Physics of Electronics:

5. Semiconductors

July – December 2009

Contents overview

- Compensation doping.
- Electrical conduction in semiconductors.
- Continuity equation.
- Semiconductor measurements.

Compensation doping

- Consider a sC with both donors and acceptors
  - Suppose all impurities are ionized ($N_a = N_a^+$ & $N_d = N_d^+$)

$$n + N_a = p + N_d \xrightarrow{np = n_i^2} n^2 - (N_d - N_a)n - n_i^2 = 0 \Rightarrow$$

$$n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\left[1 + \left(\frac{2n_i}{N_d - N_a}\right)^2\right]^{1/2} \quad p = \frac{N_a - N_d}{2} + \frac{N_d - N_a}{2}\left[1 + \left(\frac{2n_i}{N_d - N_a}\right)^2\right]^{1/2}$$

  - Extrinsic compensated $n_i \ll |N_d - N_a|$

$$\Rightarrow \begin{cases} \text{for } N_d > N_a: & n \simeq N_d - N_a \\ & p \simeq n_i^2/(N_d - N_a) \\ \text{for } N_a > N_d: & p \simeq N_a - N_d \\ & n \simeq n_i^2/(N_a - N_d) \end{cases}$$

majority carriers large (T independent)

minority carriers low (strong T dependence)

Electrical conduction in sC.

- Conductivity (response to an electric field)
  - In a sC we have e^-'s and h^+'s moving:

$$J_e = ne\mu_e \mathscr{E}$$
$$J_h = pe\mu_h \mathscr{E}$$

$$\Rightarrow \quad J = J_e + J_h = \underbrace{e(n\mu_e + p\mu_h)}_{\sigma}\mathscr{E}$$

  - In a n-type sC (n>>p):

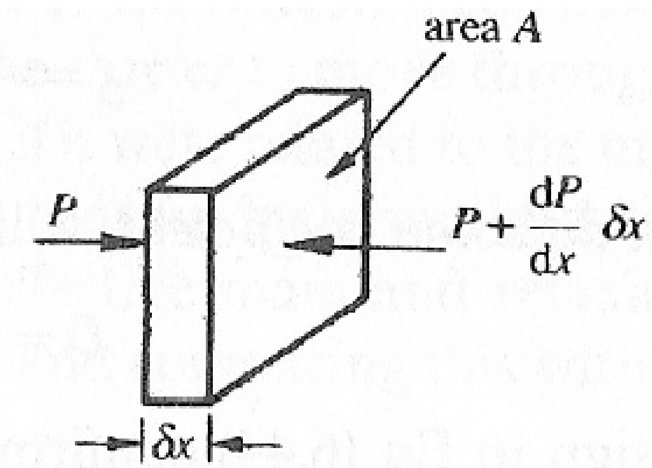
$$\sigma_n \simeq en\mu_e \simeq eN_d\mu_e$$

  - In a p-type sC (p>>n):

$$\sigma_p \simeq ep\mu_h \simeq eN_a\mu_h$$

Electrical conduction in sC.

- Diffusion of charge carriers
  - Diffusion occurs when concentration gradients are present.
  - In a **neutral gas** with gradient in the *x* direction:



Overall force on the elemental volume

$$\left[P(x) - \left(P + \frac{\mathrm{d}P}{\mathrm{d}x}\delta x\right)\right]A = -\frac{\mathrm{d}P}{\mathrm{d}x}\delta x A$$

Average force on each particle

$$F_{\mathrm{D}} = -\frac{1}{N}\frac{\mathrm{d}P}{\mathrm{d}x}$$

Electrical conduction in sC.

- Diffusion of charge carriers
  - Diffusion occurs when concentration gradients are present.
  - In a **neutral gas** with gradient in the *x* direction:

    - v_D due to an external force (see calculations in Chap 4):

$$v_D = F\tau_r / m$$

    - v_D due to F_D:

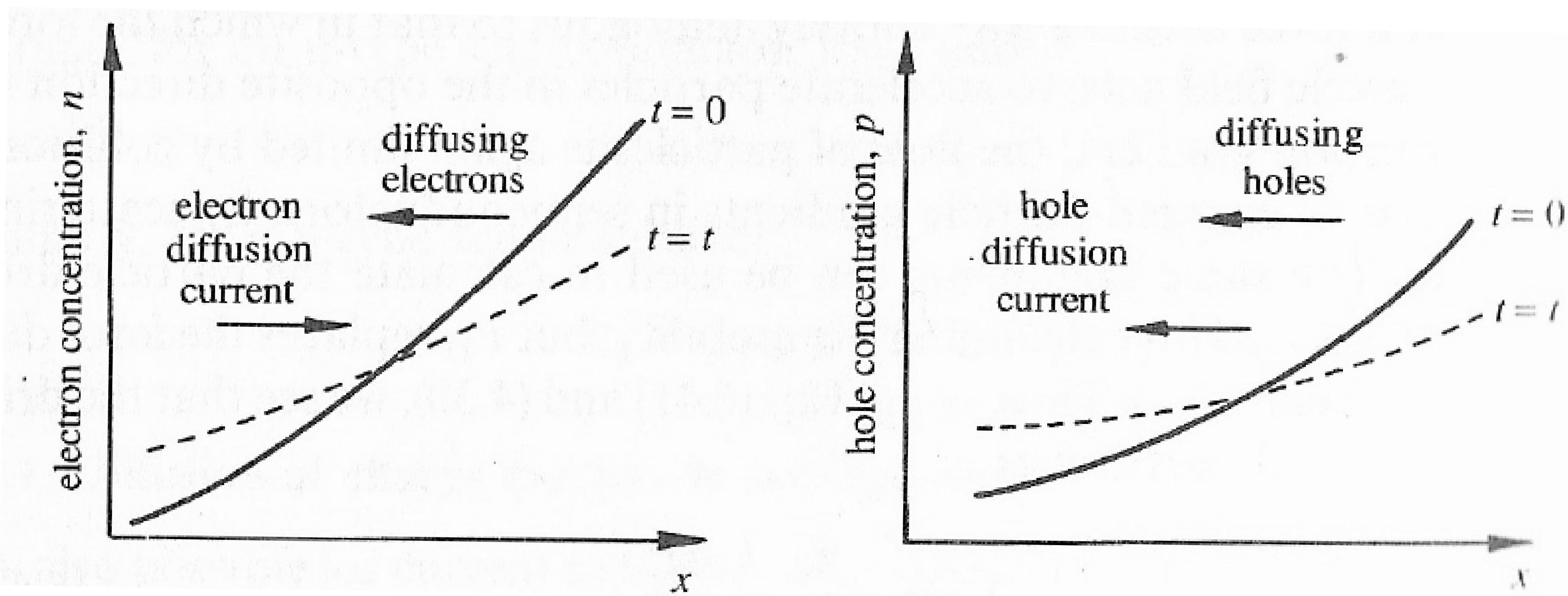
$$\Rightarrow \quad v_D = -\frac{\tau_r}{M}\frac{1}{N}\frac{dP}{dx} \quad \xrightarrow{P = NkT} \quad v_D = \overbrace{\frac{\tau_r kT}{M}}^{D}\frac{1}{N}\frac{dN}{dx}$$

Electrical conduction in sC.

- Diffusion of charge carriers
  - In a sC with a concentration gradient of the e^- and h^+:

$$v_{\mathrm{De}} = -D_e \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}x} \qquad v_{\mathrm{D_h}} = -D_h \frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}x}$$

$$\Rightarrow J_{\mathrm{De}} = -nev_{\mathrm{D}} = eD_e\,\mathrm{d}n/\mathrm{d}x \qquad \Rightarrow J_{\mathrm{Dh}} = -eD_h\,\mathrm{d}p/\mathrm{d}x$$



Electrical conduction in sC.

- Diffusion of charge carriers
  - Diffusion coefficient is related with mobility:

$$D = \frac{\tau_r kT}{M} \quad \Rightarrow \quad \mu = e\tau_r/m \quad D_e = (kT/e)\mu_e \quad D_h = (kT/e)\mu_h \quad \Rightarrow \quad \boxed{D_e/\mu_e = D_h/\mu_h = kT/e}$$

- Total current flow

$$J_e = ne\mu_e\mathscr{E} + eD_e\nabla n$$

$$J_h = pe\mu_h\mathscr{E} - eD_h\nabla p \quad \xrightarrow{J_h = 0} \quad \mathscr{E} = \frac{D_h \nabla p}{p\,\mu_h}$$

current can be compensated at a particular electrical field

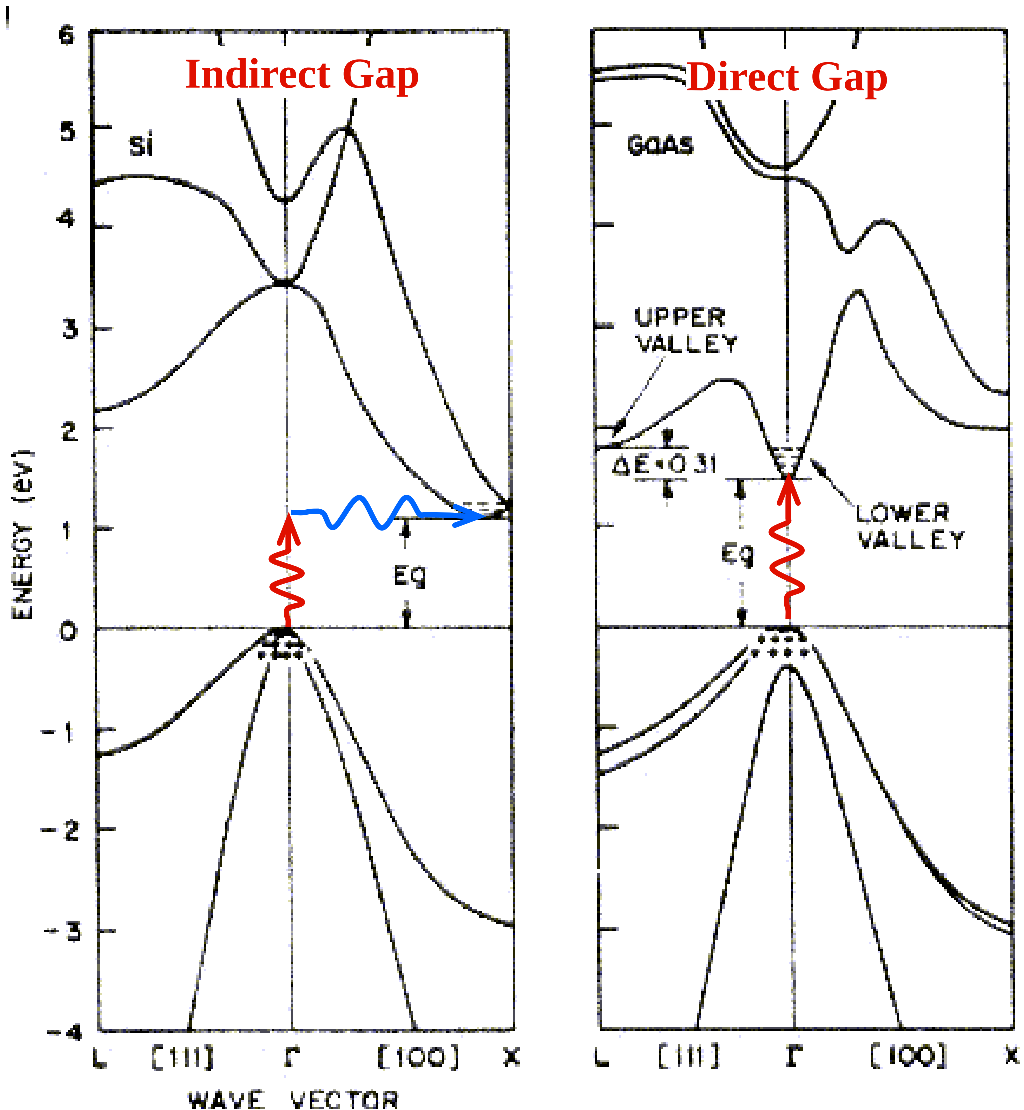
response to electrical field

diffusion

Electron Processes in Real sC

- Direct and indirect gap sC

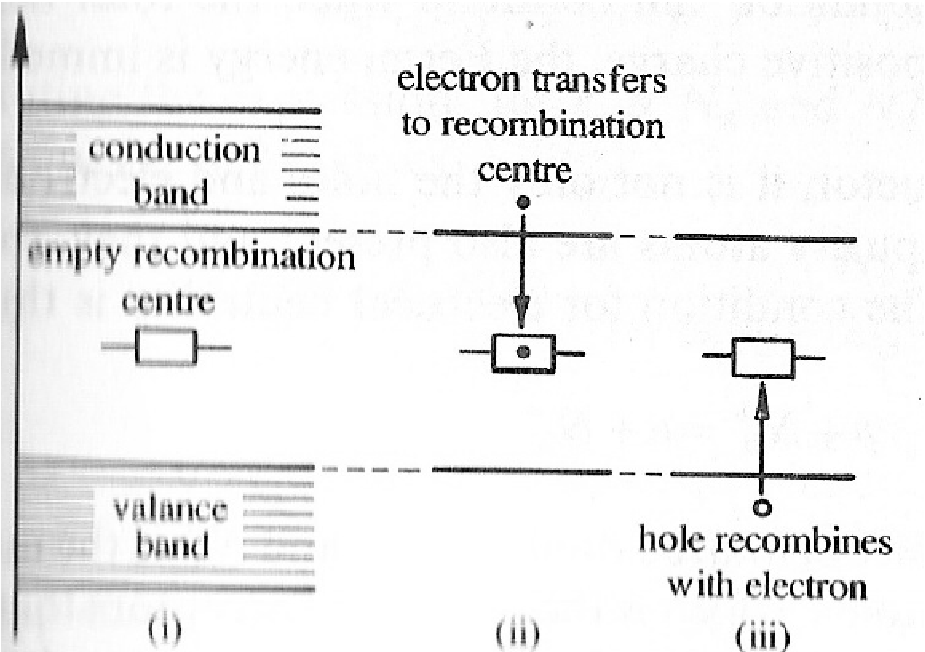
The minimum separation cannot be reached optically.



The minimum separation can be reached optically (by means of a photon).

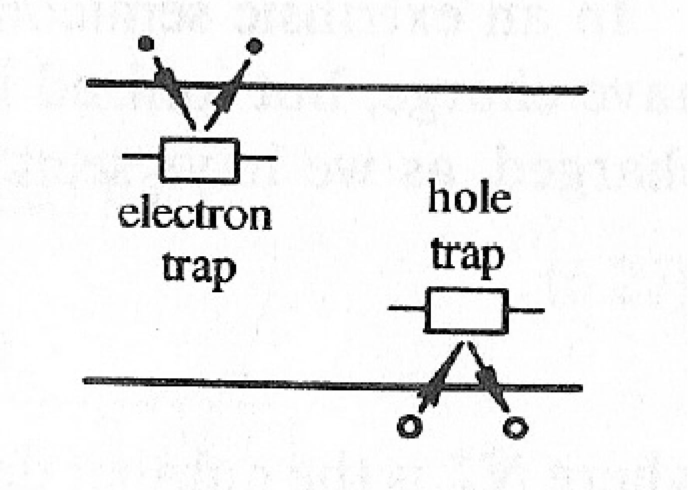
Electron Processes in Real sC

- Recombination
  - Permanent loss of a carrier.
  - Due to conservation of momentum
    - Possible in direct-gap sC's.
    - Marginally probable in indirect-gap sC's. Intermediate steps are required.



Electron Processes in Real sC

- Trapping
  - Temporary removal of a carrier on localized states



- Localized states
  - Lattice defects (dislocations, vacancies).
  - Impurities.

Continuity Equation for Minority Carriers

- General considerations:
  - Describes how charge density varies in time.
  - Two main causes for the variation:
    - Rate of generation and loss
    - Drift of carriers
- I.CASE: No current flow (no gradient & no $\mathscr{E}$)
  - Let's consider a **p-type** material:

$$dn/dt = G - R$$

Variation = Generation – Recombination

In this case, only
T **dependent**

$$\boldsymbol{R = r\, n(t)\, p(t)}$$

Continuity Equation for Minority Carriers

- I.CASE: No current flow (no gradient & no $\mathscr{E}$)
  - Consider first equilibrium:

$$\mathrm{d}n/\mathrm{d}t = 0 \;\Rightarrow\; G(T) = rn_0 p_0 = rn_i^2$$

  - Now consider variations from the equilibrium:

$$n = n_0 + \delta n \;\Leftrightarrow\; p = p_0 + \delta p \quad \textbf{with } \delta n = \delta p$$

To mantain charge neutrality

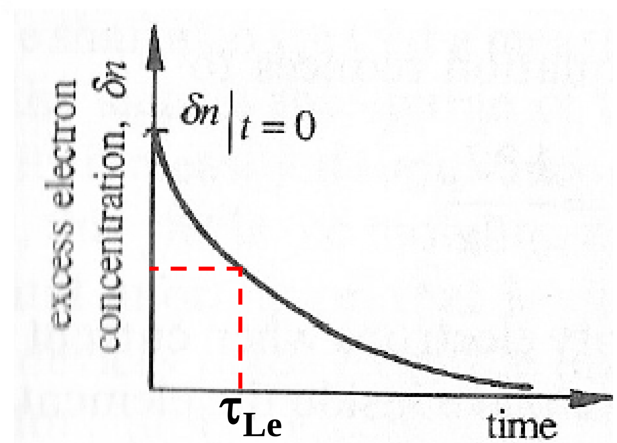
$$\Rightarrow \quad (\mathrm{d}n/\mathrm{d}t)|_{J=0} = [\mathrm{d}(\delta n)/\mathrm{d}t]|_{J=0} = G(T) - r(n_0 + \delta n)(p_0 + \delta p)$$

$p_o >> n_o$ 0

$$\Rightarrow \quad [\mathrm{d}(\delta n)/\mathrm{d}t]|_{J=0} = \cancel{-rn_0\delta p} - rp_0\delta n \cancel{- r\delta n\delta p}$$

Considering T=const.

$$\Rightarrow \quad [\mathrm{d}(\delta n)/\mathrm{d}t]|_{J=0} = -rp_0\delta n = -\delta n/\tau_{Le}$$

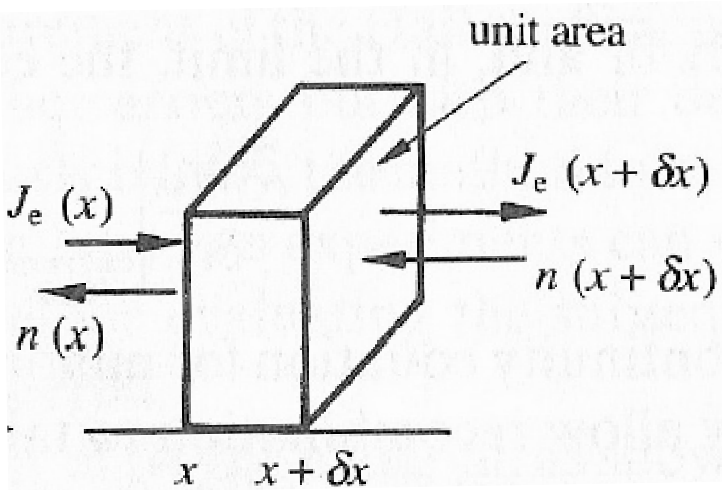


  - For a **n-type** material:

$$\mathrm{d}(\delta p)/\mathrm{d}t = -\delta p/\tau_{Lh}$$

Continuity Equation for Minority Carriers

- II.CASE: With current flow



$$J_e = -nev_D$$

$$\Rightarrow \begin{cases} n_{x+\delta x} = J_e(x+\delta x)/ev_D \\ n_x = J_e(x)/ev_D \end{cases}$$

$$\Rightarrow n_{x+\delta x} - n_x = \delta n = [J_e(x+\delta x) - J_e(x)]/ev_D$$

$$= \frac{1}{ev_D}\left(J_e(x) + \frac{\partial J_e}{\partial x}\delta x - J_e(x)\right)$$

$$\Rightarrow \delta n = \frac{1}{ev_D}\frac{\partial J_e}{\partial x}\delta x \quad \xrightarrow{v_D = \partial x/\partial t} \quad \left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{current flow}} = \frac{1}{e}\frac{\partial J_e}{\partial x}$$

Continuity Equation for Minority Carriers

- Total continuity equation

$$\left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{total}} = -\frac{\delta n}{\tau_{\text{Le}}} + \frac{1}{e}\frac{\partial J_{\text{e}}}{\partial x}$$

But $J_{\text{e}} = ne\mu_{\text{e}}\mathscr{E} + eD_{\text{e}}\nabla n$

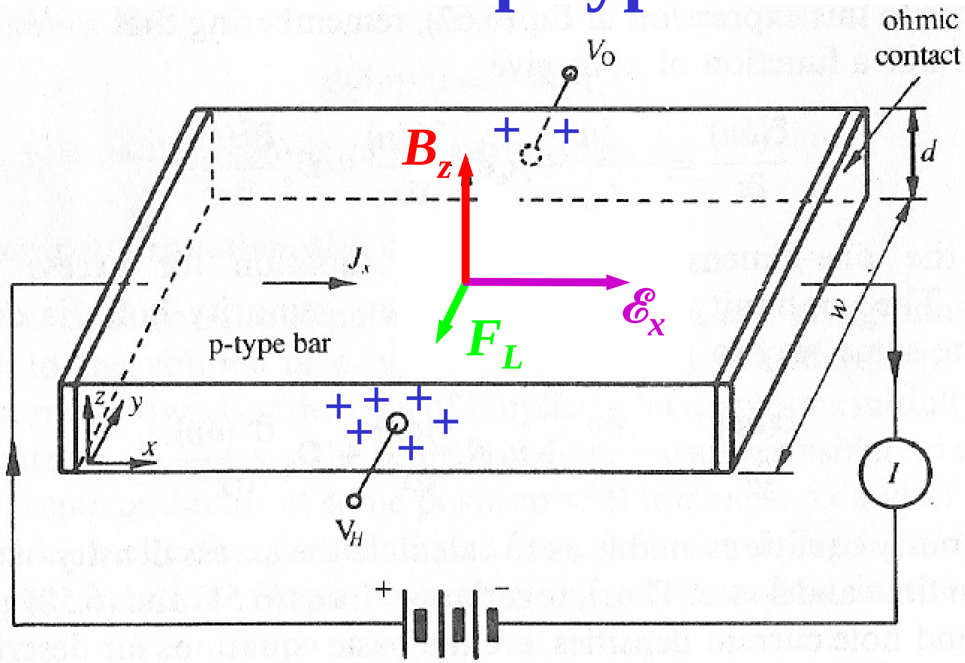
$n = n_0 + \delta n$

$$\boxed{\frac{\partial(\delta n)}{\partial t} = -\frac{\delta n}{\tau_{\text{Le}}} + \mu_{\text{e}}\mathscr{E}_x\frac{\partial(\delta n)}{\partial x} + D_{\text{e}}\frac{\partial^2(\delta n)}{\partial x^2}}$$

(idem for holes in an n-type material)

Semiconductor Measurements

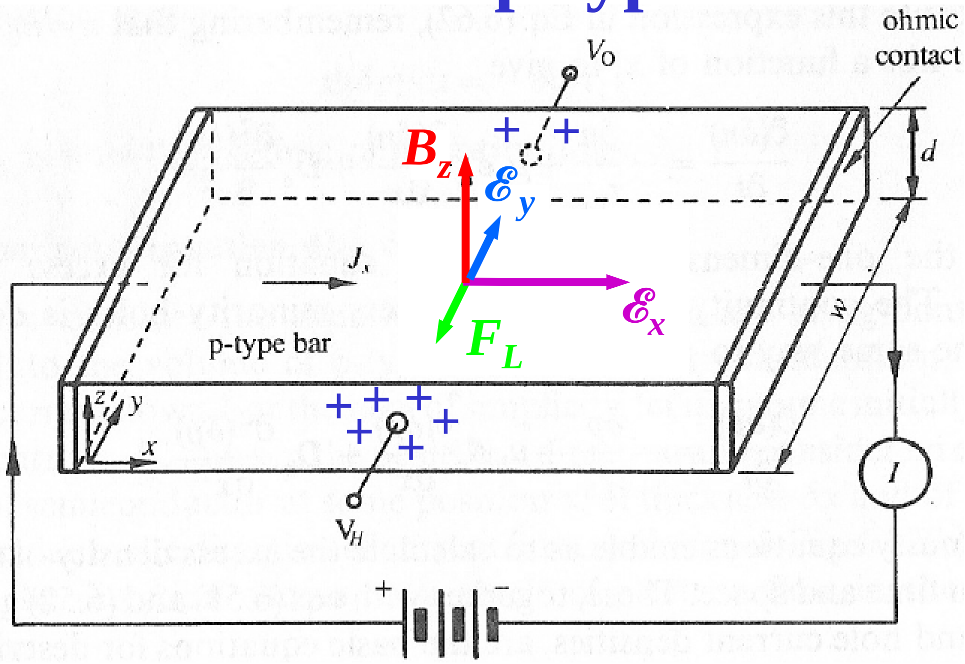
- Hall effect
  - Consider **p-type** material



$F_L = ev \times B$ ⇨ $|F_L| = e\, v_{Dx}\, B_z$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material

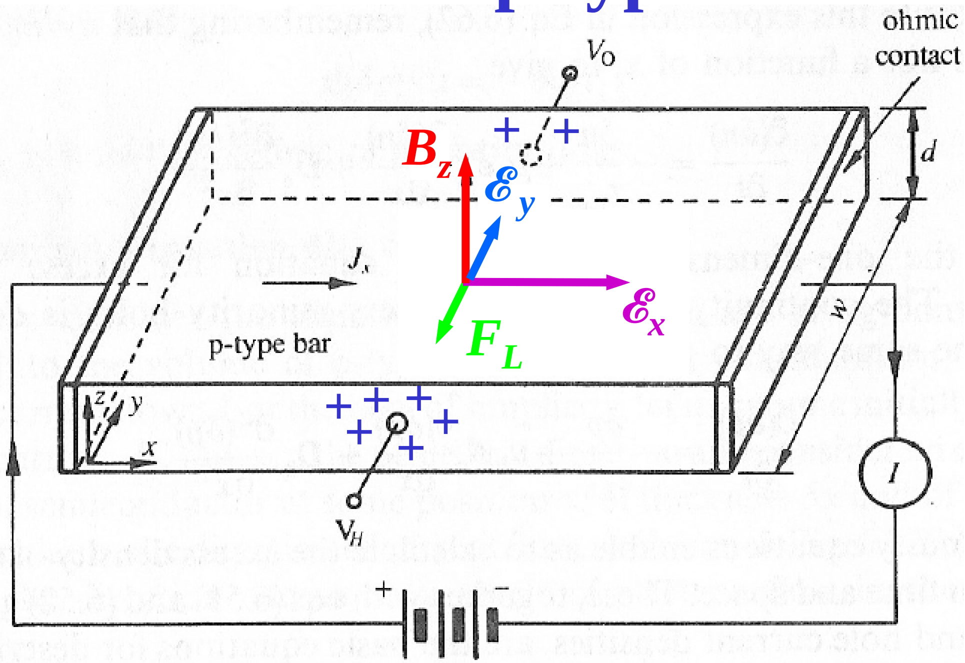


$$F_L = ev \times B \Rightarrow |F_L| = e\, v_{Dx}\, B_z$$

$$\Rightarrow e\mathcal{E}_y = F_L = ev_{Dx} B_z$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



$F_L = ev \times B$ ⇨ $|F_L| = e\, v_{Dx}\, B_z$

⇨ $e\mathscr{E}_y = F_L = e v_{Dx} B_z$ & $J_x \simeq p e v_{Dx}$ } $\mathscr{E}_y = J_x B_z / pe$

The Hall coefficient is defined:

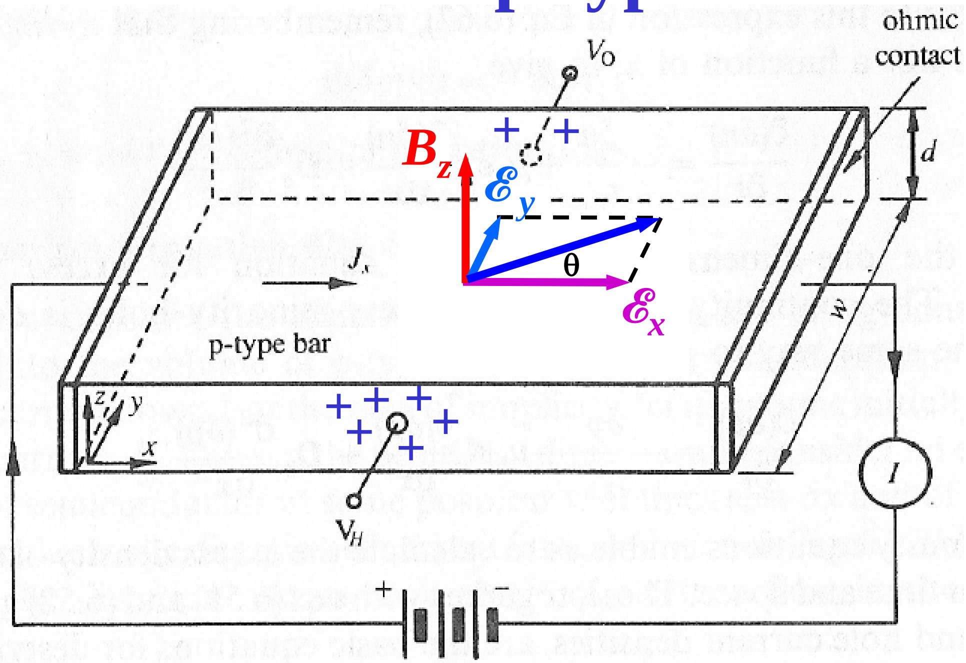
$$R_H = \mathscr{E}_y / J_x B_z = 1/pe$$

- If V_H and I are measured:

$V_H = \mathscr{E}_y w$, $I = J_x w d$ } $R_H = \frac{V_H}{wIB_z} wd = \frac{V_H d}{IB_z} = \frac{1}{pe}$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



The Hall angle is defined:

$$\tan\theta = \mathscr{E}_y / \mathscr{E}_x$$

$$\tan\theta = \frac{J_x B_z}{pe}\frac{\sigma}{J_x} = \mu_h B_z$$

$$\mu_h = R_H \sigma$$

  - For a **n-type** material:

$$R_{He} = -1/ne$$

Semiconductor Measurements

- Hall effect
  - Consider both type of carriers to be present:

$$v_{Dh} = \mu_H \mathscr{E}_x \qquad v_{De} = -\mu_H \mathscr{E}_x \quad \Rightarrow \quad F_h = -e(v_{Dh} \times B) = -ev_{Dh} B_z \qquad F_e = e(v_{De} \times B) = -ev_{De} B_z$$

    - A net current is created in the *y* direction

$$\sigma \mathscr{E}_y = e(pv_{yh} - nv_{ye})$$

    - Now, using the expressions for the Hall angle

$$v_{yh} = \mu_h \mathscr{E}_y = \mu_h(\mathscr{E}_x \tan\theta) = \mu_h(\mathscr{E}_x \mu_h B_z) \quad \& \quad v_{ye} = \mu_e^2 \mathscr{E}_x B_z$$

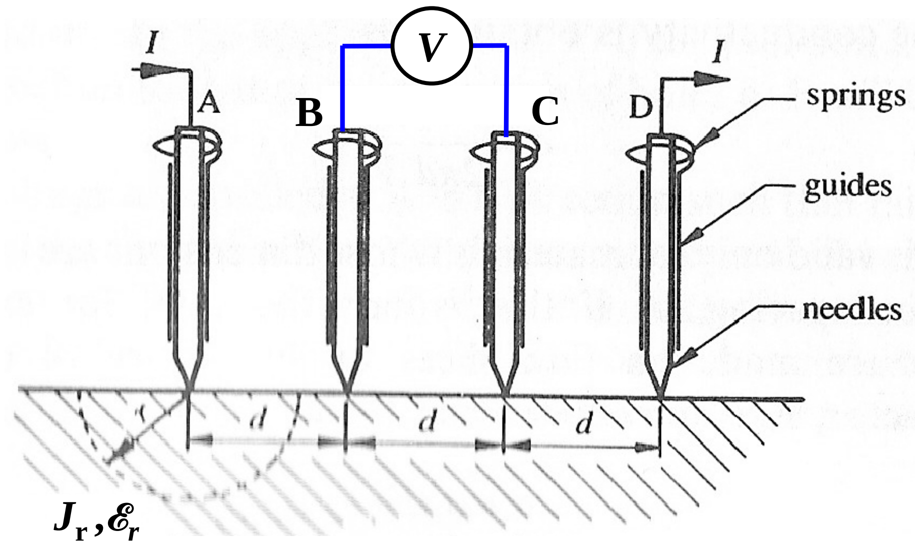
$$\Rightarrow \sigma \mathscr{E}_y = e(p\mu_h^2 - n\mu_e^2)\mathscr{E}_x B_z$$

    - Hall coefficient

$$R_H = \frac{\mathscr{E}_y}{J_x B_z} = \frac{e(p\mu_h^2 - n\mu_e^2)}{\sigma^2} \xrightarrow{\sigma = e(n\mu_e + p\mu_h)} R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

Semiconductor Measurements

- Four-point probe method for conductivity meas.
  - Consider $d << L$



For the current leaving *A*:

$$J_r = I/(2\pi r^2) \Rightarrow \mathscr{E}_r = J/\sigma = I/(2\pi\sigma r^2)$$

Potential at a distance *a* from *A*:

$$V_a = \int_{-\infty}^{a} \mathscr{E}_r \, \mathrm{d}r = -\frac{I}{2\pi\sigma}\int_{-\infty}^{a} \frac{1}{r^2}\,\mathrm{d}r = \frac{I}{2\pi\sigma a}$$

$$\Rightarrow V_{BC} = \frac{I}{2\pi\sigma d} - \frac{I}{2\pi\sigma(2d)} = \frac{I}{4\pi\sigma d}$$

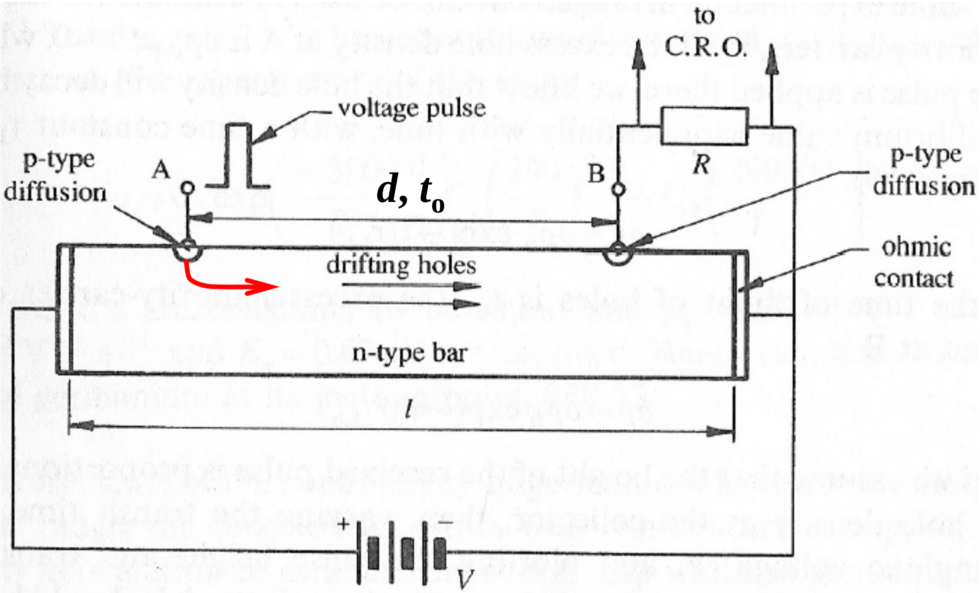
Identically for the current entering *D*, then:

$$V = 2V_{BC} = \frac{I}{(2\pi\sigma d)}$$

$$\Rightarrow \sigma = \frac{1}{2\pi d}\frac{I}{V}$$

Semiconductor Measurements

- Minority carrier life-time and mobility.



Mobility:

$$v_D = d/t_0 \Rightarrow \mu E = \mu V/l = d/t_0$$

$$\Rightarrow \boxed{\mu = ld/t_0 V}$$

Life-time:

$$d(\delta p)/dt = -\delta p/\tau_{Lh} \Rightarrow \delta p = \delta p_0 \exp(-t/\tau_{Lh})$$

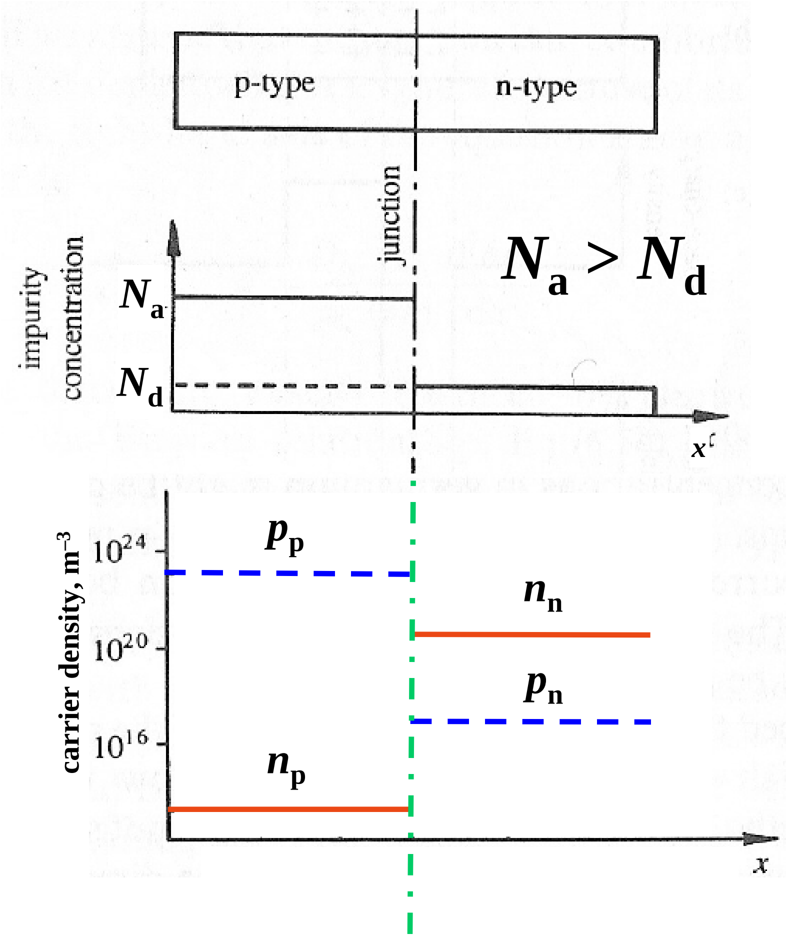
Physics of Electronics:

6. Junction Diodes

July – December 2009

Pn junction in equilibrium

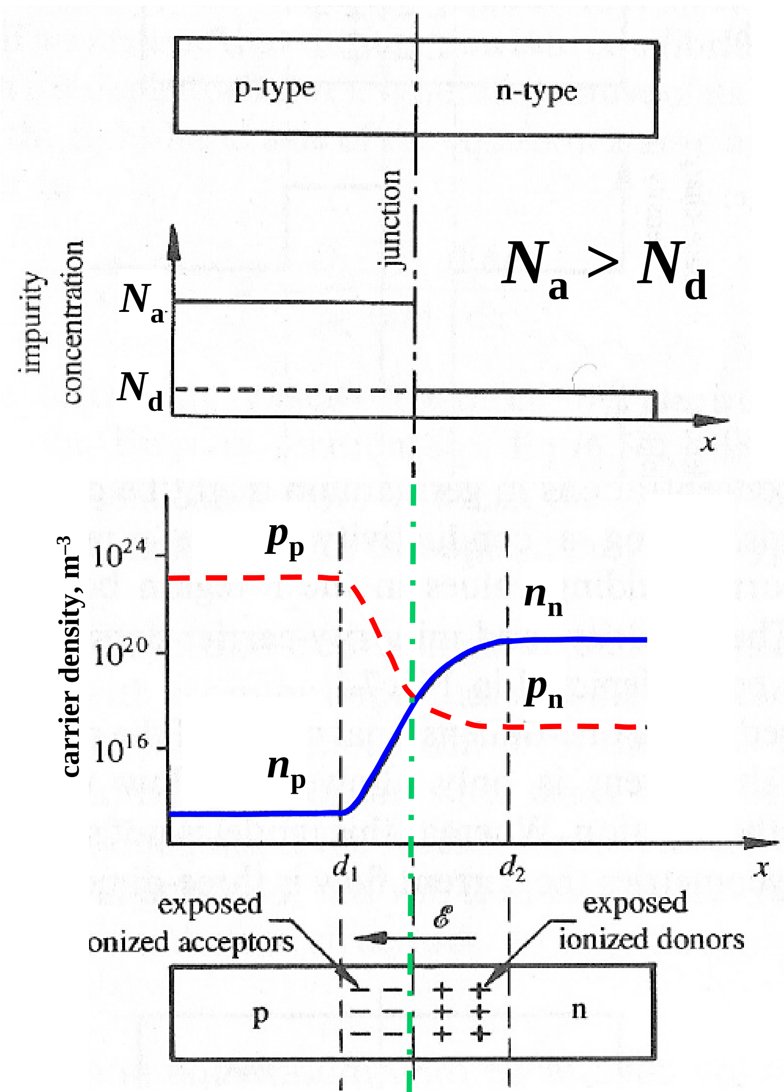
- Consider a pn junction with abrupt transition



Initially, the density of carriers change abruptly at the junction.

Pn junction in equilibrium

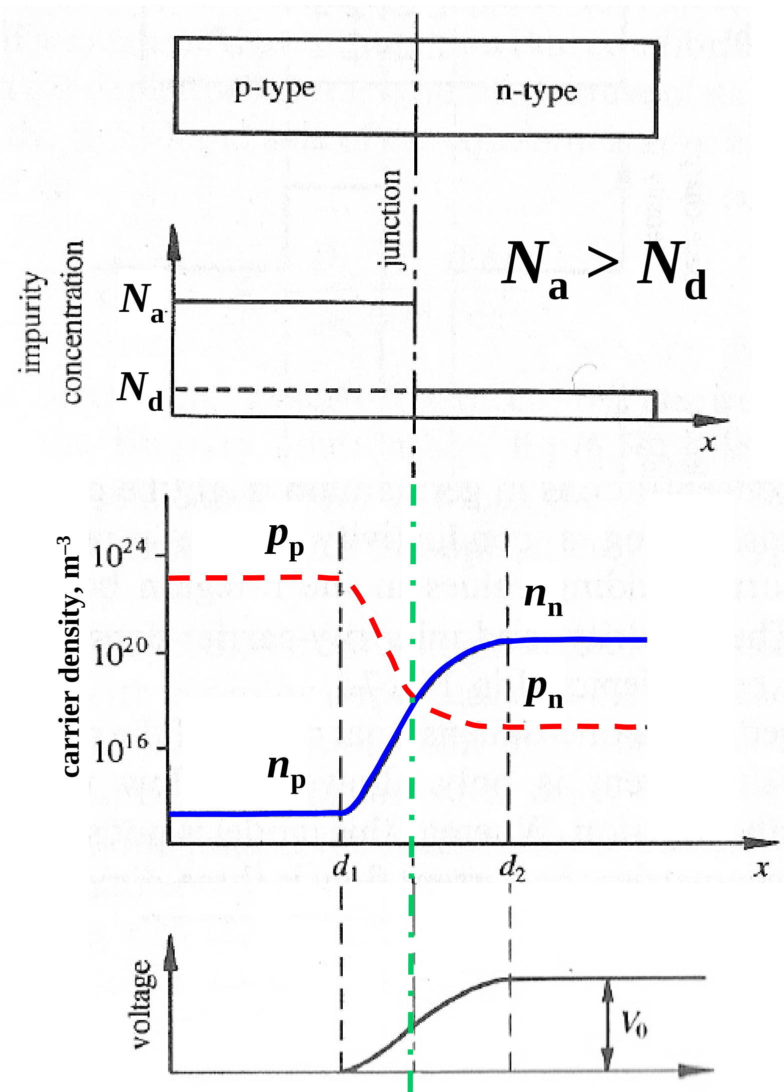
- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1µm).

Pn junction in equilibrium

- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

In equilibrium, a voltage difference is created across the depletion layer.

Pn junction in equilibrium

- Voltage across the depletion layer
  - Start with continuity eq. for holes

$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\cancelto{0}{\frac{\delta p}{\tau_{\mathrm{Lh}}}} + \mu_{\mathrm{h}}\mathscr{E}_x \frac{\partial(\delta p)}{\partial x} + D_{\mathrm{h}}\frac{\partial^2(\delta p)}{\partial x^2} \Rightarrow \mathscr{E}_x = \frac{D_{\mathrm{h}}}{\mu_{\mathrm{h}}}\frac{1}{(\delta p)}\frac{\mathrm{d}(\delta p)}{\mathrm{d}x}$$

$D_{\mathrm{h}}/\mu_{\mathrm{h}} = kT/e$

$$\Rightarrow \mathscr{E}_x = \frac{kT}{e}\frac{1}{(\delta p)}\frac{\mathrm{d}(\delta p)}{\mathrm{d}x} \Rightarrow e\int_{d_1}^{d_2} \mathscr{E}_x \, \mathrm{d}x = -kT\int_{p_{\mathrm{p}}}^{p_{\mathrm{n}}} \frac{\mathrm{d}(\delta p)}{\delta p}$$

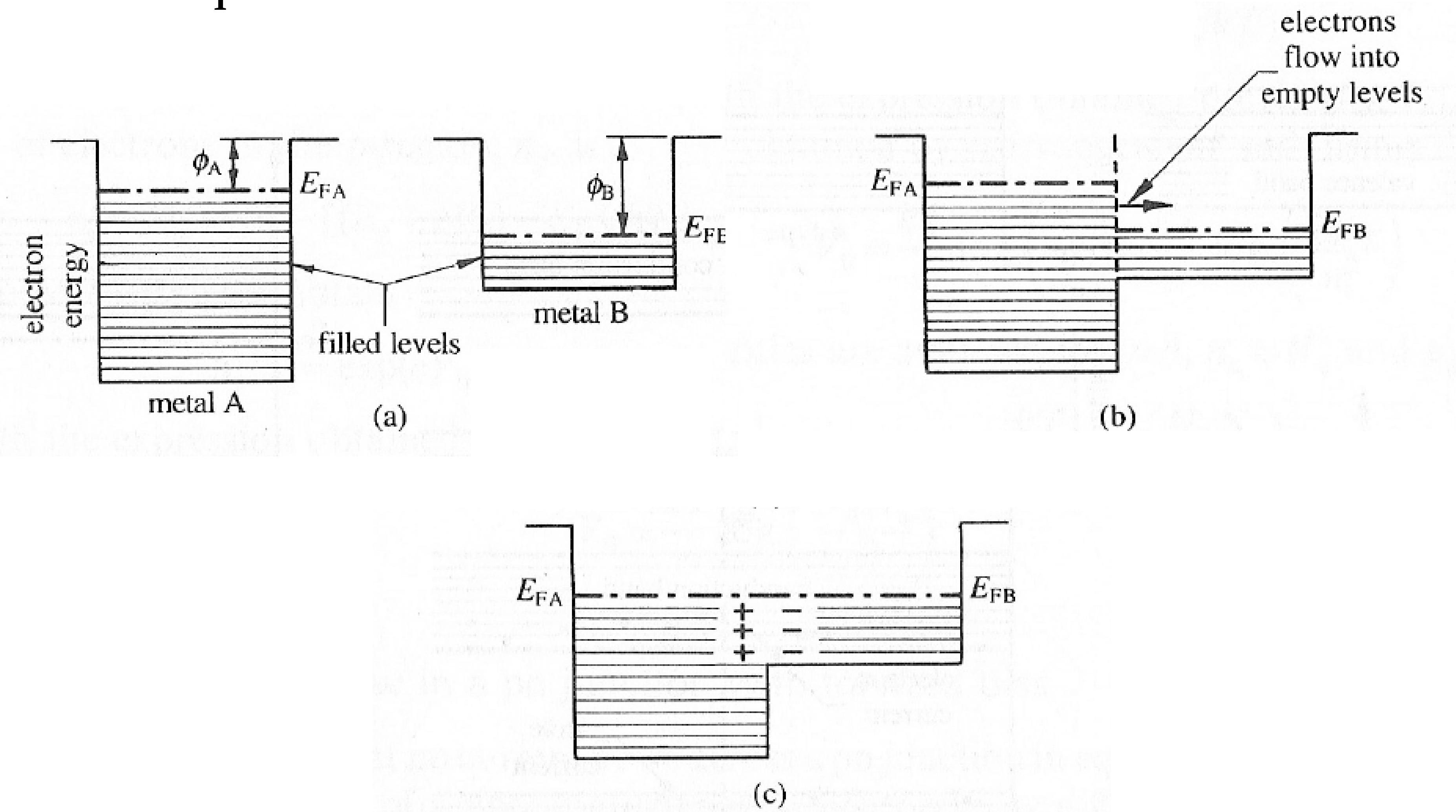
$$\Rightarrow \boxed{p_{\mathrm{p}} = p_{\mathrm{n}} \exp(eV_0/kT)}$$

  - Idem for electrons

$$\boxed{n_{\mathrm{n}} = n_{\mathrm{p}} \exp(eV_0/kT)}$$

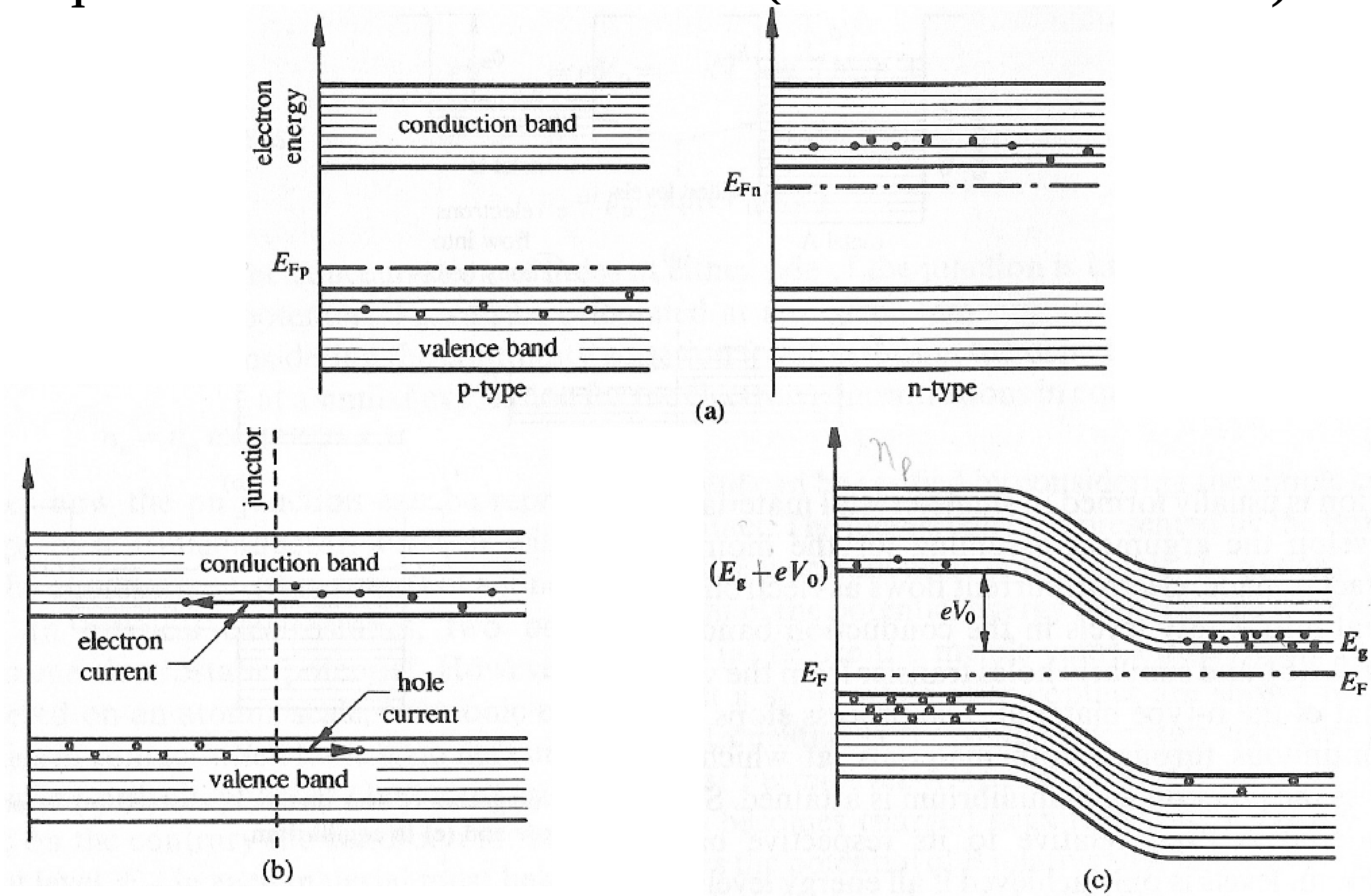
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in metals):



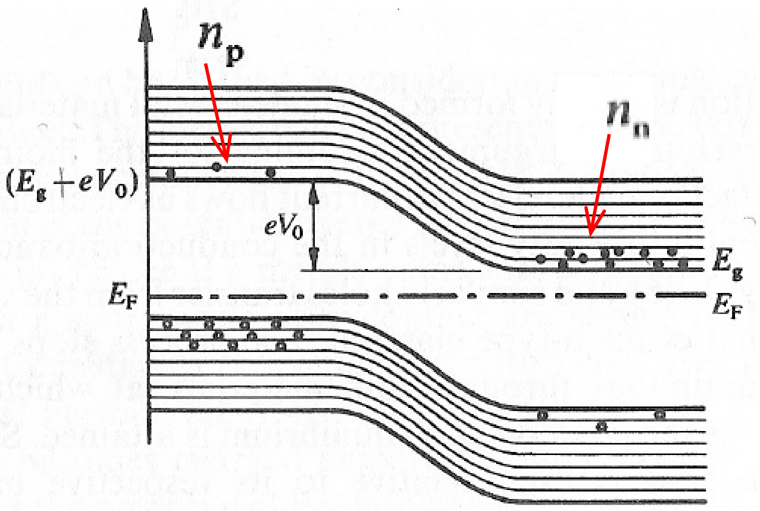
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in semiconductors)



Pn junction in equilibrium

- Number of electrons in the conduction band



$$n_n = N_c \exp[-(E_g - E_F)/kT]$$

$$n_p = N_c \exp\{-[(E_g + eV_0) - E_F]/kT\}$$

$$\Rightarrow \quad n_n/n_p = \exp(eV_0/kT)$$

- If all the impurities are ionized ($n_n \approx N_d$ and $p_p \approx N_a$):

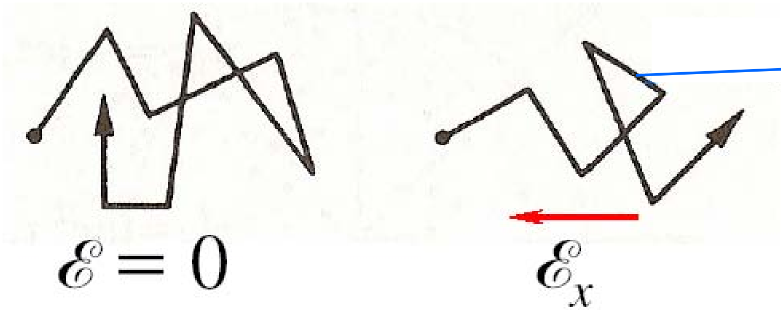
$$\boxed{V_0 \simeq \frac{kT}{e} \log_e\left(\frac{N_d N_a}{n_i^2}\right)}$$

Conclusions

- Extrinsic sC are created by adding impurities:
  - Donors (extra electrons) and acceptors (extra holes).
- Electron processes in sC were qualitatively studied.
- Components of the total current were studied.
- We have deduced the continuity equation for MINORITY carriers.
- Several measurements in sC were studied.
- The junction diode in equilibrium was studied.

Conduction Processes in a Metal

- Consider a (classical) free e^- moving in a metal.
  - There are collisions with the crystal structure:



τ_r = average time between collisions

  - The friction is assumed to be proportional to $m\,\dot{x}/\tau_r$:

$$-e\mathscr{E}_x - \frac{m}{\tau_r}\dot{x} = m\ddot{x} \quad \Rightarrow \quad -e\mathscr{E}_x = m\frac{\mathrm{d}}{\mathrm{d}t}(v_{Dx}) + \frac{m(v_{Dx})}{\tau_r}$$

$$\Rightarrow \quad v_{Dx} = \frac{-e\tau_r\mathscr{E}_x}{m}[1 - \exp(-t/\tau_r)]$$

  - At large times ($t \gg \tau_r$): $v_{Dx} = -\dfrac{eE\tau_r}{m} = \dfrac{F\tau_r}{m}$

BACK