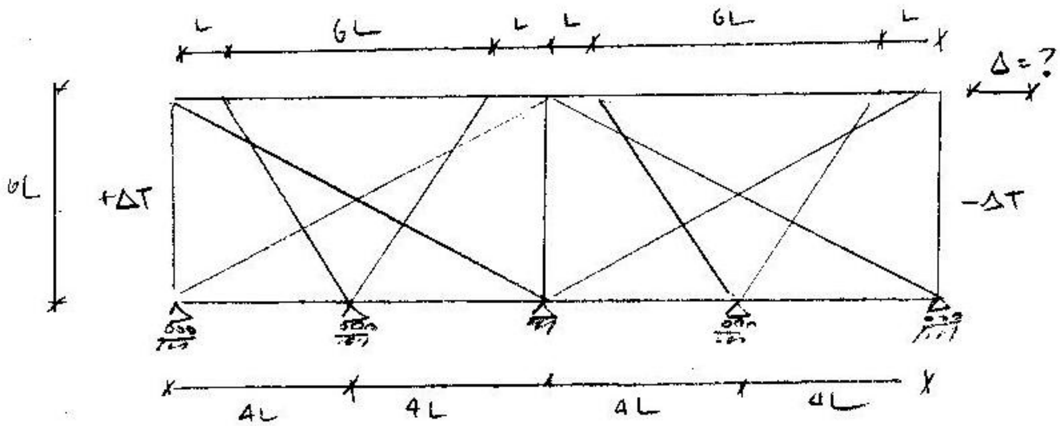
AUXILIAR 2 CI42A 2009-02

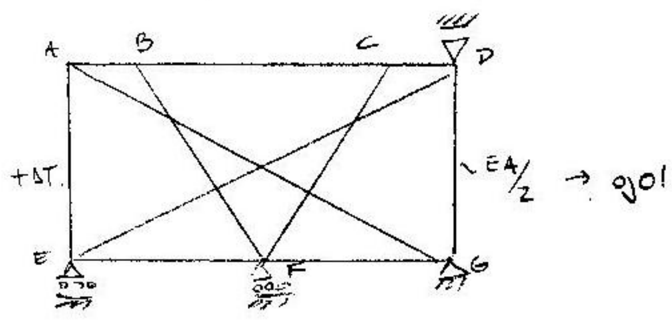
P1 Para la estructura de la fig. 1 se pide determinar:

- GIE
- Reacciones
- Diagramas de esfuerzos
- Desplazamiento horizontal Δ.



solución: Aplicando simetría

simetría planar, acciones antisimétricas.

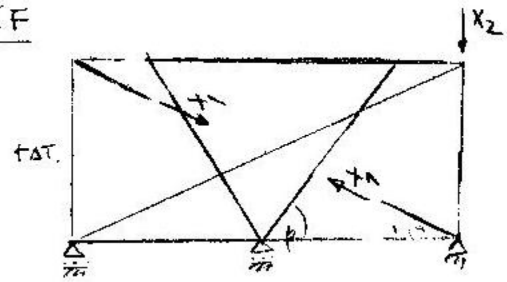


GIE estructura inicial $= 3$

GIE estructura resultante $= 2$ //

luego, liberando N_{AG} y V_D.

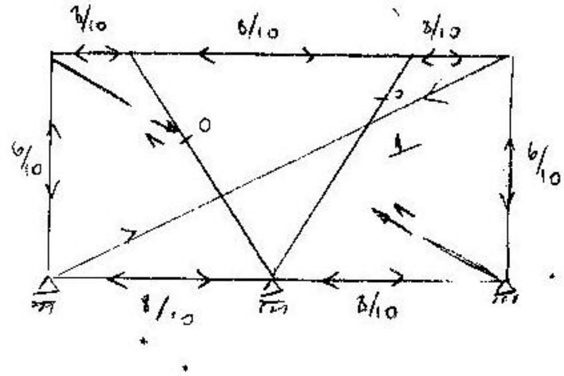
EIF



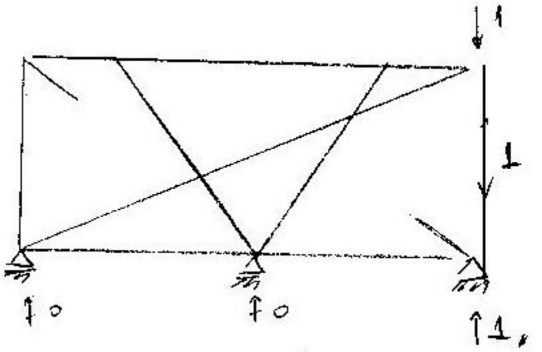
$\sin(\alpha) = \dfrac{6}{10}$, $\cos(\alpha) = \dfrac{8}{10}$

$\sin(\beta) = 6/\sqrt{45}$, $\cos\beta = 3/\sqrt{45}$

·) $N_{AG} = 1$, $X_2 = 0$



·) $X_2 = 1$, $N_{AG} = 0$



flexibilidad

$$\overset{0}{\cancel{r}} = \underset{\sum \frac{\bar{N}_i \bar{N}_i L_i}{EA}}{[f]} \{X\} + r(\Delta T)$$

$$f_{11} = -\frac{8}{10}\cdot\frac{-8}{10}(2\cdot L + 6L + 2\cdot 4L)/_{EA} + 2\cdot -1\cdot 1\cdot \frac{10L}{EA} + \frac{-6}{10}\frac{-6}{10}\frac{6L}{EA} + \frac{6}{10}\frac{6}{10}\cdot\frac{6L}{\frac{EA}{2}}$$

$$f_{11} = (10{,}24 + 20 + 2{,}16 + 4{,}32)\frac{L}{EA} \Rightarrow f_{11} = 36{,}72L/_{EA}$$

$$f_{12} = \frac{-6}{10}\cdot -1 \quad \frac{6L}{EA/_2} = \frac{72L}{10\,EA} = f_{21} \Rightarrow f_{21} = f_{12} = 7{,}2L/_{EA}$$

$$f_{22} = 1^2\, 6L/_{\frac{EA}{2}} \Rightarrow f_{22} = 12L/_{EA}$$

$$r_1(\Delta T) = \Sigma \bar{N}_i \, \varepsilon_i \cdot L_i = -\frac{6}{10} \cdot \alpha \Delta T \cdot 6L = -\frac{36}{10}\alpha\Delta T L = -\frac{18}{5}\alpha\Delta T L \quad = -3{,}6\,\alpha\Delta T L$$

$$r_2(\Delta T) = \Sigma \bar{N}_i \, \varepsilon_i \cdot L_i = 0$$

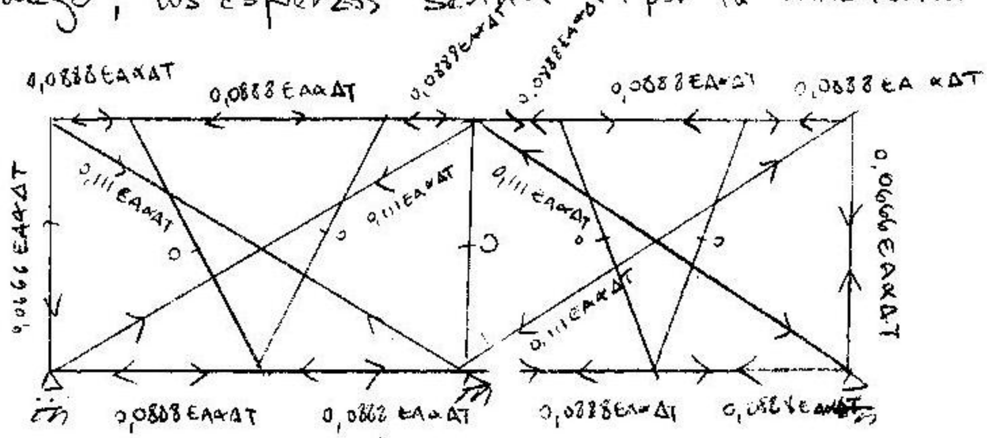
$$\Rightarrow \frac{L}{EA}\begin{bmatrix} 36{,}72 & 7{,}2 \\ 7{,}2 & 12 \end{bmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} -3{,}6\,\alpha\Delta T L \\ 0 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{EA}{L}\begin{bmatrix} 36{,}72 & 7{,}2 \\ 7{,}2 & 12 \end{bmatrix}^{-1}\begin{pmatrix} 3{,}6\,\alpha\Delta T L \\ 0 \end{pmatrix}$$

$$\Rightarrow X_1 = 0{,}111 \; EA\alpha\Delta T$$

$$X_2 = -0{,}067 \; EA\alpha\Delta T$$

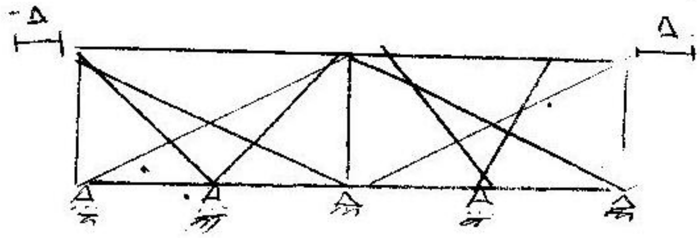
luego, los esfuerzos serán (por la antisimetría de las cargas)



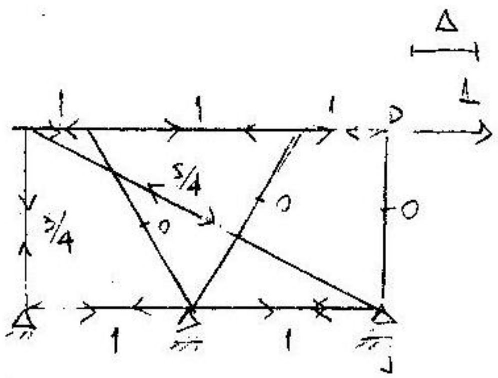
$$\frac{8}{10} \cdot 0{,}111 = 0{,}0888$$

$$\frac{6}{10} \cdot 0{,}111 = 0{,}0666.$$

Desplazamiento Δ



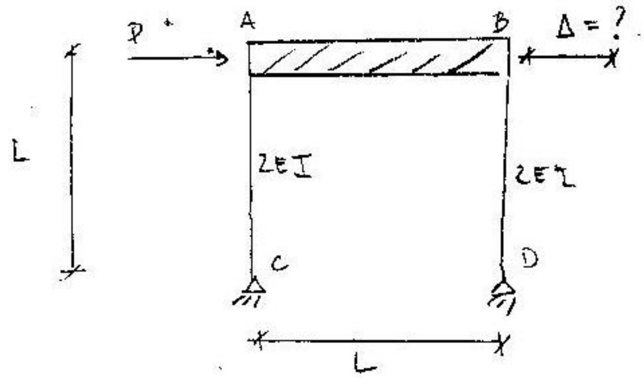
EN LA EIF, por simetría



PTV $\Rightarrow$ $1 \cdot \Delta = 2 \cdot 1 \cdot 0{,}0882 \cancel{EA} \alpha \Delta T \cdot \left(\frac{8L}{\cancel{EA}} \right) + \frac{5}{4} \cdot 0{,}111 \cancel{EA} \alpha \Delta T \; \frac{10L}{EA}$

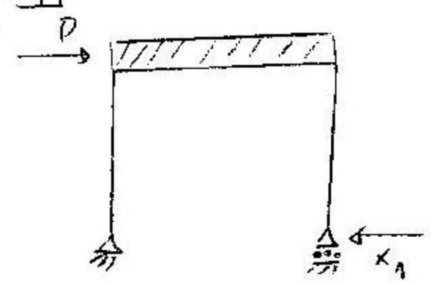
$\rightarrow \Delta = 2{,}8083 \, \alpha \Delta T L$

P2| (a) En el marco de la figura, se aplica una carga lateral P en la viga. Determine GIE, reacciones, diagramas de esfuerzos y el desplazamiento horizontal Δ.



Soluc| $GIE = \underline{1}$

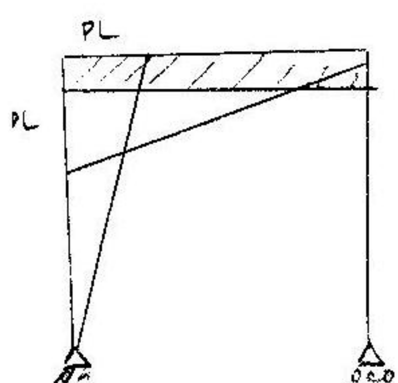
EIE|



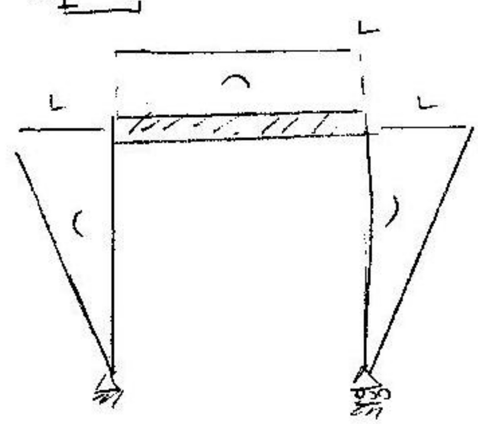
(desplazamiento horizontal nulo)

$$\cancel{r_1}^{\,0} = r_1(P) + r_1(X_1)$$

P|

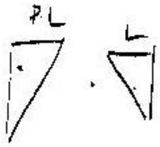


$\underline{X_1 = 1}$



$f_{11} = 2 \cdot$  $\frac{L}{2EI} +$  $\frac{L}{\cancel{EI}_{\infty}} \; 0$

$\Rightarrow f_{11} = 2\,\frac{1}{3}\,L^2\,\frac{L}{2EI} \quad \therefore \quad \boxed{f_{11} = \frac{L^3}{3EI}}$

$r_1(P) =$  $\frac{L}{2EI}$

$\Rightarrow r_1(P) = -\frac{1}{3}\,PL\,L\,\frac{L}{2EI} \quad \therefore \quad r_1(P) = -\frac{PL^3}{6EI}$

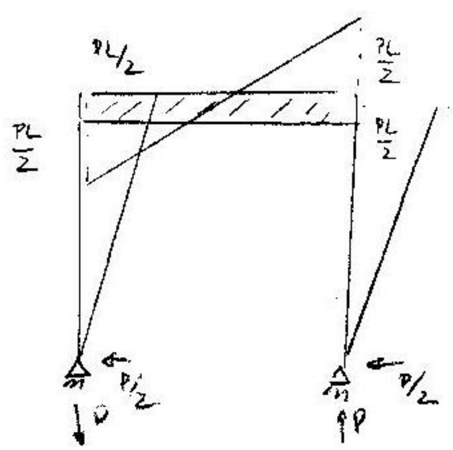
Por lo tanto,

$\Rightarrow f_{11}\,X_1 + r_1(P) = 0$

Reemplazando, $\Rightarrow X_1 = \dfrac{\frac{PL^3}{6EI}}{\frac{L^3}{3EI}} \Rightarrow \boxed{X_1 = P/2}$

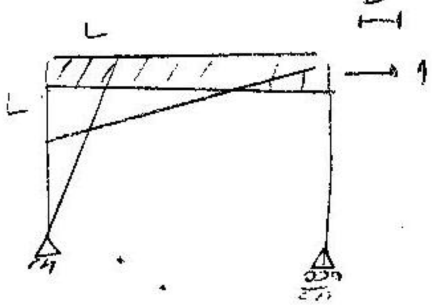
Finalmente,

Diagrama de momento y reacciones (N y Q propuestos)



Desplazamiento Δ

EIF

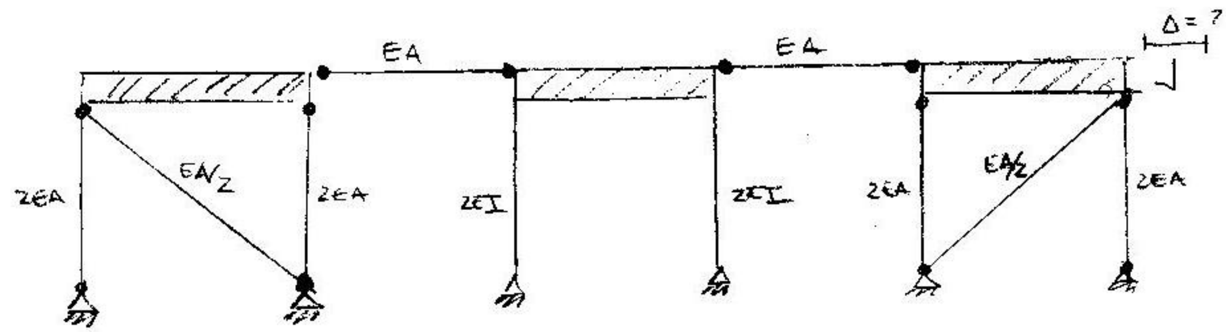


$$\Rightarrow 1\,\Delta = \left[L\right] \left[PL/2\right] \cdot \frac{L}{2EI}$$

$$\Delta = \frac{1}{3} \cdot L \cdot \frac{PL}{2} \cdot \frac{L}{2EI}$$

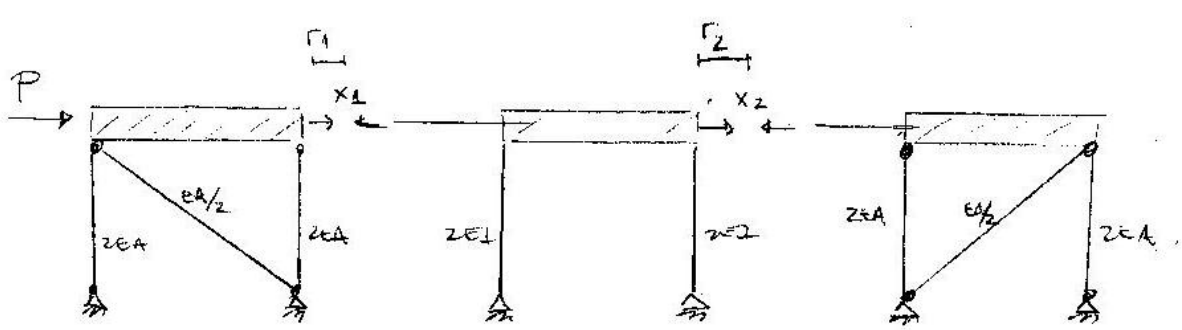
$$\boxed{\Delta = \frac{PL^3}{12EI}}$$

(b) Utilice el resultado de (a) para resolver la estructura siguiente y obtener el desplazamiento del punto J. Considere que $EA = EI/L^2$



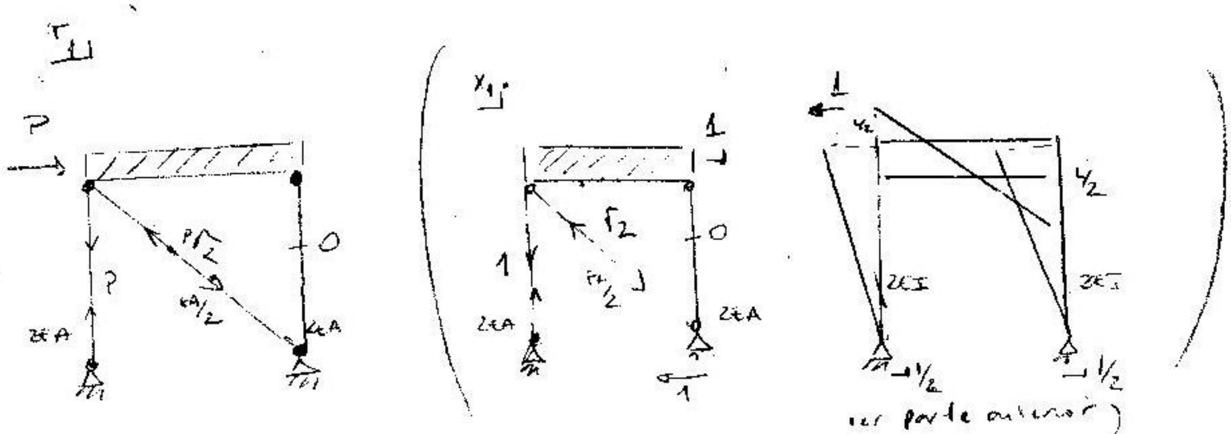
Soluc.

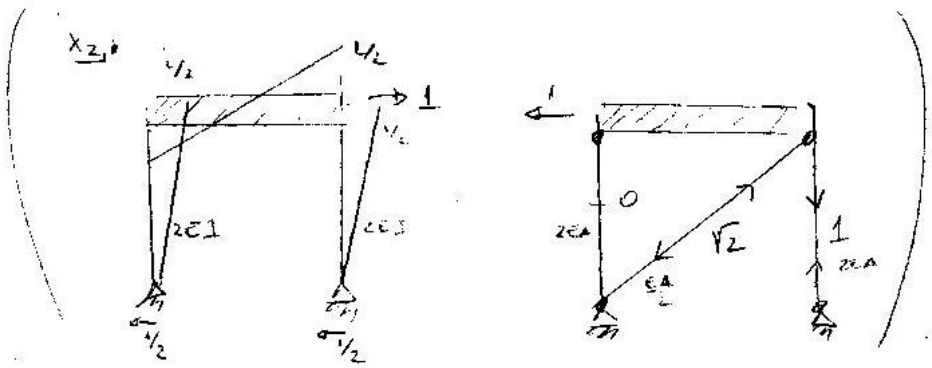
EIF



$$\underset{0}{\cancel{r_1}} = r_1(X_1) + r_1(X_2) + r_1(P)$$

r_1





$$r_1(X_1) = \frac{1^2}{2EA}L\,X_1 + \frac{(\sqrt{2})^2}{\frac{EA}{2}}\sqrt{2}L\cdot X_1 + 2\cdot\frac{1}{3}\,\frac{L}{2}\,\frac{L}{2}\,\frac{L}{2EI}X_1 + \cancel{2\cdot\frac{1}{3}\frac{L}{2}\cdot\frac{L}{2}\cdot\frac{\frac{L}{2}}{EI}}\,X_1$$

$$+ \underbrace{\frac{X_1 L}{EA}}_{\text{Biela}}$$

$$= \left(\frac{L}{2EA} + \frac{4\sqrt{2}L}{EA} + \frac{L^3}{12EI} + \frac{L}{EA}\right)X_1 = \frac{L^3}{EI}\left(\frac{1}{2} + 4\sqrt{2} + \frac{1}{12} + 1\right)X_1$$

$$r_2(X_1) = -2\cdot\frac{1}{3}\,\frac{L}{2}\,\frac{L}{2}\,\frac{L}{2EI}\cdot X_1$$

$$= -\frac{L^3}{12EI}X_1$$

$$r_1(X_2) = -2 \cdot \frac{1}{3} \cdot \frac{L}{2} \cdot \frac{L}{2} \cdot \frac{L}{2EI} \cdot X_2$$

$$= \frac{-L^3}{12EI} X_2$$

$$r_2(X_2) = \frac{1^2 \, L}{2EA} X_2 + \frac{(\sqrt{2}\cdot)^2 \cdot \sqrt{2}L}{EA/2} \cdot X_2 + 2 \cdot \frac{1}{3} \frac{L}{2} \frac{L}{2} \frac{L}{2EI} + \frac{X_2 \cdot L}{EA}$$

$$= \left(\frac{L}{2EA} + \frac{4\sqrt{2}L}{EA} + \frac{L^3}{12EI} + \frac{L}{EA}\right) X_2 = \frac{L^3}{EI}\left(\frac{1}{2} + 4\sqrt{2} + \frac{1}{12} + 1\right) X_2$$

$$r_1(P) = +P\frac{1 \, L}{2EA} + \frac{+P\sqrt{2}\,\sqrt{2}\cdot L\sqrt{2}}{EA/2} = +P\left(\frac{1}{2} + 4\sqrt{2}\right)\frac{L^3}{EI}$$

$$r_2(P) = \emptyset$$

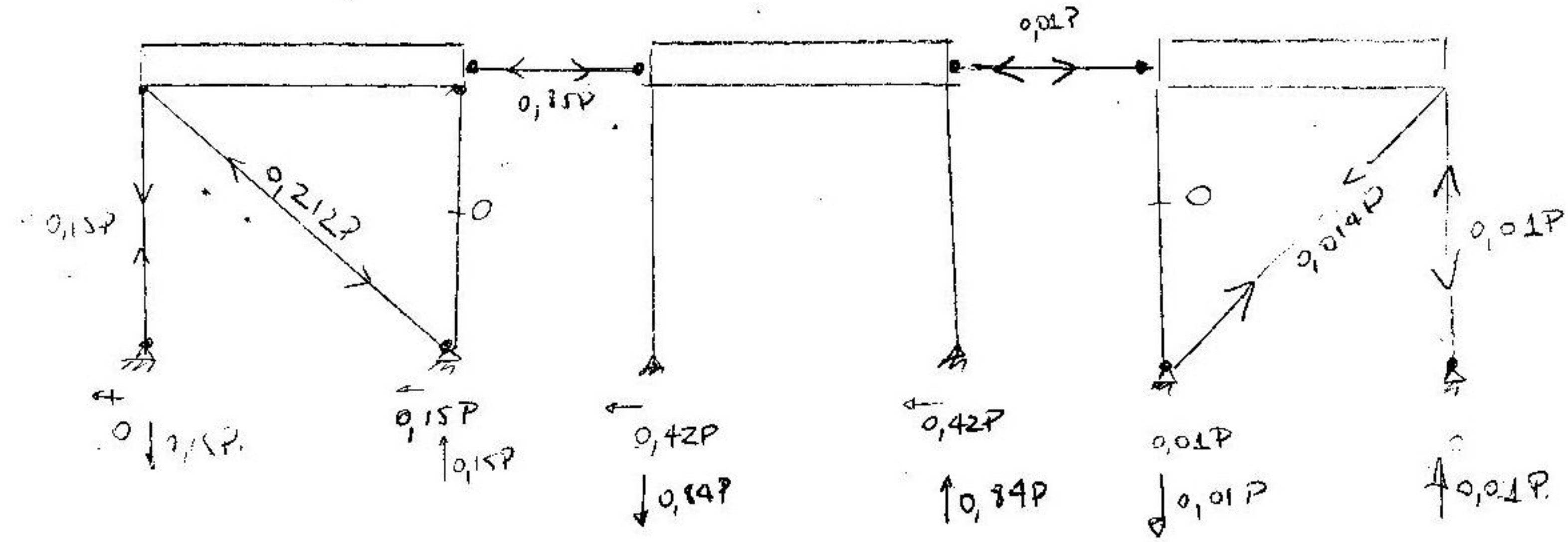
Wego,

$$\frac{L^3}{EI}\begin{bmatrix} (4\sqrt{2} + 19/12) & -1/12 \\ -1/12 & (4\sqrt{2} + 19/12) \end{bmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{L^3}{EI}\begin{pmatrix} -P(1/2 + 4\sqrt{2}) \\ 0 \end{pmatrix}$$

$$X_1 = -0{,}85P$$

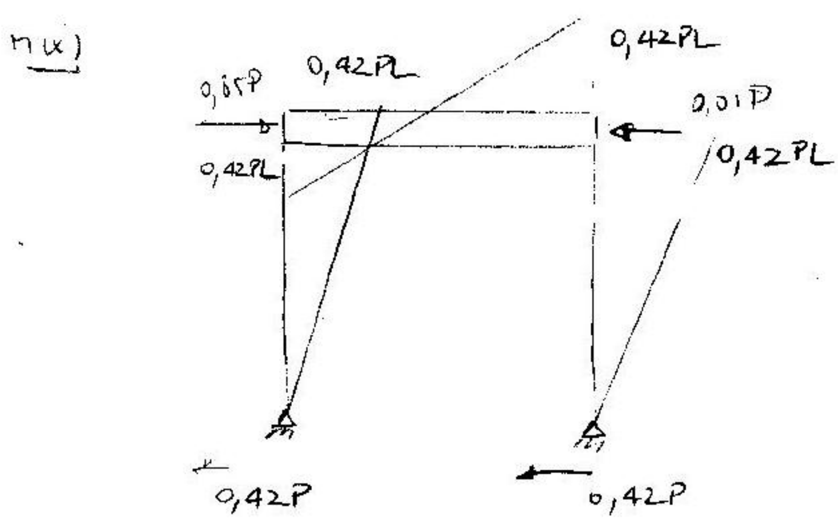
$$X_2 = -0{,}01P$$

Luego, sumando los efectos de X_1 y X_2 y P



DIAGRAMAS MARCO CENTRAL

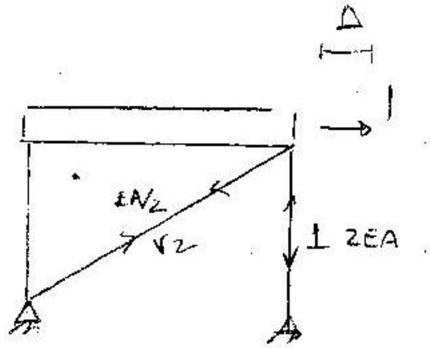
M(x)



(N y Q propuestos)

Desplazamiento Δ

EN LA EIF



$$1 \times \Delta = \frac{\sqrt{2} \cdot +0{,}014P\ \sqrt{2}L}{EA/2} + \frac{1 \ +0{,}01PL}{2EA}$$

$$\Delta = +0{,}061PL/EA$$