The Bearing Capacity of Clays

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1. Introduction

The first criterion which must be satisfied in any successful foundation design is that there should be an adequate factor of safety against a complete shear failure in the underlying soil. This is obviously a necessary condition but, in general, it is not a sufficient condition. In addition, the foundations should be designed in such a way that the settlements, and particularly the differential settlements* of the structure, remain within tolerable limits.

Except for footings or piers with a breadth of only a few feet, the settlement criterion controls the allowable pressures on sands and gravels. Consequently, methods for estimating the ultimate bearing capacity of cohesionless soils have a somewhat restricted value. In contrast, the possibility of a complete shear failure in clays is a very real one, and frequently in practice it is considered necessary, for economic reasons, to work with factors of safety against ultimate failure of not more than 3. Therefore, since these factors are of a similar magnitude to those used in structural materials such as steel and reinforced concrete, it is desirable to possess methods of calculating the ultimate bearing capacity of clays with the same order of accuracy as the methods used in structural design. But in many cases the use of a low factor of safety on the failure criterion leads to very considerable settlements, and it is necessary for the designer to be aware at least of the order of the settlements. He can then adopt a suitable type of structure which can safely withstand the deformations consequent upon the movement of the foundations. Yet the modern forms of construction involving continuous beams, portal frames, reinforced concrete shells and rigid or semi-rigid frames are sensitive to differential settlements. And these structural forms are usually more economical in materials and more elegant in design than the older forms; particularly in steel and reinforced concrete bridges. Thus it is often more satisfactory to restrict the settlements by using a higher factor of safety. This will increase the cost of the foundations, but will not necessarily increase the cost of the whole structure. Moreover, so far as buildings are concerned, the interior plastering and exterior panelling are themselves sensitive to settlement. By reducing the deformations, the occurrence of unsightly cracking in these elements of the building is also prevented, thereby reducing maintenance charges and enhancing the appearance.

2. General Considerations

On opening up the excavation, the pressure at foundation level is reduced to zero from its original value p (equal to the weight per unit area of the soil and water above this level, see Fig. 1). This release of pressure causes the soil to rise by an amount ρ_r. When the structural load becomes equal to p the original state of stress existing in the ground under the foundation, prior to excavation, is restored. The settlement taking place under the foundation pressure p is ρ_p and, if the ground were perfectly elastic and no water content changes had occurred, then ρ_p would be equal to ρ_r, and, moreover, these movements could be calculated. However, the magnitude of ρ_r is controlled by many practical factors, and even approximate estimations are difficult. But as a very rough rule it may be said that ρ_p is of the same order as ρ_r (see point b in Fig. 1).

Once the foundation pressure exceeds p the ground is subjected to stresses in excess of those existing prior to excavation, and it is the settlements resulting from these excess stresses that are calculated by the present methods of settlement analysis. Similarly, the factor of safety against ultimate failure must be expressed in terms of the so-called "nett pressure"; that is, the pressure at foundation level in excess of the original overburden p.

At the end of construction the nett settlement may be considered as being made up of two parts:

(i) the "immediate" settlement, due to deformation of the soil taking place without change in water content;
(ii) the "consolidation" settlement, due to a volume reduction caused by the extrusion of some of the pore water from the soil.

Owing to the presence of the extremely small particles of which clays are composed, the rate of consolidation is very slow and, in general, the elastic settlement is considerably the greater of the two components at the end of construction. There is, nevertheless, a small decrease in water content in the clay beneath the foundation, and this will cause a corresponding small increase in strength. But for the purpose of estimating the factor of safety against shear failure, the assumption is generally made that this increase in strength is negligible. That assumption is not only conservative but it also leads to a great simplification in the calculation. For saturated clays (and most clays are saturated) behave with respect to applied stresses as if they were purely cohesive, non-frictional materials; provided that no water content change takes place under the applied stresses. That is to say, they exhibit an angle of shearing resistance Φ equal to zero.

The assumption that $\Phi = 0$ forms the basis of all normal calculations of ultimate bearing capacity in clays. Only in special cases, with prolonged loading periods or with very silty clays, is the assumption sufficiently far from the truth to justify a more elaborate analysis.

In the course of time, however, the consolidation becomes important, and leads to the characteristic feature of foundations on clays: namely the long-continued settlements increasing, although at a decreasing rate, for years or decades after construction. The principal objects of a settlement analysis are therefore to obtain (i) a reasonable estimate of the nett "final" settlement ρ_n, corresponding to a time when consolidation is virtually complete, and (ii) at least an approximate estimate of the progress of settlement with time. The settlement at the end of construction is of minor consequence in most problems. All settlement calculations are, at the present time, based on the classical consolidation theory of Terzaghi, or on extensions of this theory.

*For the relation between average and differential settlement see the important paper by Terzaghi[1]. Limitations of space restrict the present discussion to average settlements.

3. Ultimate Bearing Capacity of Clays ($\Phi = 0$)

General

In the general case, the allowable foundation pressure may be expressed in the form[3]:—

$$q_{allowable} = \frac{1}{F}\left[c.N_c + p_o.(N_q - 1) + \frac{\gamma B}{2}.N\gamma\right] + p \qquad (1)$$

where F = the desired factor of safety.
c = apparent cohesion of the soil.
p_o = effective overburden pressure at foundation level.
p = total overburden pressure at foundation level.
γ = density of soil beneath the foundation (submerged density if foundation is below water level).
B = breadth of foundation.
N_c, N_q, N_γ = factors depending upon the angle of shearing resistance Φ of the soil, the ratio of length L to breadth B of the foundation and the ratio of the depth D to the breadth of the foundation B (see Fig. 1).
q_{nf} = the term in square brackets, is the nett ultimate bearing capacity.

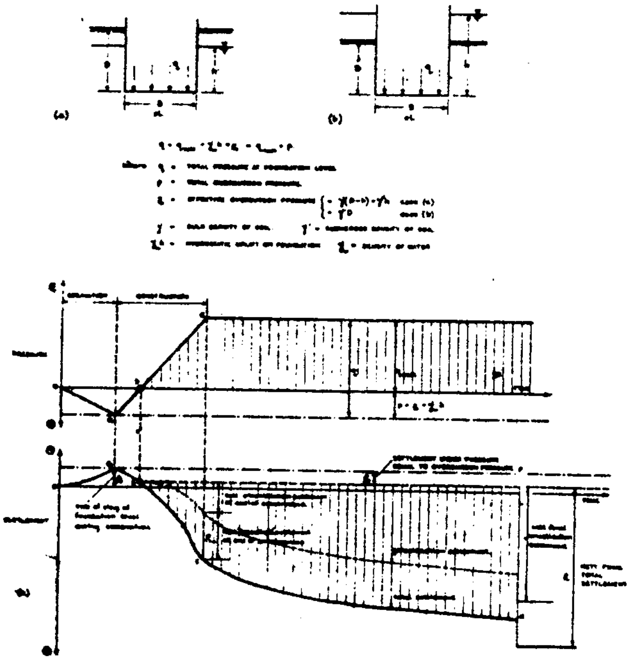


Fig. 1.—Settlement of foundations—General definitions

With the condition that $\Phi = 0$ the factors N_q and N_γ are equal to unity and zero respectively. Thus equation (1) reduces to the simple form:

$$q_{allowable} = \frac{c}{F}.N_c + p \quad \ldots \quad (2)a$$

and the ultimate bearing capacity is

$$q_f = c.N_c + p \quad \ldots \quad (2)b$$

The problem of calculating the ultimate bearing capacity of clays is therefore solved when the apparent cohesion c (usually referred to as the " shear strength ") of the clay has been determined and the factor N_c has been evaluated for the particular values of B, L and D.

Measurement of c

To determine the shear strength of the clay undisturbed samples are taken from boreholes, which should extend either to the bottom of the clay stratum or to a depth where the stresses caused by the foundation pressures are negligible. Unconfined compression tests or, preferably, undrained triaxial tests*, are then carried out on specimens cut from these cores; and if σ_1 and σ_3 are the major and minor principal stresses at failure, then

$$c = \tfrac{1}{2}(\sigma_1 - \sigma_3) \quad \ldots \quad (3)$$

In extra-sensitive clays (i.e. those very sensitive to disturbance in sampling) it is necessary to measure the shear strength directly *in situ*, by means of the vane test[3,4,5]. If only the shear strength is required then in all soft clays, including those of low or medium sensitivity, the vane test is more economical than undisturbed sampling and laboratory tests. But, in general, sampling is recommended since consolidation tests can also be carried out on the samples, and these are required for making the settlement analysis. Fortunately, disturbance is less important in its effect on the consolidation characteristics than on the shear strength of clays.

It is not possible, in this summary, to discuss in detail the procedure for estimating the value of c to be used where the strength varies appreciably with depth. It must suffice to mention that if the shear strength within a depth of approximately 2/3 B beneath foundation level does not vary by more than about ± 50 per cent. of the average strength in that depth, then this average value of c may be used in equation (2).

Value of N_c

The values suggested for the factor N_c are given in Fig. 2. As an example consider a foundation with B = 15 ft., L = 23 ft. and D = 9 ft. Then the value of N_c for a square footing with $D/B = 9/15 = 0.60$ is 7.2, from the upper curve in Fig. 2. Thus the required N_c for the actual rectangular footing with $B/L = 15/23 = 0.65$ is given by the expression

$$N_c = (0.84 + 0.16 \times 0.65)\, 7.2 = 6.8$$

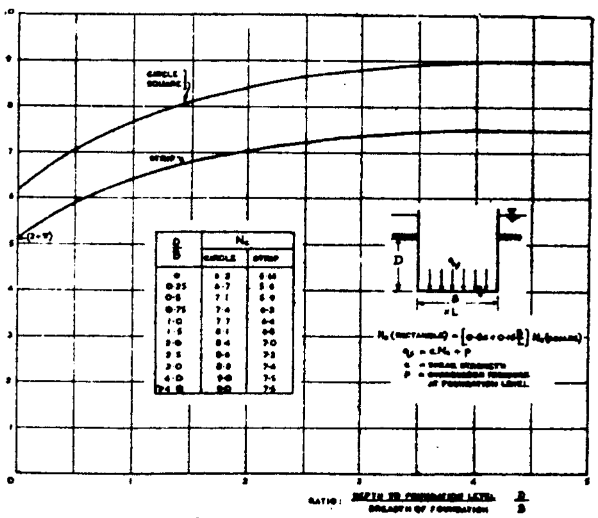


Fig. 2.—Bearing capacity factors for foundations in clay ($\Phi = 0$)

Since an important part of the research work leading to these values of N_c is of recent origin, a discussion of their derivation will be given in the following section of the paper. Before doing so, however, it will be convenient to consider the available field evidence on the ultimate bearing capacity of clays. This is assembled in Table 1, and it will at once be seen that the evidence, although limited to six cases and although

*For a recent account of triaxial testing methods, and interpretation, see a paper by Skempton and Bishop[6].

TABLE 1.—Field data on ultimate bearing capacity of clays

Location and structure	Dimensions of foundation: B ft.	Dimensions of foundation: L ft.	Dimensions of foundation: D ft.	Approx. average settlement at failure. ρ_f inches	$\frac{\rho_f}{B}$ per cent.	Nett foundation pressure at failure. q_{nf} ton/ft.²	Average shear strength of clay beneath foundation. c. ton/ft.²	Value of N_c: Actual $\frac{q_{nf}}{c}$	Value of N_c: From Fig. 2	Reference for original data
Hagalund loading tests	1.3	6.5	0 (lower limit) 1 (upper limit)	½	3	0.43	0.074 (vane) 0.067 (compr.)	5.8 6.4	5.4 6.5	Odenstad (1948)[20] Cadling and Odenstad (1950)[4]
Kippen spread footing	8	9	5.5	10	10	0.95 (with side friction) 1.15 (no side friction)	0.16	6.0 7.2	7.2	Skempton (1942)[22]
Loch Ryan screw cylinder	8	8	50	11	12[a]	1.9	0.22	8.6	9.0	Morgan (1944)[23] Skempton (1950)[24]
Newport screw cylinder	8	8	6 (in clay) 20 (total depth)	14	15[a]	2.9	0.36	8.0	7.4 8.6	Wilson (1950)[18]
Shellhaven oil tank A	25	25	0	—	—	0.84	0.135[b]	6.2	6.2	Nixon (1949)[21]
oil tank B	52	52	0	30	5	0.83	0.140[b]	5.9	6.2	Nixon (personal comm.)
Tunis warehouse	50	125	10	40	7[c]	—	—	—		Fountain (1907)[25]
Transcona grain elevator	76	195	12	140	15[c]	2.2	—	—		Allaire (1916)[26]

a : extrapolation from load-settlement curve. *b* : in a depth $z_1 = 2/3B$. *c* : failure by tilting.

subject to the usual lack of precision inherent in any field observations, provides a satisfactory confirmation of the suggested values of N_c. Further, indirect confirmation will be considered in the section on load-settlement curves.

In most cases it is possible to use Fig. 2 directly in the estimation of bearing capacity. But for some purposes it is desirable to have a set of simple rules which can easily be remembered. The following rules may be put forward :

(i) At the surface, where $D = 0$,
$N_{co} = 5$ for strip footings ;
$N_{co} = 6$ for square or circular footings.

(ii) At depths where $D/B < 2\frac{1}{2}$:
$N_{cD} = (1 + 0.2\ D/B)\ N_{co}$.

(iii) At depths where $D/B > 2\frac{1}{2}$:
$N_{cD} = 1.5\ N_{co}$.

(iv) At any depth the bearing capacity of a rectangular footing is

$$N_c \text{ (rectangle)} = \left[1 + 0.2\ B/L\right] N_c \text{ (strip)}.$$

$N_c = 5(1+.2 D/B)(1+.2 B/L)$ shallow
$= 7.5(1+.2 B/L)$ deep

4. Derivation of the Bearing Capacity Factors N_c

Theoretical Results

The analysis of the bearing capacity of strip footings on the surface ($D = 0$) is due to Prandtl[7] who showed that $N_c = 2 + \pi = 5.14$. The mechanism of failure assumed in this analysis is that the footing pushes in front of itself a "dead" wedge of clay which, in its turn, pushes the adjacent material sideways and upwards. Model tests in the laboratory indicate that this mechanism is a reasonable approximation.

When the footing is placed at a considerable depth the slip surfaces no longer rise up to ground level. Meyerhof[8] has evolved a modified form of Prandtl's analysis in which the slip surfaces curve back on to the sides of the foundation. For strip footings the corresponding value of N_c is 8.3 ; but this is, clearly, an upper limit since it involves too great a length of shear surface. It may be noted that the values of N_c for strip footings are independent of the amount of shear mobilised along the base of the footing.

For circular footings with a smooth base, on the surface of a clay, a rigorous solution has been obtained by Ishlinsky[9], N_c being 5.68. The more practical condition of a rough-based circular footing on the surface of a clay stratum has been solved by Meyerhof, using an approximate* analysis. This leads to the result $N_c = 6.2$.

For circular footings located at a considerable depth beneath the surface three solutions are available. With assumptions concerning slip surfaces similar to these mentioned above, Meyerhof finds that $N_c = 9.3$. For the reason given earlier $N_c = 9.3$ is almost certainly an upper limit. A completely different approach is that originated by Bishop, Hill and Mott[10], for metals, and extended to clays by Gibson[11] using the large-strain theory of Swainger[12]. In this analysis it is assumed that the penetration of the footing, at ultimate failure, is equivalent to expanding a spherical hole in the clay, of diameter equal to the diameter of the footing. If E is the Young's modulus of the clay, then a plastic zone is developed, of radius $\frac{B}{2}\sqrt[3]{\frac{E}{c}}$ beyond which the clay is still in the elastic state. The expression for N_c, according to Gibson, is

$$N_c = \frac{4}{3}\left[\log_e \frac{E}{c} + 1\right] + 1 \qquad \ldots \quad \ldots \quad (4)$$

For materials with stress-strain curves of the type exhibited by clays it is convenient to define E as the secant modulus at a stress equal to one-half the yield value (see Fig. 3). With this convention the range of E/c for the great majority of undisturbed clays

*An indication of the error involved in this analysis is given by the fact that the same form of solution leads to the ... $N_c = 5.71$ for the smooth base circle, as compared with Ishlinsky's $N_c = 5.68$.

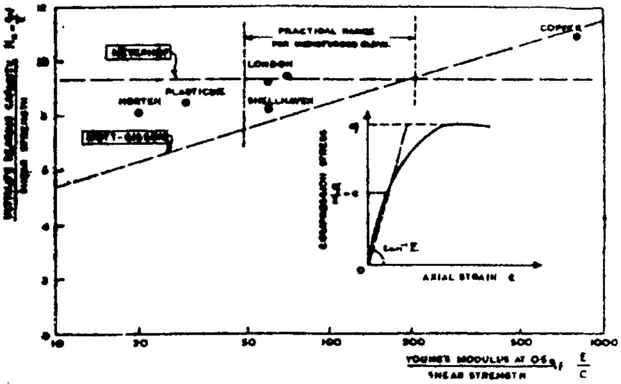


Fig. 3.—Ultimate bearing capacity factors for deeply buried circular footings in $\Phi = 0$ materials Theoretical values and laboratory tests

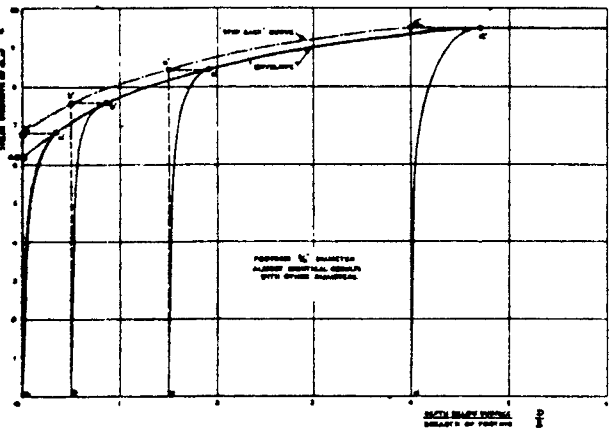


Fig. 4.—Laboratory test results for model footings in remoulded London clay

is from 50 to 200. The corresponding values of N_c in equation (4) are 7.6 and 9.4. Thus, even with this four-fold variation in E/c the change in N_c is only $\pm$ 10 per cent., and it is therefore sufficiently accurate to say that the Mott-Gibson theory leads to the result $N_c = 8.5$ for undisturbed clays.

Finally, Guthlac Wilson[13] has approached the problem of the bearing capacity of a clay loaded at depth by a rigid circular plate, by finding the foundation pressure necessary to bring about the merging of the two plastic zones originating from the edges of the footing. The result depends to a slight degree on the depth of the footing and on the original state of stress in the clay, as indicated by the coefficient of earth pressure at rest K_o, but for practical purposes N_c may be taken as 8.0, when D is greater than $4B$.

Each of these three approaches to the problem is by no means an exact analysis. And, indeed, the difficulties in the way of producing a rigorous solution for the bearing capacity factor for deep foundations are great. Yet it is remarkable that all three theories lead to values for N_c within the $\pm$ 10 per cent. range covered by the Mott-Gibson analysis for clays.

Experimental Results

The first published results obtained from model footing tests on clay, the shear strength of which was also measured, appear to be those of Golder[14]. These were carried out on footings 3 inches square and 3 inches × 18 inches long, on the surface of remoulded London clay. The tests were of a preliminary nature, but they showed that N_c was about 6.7 for the square footings and 5.2 for the long footings.

More recently, model tests have been carried out at Imperial College by Meigh[15] and Yassin[16] on both remoulded and undisturbed clays. Careful corrections were made for the effects of small decreases of water content in the clay beneath the footings, due to the diffusion of the high pore pressures set up by the load, and for the effects of different rates of strain in the loading tests and the unconfined compression tests. It was found that, if the load-settlement curves were plotted in the dimensionless form shown in Fig. 4, then these curves were almost identical for all sizes of footings used in the experiments and for all values of the shear strength of the clay under investigation. Secondly, it was found that after penetrating about four or five diameters the footings continued to settle under a constant nett pressure. The ratio of this pressure to the shear strength of the clay is clearly the value of N_c for circular footings at a considerable depth beneath the surface, and the experimental results are plotted in Fig. 3. Of the clays, "Horten" and "London"* were remoulded and "Shellhaven" was undisturbed. The value for plasticine was obtained by Meyerhof (private communication) and that for copper was determined by Bishop, Hill and Mott[10]. For simplicity, only the Mott-Gibson theory and Meyerhof's upper limit of $N_c = 9.3$ have been shown in Fig. 3. The six experimental points all lie in the zone bounded by these two theories and, for the practical range of E/c for undisturbed clays (50 to 200), it will be seen that, as previously suggested (Skempton 1950), a value of $N_c = 9.0$ is a very reasonable average from the theoretical and experimental results.

Similarly, for deep buried strip footings, $N_c = 7.5$ is a reasonable average value.

A typical relation between q/c and $\dfrac{\text{penetration}}{B}$ for a footing pushed into the clay from the surface is shown by the line O a¹ b¹ c¹ d¹ in Fig. 4. This line is also the envelope of all loading tests for footings initially

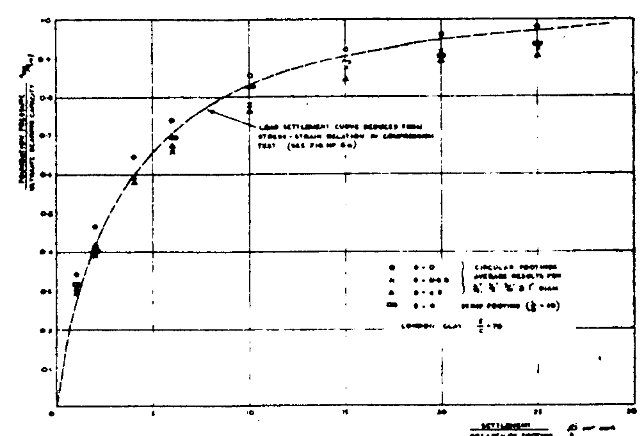


Fig. 5.—Load settlement curves for model footings in remoulded London clay

buried at any depth D; the load-settlement curves for such footings being b b¹ c¹ d¹ e, c c¹ d¹ e and d d¹ e. It is evident that, for the test starting at $D = \frac{1}{2}B$, the shear strength of the clay is progressively mobilised as the pressure is raised from zero until, at the point b¹

*Tests on model screw-cylinders, with blade diameters of two, four and six inches, by Wilson[13] also show an average value of N_c of about 9.5 for remoulded London clay. But this result is probably a little too high, since no corrections were made for pore pressure diffusion from the clay immediately under the blades. The actual strength of the clay was therefore somewhat greater than that measured by compression tests on samples taken from the bulk of clay in the test container.

on the envelope, the strength is fully mobilised. Similarly, for the test starting at $D = 1.5\ B$ the shear strength of the clay is fully mobilised at point c^1. Moreover, it will be seen that the "envelope" may be extrapolated to the axis of zero penetration at a value of $q/c = 6.2$. This is Meyerhof's value of N_c for a circular footing on the surface, and in his theory, as in that of Prandtl for a strip footing, it is tacitly assumed that failure occurs at deformations negligibly small compared with the breadth of the footing. The experimental results in Fig. 5 therefore confirm* the theoretical surface values of 6.2, and so also do the tests on strip footings; the envelope in these experiments extrapolating back to $q/c = 5.2$.

Nevertheless, since the penetrations required to mobilise full shear in the clay are, in the laboratory tests, equal to about $0.4\ B$, it is logical to take the values of q/c at the points a^1 b^1 and c^1 as the values of N_c for the appropriate foundation depths $D = 0$, $0.5\ B$ and $1.5\ B$. In this way the relation between N_c and D/B shown by the "step-back" curve in Fig. 4 is obtained. Thus, for a surface circular footing on remoulded London clay ultimate failure occurs (i.e. the full shear strength of the clay is mobilised), when $q/c = N_c = 6.8$; and similarly for any other value of D.

But, as will be seen from Table 1, ultimate failure takes place in some undisturbed clays at a penetration of only $0.1\ B$ or even less. Therefore, although the "step-back" curve in Fig. 4 is undoubtedly the logical interpretation of the particular test results expressed in that graph, yet in practice it may be an error not on the side of safety to assume that such high values of N_c can be used. Clearly, the most conservative assumption is to use the "envelope" itself, since this implies that full shear strength is mobilised after negligible penetration of the footing.

It may, of course, well be true that with more brittle clays the envelope is itself higher than that obtained for the remoulded London clay. But the tests on undisturbed Shellhaven clay did not indicate any substantial difference. Consequently the most reasonable procedure, for the present at least, until more evidence is forthcoming, is to take the average envelope from the available test data and assume that this gives the required relation between N_c and depth of the footing. This average envelope for circular footings is, in fact, that shown by the upper curve in Fig. 2. It may be noted that laboratory tests[16] (Meigh 1950) showed no significant difference between square and circular footings.

The information on strip footings is less complete, the tests so far carried out being limited to London clay. But, since the ratio of N_c for the strip to that for the circle is 0.84 both at depth and at the surface, it is unlikely that any appreciable error will be involved in the assumption that this ratio applies for all values of D/B. The ordinates of the "strip" curve in Fig. 2 are therefore simply $0.84 \times N_c$ (square).

It is further assumed that the value of N_c for a rectangular footing may be obtained by linear interpolation according to the formula:

*Cone tests approximate to the conditions implied in Meyerhof's theory but difficulties are present in carrying out cone-penetration tests with high accuracy. The shear mobilised along the surface of the cone, the high rate of strain in the early stages of the test, the dissipation of pore pressure and the depression or elevation of the clay surface during penetration, all influence the results. The most that can be said at present is that the values of N_c deduced from cone tests (in which an attempt has been made to apply these corrections) lie in the range 5.0 to 7.0 for most clays.

$$N_c\ (\text{rectangle}) = \left[0.84 + 0.16\frac{B}{L}\right] N_c\ (\text{square}) \quad (5)$$

Summary

Clearly there is scope for developing a more satisfactory theory for the bearing capacity of deep footings in clay, but the semi-empirical values of 9.0 and 7.5 for circular and strip footings are probably sufficiently accurate for practical purposes. Also the interpolation formula, equation (5), requires experimental and theoretical investigation. More important, the values of N_c given in Fig. 2 are probably somewhat conservative, and future work may lead to improvement in this respect. Nevertheless, the comparison of the bearing capacity factors as given in Fig. 2, with the available field data, in Table 1, is decidedly encouraging.

5. Load-Settlement Curves

In Fig. 5 some of the observed points on the individual load-settlement curves aa^1, bb^1 and dd^1 (shown in Fig. 4) are plotted with a common origin; the ordinates being expressed as the ratio of the pressure q to the ultimate bearing capacity q_f, as represented by points a^1, b^1 etc. The results of a typical test on a strip footing ($B/L = 0.1$) are also plotted in the same manner. As a rough approximation, all the points lie on the same curve, and it is interesting to examine the measure of agreement between these experimental points and the load-settlement curve as predicted from simple theoretical considerations.

Now, from the theory of elasticity it is known that the mean settlement of a foundation, of breadth B, on the surface of a semi-infinite solid is given by the expression

$$\rho = q.B.I_\rho . \frac{1-\mu^2}{E} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where q = foundation pressure.
I_ρ = influence value depending upon the shape and rigidity of the foundation.
μ = Poisson's ratio of the solid.
E = Young's modulus of the solid.

For the present purpose equation (5) is more conveniently written in the form

$$\frac{\rho}{B} = \frac{q}{q_f} . \frac{q_f}{c} . I_\rho \frac{1-\mu^2}{E/c} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

In saturated clays with no water content change under applied stress (the $\Phi = 0$ condition) Poisson's ratio is equal to $\frac{1}{2}$, and for a rigid circular footing on the surface $I_\rho = \pi/4$. Moreover, from the experiments previously described, $q_f/c = 6.8$. Thus for the model tests with circular footings on the surface

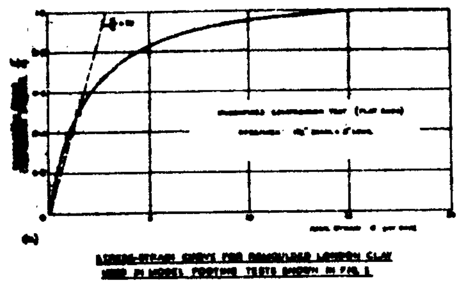
$$\frac{\rho_1}{B} = \frac{4}{E/c} . \frac{q}{q_f} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

With footings buried at some depth below the surface the influence value I_ρ decreases (Fox[17]), but the bearing capacity factor $N_c = q_{d1}/c$ increases as shown in Fig. 2, and to a first approximation the product $I_\rho . N_c$ remains constant. Therefore equation (7) holds good for all the circular footing tests.

Further, in an undrained compression test the axial strain under a deviator stress $(\sigma_1 - \sigma_3)$ is given by the expression

$$\varepsilon = \frac{(\sigma_1 - \sigma_3)}{E} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where E is the secant Young's modulus at the stress $(\sigma_1 - \sigma_3)$.



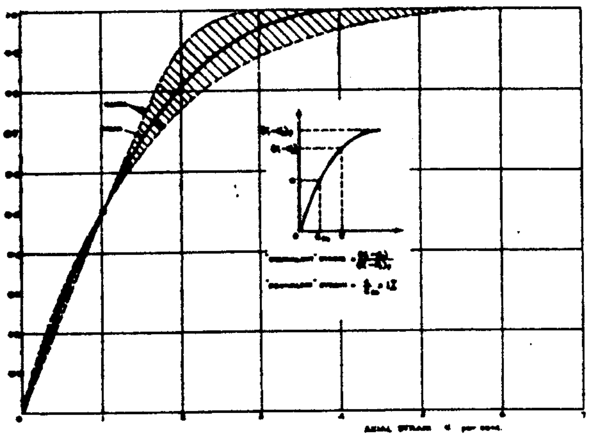


Fig. 6.—"Equivalent" stress strain curves for undisturbed clays (E/c = 100)

As before, equation (8) is more conveniently written in the form

$$\epsilon = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f} \cdot \frac{(\sigma_1 - \sigma_3)_f}{c} \cdot \frac{1}{E/c} \quad \ldots \quad \ldots \quad (9)$$

In saturated clays with no water content change under applied stress $\frac{(\sigma_1 - \sigma_3)_f}{c} = 2.0$. Thus

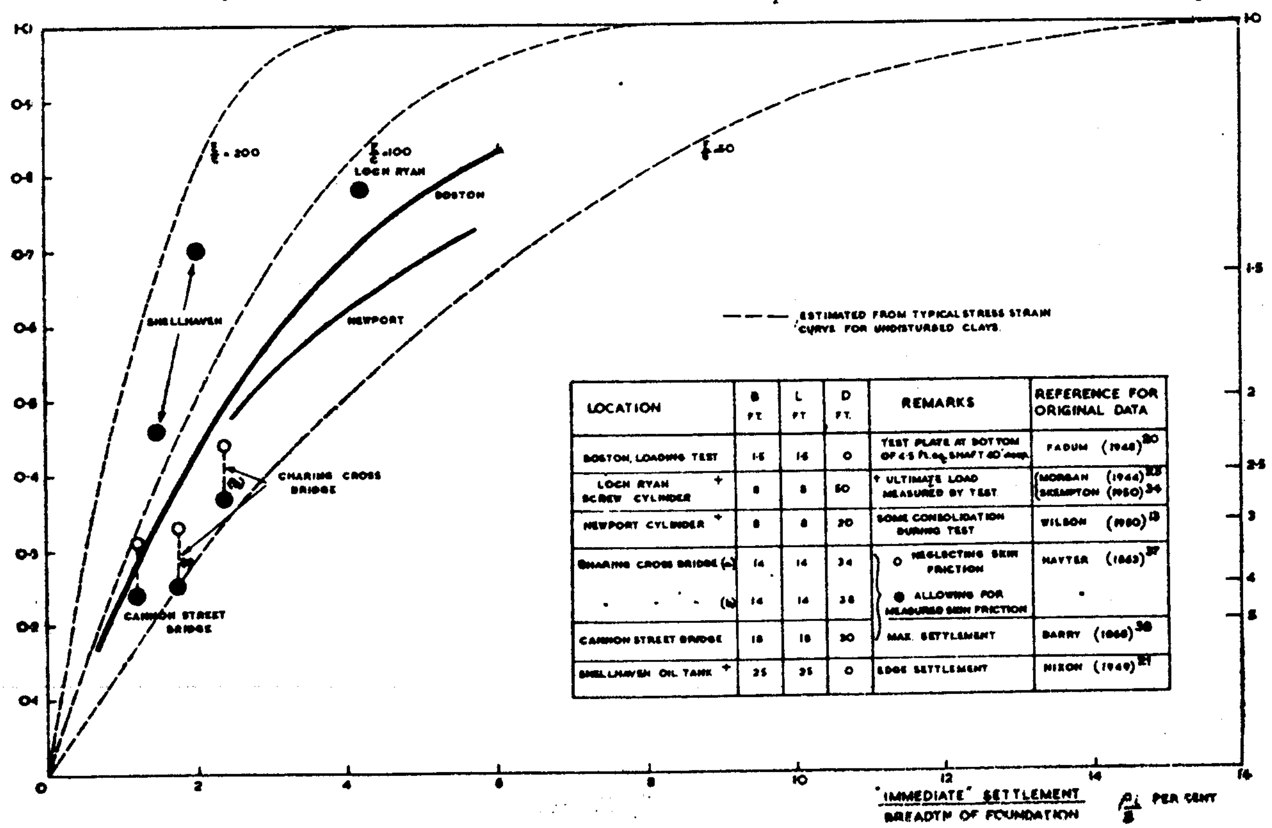
$$\epsilon = \frac{2}{E/c} \cdot \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

From a comparison of equations (7) and (10) it will therefore be seen that, for the same ratio of applied stress to ultimate stress, the strain in the loading tests is related to that in the compression test by the equation

$$\frac{\rho_1}{B} = 2 . \epsilon \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

The average stress-strain curve for all the compression tests carried out on the remoulded London clay used in the model loading tests is shown in Fig. 6 (a). From this curve the values of ρ_1/B can immediately be calculated from equation (11); and the result is shown by the dotted line in Fig. 5. The agreement with the experimental points is moderately good except for high values of q/q_f. But the simple theory leading to equation (11) cannot be expected to yield accurate results in this range, since at loads near the ultimate bearing capacity a considerable zone of the clay beneath the footing is subjected to strains greater than those at the ultimate stress in the compression test.

The container in which the circular footings were tested had a depth of at least $8B$. Theoretically[8] the settlements should therefore be about 7 per cent. less than the values calculated from equation (11). This is of no consequence, in view of the very approximate nature of the derivation of the strain relationship. However, the container in which the strip footings were tested had a depth of about $6B$. This is adequate for investigating ultimate failure; but the settlements would be 30 per cent. less than the values calculated from the theory of semi-infinite elastic solids, and the corresponding value of $I_\rho . N_c$ is only about 20 per cent. greater than that used in equation (7), whereas, on the assumption of a semi-infinite solid, the product



LOCATION	B FT.	L FT.	D FT.	REMARKS	REFERENCE FOR ORIGINAL DATA
BOSTON, LOADING TEST	1·5	1·5	0	TEST PLATE AT BOTTOM OF 4·5 ft. sq. SHAFT 40' deep	FADUM (1948)[20]
LOCH RYAN SCREW CYLINDER +	8	8	50	+ ULTIMATE LOAD MEASURED BY TEST	MORGAN (1944)[23] SKEMPTON (1950)[34]
NEWPORT CYLINDER +	8	8	20	SOME CONSOLIDATION DURING TEST	WILSON (1950)[13]
CHARING CROSS BRIDGE (a)	14	14	34	O NEGLECTING SKIN FRICTION	HAYTER (1863)[37]
" " " (b)	14	14	38	● ALLOWING FOR MEASURED SKIN FRICTION	"
CANNON STREET BRIDGE	18	18	30	MAX. SETTLEMENT	BARRY (1866)[38]
SHELLHAVEN OIL TANK +	25	25	0	EDGE SETTLEMENT	NIXON (1949)[21]

Fig. 7.—"Immediate" settlements in field loading tests on saturated clays (Φ = 0)

$I_\rho . N_c$ is about 65 per cent. greater for the 10 : 1 strip than for the circle. Hence the observed fact of roughly equal settlements, at the same factors of safety, for the two types of footing, which might at first glance seem to be anomalous, is accounted for within the limits of accuracy of the few tests carried out on strips.

In applying the foregoing conceptions to full-scale foundations it is necessary to take into account the probability that the great majority of the settlement is due to strains in the clay within a depth of not more than about $4B$ below the base of the footing. At greater depths the shear stresses are less than about 5 per cent. of the nett foundation pressure, and the corresponding value of E/c is typically 50 to 80 per cent. greater than that calculated at $\sigma = \frac{1}{2}\sigma_f$. Moreover, the strength of the clay usually increases appreciably with depth. Thus the strains at relatively great depths are one-half, or even less, of those according to simple elastic theory, with a shear strength, in equation (6), equal to that within a depth of 2/3 B beneath the footing.

From the values of I_ρ given by Terzaghi[2] and Timoshenko[18] the following results are obtained for the mean settlement of uniformly loaded areas, if strains below $4B$ are neglected.

TABLE 2

L/B	I_ρ	N_c	$\frac{3}{4} N_c . I_\rho$	$\frac{\rho_1}{B . \varepsilon}$
circle	0.73	6.2	3.4	1.7
1 : 1	0.82	6.2	3.8	1.9
2 : 1	1.00	5.7	4.3	2.1
5 : 1	1.22	5.4	4.9	2.4
10 : 1	1.26	5.3	5.0	2.5

Thus, to a degree of approximation ($\pm$ 20 per cent.) comparable with the accuracy of the assumptions, it may be taken that equation (11) applies to a circular or any rectangular footing.

In order to investigate this relationship in practice, it is necessary to know the shape of the stress-strain curves for undisturbed clays, and to compare the calculated settlements with field observations. For this purpose the stress-strain curves of a number of clays were plotted in the form shown in Fig. 6 (b) and, apart from a few exceptional cases, all the "equivalent" stress-strain curves were found to lie within the shaded zone shown in this graph. The load-settlement curve calculated from equation (11) and from the average equivalent stress-strain curve indicated by the solid line in Fig. 6 (b), is plotted in Fig. 7. This load-settlement curve is therefore a crude estimate of the theoretical curve for undisturbed clays with $E/c = 100$. The settlements at any given factor of safety ($= q_{nf}/q_n$) will be inversely proportional to E/c, and the curves for $E/c = 50$ and 200 are also shown in Fig. 7.

The author is aware of loading tests at six sites for which sufficient data are available to enable the results to be plotted in Fig. 7. Three of these tests were taken to failure, and q_n/q_{nf} is therefore known directly. In the other three cases q_{nf} has been calculated from Fig. 2 and the shear strength of the clay. The most valuable tests were those carried out by Sir John Hawkshaw on the piers of his bridges over the Thames at Charing Cross and Cannon Street. The former is only a few hundred yards away from Waterloo Bridge, where extensive investigations were recently made on the London clay[19]. Each of the cylinders forming the piers of Charing Cross Bridge were loaded with 450 tons or 700 tons, before building the deck, and the settlements were observed. In addition, the skin friction was measured during the sinking of the cylinders. Similarly at Cannon Street Bridge the cylinders were test-loaded with 850 tons. For undisturbed London clay $E/c = 50$ and the stress-strain curve is closely similar to that shown by the full line in Fig. 6 (b). It is therefore interesting to note the reasonable degree of comparison between the field observations and the approximate theoretical load-settlement curve in these cases. The clay at Boston[22] had an E/c of about 40 or 50, whereas the loading test indicates a value of the order 80. This discrepancy may be due partly to the fact that the test was carried out at the bottom of a 40 ft. shaft, and the clay had therefore been considerably "pre-stressed": the test being, in effect, a re-loading of the clay. At Shellhaven[23], it is difficult to make any direct comparison, since the oil tank rested on a 5 ft. crust of hard clay overlying soft clay. The crust had little effect on the ultimate failure of the tank, but it would appreciably reduce the settlements. Moreover, the soft clay is extra-sensitive and the laboratory value of $E/c = 80$ may well be considerably too low on account of sampling disturbance[21]. For the tests on the screw cylinder at Newport[11] the results agree reasonably well with the actual E/c for the clay, which was about 60. No value of E/c is available for the clay beneath the cylinder tested by Morgan[22] but the load-settlement result indicates about 90 and this is of the order often measured in normally consolidated silty clays.

Summarising this field evidence, it may therefore be said that none of the data is seriously at variance with the approximate theory expressed by equation (11) and Table 2, while the tests on the Thames bridges appear to confirm this theory and also, by implication, the bearing capacity factors given in Fig. 2 for circular foundations at a depth of about $1\frac{1}{2}B$ to $2\frac{1}{2}B$.

6. Factor of Safety

As a minimum requirement for the stability criterion it is usual to specify a factor of safety of not less than 2. But, for general purposes, experience has indicated that it is desirable to use a factor of safety of 3 (Terzaghi and Peck[24]). Thus, quite distinct from any settlement criteria, the allowable nett pressure should not exceed one-third of the nett pressure causing ultimate failure. Yet with a factor of safety of 3, although there can be no possibility of complete failure, or even of any appreciable over-stressing* in the clay, the settlements may be excessive. Consequently, it is necessary to give at least a brief consideration to the settlement problem if the subject of bearing capacity is to be seen in proper perspective.

7. Final Settlement

Where the clay exists as a relatively thin layer beneath the foundation, or where the foundation rests on sand or gravel underlain by clay, the "immediate" settlements are small, owing to the lateral restraint imposed on the clay by the adjacent rigid or comparatively rigid materials. In such cases the final settlement, and also the rate of settlement, can be calculated with sufficient accuracy from Terzaghi's theory of one-dimensional consolidation. The procedure for calculating settlements by this theory can be found in the standard text-books, such as Terzaghi and Peck[24], and need not be considered further in this paper.

*If the nett foundation pressure is one-third of that causing ultimate failure, the maximum shear stress in the clay does not exceed about 65 per cent. of the shear strength. Thus, a factor of safety of 3 on ultimate failure corresponds to a factor of safety of at least $1\frac{1}{2}$ on over-stressing (neglecting isolated stress concentrations.)

Where the foundation rests directly on a relatively thick bed of clay the problem is more complicated. As a first approximation, however, the nett final settlement (*including both* "immediate" and "consolidation" settlement) may be calculated from the equation

$$\rho_n = \int_0^{z_1} m_v \, . \, \sigma_z \, . \, dz \quad \ldots \quad (12)$$

$$= m_v \, . \, q_n . B \, . \, I_\rho \quad \ldots \quad (13)$$

where m_v is the compressibility of the clay at a depth z beneath the foundation as measured in oedometer tests on undisturbed samples; the compressibility being determined over the range of pressure from p_o, the original effective overburden pressure at depth z, to $(p_o + \sigma_z)$ where σ_z is the increment of vertical pressure set up at this depth by the nett foundation pressure. Also, in these equations z_1 is the maximum depth of the clay beneath the foundation or, if the clay is very thick, z_1 is some depth such as $4B$ beneath which the settlements are negligible, and I_ρ is the influence value for settlements in a depth z_1.

If the clay structure was elastic then this conventional method would underestimate the final settlement, since it implies the assumption that Poisson's ratio μ_s is zero. But the compressibility C_c of the clay structure is greater than the expansibility C_s (both expressed in terms of effective stress) and if this fact is taken into account[35] it is found that the conventional method leads to final settlements, which may be either lower or higher than those calculated from more comprehensive theory; but not differing by more than ± 40 per cent., as shown* in Table 3. The "theoretical" final settlements have, so far, only been evaluated for the centre of a uniformly loaded circular footing, and the determination of C_c, C_s and μ_s for the clay structure is experimentally a difficult matter. The purpose of the theory is therefore not to provide a method of settlement analysis, but merely to enable the order of error in the conventional analysis to be examined.

Since, in practice, structural design often does not justify an attempt to predict settlements with an accuracy greater than that implied by the results in Table 3, it may be concluded that the conventional method (equations 12 and 13) is adequate for estimating the final settlement of foundations on deep beds of clay. Field observations justify this conclusion[27, 28, 29].

In order to obtain a relationship between final settlement (from the conventional method) and factor of safety against ultimate failure, equation (13) may be written in the form

$$\frac{\rho_n}{B} = \frac{q_n}{q_{nf}} \, . \, \frac{q_{nf}}{c} \, m_v \, . c . I_\rho \quad \ldots \quad (14)$$

or, if $K_v = 1/m_v$, where K_v = modulus of compressibility as measured in oedometer,

$$\frac{\rho_n}{B} = \frac{q_n}{q_{nf}} \, . \, \frac{q_{nf}}{c} \, . \, \frac{I_\rho}{K_v/c} \quad \ldots \quad (15)$$

and equation (15) is analogous to the corresponding equation (6) for "immediate" settlements; except that equation (15) cannot be expected to hold good for values of q_n/q_{nf} of more than about 0.5, since at greater values of this ratio the clay will be overstressed.

Values of I_ρ can be found from data given by Terzaghi[9] and Timoshenko[18], and values of q_{nf}/c are given in Fig. 2. From these values it can be shown that the order of the average nett final settlement is given by the expression

$$\frac{\rho_n}{B} = \frac{5}{K_v/c} \, . \, \frac{q_n}{q_{nf}} \quad \ldots \quad (16)$$

Equation (16) enables a study to be made of the relationship between the factor of safety against ultimate failure and the average nett final settlement of a foundation on a deep bed of clay. In evaluating equation (16) it is, however, essential to know the value of the ratio K_v/c. A preliminary examination of the published data indicates that for over-consolidated clays K_v/c lies approximately in the range from 70 to 200, while for normally-consolidated clays the range is approximately from 25 to 80. In each class K_v/c tends to be higher for clays with a lower liquid limit. These values must be taken as being only indicative, but they enable certain interesting deductions to be made. In order to clarify the basis of these deductions, equation (16) has been plotted in Fig. 8 for several typical values of K_v/c. Also on this graph points have been plotted representing the results of field observations on ten structures.

The first inference from Fig. 8 is that the field observations in the six cases where K_v/c is known, agree roughly with equation (16.) The second inference is that, for any given clay, the settlement is approximately proportional to the width B, at the same factor of safety. This result was first predicted by Terzaghi[30], and there is considerable supporting evidence from loading tests. But the observations summarised in Fig. 8 show that it holds good also for the final settlement of large foundations. It therefore follows that, conversely, the allowable nett foundation pressure on any given clay will decrease in direct proportion to the foundation width, if it is required to restrict the settlement to some specified magnitude.†

The factors of safety corresponding to various settlements for several typical values of K_v/c are given

TABLE 3

$\frac{C_s}{C_c} = \lambda$	Conventional final settlement / theoretical final settlement		
	$\mu_s = 0.3$	$\mu_s = 0.35$	$\mu_s = 0.4$
0.1	1.4	1.4	1.3
0.25	1.2	1.1	0.9
0.5	1.0	1.9	0.7
1.0 (elastic)	0.8	0.7	0.6

TABLE 4

Nett settlement ρ_n inches	Width B ft.	Factor of safety			
		$\frac{K_v}{c} = 200$	$\frac{K_v}{c} = 100$	$\frac{K_v}{c} = 50$	$\frac{K_v}{c} = 25$
1	5	(3)	3	6	12
	10	3	6	12	24
	20	6	12	24	48
	40	12	24	48	96
3	5	(3)	(3)	(3)	4
	10	(3)	(3)	4	8
	20	(3)	4	8	16
	40	4	8	16	32
6	5	(3)	(3)	(3)	(3)
	10	(3)	(3)	(3)	4
	20	(3)	(3)	4	8
	40	(3)	4	8	16

*The few tests at present available show that the compressibility ratio λ lies in the range 0.1 to 0.5 (Skempton[34]).

†On this point see an excellent general treatment by Taylor[31].

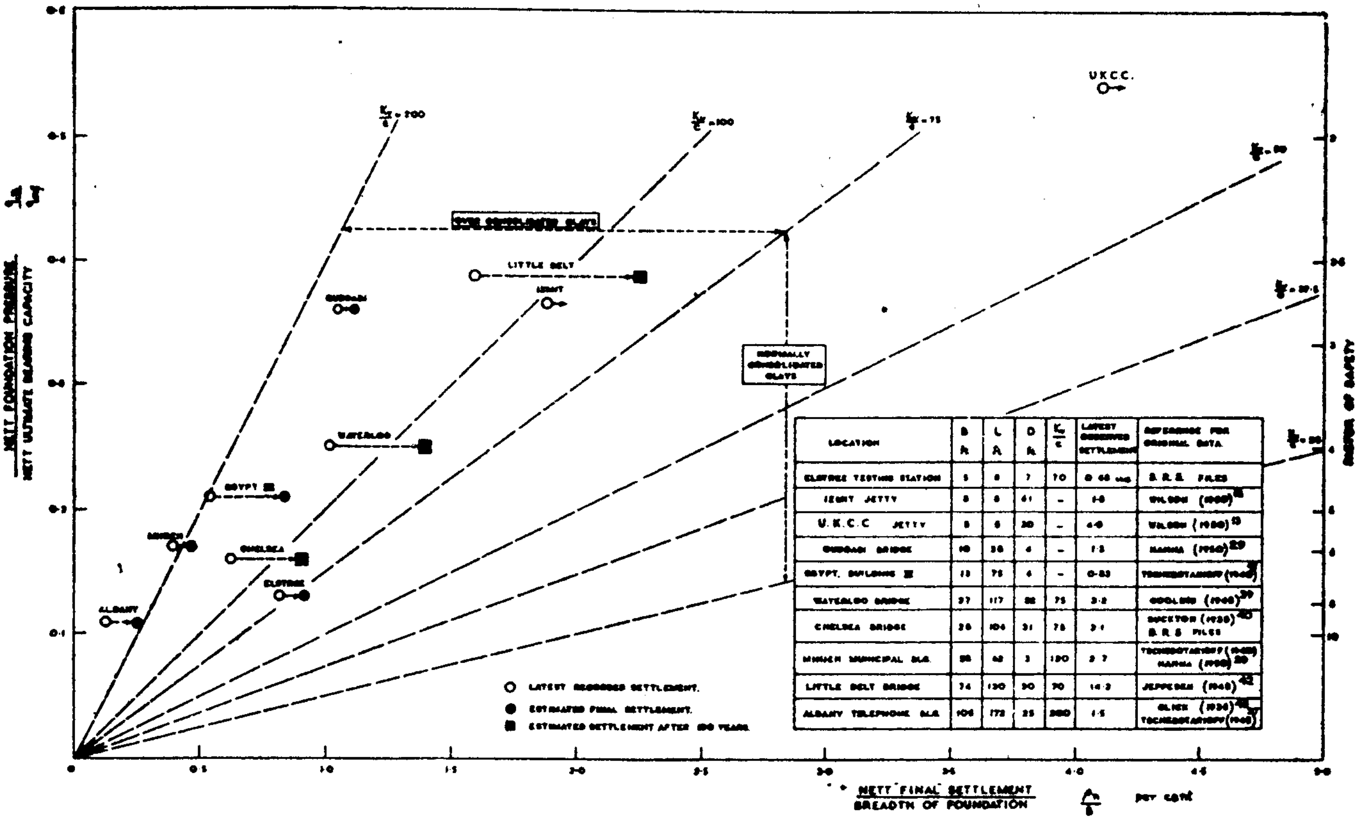


Fig. 8.—" Final " settlements of foundations in saturated clays

in Table 4. Where the factor of safety as given by equation (16) is less than 3, the stability criterion controls the design. These cases are distinguished in Table 4 by the number 3 in brackets. If it is desired to limit the average settlement to one inch, then it will be seen that the stability criterion is relevant only for small footings on over-consolidated clays. In all other cases the design is governed by settlement considerations. With a limiting average settlement of three inches, the stability criterion applies to all footings on over-consolidated clays and to small footings on most normally-consolidated clays. But for raft foundations the settlement criterion is still of controlling importance in all clays except those which are over-consolidated, with high values of K_v/c.

Settlements of more than three inches are not usually tolerated in buildings, but in bridge design settlements of six inches or more are often permissible, especially where provision exists for maintaining the correct elevation of the deck by means of jacks (as at Waterloo Bridge and elsewhere). In such cases, the factor of safety depends upon stability considerations in all clays except those with a very low value of K_v/c, unless the piers are unusually wide. The importance of width in controlling the design of foundations on clay is therefore clearly demonstrated, and also the inter-dependence of the two criteria. But a further inference may be made from an examination of Table 4, namely that the factors of safety necessary to limit the settlements to a few inches on normally-consolidated clays, with all but the smallest footings, are so large as to be outside practical possibility. Therefore, unless settlements of many inches, or even a few feet can be tolerated, it is not feasible to found directly on such clays, especially if the liquid limit is high. This point has previously been made by Terzaghi and Peck[34], but it requires re-emphasis, since there appears to be an increasing tendency to accept a factor of safety of 3 as being adequate for the design of footings of any clay. Table 5 shows that this is not even approximately correct for clays with low values of K_v/c if the settlements are to be restricted to a reasonably small magnitude.

Conclusion

In conclusion, it may be said that, so far as the present evidence is concerned, the values of N_c given in Fig. 2 are sufficiently accurate for the determination of the ultimate bearing capacity of deep beds of relatively homogeneous clay. A factor of safety of at least 3 is desirable in estimating allowable bearing capacity. But in many cases the foundation design will be controlled by settlement considerations, and the engineer may be compelled to use factors of safety very considerably greater than 3, in order to restrict the settlement to a magnitude compatible with structural requirements.

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