Physics of Electronics

Problem Set 2:
Chapters 04 to 05

July – December 2009

Assembly of Classical Particles

1. A hypothetical gas with N molecules per cubic metre has a speed distribution function

$$f(v) = Cv^2 \qquad \text{for } v_0 > v > 0$$
$$f(v) = 0 \qquad \text{for } v > v_0$$

Find the mean-square fluctuation of the speeds, which is defined as the mean-square speed minus the square of the mean speed.

2. At $T = 0$ K the electron energy levels in a metal are all occupied for $E < E_F$ and are empty for $E > E_F$. The energy distribution is then of the form

$$\Delta N/N = CE^{1/2}\Delta E \qquad \text{for } E < E_F$$
$$\Delta N/N = 0 \qquad \text{for } E > E_F$$

where C is a constant. Find (a) the average electron energy under these conditions and (b) the percentage of the total number of electrons with energies between $0.1E_F$ and $0.2E_F$.

Conduction in Metals

3. The Fermi level in copper at 0 K is 7.0 eV. Estimate the number of free electrons per unit volume in copper at this temperature.

4. Calculate the Fermi energy at 0 K in copper given that there is one conduction electron per atom, that the density of copper is 8920 kg m^{-3} and its atomic weight is 63.54.

5. Find the wave functions and their energies for a 2D electron gas.

6. Find the density of states for a 2D electron gas.

7. ***Liquid He^3.*** The atom He^3 has spin $\frac{1}{2}$ and is a fermion. The density of liquid He^3 is 0.081 g cm^{-3} near absolute zero. Calculate the Fermi energy ϵ_F and the Fermi temperature T_F.

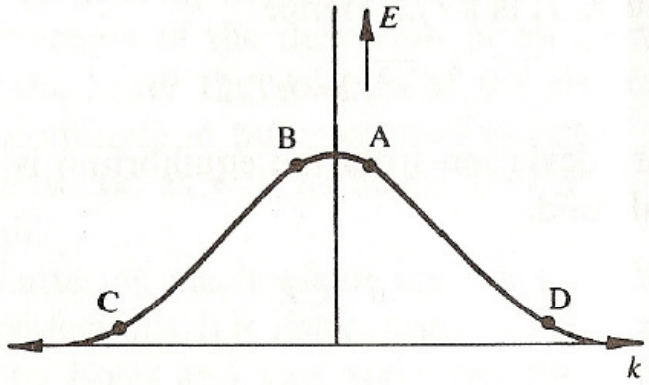
8. ***Frequency dependence of the electrical conductivity.*** Use the equation $m(dv/dt + v/\tau) = -eE$ for the electron drift velocity v to show that the conductivity at frequency ω is

$$\sigma(\omega) = \sigma(0)\left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2}\right), \qquad (62)$$

where $\sigma(0) = ne^2\tau/m$.

Electrons in Solids

9. The E–k diagram for an energy band in a particular material is as shown. If



an electric field is applied to the material in the negative k direction (force in the positive direction), find (a) the polarity of the effective masses of the four wavepackets made up of groups of states near A, B, C and D, (b) the direction of the velocity of each of the four wavepackets and (c) the direction of the acceleration of each. What are the physical consequences of these results?

Electrons in Solids

10. ***Kronig-Penney model.*** (a) For the delta-function potential and with $P \ll 1$, find at $k = 0$ the energy of the lowest energy band. (b) For the same problem find the band gap at $k = \pi/a$.

11. The conductivity of a metal having n free electrons per unit volume is given by Eq. (4.37) and the Fermi energy by Eq. (4.22). Consider a metal with a simple-cubic lattice structure of side 0.2 nm and one free conduction electron per atom. Assuming that the mean free path for electron collisions with the lattice is 100 lattice constants, find the relaxation time for an electron with the Fermi energy.

$$E_{F0} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3} = 3.65 \times 10^{-19}\, n^{2/3}\ \text{eV} \qquad (4.22)$$

$$\sigma = \frac{ne^2\tau_{rF}}{m} = \frac{ne^2\bar{l}_F}{mv_F} \qquad (4.37)$$