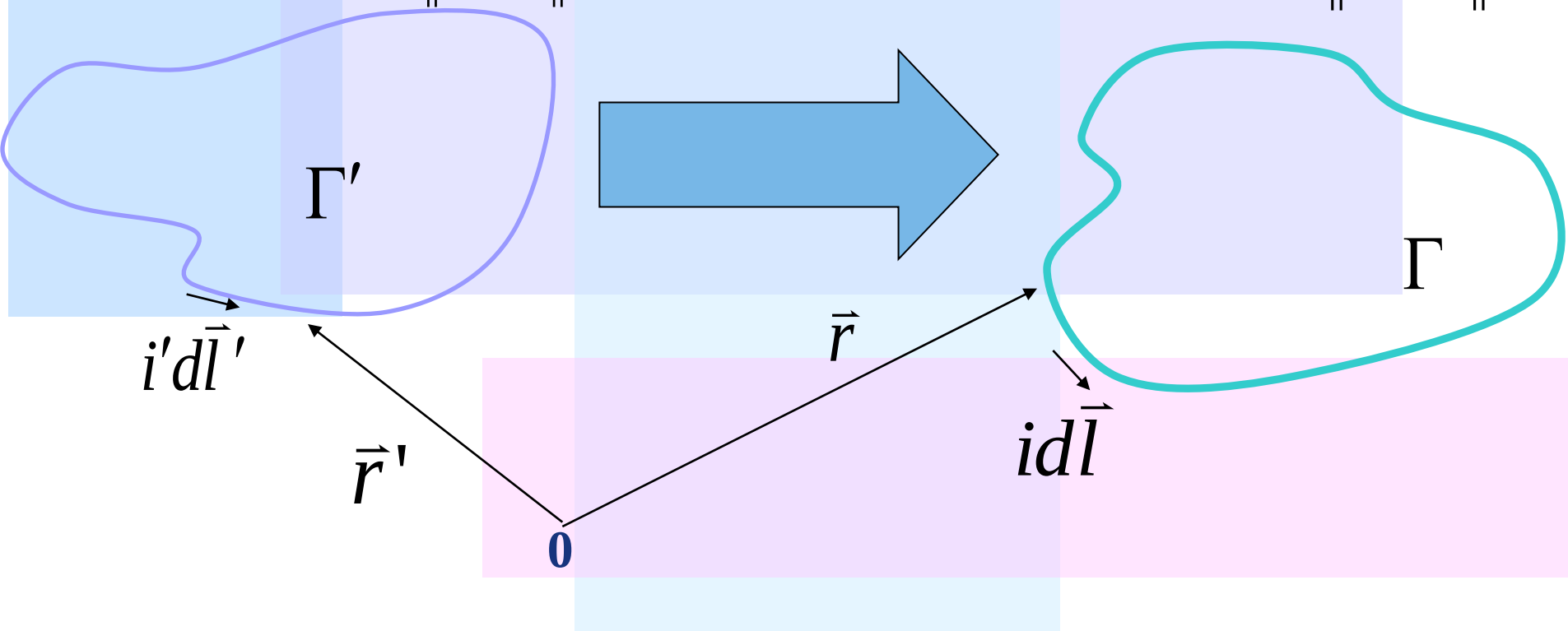
Fuerza que ejerce circuito Γ' sobre circuito Γ





Ley de Biot y Savart

$$\vec{F} = \frac{\mu_0}{4\pi} \oint_{\Gamma} \oint_{\Gamma'} \frac{I' Id\vec{l} \times (d\vec{l}' \times (\vec{r} - \vec{r}'))}{\|\vec{r} - \vec{r}'\|^3} \rightarrow d\vec{F} = \frac{Id\vec{l} \times \mu_0}{4\pi} \oint_{\Gamma'} \frac{I' d\vec{l}' \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$



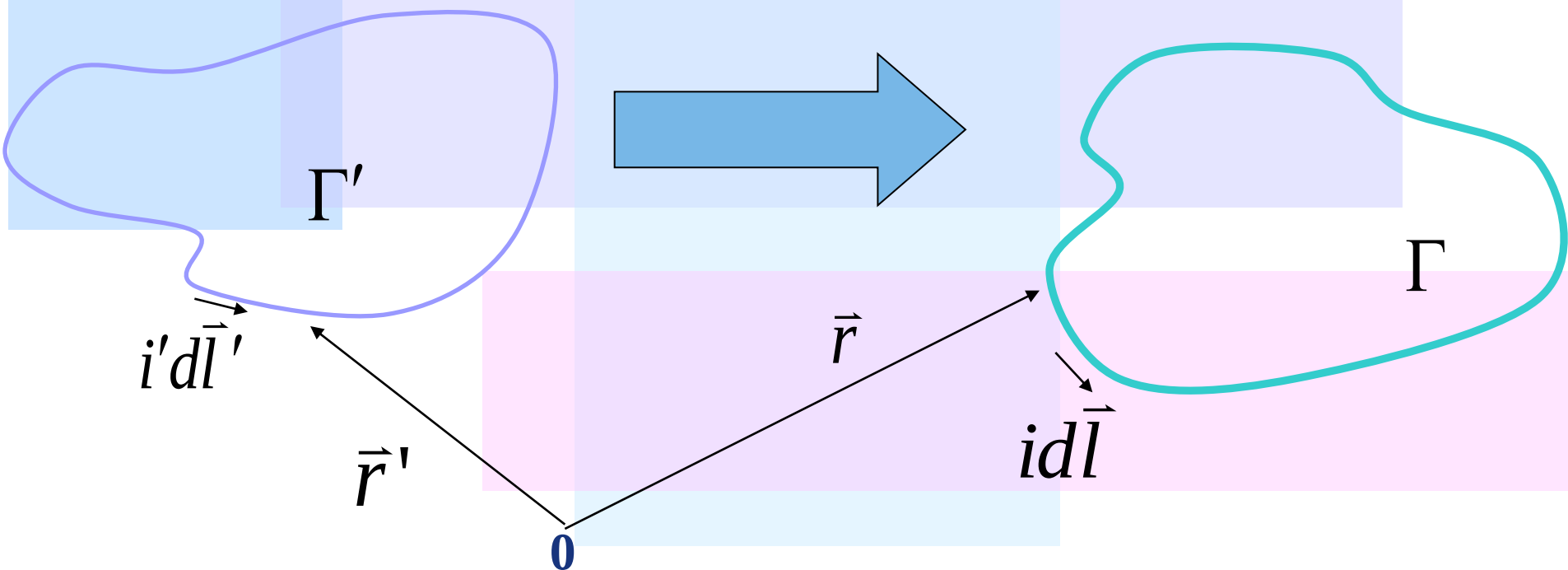


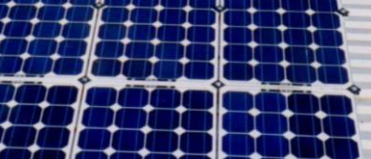
Ley de Biot y Savart

$$d\vec{F} = \frac{Id\vec{l} \times \mu_0}{4\pi} \oint_{\Gamma'} \frac{I' d\vec{l}' \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$

$$\therefore d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$

Campo magnético
producido por circuito Γ'



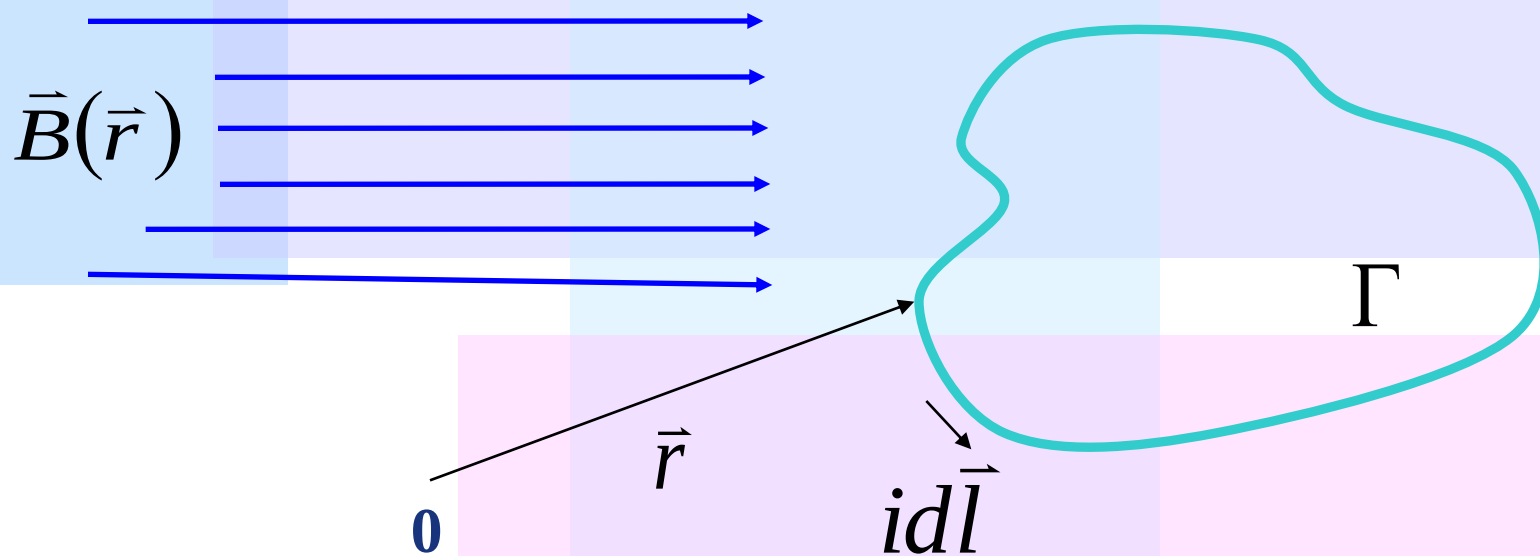


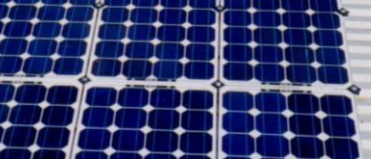
Ley de Biot y Savart

En general, un circuito en presencia de un campo magnético experimenta una fuerza dada por la ecuación

$$d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$

$$\therefore \vec{F} = \oint_{\Gamma} d\vec{F} = \oint_{\Gamma} Id\vec{l} \times \vec{B}(\vec{r})$$

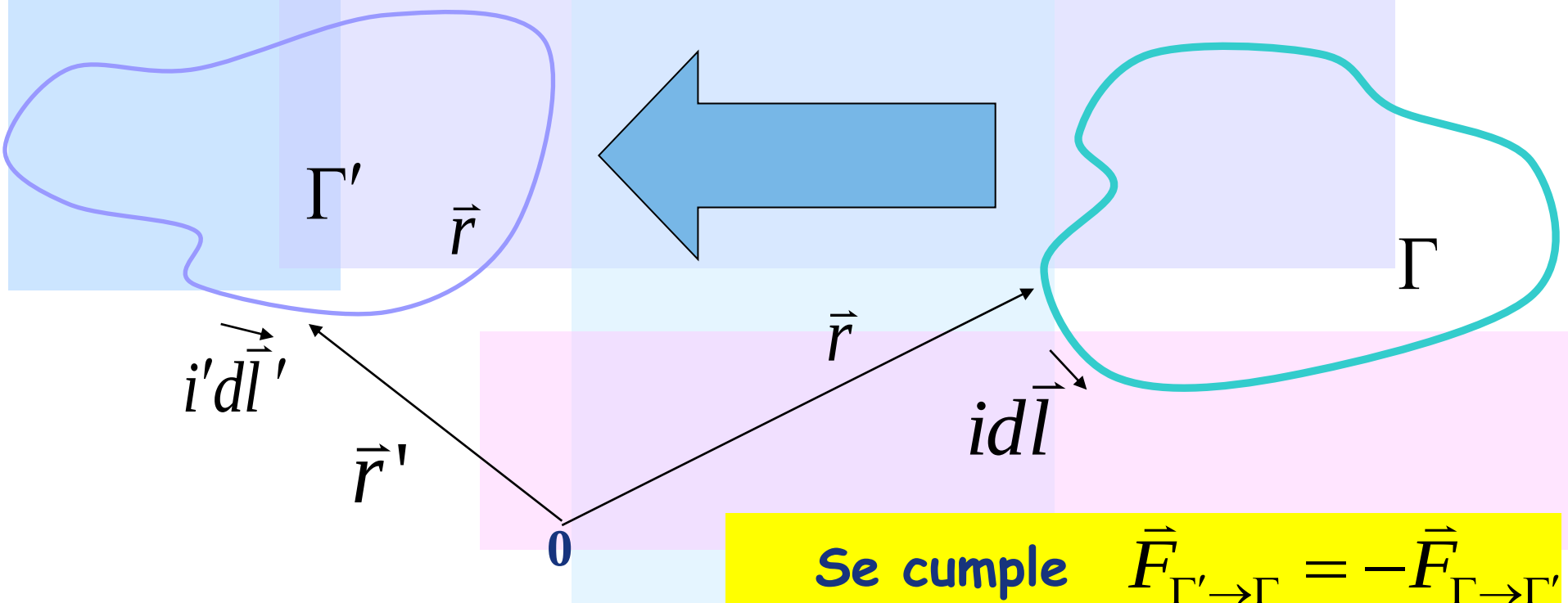




Ley de Biot y Savart

Fuerza que ejerce circuito Γ sobre circuito Γ'

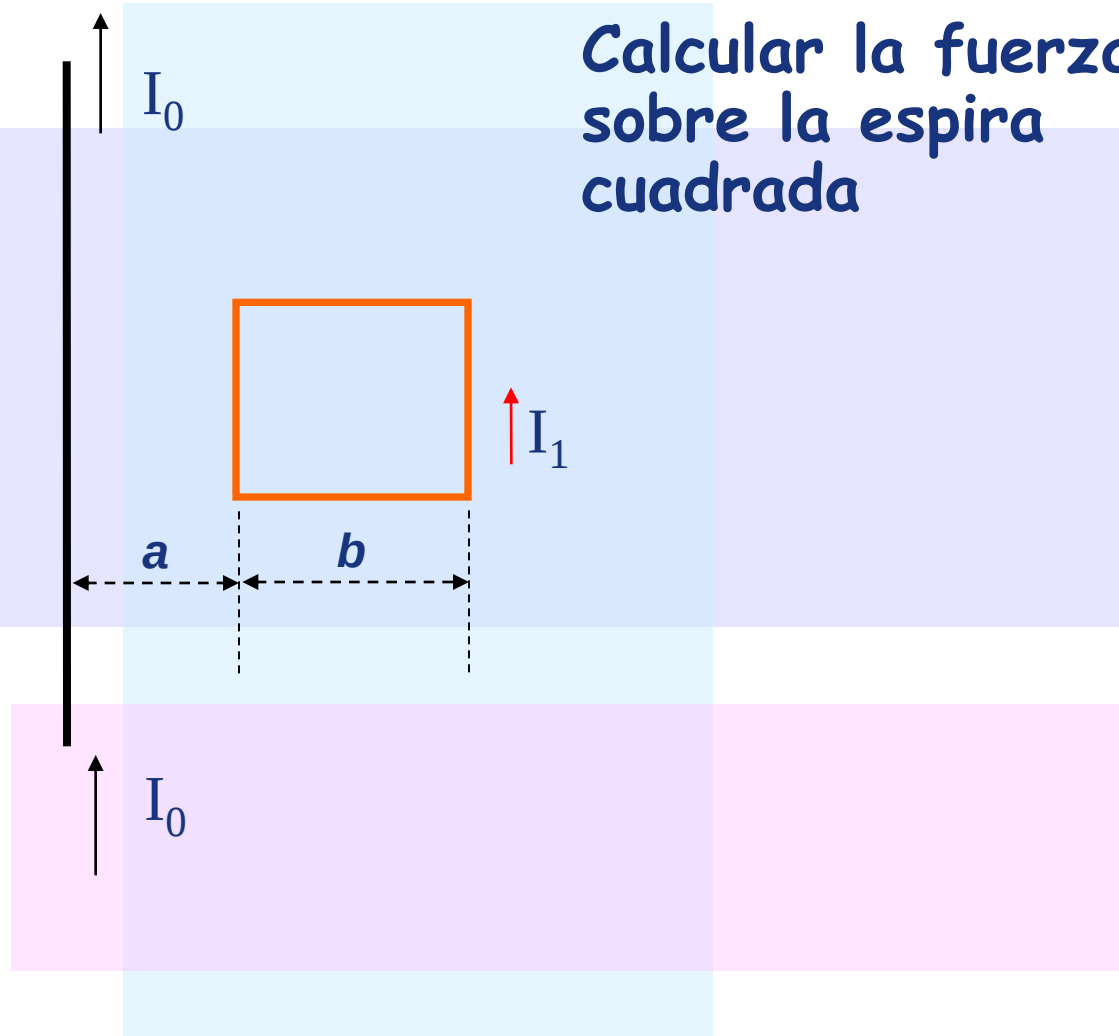
$$\vec{F}_{\Gamma \rightarrow \Gamma'} = \frac{\mu_0}{4\pi} \oint_{\Gamma'} \oint_{\Gamma} \frac{II' d\vec{l}' \times (d\vec{l} \times (\vec{r} - \vec{r}'))}{\|\vec{r} - \vec{r}'\|^3} = - \frac{\mu_0}{4\pi} \oint_{\Gamma} \oint_{\Gamma'} \frac{I'I d\vec{l} \times (d\vec{l}' \times (\vec{r} - \vec{r}'))}{\|\vec{r} - \vec{r}'\|^3}$$





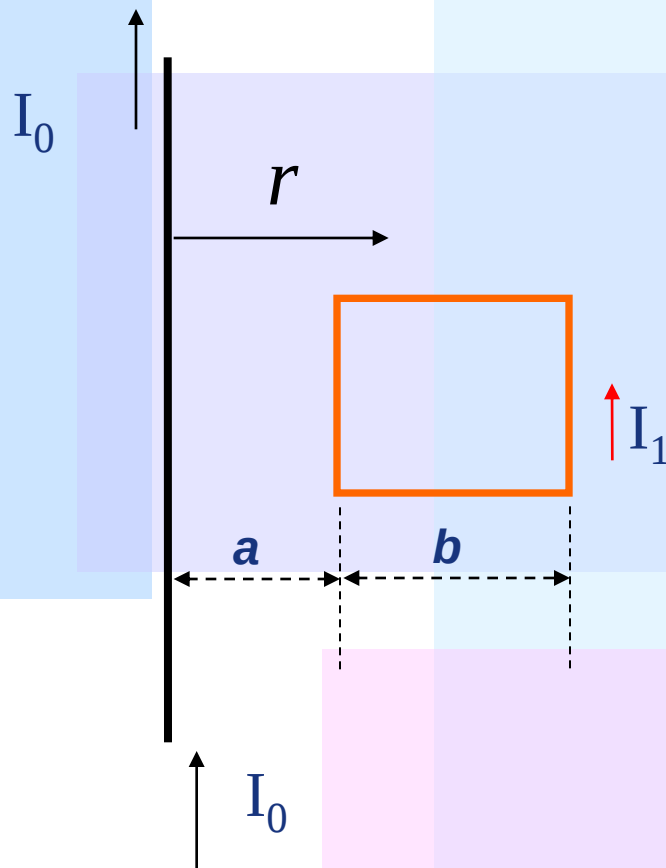
Ejemplo

Calcular la fuerza
sobre la espira
cuadrada





Ejemplo



Campo producido por el conductor infinito es

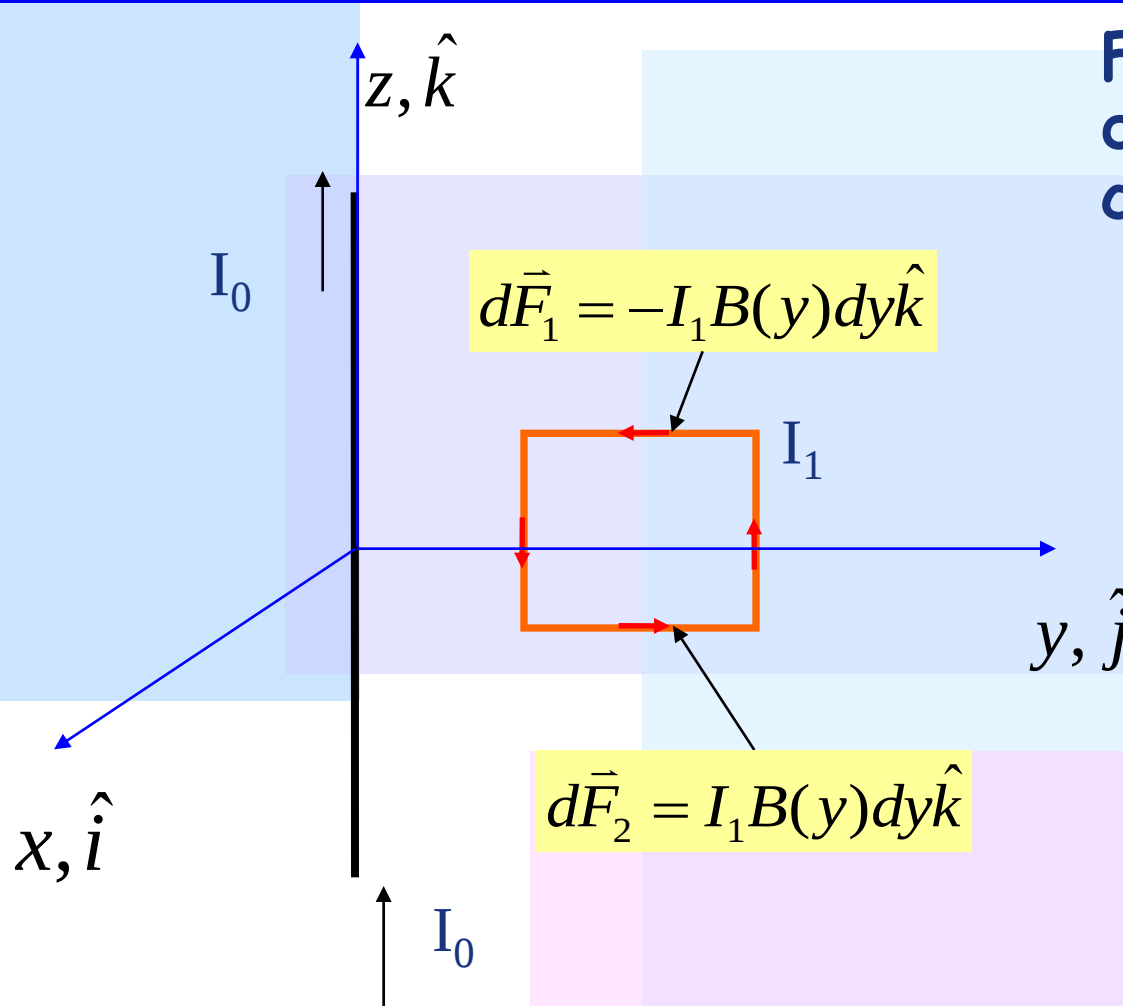
$$\vec{B} = \frac{\mu_0 I_0}{2\pi r} \hat{\theta}$$

Fuerza sobre elemento de corriente de espira cuadrada

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$



Ejemplo



Fuerza sobre elemento
de corriente de espira
cuadrada

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

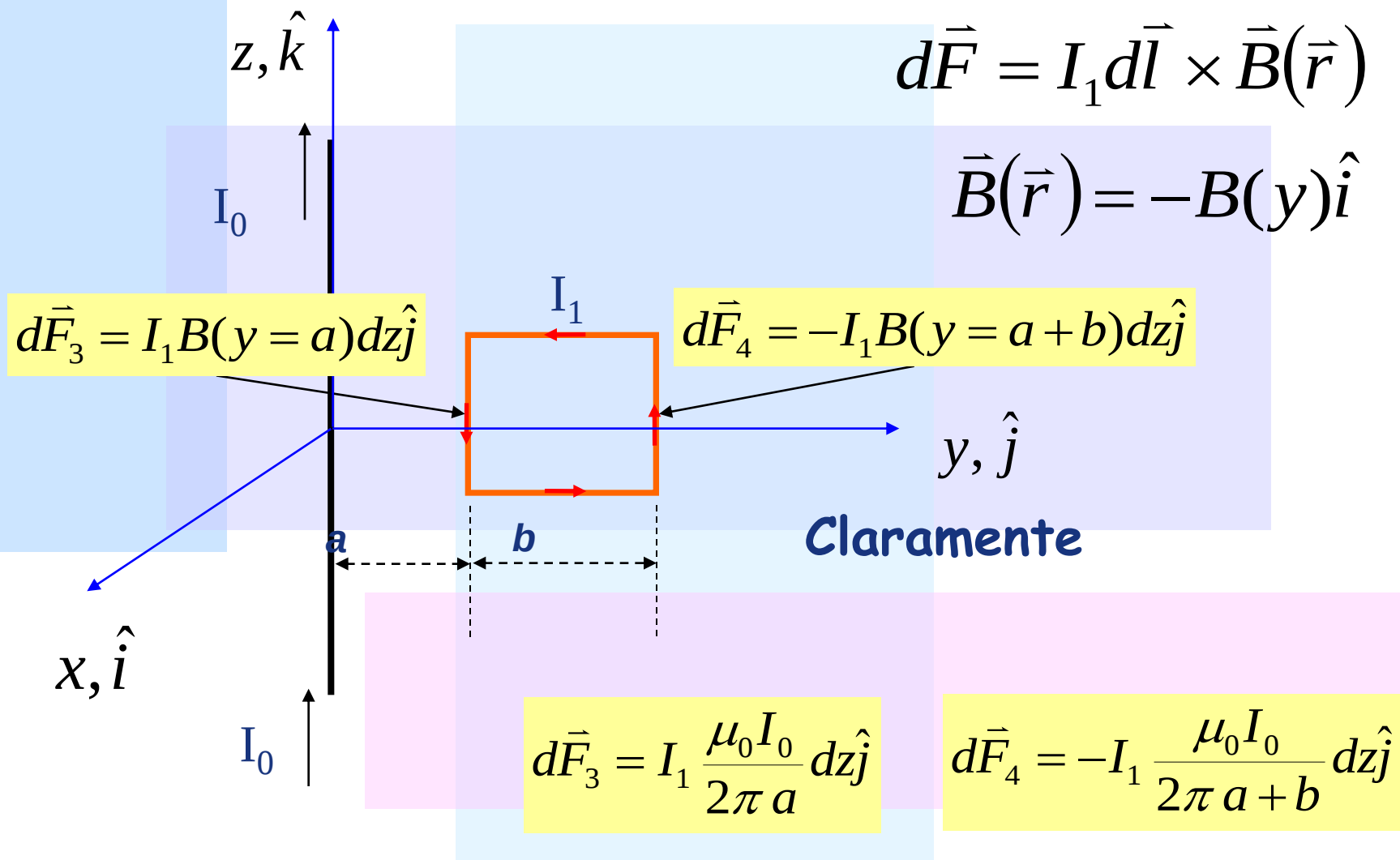
$$\vec{B}(\vec{r}) = -B(y)\hat{i}$$

Claramente

$$d\vec{F}_1 = -d\vec{F}_2$$

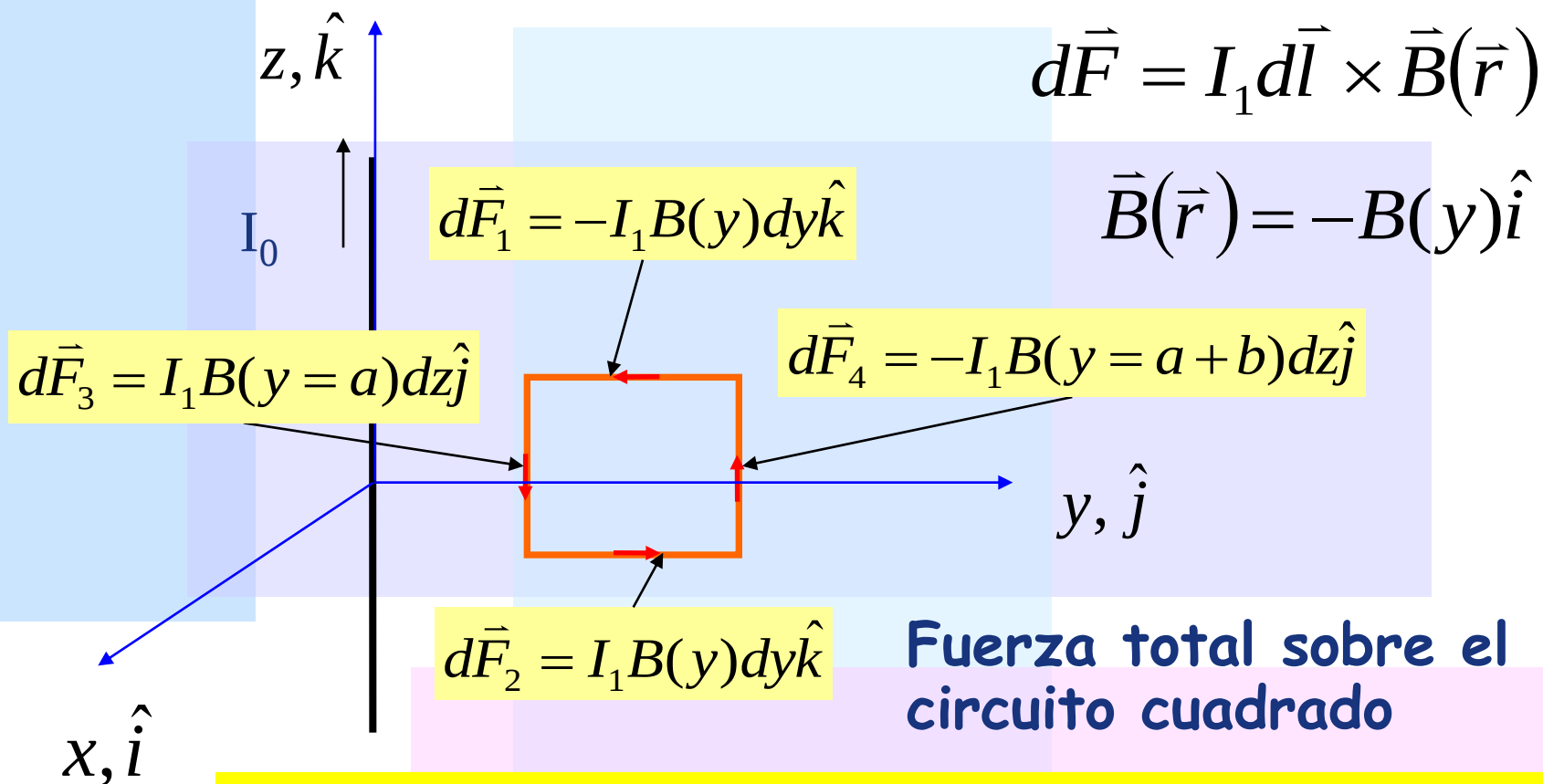


Ejemplo





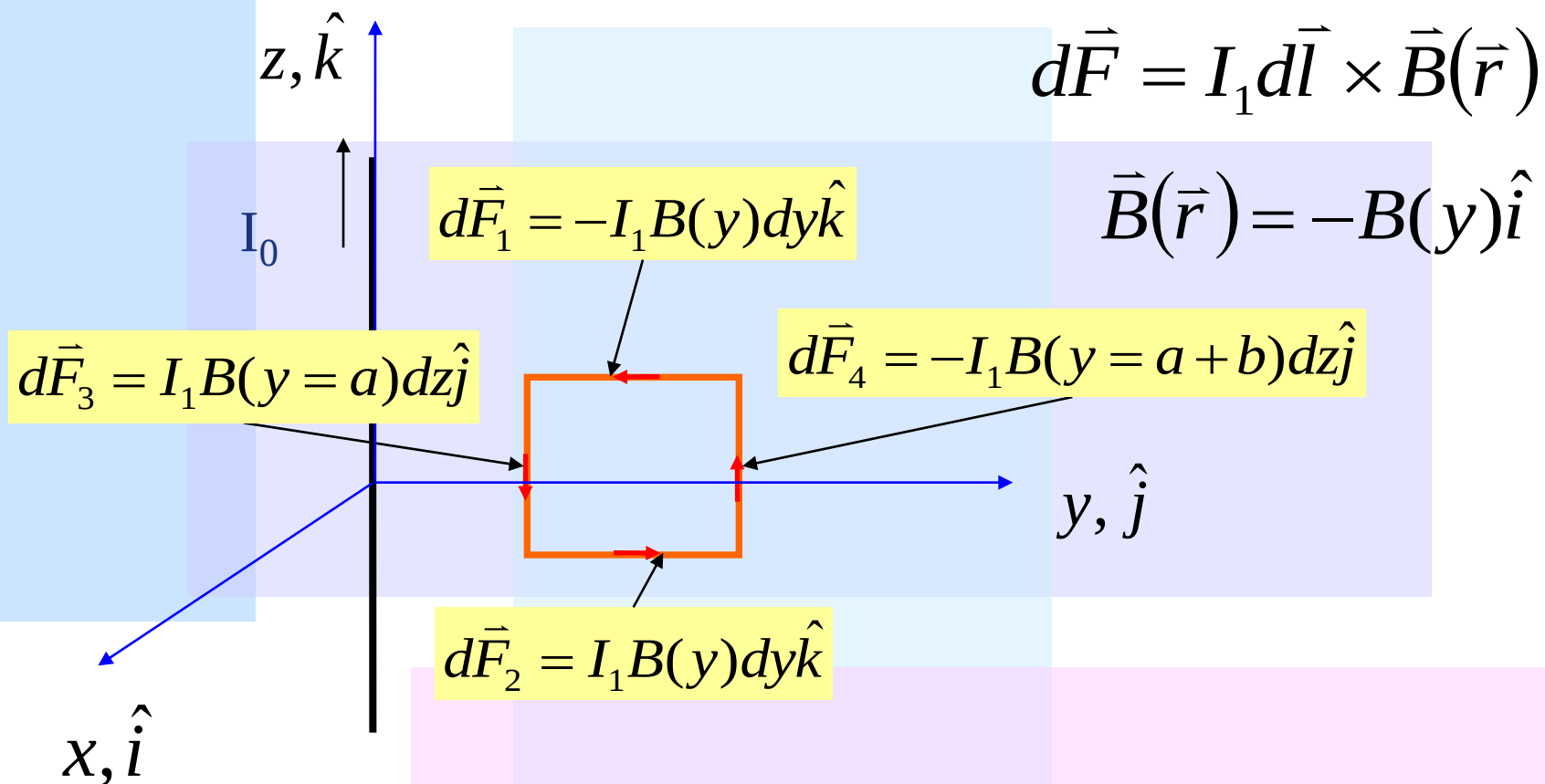
Ejemplo



$$\therefore \vec{F} = \oint_{\Gamma} d\vec{F} = \int_{y=a+b}^{y=a} d\vec{F}_1 + \int_{z=b/2}^{z=-b/2} d\vec{F}_3 + \int_{y=a}^{y=a+b} d\vec{F}_2 + \int_{z=-b/2}^{z=b/2} d\vec{F}_4$$



Ejemplo



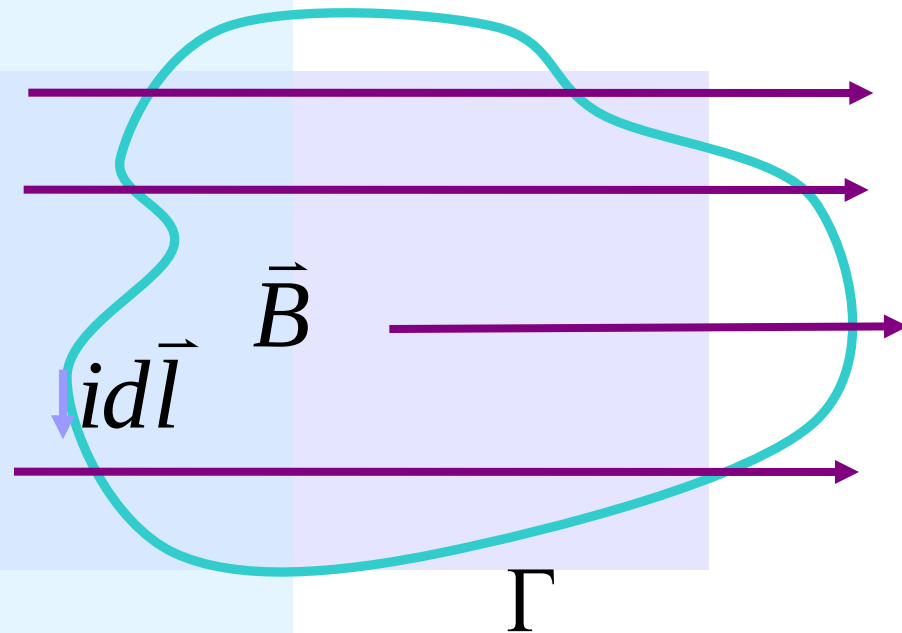
$$\vec{F} = \oint_{\Gamma} d\vec{F} = \int_{z=b/2}^{z=-b/2} \frac{\mu_0 I_1 I_0 \hat{j}}{2\pi a} dz - \int_{z=-b/2}^{z=b/2} \frac{\mu_0 I_1 I_0 \hat{j}}{2\pi(a+b)} dz = \frac{\mu_0 I_1 I_0 b^2}{2\pi(a+b)} \hat{j}$$



Torque Magnético

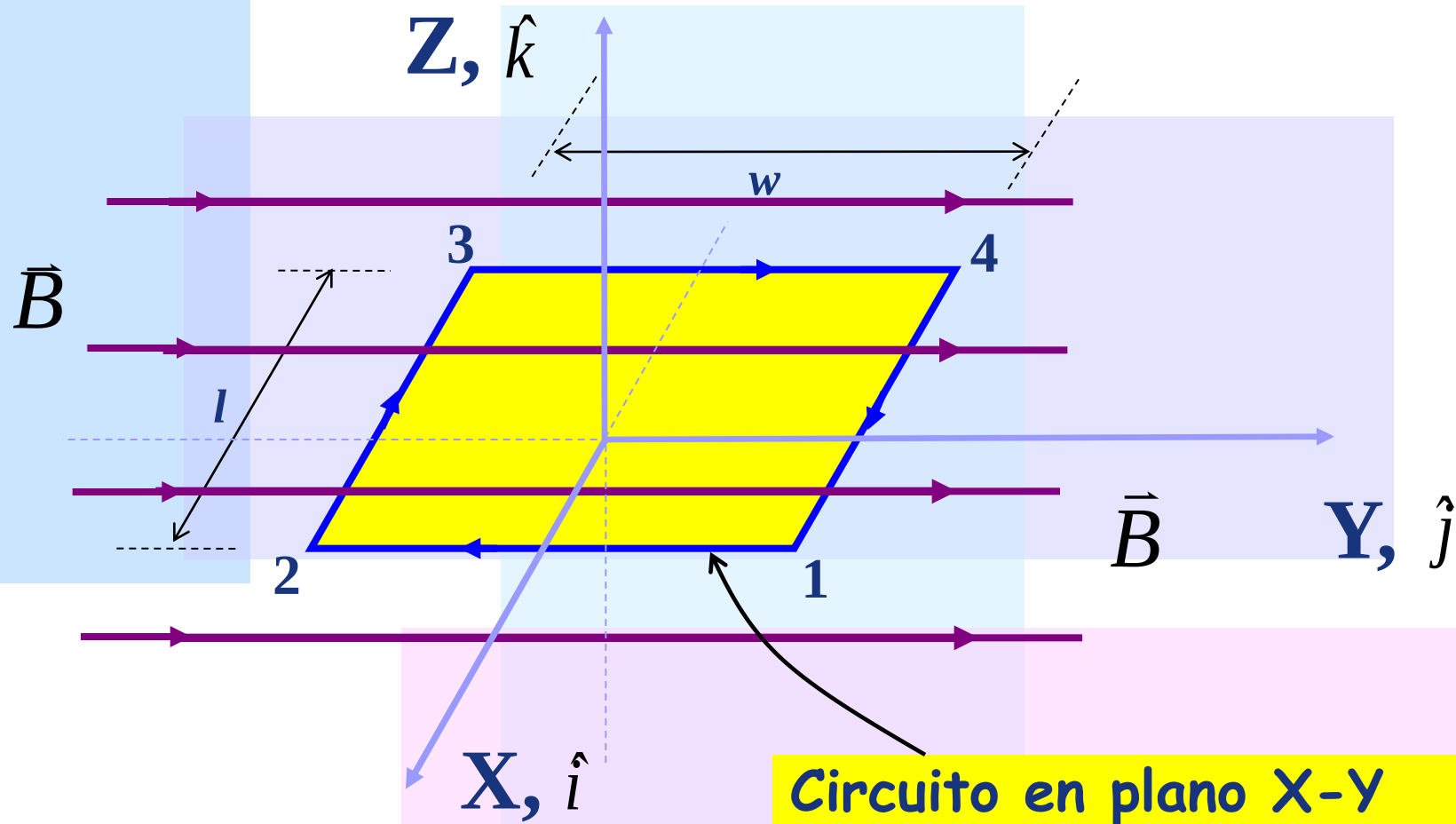
Ley de Biot y Savart

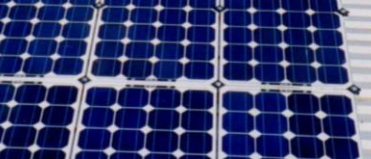
$$\therefore d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$



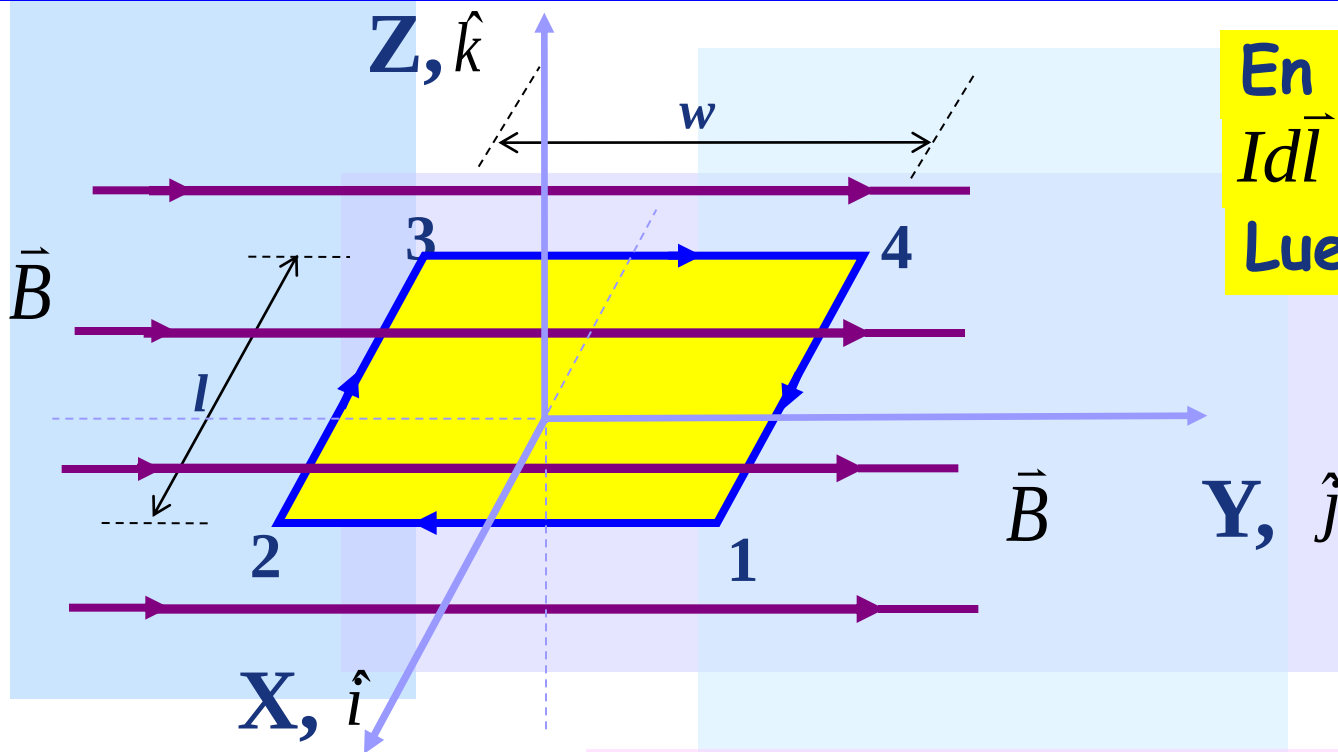


Torque Magnético





Torque Magnético



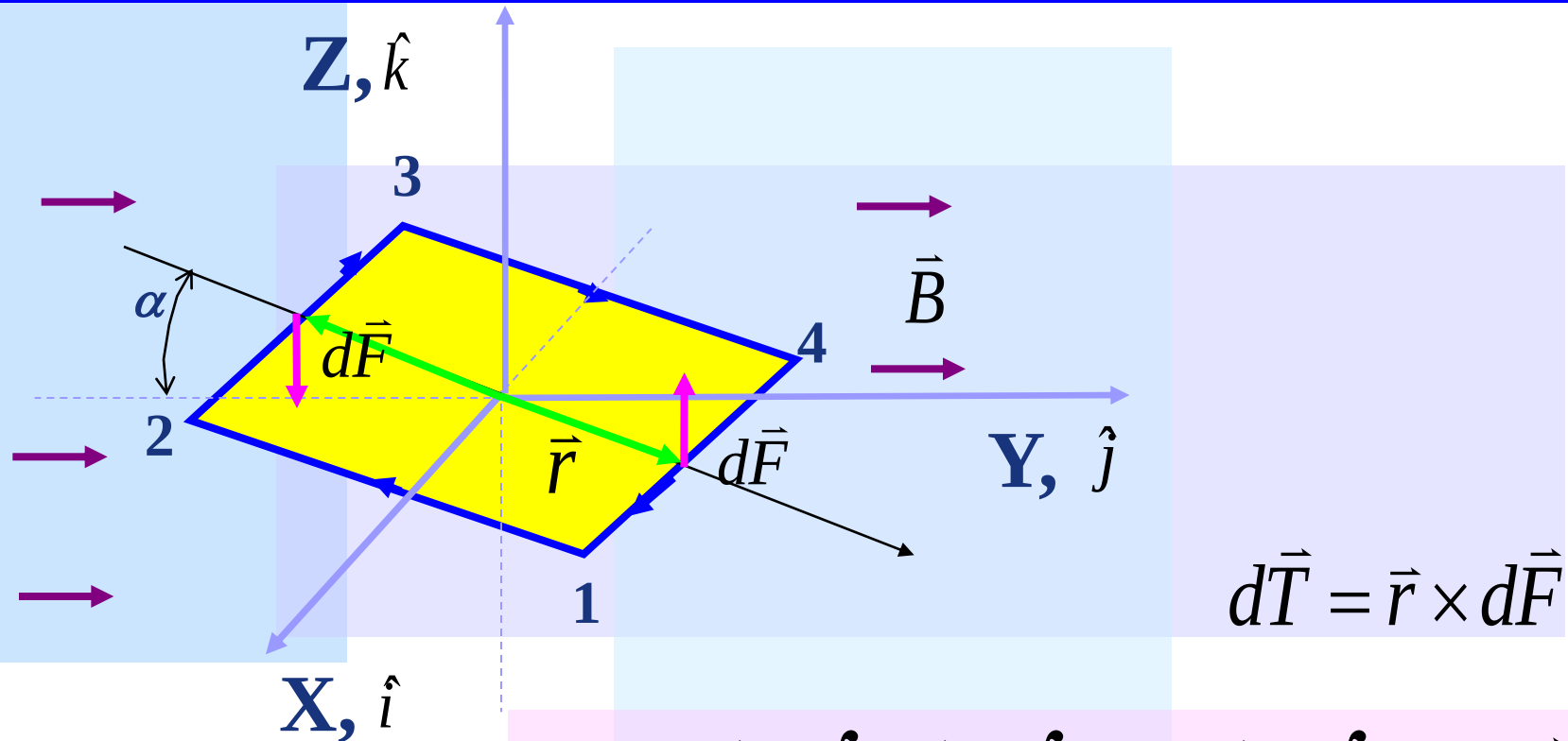
En lados 1-2 y 3-4
 $I d\vec{l}$ es paralelo a $\vec{B}$
Luego $F=0$

$$\vec{F} = I \int_2^3 d\vec{l} \times \vec{B} + I \int_4^1 d\vec{l} \times \vec{B} \Rightarrow \vec{F} = I \int_2^3 dx(-\hat{i}) \times \vec{B} + I \int_4^1 dx(\hat{i}) \times \vec{B}$$

$\therefore \vec{F} = 0$ Fuerza neta nula sobre el circuito si $\vec{B}$ constante



Torque Magnético

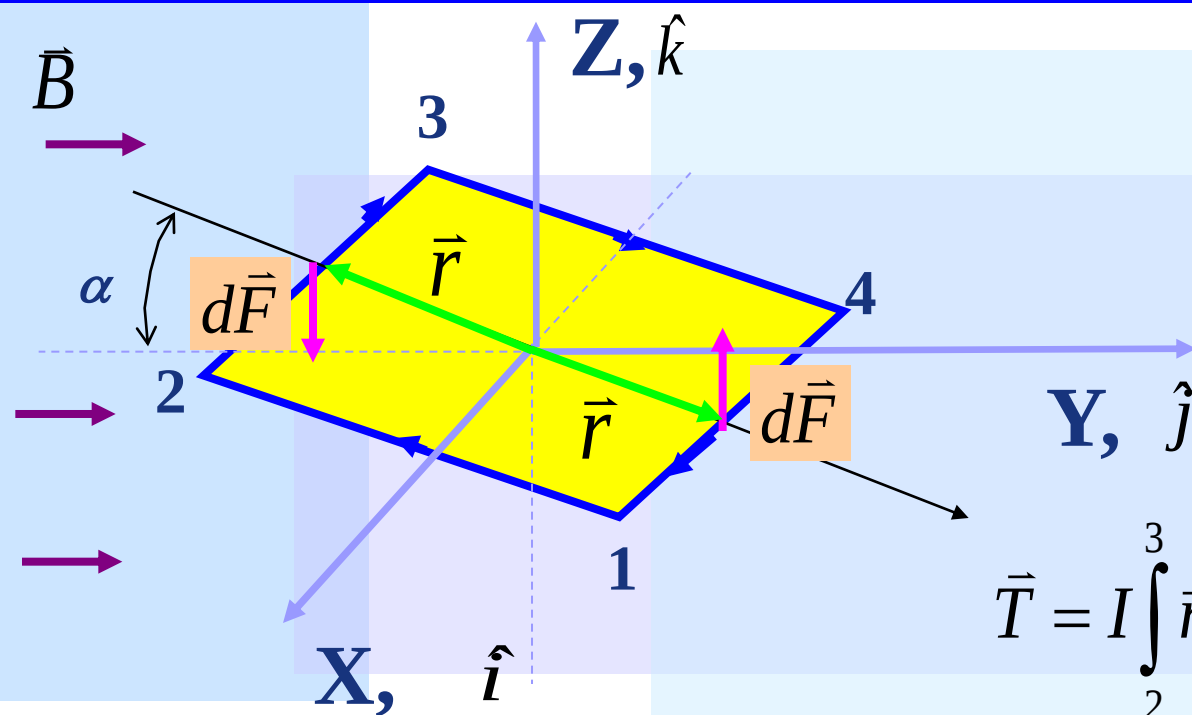


$$\vec{T} = \oint_C d\vec{T} = \oint_C \vec{r} \times d\vec{F} = \oint_C \vec{r} \times i d\vec{l} \times \vec{B}$$

Torque neto no nulo sobre el circuito



Torque Magnético



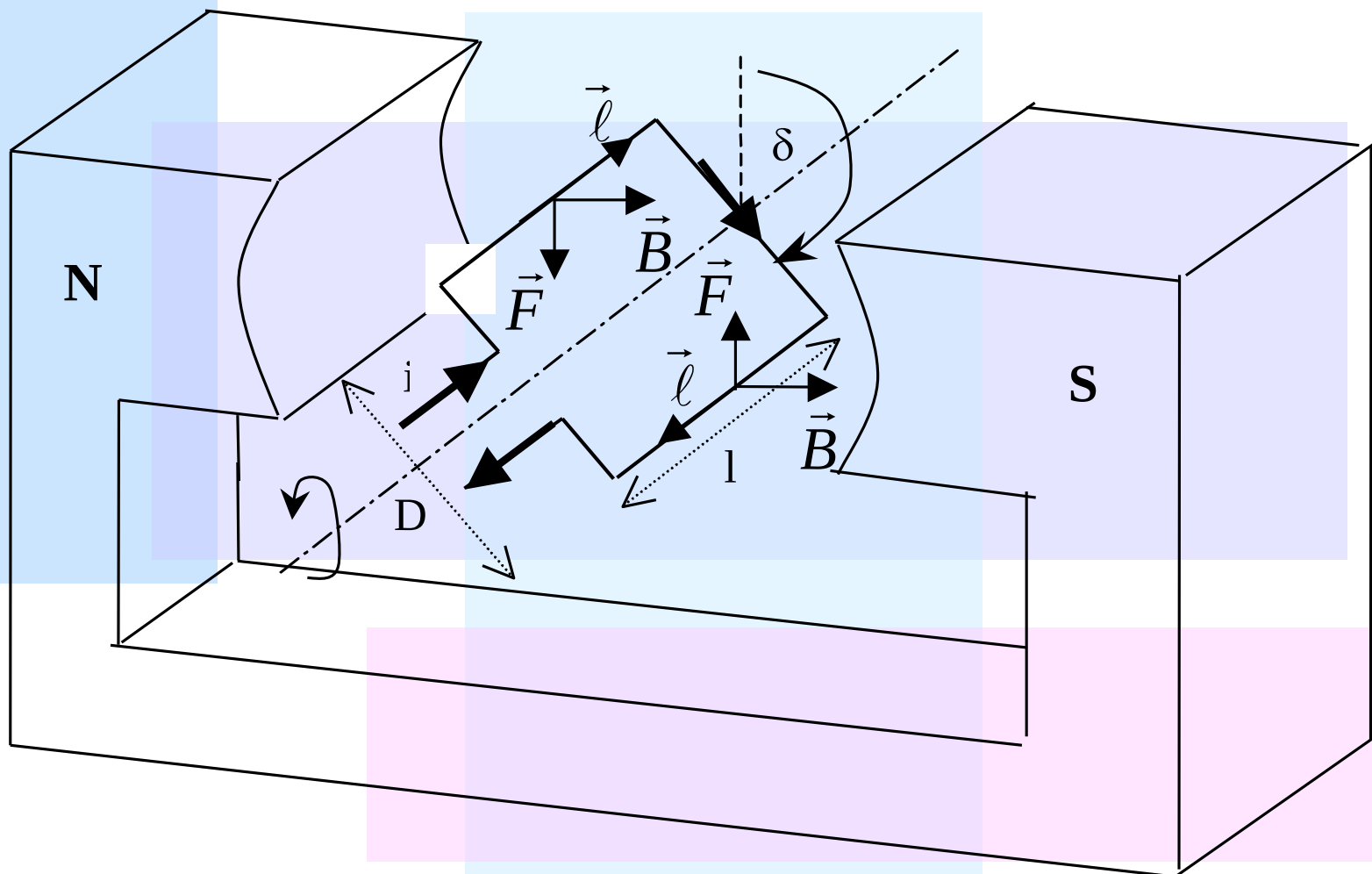
$$\vec{T} = I \int_2^3 \vec{r} \times dx \hat{i} \times \vec{B} + I \int_4^1 \vec{r} \times dx \hat{i} \times \vec{B}$$

$$\vec{T} = \frac{Iwl}{2} \cos \alpha \hat{i} + \frac{Iwl}{2} \cos \alpha \hat{i}$$

Torque neto sobre el circuito $\therefore \vec{T} = Iwl \cos \alpha \hat{i}$

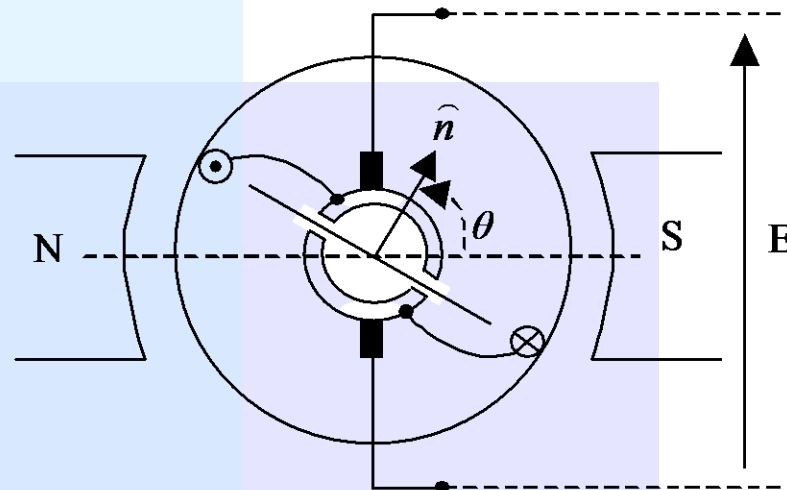
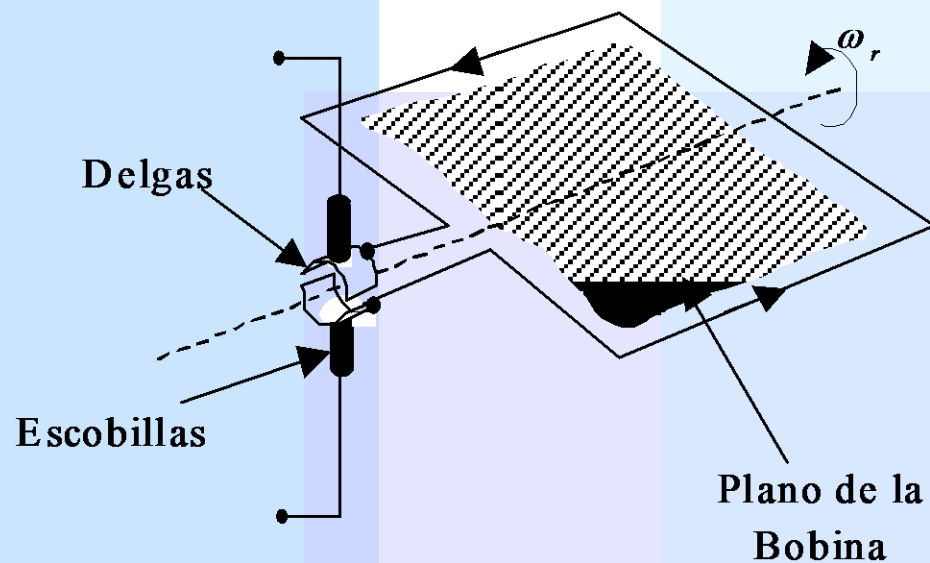


Motor elemental





Motor elemental

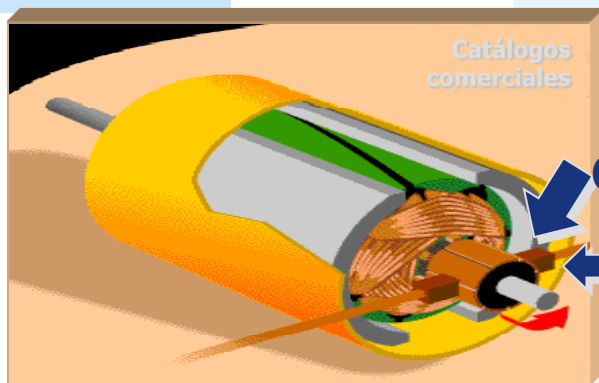




Motores



Motor de CC de 6000 kW fabricado por ABB



Escobillas

**Colector
real**

