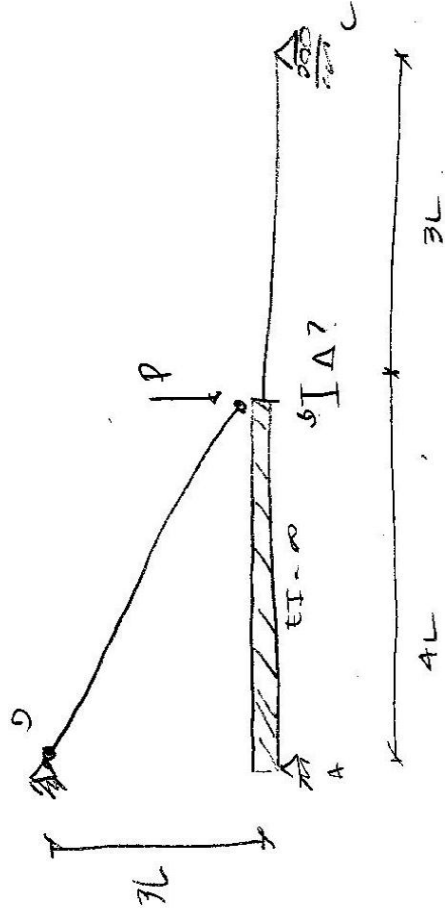


P2 et 2009 -02

1

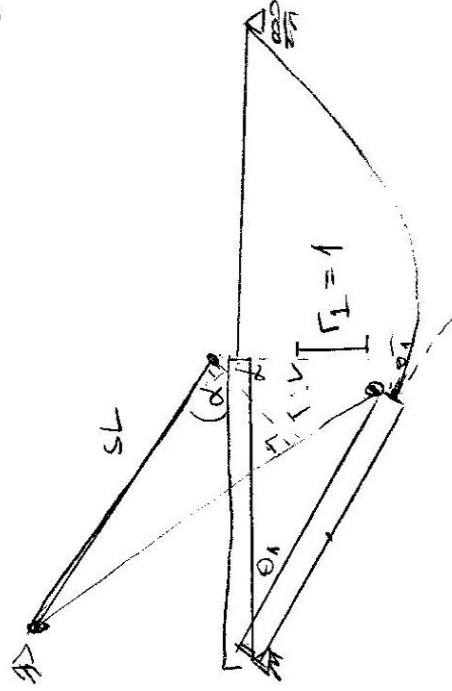


$$GIC = 1$$

$$\cos \alpha = 4/5$$

$$\sin \alpha = 3/5$$

Para el elemento en flexion



$$1 \int \dots = \int 1 \dots + \dots$$

Por comp. de def $\Rightarrow \theta_1 \cdot 4L = 1$

$$\Rightarrow \theta_1 = 1/4L$$

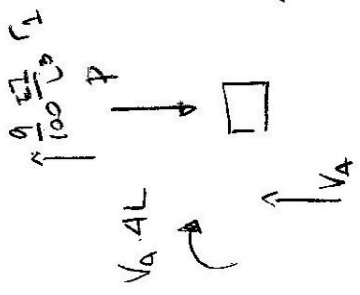
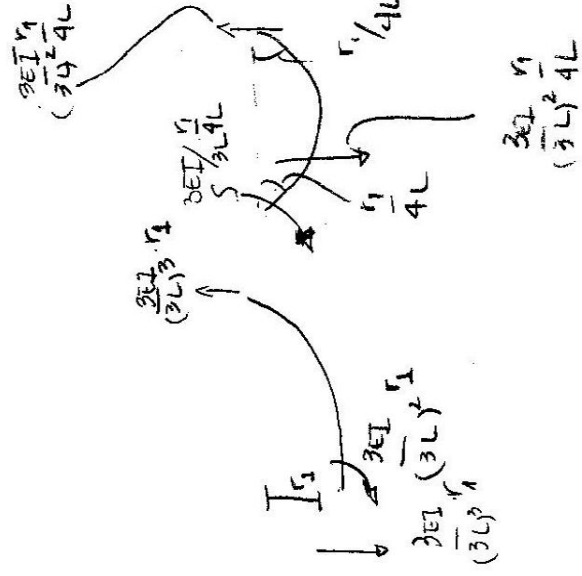
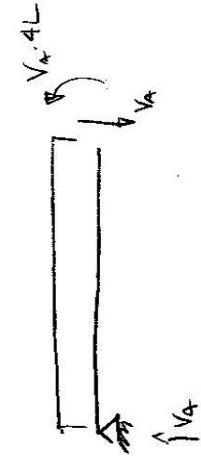
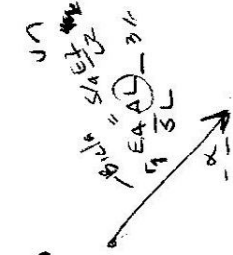
ΔL = 1 · sin α = 3/5 //

Wego, hacen do

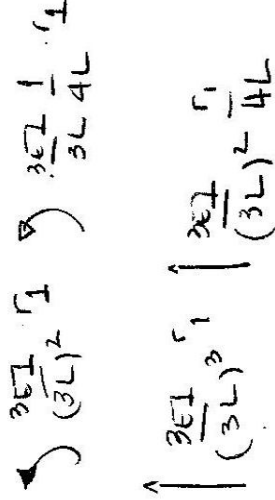
DEL

encl vudo

2



NUDO =>



$$\sum F_y = 0 \Rightarrow p = V_A + \frac{3EI}{(3L)^3} r_1 + \frac{3EI}{(3L)^2} \frac{r_1}{4L} + \frac{9}{100} \frac{EI}{L^3} r_1 \quad (1)$$

$$\sum \mathcal{M}_0 = 0 \Rightarrow V_A \cdot 4L = \frac{3EI}{9L^2} r_1 + \frac{3EI}{12L^2} \cdot r_1 \quad (2)$$

$$(1) \wedge (2) \Rightarrow \boxed{r_1 = 2,324 \frac{pL^2}{EI}}$$

$$\boxed{V_A = 0,3389P}$$

③

Análogamente, haciendo por el método tradicional

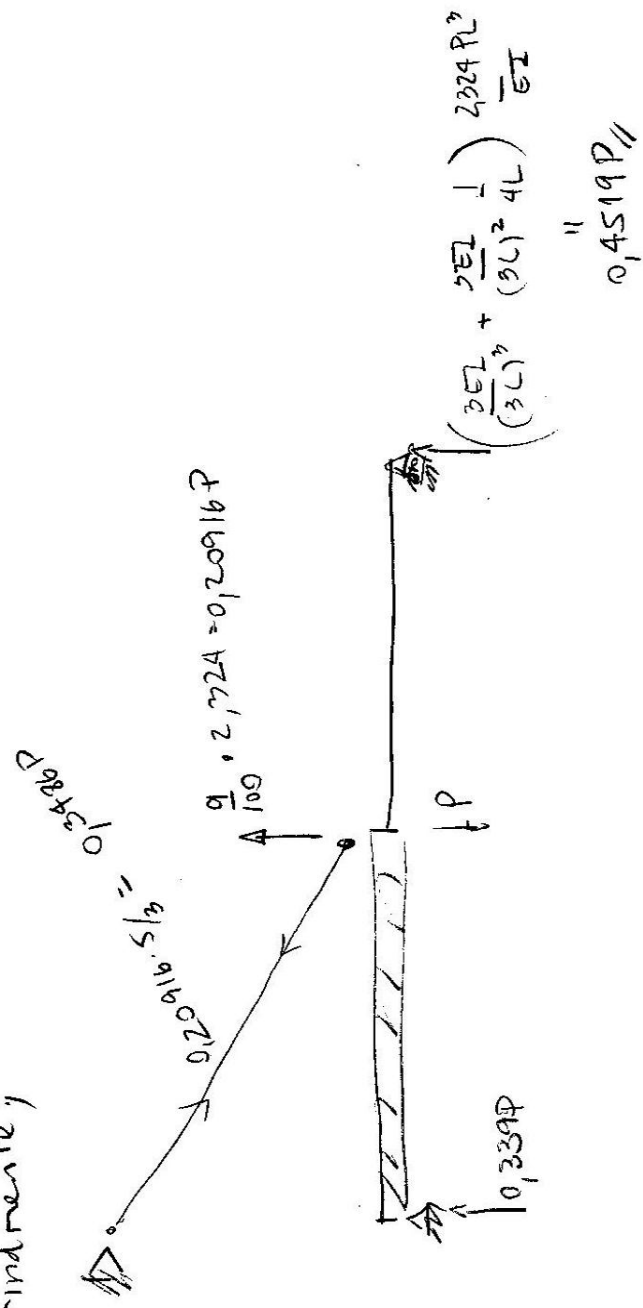
$$K_{11} = \frac{3EI}{(3L)^3} + \frac{3EI}{(3L)^2 \cdot 4L} + \frac{1}{100} \frac{EI}{L^3}$$

con el valor de cargas $R = P - V_A$

$$\Rightarrow K_{11} \cdot r_L = (P - V_A)$$

y luego para obtener V_A se hace equilibrio de momentos!

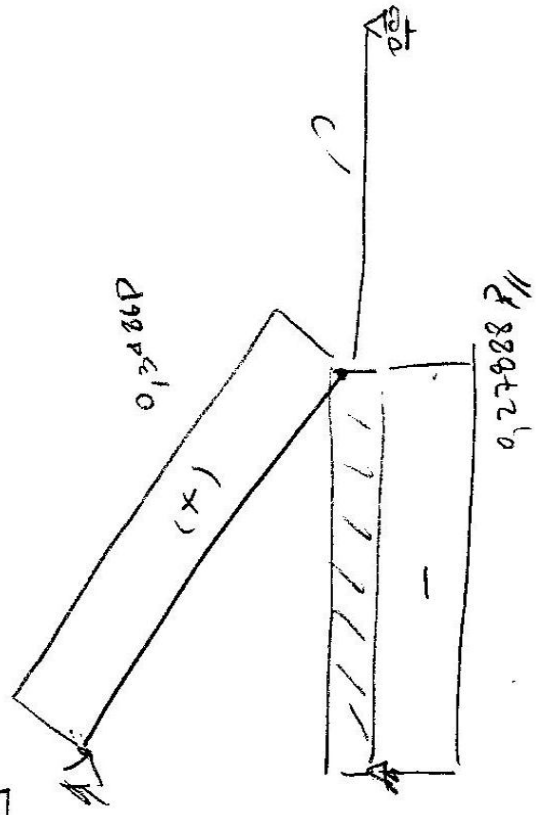
Finalmente,



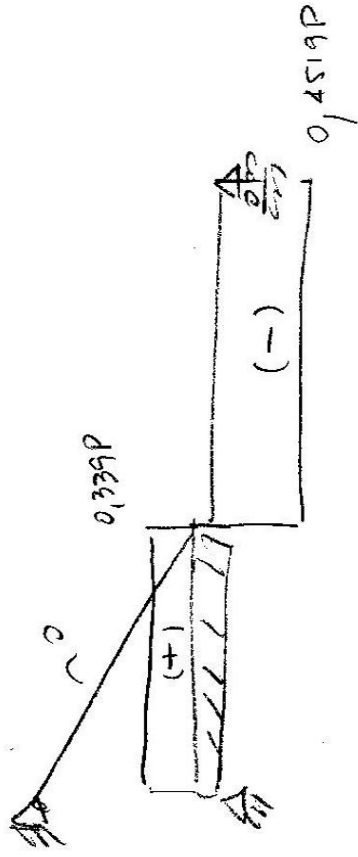
que cumple el equilibrio de fuerzas y momentos

Diagramas

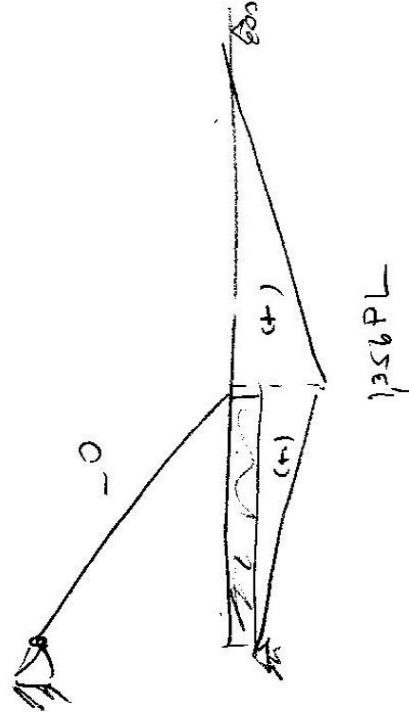
N(k)



Q(k)

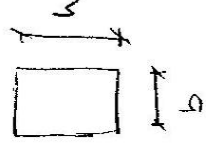
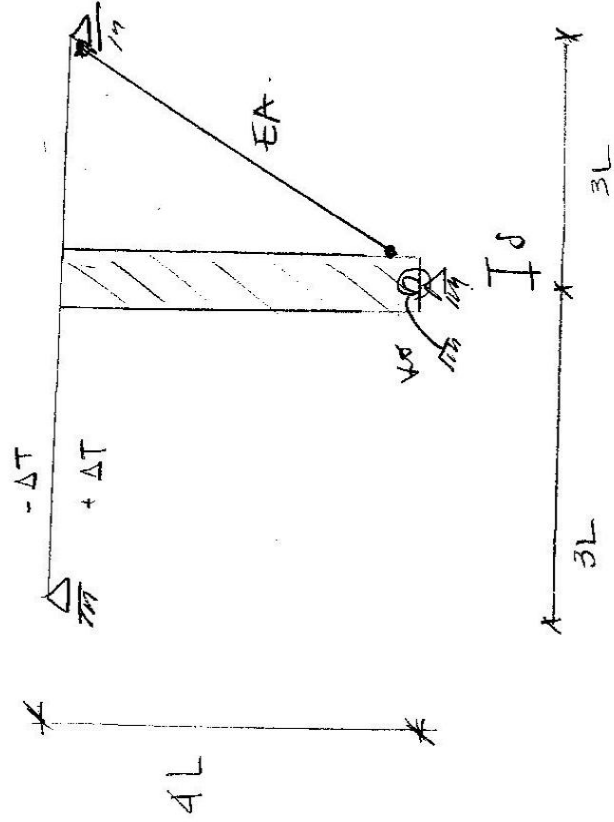


M(k)



②

P21 Determine GIE , GIC y diagramas de esfuerzos



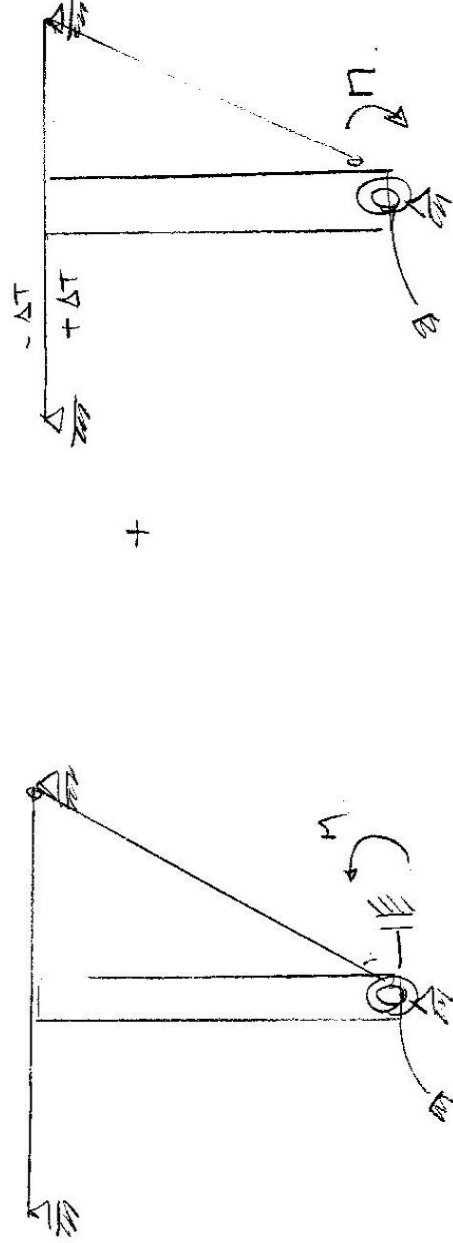
$$\phi = \frac{\alpha \Delta T}{\frac{h}{2}} = \frac{2\alpha \Delta T}{h}$$

$$EA = EI/L^2$$

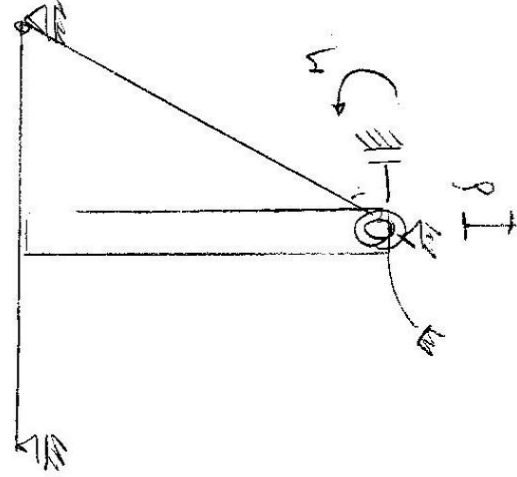
$$K_0 = EI/L$$

$$GIE = 3$$

$$GIC = 1 \checkmark \Rightarrow \text{Rigidez}$$

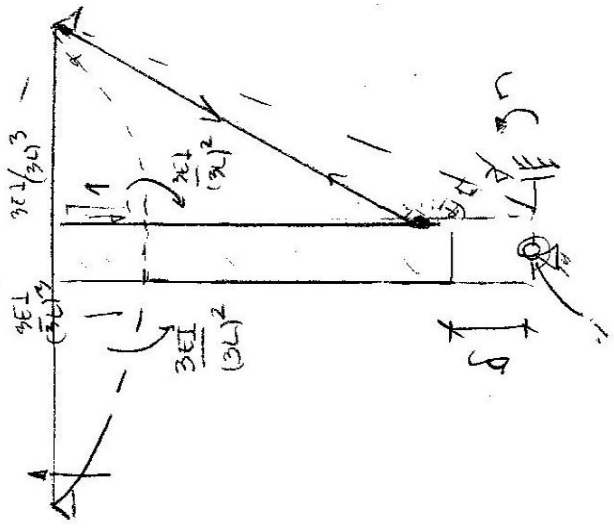


②



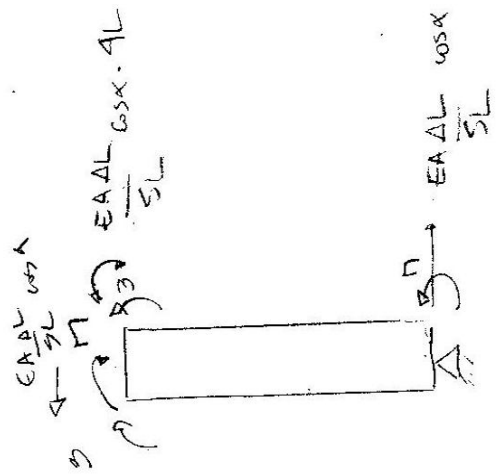
①

⑥



$$\Delta L = \delta \sin \alpha \Rightarrow \Delta L = 4/5 \delta$$

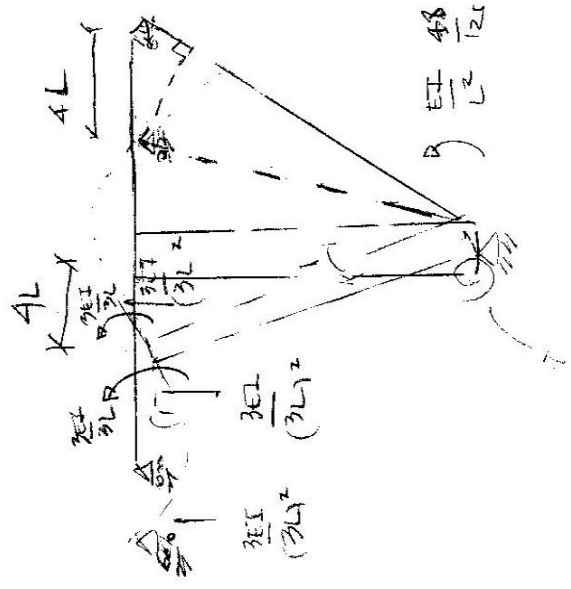
$$F_{ura} B_{ula} = EA \frac{4/5 \delta}{5L} = \frac{4EA\delta}{25L}$$



$$h = \frac{EA \Delta L}{5L} \cos \alpha \cdot 4L$$

$$h = EA \frac{4/5 \delta}{5L} \cdot \frac{3}{5} \cdot 4L$$

$$h = \frac{48}{125} \frac{EI}{L^2} \delta$$



$$\Delta L = 4L \cos \alpha$$

$$K_{11} = \frac{3EI}{3L} \times 2 + \left(\frac{EI}{L} \right) \times \left(\frac{EI}{L^2} \right) \times 4L$$

$$\Rightarrow K_{11} = \frac{415EI}{L}$$

$$R = m //$$

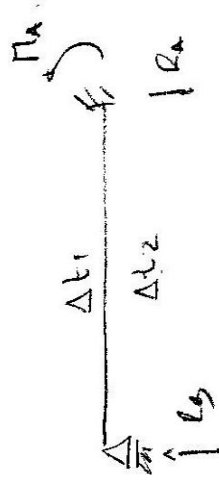
$$\Rightarrow k_{11} \cdot r_1 = R$$

$$r_1 = \frac{48}{125} \frac{EI}{L^2} \delta$$

$$\frac{4,15 EI}{L}$$

$$r_1 = 0,092 \frac{\delta}{L}$$

Para la temperatura, solo cambia el vector de carga >



$$P_n = \frac{3EI \alpha (\Delta t_1 - \Delta t_2)}{2h}$$

$$P_{02} = P_n = \frac{3EI \alpha (\Delta t_1 - \Delta t_2)}{2h \cdot L}$$

$$\text{para } \Delta t_1 = -\Delta T, \quad \Delta t_2 = \Delta T$$

$$\Rightarrow P_n = \frac{3EI \alpha (-\Delta T - \Delta T)}{2h}$$

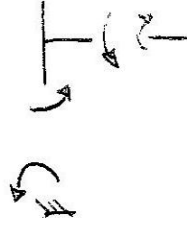
$$P_n = \frac{-3\alpha EI \Delta T}{h}$$

End work,

$$\frac{3EI \alpha \Delta T}{h}$$



$$R = \frac{48}{125} \frac{EI}{L^2} \delta + \frac{3\alpha EI \Delta T}{h}$$

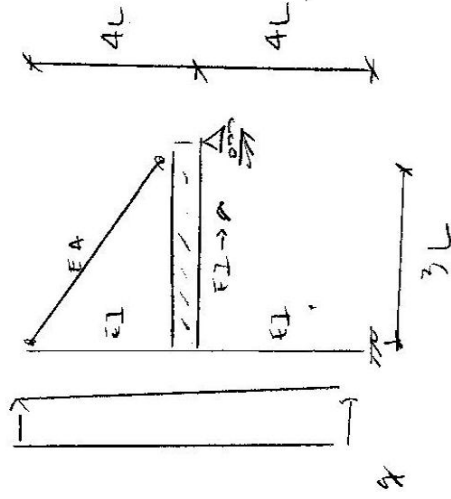


$$\frac{3\alpha EI \Delta T}{h}$$

(8)

P31 Para la estructura de la Fig, determine GIC, REACCIONES Y

DIAGRAMAS DE EFUERZOS



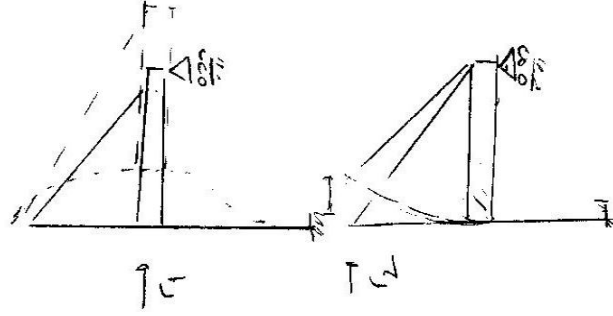
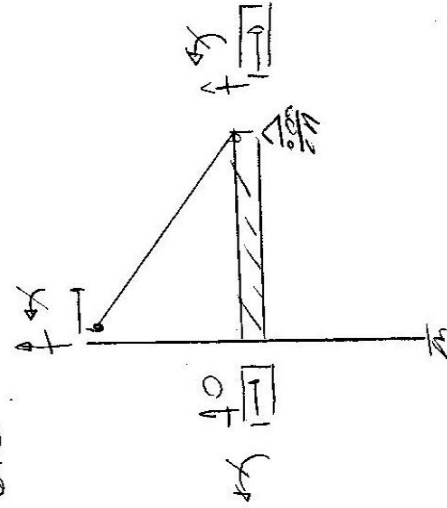
Considere

$$EA = EI/L^2$$

(columna axialmente rígida y viga)

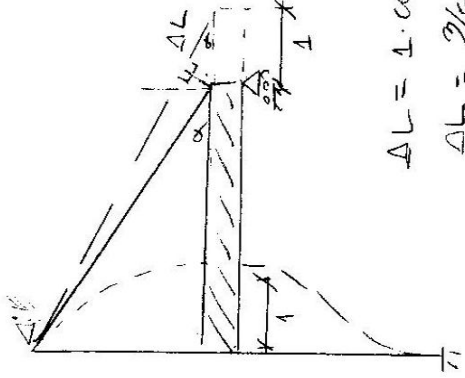
Soluc

GIC



Matriz de rigidez

$$r_1 = 1, r_2 = 0$$



$$\Delta L = L \cdot \cos \alpha$$

$$\Delta L = 3/5$$

End node 1

$$k_{11} = \frac{3EI}{(4L)^3} \left[\begin{array}{c} \rightarrow \\ \left[\begin{array}{cc} \frac{3EI}{(4L)^3} & \frac{EA \Delta L \cos \alpha}{SL} \\ \frac{12EI}{(4L)^3} & \frac{3EI}{(4L)^3} \end{array} \right] \end{array} \right]$$

$$9/125 \frac{EI}{L^3} = \frac{18}{125} \frac{EI}{L^3}$$

$$k_{21} = \frac{3EI}{(4L)^3}$$

9

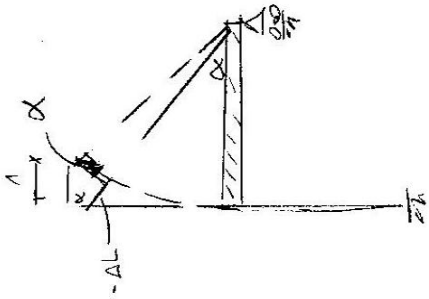
$$\begin{array}{c} \frac{EA \Delta L \sin \alpha}{SL} \rightarrow \\ \frac{EA \Delta L \cos \alpha}{SL} \rightarrow \\ \frac{3EI}{(4L)^3} \rightarrow \\ \frac{3EI}{(4L)^3} \rightarrow \\ \frac{3EI}{(4L)^3} \rightarrow \\ \frac{12EI}{(4L)^3} \rightarrow \\ \frac{6EI}{(4L)^3} \rightarrow \\ \frac{EA \Delta L \cos \alpha}{SL} \rightarrow \\ \frac{EA \Delta L \sin \alpha}{SL} \rightarrow \end{array}$$

$$k_{11} = \frac{15EI}{64L^3} + \frac{2EI \cdot 3/5 \cdot 3/5}{5L^3}$$

$$k_{11} = 0,178 \frac{EI}{L^3}$$

$$k_{21} = -0,191 \frac{EI}{L^3}$$

$$r_2 = 1, r_1 = 0$$



$$\Delta L = L \cos \alpha = \frac{3}{5} L$$

$$\frac{12 EI}{L^3} \Delta L$$

$$\frac{9 EA}{125 L}$$

$$\frac{3EI}{(4L)^3}$$

$$\frac{EA \Delta L \sin \alpha}{5L}$$

$$\frac{EA \Delta L \cos \alpha}{5L}$$

$$\frac{3EI}{(4L)^3}$$

$$\frac{3EI}{(4L)^2}$$

$$EA \cdot \frac{3}{5} \cdot \frac{3}{5}$$

$$\frac{9 EA}{125 L}$$

$$\frac{12 EI}{125 L^3}$$

$$\frac{3EI}{(4L)^3}$$

$$k_{22}$$

$$k_{22} = 0.191 \frac{EI}{L^3}$$

$$\frac{3EI}{(4L)^3}$$

$$\frac{9 EA}{125 L}$$

$$k_{12}$$

$$k_{12} = 0.191 \frac{EI}{L^3}$$

Proposedo $\Rightarrow$ $EL \gg b_0$