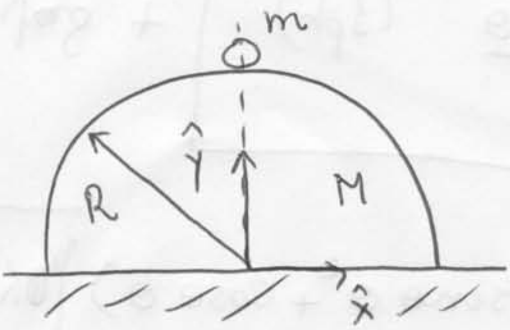


$t=0$

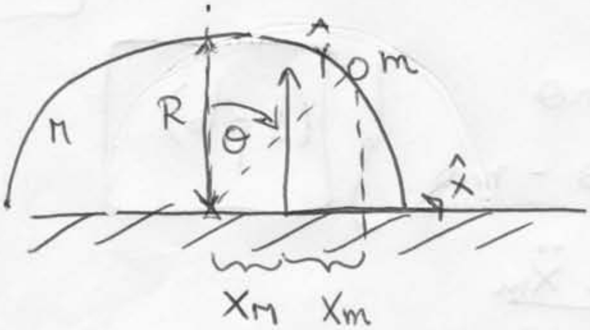
$$\sum \vec{F}_{ext} = (m+M)\,\vec{g} + \vec{N}_p$$

$\downarrow$ peso $\qquad \downarrow$ normal con el piso

Ambas fuerzas son verticales

$$\Rightarrow \sum F^x_{ext} = 0 \Rightarrow a^x_{cm} = 0$$

$$\Rightarrow V^x_{cm} = cte.$$



como en $t=0$ m y M quieto

$$\Rightarrow V^x_{cm}\big|_{t=0} = 0 = V^x_{cm} \quad \forall t.$$

$$\Rightarrow X_{cm} = cte. \quad \left(\begin{matrix} \text{en } t=0 \\ X_{cm}=0 \end{matrix}\right)$$

$$\Rightarrow \boxed{X_{cm} = 0 \quad \forall t. \ (0{,}5)}$$

$$X_{cm} = \frac{M\cdot -X_M + m(R\operatorname{sen}\theta - X_M)}{(m+M)}$$

pero $\quad X_{cm} = 0 \Rightarrow M X_M = m(R\operatorname{sen}\theta - X_M)$

$$\Rightarrow \boxed{X_M = \frac{m R\operatorname{sen}\theta}{(m+M)} \quad (0.5)}$$

$$\Rightarrow X_m = R\operatorname{sen}\theta - X_M = R\operatorname{sen}\theta\left(1 - \frac{m}{(m+M)}\right)$$

$$\Rightarrow \boxed{\boxed{X_m = \frac{M R\operatorname{sen}\theta}{(m+M)}} \quad (1)}$$

$y_m = R\cos\theta$ $\qquad \boxed{\dot{X}_M = \frac{mR\cos\theta\,\dot{\theta}}{(m+M)} \quad (1\text{ pto})}$ + gráfico

Entonces:

$$\dot{X}_m = \frac{MR\cos\theta\,\dot{\theta}}{(m+M)} \Rightarrow \ddot{X}_m = \frac{MR}{(m+M)}\left(-\sin\theta\,\dot{\theta}^2 + \cos\theta\,\ddot{\theta}\right) \quad \boxed{(0.5)}$$

$$\dot{y}_m = -R\sin\theta\,\dot{\theta} \Rightarrow \ddot{y}_m = -R\cos\theta\,\dot{\theta}^2 - R\sin\theta\,\ddot{\theta} \quad \boxed{(0.5)}$$

$$\left.\begin{aligned} (1)\quad & m\ddot{x}_m = N\sin\theta \\ (2)\quad & m\ddot{y}_m = N\cos\theta - mg \end{aligned}\right\} \quad \boxed{(1\text{ pto})}$$

$$(1) \Rightarrow N = \frac{m\ddot{x}_m}{\sin\theta}$$

$$\Rightarrow m\ddot{y}_m = m\frac{\ddot{X}_m}{\sin\theta}\cos\theta - mg$$

$$\Rightarrow -R\left(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}\right)\sin\theta = \frac{MR}{(m+M)}\left(-\sin\theta\,\dot{\theta}^2 + \cos\theta\,\ddot{\theta}\right)\cos\theta - g\sin\theta$$

$$\Rightarrow -mR\left(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}\right)\sin\theta - MR\ddot{\theta} = -g\sin\theta\,(m+M)$$

$$\Rightarrow R\left(M + m\sin^2\theta\right)\ddot{\theta} + mR\cos\theta\sin\theta\,\dot{\theta}^2 = g\sin\theta\,(m+M)$$

$$\Rightarrow \ddot{\theta} = \frac{(m+M)g\sin\theta - mR\cos\theta\sin\theta\,\dot{\theta}^2}{\left(M + m\sin^2\theta\right)R}$$

$$E_{inicial} = mgR + \frac{4R}{3\pi}$$

$$E_{final} = \frac{1}{2} m \dot{x}_m^2 + \frac{1}{2} M \dot{x}_M^2 + mgR\cos\theta + \frac{4R}{3\pi}$$

$$\Rightarrow E_{inicial} = E_{final}$$

$$\Rightarrow \frac{1}{2} m \frac{M^2 R^2 \cos^2\theta\, \dot{\theta}^2}{(m+M)^2} + \frac{1}{2} M \frac{m^2 R^2 \cos^2\theta\, \dot{\theta}^2}{(m+M)^2}$$

$$+ mgR\cos\theta + \cancel{\frac{4R}{3\pi}} = mgR + \cancel{\frac{4R}{3\pi}}$$

$$\Rightarrow \frac{R^2\cos^2\theta\, \dot{\theta}^2}{2(m+M)} (mM^2 + Mm^2) = mgR(1-\cos\theta)$$

$$\Rightarrow \frac{\cancel{m} M R^{\cancel{2}} \cos^2\theta\, \dot{\theta}^2}{2(m+M)} = \cancel{m}g\cancel{R}(1-\cos\theta)$$

$$\Rightarrow \dot{\theta}^2 = \frac{g(1-\cos\theta)\cdot 2\cdot (m+M)}{MR\cos^2\theta}$$

$$N = 0 \Rightarrow m\ddot{x}_m = 0 \Rightarrow \ddot{x}_m = 0.$$

$$\Rightarrow \cos\theta^* \ddot{\theta}^* = \sin\theta^* \dot{\theta}^{*2}$$

$$\Rightarrow \cos\theta \left[\frac{(m+M)g\cancel{\sin\theta} - mR\cos\theta\cancel{\sin\theta}\,\dot{\theta}^2}{(M+m\sin^2\theta)R} \right] = \cancel{\sin\theta}\, \dot{\theta}^2$$

$$\Rightarrow (m+M)g\cos\theta - mR\cos^2\theta\,\dot{\theta}^2 = (M + m\sin^2\theta)\,\dot{\theta}^2 R$$

$$\Rightarrow (m+M)g\cos\theta = MR\dot{\theta}^2 + mR\dot{\theta}^2$$

$$\Rightarrow \cancel{(m+M)}\,g\cos\theta = \cancel{(m+M)}\,R\dot{\theta}^2$$

$$g\cos\theta = \frac{\cancel{R}\,g(1-\cos\theta)\cdot 2(m+M)}{M\cancel{R}\cos^2\theta}$$

$$\Rightarrow M\cancel{g}\cos^3\theta = \cancel{g}(1-\cos\theta)\cdot 2(m+M)$$

$$\Rightarrow M\cos^3\theta + 2(m+M)\cos\theta - 2(m+M) = 0.$$