[P2/C2/EL3001
2009/02
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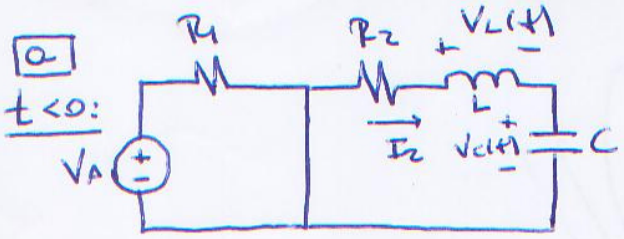
a)

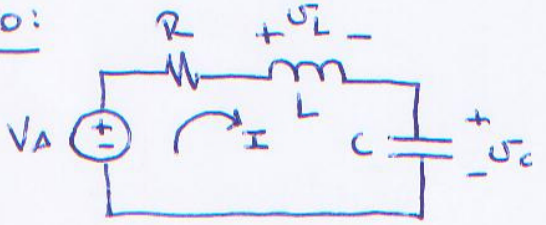
$t<0$:



t grande $\Rightarrow I_L = 0 \Rightarrow V_{R_2} = 0 \Rightarrow v_c = 0$

$$\boxed{\begin{matrix} i_L(0) = 0 \\ v_c(t) = 0 \end{matrix}} \text{ (condiciones iniciales)}$$

$t>0$:



$R = R_1 + R_2$

$I = i_L = i_C$

$$\Rightarrow i_L = C\frac{\partial v_c}{\partial t} \Rightarrow \boxed{\frac{\partial v_c}{\partial t}(0) = i_L(0) = 0}$$

LVK: $V_A = V_R + v_L + v_c$

$$V_A = RC\frac{\partial v_c}{\partial t} + LC\frac{\partial^2 v_c}{\partial t^2} + v_c \Rightarrow \boxed{\frac{\partial^2 v_c}{\partial t^2} + \frac{R}{L}\frac{\partial v_c}{\partial t} + \frac{v_c}{LC} = \frac{V_A}{LC}}$$

- $v_c(t) = v_{cp}(t) + v_{cH}(t)$
- $v_{cp}(t)$: $\frac{v_c}{LC} = \frac{V_A}{LC} \Rightarrow \boxed{v_{cp}(t) = V_A}$
- $v_{cH}(t)$: $\frac{\partial^2 v_c}{\partial t^2} + \frac{R}{L}\frac{\partial v_c}{\partial t} + \frac{v_c}{LC} = 0 \Rightarrow \boxed{s_{1,2} = \frac{-R}{2L} \pm \frac{1}{2}\sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}$

Reemplazando valores: $\boxed{s_{1,2} = -2500 \pm 500\sqrt{17}} \Rightarrow$ Respuesta sobreamortiguada.

- $\boxed{v_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + V_A}$ $(t \geq 0)$ ¿K_1, K_2?

$$\left.\begin{matrix} v_c(0) = 0 = K_1 + K_2 + V_A \\ \frac{\partial v_c}{\partial t}(0) = 0 = s_1 K_1 + s_2 K_2 \end{matrix}\right\} \Rightarrow \boxed{K_1 = \frac{s_2 V_A}{s_1 - s_2}, \quad K_2 = \frac{-s_1 V_A}{s_1 - s_2}}$$

$$\Rightarrow \boxed{v_c(t) = \left\{\frac{s_2 V_A}{s_1 - s_2} e^{s_1 t} - \frac{s_1 V_A}{s_1 - s_2} e^{s_2 t} + V_A\right\} u(t)}$$ y Reemplazando valores...

$$\Rightarrow \boxed{v_c(t) = \left\{-11{,}063\, e^{-438{,}45t} + 1{,}063\, e^{-4561{,}56t} + 10\right\} u(t)}$$ //

b) i) • $V_{out} = v_{C2}$ (1)

• $V_{in} = R_1 \cdot i_{C1} + v_{C1} + V_{out}$ (2) $i_{C1} = C_1 \frac{\partial v_{C1}}{\partial t}$

• $v_{C1} = R_2 C_2 \frac{\partial v_{C2}}{\partial t}$ (3)

(3) en (2) $\Rightarrow$ $V_{in} = R_1 C_1 R_2 C_2 \frac{\partial^2 v_{C2}}{\partial t^2} + R_2 C_2 \frac{\partial v_{C2}}{\partial t} + V_{out}$

Y usando (1) y Reordenando términos:

$$\frac{\partial^2 V_{out}}{\partial t^2} + \frac{1}{R_1 C_1} \frac{\partial V_{out}}{\partial t} + \frac{V_{out}}{R_1 C_1 R_2 C_2} = \frac{V_{in}}{R_1 C_1 R_2 C_2}$$ //

ii) de la RenC:

$$S_{1,2} = \frac{-1}{2R_1C_1} \pm \frac{1}{2}\sqrt{\underbrace{\frac{1}{R_1^2 C_1^2} - \frac{4}{R_1 C_1 R_2 C_2}}_{\Delta}}$$

Y analizando Δ, se ve que:

$R_2 C_2 > 4 R_1 C_1$ $\Rightarrow$ Respuesta sobreamortiguada

$R_2 C_2 = 4 R_1 C_1$ $\Rightarrow$ Respuesta críticamente amortiguada

$R_2 C_2 < 4 R_1 C_1$ $\Rightarrow$ Respuesta subamortiguada. //