AUXILIAR 1 CI42A 2009-02 (1)

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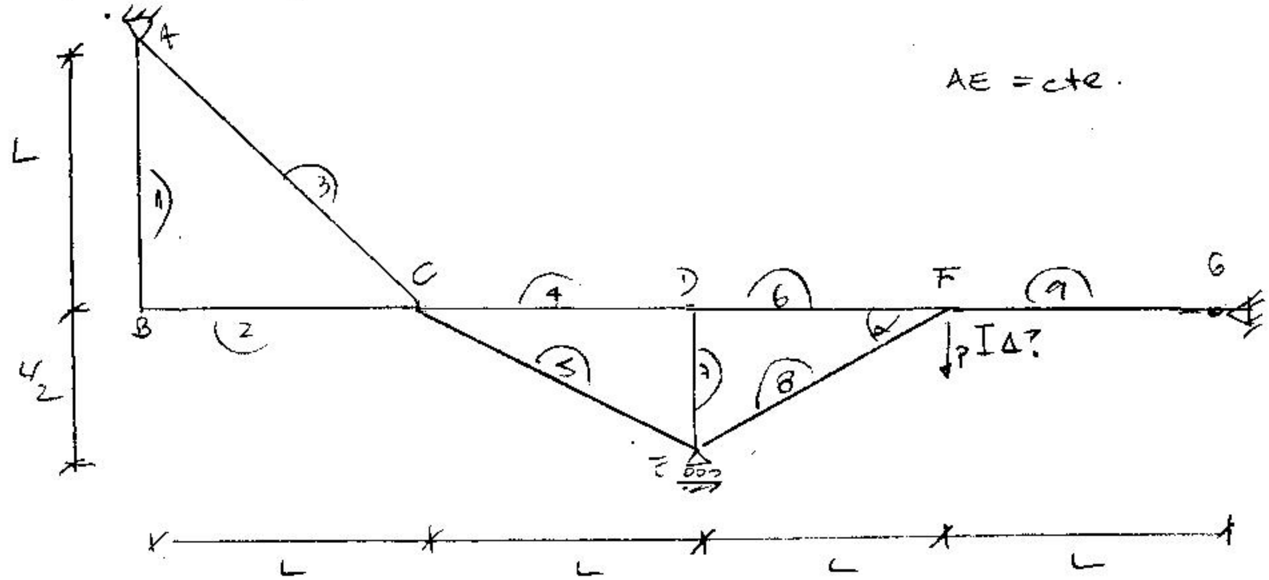
P1) PARA EL ENREJADO DE LA FIGURA, DETERMINE REACCIONES, ESFUERZOS INTERNOS Y EL DESPLAZAMIENTO Δ EN EL NODO F. DIBUJE LA DEFORMADA APROXIMADA



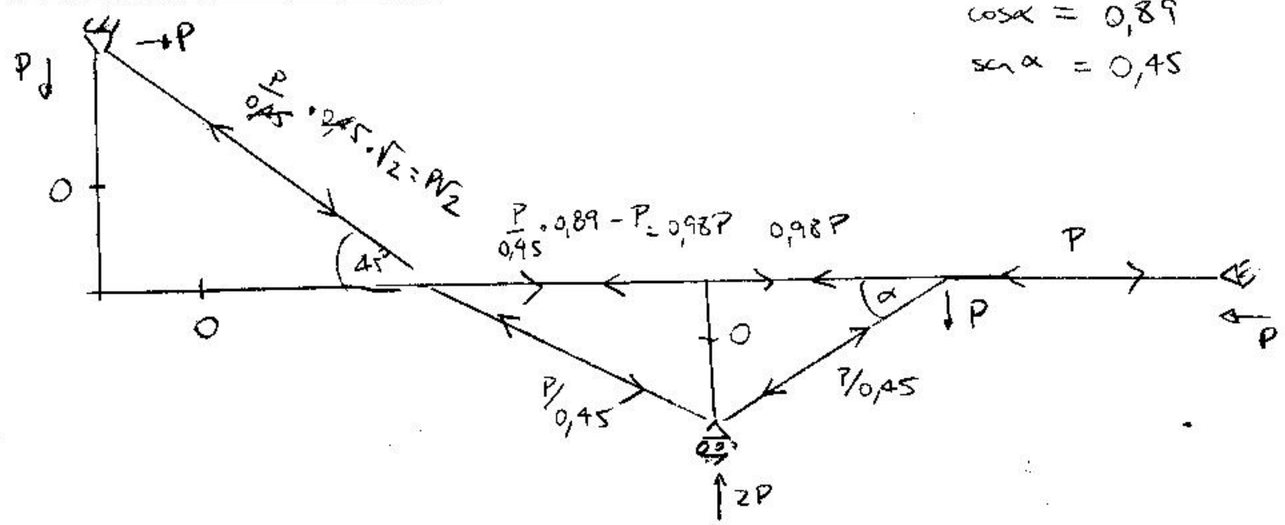
SOLUC. POR CASTIGLIANO $\frac{\partial U_i}{\partial P} = \Delta$

$\alpha = 26{,}56^\circ$

$\cos\alpha = 0{,}89$

$\sin\alpha = 0{,}45$

ESF INTERNOS Y REACCIONES.



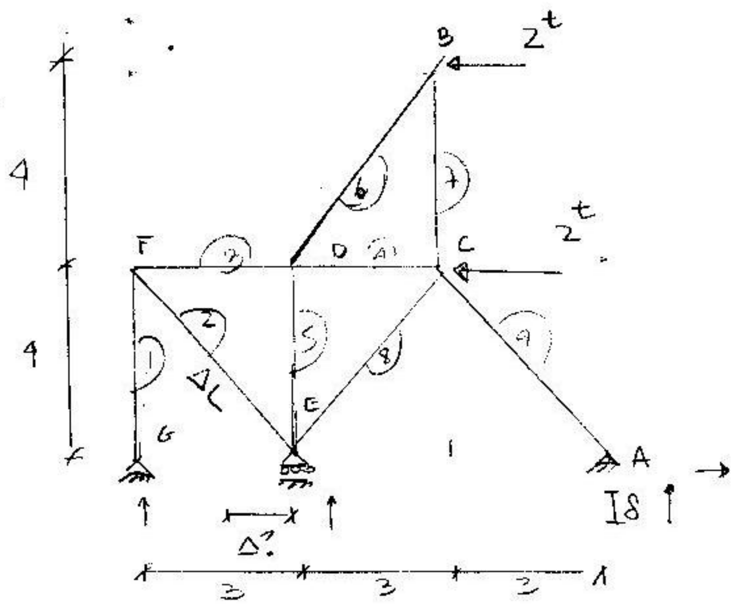
$$U_i = \int \frac{1}{2} \frac{N_i^2}{EA} dx \underset{N=cte}{=} \frac{1}{2} \frac{N_i^2}{EA} L_i$$

BARRA	N_i	N_i^2	L_i	$\frac{L_i}{EA}$	$\frac{N_i^2 L_i}{EA}$
1	0	0	L	L/EA	0
2	0	0	L	L/EA	0
3	$-P\sqrt{2}$	$2P^2$	$L\sqrt{2}$	$L\sqrt{2}/EA$	$2\sqrt{2}P^2L/EA$
4	$0{,}98P$	$0{,}96P^2$	L	L/EA	$0{,}96P^2L/EA$
5	$-P/0{,}45$	$4{,}938P^2$	$1{,}118L$	$1{,}118L/EA$	$5{,}52P^2L/EA$
6	$0{,}98P$	$0{,}96P^2$	L	L/EA	$0{,}96P^2L/EA$
7	0	0	$L/2$	$L/2EA$	0
8	$-P/0{,}45$	$4{,}938P^2$	$1{,}118L$	$1{,}118L/EA$	$5{,}52P^2L/EA$
9	$-P$	P^2	L	L/EA	P^2L/EA

$$\frac{1}{2} \Sigma \frac{N_i^2 L_i}{EA} = \frac{16{,}7884P^2L}{EA} \cdot \frac{1}{2}$$

$$\Delta = \frac{\partial U_i}{\partial P} = \frac{16{,}7884PL}{EA} //$$

P2) PARA EL ENREJADO DE LA FIGURA, DETERMINE REACCIONES Y ESFUERZOS INTERNOS EN LAS BARRAS. ADEMÁS, DETERMINE EL DESPLAZAMIENTO HORIZONTAL Δ EN EL APOYO E.



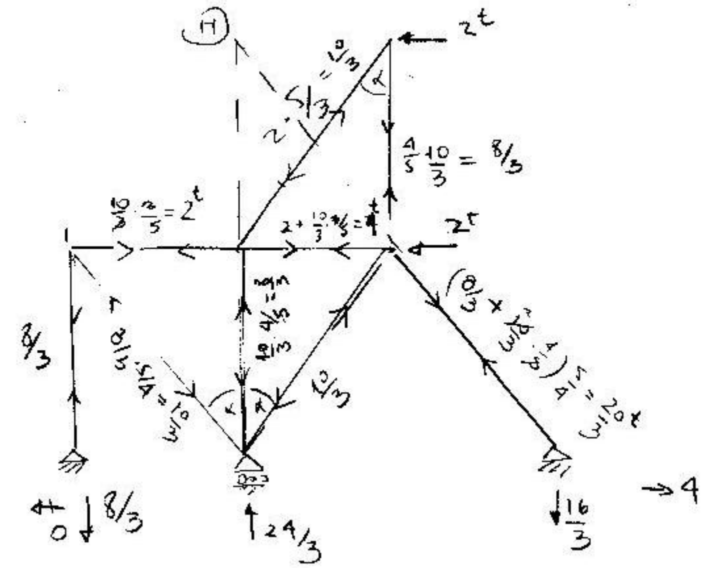
DIMENSIONES EN [m]

$EA = 40000 \; T/m^2$

$\delta = 1 \; cm$

$\Delta_L = 2 \; cm$

SOLUC. REACCIONES Y ESF. INTERNOS

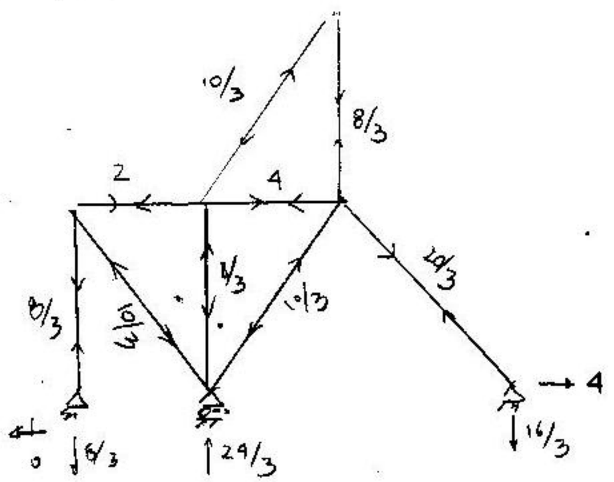


$\cos(\alpha) = 4/5 \;, \quad \sin(\alpha) = 3/5$

$\Sigma M_{H} = 0$

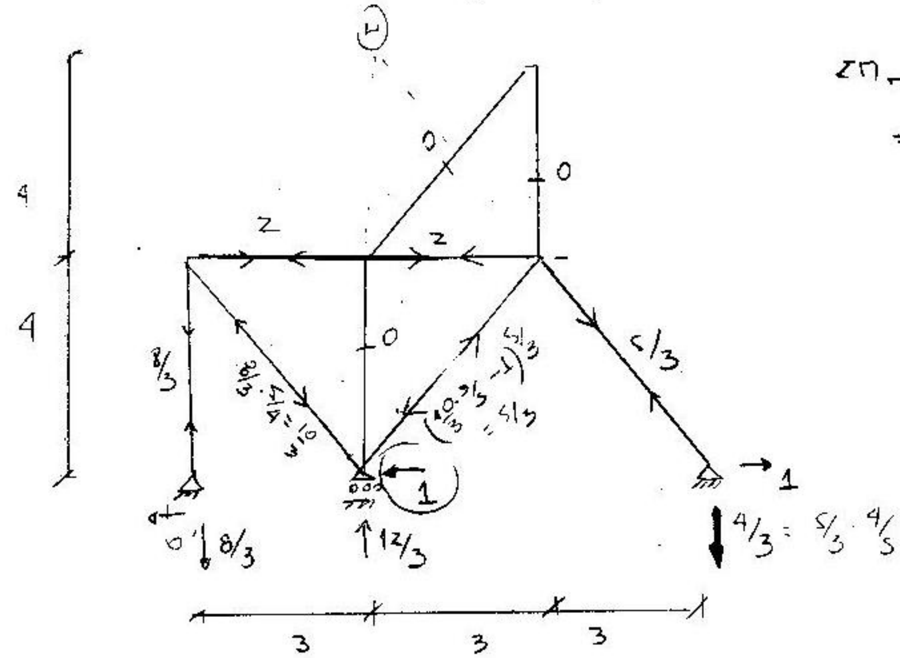
$\Rightarrow \; 2 \cdot 4 = -3 \cdot V_G \;\Rightarrow V_G = -8/3^t$

Resumen,



CARGA UNITARIA

$$\left(1 \times \Delta^{i} = \delta W_E = \delta U_i = \sum_{barras} \overline{N_i}\, \frac{N_i\, l_i}{AE} + \overline{N_i}\, \alpha \Delta T l_i + \overline{N_i}\, \Delta l_{MONT} \right.$$



$\Sigma M_H = 0$

$\Rightarrow \; V_6 \cdot 8 = -1 \cdot 8 \quad \Rightarrow V_6 = -8/3 \; t$

→← : tracción

←→ : compresión

$$\Delta = \Delta_{CARGAS} + \Delta_{\Delta L} + \Delta_{\delta}$$

1. $\Delta_{CARGAS} \Rightarrow \quad 1\, \Delta_{cargas} = \sum \overline{N_i}\, \frac{N_i\, l_i}{AE}$

BARRA	N_i	$\bar{N}_i$	l_i	l_i/EA	$\frac{N_i \bar{N}_i l_i}{EA}$
1	$8/3$	$8/3$	4	0,0001	0,000711
2	$-10/3$	$-10/3$	5	0,000125	0,001389
3	2	2	3	0,000075	0,0003
4	4	2	3	0,000075	0,0006
5	$-8/3$	0	4	—	
6	$-10/3$	0	5	—	
7	$-8/3$	0	4	—	
8	$-10/3$	$-5/3$	5	0,000125	0,000694
9	$20/3$	$5/3$	5	0,000125	0,001389

$$\sum \frac{N_i \bar{N}_i l_i}{EA} = 0,005083 \text{ m} \approx 5,1 \text{ mm}$$

$(\delta W_E = \delta U_i$, pero $\delta U_i = 0$ (no hay energía interna de deformación).

·) Δ_{δ} PTV $= 1 \cdot \delta W_E = 1 \cdot \Delta + \frac{4}{3}\delta$; $\delta W_E = 0$

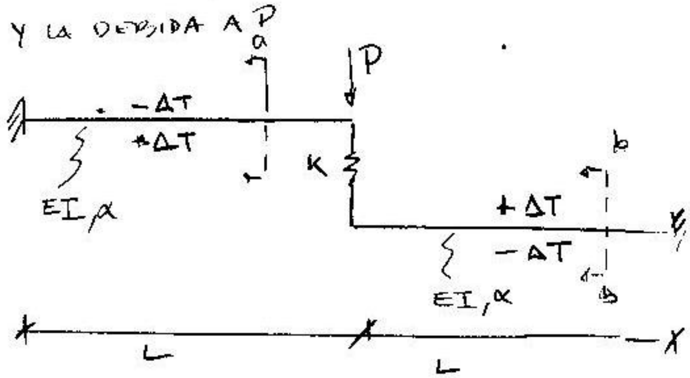
$$\Delta_\delta = -\frac{4}{3}\cdot\delta = -\frac{4}{3}\cdot 10 = -13,33 \text{ mm}$$

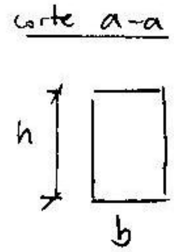
$\sum \bar{N}_i \cdot \Delta L$ (sólo generan trabajo las barras con defectos de fabricación. El resto sólo se desplaza)

·) $\Delta_{\Delta L}$ $\Delta_{\Delta L} = -\frac{10}{3}\cdot\Delta L = -\frac{10}{3}\cdot 20 = -\frac{200}{3} = -66,67$ mm

$$\Delta_f = 5,1 - 13,33 - 66,67 = -74,9 \text{ mm} = -7,49 \text{ cm}$$

P3| Determine el alargamiento (o acortamiento) del resorte de la estructura de la figura. Considere sólo def por flexión en las barras. Dibuje la deformada por temp. y la debida a P





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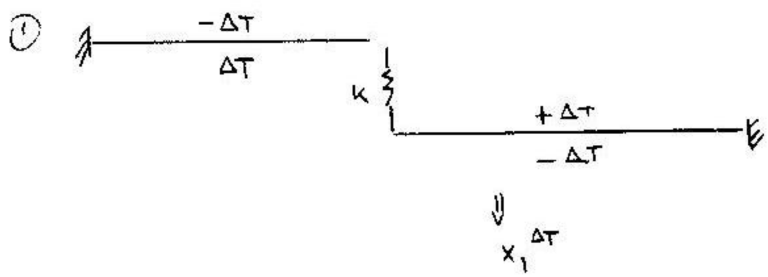

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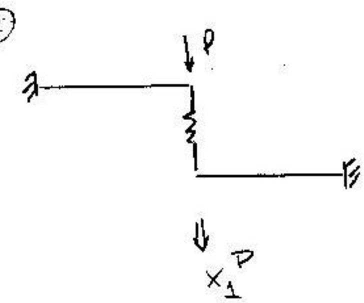
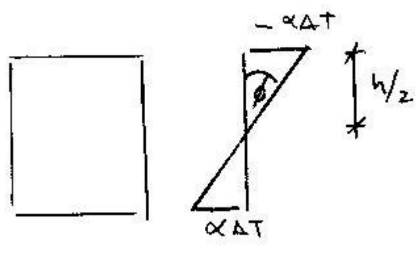


$$X_1 = X_1^{\Delta T} + X_1^P$$

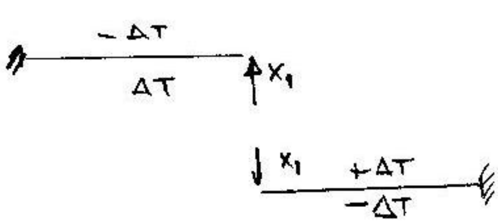
① Sección a-a



$$\phi^{a-a} = \frac{\alpha\Delta T}{h/2} = \frac{2\alpha\Delta T}{h}$$

$$\Rightarrow \underline{b-b} \Rightarrow \phi^{b-b} = \frac{-2\alpha\Delta T}{h}$$

Menabrea



$M(x)$

$M(x) = X_1(L-x)$

$M(x) = X_1(x-L)$

$\phi = M/EI$

$\Rightarrow \phi = \dfrac{X_1(L-x)}{EI}$

$$U_i^{\Delta T} = \int_0^L EI\phi^2 dx + \frac{1}{2}\frac{X_1^2}{K} = \frac{1}{2}\int_0^L EI\left(\frac{X_1(L-x)}{EI} + \frac{2\alpha\Delta T}{h}\right)^2 + \frac{1}{2}\int_0^L EI\left(\frac{X_1(x-L)}{EI} - \frac{2\alpha\Delta T}{h}\right)^2$$

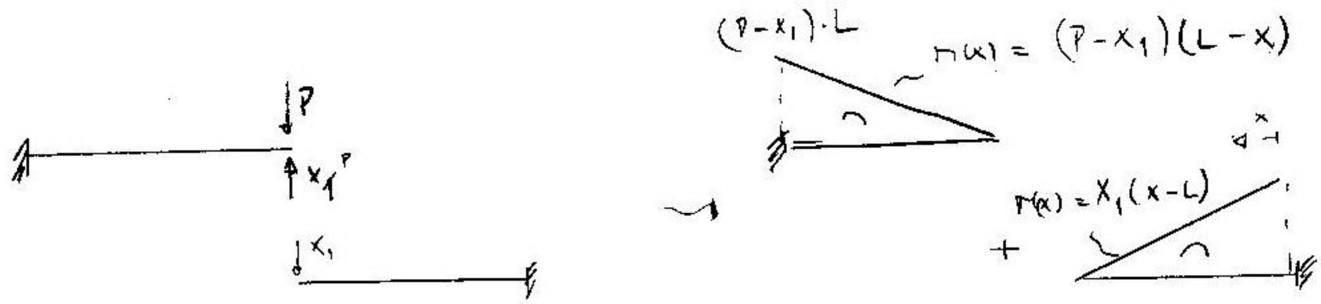
$$+ \frac{1}{2} X_1^2/K.$$

$$U_i^{\Delta T} = \frac{2L}{3EI}\left[12\left(\frac{\alpha\Delta T}{h}\right)^2 (EI)^2 + 6\left(\frac{\alpha\Delta T}{h}\right)(EI)\cdot X_1 + X_1^2 L^2\right]\frac{1}{2} + \frac{1}{2}\frac{X_1^2}{K}$$

$$\frac{\partial U_i^{\Delta T}}{\partial X_1} = \frac{X_1(2kL^3 + 3EI) + 6\left(\frac{\alpha\Delta T}{h}\right)^2 kL^2 EI}{3kEI} = \emptyset$$

$$\Rightarrow X_1^{\Delta T} = -\frac{6\left(\frac{\alpha\Delta T}{h}\right)\cdot K\cdot L^2\cdot EI}{4KL^3 + 3EI} \quad \Rightarrow \text{Tracción!}$$

2



$$U_i = \frac{1}{2}\int_0^L \frac{m^2}{EI}dx + \frac{1}{2}\frac{X_1^2}{K} \Rightarrow U_i = \frac{1}{2}\int_0^L \left(\frac{(P-X_1)(L-x)}{EI}\right)^2 dx + \frac{1}{2}\int_0^L \left(\frac{X_1(x-L)}{EI}\right)^2 dx$$

$$+ \frac{1}{2} X_1^{P\,2}/K$$

$$U_i = L^3\left(P^2 - 2P\cdot X_1 + 2X_1^2\right)/6EI + \frac{1}{2}\frac{X_1^{P\,2}}{K}$$

(5)

$$\frac{\partial U_i}{\partial X_1^P} = 0 \Rightarrow \frac{X_1(2KL^3 + 3EI) - KL^3 \cdot P}{3KEI} = 0$$

$$\Rightarrow X_1^P = \frac{PKL^3}{2KL^3 + 3EI} \quad // \quad \text{(compresión)}$$

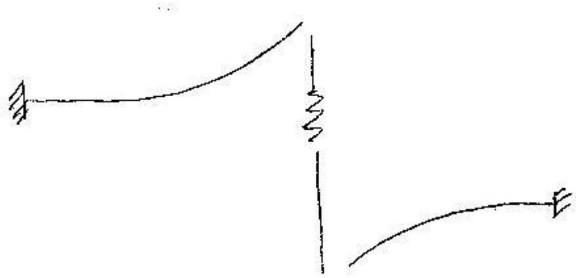
$$\therefore X_1 = \frac{PKL^3}{2KL^3 + 3EI} - \frac{6\left(\frac{\alpha\Delta T}{h}\right) K \cdot L^2 EI}{4KL^3 + 3EI}$$

→ Deformación en el resorte.

$$\Delta = \frac{X_1}{K} = \frac{\dfrac{PKL^3}{2KL^3+3EI} - \dfrac{6\left(\frac{\alpha\Delta T}{h}\right) KL^2 EI}{4KL^3 + 3EI}}{K}$$

$$\boxed{\Delta = \frac{PL^3}{2KL^3+3EI} - \frac{6\left(\frac{\alpha\Delta T}{h}\right) L^2 EI}{4KL^3 + 3EI}}$$

Deformada por t^o



Deformada por P