Physics of Electronics:

5. Semiconductors

July – December 2009

Contents overview

- Electrical conduction in semiconductors.
- Continuity equation.
- Semiconductor measurements.
- Diodes

Electrical conduction in sC.

- Current Flow = response to $\mathscr{E}$ + diffusion

$$J_e = ne\mu_e \mathscr{E} + eD_e \nabla n$$
$$J_h = pe\mu_h \mathscr{E} - eD_h \nabla p$$

- Diffusion and mobility:

$$D = \frac{\tau_r kT}{M}$$
$$\mu = e\tau_r / m$$

$\Rightarrow$

$$D_e = (kT/e)\mu_e$$
$$D_h = (kT/e)\mu_h$$

$\Rightarrow$

$$\boxed{D_e/\mu_e = D_h/\mu_h = kT/e}$$

Continuity Equation for Minority Carriers

- I.CASE: No current flow (no gradient & no $\mathscr{E}$)
  - Let's consider a **p-type** material:

Variation of minority carriers = Generation – Recombination

$$\frac{\mathrm{d}n}{\mathrm{d}t} = G(T) - rn(t)p(t)$$

- Consider first equilibrium:

$$\mathrm{d}n/\mathrm{d}t = 0 \Rightarrow G(T) = rn_0 p_0 = rn_i^2$$

- Now consider variations from the equilibrium:

$$\boldsymbol{n = n_0 + \delta n} \Leftrightarrow p = p_0 + \delta p \quad \textbf{with } \delta n = \delta p$$

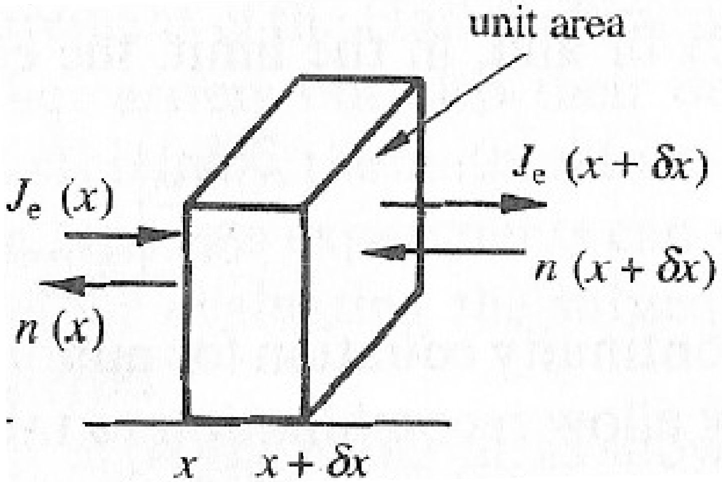
$$\Rightarrow [\mathrm{d}(\delta n)/\mathrm{d}t]|_{J=0} = -rp_0\delta n = -\delta n/\tau_{\mathrm{Le}}$$

  - For a **n-type** material:

$$\mathrm{d}(\delta p)/\mathrm{d}t = -\delta p/\tau_{\mathrm{Lh}}$$

Continuity Equation for Minority Carriers

- II.CASE: With current flow



$$J_e = -nev_D$$

$$\begin{cases} n_{x+\delta x} = J_e(x+\delta x)/ev_D \\ n_x = J_e(x)/ev_D \end{cases}$$

$$n_{x+\delta x} - n_x = \delta n = [J_e(x+\delta x) - J_e(x)]/ev_D$$

$$= \frac{1}{ev_D}\left(J_e(x) + \frac{\partial J_e}{\partial x}\delta x - J_e(x) \right)$$

$$\delta n = \frac{1}{ev_D}\frac{\partial J_e}{\partial x}\delta x \qquad v_D = \partial x/\partial t \qquad \left.\frac{\partial(\delta n)}{\partial t}\right|_{\substack{\text{current}\\ \text{flow}}} = \frac{1}{e}\frac{\partial J_e}{\partial x}$$

Continuity Equation for Minority Carriers

- II.CASE: Total continuity equation

$$\left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{total}} = -\frac{\delta n}{\tau_{\text{Le}}} + \frac{1}{e}\frac{\partial J_{\text{e}}}{\partial x}$$

But $J_{\text{e}} = ne\mu_{\text{e}}\mathscr{E} + eD_{\text{e}}\nabla n$

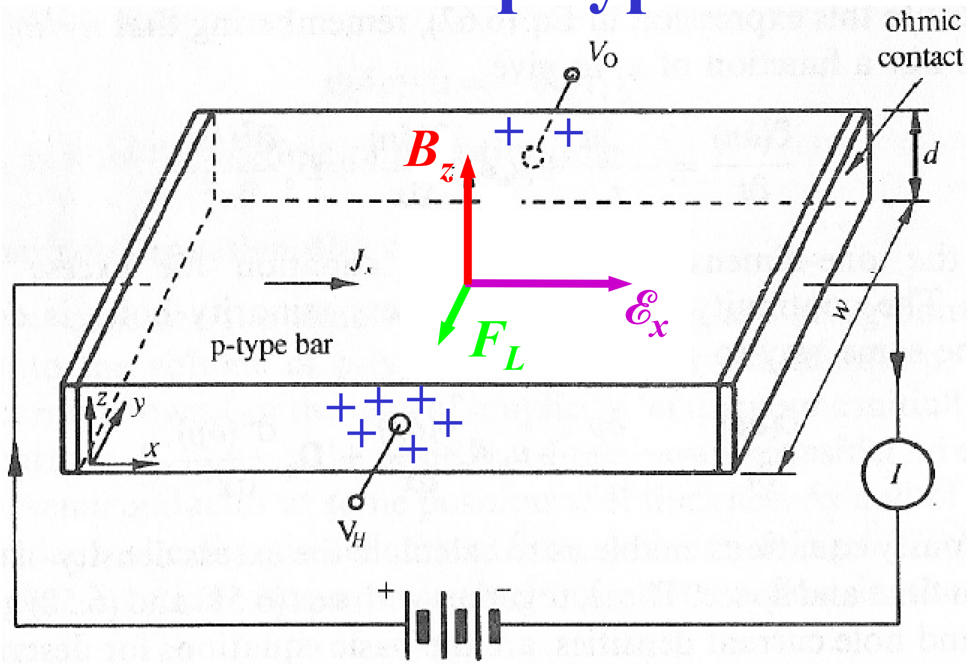
$n = n_0 + \delta n$

$$\boxed{\frac{\partial(\delta n)}{\partial t} = -\frac{\delta n}{\tau_{\text{Le}}} + \mu_{\text{e}}\mathscr{E}_x\frac{\partial(\delta n)}{\partial x} + D_{\text{e}}\frac{\partial^2(\delta n)}{\partial x^2}}$$

(idem for holes in an n-type material)

Semiconductor Measurements

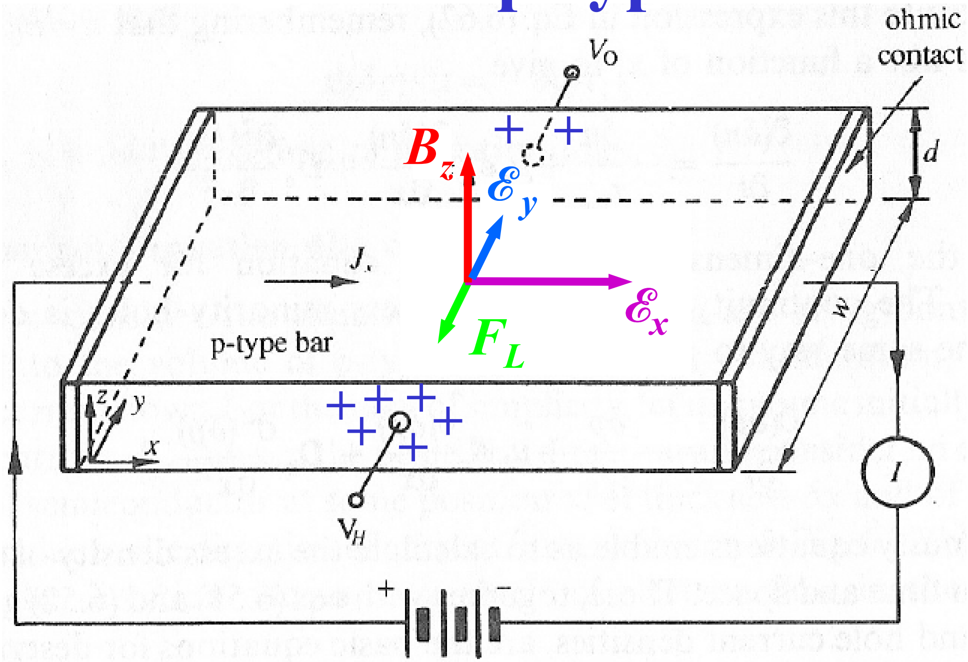
- Hall effect
  - Consider **p-type** material



$$\mathbf{F}_L = e\,\mathbf{v} \times \mathbf{B} \Rightarrow F_L = e\,v_{Dx} B_z$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



$$\mathbf{F}_L = e\,\mathbf{v}\times\mathbf{B} \Rightarrow F_L = e\,v_{Dx}B_z$$

$$\Rightarrow e\mathscr{E}_y = F_L = ev_{Dx}B_z$$

$$\& \quad J_x \simeq pev_{Dx}$$

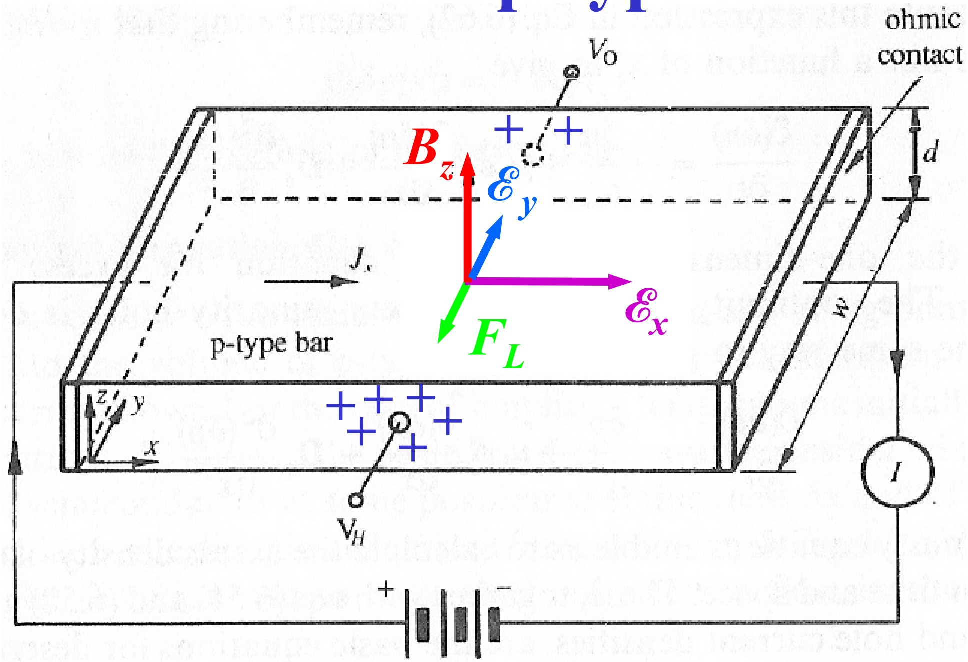
$$\mathscr{E}_y = J_xB_z/pe$$

The **Hall coefficient** is defined:

$$R_H = \mathscr{E}_y/J_xB_z = 1/pe$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



$$\mathbf{F}_L = e\,\mathbf{v}\times\mathbf{B} \Rightarrow F_L = e\,v_{Dx}B_z$$

$$\Rightarrow \left.\begin{array}{l} e\mathscr{E}_y = F_L = ev_{Dx}B_z \\ \& \;\; J_x \simeq pev_{Dx} \end{array}\right\} \mathscr{E}_y = J_x B_z / pe$$

The **Hall coefficient** is defined:

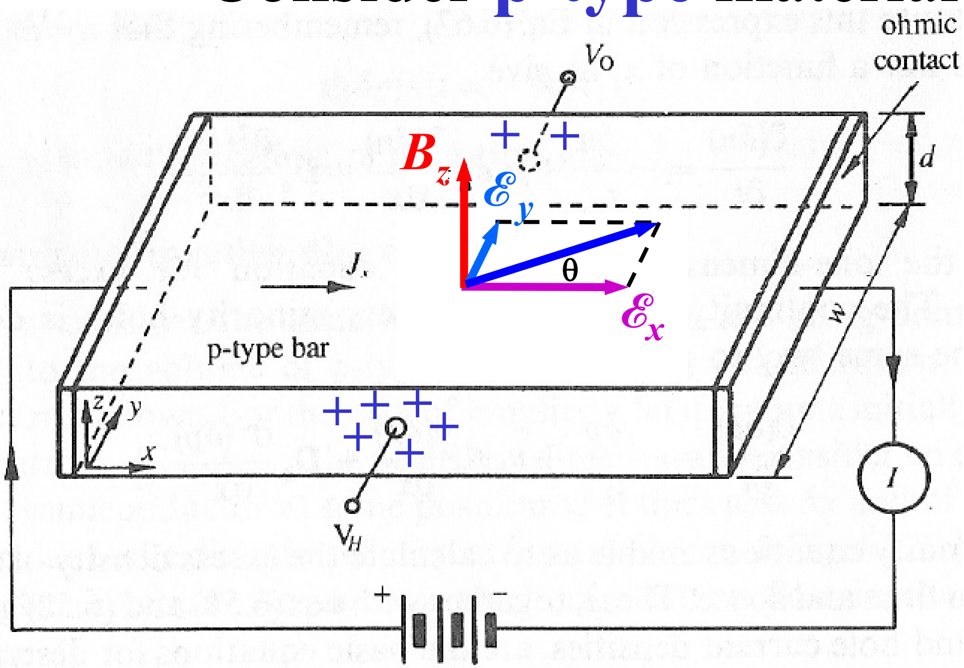
$$R_H = \mathscr{E}_y / J_x B_z = 1/pe$$

- If V_H and I are measured:

$$\left.\begin{array}{l} V_H = \mathscr{E}_y w \\ I = J_x\, w\, d \end{array}\right\} R_H = \frac{V_H}{wIB_z} wd = \frac{V_H d}{IB_z} = \frac{1}{pe}$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



The **Hall angle** is defined:

$$\tan\theta = \mathscr{E}_y/\mathscr{E}_x$$

$$\tan\theta = \frac{J_x B_z}{pe}\frac{\sigma}{J_x} = R_H B_z \sigma$$

$$\tan\theta = \frac{J_x B_z}{pe}\frac{\sigma}{J_x} = \mu_h B_z$$

$$\mu_h = R_H \sigma$$

  - For a **n-type** material:

$$R_{He} = -1/ne$$

Semiconductor Measurements

- Hall effect
  - Consider **both type of carriers** to be present:

$$v_{Dh} = \mu_H \mathscr{E}_x \qquad v_{De} = -\mu_H \mathscr{E}_x \quad \Rightarrow \quad F_h = -e(v_{Dh} \times B) = -ev_{Dh}B_z \qquad F_e = e(v_{De} \times B) = -ev_{De}B_z$$

- A net current is created in the *y* direction

$$\sigma \mathscr{E}_y = e(pv_{yh} - nv_{ye})$$

- Now, using the expressions for the Hall angle

$$v_{yh} = \mu_h \mathscr{E}_y = \mu_h(\mathscr{E}_x \tan\theta) = \mu_h(\mathscr{E}_x \mu_h B_z) \quad \& \quad v_{ye} = \mu_e^2 \mathscr{E}_x B_z$$

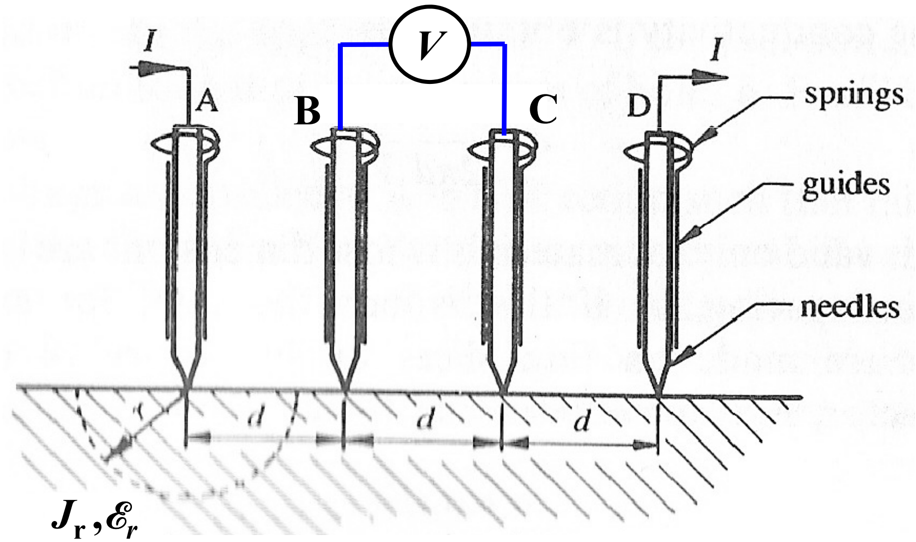
$$\Rightarrow \quad \sigma \mathscr{E}_y = e(p\mu_h^2 - n\mu_e^2)\mathscr{E}_x B_z$$

- Hall coefficient

$$R_H = \frac{\mathscr{E}_y}{J_x B_z} = \frac{e(p\mu_h^2 - n\mu_e^2)}{\sigma^2} \quad \xrightarrow{\sigma = e(n\mu_e + p\mu_h)} \quad R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

Semiconductor Measurements

- Four-point probe method for conductivity meas.
  - Consider $d << L$



For the current leaving *A*:

$$J_r = I/(2\pi r^2) \Rightarrow \mathscr{E}_r = J/\sigma = I/(2\pi\sigma r^2)$$

Potential at a distance *a* from *A*:

$$V_a = \int_{-\infty}^{a} \mathscr{E}_r \, dr = -\frac{I}{2\pi\sigma}\int_{-\infty}^{a}\frac{1}{r^2}\, dr = \frac{I}{2\pi\sigma a}$$

$$\Rightarrow V_{BC} = \frac{I}{2\pi\sigma d} - \frac{I}{2\pi\sigma(2d)} = \frac{I}{4\pi\sigma d}$$

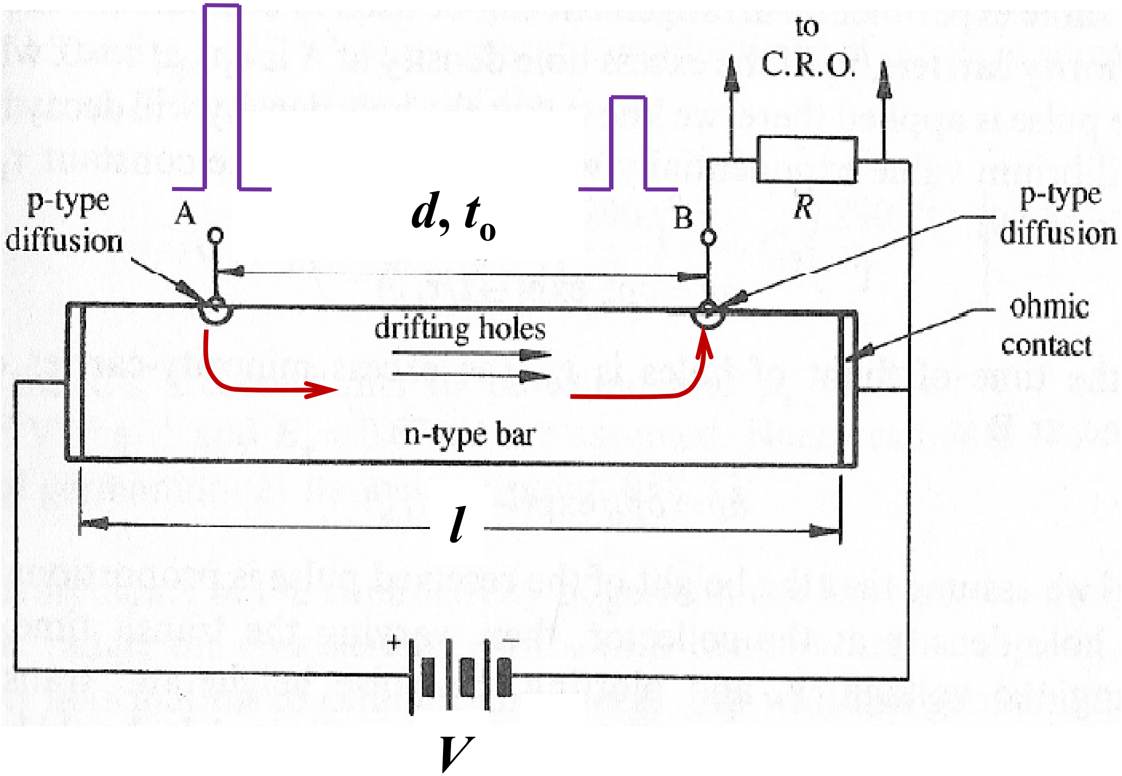
Identically for the current entering *D*, then:

$$V = 2V_{BC} = \frac{I}{(2\pi\sigma d)}$$

$$\Rightarrow \sigma = \frac{1}{2\pi d}\frac{I}{V}$$

Semiconductor Measurements

- Minority carrier life-time and mobility.



Mobility:

$v_D = d/t_0$ ➡ $\mu E = \mu V/l = d/t_0$

➡ $\boxed{\mu = ld/t_0 V}$

Life-time:

$\mathrm{d}(\delta p)/\mathrm{d}t = -\delta p/\tau_{\mathrm{Lh}}$

➡ $\delta p = \delta p_0 \exp(-t/\tau_{\mathrm{Lh}})$

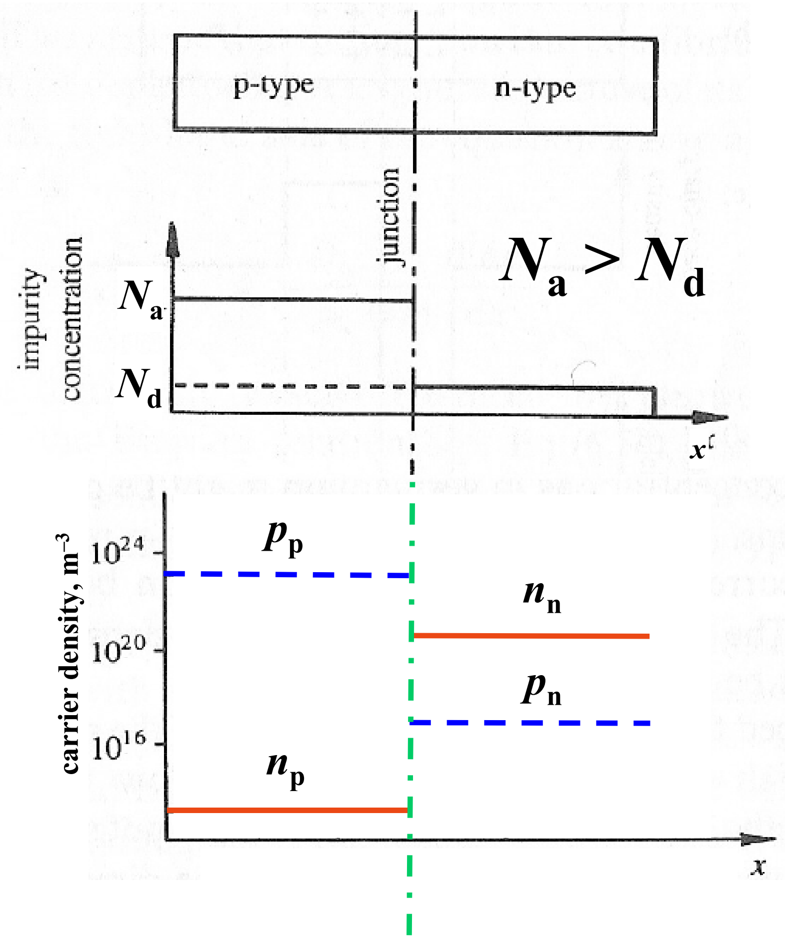
Physics of Electronics:

6. Junction Diodes

July – December 2009

Pn junction in equilibrium

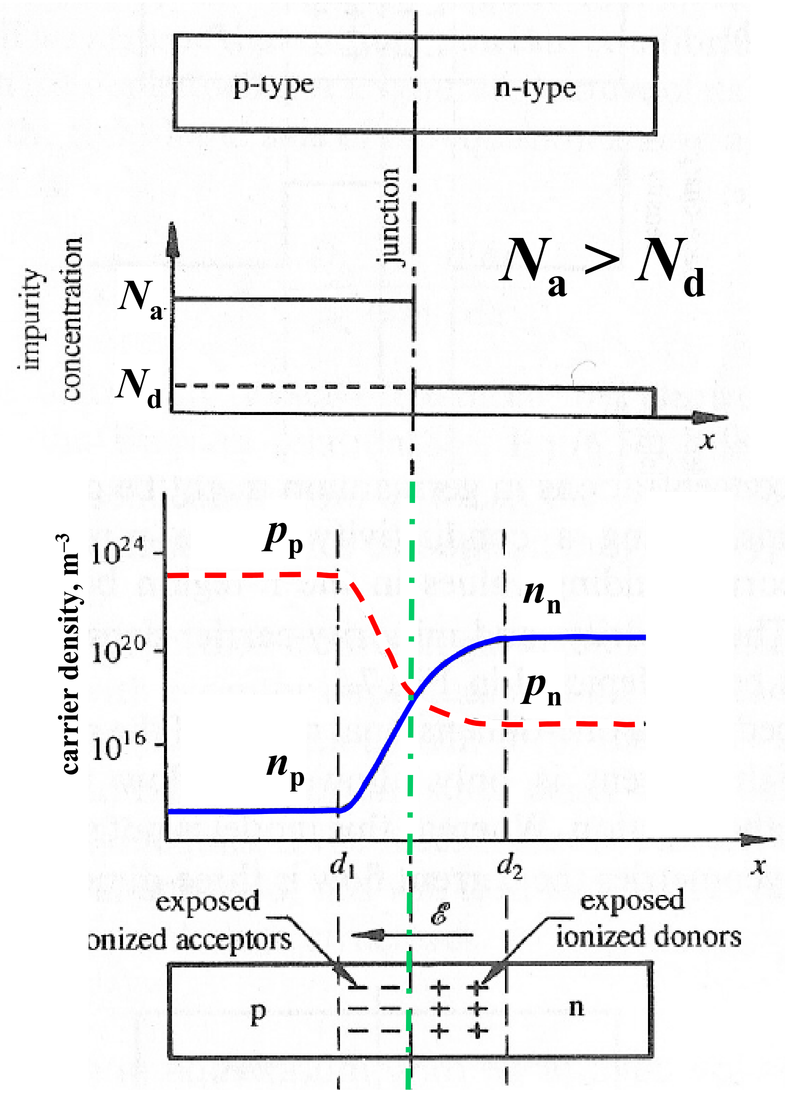
- Consider a pn junction with abrupt transition



Initially, the density of carriers change abruptly at the junction.

Pn junction in equilibrium

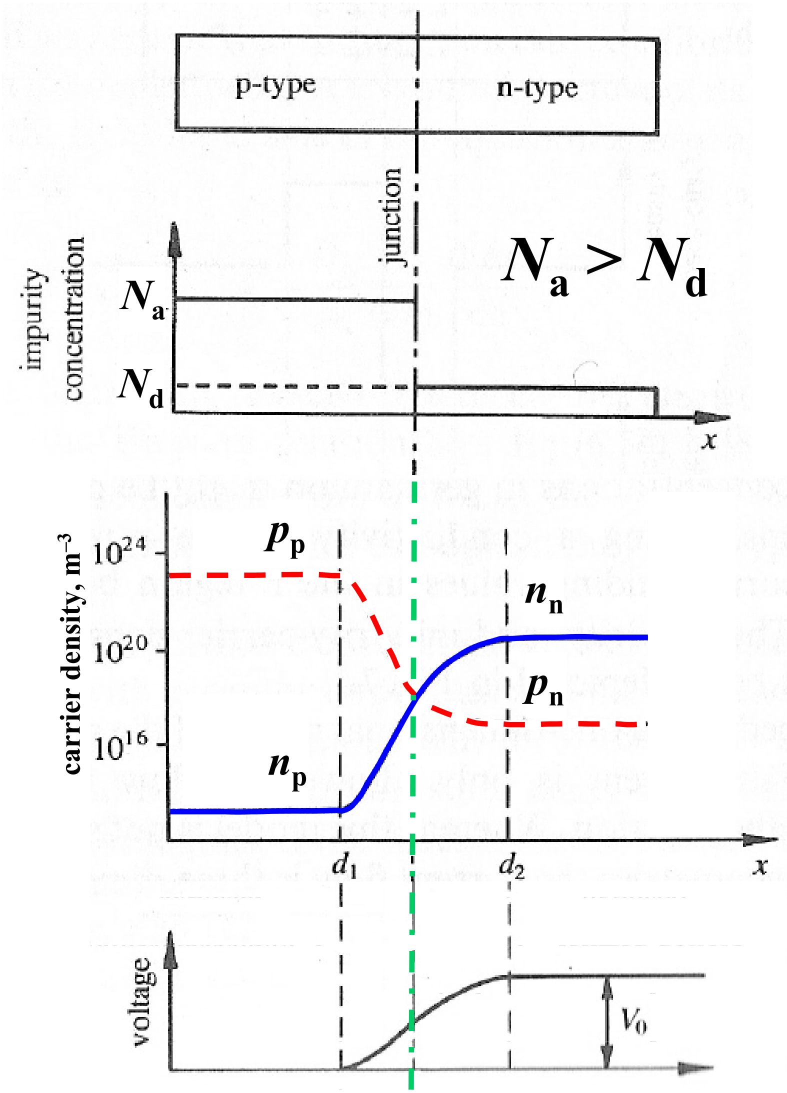
- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

Pn junction in equilibrium

- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

In equilibrium, a voltage difference is created across the depletion layer.

Pn junction in equilibrium

- Voltage across the depletion layer
  - Start with continuity eq. for holes

$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\cancelto{0}{\frac{\delta p}{\tau_{\mathrm{Lh}}}} + \mu_{\mathrm{h}} \mathscr{E}_x \frac{\partial(\delta p)}{\partial x} + D_{\mathrm{h}} \frac{\partial^2(\delta p)}{\partial x^2} \Rightarrow \mathscr{E}_x = \frac{D_{\mathrm{h}}}{\mu_{\mathrm{h}}} \frac{1}{(\delta p)} \frac{\mathrm{d}(\delta p)}{\mathrm{d}x}$$

$$D_{\mathrm{h}}/\mu_{\mathrm{h}} = kT/e$$

$$\Rightarrow \mathscr{E}_x = \frac{kT}{e} \frac{1}{(\delta p)} \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} \Rightarrow e\int_{d_1}^{d_2} \mathscr{E}_x \,\mathrm{d}x = -kT \int_{p_p}^{p_n} \frac{\mathrm{d}(\delta p)}{\delta p}$$

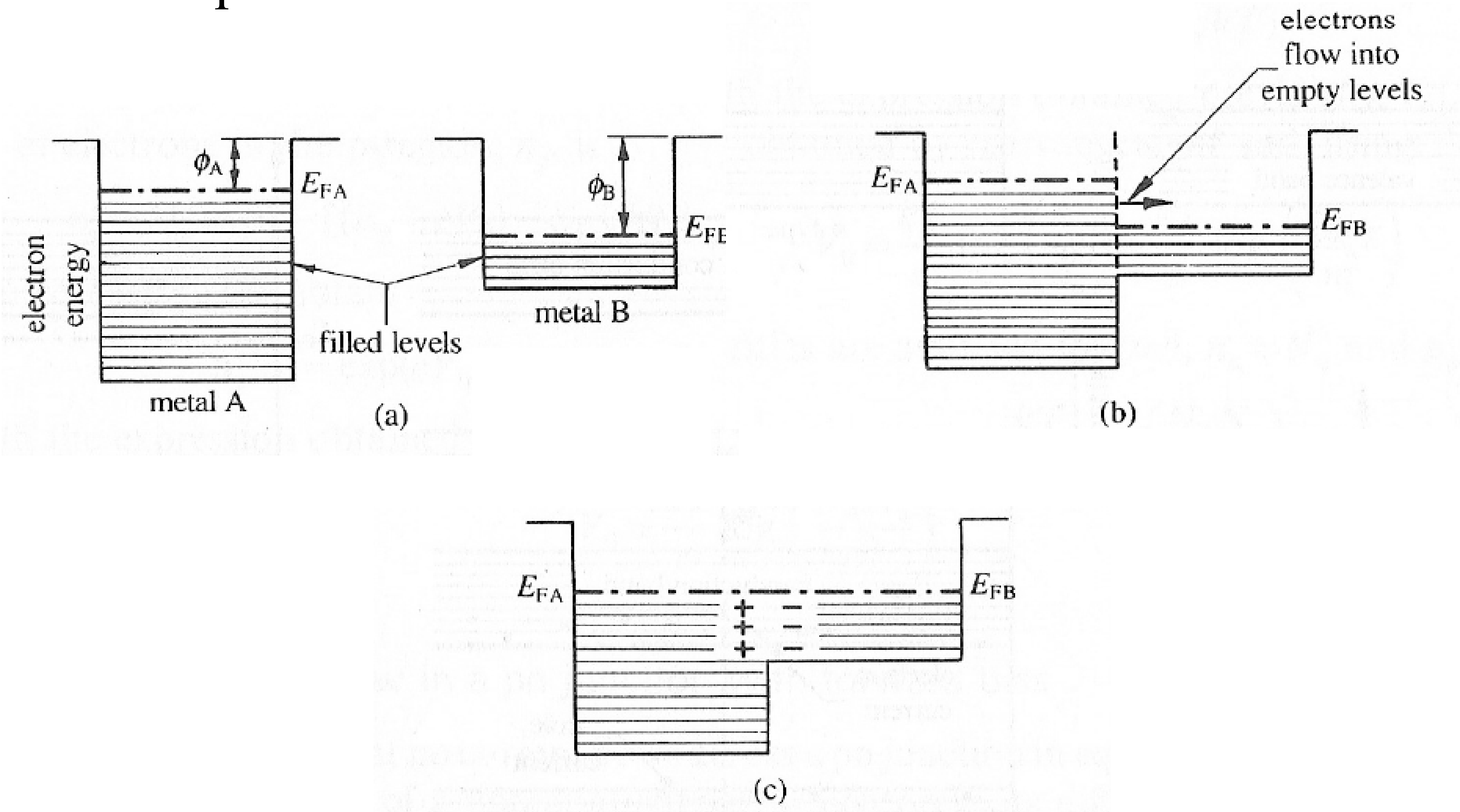
$$\Rightarrow \boxed{p_{\mathrm{p}} = p_{\mathrm{n}} \exp(eV_0/kT)}$$

  - Idem for electrons

$$\boxed{n_{\mathrm{n}} = n_{\mathrm{p}} \exp(eV_0/kT)}$$

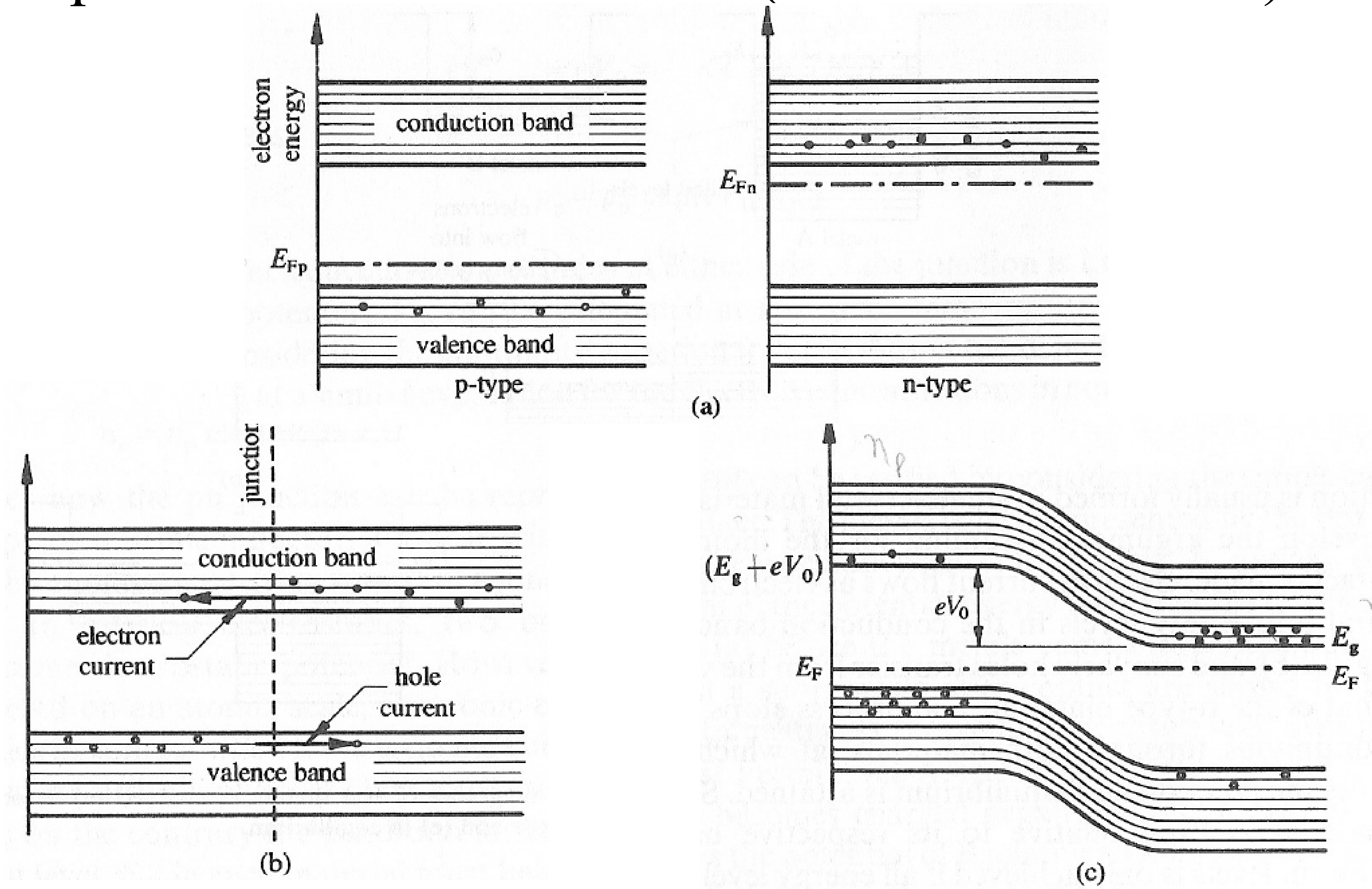
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in metals):



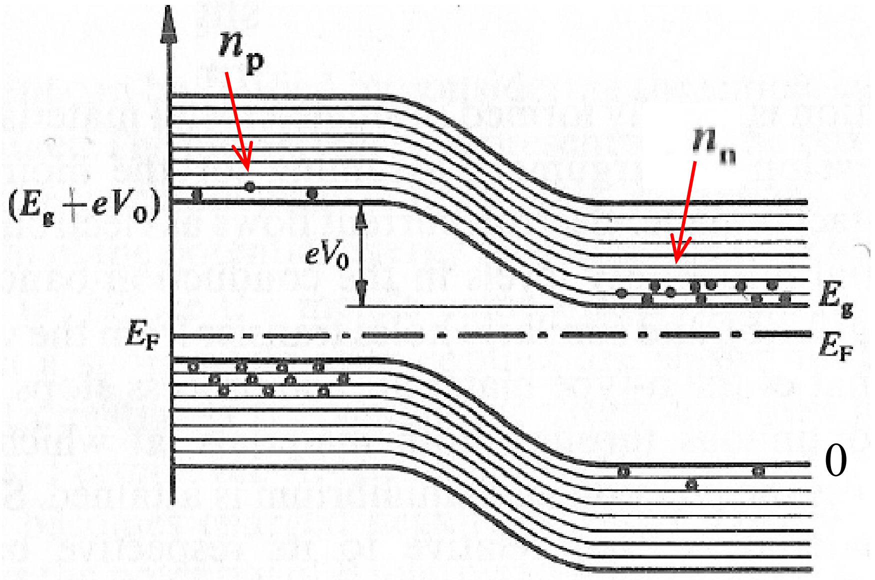
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in semiconductors)



Pn junction in equilibrium

- Number of electrons in the conduction band



$$n_n = N_c \exp\left[-\left(E_g - E_F\right)/kT\right]$$
$$n_p = N_c \exp\left[-\left(\left\{E_g + eV_0\right\} - E_F\right)/kT\right]$$

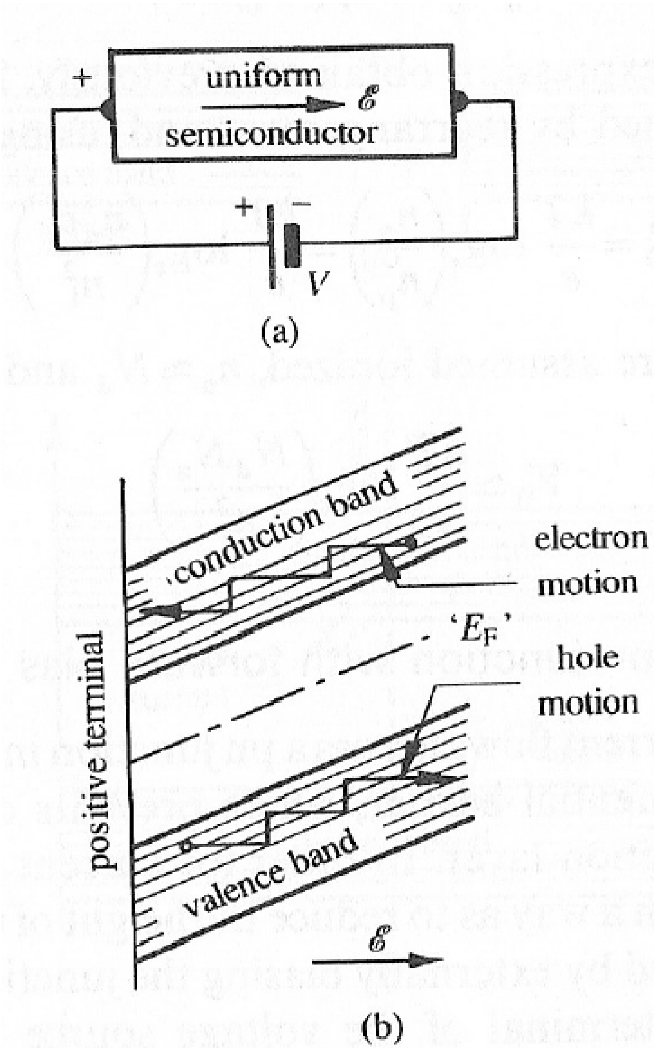
$\Rightarrow$ $n_n/n_p = \exp(eV_0/kT)$

- If all the impurities are ionized ($n_n \approx N_d$ and $p_p \approx N_a$):

$$\boxed{V_0 \simeq \frac{kT}{e}\log_e\left(\frac{N_d N_a}{n_i^2}\right)}$$

Pn junction with forward bias

- Consider a uniform sC biased with a voltage V:

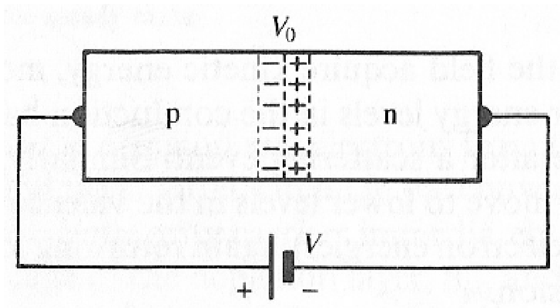


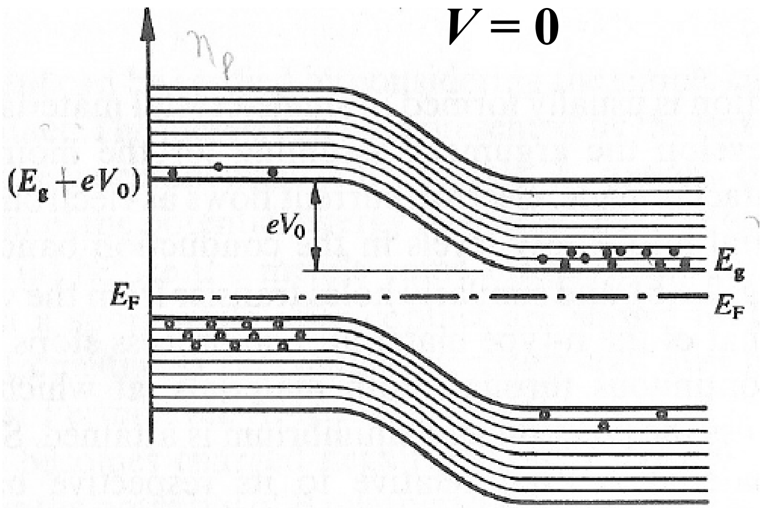
The whole band structure tilts (one end of the sC has more energy than the other) .

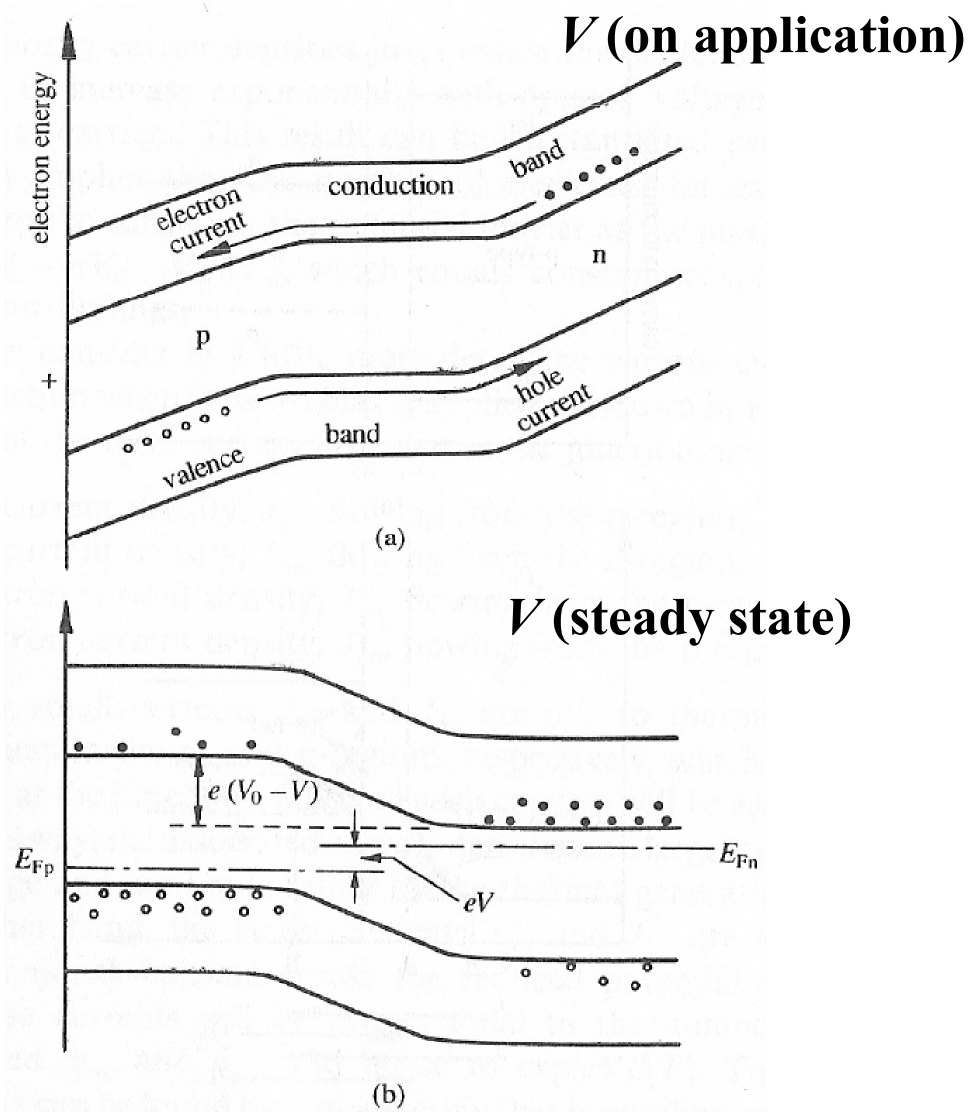
Electrons and holes move in the field (in opposite directions.

Pn junction with forward bias

- Consider a junction biased with a voltage V:

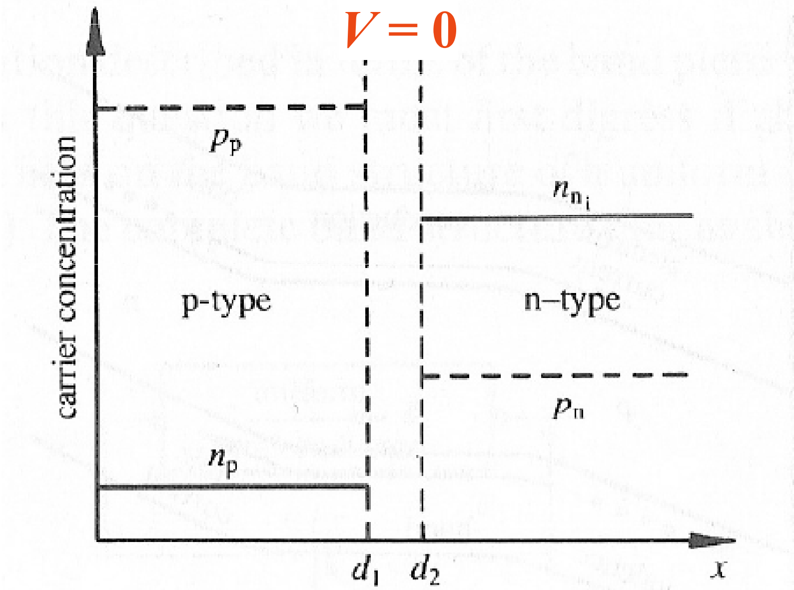


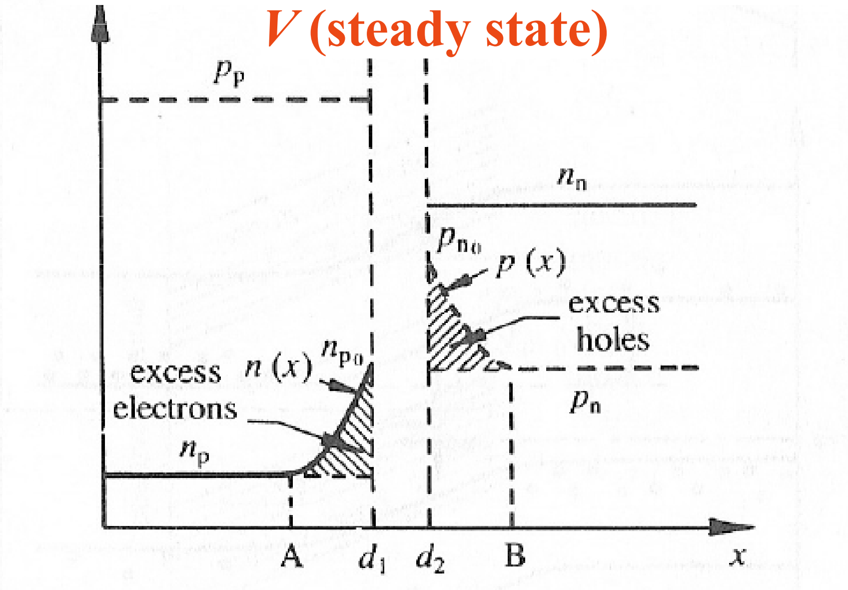




Pn junction with forward bias

- Excess carriers in a biased junctions:





$$p_p = p_n \exp(eV_0/kT)$$

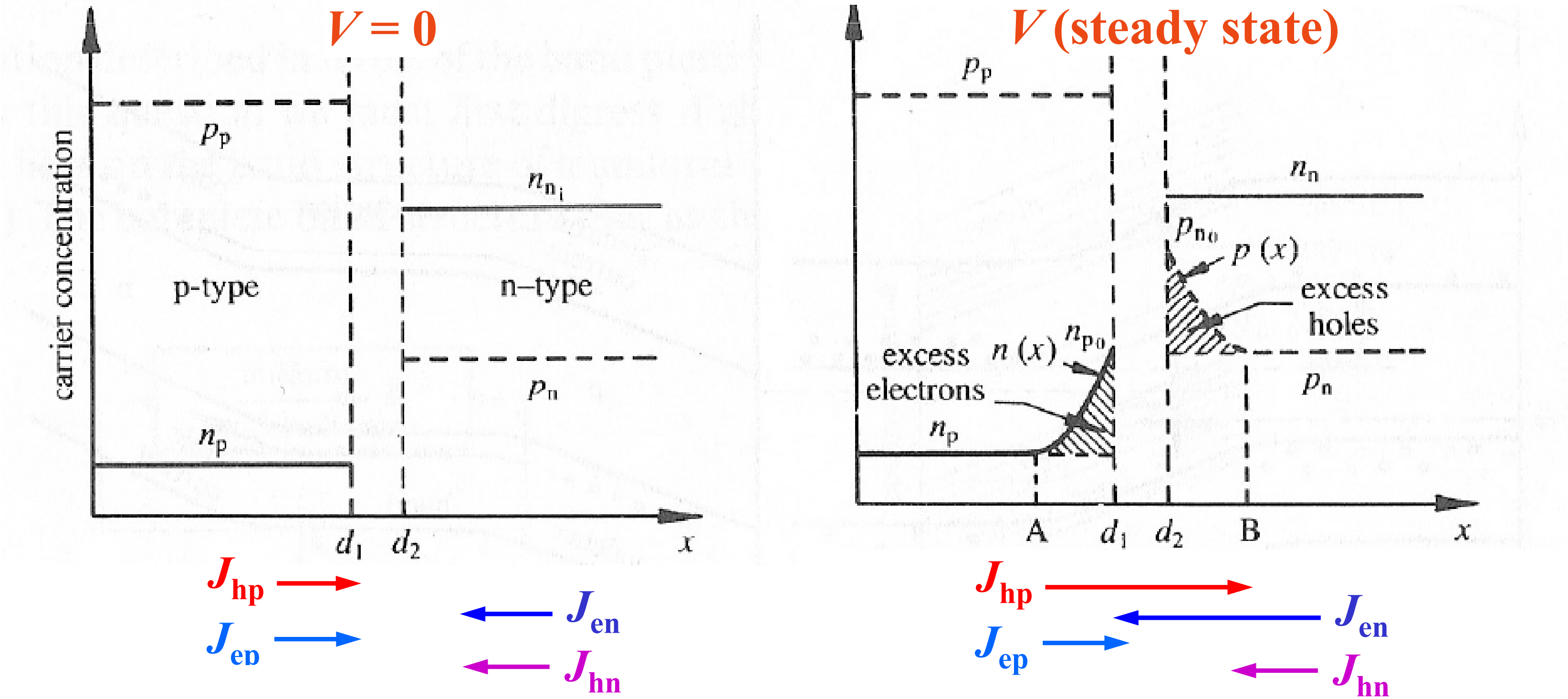
$$p_{n0} = p_p \exp[e(V - V_0)/kT]$$

$$p_{n0} = p_n \exp(eV/kT)$$

$$n_{p0} = n_p \exp(eV/kT)$$

Pn junction with forward bias

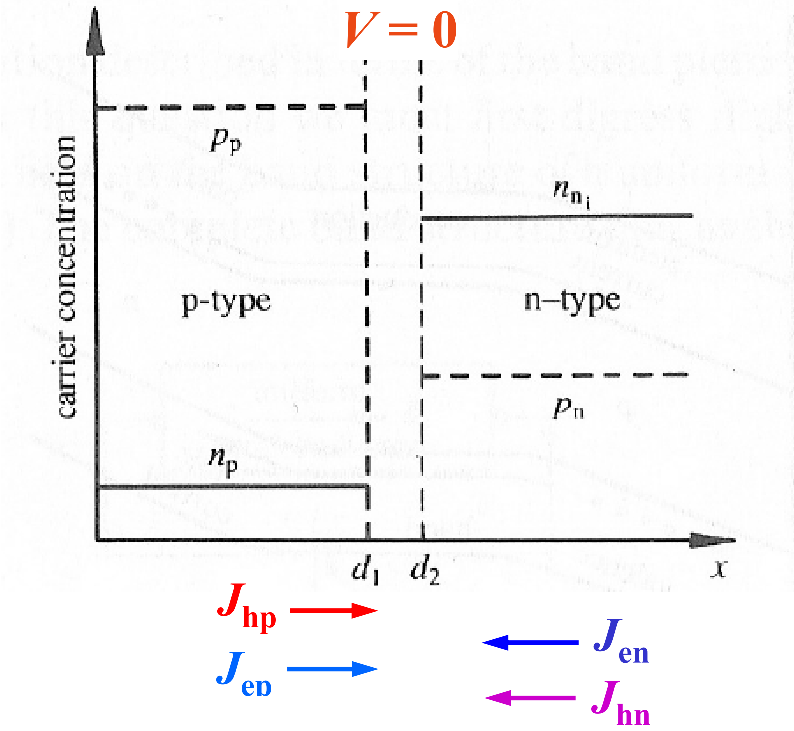
- Excess carriers in a biased junctions:

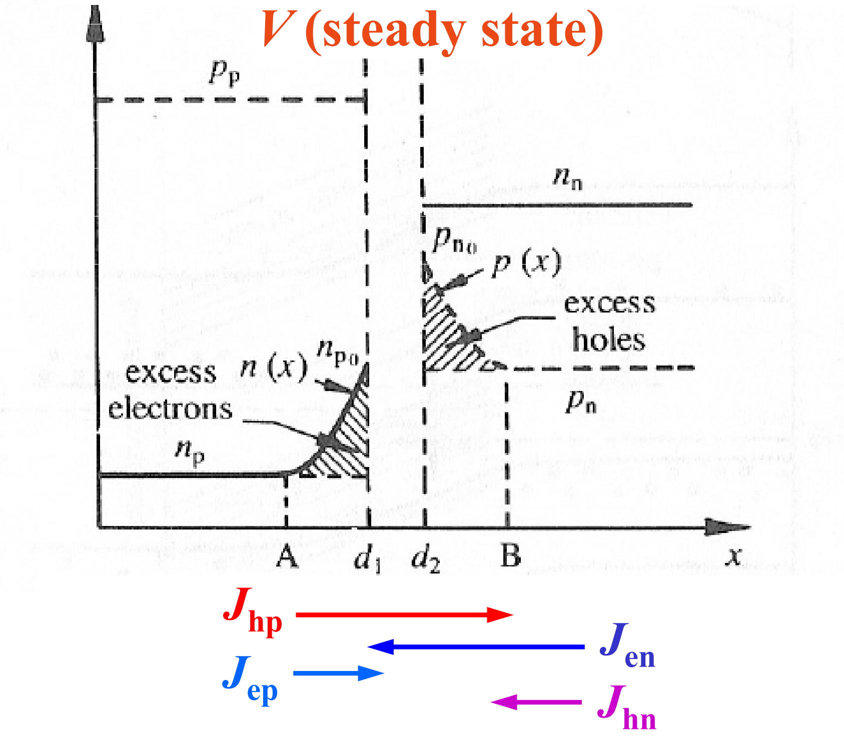


(a) a hole current density, J_{hp}, flowing from the p-region;
(b) a hole current density, J_{hn}, flowing from the n-region;
(c) an electron current density, J_{en}, flowing from the n-region;
(d) an electron current density, J_{ep}, flowing from the p-region.

Pn junction with forward bias

- Excess carriers in a biased junctions:





$$J_{hp} \propto p_{n0} = p_n \exp(eV/kT)$$

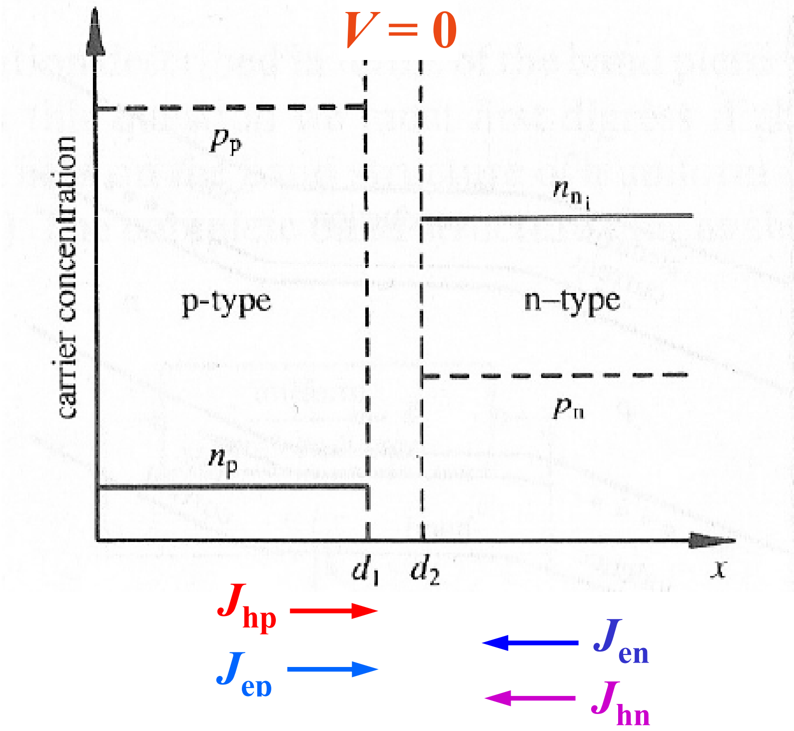
$$J_{hp} = J_{hn} \quad \Rightarrow \quad J_{hp} = J_{hn} \exp(eV/kT)$$

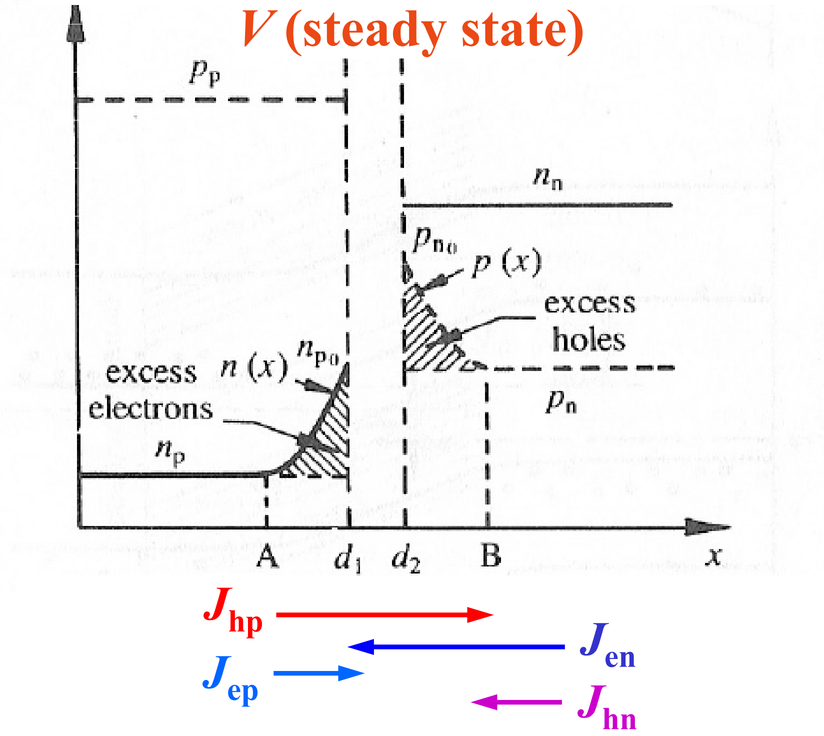
$$J_{en} \propto n_{p0} = n_p \exp(eV/kT)$$

$$J_{en} = J_{ep} \quad \Rightarrow \quad J_{en} = J_{ep} \exp(eV/kT)$$

Pn junction with forward bias

- Excess carriers in a biased junctions:





$$J_{hp} = J_{hn}$$

$$J_{en} = J_{ep}$$

Total currents:

$$J_h = J_{hp} - J_{hn} = J_{hn}[\exp(eV/kT) - 1]$$

$$J_e = J_{en} - J_{ep} = J_{ep}[\exp(eV/kT) - 1]$$

$$\boxed{J = J_0[\exp(eV/kT) - 1]}$$

Diode equation

$$J_0 = J_{hn} + J_{ep}$$

Conclusions

- Extrinsic sC are created by adding impurities:
  - Donors (extra electrons) and acceptors (extra holes).
- Electron processes in sC were qualitatively studied.
- Components of the total current were studied.
- We have deduced the continuity equation for MINORITY carriers.
- Several measurements in sC were studied.
- The junction diode was studied.