Physics of Electronics:

6. Junction Diodes

July – December 2009

Contents overview

- Continuity equation.
- Semiconductor measurements.
- Junction diode in equilibrium.
- Biased junction diode.
- Electron-hole efficiencies.
- Diode capacitance.

Continuity Equation for Minority Carriers

- For a **p-type** material ($n = n_0 + \delta n$):

$$\left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{total}} = -\frac{\delta n}{\tau_{\text{Le}}} + \frac{1}{e}\frac{\partial J_{\text{e}}}{\partial x}$$

Change due to current flow

Change due to generation and recombination

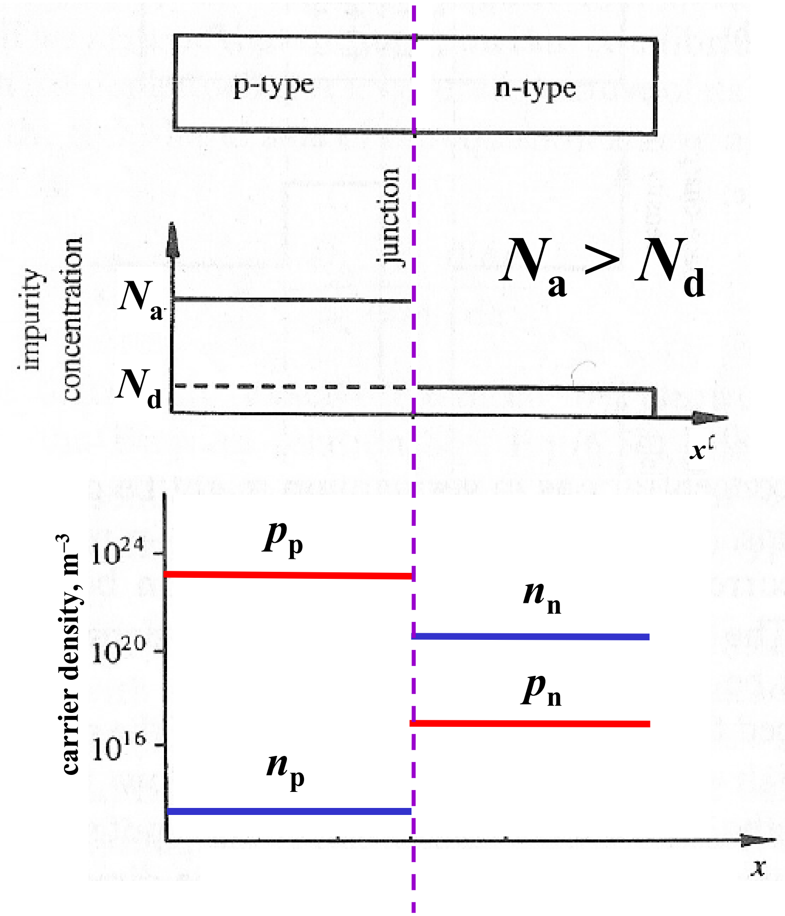
But $J_{\text{e}} = ne\mu_{\text{e}}\mathscr{E} + eD_{\text{e}}\nabla n$

$$\boxed{\frac{\partial(\delta n)}{\partial t} = -\frac{\delta n}{\tau_{\text{Le}}} + \mu_{\text{e}}\mathscr{E}_x\frac{\partial(\delta n)}{\partial x} + D_{\text{e}}\frac{\partial^2(\delta n)}{\partial x^2}}$$

(idem for holes in an n-type material)

Pn junction in equilibrium

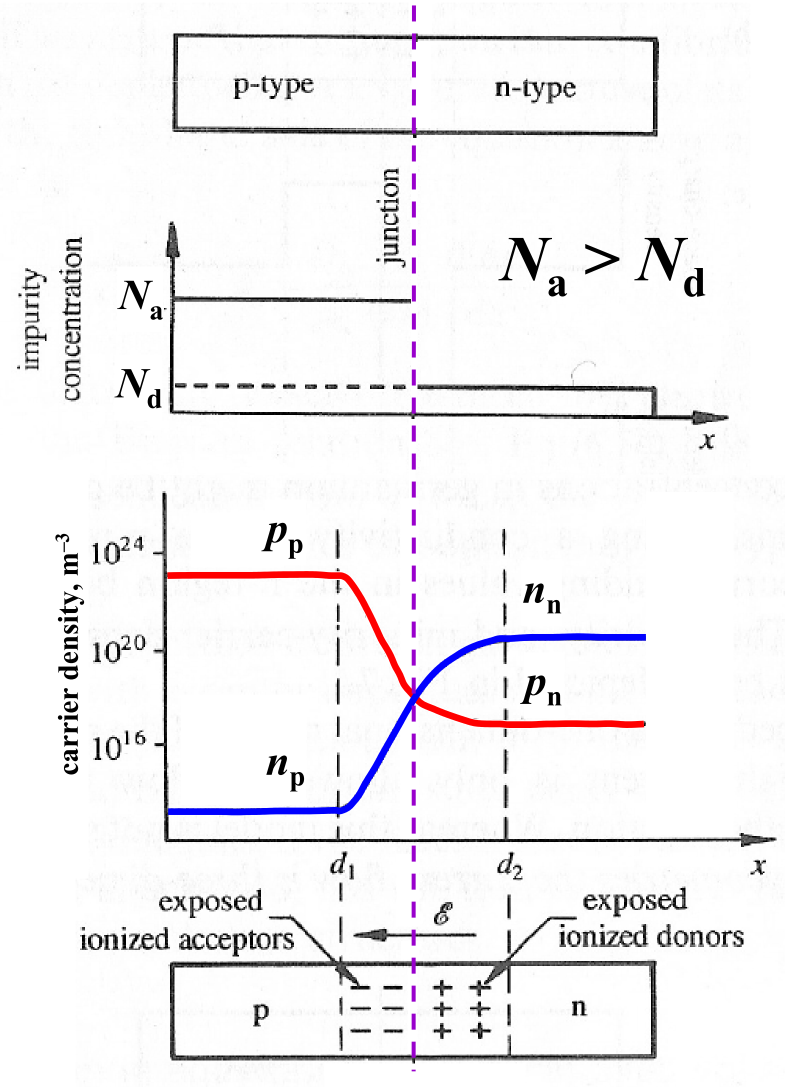
- Consider a pn junction with abrupt transition



Initially, the density of carriers change abruptly at the junction.

Pn junction in equilibrium

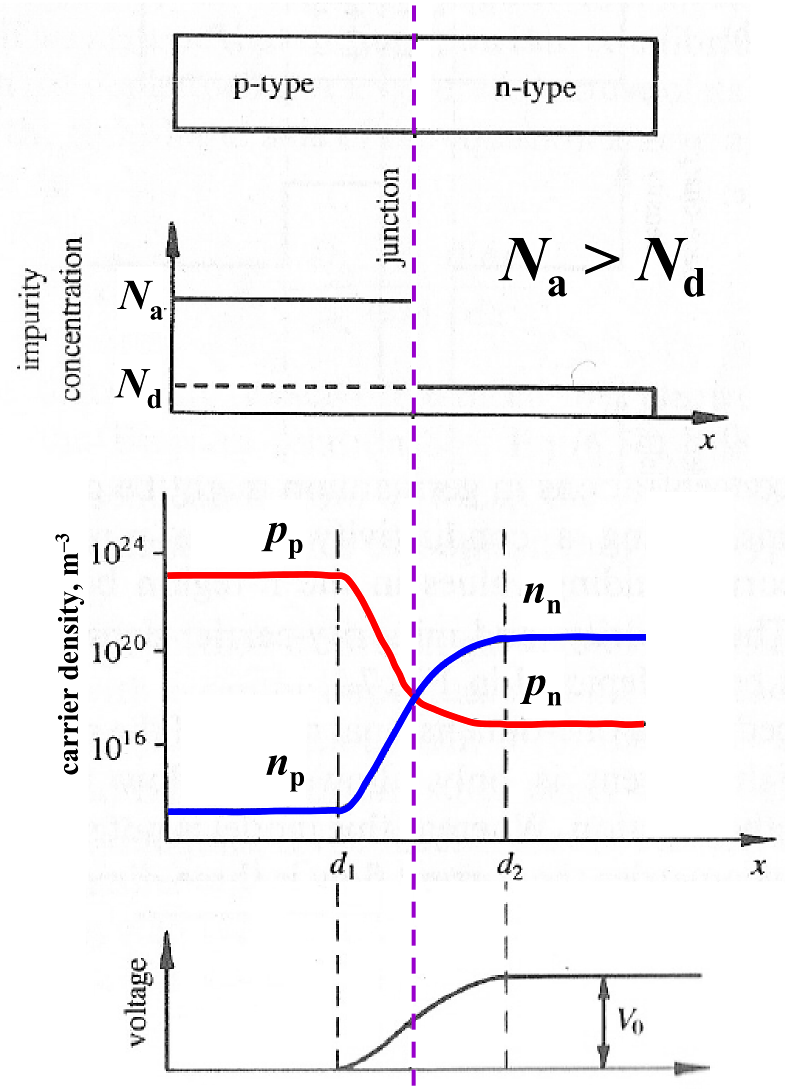
- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

Pn junction in equilibrium

- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

In equilibrium, a voltage difference is created across the depletion layer.

Pn junction in equilibrium

- Voltage across the depletion layer
  - Start with continuity eq. for holes

$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\cancelto{0}{\frac{\delta p}{\tau_{Lh}}} + \mu_h \mathscr{E}_x \frac{\partial(\delta p)}{\partial x} + D_h \frac{\partial^2(\delta p)}{\partial x^2} \Rightarrow \mu_h \mathscr{E}_x \frac{\mathrm{d}p}{\mathrm{d}x} = -D_h \frac{\mathrm{d}^2 p}{\mathrm{d}x^2}$$

$$D_h/\mu_h = kT/e \Rightarrow \mathscr{E}_x = \frac{kT}{e}\frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}x} \Rightarrow e\int_{d_1}^{d_2} \mathscr{E}_x \mathrm{d}x = -kT\int_{p_p}^{p_n} \frac{\mathrm{d}(\delta p)}{\delta p}$$

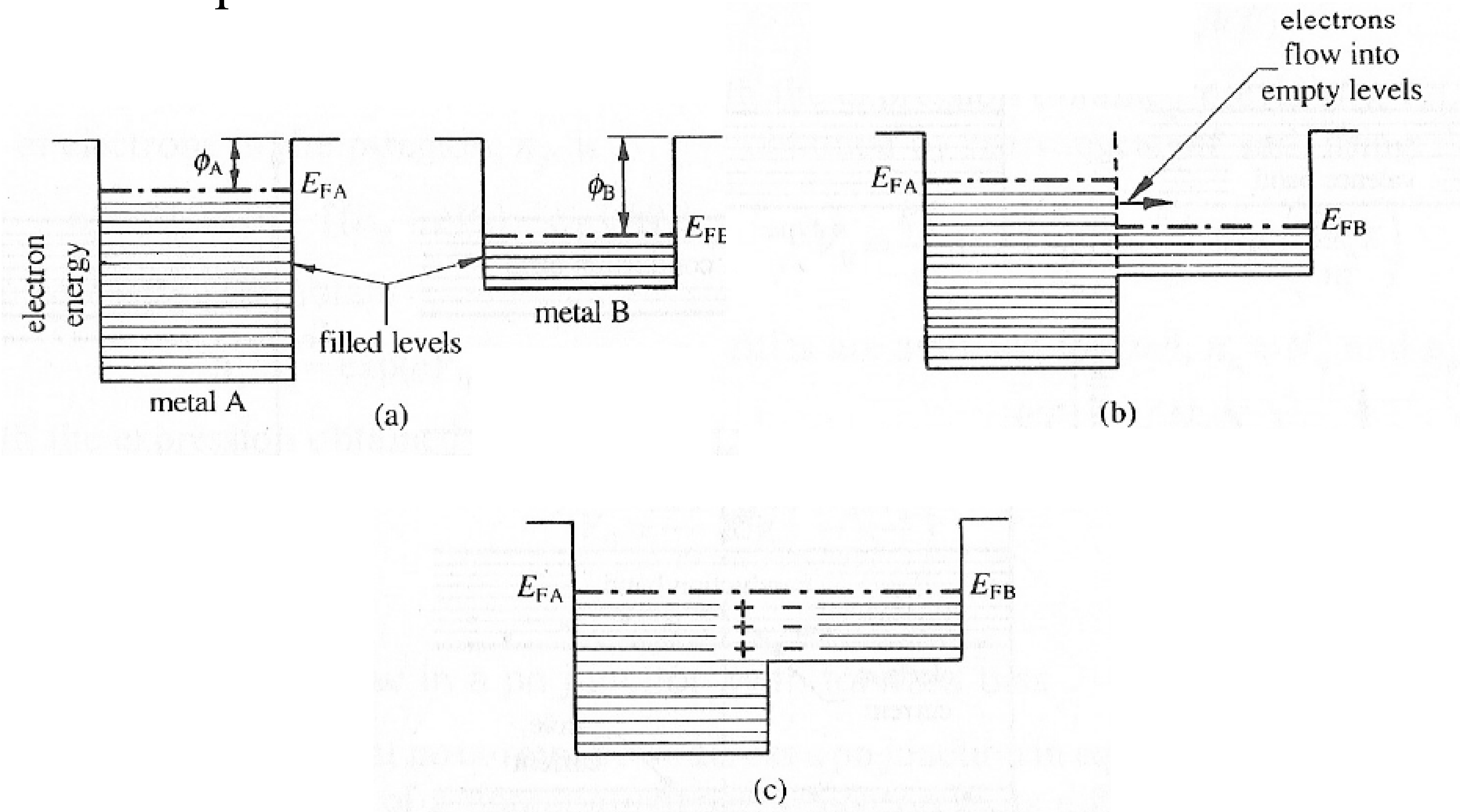
$$eV\Big|_{d_1}^{d_2} = -kT \ln p\Big|_{p_p}^{p_n} \Rightarrow \boxed{p_p = p_n \exp(eV_0/kT)}$$

  - Idem for electrons

$$\boxed{n_n = n_p \exp(eV_0/kT)}$$

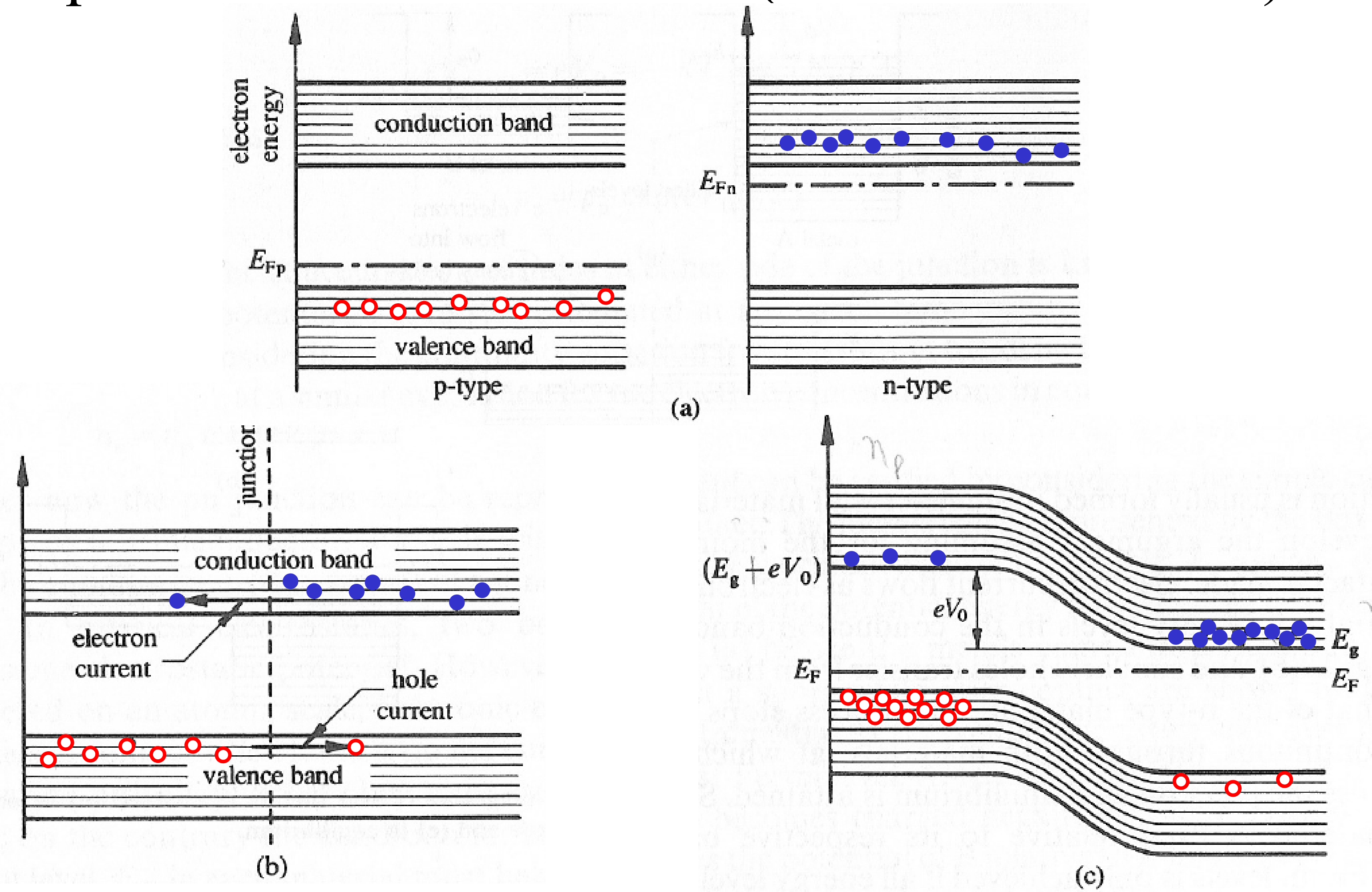
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in metals):



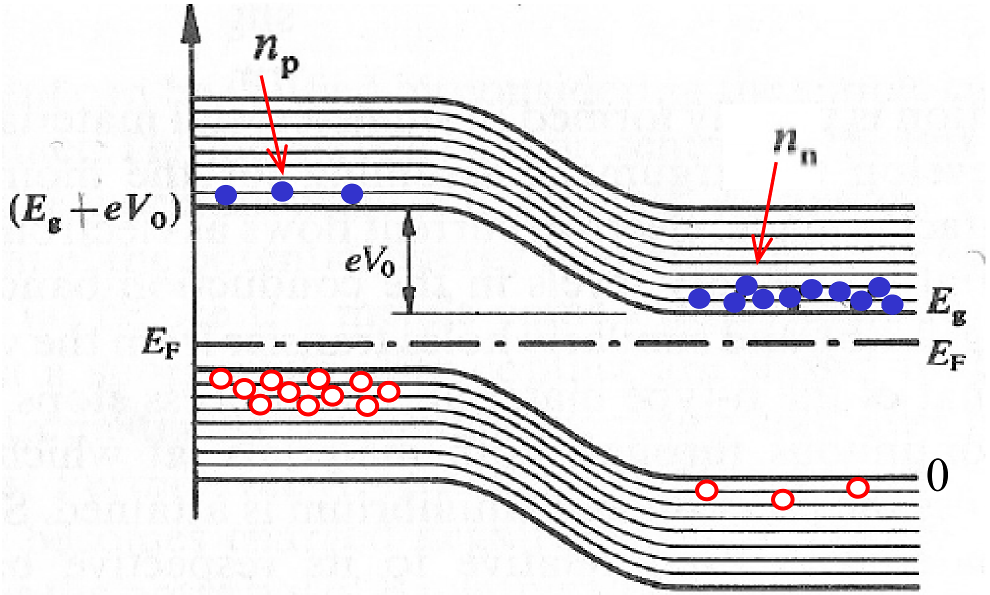
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in semiconductors)



Pn junction in equilibrium

- Number of electrons in the conduction band



$$n_n = N_c \exp\left[-\left(E_g - E_F\right)/kT\right]$$

$$n_p = N_c \exp\left[-\left(\left\{E_g + eV_0\right\} - E_F\right)/kT\right]$$

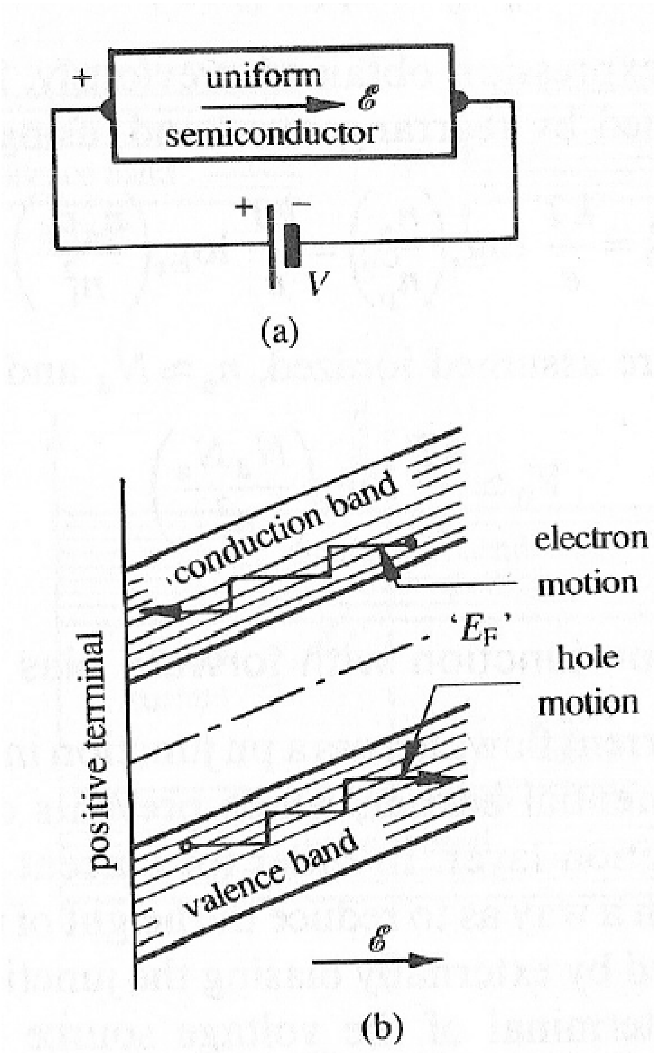
$$\Rightarrow \quad n_n/n_p = \exp(eV_0/kT)$$

- If all the impurities are ionized ($n_n \approx N_d$ and $p_p \approx N_a$):

$$\boxed{V_0 \simeq \frac{kT}{e} \log_e\left(\frac{N_d N_a}{n_i^2}\right)}$$

Pn junction with forward bias

- Consider a uniform sC biased with a voltage V:

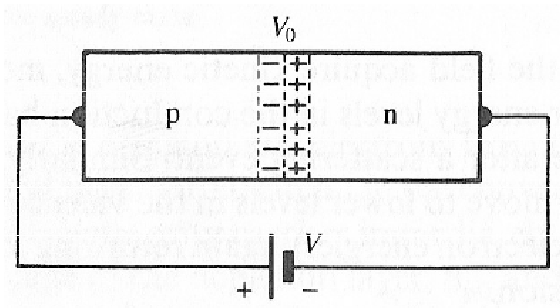


The whole band structure tilts (one end of the sC has more energy than the other) .

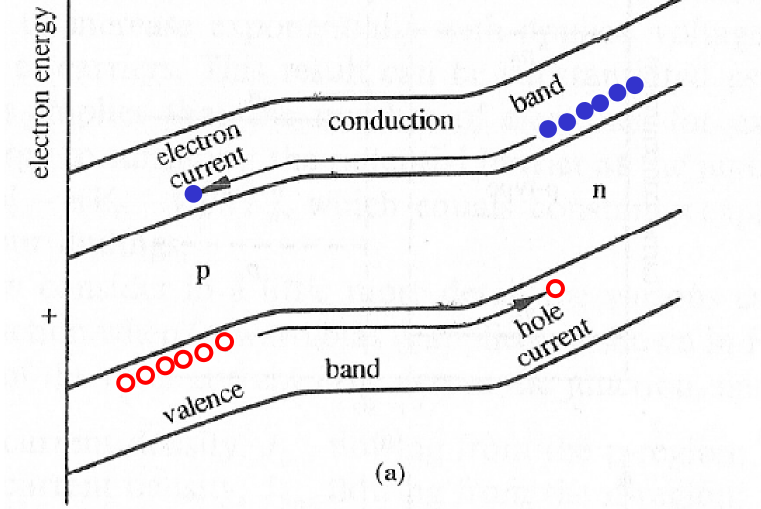
Electrons and holes move in the field (in opposite directions.

Pn junction with forward bias

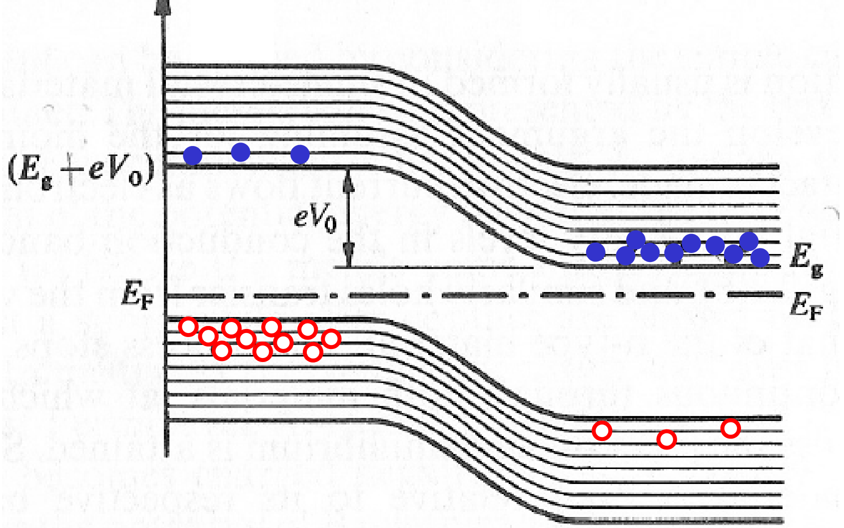
- Consider a junction biased with a voltage V:



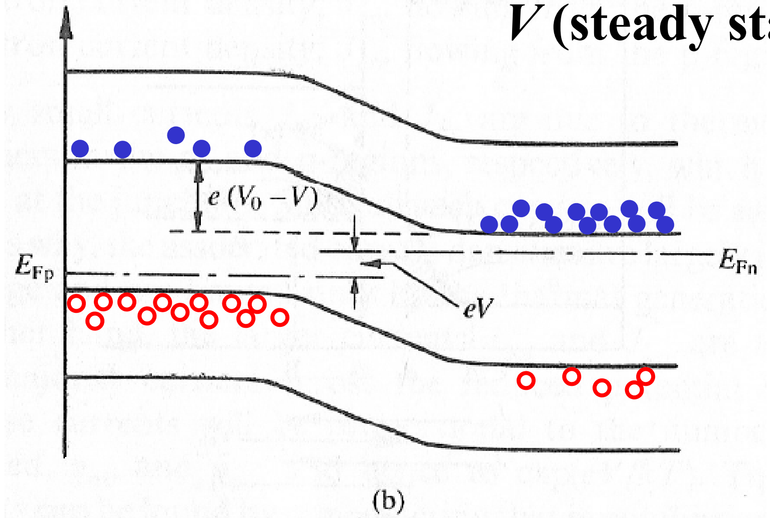
V (on application)



$V = 0$

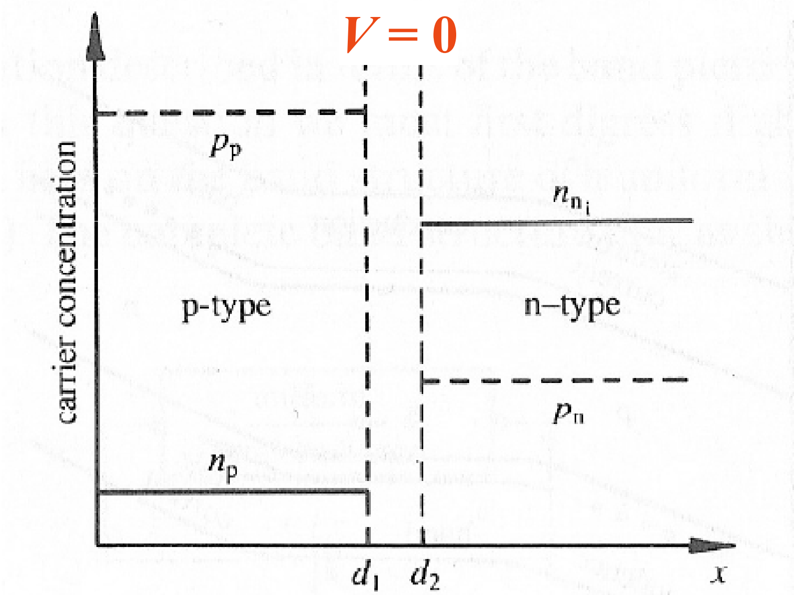


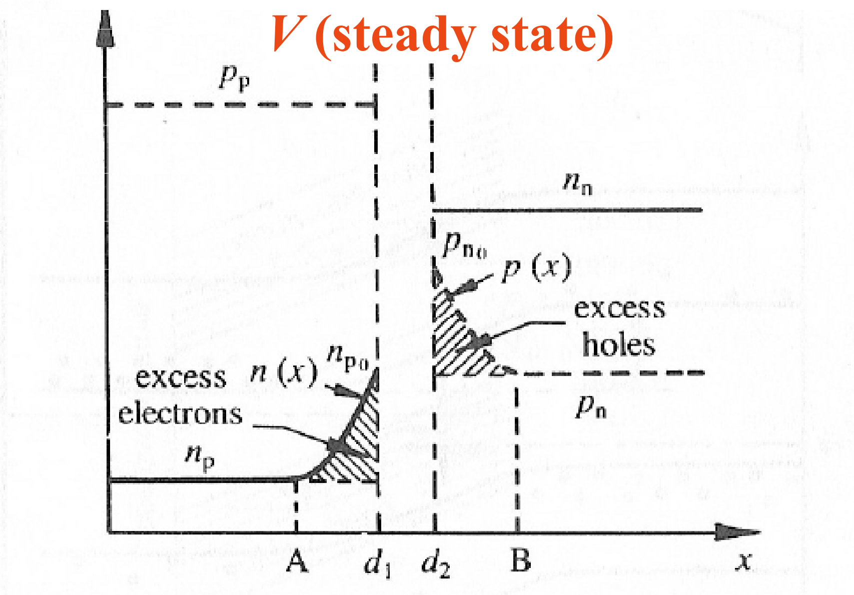
V (steady state)



Pn junction with forward bias

- Excess carriers in a biased junctions:





$$p_p = p_n \exp(eV_0/kT)$$

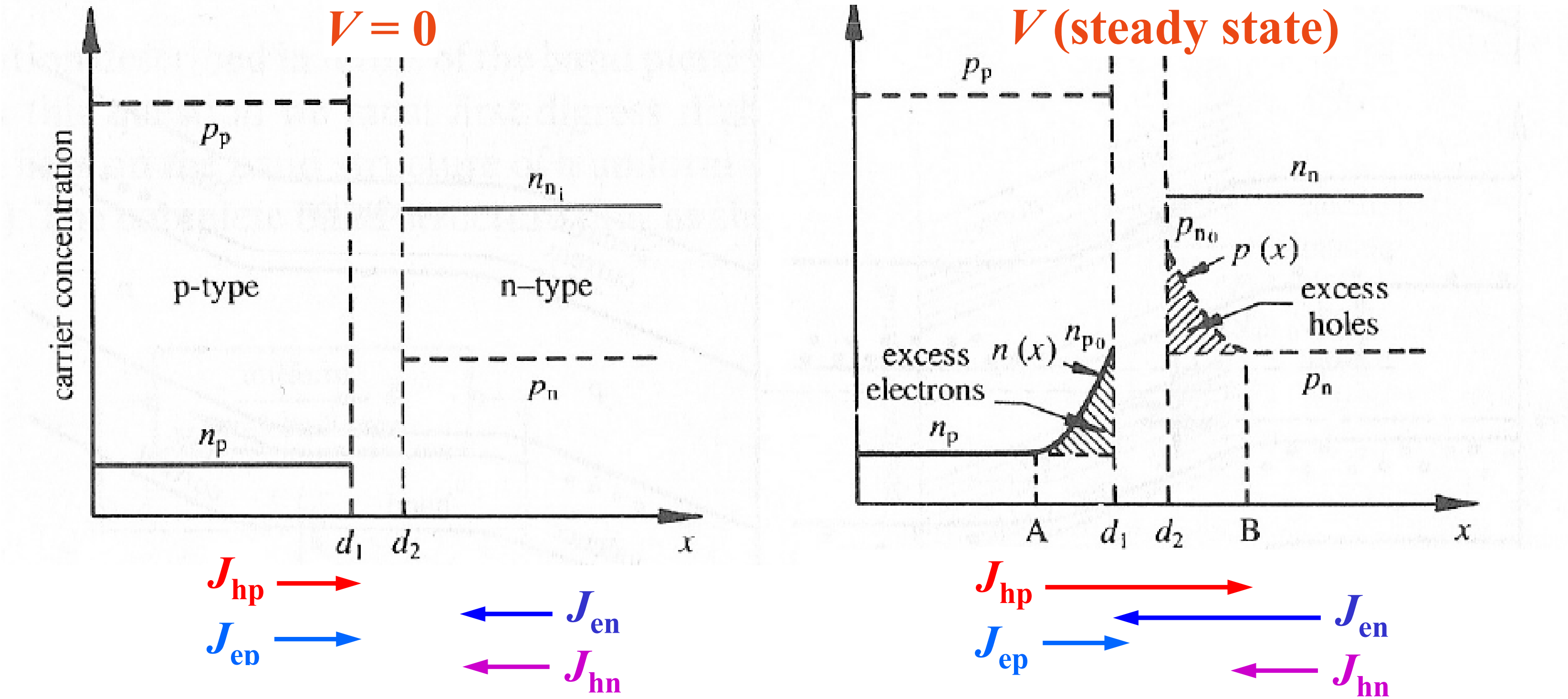
$$p_{n0} = p_p \exp[e(V - V_0)/kT]$$

$$p_{n0} = p_n \exp(eV/kT)$$

$$n_{p0} = n_p \exp(eV/kT)$$

Pn junction with forward bias

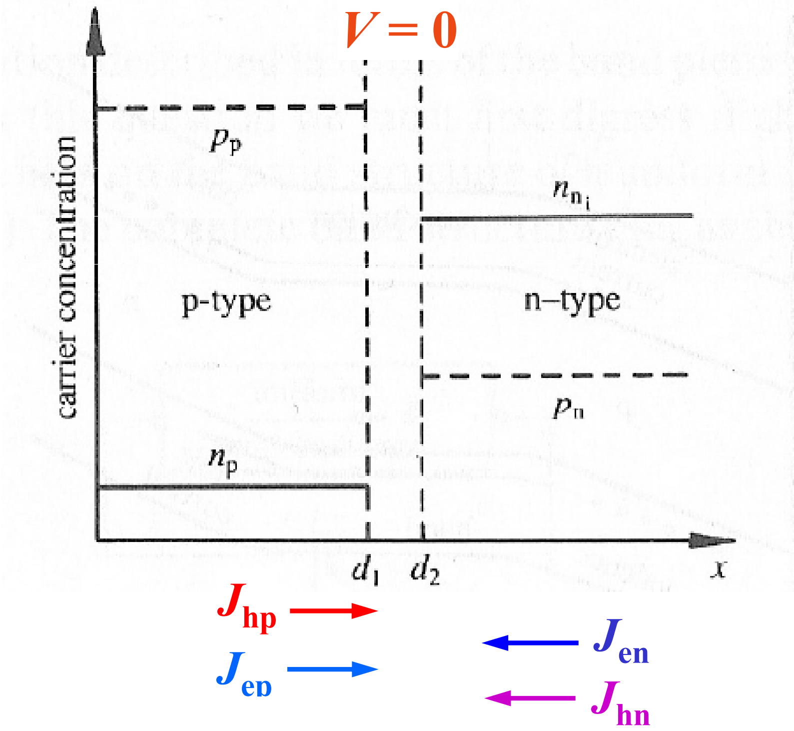
- Excess carriers in a biased junctions:

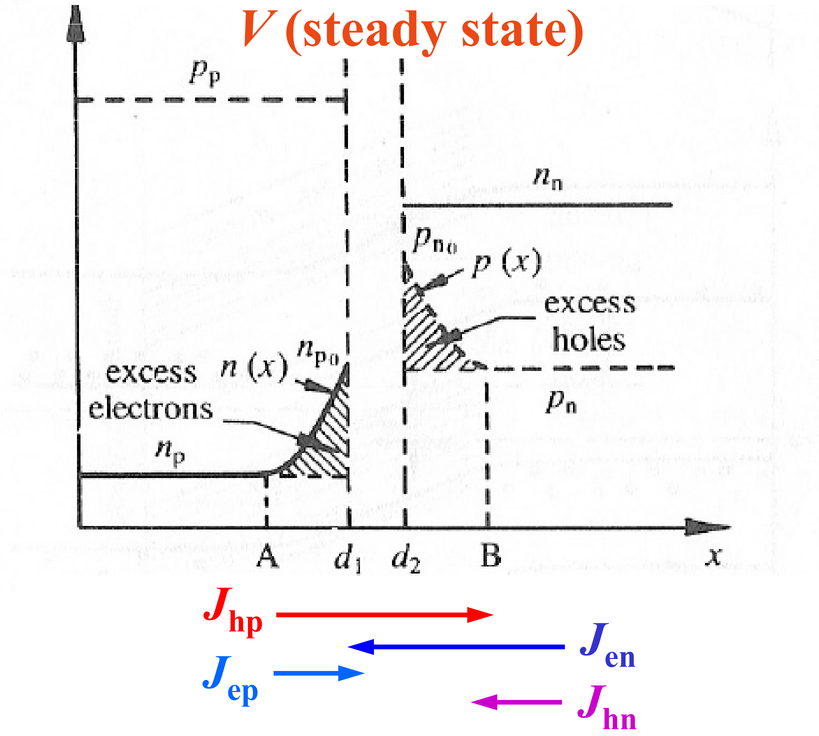


(a) a hole current density, J_{hp}, flowing from the p-region;
(b) a hole current density, J_{hn}, flowing from the n-region;
(c) an electron current density, J_{en}, flowing from the n-region;
(d) an electron current density, J_{ep}, flowing from the p-region.

Pn junction with forward bias

- Excess carriers in a biased junctions:





$$J_{hp} \propto p_{n0} = p_n \exp(eV/kT)$$

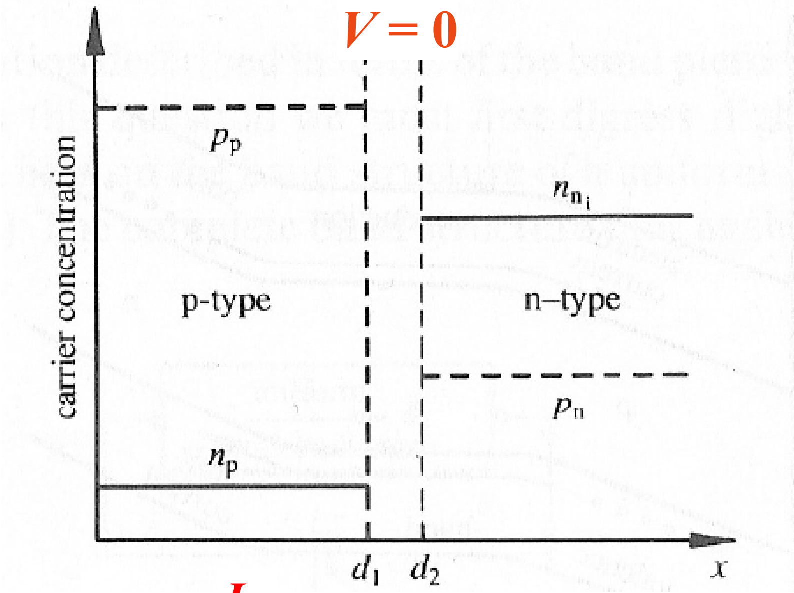
$$J_{hp} = J_{hn} \quad \Rightarrow \quad J_{hp} = J_{hn} \exp(eV/kT)$$

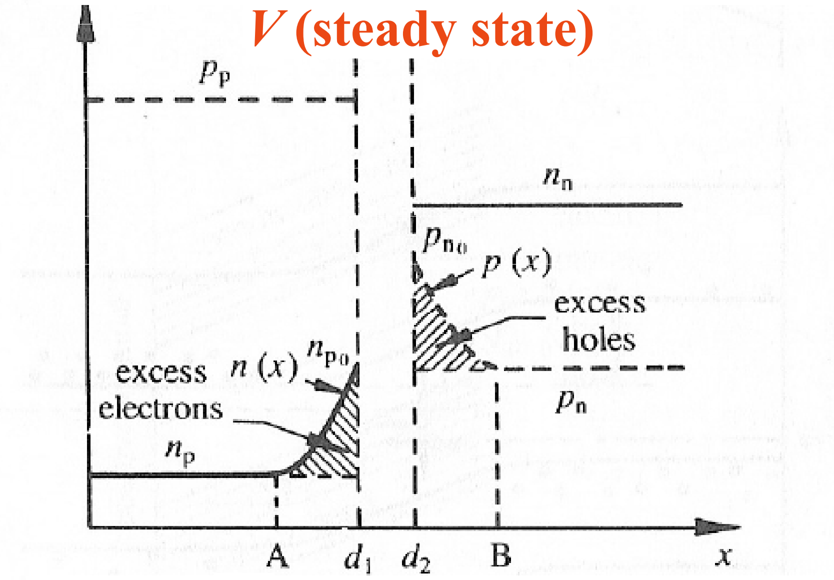
$$J_{en} \propto n_{p0} = n_p \exp(eV/kT)$$

$$J_{en} = J_{ep} \quad \Rightarrow \quad J_{en} = J_{ep} \exp(eV/kT)$$

Pn junction with forward bias

- Excess carriers in a biased junctions:





$J_{hp} \rightarrow$ $\leftarrow J_{en}$

$J_{ep} \rightarrow$ $\leftarrow J_{hn}$

$J_{hp} \rightarrow$ $\leftarrow J_{en}$

$J_{ep} \rightarrow$ $\leftarrow J_{hn}$

$$J_{hp} = J_{hn}$$

$$J_{en} = J_{ep}$$

Total currents:

$$J_h = J_{hp} - J_{hn} = J_{hn}[\exp(eV/kT) - 1]$$

$$J_e = J_{en} - J_{ep} = J_{ep}[\exp(eV/kT) - 1]$$

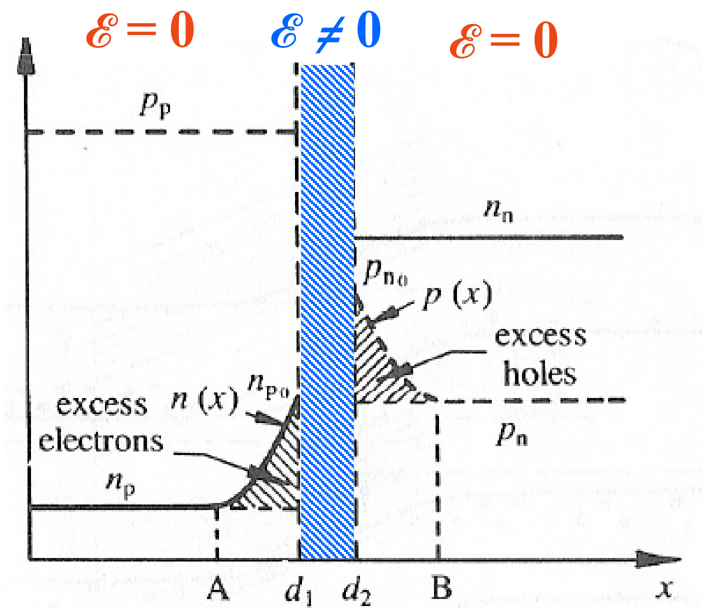
$$\Rightarrow \boxed{J = J_0[\exp(eV/kT) - 1]}$$ **Diode equation**

$$J_0 = J_{hn} + J_{ep}$$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for h^+ at $x \geq d_2$

V **(steady state)**



$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\frac{\delta p}{\tau_{Lh}} + \cancelto{0}{\mu_h \mathscr{E}_x \frac{\partial(\delta p)}{\partial x}} + D_h \frac{\partial^2(\delta p)}{\partial x^2}$$

$$\Rightarrow \quad \frac{d^2(\delta p)}{dx^2} = \frac{\delta p}{\tau_{Lh} D_h} = \frac{\delta p}{L_h^2}$$

$$\Rightarrow \quad \delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

$$\Rightarrow \quad p(x) = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h) + p_n$$

To find the constants:

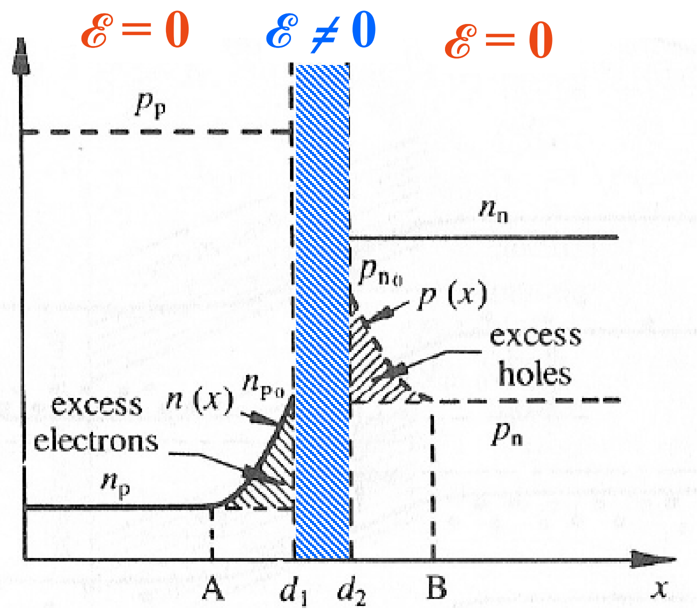
- $x = \infty \Rightarrow p(\infty) = p_n \Rightarrow C_2 = 0$
- $x = d_2 \Rightarrow p(0) = p_{n0} \Rightarrow C_1 = p_{n0} - p_n$

$$\Rightarrow \quad p(x) = (p_{n0} - p_n) \exp(-x/L_h) + p_n$$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for h^+ at $x \geq d_2$

V **(steady state)**



- Since $\mathscr{E} = 0$, there is only diffusion:

$$J_{Dh} = -eD_h \mathrm{d}p/\mathrm{d}x$$

$$\Rightarrow \quad J_h = -eD_h[-(1/L_h)(p_{n0} - p_n)\exp(-x/L_h)]$$

$x = 0$

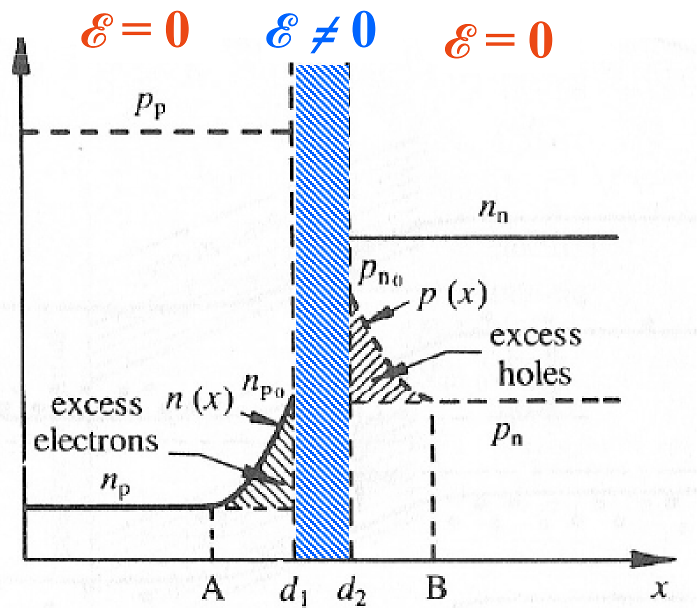
$$\Rightarrow \quad J_h|_{d_2} = (eD_h/L_h)(p_{n0} - p_n)$$

But $p_{n0} = p_n \exp(eV/kT)$ $\Rightarrow$ $J_h|_{d_2} = (eD_h/L_h)p_n\,[\exp(eV/kT) - 1]$

Pn junction with forward bias

- Diode equation:
  – Continuity eq. for e^- at $x \leq d_1$:

V (steady state)



$$J_e|_{d_1} = (eD_e n_p/L_e)\,[\exp(eV/kT) - 1]$$

Total current is then:

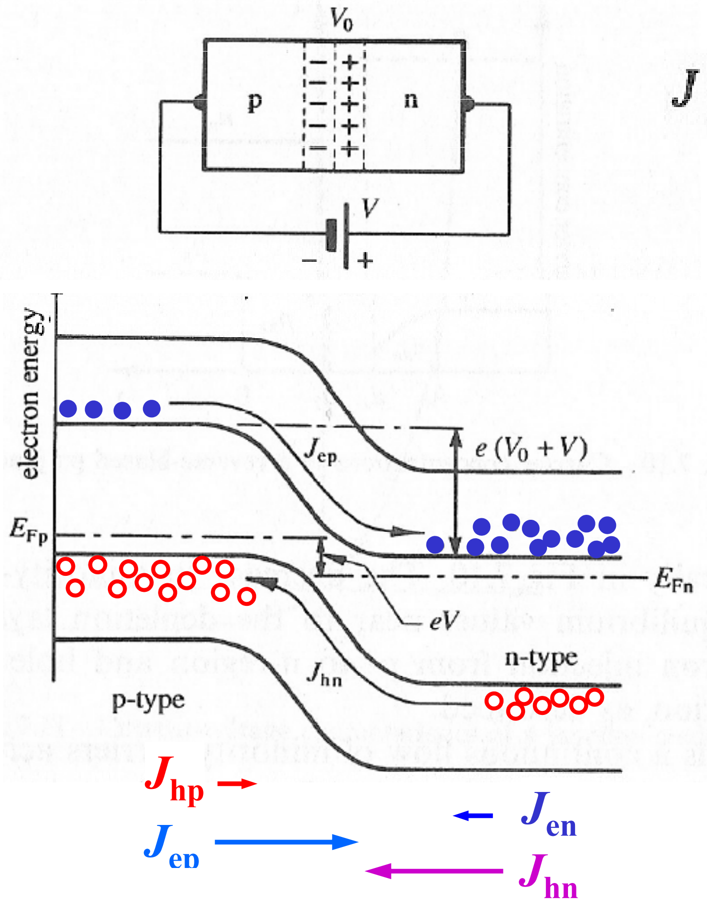
$$\boxed{J = J_h + J_e = e\underbrace{\left(\frac{D_h p_n}{L_h} + \frac{D_e n_p}{L_e}\right)}_{J_0}[\exp(eV/kT) - 1]}$$

For $V >> kT$:

$$J \simeq J_0 \exp(eV/kT)$$

Pn junction with reversed bias

- Currents through the junction :
  - Applying diode equation (with $-V$):



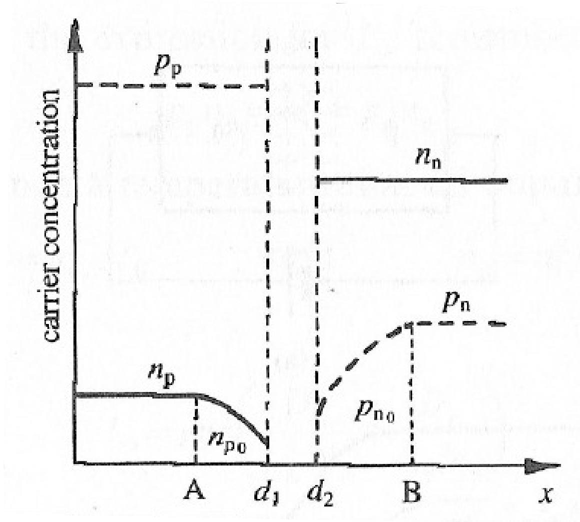
$$J = J_0[\exp(-eV/kT) - 1] = -J_0[1 - \exp(-eV/kT)]$$

For $V \gg kT$: $\quad J \approx -J_0$

$$J = \cancelto{0}{J_{hp}} - J_{hn} + \cancelto{0}{J_{en}} - J_{ep} \approx -J_0$$

Pn junction with reversed bias

- Minority carriers:
  - As before (with $-V$):



$$n_{p0} = n_p \exp(-eV/kT)$$

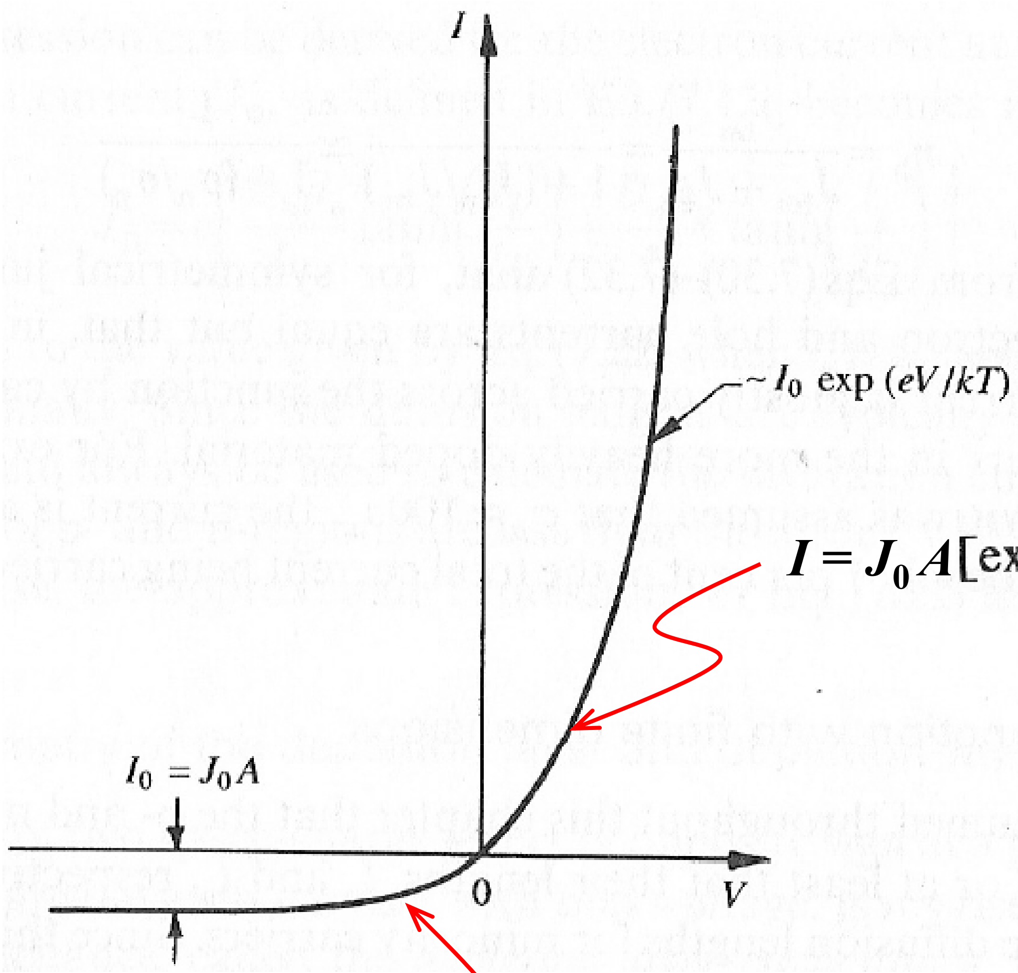
$$p_{n0} = p_n \exp(-eV/kT)$$

  - Applying continuity eq. as before

$$\boldsymbol{p(x) = p_n\{[\exp(-eV/kT) - 1]\exp(-x/L_h) + 1\}}$$

I-V curve of a junction diode

- Applying the results for the current density:



$$I = J_0 A[\exp(eV/kT) - 1]$$

$$I = -J_0 A[1 - \exp(-eV/kT)]$$

Electron-holes efficiencies

- Relative hole and electron currents:
  - Excess carriers:

$$\left.\begin{aligned} J_h &= J_{hp} - J_{hn} = J_{hn}\,[\exp(eV/kT) - 1] \\ J_e &= J_{en} - J_{ep} = J_{ep}\,[\exp(eV/kT) - 1] \end{aligned}\right\} \qquad \frac{J_h}{J_e} = \frac{J_{hn}}{J_{ep}}$$

  - Continuity equations:

$$\left.\begin{aligned} J_h|_{d_2} &= (eD_h/L_h)p_n\,[\exp(eV/kT) - 1] \\ J_e|_{d_1} &= (eD_e n_p/L_e)\,[\exp(eV/kT) - 1] \end{aligned}\right\} \qquad \frac{J_h}{J_e} = \frac{J_{hn}}{J_{ep}} = \frac{D_h}{L_h N_d}\,\frac{L_e N_a}{D_e}$$

  - Einstein ($D_e/\mu_e = D_h/\mu_h = kT/e$) and conductivity ($\sigma = e(n\mu_e + p\mu_h)$) eqs.:

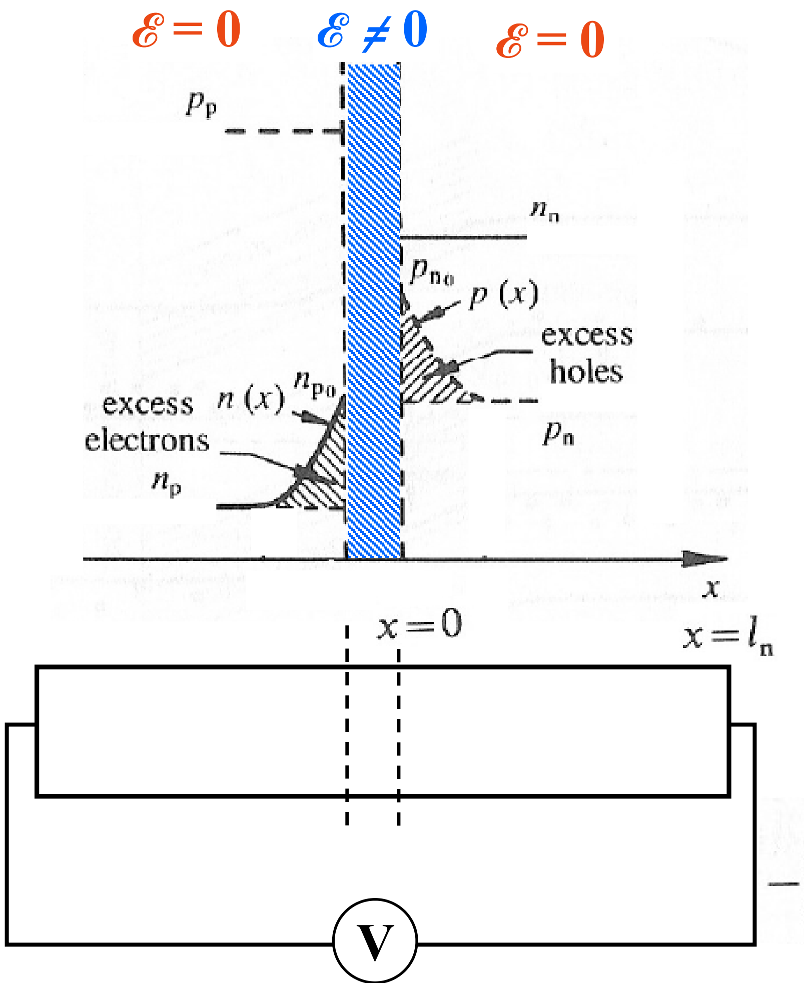
$$\frac{J_h}{J_e} = \frac{L_e}{L_h}\,\frac{\mu_h N_a}{\mu_e N_d} \quad \Rightarrow \quad \boxed{J_h/J_e \simeq \sigma_p/\sigma_n}$$

  - Hole efficiency:

$$\eta_h \equiv \frac{J_{hn}}{J_{hn} + J_{ep}} = \frac{1}{1 + (J_{ep}/J_{hn})} \simeq \frac{1}{1 + (\sigma_n/\sigma_p)} \qquad \eta_h \equiv \frac{J_{ep}}{J_{hn} + J_{ep}} = \frac{1}{1 + (J_{hn}/J_{ep})} \simeq \frac{1}{1 + (\sigma_p/\sigma_n)}$$

Pn junction with finite dimensions

- Current in a finite junction:
  - From the continuity eq. we obtained



$$\delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

New border conditions:

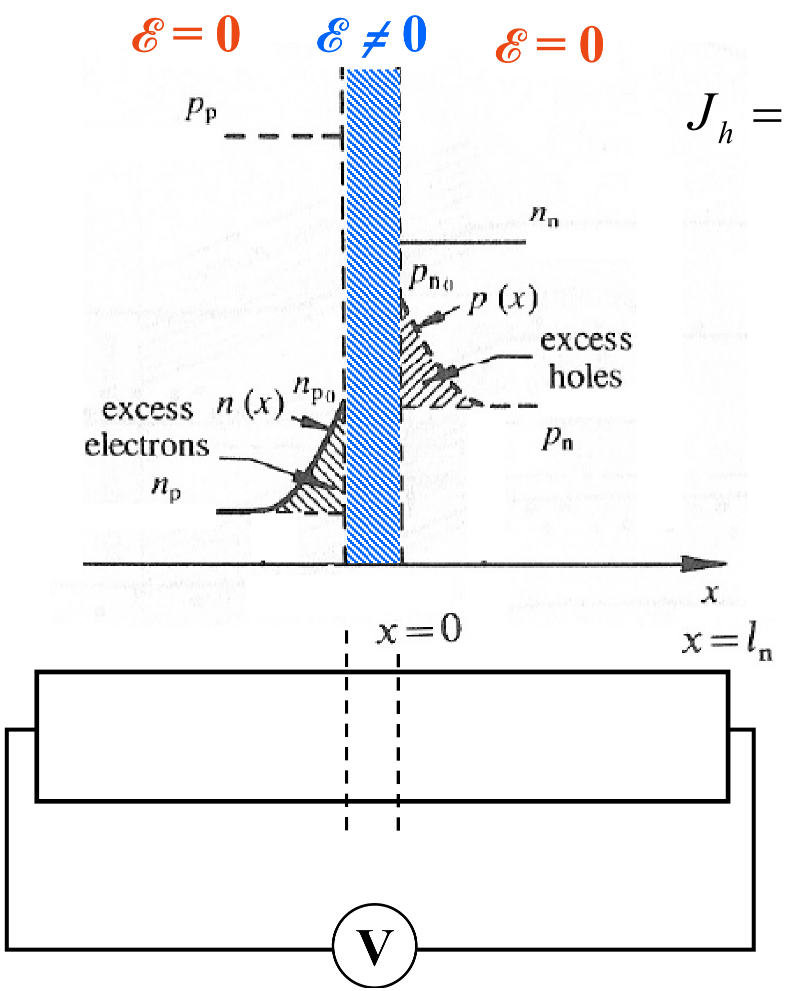
$p = p_{n0} = p_n \exp(eV/kT)$ when $x=0$

$p = 0$ when $x = l_n$

$$p(x) = \left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1-\exp(-2l_n/L_h)}\right)\exp(-x/L_h) - \left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1-\exp(-2l_n/L_h)} - (p_{n0}-p_n)\right)\exp(+x/L_h) + p_n$$

Pn junction with finite dimensions

- Current in a finite junction:
  - Currents:



For holes:

$$J_h = -eD_h\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)\Bigg|_{x=0} = \frac{eD_h}{L_h}\,p_n\tanh\left(\frac{l_n}{L_h}\right)\left[\exp\left(eV/kT\right)-1\right]$$

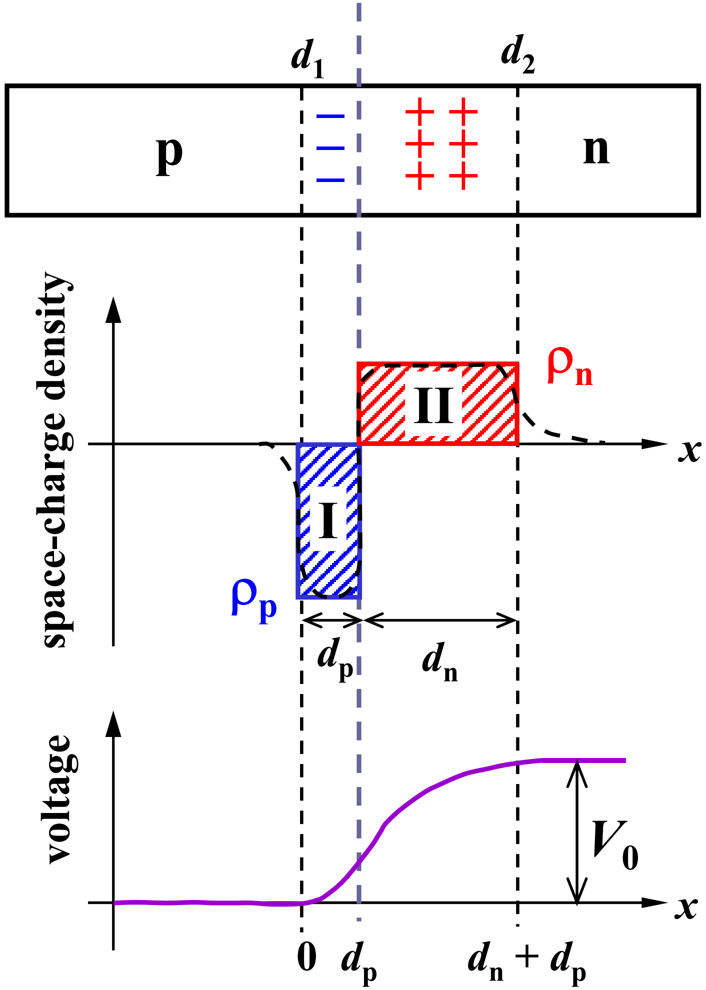
Idem for the electrons. Therefore the saturation current is:

$$J_0 = e\left[\frac{D_h p_n}{L_h}\underbrace{\tanh\left(\frac{l_n}{L_h}\right)}_{}+\frac{D_e n_p}{L_e}\underbrace{\tanh\left(\frac{l_p}{L_e}\right)}_{}\right]$$

$\rightarrow 0$ when $L \gg l$
($l \sim 1$ mm)

Depletion-layer capacitance

- The depletion layer forms a capacitance (important for high frequency applications)



Using Poisson relation ($\partial^2 V/\partial x^2 = -\rho/\epsilon$):

I:

$\rho_p \simeq -N_a e \Rightarrow \partial^2 V_1/\partial x^2 = eN_a/\epsilon$

$V = 0$ & $\partial V/\partial x = 0$ at $x = 0$ $\Rightarrow V_1(x) = eN_a x^2/2\epsilon$

II:

$\rho_n \simeq N_d e \Rightarrow \partial^2 V_2/\partial x^2 = -eN_d/\epsilon$

$\Rightarrow V_2 = -eN_d x^2/2\epsilon + C_1 x + C_2$

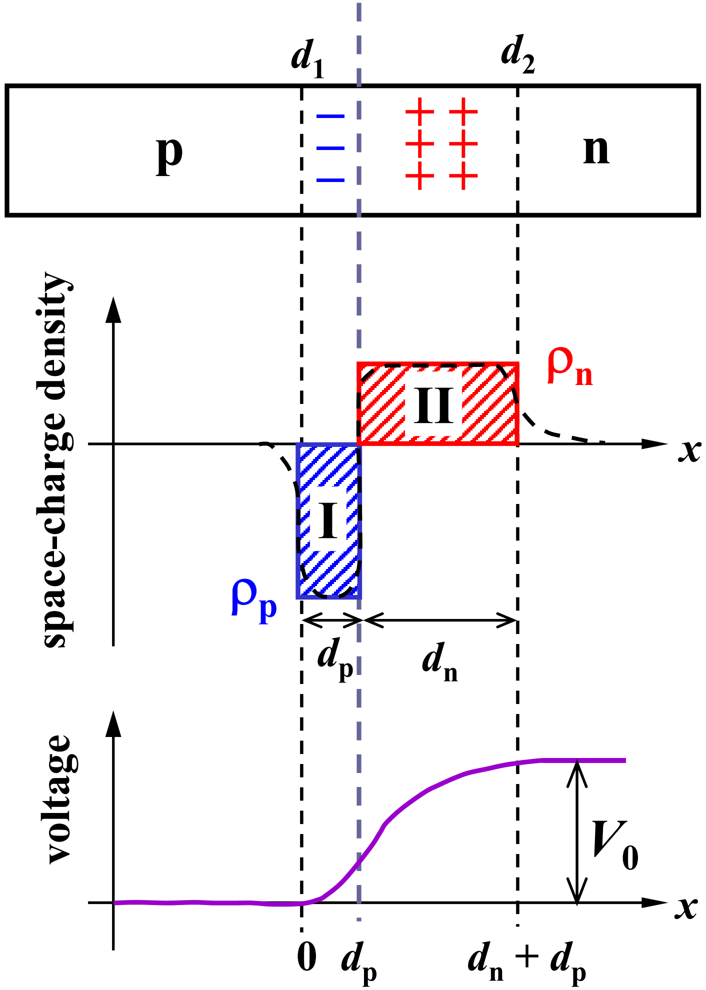
$(\partial V_1/\partial x)|_{d_p} = (\partial V_2/\partial x)|_{d_p}$, $V_1|_{d_p} = V_2|_{d_p}$ $\Rightarrow$

$$C_1 = ed_p(N_a + N_d)/\epsilon$$

$$C_2 = -\frac{ed_p^2(N_a + N_d)}{2\epsilon}$$

Depletion-layer capacitance

- Expressions for the depletion layer:



Applying $\mathscr{E} = 0$ at $x \geq d_n + d_p$:

$(\partial V_2/\partial x)|_{d_n+d_p} = 0 \Rightarrow \boxed{d_p/d_n = N_d/N_a}$

Applying $V = V_0$ at $x \geq d_n + d_p$ & using previous relation:

$$V_2\big|_{d_n+d_p} = V_0 \Rightarrow \boxed{d_p = \left(\frac{2\epsilon V_0 N_d}{eN_a(N_a+N_d)}\right)^{1/2}}$$

$$\Rightarrow \boxed{d_n = \left(\frac{2\epsilon V_0 N_a}{eN_d(N_a+N_d)}\right)^{1/2}}$$

This expresions are still applicable for a biased junction $V_0 \leftrightarrow V_0 - V$

Depletion-layer capacitance

- Capacitance of the depletion layer:
  - Charge per unit area accumulated at the depletion layer

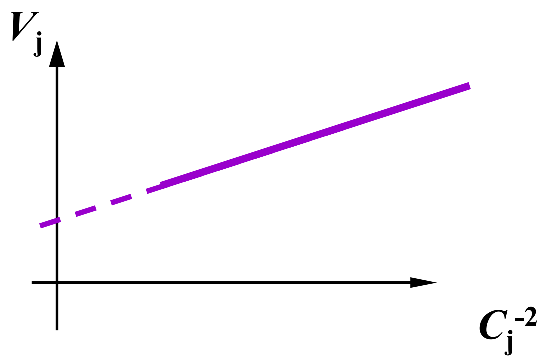
$$Q_j = eN_d d_n = eN_a d_p \xrightarrow{V_j = V_0 \pm V} Q_j = \left(\frac{2\epsilon e V_j N_a N_d}{N_a + N_d}\right)^{1/2}$$

  - The capacitance per unit area ($C_j = dQ_j/dV_j$) is then:

$$C_j = \left(\frac{\epsilon e N_a N_d}{2(N_a + N_d)}\right)^{1/2} \frac{1}{V_j^{1/2}} \Rightarrow \boxed{C_j = \epsilon/(d_p + d_n)}$$

$N_a \gg N_d$:

$$V_j = V_0 + V \simeq \frac{1}{C_j^2}\left(\frac{\epsilon e N_d}{2}\right)^{1/2}$$



Conclusions

- A junction diode is formed by “joining” a p and an n semiconductors.
- This junction generates a potential characteristic of the junction.
- We have studied the diode from the point of view of band structure.
- The diode equation was calculated using the continuity equation.
- The diode forms a natural capacitance that was calculated.