Physics of Electronics

Problem Set 1:
Chapters 01 to 02

Due on August 10th.

Introduction to Q.M.

Problem 1.7 At time $t = 0$ a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} Ax/a, & \text{if } 0 \le x \le a, \\ A(b-x)/(b-a), & \text{if } a \le x \le b, \\ 0, & \text{otherwise,} \end{cases}$$

where A, a, and b are constants.

(a) Normalize Ψ (that is, find A in terms of a and b).

(b) Sketch $\Psi(x,0)$ as a function of x.

(c) Where is the particle most likely to be found, at $t = 0$?

(d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.

(e) What is the expectation value of x?

$*$**Problem 1.8** Consider the wave function

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t},$$

where A, λ, and ω are positive real constants.

(a) Normalize Ψ.

(b) Determine the expectation values of x and x^2.

(c) Find the standard deviation of x. Sketch the graph of $|\Psi|^2$, as a function of x, and mark the points $(\langle x\rangle + \sigma)$ and $(\langle x\rangle - \sigma)$ to illustrate the sense in which σ represents the "spread" in x. What is the probability that the particle would be found outside this range?

Introduction to Q.M.

*__Problem 1.14__ A particle of mass m is in the state

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]},$$

where A and a are positive real constants.

(a) Find A.

(b) For what potential energy function $V(x)$ does Ψ satisfy the Schrödinger equation?

(c) Calculate the expectation values of x, x^2, p, and p^2.

(d) Find σ_x and σ_p. Is their product consistent with the uncertainty principle?

Problem 2.44 (*Attention*: This is a strictly qualitative problem—no calculations allowed!) Consider the "double square well" potential (Figure 2.17). Suppose the depth V_0 and the width a are fixed, and great enough so that several bound states occur.

(a) Sketch the ground-state wave function ψ_1 and the first excited state ψ_2, (i) for the case $b = 0$, (ii) for $b \approx a$, and (iii) for $b \gg a$.

(b) Qualitatively, how do the corresponding energies (E_1 and E_2) vary, as b goes from 0 to ∞? Sketch $E_1(b)$ and $E_2(b)$ on the same graph.

(c) The double well is a very primitive one-dimensional model for the potential experienced by an electron in a diatomic molecule (the two wells represent the attractive force of the nuclei). If the nuclei are free to move, they will adopt the configuration of minimum energy. In view of your conclusions in (b), does the electron tend to draw the nuclei together, or push them apart? (Of course, there is also the internuclear repulsion to consider, but that's a separate problem.)

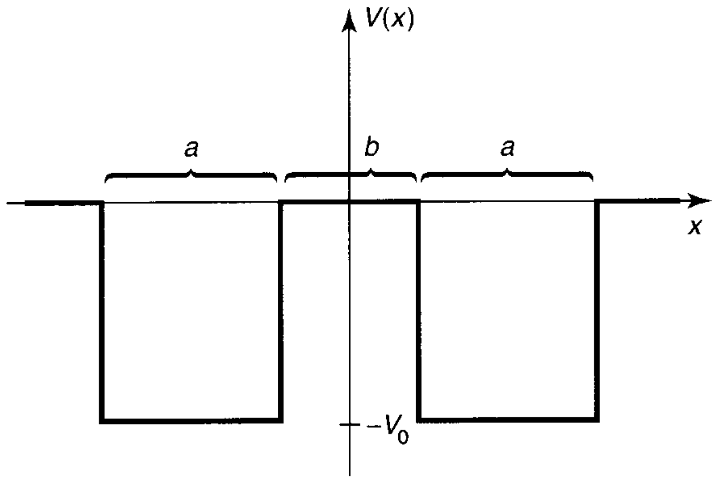

Figure 2.17: The double square well (Problem 2.44).

Introduction to Q.M.

Problem *Consider the delta function potential:*

$$V(x) = -\alpha\delta(x)$$

(a) *Demonstrate that there is exactly only one bound state given by:*

$$\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2}; \quad E = -\frac{m\alpha^2}{2\hbar^2}$$

(b) *Demonstrate that the reflection and transmission coefficients for unbound (or scattering) states are given by:*

$$R = \frac{1}{1 + (2\hbar^2 E/m\alpha^2)}, \quad T = \frac{1}{1 + (m\alpha^2/2\hbar^2 E)}.$$

Problem *Consider the finite well:*

$$V(x) = \begin{cases} 0 & \text{if } |x| > a \\ -V_0 & \text{if } |x| < a \end{cases}$$

(a) *Find the wavefunctions. Express them in terms of the integration constants if they are different from zero.*
(b) *Demonstrate that their corresponding energies are given by:*

$$k = l\tan(la); \quad k = -l\cot(la)$$

where:

$$k = \sqrt{-2mE}/\hbar; \quad l = \sqrt{2m(E+V_0)}/\hbar$$

(b) *Use your preferred mathematical program to calculate the energies and compare them with the infinite well. In the same fashion, draw the three lowest energy states.*

Electronic Structure of Atoms

Problem 2.4 Solve the time-independent Schrödinger equation with appropriate boundary conditions for an infinite square well centered at the origin [$V(x) = 0$, for $-a/2 < x < +a/2$; $V(x) = \infty$ otherwise]. Check that your allowed energies are consistent with mine (Equation 2.23), and confirm that your ψ's can be obtained from mine (Equation 2.24) by the substitution $x \to x - a/2$.

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}. \quad \textbf{(Eq. 2.23)} \qquad \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right). \quad \textbf{(Eq. 2.24)}$$

Problem 4.3 Use Equations 4.27, 4.28, and 4.32 to construct Y_0^0 and Y_2^1. Check that they are normalized and orthogonal.

$$P_l^m(x) \equiv (1-x^2)^{|m|/2}\left(\frac{d}{dx}\right)^{|m|} P_l(x) \quad \textbf{(Eq. 4.27)} \qquad P_l(x) \equiv \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2-1)^l. \quad \textbf{(Eq. 4.28)}$$

$$Y_l^m(\theta, \phi) = \epsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}\, e^{im\phi} P_l^m(\cos\theta), \quad \textbf{(Eq. 4.32)}$$

Problem 4.13

(a) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius a.

(b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen. *Hint*: This requires no new integration—note that $r^2 = x^2 + y^2 + z^2$, and exploit the symmetry of the ground state.

(c) Find $\langle x^2 \rangle$ in the state $n = 2$, $l = 1$, $m = 1$. *Hint*: This state is *not* symmetrical in x, y, z. Use $x = r \sin\theta \cos\phi$.

(d) *Sketch the shape of the probability density in this state (find zeros and maxima).*