Physics of Electronics:

6. Junction Diodes

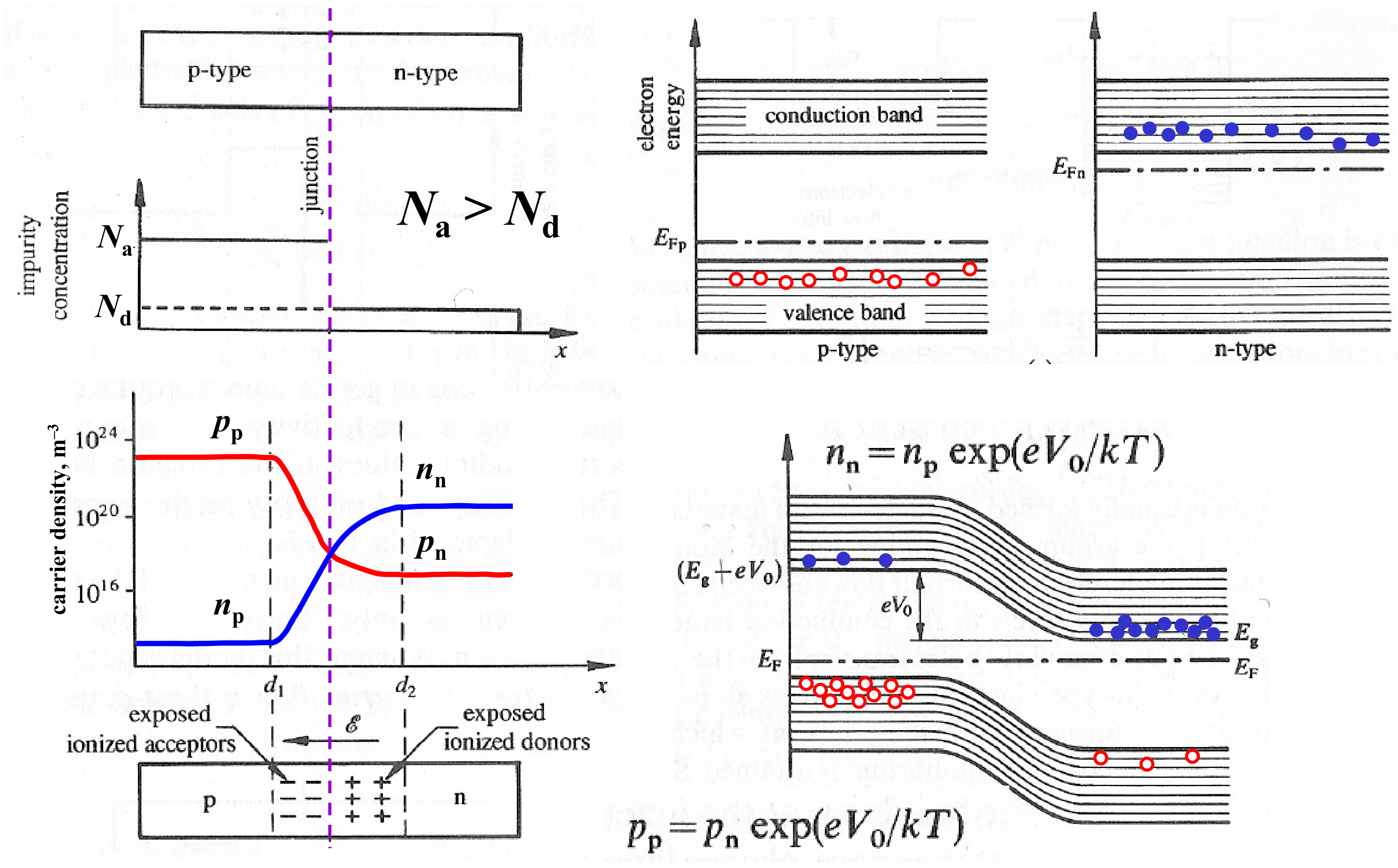
July – December 2009

Contents overview

- Junction diode in equilibrium.
- Biased junction diode.
- Electron-hole efficiencies.
- Diode capacitance.

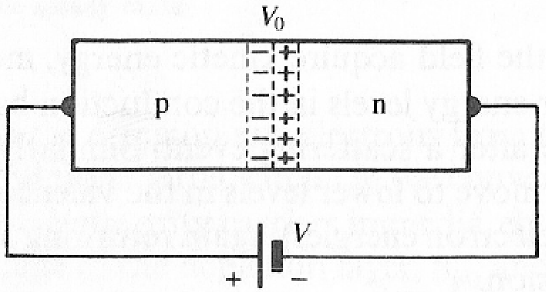
Pn junction in equilibrium

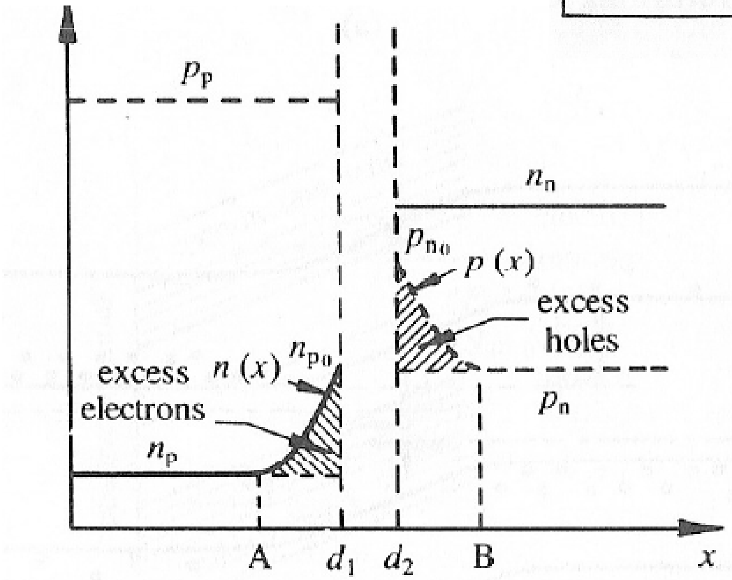
- Consider a pn junction with abrupt transition

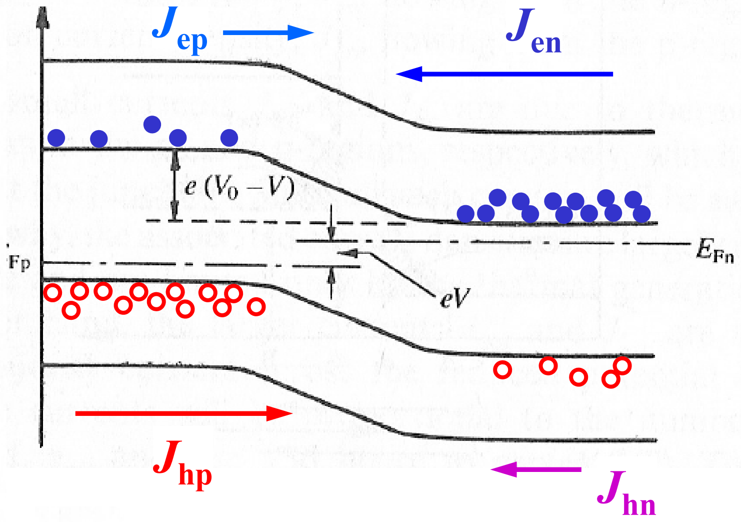


Pn junction with forward bias

- Consider a junction biased with a voltage V:







$J_{hp} \propto p_{n0}$

$$p_{n0} = p_n \exp(eV/kT) \quad \Rightarrow \quad J_{hp} = J_{hn} \exp(eV/kT)$$

$$\boxed{J = J_0[\exp(eV/kT) - 1]}$$

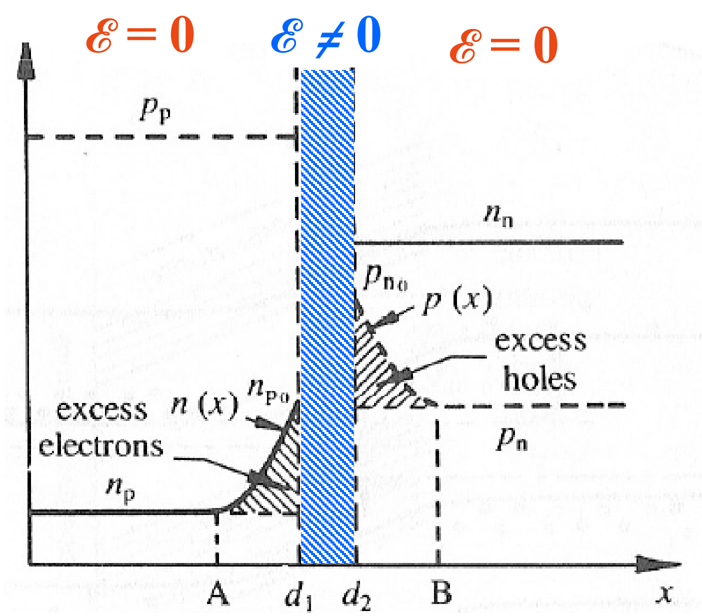
$J_{en} \propto n_{p0}$

$$n_{p0} = n_p \exp(eV/kT) \quad \Rightarrow \quad J_{en} = J_{ep} \exp(eV/kT)$$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for h^+ at $x \geq d_2$

V (steady state)



$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\frac{\delta p}{\tau_{Lh}} + \cancelto{0}{\mu_h \mathscr{E}_x \frac{\partial(\delta p)}{\partial x}} + D_h \frac{\partial^2(\delta p)}{\partial x^2}$$

$$\Rightarrow \quad \frac{d^2(\delta p)}{dx^2} = \frac{\delta p}{\tau_{Lh} D_h} = \frac{\delta p}{L_h^2}$$

$$\Rightarrow \quad \delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

$$\Rightarrow \quad p(x) = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h) + p_n$$

To find the constants:

- $x = \infty \Rightarrow p(\infty) = p_n \Rightarrow C_2 = 0$
- $x = d_2 \Rightarrow p(0) = p_{n0} \Rightarrow C_1 = p_{n0} - p_n$

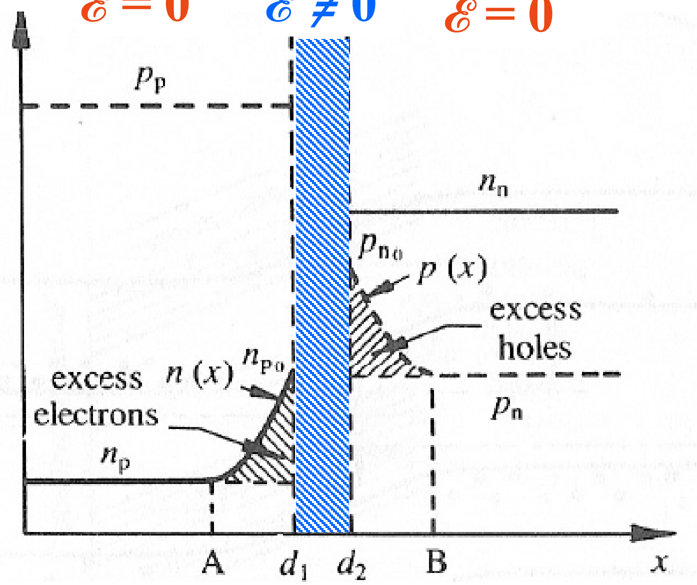
$$\Rightarrow \quad p(x) = (p_{n0} - p_n)\exp(-x/L_h) + p_n$$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for h^+ at $x \geq d_2$

V (steady state)

$\mathscr{E} = 0$ $\mathscr{E} \neq 0$ $\mathscr{E} = 0$



- Since $\mathscr{E} = 0$, there is only diffusion:

$$J_{Dh} = -eD_h \mathrm{d}p/\mathrm{d}x$$

$$\Rightarrow J_h = -eD_h[-(1/L_h)(p_{n0} - p_n)\exp(-x/L_h)]$$

$x = 0$

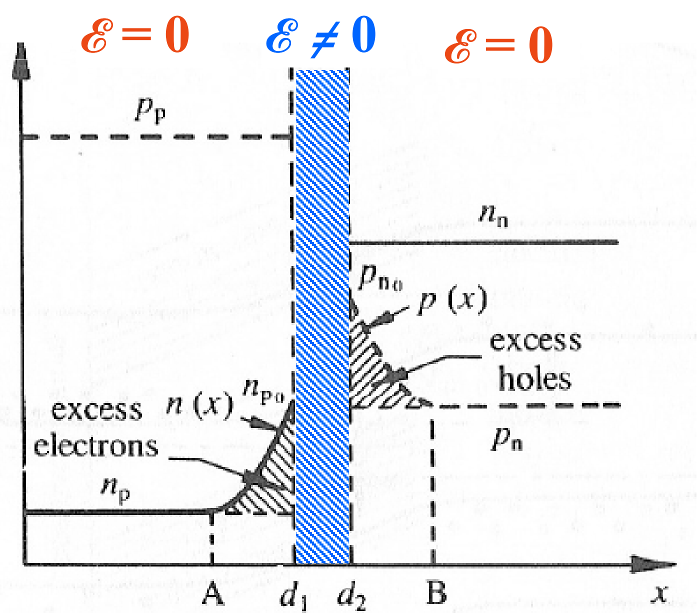
$$\Rightarrow J_h|_{d_2} = (eD_h/L_h)(p_{n0} - p_n)$$

But $p_{n0} = p_n \exp(eV/kT)$ $\Rightarrow$ $J_h|_{d_2} = (eD_h/L_h)p_n\,[\exp(eV/kT) - 1]$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for e^- at $x \le d_1$:

V (steady state)



$$J_e|_{d_1} = (eD_e n_p/L_e)\,[\exp(eV/kT) - 1]$$

Total current is then:

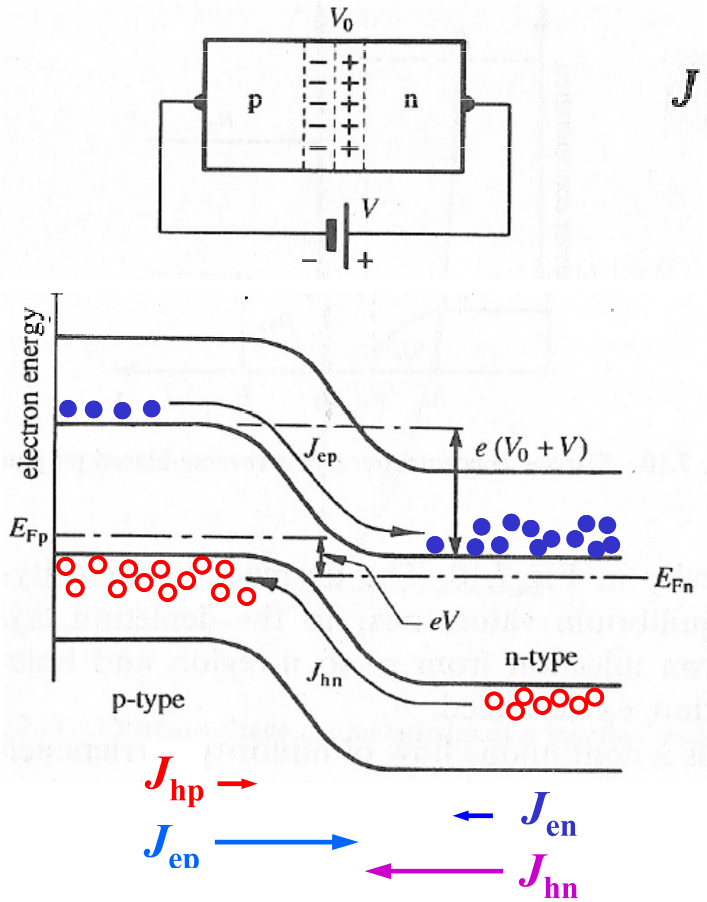
$$\boxed{J = J_h + J_e = e\underbrace{\left(\frac{D_h p_n}{L_h} + \frac{D_e n_p}{L_e}\right)}_{J_0}[\exp(eV/kT) - 1]}$$

For $V >> kT$:

$$J \simeq J_0 \exp(eV/kT)$$

Pn junction with reversed bias

- Currents through the junction :
  - Applying diode equation (with $-V$):



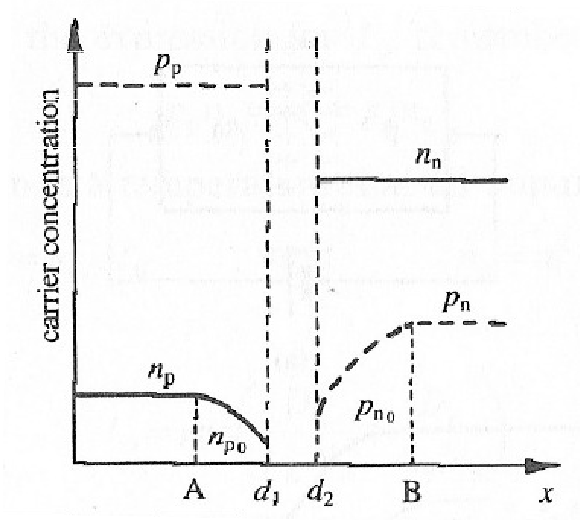
$$J = J_0[\exp(-eV/kT) - 1] = -J_0[1 - \exp(-eV/kT)]$$

For $V \gg kT$: $\quad J \approx -J_0$

$$J = \cancelto{0}{J_{hp}} - J_{hn} + \cancelto{0}{J_{en}} - J_{ep} \approx -J_0$$

Pn junction with reversed bias

- Minority carriers:
  - As before (with $-V$):



$$n_{p0} = n_p \exp(-eV/kT)$$

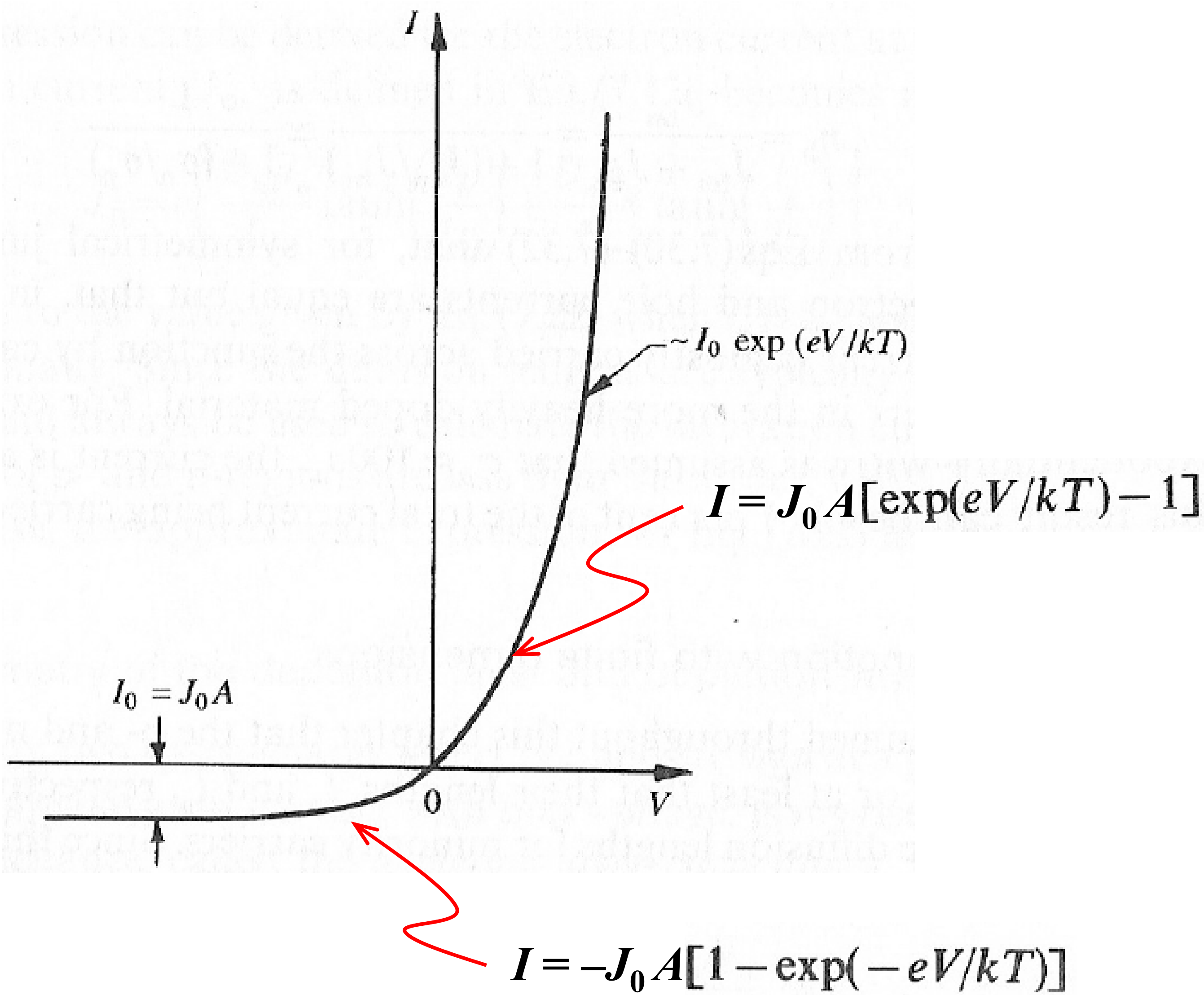
$$p_{n0} = p_n \exp(-eV/kT)$$

  - Applying continuity eq. as before

$$\boldsymbol{p(x) = p_n\{[\exp(-eV/kT) - 1]\exp(-x/L_h) + 1\}}$$

I-V curve of a junction diode

- Applying the results for the current density:



Electron-holes efficiencies

- Relative hole and electron currents:
  - Excess carriers:

$$J_h = J_{hp} - J_{hn} = J_{hn}\,[\exp(eV/kT) - 1]$$

$$J_e = J_{en} - J_{ep} = J_{ep}\,[\exp(eV/kT) - 1]$$

$$\frac{J_h}{J_e} = \frac{J_{hn}}{J_{ep}}$$

  - Continuity equations:

$$J_h|_{d_2} = (eD_h/L_h)p_n\,[\exp(eV/kT) - 1]$$

$$J_e|_{d_1} = (eD_e n_p/L_e)\,[\exp(eV/kT) - 1]$$

$$\frac{J_h}{J_e} = \frac{J_{hn}}{J_{ep}} = \frac{D_h}{L_h N_d}\,\frac{L_e N_a}{D_e}$$

Electron-holes efficiencies

- Relative hole and electron currents:
  - Einstein equation ($D_e/\mu_e = D_h/\mu_h = kT/e$) and conductivity equation ($\sigma = e(n\mu_e + p\mu_h)$):

$$\frac{J_h}{J_e} = \frac{L_e}{L_h}\frac{\mu_h N_a}{\mu_e N_d} \Rightarrow \boxed{J_h/J_e \simeq \sigma_p/\sigma_n}$$

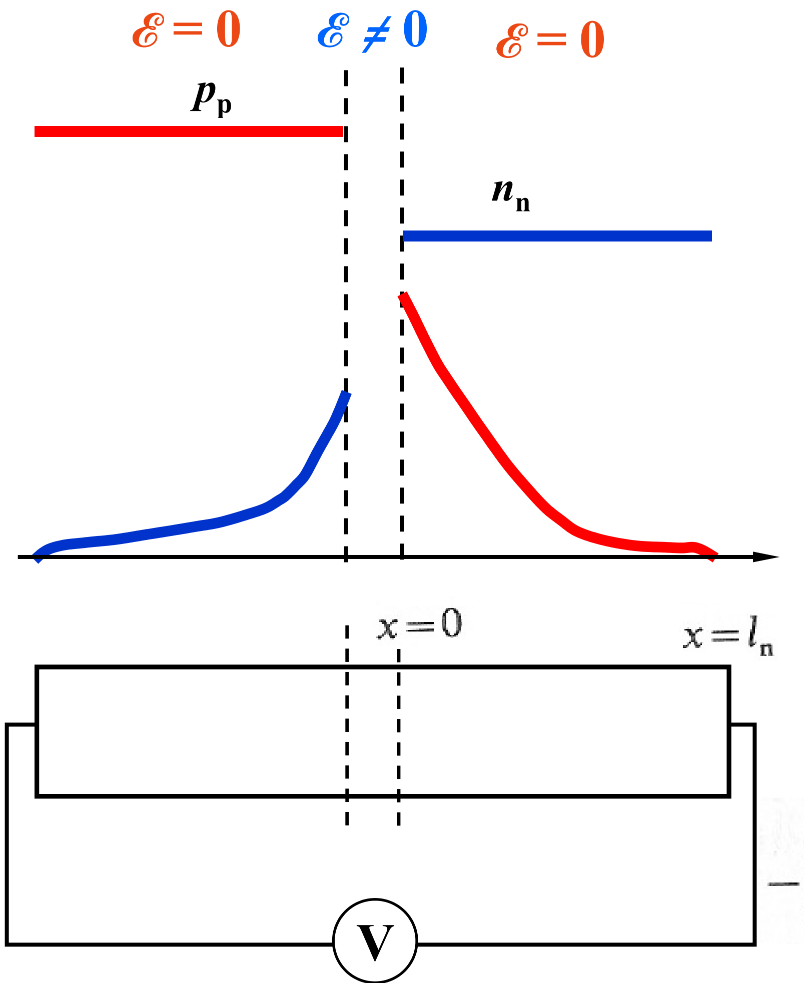
  - Hole and electron efficiencies:

$$\eta_h \equiv \frac{J_{hn}}{J_{hn} + J_{ep}} = \frac{1}{1+(J_{ep}/J_{hn})} \simeq \frac{1}{1+(\sigma_n/\sigma_p)}$$

$$\eta_e \equiv \frac{J_{ep}}{J_{hn} + J_{ep}} = \frac{1}{1+(J_{hn}/J_{ep})} \simeq \frac{1}{1+(\sigma_p/\sigma_n)}$$

Pn junction with finite dimensions

- Current in a finite junction:
  - From the continuity eq. we obtained



$$\delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

New border conditions:

$p = p_{n0} = p_n \exp(eV/kT)$ when $x=0$

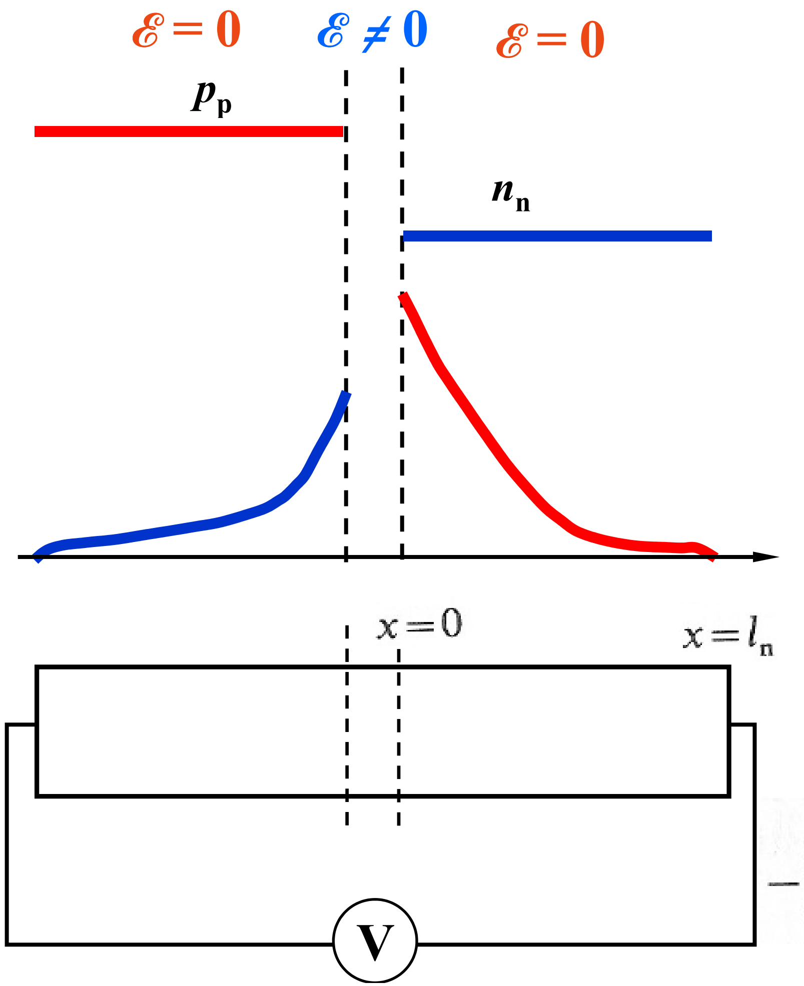
$p = 0$ when $x = l_n$

$$p(x) = \left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1-\exp(-2l_n/L_h)}\right)\exp(-x/L_h)$$

$$-\left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1-\exp(-2l_n/L_h)} - (p_{n0}-p_n)\right)\exp(+x/L_h) + p_n$$

Pn junction with finite dimensions

- Current in a finite junction:
  - From the continuity eq. we obtained



$$\delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

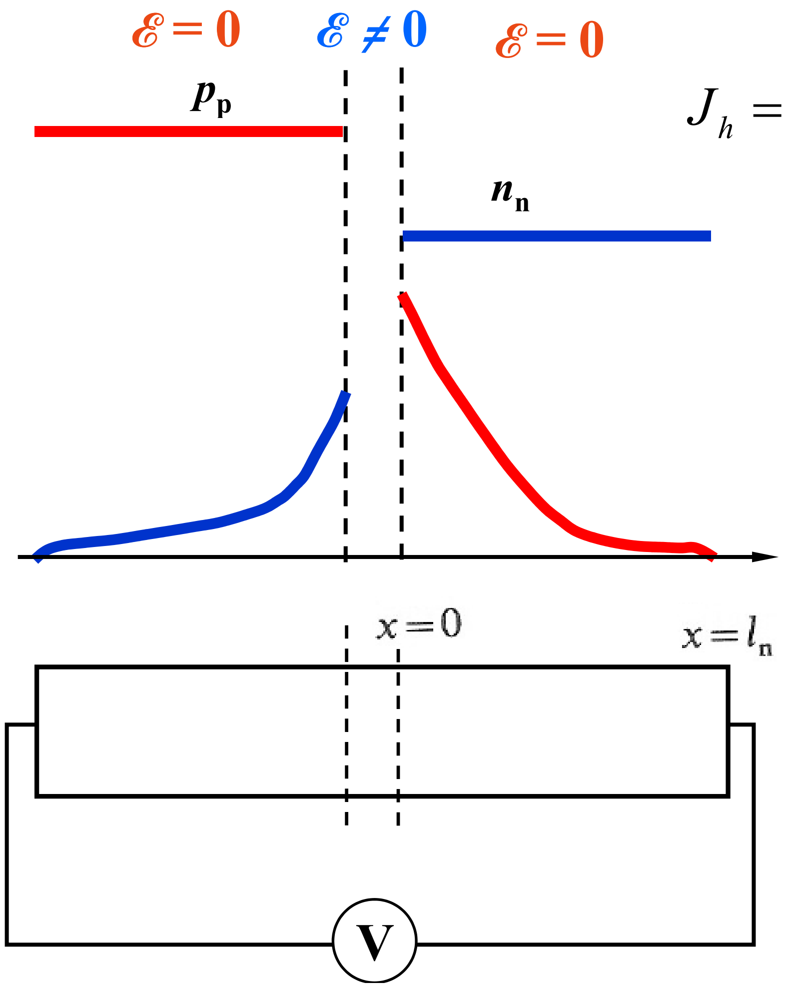
New border conditions:

$$p = p_{n0} = p_n \exp(eV/kT) \qquad \text{when } x = 0$$
$$p = 0 \qquad \text{when } x = l_n$$

$$p(x) = \left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1 - \exp(-2l_n/L_h)}\right)\exp(-x/L_h)$$
$$-\left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1 - \exp(-2l_n/L_h)} - (p_{n0} - p_n)\right)\exp(+x/L_h) + p_n$$

Pn junction with finite dimensions

- Current in a finite junction:
  - Currents:



For holes:

$$J_h = -eD_h\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)\bigg|_{x=0} = \frac{eD_h}{L_h}\,p_n\tanh\left(\frac{l_n}{L_h}\right)\left[\exp(eV/kT)-1\right]$$

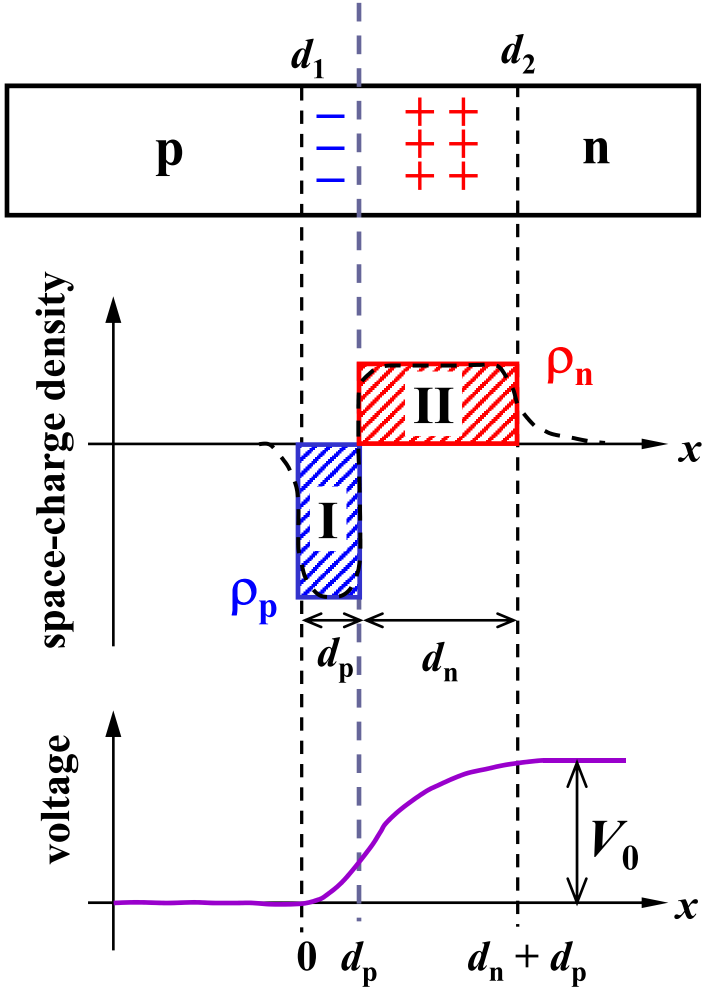
Idem for the electrons. Therefore the saturation current is:

$$J_0 = e\left[\frac{D_h p_n}{L_h}\underbrace{\tanh\left(\frac{l_n}{L_h}\right)}+\frac{D_e n_p}{L_e}\underbrace{\tanh\left(\frac{l_p}{L_e}\right)}\right]$$

→1 when $l \gg L$
($L \sim 1$ mm)

Depletion-layer capacitance

- The depletion layer forms a capacitance (important for high frequency applications)



Using Poisson relation ($\partial^2 V/\partial x^2 = -\rho/\epsilon$):

I:

$$\rho_p \simeq -N_a e \quad \Rightarrow \quad \partial^2 V_1/\partial x^2 = eN_a/\epsilon$$

$V = 0$ & $\partial V/\partial x = 0$ at $x = 0$

$$\Rightarrow \quad V_1(x) = eN_a x^2/2\epsilon$$

II:

$$\rho_n \cong N_d e \quad \Rightarrow \quad \partial^2 V_2/\partial x^2 = -eN_d/\epsilon$$

$$\Rightarrow \quad V_2 = -eN_d x^2/2\epsilon + C_1 x + C_2$$

$$(\partial V_1/\partial x)|_{d_p} = (\partial V_2/\partial x)|_{d_p}$$

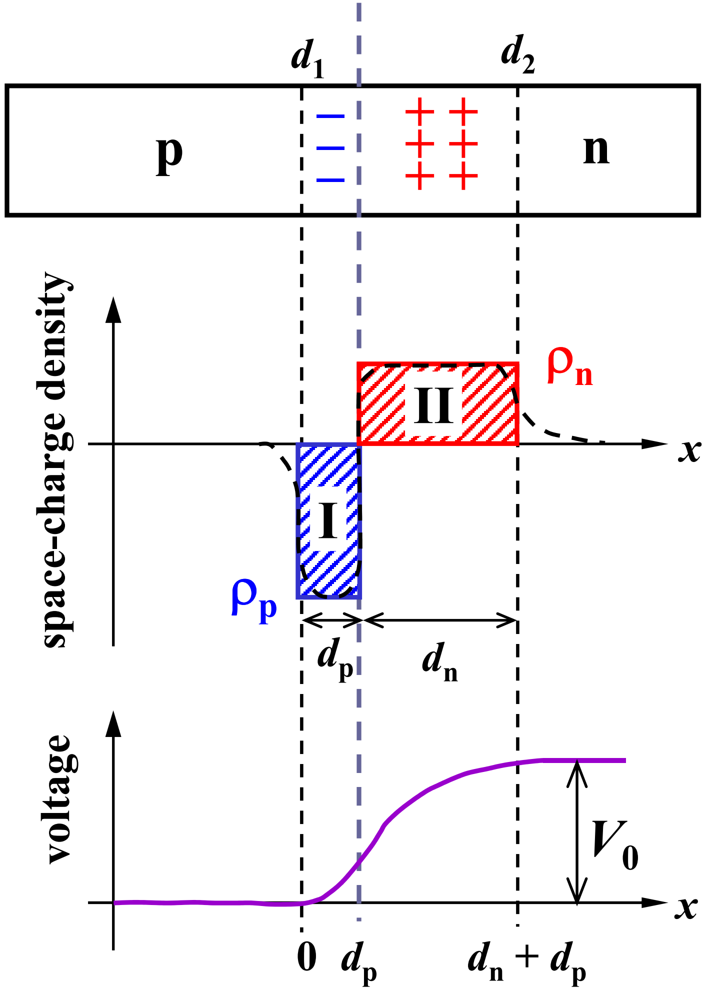
$$V_1|_{d_p} = V_2|_{d_p}$$

$$\Rightarrow \quad C_1 = ed_p(N_a + N_d)/\epsilon$$

$$C_2 = -\frac{ed_p^2(N_a + N_d)}{2\epsilon}$$

Depletion-layer capacitance

- Expressions for the depletion layer:



Applying $\mathscr{E} = 0$ at $x \geq d_n + d_p$:

$$(\partial V_2/\partial x)|_{d_n + d_p} = 0 \Rightarrow \boxed{d_p/d_n = N_d/N_a}$$

Applying $V = V_0$ at $x \geq d_n + d_p$ & using previous relation:

$$V_2\big|_{d_n + d_p} = V_0 \Rightarrow \boxed{d_p = \left(\frac{2\epsilon V_0 N_d}{eN_a(N_a + N_d)}\right)^{1/2}}$$

$$\Rightarrow \boxed{d_n = \left(\frac{2\epsilon V_0 N_a}{eN_d(N_a + N_d)}\right)^{1/2}}$$

This expresions are still applicable for a biased junction $V_0 \leftrightarrow V_0 - V$

Depletion-layer capacitance

- Capacitance of the depletion layer:
  - Charge per unit area accumulated at the depletion layer

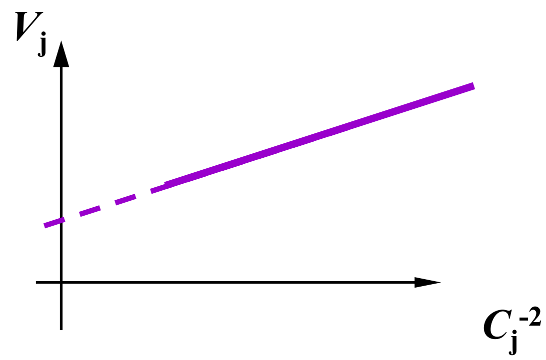
$$Q_j = eN_d d_n = eN_a d_p \xrightarrow{V_j = V_0 \pm V} Q_j = \left(\frac{2\epsilon e V_j N_a N_d}{N_a + N_d}\right)^{1/2}$$

  - The capacitance per unit area ($C_j = dQ_j/dV_j$) is then:

$$C_j = \left(\frac{\epsilon e N_a N_d}{2(N_a + N_d)}\right)^{1/2} \frac{1}{V_j^{1/2}} \Longrightarrow \boxed{C_j = \epsilon/(d_p + d_n)}$$

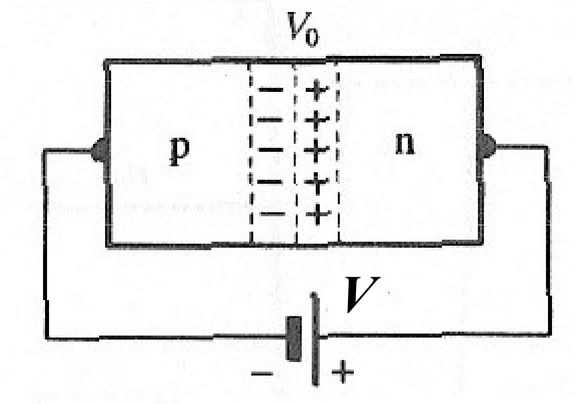
$$\Big\Downarrow N_a \gg N_d:$$

$$V_j = V_0 + V \simeq \frac{1}{C_j^2}\left(\frac{\epsilon e N_d}{2}\right)^{1/2}$$



Equivalent circuit

- Reversed biased junction:



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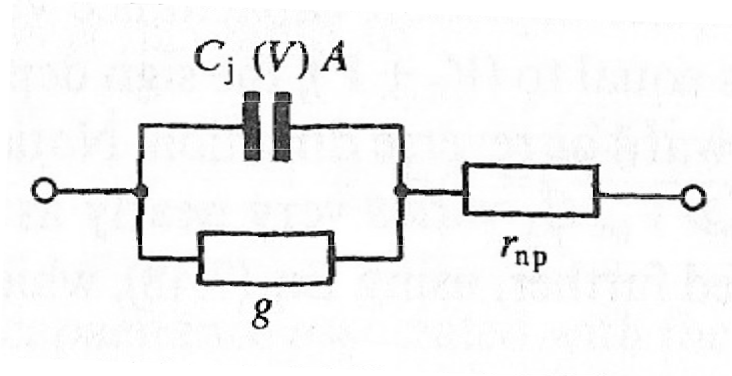
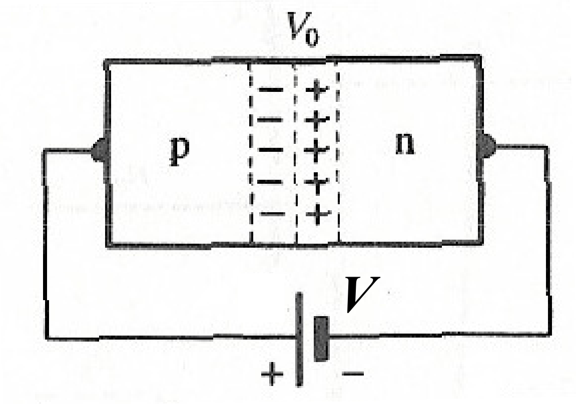


r_{np} :	Resistance of the junction outside depletion layer
$C_j(V)$:	Depletion layer capacitance
g :	Conductance of the reverse flow

Equivalent circuit

- Forward biased junction:



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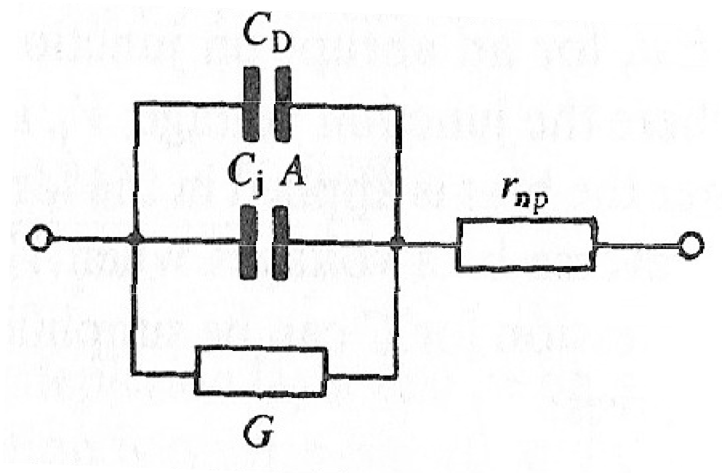
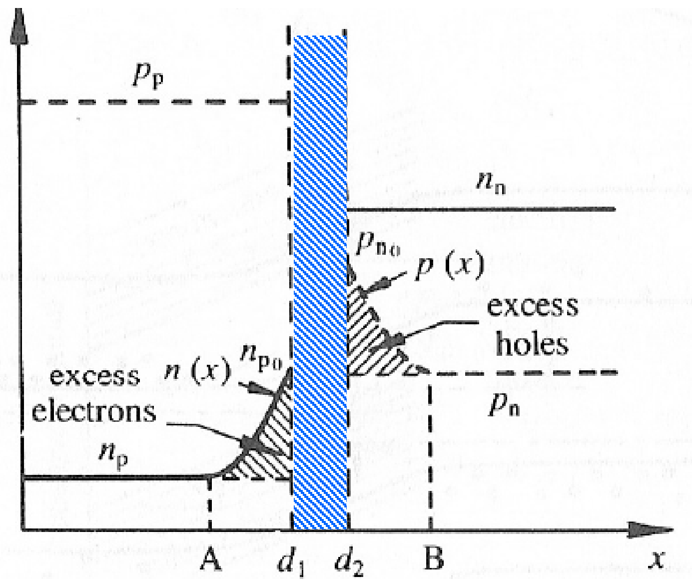
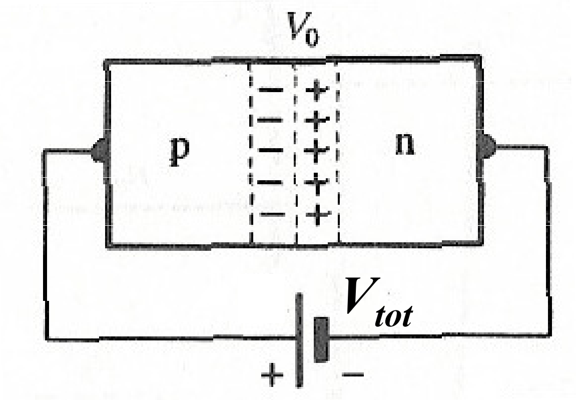




r_{np} :	Resistance of the junction outside depletion layer
$C_j(V)$:	Depletion layer capacitance
G :	Conductance of the forward flow
C_D :	Diffusion capacitance (due to excess holes and excess electrons)

Equivalent circuit

- Forward biased junction with an AC signal:



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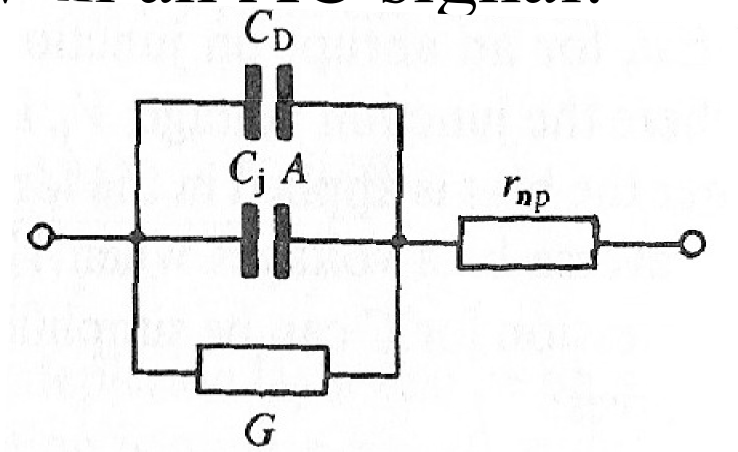


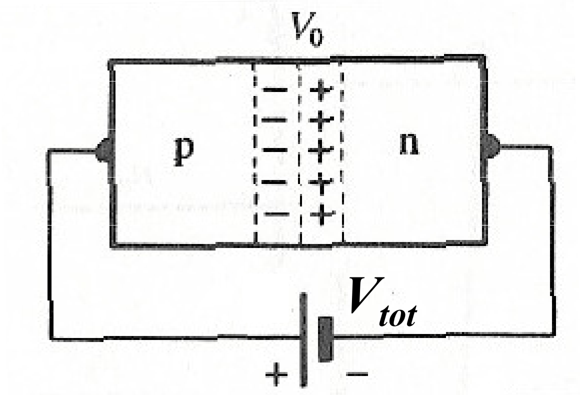
$$\left.\begin{array}{l} V_{tot} = V + V_1 \exp(i\omega t) \\ p_{n0} = p_n \exp(eV/kT) \end{array}\right\} \Rightarrow p_{n0} = p_n \exp\left(\frac{e}{kT}[V + V_1 \exp(j\omega t)]\right)$$

$$V_1 \ll V \Rightarrow p_{n0} = \underbrace{p_n \exp\left(\frac{eV}{kT}\right)}_{\text{DC}} \underbrace{\left(1 + \frac{eV_1}{kT}\exp(j\omega t)\right)}_{\text{AC}}$$

$$\Rightarrow \delta p = p(x,t) - p_n = \overbrace{p_0(x)}^{\text{DC}} + \overbrace{p_1(x)\exp(j\omega t)}^{\text{AC}} - p_n$$

Equivalent circuit

- Forward biased junction with an AC signal:



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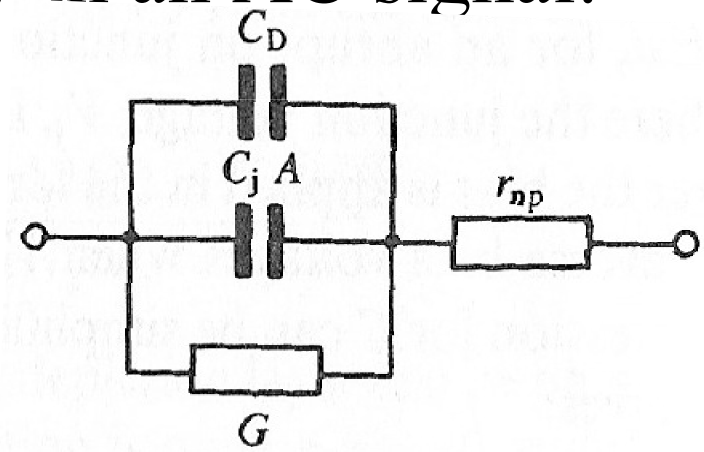


$$\frac{\partial(\delta n)}{\partial t} = -\frac{\delta n}{\tau_{Le}} + \mu_e \mathscr{E}_x \frac{\partial(\delta n)}{\partial x} + D_e \frac{\partial^2(\delta n)}{\partial x^2} \Rightarrow \mathrm{j}\omega p_1 = \frac{-p_1}{\tau_{Lh}} + D_h \frac{\partial^2 p_1}{\partial x^2}$$

$$\Rightarrow \frac{\partial^2 p_1}{\partial x^2} = \frac{1+\mathrm{j}\omega\tau_{Lh}}{L_h^2} p_1 \Rightarrow p_1 = C \exp\left(-\frac{(1+\mathrm{j}\omega\tau_{Lh})^{1/2} x}{L_h} \right)$$

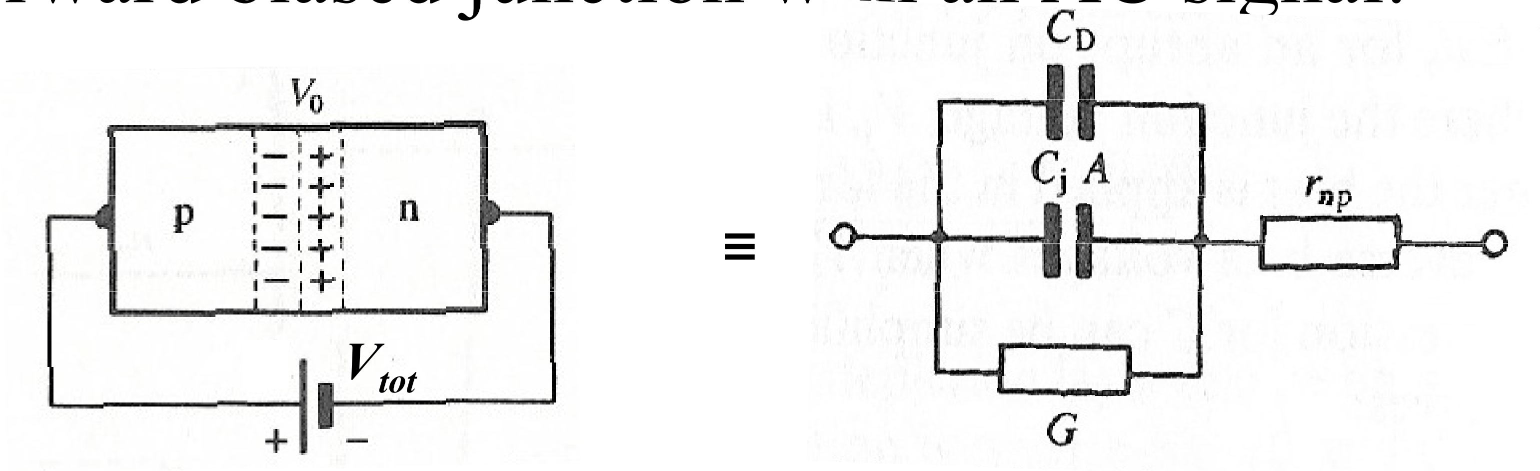
But:

$$p_1(x=0) = p_{n0}$$
$$= p_n \exp(eV/kT)$$

$$\Rightarrow p_1 = p_n \left(\frac{eV_1}{kT}\right) \exp\left(\frac{eV}{kT}\right) \exp\left(-\frac{(1+\mathrm{j}\omega\tau_{Lh})^{1/2} x}{L_h} \right) \exp(\mathrm{j}\omega t)$$

Equivalent circuit

- Forward biased junction with an AC signal:



  – Alternating currents of holes injected into n-region:

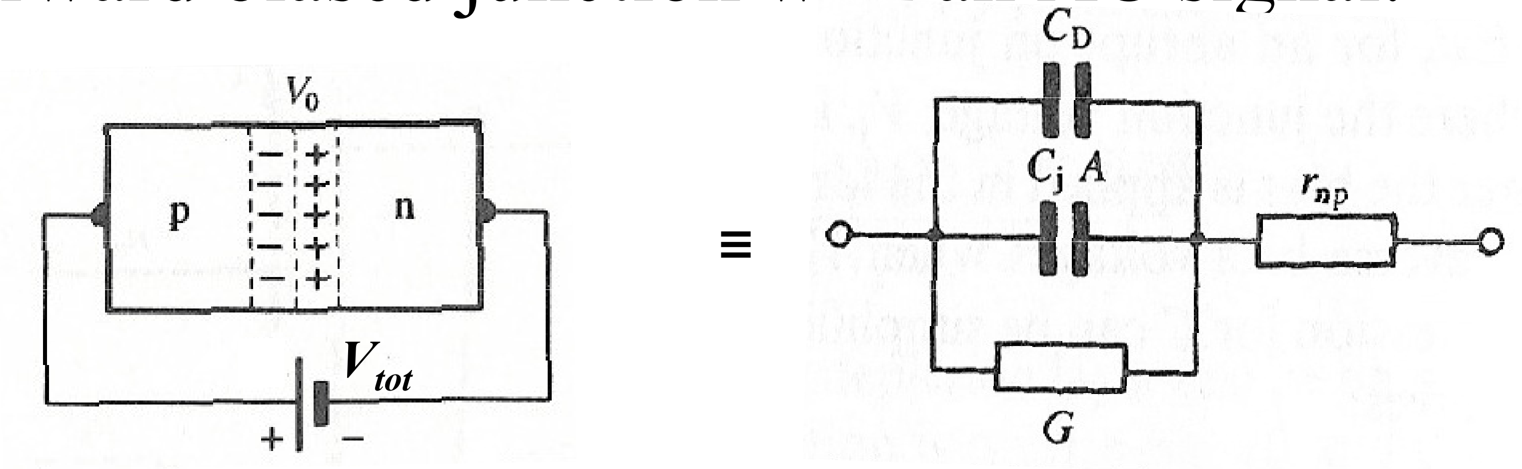
$$J_{h1} = -eD_h \left(\frac{\mathrm{d}p(x)}{\mathrm{d}x} \right)\bigg|_{x=0} = \frac{(1+i\omega\tau_{Lh})^{1/2}}{L_h} \frac{p_n D_h e^2 V_1}{kT} \exp\left(\frac{eV}{kT}\right) \exp(i\omega t)$$

  – Alternating currents of electrons injected into p-region:

$$J_{e1} = -eD_e \left(\frac{\mathrm{d}n(x)}{\mathrm{d}x} \right)\bigg|_{x=0} = \frac{(1+i\omega\tau_{Le})^{1/2}}{L_e} \frac{n_p D_e e^2 V_1}{kT} \exp\left(\frac{eV}{kT}\right) \exp(i\omega t)$$

Equivalent circuit

- Forward biased junction with an AC signal:



  – Total alternating currents of minority injected carriers:

$$J_1 = J_{h1} + J_{e1}$$

  – Admittance ($Y_1 = J_1/V_1$) for $\omega t \ll 1$:

$$Y_1 \cong \underbrace{\frac{e^2}{kT}\exp\left(\frac{eV}{kT}\right)\left(\frac{D_h p_n}{L_h} + \frac{D_e n_p}{L_e}\right)}_{G} + \underbrace{\frac{j\omega e^2}{2kT}\left(\frac{D_h p_n \tau_{Lh}}{L_h} + \frac{D_e n_p \tau_{Le}}{L_e}\right)}_{C_D}$$

Conclusions

- The diode equation was calculated using the continuity equation.
- The diode forms a natural capacitance that was calculated.
- The equivalent circuit of the diode was studied.