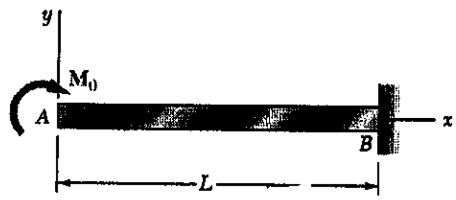
CHAPTER 9

PROBLEM 9.1



9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

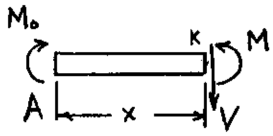
$$\circlearrowleft \Sigma M_K = 0 \qquad -M_0 + M = 0$$

$$M = M_0$$

$[x = L,\ y = 0]$

$[x = L,\ \frac{dy}{dx} = 0]$

$$EI\frac{d^2y}{dx^2} = M = M_0$$



$$EI\frac{dy}{dx} = M_0 x + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \qquad 0 = M_0 L + C_1 \qquad C_1 = -M_0 L$$

$$EIy = \frac{1}{2}M_0 x^2 + C_1 x + C_2$$

$$[x = L,\ y = 0] \qquad 0 = \frac{1}{2}M_0 L^2 - M_0 L^2 + C_2$$

$$C_2 = \frac{1}{2}M_0 L^2$$

(a) Elastic curve $\qquad y = \frac{M_0}{2EI}(x^2 - 2Lx + L^2)$

$$= \frac{M_0}{2EI}(L - x)^2 \quad ◀$$

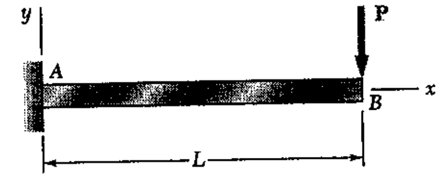
(b) y @ $x = 0$ $\qquad y_A = \frac{M_0}{2EI}(L - 0)^2 = \frac{M_0 L^2}{2EI} \uparrow \quad ◀$

(c) $\frac{dy}{dx}$ @ $x = 0$ $\qquad \frac{dy}{dx} = -\frac{M_0}{EI}(L - x) = -\frac{M_0}{EI}(L - 0) = -\frac{M_0 L}{EI}$

$$\theta_A = \frac{M_0 L}{EI} \quad ◀$$

PROBLEM 9.2

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



SOLUTION

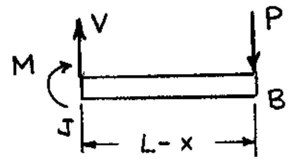
$+\circlearrowleft \Sigma M_J = 0 \qquad -M - P(L-x) = 0$

$M = -P(L-x)$

$[x=0,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$

$EI\frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$



$EI\frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 = -0 + 0 + C_1$

$C_1 = 0$

$EIy = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$

$[x=0,\ y=0] \qquad 0 = -0 + 0 + 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve $\qquad y = -\frac{Px^2}{6EI}(3L-x)$

$\frac{dy}{dx} = -\frac{Px}{2EI}(2L-x)$

(b) y @ $x = L$ $\qquad y_B = -\frac{PL^2}{6EI}(3L-L) = -\frac{PL^3}{3EI}$

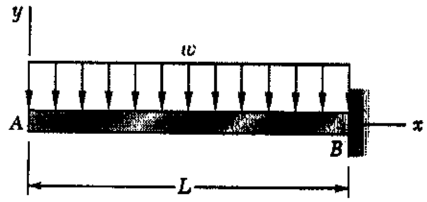
$y_B = \frac{PL^3}{3EI}\downarrow$

(c) $\frac{dy}{dx}$ @ $x = L$ $\qquad \left.\frac{dy}{dx}\right|_B = -\frac{PL}{2EI}(2L-L) = -\frac{PL^2}{2EI}$

$\theta_B = \frac{PL^2}{2EI}$

PROBLEM 9.3

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



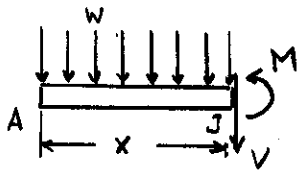
SOLUTION

$\circlearrowleft \Sigma M_J = 0 \qquad (wx)\frac{x}{2} + M = 0$

$$M = -\frac{1}{2}wx^2$$

$[x = L,\ y = 0]$

$[x = L,\ \frac{dy}{dx} = 0]$



$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$[x = L,\ \frac{dy}{dx} = 0.] \qquad 0 = -\frac{1}{6}wL^3 + C_1$

$$C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$[x = L,\ y = 0] \qquad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$

$$C_2 = (\frac{1}{24} - \frac{1}{6})wL^4 = -\frac{3}{24}wL^4$$

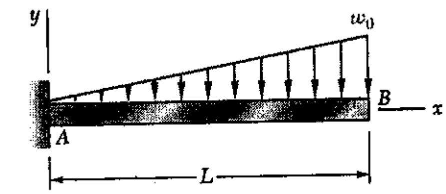
(a) Elastic curve $\qquad y = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4)$ ◄

(b) y @ $x = 0 \qquad y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI} \qquad y_A = \frac{wL^4}{8EI}\downarrow$ ◄

(c) $\frac{dy}{dx}$ @ $x = 0 \qquad \left.\frac{dy}{dx}\right|_A = \frac{wL^3}{6EI} \qquad \theta_A = \frac{wL^3}{6EI}$ ◄

PROBLEM 9.4

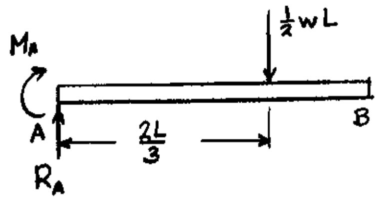
9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



$[x=0,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$

SOLUTION



$+\uparrow \Sigma F_y = 0$

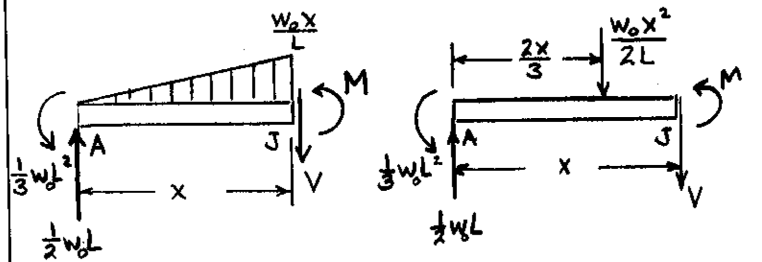
$R_A - \frac{1}{2}wL = 0$

$R_A = \frac{1}{2}w_0 L$

$\circlearrowleft \Sigma M_A = 0$

$-M_A - \frac{2L}{3}\cdot\frac{wL}{2} = 0$

$M_A = -\frac{1}{3}w_0 L^2$



$\circlearrowleft \Sigma M_J = 0 \qquad \frac{1}{3}w_0L^2 - \frac{1}{2}w_0Lx + \frac{w_0x^2}{2L}\cdot\frac{x}{3} + M = 0$

$$M = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{dy}{dx} = -\frac{1}{3}w_0L^2x + \frac{1}{4}w_0Lx^2 - \frac{w_0x^4}{24L} + C_1$$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 = -0 + 0 - 0 + C_1 \qquad C_1 = 0$

$$EIy = -\frac{1}{6}w_0L^2x^2 + \frac{1}{12}w_0Lx^3 - \frac{w_0x^5}{120L} + C_2$$

$[x=0,\ y=0] \qquad 0 = -0 + 0 - 0 + 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve $\qquad y = -\frac{w_0}{EIL}\left(\frac{1}{6}L^3x^2 - \frac{1}{12}Lx^4 + \frac{1}{120}x^5\right)$ ◄

(b) y @ $x = L \qquad y_B = -\frac{w_0L^4}{EI}\left(\frac{1}{6} - \frac{1}{12} + \frac{1}{120}\right) = -\frac{11}{120}\frac{w_0L^4}{EI}$

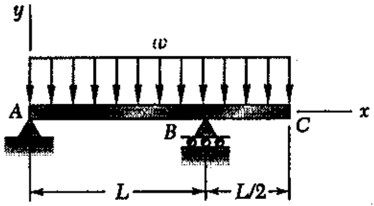
$y_B = \frac{11}{120}\frac{w_0L^4}{EI} \downarrow$ ◄

(c) $\frac{dy}{dx}$ @ $x = L \qquad \left.\frac{dy}{dx}\right|_B = -\frac{w_0L^3}{EI}\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{24}\right) = -\frac{1}{8}\frac{w_0L^3}{EI}$

$\theta_B = \frac{1}{8}\frac{w_0L^3}{EI}$ ◄

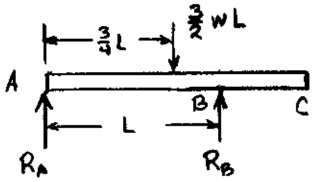
PROBLEM 9.5

9.5 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the slope at *A*, (*c*) the slope at *B*.



$[x=0, y=0]$ $[x=L, y=0]$

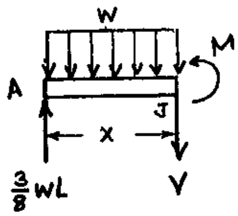
SOLUTION



$+\circlearrowleft \Sigma M_B = 0$

$-R_A L + (\frac{3}{2}wL)(\frac{1}{4}L) = 0$

$R_A = \frac{3}{8}wL$



For portion AB only $(0 \le x < L)$

$+\circlearrowleft \Sigma M_J = 0 \qquad -\frac{3}{8}wLx + (wx)\frac{x}{2} + M = 0$

$M = \frac{3}{8}wLx - \frac{1}{2}wx^2$

$$EI\frac{d^2y}{dx^2} = \frac{3}{8}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{3}{16}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$EI\,y = \frac{1}{16}wLx^3 - \frac{1}{24}wx^4 + C_1x + C_2$$

$[x=0, y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0] \qquad 0 = \frac{1}{16}wL^3 - \frac{1}{24}wL^4 + C_1L \qquad C_1 = -\frac{1}{48}wL^3$

(a) Elastic curve

$$y = \frac{w}{EI}\left(\frac{1}{16}Lx^3 - \frac{1}{24}x^4 - \frac{1}{48}L^3x\right)$$

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{3}{16}Lx^2 - \frac{1}{6}x^3 - \frac{1}{48}L^3\right)$$

(b) $\frac{dy}{dx}$ @ $x = 0$ $\qquad \left.\frac{dy}{dx}\right|_A = \frac{w}{EI}\left(0 - 0 - \frac{1}{48}L^3\right) = -\frac{wL^3}{48EI}$

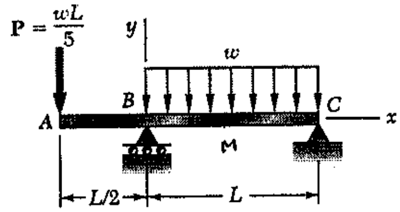
$$\theta_A = \frac{wL^3}{48EI}$$

(c) $\frac{dy}{dx}$ @ $x = L$ $\qquad \left.\frac{dy}{dx}\right|_B = \frac{w}{EI}\left(\frac{3}{16}L^3 - \frac{1}{6}L^3 - \frac{1}{48}L^3\right) = 0$

$$\theta_B = 0$$

PROBLEM 9.6

9.6 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *BC* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.



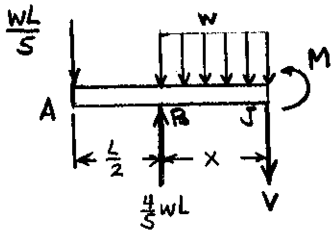
$x = -\frac{L}{2}$ $\begin{bmatrix} x=0 \\ y=0 \end{bmatrix}$ $\begin{bmatrix} x=L \\ y=0 \end{bmatrix}$

SOLUTION

Using ABC as a free body

$$\circlearrowleft \Sigma M_C = 0 \qquad \left(\frac{wL}{5}\right)\left(\frac{3L}{2}\right) - R_B L + (wL)\left(\frac{L}{2}\right) = 0$$

$$R_B = \frac{4}{5} wL$$



For portion BC only $\quad 0 < x < L$

$$\circlearrowleft \Sigma M_J = 0 \qquad \frac{wL}{5}\left(\frac{L}{2} + x\right) - \frac{4}{5} wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5} wLx - \frac{1}{2} wx^2 - \frac{1}{10} wL^2$$

$$EI \frac{d^2y}{dx^2} = \frac{3}{5} wLx - \frac{1}{2} wx^2 - \frac{1}{10} wL^2$$

$$EI \frac{dy}{dx} = \frac{3}{10} wLx^2 - \frac{1}{6} wx^3 - \frac{1}{10} wL^2 x + C_1$$

$$EI\, y = \frac{1}{10} wLx^3 - \frac{1}{24} wx^4 - \frac{1}{20} wL^2x^2 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\qquad 0 = 0 - 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\qquad 0 = \left(\frac{1}{10} - \frac{1}{24} - \frac{1}{20}\right) wL^4 + C_1 L + 0 \qquad C_1 = -\frac{1}{120} wL^3$

(a) Elastic curve $\qquad y = \frac{w}{EI}\left(\frac{1}{10} Lx^3 - \frac{1}{24} x^4 - \frac{1}{20} L^2x^2 - \frac{1}{120} L^3 x\right)$ ◄

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{3}{10} Lx^2 - \frac{1}{6} x^3 - \frac{1}{10} L^2 x - \frac{1}{120} L^3\right)$$

(b) y @ $x = \frac{L}{2}$

$$y_M = \frac{w}{EI}\left\{\frac{1}{10} L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20} L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120} L^3\left(\frac{L}{2}\right)\right\}$$

$$= \frac{wL^4}{EI}\left\{\frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240}\right\} = -\frac{13\, wL^4}{1920\, EI}$$

$$y_M = \frac{13\, wL^4}{1920\, EI} \downarrow$$ ◄

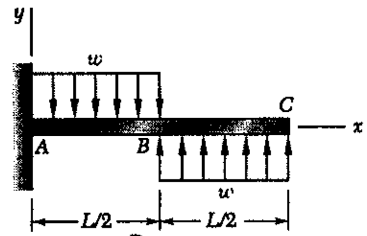
(c) $\frac{dy}{dx}$ @ $x = 0$

$$\left.\frac{dy}{dx}\right|_B = \frac{w}{EI}\left(0 - 0 - 0 - \frac{1}{120} L^3\right) = -\frac{wL^3}{120\, EI}$$

$$\theta_B = \frac{wL^3}{120\, EI}$$ ◄

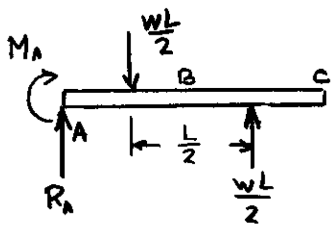
PROBLEM 9.7

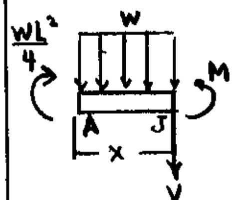
9.7 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



$[x = 0,\ y = 0]$

$[x = 0,\ \frac{dy}{dx} = 0]$





SOLUTION

Using ABC as a free body

$+\uparrow \Sigma F_y = 0 \qquad R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \qquad R_A = 0$

$+\circlearrowleft M_A = 0 \qquad -M_A + \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$

Using AJ as a free body (Portion AB only)

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{wL^2}{4} + (wx)\frac{x}{2} + M = 0$

$$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$$

$[x = 0,\ \frac{dy}{dx} = 0] \qquad 0 = 0 - 0 + C_1 \qquad C_1 = 0$

$$EIy = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$$

$[x = 0,\ y = 0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve

$$y = \frac{w}{EI}\left(\frac{1}{8}L^2x^2 - \frac{1}{24}x^4\right)$$

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{1}{4}L^2x - \frac{1}{6}x^3\right)$$

(b) y at $x = \frac{L}{2}$

$$y_B = \frac{w}{EI}\left\{\frac{1}{8}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{24}\left(\frac{L}{2}\right)^4\right\} = \frac{wL^4}{EI}\left\{\frac{1}{32} - \frac{1}{384}\right\}$$

$$= \frac{11\,wL^4}{384\,EI} \qquad y_B = \frac{11\,wL^4}{384\,EI}\ \uparrow$$

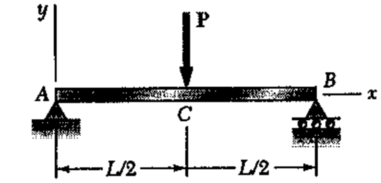
(c) $\frac{dy}{dx}$ at $x = \frac{L}{2}$

$$\theta_B = \frac{w}{EI}\left\{\frac{1}{4}L^2\left(\frac{L}{2}\right) - \frac{1}{6}\left(\frac{L}{2}\right)^3\right\} = \frac{wL^3}{EI}\left\{\frac{1}{8} - \frac{1}{48}\right\}$$

$$= \frac{5\,wL^3}{48\,EI} \qquad \theta_B = \frac{5\,wL^3}{48\,EI}$$

PROBLEM 9.8

9.8 For the beam shown with load **P**, determine (*a*) the equation of the elastic curve for portion *AC* of the beam, (*b*) the slope at *A*, (*c*) the deflection at *C*.



$[x=0,\ y=0]$ $\quad [x=\frac{L}{2},\ \frac{dy}{dx}=0]$

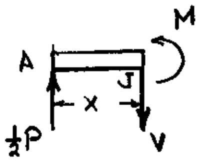
SOLUTION

Because of symmetry $\frac{dy}{dx}=0$ at $x=\frac{L}{2}$.

Reaction at A $\quad R_A=\frac{1}{2}P$

For portion AC only, using free body AJ

$$+\circlearrowleft M_J=0 \qquad -\tfrac{1}{2}Px+M=0$$



$$M=\tfrac{1}{2}Px$$

$$EI\frac{d^2y}{dx^2}=\tfrac{1}{2}Px$$

$$EI\frac{dy}{dx}=\tfrac{1}{4}Px^2+C_1$$

$[x=\frac{L}{2},\ \frac{dy}{dx}=0] \qquad 0=\frac{1}{4}P\left(\frac{L}{2}\right)^2+C_1 \qquad C_1=-\frac{1}{16}PL^2$

$$EIy=\tfrac{1}{12}Px^3+C_1x+C_2$$

$[x=0,\ y=0] \qquad 0=0+0+C_2 \qquad C_2=0$

(a) Elastic curve $\qquad y=\frac{P}{EI}\left(\frac{1}{12}x^3-\frac{1}{16}L^2x\right)$ ◄

$$\frac{dy}{dx}=\frac{P}{EI}\left(\tfrac{1}{4}x^2-\tfrac{1}{16}L^2\right)$$

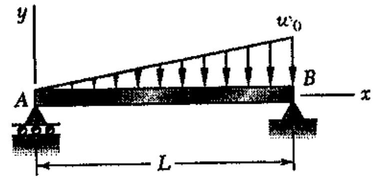
(b) $\frac{dy}{dx}$ at $x=0$ $\qquad \theta_A=\frac{P}{EI}\left(0-\frac{1}{16}L^2\right)=-\frac{PL^2}{16EI}$ or $\frac{PL^2}{16EI}$ ◄

(c) y at $x=\frac{L}{2}$ $\qquad y_C=\frac{P}{EI}\left\{\frac{1}{12}\left(\frac{L}{2}\right)^3-\frac{1}{16}L^2\left(\frac{L}{2}\right)\right\}=\frac{PL^3}{EI}\left\{\frac{1}{96}-\frac{1}{32}\right\}$

$$=-\frac{PL^3}{48EI} \quad \text{or} \quad \frac{PL^3}{48EI}\downarrow$$ ◄

PROBLEM 9.9

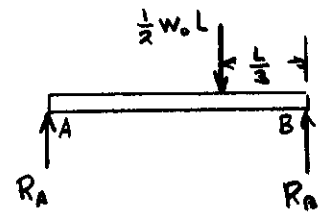
9.9 and 9.10 For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W18 × 50 rolled shape and that w_0 = 4.5 kips/ft, L = 18 ft, and $E = 29 \times 10^6$ psi.



$[x=0, y=0]$ $[x=L, y=0]$

SOLUTION

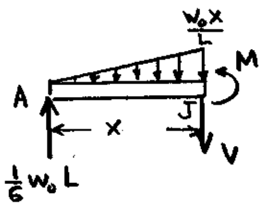
Using entire beam as a free body



$\circlearrowleft\Sigma M_B = 0 \qquad -R_A L + (\tfrac{1}{2}w_0 L)(\tfrac{L}{3}) = 0$

$R_A = \frac{1}{6}w_0 L$

Using AJ as a free body $\circlearrowleft\Sigma M_J = 0$



$-\frac{1}{6}w_0Lx + (\frac{1}{2}\frac{w_0x^2}{L})(\frac{x}{3}) + M = 0$

$M = \frac{1}{6}w_0Lx - \frac{1}{6}\frac{w_0}{L}x^3$

$$EI\frac{d^2y}{dx^2} = \frac{1}{6}w_0Lx - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{12}w_0Lx^2 - \frac{1}{24}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{36}w_0Lx^3 - \frac{1}{120}\frac{w_0}{L}x^5 + C_1x + C_2$$

$[x=0, y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0] \qquad 0 = \frac{1}{36}w_0L^4 - \frac{1}{120}w_0L^4 + C_1L + 0 \qquad C_1 = -\frac{7w_0L^3}{360}$

Elastic curve $\quad y = \frac{w_0}{EI}\left\{\frac{1}{36}Lx^3 - \frac{1}{120}\frac{x^5}{L} - \frac{7}{360}L^3x\right\}$

$$\frac{dy}{dx} = \frac{w_0}{EI}\left\{\frac{1}{12}Lx^2 - \frac{1}{24}\frac{x^4}{L} - \frac{7}{360}L^3\right\}$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$

$15x_m^4 - 30L^2x_m^2 + 7L^4 = 0 \qquad x_m^2 = \frac{30L^2 - \sqrt{900L^4 - 420L^4}}{30}$

$x_m^2 = (1 - \sqrt{\frac{8}{15}})L^2 = 0.2697L^2 \qquad x_m = 0.5193\ L$ ◄

$y_m = \frac{w_0}{EI}\left\{\frac{1}{36}L(0.5193L)^3 - \frac{1}{120}\frac{(0.5193L^5)}{L} - \frac{7}{360}L^3(0.5193L)\right\}$

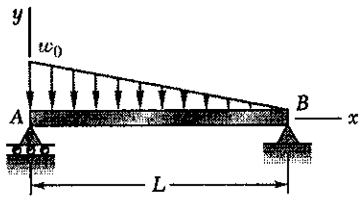
$= -0.00652\frac{w_0L^4}{EI}$ or $0.00652\frac{w_0L^4}{EI}\downarrow$ ◄

Data: $w_0 = 4.5$ kips/ft $= \frac{4500}{12} = 375$ lb/in, $\quad L = 18$ ft $= 216$ in,

$I = 800\ \text{in}^4$ for W18×50

$y_m = \frac{(0.00652)(375)(216)^4}{(29\times10^6)(800)} = 0.229$ in. ◄

PROBLEM 9.10



9.9 and 9.10 For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W18 × 50 rolled shape and that w_0 = 4.5 kips/ft, L = 18 ft, and $E = 29 \times 10^6$ psi.

SOLUTION

$[x=0, y=0]$ $[x=L, y=0]$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L-x)$$

$$V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + C_V = \frac{dM}{dx}$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_V x + C_M$$

$[x=0, M=0]$ $\quad 0 = 0 + 0 + 0 + C_M \qquad C_M = 0$

$[x=L, M=0]$ $\quad 0 = -\frac{w_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{6}L^3\right) + C_V L \qquad C_V = \frac{1}{3}w_0 L$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L}\left(\frac{1}{3}L^2x - \frac{1}{2}Lx^2 + \frac{1}{6}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{6}L^2x^2 - \frac{1}{6}Lx^3 + \frac{1}{24}x^4\right) + C_1$$

$$EIy = \frac{w_0}{L}\left(\frac{1}{18}L^2x^3 - \frac{1}{24}Lx^4 + \frac{1}{120}x^5\right) + C_1x + C_2$$

$[x=0, y=0]$ $\quad 0 = 0 - 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0]$ $\quad 0 = \frac{w_0}{L}\left\{\frac{1}{18}L^5 - \frac{1}{24}L^5 + \frac{1}{120}L^5\right\} + C_1L + 0 \qquad C_1 = -\frac{1}{45}w_0L^3$

$$y = \frac{w_0}{EIL}\left(\frac{1}{18}L^2x^3 - \frac{1}{24}Lx^4 + \frac{1}{120}x^5 - \frac{1}{45}L^4x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{6}L^2x^2 - \frac{1}{6}Lx^3 + \frac{1}{24}x^4 - \frac{1}{45}L^4\right)$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$.

$$f = \frac{1}{6}L^2x_m^2 - \frac{1}{6}Lx_m^3 + \frac{1}{24}x_m^4 - \frac{1}{45}L^4 = 0$$

Let $z = \frac{x_m}{L}$ $\qquad f(z) = \frac{1}{6}z^2 - \frac{1}{6}z^3 + \frac{1}{24}z^4 - \frac{1}{45}$

$$\frac{df}{dz} = \frac{1}{3}z - \frac{1}{2}z^2 + \frac{1}{6}z^3$$

By Newton-Raphson method $\quad z = z_0 - \frac{f(z_0)}{df/dz}$

$z = \cancel{0.5}, \cancel{0.4805}, 0.4807, 0.4807 \qquad x_m = 0.4807\ L$

$$y_m = \frac{w_0L^4}{EI}\left\{\frac{1}{18}(0.4807)^3 - \frac{1}{24}(0.4807)^4 + \frac{1}{120}(0.4807)^5 - \frac{1}{45}(0.4807)\right\}$$

$$= -0.00652\frac{w_0L^4}{EI} \qquad y_m = 0.00652\frac{w_0L^4}{EI}\downarrow$$

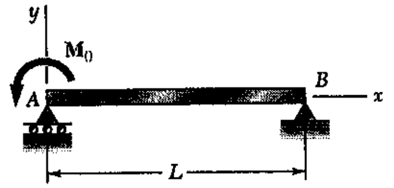
Data: $w_0 = 4.5$ kips/ft $= \frac{4500}{12} = 375$ lb/in, $\quad L = 18$ ft $= 216$ in.

$I = 800$ in^4 for W 18×50

$$y_m = \frac{(0.00652)(375)(216)^4}{(29\times10^6)(800)} = 0.229 \text{ in.}$$

PROBLEM 9.11

9.11 (*a*) Determine the location and magnitude of the maximum deflection of beam *AB*. (*b*) Assuming that beam *AB* is a W360 × 64, $L = 3.5$ m and $E = 200$ GPa, calculate the maximum allowable value of the applied moment $\mathbf{M}_0$ if the maximum deflection is not to exceed 1 mm.



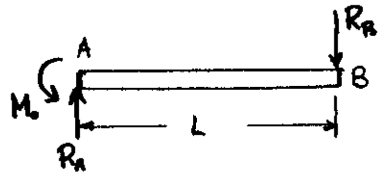
$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

SOLUTION

Using entire beam as a free body

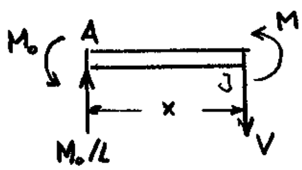
$$\circlearrowleft \Sigma M_B = 0 \qquad M_0 - R_A L = 0 \qquad R_A = \frac{M_0}{L}$$

Using portion AJ



$$+\circlearrowleft \Sigma M_J = 0 \qquad M_0 - \frac{M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(x - L)$$



$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\tfrac{1}{2}x^2 - Lx\right) + C_1$$

$$EIy = \frac{M_0}{L}\left(\tfrac{1}{6}x^3 - \tfrac{1}{2}Lx^2\right) + C_1x + C_2$$

$[x=0, y=0]$ $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0]$ $\qquad 0 = \frac{M_0}{L}\left(\tfrac{1}{6}L^3 - \tfrac{1}{2}L^3\right) + C_1L + 0 \qquad C_1 = \tfrac{1}{3}M_0L$

$$y = \frac{M_0}{EIL}\left(\tfrac{1}{6}x^3 - \tfrac{1}{2}Lx^2 + \tfrac{1}{3}L^2x\right) \qquad \frac{dy}{dx} = \frac{M_0}{EIL}\left(\tfrac{1}{2}x^2 - Lx + \tfrac{1}{3}L^2\right)$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$

$$\tfrac{1}{2}x_m^2 - Lx_m + \tfrac{1}{3}L^2 = 0 \qquad x_m = L - \sqrt{L^2 - (4)(\tfrac{1}{2})(\tfrac{1}{3}L^2)} = \left(1 - \sqrt{\tfrac{1}{3}}\right)L$$

$$= 0.42265\,L$$

$$y_m = \frac{M_0L^2}{EI}\left\{(\tfrac{1}{6})(0.42265)^3 - (\tfrac{1}{2})(0.42265)^2 + (\tfrac{1}{3})(0.42265)\right\} = 0.06415\,\frac{M_0L^2}{EI}\uparrow$$

Solving for M_0 $\qquad M_0 = \dfrac{EI\,y_m}{0.06415\,L^2}$

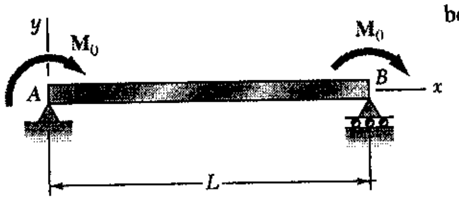
Data: $\quad E = 200 \times 10^9$ Pa, $\quad I = 178 \times 10^6\ \text{mm}^4 = 178 \times 10^{-6}\ \text{m}^4$

$\qquad L = 3.5$ m, $\quad y_m = 1$ mm $= 10^{-3}$ m

$$M_0 = \frac{(200\times10^9)(178\times10^{-6})(10^{-3})}{(0.06415)(3.5)^2} = 45.3 \times 10^3\ \text{N}\cdot\text{m} = 45.3\ \text{kN}\cdot\text{m}$$

PROBLEM 9.12

9.12 (*a*) Determine the location and magnitude of the maximum absolute deflection in *AB* between *A* and the center of the beam. (*b*) Assuming that beam *AB* is a W460 × 113, M_0 = 224 kN·m and E = 200 GPa, determine the maximum allowable length L of the beam if the maximum deflection is not to exceed1.2 mm.



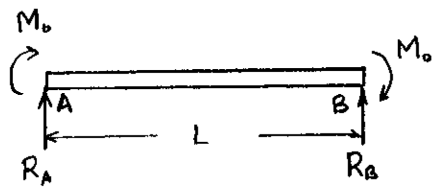
$[x=0,\ y=0]$ $\qquad$ $[x=L,\ y=0]$

SOLUTION

Using AB as a free body

$$\Sigma M_B = 0 \qquad -2M_0 - R_A L = 0$$

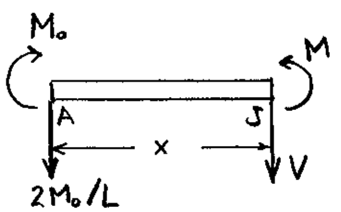
$$R_A = -\frac{2M_0}{L}$$



Using portion AJ as a free body

$$\Sigma M_J = 0 \qquad -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$



$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EI\,y = \frac{M_0}{L}\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{3}x^3\right) + C_1 x + C_2$$

$$[x=0,\ y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x=L,\ y=0] \qquad 0 = \frac{M_0}{L}\left(\tfrac{1}{2}L^3 - \tfrac{1}{3}L^3\right) + C_1 L + 0 \qquad C_1 = -\tfrac{1}{6}M_0 L^2$$

$$y = \frac{M_0}{EIL}\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{3}x^3 - \tfrac{1}{6}L^2 x\right) \qquad \frac{dy}{dx} = \frac{M_0}{EIL}\left(Lx - x^2 - \tfrac{1}{6}L^2\right)$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$

$$x_m^2 - Lx_m - \tfrac{1}{6}L^2 = 0 \qquad x_m = \frac{L - \sqrt{L^2 - (4)(\frac{1}{6}L^2)}}{2} = \tfrac{1}{2}\left(1 - \tfrac{\sqrt{3}}{3}\right)L = 0.21132\ L$$

$$y_m = \frac{M_0 L^2}{EI}\left\{(\tfrac{1}{2})(0.21132)^2 - (\tfrac{1}{3})(0.21132)^3 - (\tfrac{1}{6})(0.21132)\right\} = -0.0160375\frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375\frac{M_0 L^2}{EI}$$

Solving for L $\qquad L = \left\{\dfrac{EI\,|y_m|}{0.0160375 M_0}\right\}^{1/2}$

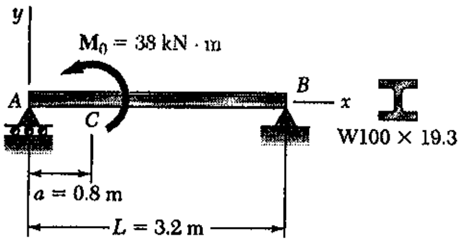
Data: $E = 200 \times 10^9$ Pa, $\quad I = 556 \times 10^6\ \text{mm}^4 = 556 \times 10^{-6}\ \text{m}^4$

$|y_m| = 1.2\ \text{mm} = 1.2 \times 10^{-3}\ \text{m}, \qquad M_0 = 224 \times 10^3\ \text{N·m}$

$$L = \left\{\frac{(200\times10^9)(556\times10^{-6})(1.2\times10^{-3})}{(0.0160375)(224\times10^3)}\right\}^{1/2} = 6.09\ \text{m}$$

PROBLEM 9.13

9.13 and 9.14 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



$[x=0, y=0]$ $[x=L, y=0]$

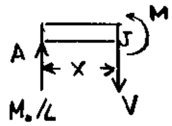
$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

SOLUTION

Reactions: $R_A = M_0/L \uparrow$, $R_B = M_0/L \downarrow$

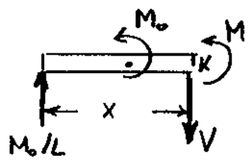
$0 < x < a$ $\quad \circlearrowleft \Sigma M_J = 0$



$$-\frac{M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}x$$

$a < x < L$ $\quad + \circlearrowright M_K = 0$



$$-\frac{M_0}{L}x + M_0 + M = 0$$

$$M = \frac{M_0}{L}(x - L)$$

$0 < x < a$	$a < x < L$
$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}x$	$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x-L)$
$EI\frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2) + C_1 \qquad (1)$	$EI\frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2 - Lx) + C_3 \qquad (3)$
$EIy = \frac{M_0}{L}(\frac{1}{6}x^3) + C_1x + C_2 \qquad (2)$	$EIy = \frac{M_0}{L}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2) + C_3x + C_4 \qquad (4)$

$[x=0, y=0]$ Eq. (2): $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) & (3): $\quad \frac{M_0}{L}(\frac{1}{2}a^2) + C_1 = \frac{M_0}{L}(\frac{1}{2}a^2 - La) + C_3$

$$C_3 = C_1 + M_0 a$$

$[x=a, y=y]$ Eqs (2) & (4) $\quad \frac{M_0}{L}(\cancel{\frac{1}{6}a^3}) + \cancel{C_1 a} = \frac{M_0}{L}(\cancel{\frac{1}{6}a^3} - \frac{1}{2}La^2) + (\cancel{C_1} + M_0a)a + C_4$

$$C_4 = -\frac{1}{2}M_0a^2$$

$[x=L, y=0]$ Eq (4) $\quad \frac{M_0}{L}(\frac{1}{6}L^3 - \frac{1}{2}L^3) + (C_1 + M_0a)L - \frac{1}{2}M_0a^2 = 0$

$$C_1 = \frac{M_0}{L}(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)$$

Elastic curve for $0 < x < a$ $\qquad y = \frac{M_0}{EIL}\left[\frac{1}{6}x^3 + (\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)x\right]$

Make $x = a$ $\qquad y_C = \frac{M_0}{EIL}\left[\frac{1}{6}a^3 + \frac{1}{3}L^2a + \frac{1}{2}a^3 - a^2L\right] = \frac{M_0}{EIL}\left[\frac{2}{3}a^3 + \frac{1}{3}L^2a - La^2\right]$

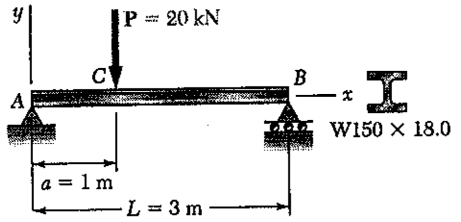
Data: $E = 200 \times 10^9$ Pa, $I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6}$ m, $M_0 = 38 \times 10^3$ N·m

$$y_C = \frac{(38 \times 10^3)[(2)(0.8)^3/3 + (3.2)^2(0.8)/3 - (3.2)(0.8)^2]}{(200 \times 10^9)(4.77 \times 10^{-6})(3.2)} = 12.75 \times 10^{-3} \text{ m}$$

$$= 12.75 \text{ mm}$$

PROBLEM 9.14

9.13 and 9.14 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



$[x=0, y=0]$ $[x=L, y=0]$
$[x=a, y=y]$
$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

SOLUTION

Let $b = L - a$

Reactions: $R_A = \frac{Pb}{L}\uparrow$, $R_B = \frac{Pa}{L}\uparrow$

Bending moments

$0 < x < a \qquad M = \frac{Pb}{L}x$

$a < x < L \qquad M = \frac{P}{L}[bx - L(x-a)]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EI\,y = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x-a)^2\right] + C_3 \qquad (2)$$

$$EIy = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x-a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x=0, y=0]$ Eq (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) and (3) $\quad \frac{P}{L}\left(\cancel{\frac{1}{2}ba^2}\right) + C_1 = \frac{P}{L}\left[\cancel{\frac{1}{2}ba^2} + 0\right] + C_3 \therefore C_3 = C_1$

$[x=a, y=y]$ Eqs (2) and (4) $\quad \frac{P}{L}\left(\cancel{\frac{1}{6}ba^3}\right) + C_1a + C_2$
$= \frac{P}{L}\left[\cancel{\frac{1}{6}ba^3} + 0\right] + \cancel{C_1a} + C_4 \qquad C_4 = C_2 = 0$

$[x=L, y=0]$ Eq. (4) $\quad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3L = 0$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq (2)

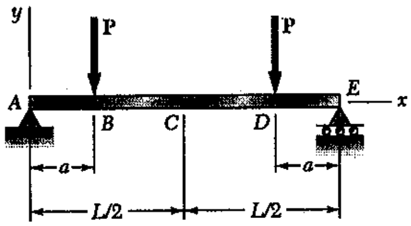
$$y_c = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 20\times10^3$ N, $E = 200\times10^9$ Pa, $I = 9.17\times10^6\ \text{mm}^4 = 9.17\times10^{-6}\ \text{m}^4$
$L = 3$ m, $a = 1$ m, $b = 2$ m

$$y_c = \frac{(20\times10^3)[(2)(1)^3 + (2)^3(1) - (3)^2(1)(2)]}{(6)(200\times10^9)(9.17\times10^{-6})(3)} = -4.85\times10^{-3}\ \text{m}$$

i.e $\quad 4.85$ mm $\downarrow$

PROBLEM 9.15



9.15 Knowing that beam *AE* is an S200 × 27.4 rolled shape and that $P = 17.5$ kN, $L = 2.5$ m, $a = 0.8$ m and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion *BD*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Consider portion ABC only, and consider symmetry about C.

Reactions $R_A = R_E = P$

Boundary conditions: $[x=0,\ y=0]$, $[x=a,\ y=y]$, $[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$, $[x=\frac{L}{2},\ \frac{dy}{dx}=0]$

$\underline{0 < x < a}$

$$EI\frac{d^2y}{dx^2} = M = Px$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}Px^2 + C_1 \qquad (1)$$

$$EIy = \tfrac{1}{6}Px^3 + C_1x + C_2 \qquad (2)$$

$$[x=0,\ y=0] \rightarrow C_2 = 0$$

$\underline{a < x < L-a}$

$$EI\frac{d^2y}{dx^2} = M = Pa$$

$$EI\frac{dy}{dx} = Pax + C_3$$

$$EIy = \tfrac{1}{2}Pax^2 + C_3x + C_4$$

$$[x=\tfrac{L}{2},\ \tfrac{dy}{dx}=0] \rightarrow C_3 = -\tfrac{1}{2}PaL$$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}]$ $\quad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \qquad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$

$[x=\frac{L}{2},\ y=y]$ $\quad \frac{1}{6}Pa^3 + (\frac{1}{2}Pa^2 - \frac{1}{2}PaL)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$

$$C_4 = \tfrac{1}{6}Pa^3$$

(a) Elasic curve for portion BD

$$y = \frac{1}{EI}\left(\tfrac{1}{2}Pax^2 + C_3x + C_4\right)$$

$$= \frac{P}{EI}\left(\tfrac{1}{2}ax^2 - \tfrac{1}{2}aLx + \tfrac{1}{6}a^3\right)$$

For deflection at C set $x = \frac{L}{2}$

$$y_c = \frac{P}{EI}\left(\tfrac{1}{8}aL^2 - \tfrac{1}{4}aL^2 + \tfrac{1}{6}a^3\right) = -\frac{Pa}{EI}\left(\tfrac{1}{8}L^2 - \tfrac{1}{6}a^2\right)$$

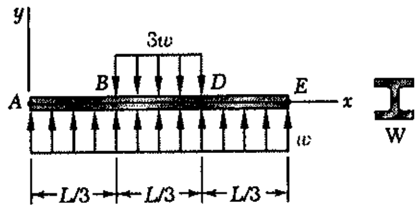
Data: $I = 23.9\times10^6\ \text{mm}^4 = 23.9\times10^{-6}\ \text{m}^4$, $E = 200\times10^9$ Pa

$P = 17.5\times10^3$ N, $L = 2.5$ m, $a = 0.8$ m

(b) $$y_c = -\frac{(17.5\times10^3)(0.8)}{(200\times10^9)(23.9\times10^6)}\left\{\frac{2.5^2}{8} - \frac{0.8^2}{6}\right\} = -1.976\times10^{-3}\ \text{m}$$

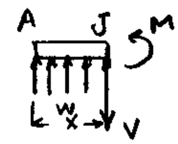
$$y_c = 1.976\ \text{mm} \downarrow$$

PROBLEM 9.16



9.16 Uniformly distributed loads are applied to beam AE as shown. (*a*) Selecting the x axis through the centers A and E of the end sections of the beam, determine the equation of the elastic curve for portion AB of the beam. (*b*) Knowing that the beam is a W200 × 35.9 rolled shape and that $L = 3$ m, $w = 5$ kN/m, and $E = 200$ GPa, determine the distance of the center of the beam from the x axis.

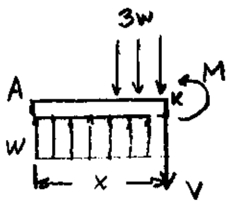
SOLUTION



$0 < x < \frac{L}{3}$ $\quad \circlearrowleft \Sigma M_J = 0$

$-(wx)(\frac{x}{2}) + M = 0$

$M = \frac{1}{2}wx^2$



$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{6}wx^3 + C_1$

$EIy = \frac{1}{24}wx^4 + C_1x + C_2$

$\frac{L}{3} \le x \le \frac{2L}{3}$ $\quad +\circlearrowleft \Sigma M_K = 0$ $\quad -(wx)(\frac{x}{2}) + 3w(x - \frac{L}{3})(\frac{x-\frac{L}{3}}{2}) + M = 0$

$M = \frac{1}{2}wx^2 - \frac{3}{2}w(x - \frac{L}{3})^2$

$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{3}{2}w(x - \frac{L}{3})^2$

$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{1}{2}w(x - \frac{L}{3})^3 + C_3$

$EIy = \frac{1}{24}wx^4 - \frac{1}{8}w(x - \frac{L}{3})^4 + C_3x + C_4$

$[x = 0, y = 0]$ $\quad 0 = 0 + 0 + C_2$ $\quad C_2 = 0$

$[x = \frac{L}{3}, \frac{dy}{dx} = \frac{dy}{dx}]$ $\quad \cancel{\frac{1}{6}w(\frac{L}{3})^3} + C_1 = \cancel{\frac{1}{6}w(\frac{L}{3})^3} + 0 + C_3$ $\quad C_1 = C_3$

$[x = \frac{L}{3}, y = y]$ $\quad \cancel{\frac{1}{24}w(\frac{L}{3})^4} + \cancel{C_1\frac{L}{3}} + C_2 = \cancel{\frac{1}{24}w(\frac{L}{3})^4} + 0 + \cancel{C_3\frac{L}{3}} + C_4$

$C_4 = C_2 = 0$

Symmetry boundary condition $[x = \frac{L}{2}, \frac{dy}{dx} = 0]$

$\frac{1}{6}w(\frac{L}{2})^3 - \frac{1}{2}w(\frac{L}{2} - \frac{L}{3})^3 + C_3 = 0$ $\quad C_3 = -(\frac{1}{48} - \frac{1}{432})wL^3 = -\frac{1}{54}wL^3$

(a) Elastic curve for portion AB

$y = \frac{1}{EI}\left\{\frac{1}{24}wx^4 + C_1x + C_2\right\} = \frac{w}{EI}\left(\frac{1}{24}x^4 - \frac{1}{54}L^3x\right)$ ◄

(b) Deflection at center $\quad y_c = \frac{1}{EI}\left\{\frac{1}{24}w(\frac{L}{2})^4 - \frac{1}{8}w(\frac{L}{2} - \frac{L}{3})^4 - \frac{1}{54}wL^3(\frac{L}{2}) + 0\right\}$

$= \frac{wL^4}{EI}\left\{\frac{1}{384} - \frac{1}{10368} - \frac{1}{108}\right\} = -\frac{35}{5184}\frac{wL^4}{EI}$

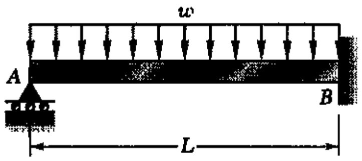
Data: $I = 34.4 \times 10^6\ mm^4 = 34.4 \times 10^{-6}\ m^4$, $E = 200 \times 10^9$ Pa, $L = 3$ m

$w = 5 \times 10^3$ N/m

$y_c = -\frac{35}{5184}\frac{(5 \times 10^3)(3)^4}{(200 \times 10^9)(34.4 \times 10^{-6})} = -397 \times 10^{-6}$ m, ie 0.397 mm ↓

PROBLEM 9.17

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.

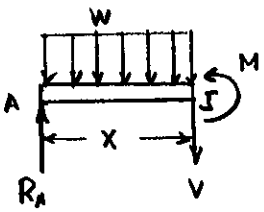


$[x=0, y=0]$ $[x=L, y=0]$ $[x=L, \frac{dy}{dx}=0]$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left.



$\circlearrowleft \Sigma M_J = 0 \qquad -R_A x + (wx)\frac{x}{2} + M = 0$

$M = -\frac{1}{2}wx^2 + R_A x$

$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + R_A x$

$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}R_A x^2 + C_1$

$EI\,y = -\frac{1}{24}wx^4 + \frac{1}{6}R_A x^3 + C_1 x + C_2$

$[x=0, y=0] \qquad 0 = -0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, \frac{dy}{dx}=0] \qquad 0 = -\frac{1}{6}wL^3 + \frac{1}{2}R_A L^2 + C_1 \qquad C_1 = \frac{1}{6}wL^3 - \frac{1}{2}R_A L^2$

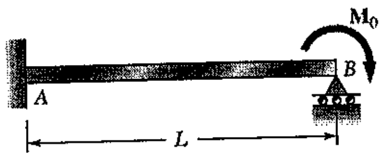
$[x=L, y=0] \qquad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}R_A L^3 + (\frac{1}{6}wL^3 - \frac{1}{2}R_A L^2)L + 0$

$(\frac{1}{2}-\frac{1}{6})R_A = (\frac{1}{6}-\frac{1}{24})wL \qquad \frac{1}{3}R_A = \frac{1}{8}wL$

$R_A = \frac{3}{8}wL \uparrow$

PROBLEM 9.18

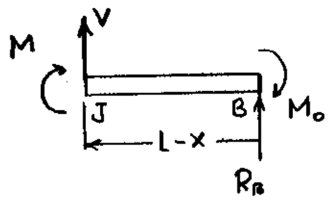
9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$ $[x=L, y=0]$
$[x=0, \frac{dy}{dx}=0]$

SOLUTION

Reactions are statically indeterminate.
Boundary condition are shown at left.
Using free body JB



$$\circlearrowleft \Sigma M_J = 0 \qquad -M + R_B(L-x) - M_0 = 0$$

$$M = -M_0 + R_B(L-x)$$

$$EI\frac{d^2y}{dx^2} = -M_0 + R_B(L-x)$$

$$EI\frac{dy}{dx} = -M_0x + R_B(Lx - \tfrac{1}{2}x^2) + C_1$$

$$EI\,y = -\tfrac{1}{2}M_0x^2 + R_B(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3) + C_1x + C_2$$

$[x=0, y=0]$ $\quad 0 = -0 + 0 - 0 + 0 + C_2 \qquad C_2 = 0$

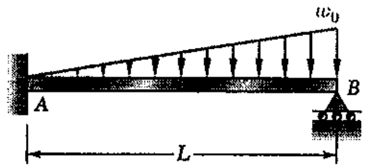
$[x=0, \frac{dy}{dx}=0]$ $\quad 0 = -0 + 0 - 0 + C_1 \qquad C_1 = 0$

$[x=L, y=0]$ $\quad 0 = -\tfrac{1}{2}M_0L^2 + R_B(\tfrac{1}{2}L^3 - \tfrac{1}{6}L^3)$

$$\tfrac{1}{3}R_B = \tfrac{1}{2}\frac{M_0}{L} \qquad R_B = \tfrac{3}{2}\frac{M_0}{L}\uparrow$$

PROBLEM 9.19

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



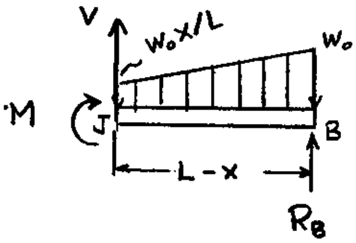
$[x=0, y=0]$ $[x=L, y=0]$
$[x=0, \frac{dy}{dx}=0]$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body JB $\circlearrowleft \Sigma M_J = 0$



$$-M + R_B(L-x) + \tfrac{1}{2} w_0 (L-x) \tfrac{2}{3}(L-x) + \tfrac{1}{2}\frac{w_0 x}{L}(L-x)\tfrac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}\left[2L(L-x)^2 + x(L-x)^2\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}\left[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{dy}{dx} = R_B(Lx - \tfrac{1}{2}x^2) - \frac{w_0}{6L}\left(\tfrac{1}{4}x^4 - \tfrac{3}{2}L^2x^2 + 2L^3x\right) + C_1$$

$$EIy = R_B\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) - \frac{w_0}{6L}\left(\tfrac{1}{20}x^5 - \tfrac{1}{2}L^2x^3 + L^3x^2\right) + C_1x + C_2$$

$[x=0, y=0] \longrightarrow C_2 = 0$

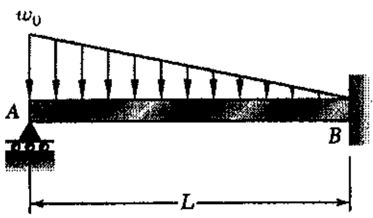
$[x=0, \frac{dy}{dx}=0] \longrightarrow C_1 = 0$

$[x=L, y=0] \quad 0 = R_B L^3\left(\tfrac{1}{2} - \tfrac{1}{6}\right) - \frac{w_0 L^4}{6}\left(\tfrac{1}{20} - \tfrac{1}{2} + 1\right)$

$\tfrac{1}{3}R_B = \left(\tfrac{1}{6}\right)\left(\tfrac{11}{20}\right) w_0 L \qquad R_B = \tfrac{11}{40} w_0 L \uparrow$

PROBLEM 9.20

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.

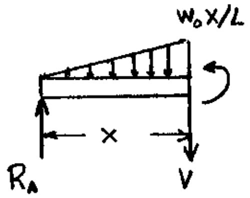


SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left.

$[x=0, y=0]$ $\quad$ $[x=L, y=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$



$$w = \frac{w_0}{L}(L-x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L-x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\frac{1}{6}Lx^3 - \frac{1}{24}x^4\right) + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = -\frac{w_0}{L}\left(\frac{1}{24}Lx^4 - \frac{1}{120}x^5\right) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0, y=0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, \frac{dy}{dx}=0]$ $\qquad -\frac{w_0}{L}\left(\frac{1}{6}L^4 - \frac{1}{24}L^4\right) + \frac{1}{2}R_A L^2 + C_1 = 0$

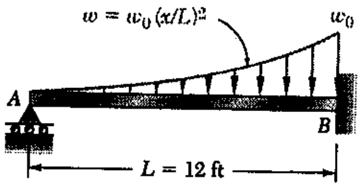
$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$[x=L, y=0]$ $\qquad -\frac{w_0}{L}\left(\frac{1}{24}L^4 - \frac{1}{120}L^4\right) + \frac{1}{6}R_A L^3 + \left(\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$

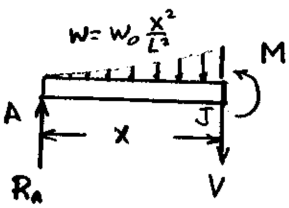
$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{8} - \frac{1}{24} + \frac{1}{120}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L \qquad R_A = \frac{11}{40}w_0 L \uparrow$$

PROBLEM 9.21



$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$ $\quad$ $[x=L,\ \frac{dy}{dx}=0]$



9.21 For the beam shown, determine the reaction at the roller support when $w_0 = 6$ kips/ft.

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left

$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2}\frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L^2}\frac{x^5}{60} + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = -\frac{w_0}{L^2}\frac{x^6}{360} + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0,\ y=0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L,\ \frac{dy}{dx}=0]$ $\quad -\frac{1}{60}w_0 L^3 + \frac{1}{2}R_A L^2 + C_1 = 0 \qquad C_1 = \frac{1}{60}w_0 L^3 - \frac{1}{2}R_A L^2$

$[x=L,\ y=0]$ $\quad -\frac{1}{360}w_0 L^4 + \frac{1}{6}R_A L^3 + \left(\frac{1}{60}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{60} - \frac{1}{360}\right)w_0 L$$

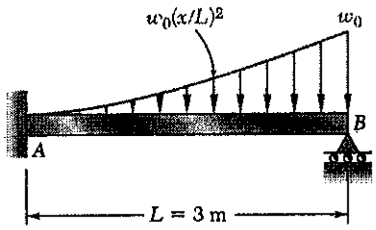
$$\frac{1}{3}R_A = \frac{1}{72}w_0 L \qquad R_A = \frac{1}{18}w_0 L$$

Data: $w_0 = 6$ kips/ft, $L = 12$ ft

$$R_A = \frac{1}{18}(6)(12) = 3 \text{ kips} \uparrow$$

PROBLEM 9.22

9.22 For the beam shown, determine the reaction at the roller support when $w_0 = 15$ kN/m.



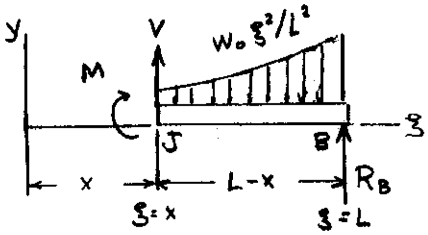
SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left.

$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$



Using free body JB $\quad \circlearrowleft \Sigma M_J = 0$

$$-M + \int_x^L \frac{w_0}{L^2}\xi^2(\xi - x)\,d\xi + R_B(L-x) = 0$$

$$M = \frac{w_0}{L^2}\int_x^L \xi^2(\xi - x)\,d\xi - R_B(L-x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}\xi^4 - \frac{1}{3}x\xi^3\right)\Big|_x^L - R_B(L-x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L-x)$$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L-x)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4x - \frac{1}{6}L^3x^2 + \frac{1}{60}x^5\right) - R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EI\,y = \frac{w_0}{L^2}\left(\frac{1}{8}L^4x^2 - \frac{1}{18}L^3x^3 + \frac{1}{360}x^6\right) - R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1x + C_2$$

$[x=0,\ \frac{dy}{dx}=0]$ $\qquad 0 = 0 + 0 + C_1 \qquad C_1 = 0$

$[x=0,\ y=0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L,\ y=0]$ $\qquad \left(\frac{1}{8} - \frac{1}{18} + \frac{1}{360}\right)w_0L^4 - \left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = 0$

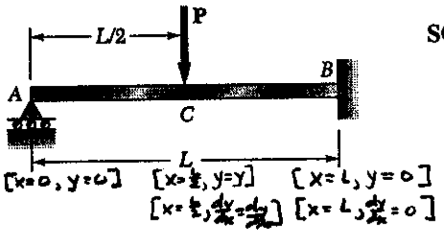
$$\frac{13}{180}w_0L^4 - \frac{1}{3}R_BL^3 = 0 \qquad R_B = \frac{13}{60}w_0L$$

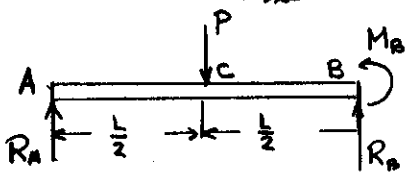
Data: $w_0 = 15\times10^3$ N/m $\qquad L = 3$ m

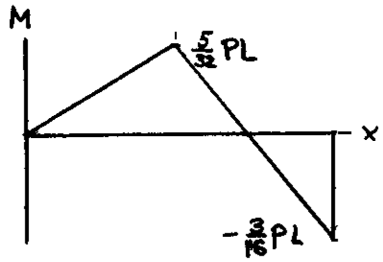
$$R_B = \frac{13}{60}(15\times10^3)(3) = 9.75\times10^3 \text{ N} = 9.75 \text{ kN} \uparrow$$

PROBLEM 9.23

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.







SOLUTION

Reactions are statically indeterminate

$0 < x < \frac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = R_A x \qquad (1)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \qquad (2)$$

$$EI\,y = \frac{1}{6}R_A x^3 + C_1 x + C_2 \qquad (3)$$

$\frac{L}{2} < x < L$

$$EI\frac{d^2y}{dx^2} = M = R_A x - P\left(x - \frac{L}{2}\right) \qquad (4)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{2}P\left(x-\frac{L}{2}\right)^2 + C_3 \qquad (5)$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{6}P\left(x-\frac{L}{2}\right)^3 + C_3 x + C_4 \qquad (6)$$

$[x=0,\ y=0]$ $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}]$ $\quad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + 0 + C_3 \qquad C_3 = C_1$

$[x=\frac{L}{2},\ y=y]$ $\quad \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + C_1\frac{L}{2} + \cancel{C_2} = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + 0 + C_3\frac{L}{2} + C_4$

$C_4 = 0$

$[x=L,\ \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A L^2 - \frac{1}{2}P\left(\frac{L}{2}\right)^2 + C_3 = 0 \qquad C_3 = \frac{1}{8}PL^2 - \frac{1}{2}R_A L^2$

$[x=L,\ y=0]$ $\quad \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{L}{2}\right)^3 + \left(\frac{1}{8}PL^2 - \frac{1}{2}R_A L^2\right)L + 0 = 0$

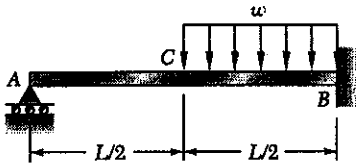
$\left(\frac{1}{2} - \frac{1}{6}\right)R_A L^3 = \left(\frac{1}{8} - \frac{1}{48}\right)PL^3 \qquad \frac{1}{3}R_A = \frac{5}{48}P \qquad R_A = \frac{5}{16}P \uparrow$ ◄

From (1), with $x = \frac{L}{2}$ $\qquad M_C = R_A\frac{L}{2} = \frac{5}{32}PL$ ◄

From (4), with $x = L$ $\qquad M_B = R_A L - \frac{1}{2}PL = -\frac{3}{16}PL$ ◄

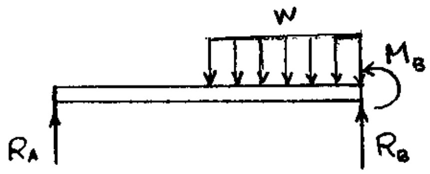
PROBLEM 9.24

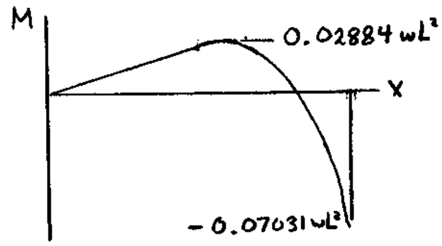
9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Reactions are statically indeterminate.





$0 < x < \frac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = R_A x \qquad (1)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \qquad (2)$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2 \qquad (3)$$

$\frac{L}{2} < x < L$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}w\left(x-\frac{L}{2}\right)^2 \qquad (4)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}w\left(x-\frac{L}{2}\right)^3 + C_3 \qquad (5)$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}w\left(x-\frac{L}{2}\right)^4 + C_3 x + C_4 \qquad (6)$$

$[x=0,\ y=0]$ $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}]$ $\quad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + 0 + C_3 \qquad C_1 = C_3$

$[x=\frac{L}{2},\ y=y]$ $\quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \cancel{C_1\frac{L}{2}} + C_2 = \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - 0 + \cancel{C_3\frac{L}{2}} + C_4 \qquad C_2 = C_4 = 0$

$[x=L,\ \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_3 = 0 \qquad C_3 = \frac{1}{48}wL^3 - \frac{1}{2}R_A L^2$

$[x=L,\ y=0]$ $\quad \frac{1}{6}R_A L^3 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\frac{1}{48}wL^3 - \frac{1}{2}R_A L^2\right)L + 0 = 0$

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3 = \left(\frac{1}{48}-\frac{1}{384}\right)wL^4 \qquad \frac{1}{3}R_A = \frac{7}{384}wL \qquad R_A = \frac{7}{128}wL\uparrow$$

From (1), with $x = \frac{L}{2}$ $\quad M_C = R_A\left(\frac{L}{2}\right) = \frac{7}{256}wL^2 = 0.02734\,wL^2$

From (4), with $x = L$ $\quad M_B = R_A L - \frac{1}{2}w\left(\frac{L}{2}\right)^2 = \left(\frac{7}{128}-\frac{1}{8}\right)wL - \frac{9}{128}wL^2$

$$= -0.07031\,wL$$

Location of maximum positive M

$\frac{L}{2} < x < L \qquad V_m = R_A - w\left(x_m - \frac{L}{2}\right) = 0 \qquad x_m - \frac{L}{2} = \frac{R_A}{w} = \frac{7}{128}L$

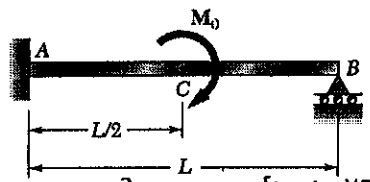
$$x_m = \frac{L}{2} + \frac{7}{128}L = \frac{71}{128}L$$

From (4), with $x = x_m$ $\quad M_m = R_A x_m - \frac{1}{2}w\left(x_m - \frac{L}{2}\right)^2$

$$= \left(\frac{7}{128}wL\right)\left(\frac{71}{128}L\right) - \frac{1}{2}w\left(\frac{7}{128}L\right)^2 = 0.02884\,wL^2$$

PROBLEM 9.26

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$

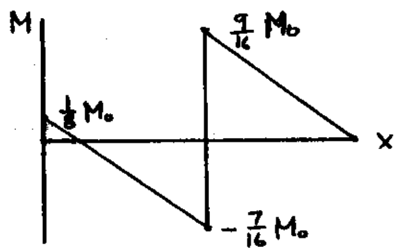
SOLUTION

Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B = 0 \qquad R_A = -R_B$

$\circlearrowleft \Sigma M_A = 0 \qquad -M_A - M_0 + R_B L = 0$

$$M_A = R_B L - M_0$$



$0 < x < \frac{L}{2}$

$M = R_A x + M_A = -M_0 + R_B L - R_B x$

$EI\frac{d^2y}{dx^2} = -M_0 + R_B(L-x)$

$EI\frac{dy}{dx} = -M_0 x + R_B(Lx - \frac{1}{2}x^2) + C_1$

$EI\,y = -\frac{1}{2}M_0 x^2 + R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_1 x + C_2$

$\frac{L}{2} < x < L \qquad M = R_B(L-x)$

$EI\frac{d^2y}{dx^2} = R_B(L-x)$

$EI\frac{dy}{dx} = R_B(Lx - \frac{1}{2}x^2) + C_3$

$EI\,y = R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_3 x + C_4$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \qquad -M_0\frac{L}{2} + \cancel{R_B(\frac{1}{2}L^2 - \frac{1}{6}L^2)} = \cancel{R_B(\frac{1}{2}L^2 - \frac{1}{6}L^2)} + C_3 \qquad C_3 = -\frac{M_0 L}{2}$

$[x=\frac{L}{2}, y=y] \qquad -\frac{1}{2}M_0(\frac{L}{2})^2 + \cancel{R_B(\frac{1}{8}L^3 - \frac{1}{48}L^3)} = \cancel{R_B(\frac{1}{8}L^3 - \frac{1}{48}L^3)} + C_3\frac{L}{2} + C_4$

$C_4 = -\frac{1}{8}M_0L^2 - \frac{1}{2}C_3 L = (-\frac{1}{8} + \frac{1}{4})M_0L^2 = \frac{1}{8}M_0L^2$

$[x=L, y=0] \qquad R_B(\frac{1}{2}L^3 - \frac{1}{6}L^3) + \frac{M_0L}{2}L + \frac{1}{8}M_0L^2 = 0$

$(\frac{1}{2} - \frac{1}{6})R_BL^3 = (\frac{1}{2} - \frac{1}{8})M_0L^2 \qquad \frac{1}{3}R_B = \frac{3}{8}\frac{M_0}{L}$

$$R_B = \frac{9}{8}\frac{M_0}{L}\uparrow$$ ◄

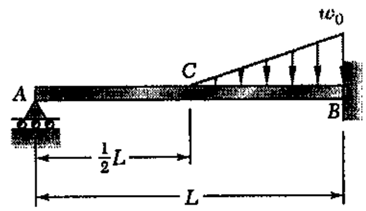
$M_A = \frac{9}{8}M_0 - M_0 = \frac{1}{8}M_0$ ◄

$M_{C^-} = -M_0 + \frac{9}{8}\frac{M_0}{L}\frac{L}{2} = -\frac{7}{16}M_0$ ◄

$M_{C^+} = R_B(L - \frac{L}{2}) = \frac{9}{8}\frac{M_0}{L}(\frac{L}{2}) = \frac{9}{16}M_0$ ◄

PROBLEM 9.25

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

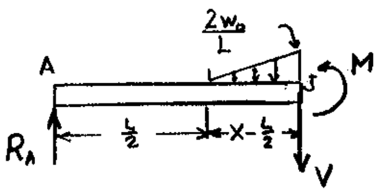
Reactions are statically indeterminate.

$0 \le x \le \frac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = R_A x \qquad (1)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \qquad (2)$$

$$EI\, y = \frac{1}{6}R_A x^3 + C_1 x + C_2 \qquad (3)$$



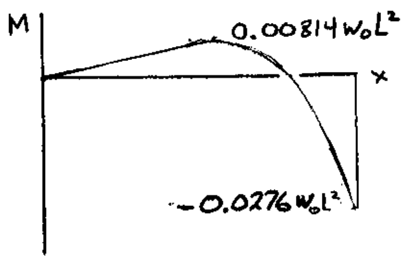
$\frac{L}{2} \le x \le L \qquad \circlearrowleft \Sigma M_J = 0$

$$-R_A x + \frac{1}{2}\frac{2w_0}{L}\left(x-\frac{L}{2}\right)\left(x-\frac{L}{2}\right)\frac{1}{3}\left(x-\frac{L}{2}\right) + M = 0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{3}\frac{w_0}{L}\left(x-\frac{L}{2}\right)^3 \qquad (4)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}\left(x-\frac{L}{2}\right)^4 + C_3 \qquad (5)$$

$$EI\, y = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}\left(x-\frac{L}{2}\right)^5 + C_3 x + C_4 \qquad (6)$$



[$x = 0$, $y = 0$] $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

[$x = \frac{L}{2}$, $\frac{dy}{dx} = \frac{dy}{dx}$] $\quad \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 + C_1 = \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 - 0 + C_3 \qquad C_1 = C_3$

[$x = \frac{L}{2}$, $y = y$] $\quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \cancel{C_1\frac{L}{2}} + C_2 = \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - 0 + \cancel{C_3\frac{L}{2}} + C_4 \qquad C_4 = C_2 = 0$

[$x = L$, $\frac{dy}{dx} = 0$] $\quad \frac{1}{2}R_A L^2 - \frac{1}{12}\frac{w_0}{L}\left(\frac{L}{2}\right)^4 + C_3 = 0 \qquad C_3 = \frac{1}{192}w_0 L^3 - \frac{1}{2}R_A L^2$

[$x = L$, $y = 0$] $\quad \frac{1}{6}R_A L^3 - \frac{1}{60}\frac{w_0}{L}\left(\frac{L}{2}\right)^5 + \frac{1}{96}w_0 L^4 - \frac{1}{2}R_A L^3 + 0 = 0$

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3 = \left(\frac{1}{192}-\frac{1}{1920}\right)w_0 L^4 \qquad \frac{1}{3}R_A = \frac{3}{640}w_0 L \qquad R_A = \frac{9}{640}w_0 L \uparrow$$

From (1), with $x = \frac{L}{2}$ $\qquad M_C = R_A\frac{L}{2} = \frac{9}{1280}w_0 L^2 = 0.007031\ w_0 L^2$

From (4), with $x = L$ $\qquad M_B = \frac{9}{640}w_0 L^2 - \frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 = -\frac{53}{1920}w_0 L^2$

$$= -0.02761\ w_0 L^2$$

Location of maximum positive M in portion CB.

$$\frac{dM}{dx} = R_A - \frac{w_0}{L}\left(x_m - \frac{L}{2}\right)^2 = 0 \qquad x_m - \frac{L}{2} = \sqrt{\frac{R_A L}{w_0}} = \sqrt{\frac{9}{640}}\,L = 0.1186\,L$$

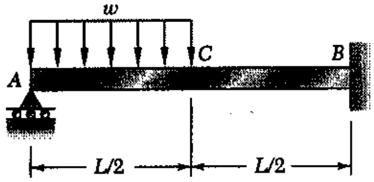
$$x_m = 0.5L + 0.1186\,L = 0.6186\,L$$

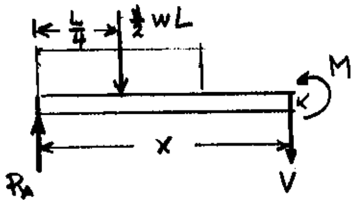
From (4), with $x = x_m$ $\qquad M_m = R_A(0.6186\,L) - \frac{1}{3}\frac{w_0}{L}(0.1186\,L)^3$

$$= 0.008143\ w_0 L^2$$

PROBLEM 9.27

9.27 **and** *9.28* Determine the reaction at the roller support and the deflection at point C.





SOLUTION

Reactions are statically indeterminate.

$0 < x < \frac{L}{2}$

$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2} w x^2$

$EI\frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w x^3 + C_1$

$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} w x^4 + C_1 x + C_2$

$\frac{L}{2} < x < L$ (See free body diagram)

$\circlearrowright \Sigma M_K = 0 \qquad -R_A x + \frac{1}{2} wL\left(x - \frac{L}{4}\right) + M = 0$

$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2} wL\left(x - \frac{1}{4}L\right)$

$EI\frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{4} wL\left(x - \frac{L}{4}\right)^2 + C_3$

$EI y = \frac{1}{6} R_A x^3 - \frac{1}{12} wL\left(x - \frac{L}{4}\right)^3 + C_3 x + C_4$

$[x = 0,\ y = 0] \qquad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}] \qquad \cancel{\frac{1}{2} R_A\left(\frac{L}{2}\right)^2} - \frac{1}{6} w\left(\frac{L}{2}\right)^3 + C_1 = \cancel{\frac{1}{2} R_A\left(\frac{L}{2}\right)^2} - \frac{1}{4} wL\left(\frac{L}{4}\right)^2 + C_3$

$C_1 = C_3 + \frac{1}{48} wL^3 - \frac{1}{64} wL^3 = C_3 + \frac{1}{192} wL^3$

$[x = \frac{L}{2},\ y = y] \qquad \cancel{\frac{1}{6} R_A\left(\frac{L}{2}\right)^3} - \frac{1}{24} w\left(\frac{L}{2}\right)^4 + \left(\cancel{C_3} + \frac{1}{192} wL^3\right)\frac{L}{2} = \cancel{\frac{1}{6} R_A\left(\frac{L}{2}\right)^3} - \frac{1}{12} wL\left(\frac{L}{4}\right)^3 + \cancel{C_3 \frac{L}{2}} + C_4$

$C_4 = -\frac{1}{384} wL^4 + \frac{1}{384} wL^4 + \frac{1}{768} wL^4 = \frac{1}{768} wL^4$

$[x = L,\ \frac{dy}{dx} = 0] \qquad \frac{1}{2} R_A L^2 - \frac{1}{4} wL\left(\frac{3L}{4}\right)^2 + C_3 = 0 \qquad C_3 = \frac{9}{64} wL^3 - \frac{1}{2} R_A L^2$

$[x = L,\ y = 0] \qquad \frac{1}{6} R_A L^3 - \frac{1}{12} wL\left(\frac{3L}{4}\right)^3 + \left(\frac{9}{64} wL^3 - \frac{1}{2} R_A L^2\right)L + \frac{1}{768} wL^4 = 0$

$\left(\frac{1}{2} - \frac{1}{6}\right) R_A L^3 = \left(\frac{9}{64} - \frac{27}{768} + \frac{1}{768}\right) wL^4 \qquad \frac{1}{3} R_A = \frac{41}{384} wL \qquad R_A = \frac{41}{128} wL \uparrow$

$C_3 = \frac{9}{64} wL^3 - \frac{1}{2}\cdot\frac{41}{128} wL^3 = -\frac{5}{256} wL^3$

$C_1 = -\frac{5}{256} wL^3 + \frac{1}{192} wL^3 = -\frac{11}{768} wL^3$

Deflection at C (y at $x = \frac{L}{2}$)

$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\cdot\left(\frac{1}{2}\right)^3 - \frac{1}{24}\cdot\left(\frac{1}{2}\right)^4 - \frac{11}{768}\cdot\frac{1}{2} + 0\right\}$

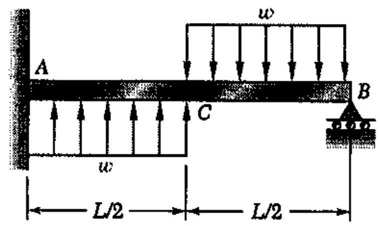
$= \left(\frac{41}{6144} - \frac{1}{384} - \frac{11}{1536}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI} \qquad y_C = \frac{19}{6144}\frac{wL^4}{EI} \downarrow$

or $y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\left(\frac{1}{2}\right)^3 - \frac{1}{12}\cdot\left(\frac{1}{4}\right)^3 - \frac{5}{256}\cdot\frac{1}{2} + \frac{1}{768}\right\}$

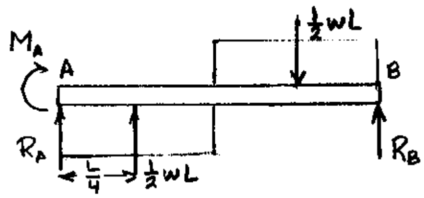
$= \left(\frac{41}{6144} - \cancel{\frac{1}{768}} - \frac{5}{512} + \cancel{\frac{1}{768}}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI}$

PROBLEM 9.28

9.27 and 9.28 Determine the reaction at the roller support and the deflection at point C.



$[x=0,\ y=0]$ $[x=L,\ y=0]$
$[x=0,\ \frac{dy}{dx}=0]$
$[x=\frac{L}{2},\ y=y]$
$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}]$



SOLUTION

Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0 \qquad R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \qquad R_A = -R_B$

$\circlearrowleft \Sigma M_A = 0 \qquad -M_A - (\frac{1}{2}wL)\frac{L}{2} + R_B L = 0$

$M_A = R_B L - \frac{1}{4}wL^2$

From A to C $\qquad 0 < x \le \frac{L}{2}$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$

$EI\, y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$

From C to B $\qquad \frac{L}{2} \le x < L$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wL(x - \frac{L}{4}) - \frac{1}{2}w(x - \frac{L}{2})^2$

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}wL(x - \frac{L}{4})^2 - \frac{1}{6}w(x - \frac{L}{2})^3 + C_3$

$EI\, y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{12}wL(x - \frac{L}{4})^3 - \frac{1}{24}(x - \frac{L}{2})^4 + C_3 x + C_4$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0,\ y=0] \qquad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}] \qquad \cancel{M_A\frac{L}{2}} - \cancel{\frac{1}{2}R_A(\frac{L}{2})^2} + \frac{1}{6}w(\frac{L}{2})^3 = \cancel{M_A\frac{L}{2}} + \cancel{\frac{1}{2}R_A(\frac{L}{2})^2} + \frac{1}{4}wL(\frac{L}{4})^2 - 0 + C_3$

$C_3 = (\frac{1}{48} - \frac{1}{64})wL^3 = \frac{1}{192}wL^3$

$[x=\frac{L}{2},\ y=y] \qquad \cancel{\frac{1}{2}M_A(\frac{L}{2})^2} + \cancel{\frac{1}{6}R_A(\frac{L}{2})^3} + \frac{1}{24}w(\frac{L}{2})^4$

$= \cancel{\frac{1}{2}M_A(\frac{L}{2})^2} + \cancel{\frac{1}{6}R_A(\frac{L}{2})^3} + \frac{1}{12}wL(\frac{L}{4})^3 - 0 + \frac{1}{192}wL^3(\frac{L}{2}) + C_4$

$C_4 = (\frac{1}{384} - \frac{1}{768} + \frac{1}{384})wL^4 = -\frac{1}{768}wL^4$

$[x=L,\ y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 + \frac{1}{12}wL(\frac{3L}{4})^3 - \frac{1}{24}w(\frac{L}{2})^4 + \frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$

$\frac{1}{2}(R_B L - \frac{1}{4}wL^2)L^2 + \frac{1}{6}(-R_B)L^3 + (\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768})wL^4 = 0$

$(\frac{1}{2} - \frac{1}{6})R_B L^3 = -(\frac{1}{8} - \frac{7}{192})wL^4 \qquad \frac{1}{3}R_B = \frac{17}{192}wL \qquad R_B = \frac{17}{64}wL \uparrow$

$R_A = -R_B = -\frac{17}{64}wL$

$M_A = R_B L - \frac{1}{4}wL^2 = (\frac{17}{64} - \frac{1}{4})wL^2 = \frac{1}{64}wL^2$

(b) Deflection at C $\qquad$ (y at $x = \frac{L}{2}$)

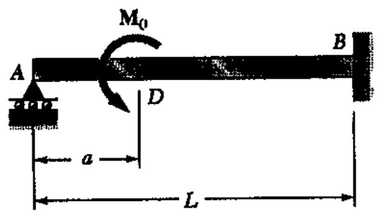
$EI y_C = \frac{1}{2}M_A(\frac{L}{2})^2 + \frac{1}{6}R_A(\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4 = \frac{1}{2}(\frac{1}{64}wL^2)(\frac{L}{2})^2 + \frac{1}{6}(-\frac{17}{64}wL)(\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4$

$= (\frac{1}{512} - \frac{17}{3072} + \frac{1}{384})wL^4 = -\frac{1}{1024}wL^4$

$y_C = \frac{1}{1024}\frac{wL^4}{EI} \downarrow$

PROBLEM 9.29

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point D, knowing that a is equal to $L/3$.



$[x=0, y=0]$ $[x=L, y=0]$ $[x=L, \frac{dy}{dx}=0]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

$[x=a, y=y]$

SOLUTION

Reactions are statically indeterminate.

$0 < x < a \qquad M = R_A x$

$$EI\frac{d^2y}{dx^2} = M = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$a < x < L \qquad M = R_A x - M_0$

$$EI\frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0(x-a) + C_3$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0(x-a)^2 + C_3 x + C_4$$

$[x=0, y=0] \qquad 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}] \qquad \cancel{\frac{1}{2}R_A a^2} + C_1 = \cancel{\frac{1}{2}R_A a^2} - 0 + C_3 \qquad C_1 = C_3$

$[x=a, y=y] \qquad \frac{1}{6}R_A a^3 + C_1 a + C_2 = \frac{1}{6}R_A a^3 + 0 + C_3 a + C_4 \qquad C_2 = C_4 = 0$

$[x=L, \frac{dy}{dx}=0] \qquad \frac{1}{2}R_A L^2 - M_0(L-a) + C_3 = 0 \qquad C_3 = M_0(L-a) - \frac{1}{2}R_A L^2$

$[x=L, y=0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L-a)^2 + [M_0(L-a) - \frac{1}{2}R_A L^2]L + 0 = 0$

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3 = M_0\left[(L-a)L - \frac{1}{2}(L-a)^2\right]$$

$$\frac{1}{3}R_A L^3 = M_0\left[L^2 \cancel{- aL} - \frac{1}{2}L^2 \cancel{+ La} - \frac{1}{2}a^2\right] = \frac{1}{2}M_0(L^2-a^2)$$

$$R_A = \frac{3}{2}\frac{M_0}{L^3}(L^2-a^2) = \frac{3}{2}\frac{M_0}{L^3}\left[L^2 - \left(\frac{L}{3}\right)^2\right] = \frac{4}{3}\frac{M_0}{L} \uparrow$$

Deflection at D (y at $x = a = \frac{L}{3}$)

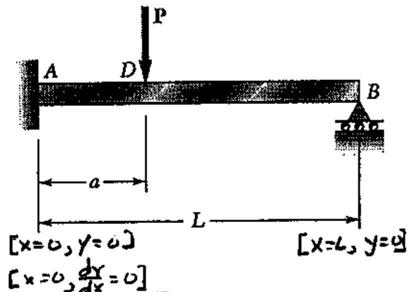
$$y_D = \frac{1}{EI}\left\{\frac{1}{6}R_A\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right)\right\} = \frac{1}{EI}\left\{\frac{1}{6}\left(\frac{4}{3}\frac{M_0}{L}\right)\left(\frac{L}{3}\right)^3 + C_3\left(\frac{L}{3}\right)\right\}$$

$$= \frac{1}{EI}\left\{\frac{4}{486}M_0 L^2 + \left[M_0\left(L - \frac{L}{3}\right) - \frac{1}{2}\cdot\frac{4}{3}\frac{M_0}{L}L^2\right]\frac{L}{3}\right\}$$

$$= \frac{M_0 L^2}{EI}\left(\frac{4}{486} + \frac{2}{9} - \frac{4}{18}\right) = \frac{2}{243}\frac{M_0 L^2}{EI} \uparrow$$

PROBLEM 9.30

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point *D*, knowing that *a* is equal to *L*/3.



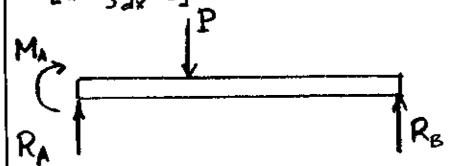
SOLUTION

Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - P = 0 \qquad R_A = P - R_B$

$+\circlearrowleft \Sigma M_A = 0 \qquad -M_A - Pa - R_B L = 0$

$M_A = R_B L - Pa$



$0 < x < a \qquad M = M_A + R_A x$

$EI \frac{d^2y}{dx^2} = M = M_A + R_A x$

$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$

$EI\, y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$

$a < x < L \qquad M = M_A + R_A x - P(x-a)$

$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P(x-a)$

$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P (x-a)^2 + C_3$

$EI\, y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P (x-a)^3 + C_3 x + C_4$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \qquad \cancel{M_A a} + \cancel{\frac{1}{2} R_A a^2} + C_1 = \cancel{M_A a} + \cancel{\frac{1}{2} R_A a^2} - 0 + C_3 \qquad C_3 = C_1 = 0$

$[x=a, y=y] \qquad \cancel{\frac{1}{2} M_A a^2} + \cancel{\frac{1}{6} R_A a^3} + \cancel{C_1 a} + C_2$
$= \cancel{\frac{1}{2} M_A a^2} + \cancel{\frac{1}{6} R_A a^3} - 0 + \cancel{C_3 a} + C_4 \qquad C_4 = C_2 = 0$

$[x=L, y=0] \qquad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P (L-a)^3 + 0 + 0 = 0$

$\frac{1}{2}(R_B L - Pa)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{6} P (L-a)^3 = 0$

$(\frac{1}{2} - \frac{1}{6}) R_B L^3 = P\left[\frac{1}{2} a L^2 - \frac{1}{6} L^3 + \frac{1}{6}(L-a)^3\right]$

$\frac{1}{3} R_B L^3 = P\left[\cancel{\frac{1}{2} a L^2} - \cancel{\frac{1}{6} L^3} + \cancel{\frac{1}{6} L^3} - \cancel{\frac{1}{2} L^2 a} + \frac{1}{2} L a^2 - \frac{1}{6} a^3\right]$

$= P a^2 (\frac{1}{2} L - \frac{1}{6} a)$

$R_B = \frac{Pa^2}{2L^3}(3L - a) = \frac{P(L/3)^2}{2L^3}(3L - \frac{L}{3}) = \frac{4}{27} P \uparrow$ ◄

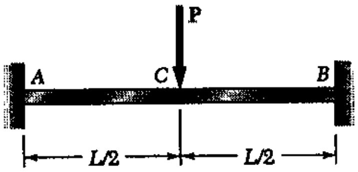
Deflection at D $\quad (y \text{ at } x = a = \frac{L}{3})$

$y_D = \frac{1}{EI}\left\{\frac{1}{2} M_A (\frac{L}{3})^2 + \frac{1}{6} R_A (\frac{L}{3})^3\right\} = \frac{1}{EI}\left\{\frac{1}{18}(R_B L - P\frac{L}{3})L^2 + \frac{1}{162}(P - R_B)L^3\right\}$

$= \frac{PL^3}{EI}\left\{\frac{1}{18}(\frac{4}{27} - \frac{1}{3}) + \frac{1}{162}(1 - \frac{4}{27})\right\} = -\frac{11}{2187}\frac{PL^3}{EI}, \qquad y_D = \frac{11}{2187}\frac{PL^3}{EI} \downarrow$ ◄

PROBLEM 9.31

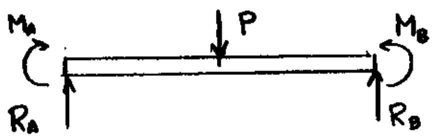
9.31 and 9.32 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

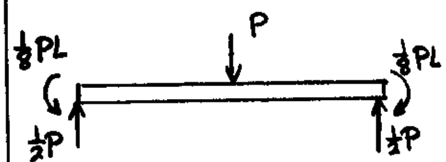


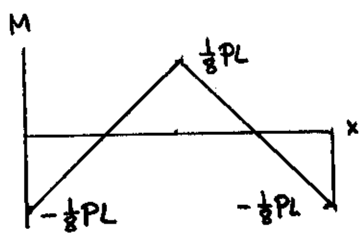
$[x=0,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$

$[x=\frac{L}{2},\ \frac{dy}{dx}=0]$







SOLUTION

By symmetry, $R_A = R_B$ and $\frac{dy}{dx} = 0$ at $x = \frac{L}{2}$.

$+\uparrow\Sigma F_y = 0 \quad R_A + R_B - P = 0 \qquad R_A = R_B = \frac{1}{2}P$ ◄

Moment reaction is statically indeterminate.

$0 < x < \frac{L}{2} \qquad M = M_A + R_A x = M_A + \frac{1}{2}Px$

$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$

$EI\frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=0] \qquad M_A\frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = 0$

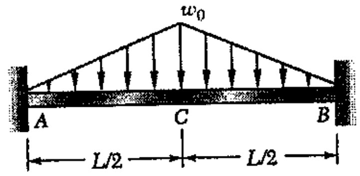
$M_A = -\frac{1}{8}PL \qquad M_A = \frac{1}{8}PL$ ↺ ◄

By symmetry $\quad M_B = M_A = \frac{1}{8}PL$ ↻ ◄

$M_C = M_A + \frac{1}{2}P\frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL$ ◄

PROBLEM 9.32

9.31 and 9.32 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

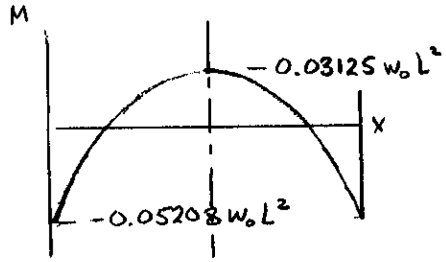


$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$ $\quad$ $[x=L,\ \frac{dy}{dx}=0]$

$[x=\frac{L}{2},\ V=0]$

$[x=\frac{L}{2},\ \frac{dy}{dx}=0]$



SOLUTION

Reactions are statically indeterminate.

Because of symmetry $\frac{dy}{dx}=0$ and $V=0$ at $x=\frac{L}{2}$.

Use portion AC of beam $\quad (0 < x \leq \frac{L}{2})$

$$\frac{dV}{dx} = -w = -2\frac{w_0}{L}x$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}x^2 + R_A \tag{1}$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{3}\frac{w_0}{L}x^3 + R_Ax + M_A \tag{2}$$

$$EI\frac{dy}{dx} = -\frac{1}{12}\frac{w_0}{L}x^4 + \frac{1}{2}R_Ax^2 + M_Ax + C_1 \tag{3}$$

$$EIy = -\frac{1}{60}\frac{w_0}{L}x^5 + \frac{1}{6}R_Ax^3 + \frac{1}{2}M_Ax^2 + C_1x + C_2 \tag{4}$$

$[x=0,\ \frac{dy}{dx}=0]$ $\quad 0 = 0+0+0+C_1 \qquad C_1 = 0$

$[x=0,\ y=0]$ $\quad 0 = 0+0+0+0+C_2 \qquad C_2 = 0$

$[x=\frac{L}{2},\ V=0]$ $\quad -\frac{w_0}{L}\left(\frac{L}{2}\right)^2 + R_A = 0 \qquad R_A = \frac{1}{4}w_0L \uparrow$

$[x=\frac{L}{2},\ \frac{dy}{dx}=0]$ $\quad -\frac{1}{12}\frac{w_0}{L}\left(\frac{L}{2}\right)^4 + \frac{1}{2}\left(\frac{1}{4}w_0L\right)\left(\frac{L}{2}\right)^2 + M_A\frac{L}{2} + 0 = 0$

$$M_A = -2\left(\frac{1}{32} - \frac{1}{192}\right)w_0L^2 = -\frac{5}{96}w_0L^2 = -0.05208\,w_0L^2$$

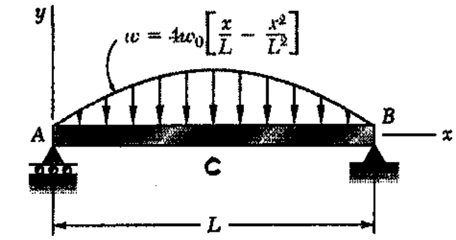
From (2), with $x=\frac{L}{2}$

$$M_C = -\frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 + \left(\frac{1}{4}w_0L\right)\left(\frac{L}{2}\right) - \frac{5}{96}w_0L^2$$

$$= \left(-\frac{1}{24} + \frac{1}{8} - \frac{5}{96}\right)w_0L^2 = \frac{1}{32}w_0L^2 = 0.03125\,w_0L^2$$

PROBLEM 9.33

9.33 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.



$[x=0,\ M=0]$ $\quad [x=L,\ M=0]$

$[x=0,\ y=0]$ $\quad [x=L,\ y=0]$

SOLUTION

Boundary conditions at A and B are noted.

$$w = \frac{w_0}{L^2}(4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}(4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left(\tfrac{4}{3}x^3 - 2Lx^2\right) + C_1$$

$$M = \frac{w_0}{L^2}\left(\tfrac{1}{3}x^4 - \tfrac{2}{3}Lx^3\right) + C_1x + C_2$$

$[x=0,\ M=0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L,\ M=0]$ $\qquad 0 = \frac{w_0}{L^2}\left(\tfrac{1}{3}L^4 - \tfrac{2}{3}L^4\right) + C_1L + 0 \qquad C_1 = \tfrac{1}{3}w_0L^2$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left(\tfrac{1}{3}x^4 - \tfrac{2}{3}Lx^3 + \tfrac{1}{3}L^3x\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\tfrac{1}{15}x^5 - \tfrac{1}{6}Lx^4 + \tfrac{1}{6}L^3x^2\right) + C_3$$

$$EI\,y = \frac{w_0}{L^2}\left(\tfrac{1}{90}x^6 - \tfrac{1}{30}Lx^5 + \tfrac{1}{18}L^3x^3\right) + C_3x + C_4$$

$[x=0,\ y=0]$ $\qquad 0 = 0 + 0 + 0 + 0 + C_4 \qquad C_4 = 0$

$[x=L,\ y=0]$ $\qquad 0 = \frac{w_0}{L^2}\left(\tfrac{1}{90}L^6 - \tfrac{1}{30}L^6 + \tfrac{1}{18}L^6\right) + C_3L + 0 \qquad C_3 = -\tfrac{1}{30}w_0L^3$

(a) Elastic curve:

$$y = \frac{w_0}{EIL^2}\left(\tfrac{1}{90}x^6 - \tfrac{1}{30}Lx^5 + \tfrac{1}{18}L^3x^3 - \tfrac{1}{30}L^5x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2}\left(\tfrac{1}{15}x^5 - \tfrac{1}{6}Lx^4 + \tfrac{1}{6}L^3x^2 - \tfrac{1}{30}L^5\right)$$

(b) Slope at end A. Set $x=0$ in $\frac{dy}{dx}$ $\qquad \left.\frac{dy}{dx}\right|_A = -\frac{1}{30}\frac{w_0L^3}{EI}$

$$\theta_A = \frac{1}{30}\frac{w_0L^3}{EI}$$

(c) Deflection at midpoint. Set $x = \frac{L}{2}$ in y

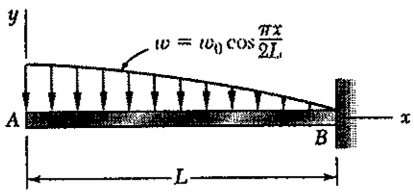
$$y_C = \frac{w_0L^4}{EI}\left\{\left(\tfrac{1}{90}\right)\left(\tfrac{1}{2}\right)^6 - \left(\tfrac{1}{30}\right)\left(\tfrac{1}{2}\right)^5 + \tfrac{1}{18}\left(\tfrac{1}{2}\right)^3 - \tfrac{1}{30}\left(\tfrac{1}{2}\right)\right\}$$

$$= \frac{w_0L^4}{EI}\left\{\frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60}\right\} = -\frac{61}{5760}\frac{w_0L^4}{EI}$$

$$y_C = \frac{61}{5760}\frac{w_0L^4}{EI}\ \downarrow$$

PROBLEM 9.34

9.34 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at the free end, (*c*) the deflection at the free end.



$[x=0, V=0]$ $[x=L, \frac{dy}{dx}=0]$

$[x=0, M=0]$ $[x=L, y=0]$

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2w_0 L}{\pi} \sin \frac{\pi x}{2L} + C_1$$

$$[x=0, V=0] \qquad 0 = 0 + C_1 \qquad C_1 = 0$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi} \sin \frac{\pi x}{2L}$$

$$M = \frac{4w_0 L^2}{\pi^2} \cos \frac{\pi x}{2L} + C_2$$

$$[x=0, M=0] \qquad C_2 = -\frac{4w_0 L^2}{\pi^2}$$

$$EI \frac{d^2y}{dx^2} = M = \frac{4w_0 L^2}{\pi^2}\left(\cos \frac{\pi x}{2L} - 1\right)$$

$$EI \frac{dy}{dx} = \frac{4w_0 L^2}{\pi^2}\left(\frac{2L}{\pi} \sin \frac{\pi x}{2L} - x\right) + C_3$$

$$[x=L, \frac{dy}{dx}=0] \qquad \frac{4w_0 L^2}{\pi^2}\left(\frac{2L}{\pi} - L\right) + C_3 = 0 \qquad C_3 = \frac{4w_0 L^3}{\pi^3}(\pi - 2)$$

$$EIy = \frac{4w_0 L^2}{\pi^2}\left[-\frac{4L^2}{\pi^2} \cos \frac{\pi x}{2L} - \frac{1}{2}x^2\right] + C_3 x + C_4$$

$$[x=L, y=0] \qquad \frac{4w_0 L^2}{\pi^2}\left(-\frac{1}{2}L^2\right) + C_3 L + C_4 = 0$$

$$C_4 = \frac{2w_0 L^4}{\pi^2} - C_3 L$$

(a) Elastic curve
$$y = \frac{w_0}{EI}\left\{-\frac{16L^4}{\pi^4}\cos \frac{\pi x}{2L} - \frac{2L^2x^2}{\pi^2} + \frac{4L^3}{\pi^3}(\pi-2)(x-L) + \frac{2L^4}{\pi^2}\right\}$$

$$y = \frac{2w_0 L^4}{\pi^4 EI}\left\{-8 \cos \frac{\pi x}{2L} - \pi^2 \frac{x^2}{L^2} + 2\pi(\pi-2)\frac{x}{L} + \pi(4-\pi)\right\}$$ ◀

(b) Slope at free end $(x=0)$

$$EI \left.\frac{dy}{dx}\right|_{x=0} = C_3 = \frac{4(\pi-2)}{\pi^3} w_0 L^3$$

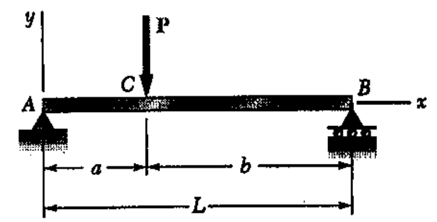
$$\left.\frac{dy}{dx}\right|_A = \frac{4(\pi-2)}{\pi^3}\frac{w_0 L^3}{EI} = 0.14727 \frac{w_0 L^3}{EI}$$ ◀

(c) Deflection at free end $(x=0)$

$$y_A = \frac{2w_0 L^4}{\pi^4 EI}\left\{-8 + \pi(4-\pi)\right\} = -0.10889 \frac{w_0 L^4}{EI}$$ ◀

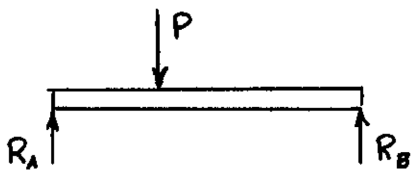
PROBLEM 9.35

9.35 thorough 9.38 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



$[x=0, M=0]$ $[x=L, M=0]$

$[x=0, y=0]$ $[x=L, y=0]$



SOLUTION

$$\circlearrowleft \Sigma M_B = 0 \qquad -R_A L + Pb = 0 \qquad R_A = \frac{Pb}{L}$$

$$\frac{dM}{dx} = V = R_A - P\langle x-a\rangle^0 = \frac{Pb}{L} - P\langle x-a\rangle^0$$

$$M = \frac{Pb}{L}x - P\langle x-a\rangle^1 + \cancel{M_A}$$

$$EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x - P\langle x-a\rangle$$

$$EI\frac{dy}{dx} = \frac{Pb}{2L}x^2 - \frac{1}{2}P\langle x-a\rangle^2 + C_1$$

$$EI\,y = \frac{Pb}{6L}x^3 - \frac{1}{6}P\langle x-a\rangle^3 + C_1 x + C_2$$

$[x=0, y=0]$ $\qquad C_2 = 0$

$[x=L, y=0]$ $\qquad \frac{Pb}{6L}L^3 - \frac{1}{6}P(L-a)^3 + C_1 L = 0$

$$C_1 = -\frac{1}{6}\frac{P}{L}(bL^2 - b^3) = -\frac{1}{6}\frac{Pb}{L}(L^2-b^2)$$

(a) Elastic curve

$$y = \frac{P}{EI}\left\{\frac{b}{6L}x^3 - \frac{1}{6}\langle x-a\rangle^3 - \frac{1}{6}\frac{b}{L}(L^2-b^2)x\right\}$$

$$y = \frac{P}{6EIL}\left\{bx^3 - L\langle x-a\rangle^3 - b(L^2-b^2)\right\}$$

(b) Slope at end A.

$$EI\left.\frac{dy}{dx}\right|_{x=0} = C_1 = -\frac{Pb}{6L}(L^2-b^2)$$

$$\theta_A = -\frac{Pb}{6EIL}(L^2-b^2)$$

(c) Deflection at C

$$EI\,y_C = \frac{Pb}{6L}a^3 + C_1 a = \frac{Pba^3}{6L} - \frac{Pb}{6L}(L^2-b^2)a$$

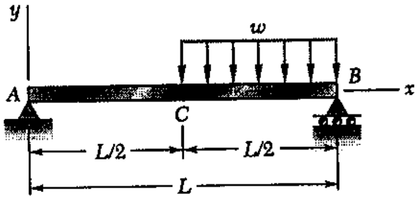
$$= \frac{Pba}{6L}(a^2 - L^2 - b^2)$$

$$y_C = -\frac{Pab}{6EIL}(L^2-a^2-b^2) = -\frac{Pab}{6EIL}\{\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}-\cancel{b^2}\}$$

$$= -\frac{Pa^2b^2}{3EIL}$$

PROBLEM 9.36

9.35 thorough 9.38 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



$[x=0, M=0]$ $[x=L, M=0]$

$[x=0, y=0]$ $[x=L, y=0]$

SOLUTION

$$\frac{dV}{dx} = -w\langle x-\tfrac{L}{2}\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x-\tfrac{L}{2}\rangle^1$$

$$M = \cancel{M_A} + R_A x - \tfrac{1}{2}w\langle x-\tfrac{L}{2}\rangle^2$$

$[x=L, M=0]$ $\quad R_A L - \frac{1}{2}w(\frac{L}{2})^2 = 0$

$$R_A = \tfrac{1}{8}wL$$

$$EI\frac{d^2y}{dx^2} = \tfrac{1}{8}wL\,x - \tfrac{1}{2}w\langle x-\tfrac{L}{2}\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{16}wL\,x^2 - \tfrac{1}{6}w\langle x-\tfrac{L}{2}\rangle^3 + C_1$$

$$EI\,y = \tfrac{1}{48}wL\,x^3 - \tfrac{1}{24}w\langle x-\tfrac{L}{2}\rangle^4 + C_1x + C_2$$

$[x=0, y=0]$ $\quad 0 = 0 + 0 + 0 + C_2 \quad\quad C_2 = 0$

$[x=L, y=0]$ $\quad \frac{1}{48}wL^4 - \frac{1}{24}w(\frac{L}{2})^4 + C_1L + 0 = 0$

$$C_1 = -\left(\tfrac{1}{48} - \tfrac{1}{24}\cdot\tfrac{1}{16}\right)wL^3 = -\tfrac{7}{384}wL^3$$

(a) Elastic curve

$$EI\,y = \tfrac{1}{48}wLx^3 - \tfrac{1}{24}w\langle x-\tfrac{L}{2}\rangle^4 - \tfrac{7}{384}wL^3x$$

$$y = \frac{w}{EI}\left\{\tfrac{1}{48}Lx^3 - \tfrac{1}{24}\langle x-\tfrac{L}{2}\rangle^4 - \tfrac{7}{384}L^3x\right\}$$

$$\frac{dy}{dx} = \frac{w}{EI}\left\{\tfrac{1}{16}Lx^2 - \tfrac{1}{6}\langle x-\tfrac{L}{2}\rangle^3 - \tfrac{7}{384}L^3\right\}$$

(b) Slope at A $\left(x=0 \text{ in } \frac{dy}{dx}\right)$

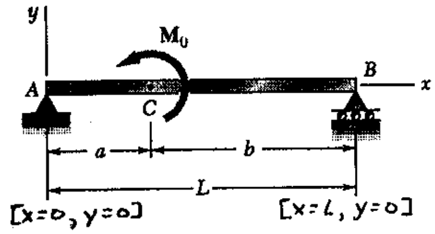
$$\theta_A = -\frac{7}{384}\frac{wL^3}{EI}$$

(c) Deflection at C $\left(x = \frac{L}{2} \text{ in } y\right)$

$$y_C = \frac{wL^4}{EI}\left\{\tfrac{1}{48}\cdot\tfrac{1}{8} - \tfrac{7}{384}\cdot\tfrac{1}{2}\right\} = \left(\tfrac{1}{384} - \tfrac{7}{768}\right)\frac{wL^4}{EI} = -\frac{5}{768}\frac{wL^4}{EI}$$

PROBLEM 9.37

9.35 thorough 9.38 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



SOLUTION

Reactions $\quad R_A = \frac{M_0}{L}\uparrow, \quad R_B = \frac{M_0}{L}\downarrow$

$0 < x < a \qquad M = R_A x$

$a < x < L \qquad M = R_A x - M_0$

Using singularity functions

$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x-a\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x-a\rangle^1 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + C_1 x + C_2$$

$[x=0, y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L-a)^2 + C_1 L + 0 = 0$

$$C_1 L = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0 b^2 \qquad C_1 = \frac{M_0}{6L}(3b^2 - L^2)$$

(a) Elastic curve

$$y = \frac{1}{EI}\left\{\frac{1}{6}\frac{M_0}{L}x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + \frac{M_0}{6L}(3b^2-L^2)x\right\}$$

$$= \frac{M_0}{6EIL}\left\{x^3 - 3L\langle x-a\rangle^2 + (3b^2-L^2)x\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL}\left\{3x^2 - 6L\langle x-a\rangle^1 + (3b^2-L^2)\right\}$$

(b) Slope at A $\quad \left(\frac{dy}{dx} \text{ at } x=0\right)$

$$\theta_A = \frac{M_0}{6EIL}\left\{0 - 0 + 3Lb^2 - L^3\right\} = \frac{M_0}{6EIL}(3b^2 - L^2)$$

(c) Deflection at C $\quad (y \text{ at } x=a)$

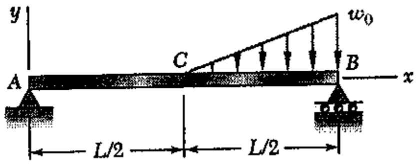
$$y_C = \frac{M_0}{6EIL}\left\{a^3 - 0 + (3b^2-L^2)a\right\} = \frac{M_0 a}{6EIL}\left\{a^2 + 3b^2 - (a+b)^2\right\}$$

$$= \frac{M_0 a}{6EIL}\left\{\cancel{a^2} + 3b^2 - \cancel{a^2} - 2ab - b^2\right\} = \frac{M_0 a}{6EIL}\left\{2b^2 - 2ab\right\}$$

$$= \frac{M_0 ab}{3EIL}(b-a)\uparrow$$

PROBLEM 9.38

9.35 thorough 9.38 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



$[x=0, y=0]$ $[x=L, y=0]$

$[x=0, M=0]$ $[x=L, M=0]$

SOLUTION

$$w = \frac{2w_0}{L}\langle x-\tfrac{L}{2}\rangle^1$$

$$\frac{dV}{dx} = -w = -\frac{2w_0}{L}\langle x-\tfrac{L}{2}\rangle^1$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\langle x-\tfrac{L}{2}\rangle^2 + R_A$$

$$M = -\frac{1}{3}\frac{w_0}{L}\langle x-\tfrac{L}{2}\rangle^3 + R_A x + M_A$$

$[x=0, M=0]$ $\quad M_A = 0$

$[x=L, M=0]$ $\quad -\frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 + R_A L + 0 = 0 \qquad R_A = \frac{1}{24}w_0 L$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{3}\frac{w_0}{L}\langle x-\tfrac{L}{2}\rangle^3 + \frac{1}{24}w_0 L x$$

$$EI\frac{dy}{dx} = -\frac{1}{12}\frac{w_0}{L}\langle x-\tfrac{L}{2}\rangle^4 + \frac{1}{48}w_0 L x^2 + C_1$$

$$EIy = -\frac{1}{60}\frac{w_0}{L}\langle x-\tfrac{L}{2}\rangle^5 + \frac{1}{144}w_0 L x^3 + C_1 x + C_2$$

$[x=0, y=0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0]$ $\quad -\frac{1}{60}\frac{w_0}{L}\left(\frac{L}{2}\right)^5 + \frac{1}{144}w_0 L^4 + C_1 L + 0 = 0$

$$C_1 = -\left(\frac{1}{144} - \frac{1}{1920}\right)w_0 L^3 = -\frac{37}{5760}w_0 L^3$$

(a) Elastic curve

$$y = \frac{w_0}{EIL}\left\{-\frac{1}{60}\langle x-\tfrac{L}{2}\rangle^5 + \frac{1}{144}L^2x^3 - \frac{37}{5760}L^4x\right\}$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left\{-\frac{1}{12}\langle x-\tfrac{L}{2}\rangle^4 + \frac{1}{48}L^2x^2 - \frac{37}{5760}L^4\right\}$$

(b) Slope at A $\quad \left(\frac{dy}{dx} \text{ at } x=0\right)$

$$\theta_A = \frac{w_0}{EIL}\left\{0 + 0 - \frac{37}{5760}L^4\right\} = -\frac{37}{5760}\frac{w_0L^3}{EI}, \qquad \theta_A = \frac{37}{5760}\frac{w_0L^3}{EI}$$

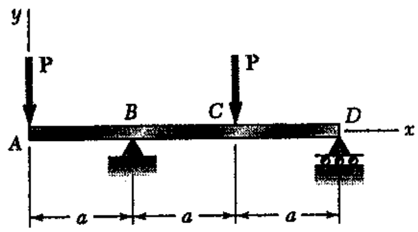
(c) Deflection at C $\quad \left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{w_0}{EIL}\left\{0 + \frac{1}{144}L^2\left(\frac{L}{2}\right)^3 - \frac{37}{5760}L^4\left(\frac{L}{2}\right)\right\}$$

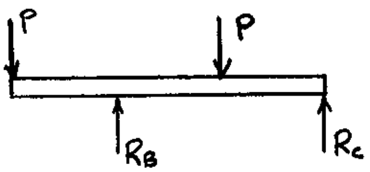
$$= \left(\frac{1}{1152} - \frac{37}{11520}\right)\frac{w_0L^4}{EI} = -\frac{3}{1280}\frac{w_0L^4}{EI}$$

PROBLEM 9.39

9.39* and *9.40 For the beam and loading shown, determine (*a*) the deflection at end *A*, (*b*) the deflection point *C*, (*c*) the slope at end *D*.



$[x=a, y=0]$ $[x=3a, y=0]$



SOLUTION

Reactions: $R_B = 2P \uparrow$, $R_D = 0$

$0 < x < a$ $V = -P$

$a < x < 2a$ $V = -P + 2P$

$2a < x < 3a$ $V = -P + 2P - P$

Using singularity functions

$$\frac{dM}{dx} = V = -P + 2P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$$

$$M = -Px + 2P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 + M_A$$

But $M = 0$ at $x = 0$ $M_A = 0$

$$EI\frac{d^2y}{dx^2} = M = -Px + 2P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 \quad (1)$$

$$EI\frac{dy}{dx} = -\tfrac{1}{2}Px^2 + P\langle x-a\rangle^2 - \tfrac{1}{2}P\langle x-2a\rangle^2 + C_1 \quad (2)$$

$$EI\,y = -\tfrac{1}{6}Px^3 + \tfrac{1}{3}P\langle x-a\rangle^3 - \tfrac{1}{6}P\langle x-2a\rangle^3 + C_1x + C_2 \quad (3)$$

$[x=a, y=0]$ $-\tfrac{1}{6}Pa^3 + 0 - 0 + C_1a + C_2 = 0$ $aC_1 + C_2 = \tfrac{1}{6}Pa^3$ (4)

$[x=3a, y=0]$ $-\tfrac{1}{6}P(3a)^3 + \tfrac{1}{3}P(2a)^3 - \tfrac{1}{6}Pa^3 + C_1(3a) + C_2 = 0$ $3aC_1 + C_2 = 2Pa^2$ (5)

Eq(5) - Eq(4) $2C_1a = \tfrac{11}{6}Pa^2$ $C_1 = \tfrac{11}{12}Pa^2$

$$C_2 = \tfrac{1}{6}Pa^2 - aC_1 = -\tfrac{3}{4}Pa^3$$

$$y = \frac{P}{EI}\left\{-\tfrac{1}{6}x^3 + \tfrac{1}{3}\langle x-a\rangle^3 - \tfrac{1}{6}\langle x-2a\rangle^3 + \tfrac{11}{12}a^2x - \tfrac{3}{4}a^3\right\}$$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\tfrac{1}{2}x^2 + \langle x-a\rangle^2 - \tfrac{1}{2}\langle x-2a\rangle^2 + \tfrac{11}{12}a^2\right\}$$

(a) Deflection at A (y at $x = 0$)

$$y_A = \frac{Pa^3}{EI}\left\{-0 + 0 - 0 + 0 - \tfrac{3}{4}\right\} = -\frac{3}{4}\frac{Pa^3}{EI} \qquad y_A = \frac{3}{4}\frac{Pa^3}{EI}\downarrow$$

(b) Deflection at C (y at $x = 2a$)

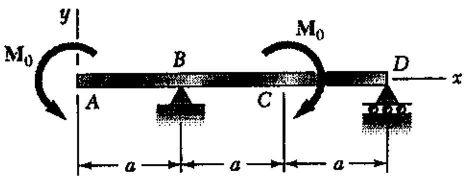
$$y_C = \frac{Pa^3}{EI}\left\{-\tfrac{1}{6}(2)^3 + \tfrac{1}{3}(1)^3 - 0 + \tfrac{11}{12}(2) - \tfrac{3}{4}\right\} = \frac{1}{12}\frac{Pa^3}{EI}\uparrow$$

(c) Slope at D ($\frac{dy}{dx}$ at $x = 3a$)

$$\theta_D = \frac{Pa^2}{EI}\left\{-\tfrac{1}{2}(3)^2 + (2)^2 - \tfrac{1}{2}(1)^2 + \tfrac{11}{12}\right\} = -\frac{1}{12}\frac{Pa^2}{EI} \qquad \theta_D = \frac{1}{12}\frac{Pa^2}{EI}$$

PROBLEM 9.40

9.39* and *9.40 For the beam and loading shown, determine (*a*) the deflection at end *A*, (*b*) the deflection point *C*, (*c*) the slope at end *D*.



$[x=a, y=0]$ $[x=3a, y=0]$

SOLUTION

Since loads self equilibrate

$R_B = 0 \qquad R_D = 0$

$0 < x \; 2a \qquad M = -M_0$

$2a < x < 3a \qquad M = -M_0 + M_0 = 0$

Using singularity functions

$$EI\frac{d^2y}{dx^2} = M = -M_0 + M_0\langle x-2a\rangle^0$$

$$EI\frac{dy}{dx} = -M_0x + M_0\langle x-2a\rangle^1 + C_1$$

$$EIy = -\tfrac{1}{2}M_0x^2 + \tfrac{1}{2}M_0\langle x-2a\rangle^2 + C_1x + C_2$$

$[x=3a, y=0] \qquad -\tfrac{1}{2}M_0(3a)^2 + \tfrac{1}{2}M_0a^2 + C_1(3a) + C_2 = 0 \qquad 3aC_1 + C_2 = 4M_0a^2$

$[x=a]\; y=0] \qquad -\tfrac{1}{2}M_0a^2 + 0 + C_1a + C_2 = 0 \qquad aC_1 + C_2 = \tfrac{1}{2}M_0a^2$

Subtracting $\quad 2aC_1 = \tfrac{7}{2}M_0a^2 \qquad C_1 = \tfrac{7}{4}M_0a$

$$C_2 = \tfrac{1}{2}M_0a^2 - aC_1 = -\tfrac{5}{4}M_0a^2$$

$$y = \frac{M_0}{EI}\left\{-\tfrac{1}{2}x^2 + \tfrac{1}{2}\langle x-2a\rangle^2 + \tfrac{7}{4}ax - \tfrac{5}{4}a^2\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{EI}\left\{-x + \langle x-a\rangle^1 + \tfrac{7}{4}a\right\}$$

(a) Deflection at A $\quad$ (y at $x = 0$)

$$y_A = \frac{M_0a^2}{EI}\left\{-0 + 0 + 0 - \tfrac{5}{4}\right\} = -\tfrac{5}{4}\frac{M_0a^2}{EI}, \qquad y_A = \tfrac{5}{4}\frac{M_0a^2}{EI}\downarrow$$ ◄

(b) Deflection at C $\quad$ (y at $x = 2a$)

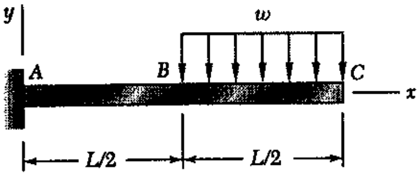
$$y_C = \frac{M_0a^2}{EI}\left\{-\tfrac{1}{2}(2)^2 + 0 + \tfrac{7}{4}(2) - \tfrac{5}{4}\right\} = \tfrac{1}{4}\frac{M_0a^2}{EI}\uparrow$$ ◄

(c) Slope at D $\quad$ ($\frac{dy}{dx}$ at $x = 3a$)

$$\theta_D = \frac{M_0a}{EI}\left\{-3 + 1 + \tfrac{7}{4}\right\} = -\tfrac{1}{4}\frac{M_0a}{EI}, \qquad \theta_D = \tfrac{1}{4}\frac{M_0a}{EI}$$ ◄

PROBLEM 9.41

9.41 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *C*.



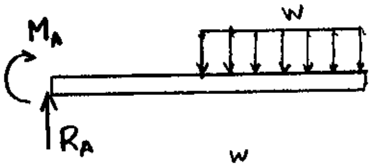
$[x=0,\ \frac{dy}{dx}=0]$
$[x=0,\ y=0]$

SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A - \frac{1}{2}wL = 0 \qquad R_A = \frac{1}{2}wL$

$\circlearrowleft \Sigma M_A = 0 \qquad -M_A - (\frac{1}{2}wL)(\frac{3}{4}L) = 0$

$$M_A = -\frac{3}{8}wL^2$$



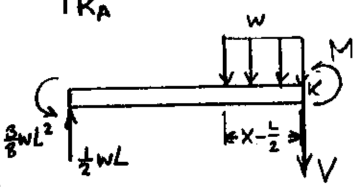
$0 < x < \frac{L}{2} \qquad M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx$

$\frac{L}{2} < x < L \qquad$ (See free body diagram.)

$\circlearrowleft \Sigma M_K = 0$

$\frac{3}{8}wL^2 - \frac{1}{2}wLx + \frac{1}{2}w(x-\frac{L}{2})^2 + M = 0$

$M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w(x-\frac{L}{2})^2$



Using singularity functions

$EI\frac{d^2y}{dx^2} = M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w\langle x-\frac{L}{2}\rangle^2$

$EI\frac{dy}{dx} = -\frac{3}{8}wL^2x + \frac{1}{4}wLx^2 - \frac{1}{6}w\langle x-\frac{L}{2}\rangle^3 + C_1$

$[x=0,\ \frac{dy}{dx}=0] \qquad -0+0-0+C_1 = 0 \qquad\qquad C_1 = 0$

$EIy = -\frac{3}{16}wL^2x^2 + \frac{1}{12}wLx^3 - \frac{1}{24}w\langle x-\frac{L}{2}\rangle^4 + C_1x + C_2$

$[x=0,\ y=0] \qquad -0+0-0+0+C_2 = 0 \qquad\qquad C_2 = 0$

(a) Elastic curve $\qquad y = \frac{w}{EI}\left\{-\frac{3}{16}L^2x^2 + \frac{1}{12}Lx^3 - \frac{1}{24}\langle x-\frac{L}{2}\rangle^4\right\}$ ◄

$$\frac{dy}{dx} = \frac{w}{EI}\left\{-\frac{3}{8}L^2x + \frac{1}{4}Lx^2 - \frac{1}{6}\langle x-\frac{L}{2}\rangle^3\right\}$$

(b) Deflection at B $\qquad$ (y at $x = \frac{L}{2}$)

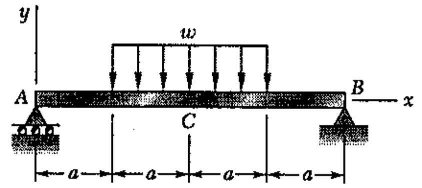
$y_B = \frac{wL^4}{EI}\left\{-\frac{3}{16}(\frac{1}{2})^2 + \frac{1}{12}(\frac{1}{2})^3 - 0\right\} = -\frac{7}{192}\frac{wL^4}{EI}, \qquad y_B = \frac{7}{192}\frac{wL^4}{EI}\downarrow$ ◄

(c) Deflection at C $\qquad$ (y at $x = L$)

$y_C = \frac{wL^4}{EI}\left\{-\frac{3}{16}(1)^2 + \frac{1}{12}(1)^3 - \frac{1}{24}(\frac{1}{2})^4\right\} = -\frac{41}{384}\frac{wL^4}{EI}, \qquad y_C = \frac{41}{384}\frac{wL^4}{EI}\downarrow$ ◄

PROBLEM 9.42

9.42 and 9.43 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.



$[x=0,\ y=0]$ $[x=0,\ M=0]$

$[x=4a,\ y=0]$ $[x=4a,\ M=0]$

SOLUTION

By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - 2wa = 0 \qquad R_A = wa$

$$w(x) = w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x-a\rangle^1 + w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \tfrac{1}{2}w\langle x-a\rangle^2 + \tfrac{1}{2}\langle x-3a\rangle^2 \qquad \text{with } M_A = 0$$

$$EI\frac{d^2y}{dx^2} = M = wax - \tfrac{1}{2}w\langle x-a\rangle^2 + \tfrac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}wax^2 - \tfrac{1}{6}w\langle x-a\rangle^3 + \tfrac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EI\,y = \tfrac{1}{6}wax^3 - \tfrac{1}{24}w\langle x-a\rangle^4 + \tfrac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$[x=0,\ y=0] \qquad 0 - 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=4a,\ y=0] \qquad \tfrac{1}{6}wa(4a)^3 - \tfrac{1}{24}w(3a)^4 + \tfrac{1}{24}(a)^4 + C_1(4a) = 0$

$$4C_1 = wa^3\left\{-\frac{64}{6} + \frac{81}{24} - \frac{1}{24}\right\} = -\frac{22}{3}wa^3 \qquad C_1 = -\frac{11}{6}wa^3$$

(a) Equation of elastic curve

$$y = \frac{w}{EI}\left\{\tfrac{1}{6}ax^3 - \tfrac{1}{24}\langle x-a\rangle^4 + \tfrac{1}{24}\langle x-3a\rangle^4 - \tfrac{11}{6}a^3x\right\}$$

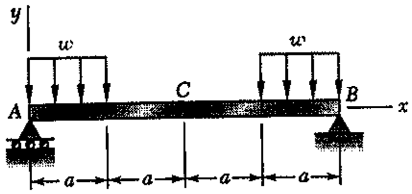
(b) Deflection at C $\quad$ (y at $x=2a$)

$$y_C = \frac{wa^4}{EI}\left\{\tfrac{1}{6}(2)^3 - \tfrac{1}{24}(1)^4 + 0 - \tfrac{11}{6}(2)\right\} = -\frac{19}{8}\frac{wa^4}{EI}$$

$$y_C = \frac{19}{8}\frac{wa^4}{EI}\downarrow$$

PROBLEM 9.43

9.42 and 9.43 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.



$[x=0, y=0]$ $[x=0, M=0]$

$[x=4a, y=0]$ $[x=4a, M=0]$

SOLUTION

By symmetry $\quad R_A = R_B$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - 2wa = 0 \qquad R_A = wa$

$$w(x) = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \tfrac{1}{2}wx^2 + \tfrac{1}{2}w\langle x-a\rangle^2 - \tfrac{1}{2}w\langle x-3a\rangle^2 \qquad \text{with } M_A = 0$$

$$EI\frac{d^2y}{dx^2} = M = wax - \tfrac{1}{2}wx^2 + \tfrac{1}{2}w\langle x-a\rangle^2 - \tfrac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}wax^2 - \tfrac{1}{6}wx^3 + \tfrac{1}{6}w\langle x-a\rangle^3 - \tfrac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EI\,y = \tfrac{1}{6}wax^3 - \tfrac{1}{24}wx^4 + \tfrac{1}{24}w\langle x-a\rangle^4 - \tfrac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$[x=0, y=0] \qquad 0 - 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=4a, y=0] \qquad \tfrac{1}{6}wa(4a)^3 - \tfrac{1}{24}w(4a)^4 + \tfrac{1}{24}w(3a)^4 - \tfrac{1}{24}wa^4 + C_1(4a) = 0$

$$4C_1 = wa^3\left\{-\frac{64}{6} + \frac{256}{24} - \frac{81}{24} + \frac{1}{24}\right\} = -\frac{10}{3}wa^3 \qquad C_1 = -\frac{5}{6}wa^3$$

(a) Equation of elastic curve

$$y = \frac{w}{EI}\left\{\frac{1}{6}ax^3 - \frac{1}{24}x^4 + \frac{1}{24}\langle x-a\rangle^4 - \frac{1}{24}\langle x-3a\rangle^4 - \frac{5}{6}a^3x\right\}$$

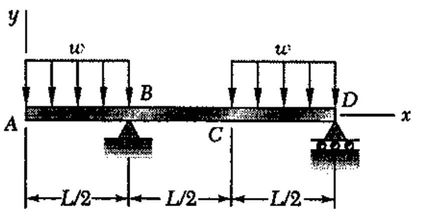
(b) Deflection at C $\qquad$ (y at $x = 2a$)

$$y_C = \frac{wa^4}{EI}\left\{\frac{1}{6}(2)^3 - \frac{1}{24}(2)^4 + \frac{1}{24}(1)^4 + 0 - \frac{5}{6}(2)\right\} = -\frac{23}{24}\frac{wa^4}{EI}$$

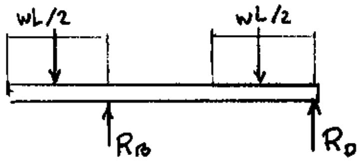
$$y_C = \frac{23}{24}\frac{wa^4}{EI}\downarrow$$

PROBLEM 9.44

9.44 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *A*, (*c*) the deflection at point *C*.



$[x=\frac{L}{2}, y=0]$ $\quad$ $[x=\frac{3L}{2}, y=0]$



SOLUTION

$$\circlearrowleft M_D = 0 \qquad \frac{wL}{2}\cdot\frac{5L}{4} - R_B L + \frac{wL}{2}\cdot\frac{L}{4} = 0$$

$$R_B = \frac{3}{4}w$$

$$w(x) = w - w\langle x-\tfrac{L}{2}\rangle^0 + w\langle x-L\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-\tfrac{L}{2}\rangle^0 - w\langle x-L\rangle^0$$

$$\frac{dM}{dx} = V = -wx + R_B\langle x-\tfrac{L}{2}\rangle^0 + w\langle x-\tfrac{L}{2}\rangle^1 - w\langle x-L\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = -\tfrac{1}{2}wx^2 + \tfrac{3}{4}wL\langle x-\tfrac{L}{2}\rangle^1 + \tfrac{1}{2}w\langle x-\tfrac{L}{2}\rangle^2 - \tfrac{1}{2}w\langle x-L\rangle^2$$

$$EI\frac{dy}{dx} = -\tfrac{1}{6}wx^3 + \tfrac{3}{8}wL\langle x-\tfrac{L}{2}\rangle^2 + \tfrac{1}{6}w\langle x-\tfrac{L}{2}\rangle^3 - \tfrac{1}{6}w\langle x-L\rangle^3 + C_1$$

$$EI\,y = -\tfrac{1}{24}wx^4 + \tfrac{1}{8}wL\langle x-\tfrac{L}{2}\rangle^3 + \tfrac{1}{24}w\langle x-\tfrac{L}{2}\rangle^4 - \tfrac{1}{24}w\langle x-L\rangle^4 + C_1x + C_2$$

$$[x=\tfrac{L}{2}, y=0] \quad -\tfrac{1}{24}w\left(\tfrac{L}{2}\right)^4 + 0 + 0 - 0 + C_1\tfrac{L}{2} + C_2 = 0$$

$$C_2 = \tfrac{1}{384}wL^4 - C_1\tfrac{L}{2}$$

$$[x=\tfrac{3L}{2}, y=0] \quad -\tfrac{1}{24}w\left(\tfrac{3L}{2}\right)^4 + \tfrac{1}{8}wL\cdot L^3 + \tfrac{1}{24}wL^4 - \tfrac{1}{24}w\left(\tfrac{L}{2}\right)^4 + C_1\tfrac{3L}{2} + \left(\tfrac{1}{384}wL^4 - C_1\tfrac{L}{2}\right) = 0$$

$$\left(\tfrac{3}{2}-\tfrac{1}{2}\right)C_1L = \left(\tfrac{1}{24}\cdot\tfrac{81}{16} - \tfrac{1}{8} - \tfrac{1}{24} + \tfrac{1}{24}\cdot\tfrac{1}{16} - \tfrac{1}{384}\right)wL^4 \qquad C_1 = \tfrac{17}{384}wL^3$$

$$C_2 = \left(\tfrac{1}{384} - \tfrac{17}{768}\right)wL^4 = -\tfrac{5}{256}wL^4$$

(a) $$y = \frac{w}{EI}\left\{-\tfrac{1}{24}x^4 + \tfrac{1}{8}L\langle x-\tfrac{L}{2}\rangle^3 + \tfrac{1}{24}\langle x-\tfrac{L}{2}\rangle^4 - \tfrac{1}{24}\langle x-L\rangle^4 + \tfrac{17}{384}L^3x - \tfrac{5}{256}L^4\right\}$$ ◄

(b) Deflection at A $\quad$ (y at $x=0$)

$$y_A = \frac{w}{EI}\left\{0+0+0+0+0-\tfrac{5}{256}L^4\right\} = -\frac{5}{256}\frac{wL^4}{EI}$$

$$y_A = \frac{5}{256}\frac{wL^4}{EI}\downarrow$$ ◄

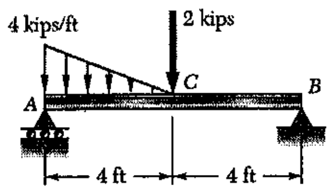
(c) Deflection at C $\quad$ (y at $x=L$)

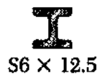
$$y_C = \frac{w}{EI}\left\{-\tfrac{1}{24}L^4 + \tfrac{1}{8}L\tfrac{L^3}{8} + \tfrac{1}{24}\tfrac{L^4}{16} - 0 + \tfrac{17}{384}L^3L - \tfrac{5}{256}L^4\right\}$$

$$= \frac{1}{768}\frac{wL^4}{EI}\uparrow$$ ◄

PROBLEM 9.45

9.45 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 29 \times 10^6$ psi.

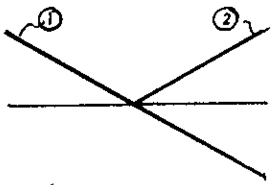


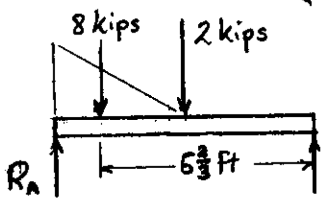


SOLUTION

Distributed loads: ① $w_1(x) = w_0 - kx$
② $w_2(x) = kx$

Data: $a = 4\text{ ft}$, $w_0 = 4\text{ kips/ft}$, $k = 1\text{ kip/ft}^2$
$P = 2\text{ kips}$.





$+\circlearrowleft \Sigma M_B = 0 \quad -8R_A + (8)(6\tfrac{2}{3}) + (2)(4) = 0 \qquad R_A = \frac{23}{3}\text{ kips}$

$$w(x) = w_0 - kx + k\langle x-4\rangle^1$$
$$= 4 - x + \langle x-4\rangle^1$$

$$\frac{dV}{dx} = -w = -4 + x - \langle x-4\rangle^1$$

$$\frac{dM}{dx} = V = \frac{23}{3} - 4x + \frac{1}{2}x^2 - \frac{1}{2}\langle x-4\rangle^2 - 2\langle x-4\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = \frac{23}{3}x - 2x^2 + \frac{1}{6}x^3 - \frac{1}{6}\langle x-4\rangle^3 - 2\langle x-4\rangle^1 \quad \text{kip}\cdot\text{ft}$$

$$EI\frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}\langle x-4\rangle^4 - \langle x-4\rangle^2 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EIy = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}\langle x-4\rangle^5 - \frac{1}{3}\langle x-4\rangle^3 + C_1x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$[x=0, y=0] \quad 0 - 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=8, y=0] \quad (\frac{23}{18})(8)^3 - \frac{1}{6}(8)^4 + \frac{1}{120}(8)^5 - \frac{1}{120}(4)^5 - \frac{1}{3}(4)^3 + C_1(8) = 0$

$C_1 = -26.844 \quad \text{kip}\cdot\text{ft}^2$

Data: $E = 29 \times 10^6\text{ psi} = 29 \times 10^3\text{ ksi} \qquad I = 22.1\text{ in}^4$

$EI = (29 \times 10^3)(22.1) = 640.9 \times 10^3\text{ kip}\cdot\text{in}^2 = 4451\text{ kip}\cdot\text{ft}^2$

(a) Slope at A $\quad (\frac{dy}{dx}$ at $x = 0)$

$$EI\theta_A = 0 + 0 + 0 + 0 + 0 - 26.844 \quad \text{kip}\cdot\text{ft}^2$$

$$\theta_A = -\frac{26.844}{4451} = -6.03 \times 10^{-3}\text{ rad}$$

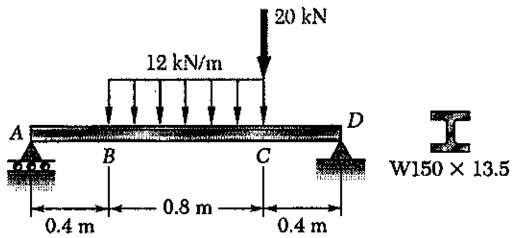
(b) Deflection at C $\quad (y$ at $x = 4\text{ ft.})$

$$EIy_C = \frac{23}{18}(4)^3 - \frac{1}{6}(4)^4 + \frac{1}{120}(4)^5 - 0 - 0 - (26.844)(4) + 0$$
$$= -59.73 \quad \text{kip}\cdot\text{ft}^3$$

$$y_C = -\frac{59.73}{4451} = -13.42 \times 10^{-3}\text{ ft}$$
$$= 0.1610\text{ in.} \downarrow$$

PROBLEM 9.46

9.46 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.



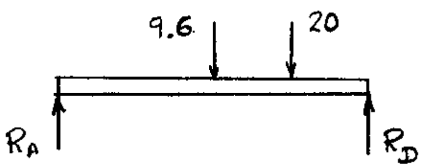
SOLUTION

Units: Forces in kN, lengths in m

$+\circlearrowleft M_D = 0$

$-1.6\,R_A + (9.6)(0.8) + (20)(0.4) = 0$

$R_A = 9.8 \quad \text{kN}$



$w(x) = 12\langle x-0.4\rangle^0 - 12\langle x-1.2\rangle^0 \quad \text{kN/m}$

$\frac{dV}{dx} = -w(x) = -12\langle x-0.4\rangle^0 + 12\langle x-1.2\rangle^0 \quad \text{kN/m}$

$\frac{dM}{dx} = V = 9.8 - 12\langle x-0.4\rangle^1 + 12\langle x-1.2\rangle^1 - 20\langle x-1.2\rangle^0 \quad \text{kN}$

$EI\frac{d^2y}{dx^2} = M = 9.8x - 6\langle x-0.4\rangle^2 + 6\langle x-1.2\rangle^2 - 20\langle x-1.2\rangle^1 \quad \text{kN}\cdot\text{m}$

$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x-0.4\rangle^3 + 2\langle x-1.2\rangle^3 - 10\langle x-1.2\rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$

$EIy = 1.63333x^3 - \frac{1}{2}\langle x-0.4\rangle^4 + \frac{1}{2}\langle x-1.2\rangle^4 - \frac{10}{3}\langle x-1.2\rangle^3 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$

$[x=0,\ y=0] \qquad 0 - 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=1.6,\ y=0] \qquad (1.63333)(1.6)^3 - \frac{1}{2}(1.2)^4 + \frac{1}{2}(0.4)^4 - \frac{10}{3}(0.4)^3 + C_1(1.6) + 0 = 0$

$C_1 = -3.4080 \quad \text{kN}\cdot\text{m}^2$

Data: $E = 200\times10^9$ Pa $\qquad I = 6.87\times10^6\ \text{mm}^4 = 6.87\times10^{-6}\ \text{mm}^4$

$EI = (200\times10^9)(6.87\times10^{-6}) = 1.374\times10^6\ \text{N}\cdot\text{m}^2 = 1374\ \text{kN}\cdot\text{m}^2$

(a) Slope at A $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$EI\frac{dy}{dx} = 0 - 0 + 0 - 0 - 3.4080 \quad \text{kN}\cdot\text{m}^2$

$\theta_A = -\frac{3.4080}{1374} = -2.48\times10^{-3}\ \text{rad} = 2.48\times10^{-3}\ \text{rad} \curvearrowleft$ ◄

(b) Deflection at C (y at $x = 1.2$ m)

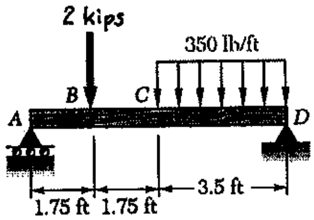
$EIy_C = (1.63333)(1.2)^3 - \frac{1}{2}(0.8)^4 + 0 - 0 - (3.4080)(1.2) + 0$

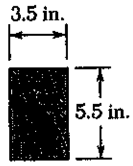
$= -1.4720 \quad \text{kN}\cdot\text{m}^3$

$y_C = -\frac{1.4720}{1374} = -1.071\times10^{-3}\ \text{m} = 1.071\ \text{mm}\downarrow$ ◄

PROBLEM 9.47

9.47 **For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 1.6 \times 10^6$ psi.**





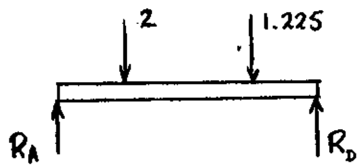
SOLUTION

Units: Forces in kips, lengths in ft.

$+\circlearrowleft \Sigma M_D = 0$

$-7R_A + (2)(5.25) + (1.225)(1.75) = 0$

$R_A = 1.80625$ kips



$w(x) = 0.350\langle x - 3.5\rangle^0$

$\frac{dV}{dx} = -w = -0.35\langle x - 3.5\rangle^0$

$\frac{dM}{dx} = V = 1.05625 - 1\langle x - 1.75\rangle^0 - 0.35\langle x - 3.5\rangle^1$

$EI\frac{d^2y}{dx^2} = M = 1.80625x - 2\langle x - 1.75\rangle^1 - 0.175\langle x - 3.5\rangle^2$ kip·ft

$EI\frac{dy}{dx} = 0.903125x^2 - 1\langle x - 1.75\rangle^2 - 0.05833\langle x - 3.5\rangle^3 + C_1$ kip·ft²

$EI\,y = 0.301042x^3 - \frac{1}{3}\langle x - 1.75\rangle^3 - 0.014583\langle x - 3.5\rangle^4 + C_1x + C_2$ kip·ft³

$[x = 0, y = 0]$ $C_2 = 0$

$[x = 7, y = 0]$ $(0.301042)(7)^3 - \frac{1}{3}(5.25)^3 - 0.014583(3.5)^4 + C_1(7) + 0 = 0$

$C_1 = -7.54779$ kip·ft²

Data: $E = 1.6 \times 10^6$ psi $= 1.6 \times 10^3$ ksi

$I = \frac{1}{12}(3.5)(5.5)^3 = 48.526$ in³

$EI = (1.6 \times 10^3)(48.526) = 77.6417$ kip·in² $= 539.18$ kip·ft²

(a) Slope at A $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$EI\frac{dy}{dx} = 0 - 0 - 0 - 7.54779$ kip·ft²

$\theta_A = -\frac{7.54779}{539.18} = -14.00 \times 10^{-3}$ rad $= 14.00 \times 10^{-3}$ rad ◄

(b) Deflection at C (y at $x = 3.5$ ft.)

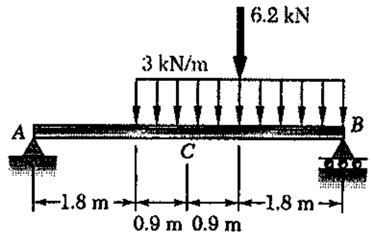
$EIy_C = (0.301042)(3.5)^3 - \frac{1}{3}(1.75)^3 - 0 - (7.54779)(3.5) + 0$

$= -15.297$ kip·ft³

$y_C = -\frac{15.297}{539.18} = -28.37 \times 10^{-3}$ ft $= 0.340$ in ↓ ◄

PROBLEM 9.48

9.48 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.



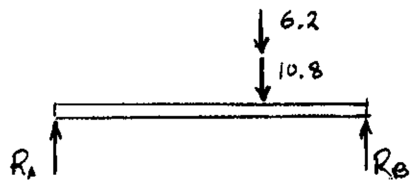
SOLUTION

Units: Forces in kN, lengths in meters.

$+\circlearrowleft \Sigma M_B = 0$

$-5.4 R_A - (1.8)(6.2 + 10.8) = 0$

$R_A = 5.6667$ kN



$w(x) = 3\langle x - 1.8\rangle^0$

$\frac{dV}{dx} = -w(x) = -3\langle x - 1.8\rangle^0$

$\frac{dM}{dx} = V = 5.6667 - 3\langle x-1.8\rangle^1 - 6.2\langle x-3.6\rangle^0$

$EI\frac{d^2y}{dx^2} = M = 5.6667x - \frac{3}{2}\langle x-1.8\rangle^2 - 6.2\langle x-3.6\rangle^1$ kN·m

$EI\frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x-1.8\rangle^3 - 3.1\langle x-3.6\rangle^2 + C_1$ kN·m²

$EIy = 0.9444x^3 - \frac{1}{8}\langle x-1.8\rangle^4 - 1.0333\langle x-3.6\rangle^3 + C_1x + C_2$ kN·m³

$[x=0, y=0]$ $\quad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=5.4, y=0]$ $\quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0333(1.8)^3 + C_1(5.4) + 0 = 0$

$C_1 = -22.535$ kN·m²

Data: $E = 200\times10^9$ Pa, $\quad I = 129\times10^6\ \text{mm}^4 = 129\times10^{-6}\ \text{m}^4$

$EI = (200\times10^9)(129\times10^{-6}) = 25.8\times10^6\ \text{N}\cdot\text{m}^2 = 25.8\times10^3\ \text{kN}\cdot\text{m}^2$

(a) Slope at A $\quad$ ($\frac{dy}{dx}$ at $x = 0$)

$EI\frac{dy}{dx} = 0 - 0 - 0 - 22.535$ kN·m²

$\theta_A = -\frac{22.535}{25.8\times10^3} = -873\times10^{-6} = 0.873\times10^{-3}$ rad ◄

(b) Deflection at C $\quad$ (y at $x = 2.7$ m)

$EIy_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 - (22.535)(2.7) + 0$

$= -42.337$ kN·m³

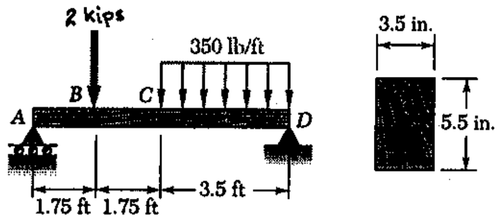
$y_C = -\frac{42.337}{25.8\times10^3} = -1.641\times10^{-3}$ m

$y_C = 1.641$ mm ↓ ◄

PROBLEM 9.49

9.49 and 9.50 For the beam and loading indicated, write a computer program and use it to calculate the slope and deflection of the beam at intervals ΔL, starting at point *A* and ending at the right-hand support.

9.49 Beam and loading of Prob. 9.47 with $\Delta L = 3.0$ in.



SOLUTION

See solution to Prob. 9.47 for the derivation of the equations used in the following.

$EI = 539.18 \quad \text{kip}\cdot\text{ft}^2$

$EI\frac{dy}{dx} = 0.903125\,x^2 - 1\langle x - 1.75\rangle^2 - 0.05833\langle x-3.5\rangle^3 - 7.54779 \quad \text{kip}\cdot\text{ft}^2$

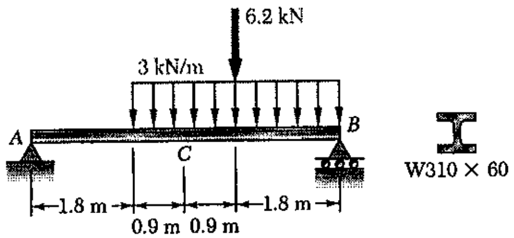
$EI\,y = 0.301042\,x^3 - \frac{1}{3}\langle x - 1.75\rangle^3 - 0.014583\langle x-3.5\rangle^4 - 7.54779\,x \quad \text{kip}\cdot\text{ft}^3$

x (in)	x (ft)	θ (10^{-3} rad)	y (10^{-3} ft)	y (in)
0	0	-14.00	0	0
3	0.25	-13.89	-3.49	-0.042
6	0.5	-13.58	-6.93	-0.083
9	0.75	-13.06	-10.26	-0.123
12	1.0	-12.32	-13.44	-0.161
15	1.25	-11.38	-16.41	-0.197
18	1.5	-10.23	-19.11	-0.229
→ 21	1.75	-8.87	-21.51	-0.258
24	2.0	-7.41	-23.54	-0.282
27	2.25	-5.98	-25.21	-0.303
30	2.5	-4.57	-26.53	-0.318
33	2.75	-3.19	-27.50	-0.330
36	3.0	-1.82	-28.13	-0.338
39	3.25	-0.48	-28.42	-0.341
→ 42	3.5	0.84	-28.37	-0.340
45	3.75	2.14	-28.00	-0.336
48	4.0	3.40	-27.30	-0.328
51	4.25	4.62	-26.30	-0.316
54	4.5	5.79	-25.00	-0.300
57	4.75	6.89	-23.41	-0.281
60	5.0	7.92	-21.56	-0.259
63	5.25	8.87	-19.46	-0.234
66	5.5	9.72	-17.13	-0.206
69	5.75	10.47	-14.61	-0.175
72	6.0	11.11	-11.91	-0.143
75	6.25	11.62	-9.06	-0.109
78	6.5	12.00	-6.11	-0.073
81	6.75	12.24	-3.07	-0.037
84	7.0	12.32	0	0

PROBLEM 9.50

9.49 and 9.50 For the beam and loading indicated, write a computer program and use it to calculate the slope and deflection of the beam at intervals ΔL, starting at point A and ending at the right-hand support.

9.50 Beam and loading of Prob. 9.48 with $\Delta L = 0.3$ m.



SOLUTION

See solution to Prob. 9.48 for the derivation of the equations used in the following.

$EI = 25.8 \times 10^3 \quad kN \cdot m^2$

$EI \frac{dy}{dx} = 2.8333 x^2 - \frac{1}{2}\langle x - 1.8\rangle^3 - 3.1\langle x - 3.6\rangle^2 - 22.535 \quad kN \cdot m^2$

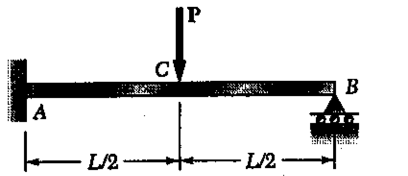
$EI\, y = 0.9444 x^3 - \frac{1}{8}\langle x - 1.8\rangle^4 - 1.03333\langle x - 3.6\rangle^3 - 22.535 x \quad kN \cdot m^3$

x (m)	θ (10^{-6} rad)	y (mm)
0	−873	0
0.3	−864	−0.261
0.6	−834	−0.516
0.9	−784	−0.759
1.2	−715	−0.985
1.5	−626	−1.187
→ 1.8	−518	−1.359
2.1	−390	−1.495
2.4	−245	−1.591
2.7	−87	−1.641
3.0	81	−1.642
3.3	257	−1.591
→ 3.6	437	−1.487
3.9	606	−1.330
4.2	753	−1.126
4.5	872	−0.882
4.8	960	−0.606
5.1	1016	−0.309
5.4	1035	0

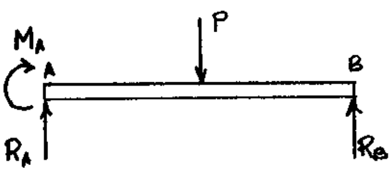
(b) Deflection at C (y at $x = \frac{L}{2}$)

PROBLEM 9.51

9.51 through 9.54 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point C.



$[x=0, y=0]$ $[x=L, y=0]$
$[x=0, \frac{dy}{dx}=0]$



SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - P = 0 \qquad R_A = P - R_B$

$+\circlearrowleft \Sigma M_A = 0 \qquad -M_A - P\frac{L}{2} + R_B L = 0 \qquad M_A = R_B L - \frac{1}{2}PL$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - P\langle x - \tfrac{L}{2}\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \tfrac{L}{2}\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - \tfrac{1}{2}P\langle x - \tfrac{L}{2}\rangle^2 + C_1$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{6}P\langle x - \tfrac{L}{2}\rangle^3 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=L, y=0] \qquad \tfrac{1}{2}M_A L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{6}P(\tfrac{L}{2})^3 + 0 + 0 = 0$

$$\tfrac{1}{2}(R_B L - \tfrac{1}{2}PL)L^2 + \tfrac{1}{6}(P - R_B)L^3 - \tfrac{1}{48}PL^3 = 0$$

$(\tfrac{1}{2} - \tfrac{1}{6})R_B L^3 = (\tfrac{1}{4} - \tfrac{1}{6} + \tfrac{1}{48})PL^3 \qquad \tfrac{1}{3}R_B = \tfrac{5}{48}P \qquad R_B = \tfrac{5}{16}P \uparrow$ ◄

$$R_A = P - \tfrac{5}{16}P = \tfrac{11}{16}P$$

$$M_A = \tfrac{5}{16}PL - \tfrac{1}{2}PL = -\tfrac{3}{16}PL$$

(b) Deflection at C $\quad$ (y at $x = \frac{L}{2}$)

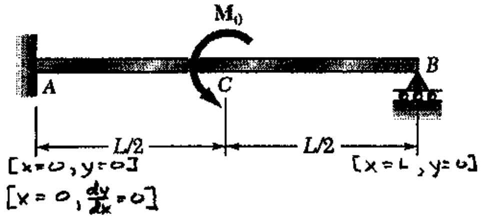
$$y_C = \frac{1}{EI}\left\{\tfrac{1}{2}M_A(\tfrac{L}{2})^2 + \tfrac{1}{6}R_A(\tfrac{L}{2})^3 + 0 + 0 + 0\right\}$$

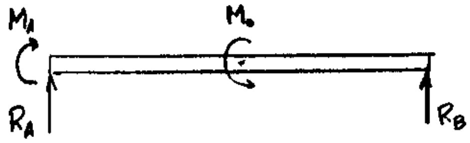
$$= \frac{PL^3}{EI}\left\{(\tfrac{1}{2})(-\tfrac{3}{16})(\tfrac{1}{4}) + (\tfrac{1}{6})(\tfrac{11}{16})(\tfrac{1}{8})\right\} = -\frac{7}{168}\frac{PL^3}{EI}$$

$y_C = \frac{7}{168}\frac{PL^3}{EI} \downarrow$ ◄

PROBLEM 9.52

9.51 through 9.54 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point C.





SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B = 0 \qquad\qquad R_A = -R_B$

$+\circlearrowleft \Sigma M_A = 0 \qquad -M_A + M_0 - R_B L = 0 \qquad\qquad M_A = M_0 + R_B L$

Reactions are statically indeterminate.

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0\langle x - \tfrac{L}{2}\rangle^0$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - M_0\langle x - \tfrac{L}{2}\rangle^1 + C_1$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{2}M_0\langle x - \tfrac{L}{2}\rangle^2 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + 0 + C_1 = 0 \qquad\qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + 0 + C_2 = 0 \qquad\qquad C_2 = 0$

$[x=L, y=L] \qquad \tfrac{1}{2}M_A L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{2}M_0(\tfrac{L}{2})^2 + 0 + 0 = 0$

$$\tfrac{1}{2}(M_0 + R_B L)L^2 - \tfrac{1}{6}R_B L^3 + \tfrac{1}{8}M_0 L^2 = 0$$

$(\tfrac{1}{2} - \tfrac{1}{6})R_B L^3 = (\tfrac{1}{8} - \tfrac{1}{2})M_0 L^2 \qquad \tfrac{1}{3}R_B = -\tfrac{3}{8}\frac{M_0}{L} \qquad R_B = -\tfrac{9}{8}\frac{M_0}{L}$

$$R_B = \tfrac{9}{8}\frac{M_0}{L}\downarrow$$

$R_A = \tfrac{9}{8}\frac{M_0}{L}$

$M_A = M_0 - \tfrac{9}{8}\frac{M_0}{L}\cdot L = -\tfrac{1}{8}M_0$

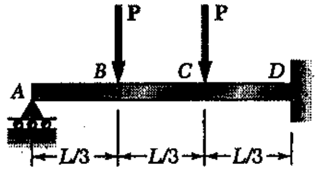
(b) Deflection at C $\quad$ (y at $x = \frac{L}{2}$)

$$y_C = \frac{1}{EI}\left\{\tfrac{1}{2}M_A(\tfrac{L}{2})^2 + \tfrac{1}{6}R_A(\tfrac{L}{2})^3\right\} = \frac{M_0L^2}{EI}\left\{(\tfrac{1}{2})(-\tfrac{1}{8})(\tfrac{1}{4}) + (\tfrac{1}{6})(\tfrac{9}{8})(\tfrac{1}{8})\right\}$$

$$= \frac{1}{128}\frac{M_0L^2}{EI}\uparrow$$

PROBLEM 9.53

9.51 through 9.54 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point C.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=0, M=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$

SOLUTION

$$\frac{dM}{dx} = V = R_A - P\langle x-\tfrac{L}{3}\rangle^0 - P\langle x-\tfrac{2L}{3}\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - P\langle x-\tfrac{L}{3}\rangle^1 - P\langle x-\tfrac{2L}{3}\rangle^1$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}R_A x^2 - \tfrac{1}{2}P\langle x-\tfrac{L}{3}\rangle^2 - \tfrac{1}{2}P\langle x-\tfrac{2L}{3}\rangle^2 + C_1$$

$$EIy = \tfrac{1}{6}R_A x^3 - \tfrac{1}{6}P\langle x-\tfrac{L}{3}\rangle^3 - \tfrac{1}{6}P\langle x-\tfrac{2L}{3}\rangle^3 + C_1 x + C_2$$

$[x=0, y=0]$ $\quad 0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=L, \frac{dy}{dx}=0]$ $\quad \tfrac{1}{2}R_A L^2 - \tfrac{1}{2}P(\tfrac{2L}{3})^2 - \tfrac{1}{2}P(\tfrac{L}{3})^2 + C_1 + 0 = 0$

$$C_1 = \tfrac{1}{2}\left[(\tfrac{4}{9}+\tfrac{1}{9})P - R_A\right]L^2 = \tfrac{1}{2}(\tfrac{5}{9}P - R_A)L^2$$

$[x=L, y=0]$ $\quad \tfrac{1}{6}R_A L^3 - \tfrac{1}{6}P(\tfrac{2L}{3})^3 - \tfrac{1}{6}P\langle\tfrac{L}{3}\rangle^3 + \tfrac{1}{2}(\tfrac{5}{9}P - R_A)L^2 L + 0 = 0$

$$(\tfrac{1}{2}-\tfrac{1}{6})R_A L^3 = \left[(\tfrac{1}{2})(\tfrac{5}{9}) - (\tfrac{1}{6})(\tfrac{8}{27}) - (\tfrac{1}{6})(\tfrac{1}{27})\right]PL^3, \qquad \tfrac{1}{3}R_A = \tfrac{2}{9}P, \qquad R_A = \tfrac{2}{3}P\uparrow$$

$$C_1 = \tfrac{1}{2}(\tfrac{5}{9}P - \tfrac{2}{3}P)L^2 = -\tfrac{1}{18}PL^2$$

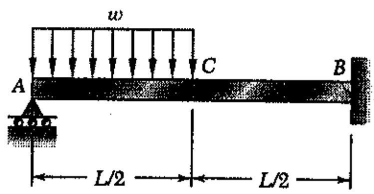
(b) Deflection at C $\quad$ (y at $x = \frac{2L}{3}$)

$$y_C = \frac{1}{EI}\left\{\tfrac{1}{6}(\tfrac{2}{3}P)(\tfrac{2L}{3})^3 - \tfrac{1}{6}P(\tfrac{L}{3})^3 - 0 - \tfrac{1}{18}PL^2(\tfrac{2L}{3})\right\}$$

$$= \frac{PL^3}{EI}\left(\frac{16}{486} - \frac{1}{162} - \frac{2}{54}\right) = -\frac{5}{486}\frac{PL^3}{EI} \qquad y_C = \frac{5}{486}\frac{PL^3}{EI}\downarrow$$

PROBLEM 9.54

9.51 through 9.54 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point C.



[x=0, y=0]
[y=0, M=0]

[x=L, y=0]
[x=L, $\frac{dy}{dx}$=0]

SOLUTION

$$w(x) = w - w\langle x-\tfrac{L}{2}\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-\tfrac{L}{2}\rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w\langle x-\tfrac{L}{2}\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \tfrac{1}{2}wx^2 + \tfrac{1}{2}w\langle x-\tfrac{L}{2}\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}R_A x^2 - \tfrac{1}{6}wx^3 + \tfrac{1}{6}w\langle x-\tfrac{L}{2}\rangle^3 + C_1$$

$$EI\,y = \tfrac{1}{6}R_A x^3 - \tfrac{1}{24}wx^4 + \tfrac{1}{24}\langle x-\tfrac{L}{2}\rangle^4 + C_1x + C_2$$

[x=0, y=0] $\quad 0+0+0+0+C_2=0 \qquad C_2 = 0$

[x=L, $\frac{dy}{dx}$=0] $\quad \tfrac{1}{2}R_AL^2 - \tfrac{1}{6}wL^3 + \tfrac{1}{6}w(\tfrac{L}{2})^3 + C_1 = 0 \qquad C_1 = \tfrac{7}{48}wL^3 - \tfrac{1}{2}R_AL^2$

[x=L, y=0] $\quad \tfrac{1}{6}R_AL^3 - \tfrac{1}{24}wL^4 + \tfrac{1}{24}w(\tfrac{L}{2})^4 + (\tfrac{7}{48}wL^3 - \tfrac{1}{2}R_AL^2)L + 0 = 0$

$$(\tfrac{1}{2}-\tfrac{1}{6})R_AL^3 = (-\tfrac{1}{24} + \tfrac{1}{24}(\tfrac{1}{16}) + \tfrac{7}{48})wL^4 \qquad \tfrac{1}{3}R_A = \tfrac{41}{384}wL \qquad R_A = \tfrac{41}{128}wL\uparrow$$

$$C_1 = \tfrac{7}{48}wL^3 - \tfrac{1}{2}(\tfrac{41}{128}wL)L^2 = -\tfrac{11}{768}wL^3$$

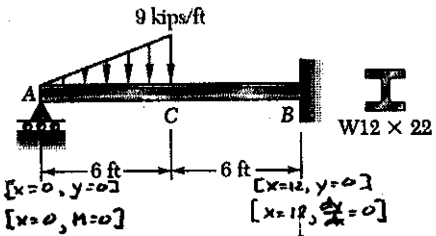
(b) Deflection at C $\quad$ (y at $x=\frac{L}{2}$)

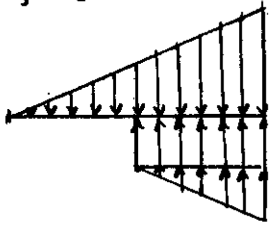
$$y_C = \frac{1}{EI}\left\{(\tfrac{1}{6})(\tfrac{41}{128}wL)(\tfrac{L}{2})^3 - \tfrac{1}{24}w(\tfrac{L}{2})^4 + 0 - \tfrac{11}{768}wL^3\tfrac{L}{2} + 0\right\}$$

$$= \frac{wL^4}{EI}\left(\frac{41}{6144} - \frac{1}{384} - \frac{11}{1536}\right) = -\frac{19}{6144}\frac{wL}{EI} \qquad y_C = \frac{19}{6144}\frac{wL^4}{EI}\downarrow$$

PROBLEM 9.55

9.55* and *9.56 For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the deflection at *C*. Use $E = 29 \times 10^6$ psi.





SOLUTION

Units: Forces in kips, lengths in ft.

$$k = \frac{9 \text{ kips/ft}}{6 \text{ ft}} = 1.5 \text{ kips/ft}^2$$

$$w(x) = 1.5x - 9\langle x-6\rangle^0 - 1.5\langle x-6\rangle^1$$

$$\frac{dV}{dx} = -w(x) = -1.5x + 9\langle x-6\rangle^0 + 1.5\langle x-6\rangle^1$$

$$\frac{dM}{dx} = V = R_A - 0.75x^2 + 9\langle x-6\rangle^1 + 0.75\langle x-6\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 0.25x^3 + 4.5\langle x-6\rangle^2 + 0.25\langle x-6\rangle^3$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}R_A x^2 - 0.0625x^4 + 1.5\langle x-6\rangle^3 + 0.0625\langle x-6\rangle^4 + C_1 \qquad \text{kip}\cdot\text{ft}^2$$

$$EIy = \tfrac{1}{6}R_A x^3 - 0.0125x^5 + 0.375\langle x-6\rangle^4 + 0.0125\langle x-6\rangle^5 + C_1x + C_2 \qquad \text{kip}\cdot\text{ft}^3$$

$[x=0, y=0]$ $\quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=12, \frac{dy}{dx}=0]$ $\quad \tfrac{1}{2}(R_A)(12)^2 - (0.0625)(12)^4 + (1.5)(6)^3 + (0.0625)(6)^4 + C_1 = 0$

$$C_1 = 891 - 72R_A = 0 \qquad \text{kip}\cdot\text{ft}^2$$

$[x=12, y=0]$ $\quad \tfrac{1}{6}R_A(12)^3 - (0.0125)(12)^5 + (0.375)(6)^4 + (0.0125)(6)^5 + (891 - 72R_A)(12) + 0 = 0$

$(864 - 288)R_A = 8164.8 \qquad R_A = 14.175 \text{ kips} \uparrow$

$$C_1 = 891 - (72)(14.175) = -129.6 \text{ kip}\cdot\text{ft}^2$$

Data: $E = 29\times10^6 \text{ psi} = 29\times10^3 \text{ ksi} \qquad I = 156 \text{ in}^4$

$EI = (29\times10^3)(156) = 4.524\times10^6 \text{ kip}\cdot\text{in}^2 = 31417 \text{ kip ft}^2$

(b) Deflection at C $\quad$ (y at $x = 6$)

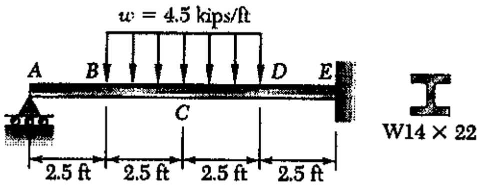
$$EIy_c = \tfrac{1}{6}(14.175)(6)^3 - (0.0125)(6)^5 + 0 + 0 - (129.6)(6) + 0$$
$$= -364.5 \text{ kip}\cdot\text{ft}^3$$

$$y_c = -\frac{364.5}{31417} = -11.60\times10^{-3} \text{ ft}$$

$$y_c = 0.1392 \text{ in} \downarrow$$

PROBLEM 9.56

9.55* and *9.56 For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the deflection at *C*. Use $E = 29 \times 10^6$ psi.



$[x=0,\ y=0]$
$[x=0,\ M=0]$

$[x=10,\ y=0]$
$[x=10,\ \frac{dy}{dx}=0]$

SOLUTION

Units: Forces in kips, lengths in ft.

$$w(x) = 4.5\langle x-2.5\rangle^0 - 4.5\langle x-7.5\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -4.5\langle x-2.5\rangle^0 + 4.5\langle x-7.5\rangle^0 \quad \text{kip/ft}$$

$$\frac{dM}{dx} = V = R_A - 4.5\langle x-2.5\rangle^1 + 4.5\langle x-7.5\rangle^1 \quad \text{kips}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 2.25\langle x-2.5\rangle^2 + 2.25\langle x-7.5\rangle^2 \quad \text{kip}\cdot\text{ft}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{2.25}{3}\langle x-2.5\rangle^3 + \frac{2.25}{3}\langle x-7.5\rangle^3 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{2.25}{12}\langle x-2.5\rangle^4 + \frac{2.25}{12}\langle x-2.5\rangle^4 + C_1 x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$[x=0,\ y=0]$ $\quad 0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=10,\ \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A(10)^2 - \frac{2.25}{3}(7.5)^3 + \frac{2.25}{3}\langle 2.5\rangle^3 + C_1 = 0$

$$C_1 = 304.69 - 50R_A \quad \text{kip}\cdot\text{ft}^2$$

$[x=10,\ y=0]$ $\quad \frac{1}{6}R_A(10)^3 - \frac{2.25}{12}(7.5)^4 + \frac{2.25}{12}(2.5)^4 + (304.69 - 50R_A)(10) + 0 = 0$

$$\left(500 - \frac{1000}{6}\right)R_A = 24609 \qquad R_A = 7.3833 \text{ kips} \uparrow$$

$$C_1 = 304.69 - (50)(7.3833) = -64.45 \quad \text{kip}\cdot\text{ft}^2$$

Data: $E = 29\times10^6$ psi $= 29\times10^3$ ksi, $\quad I = 199$ in^4

$EI = (29\times10^3)(199) = 5.771\times10^6$ kip·in^2 $= 40076$ kip·ft^2

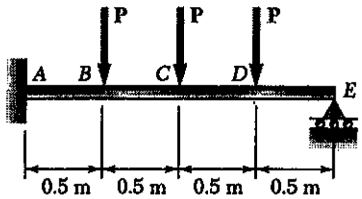
(b) Deflection at C $\quad$ (y at $x = 5$ ft)

$$EI\,y_C = \frac{1}{6}(7.3833)(5)^3 - \frac{2.25}{12}(2.5)^4 + 0 - (64.45)(5) + 0 = -175.76 \quad \text{kip}\cdot\text{ft}^3$$

$$y_C = -\frac{175.76}{40076} = -4.3856\times10^{-3} \text{ ft} \qquad y_C = -0.0526 \text{ in.}$$

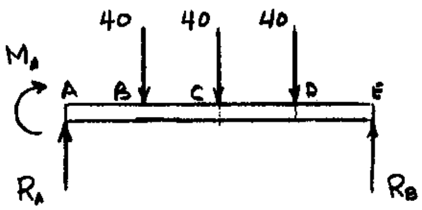
PROBLEM 9.57

9.57 For the beam shown and knowing that $P = 40$ kN, determine (*a*) the reaction at *E*, (*b*) the deflection at *C*. Use $E = 200$ GPa.



$[x=0, y=0]$ $[x=2, y=0]$

$[x=0, \frac{dy}{dx}=0]$



SOLUTION

Units: Forces in kN, lengths in m.

$+\uparrow \Sigma F_y = 0 \qquad R_A - 40 - 40 - 40 + R_E = 0$

$R_A = 120 - R_E \quad$ kN

$+\circlearrowleft M_A = 0 \qquad -M_A - 20 - 40 - 60 + 2R_B = 0$

$M_A = 2R_E - 120 \quad$ kN·m

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - 40\langle x-0.5\rangle^0 - 40\langle x-1\rangle^0 - 40\langle x-1.5\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 40\langle x-0.5\rangle^1 - 40\langle x-1\rangle^1 - 40\langle x-1.5\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - 20\langle x-0.5\rangle^2 - 20\langle x-1\rangle^2 - 20\langle x-1.5\rangle^2 + C_1$$

$$EI\,y = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{20}{3}\langle x-0.5\rangle^3 - \tfrac{20}{3}\langle x-1\rangle^3 - \tfrac{20}{3}\langle x-1.5\rangle^3 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0] \qquad 0+0+0+0+0+C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0+0+0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=2, y=0] \qquad \tfrac{1}{2}M_A(2)^2 + \tfrac{1}{6}R_A(2)^3 - \tfrac{20}{3}(1.5)^3 - \tfrac{20}{3}(1)^3 - \tfrac{20}{3}(0.5)^3 + 0 + 0 = 0$

$\tfrac{1}{2}(2R_E - 120)(2)^2 + \tfrac{1}{6}(120 - R_E)(2)^3 = 30$

$2.66667\,R_E = 30 + 240 - 160 = 110 \qquad R_E = 41.25 \text{ kN}\uparrow$

$M_A = (2)(41.25) - 120 = -37.5$ kN·m

$R_A = 120 - 41.25 = 78.75$ kN

Data: $E = 200 \times 10^9$ Pa, $I = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$

$EI = (200\times10^9)(45.5\times10^{-6}) = 9.10 \times 10^6 \text{ N}\cdot\text{m}^2 = 9100 \text{ kN}\cdot\text{m}^2$

(b) Deflection at C $\quad$ (y at $x = 1$ m)

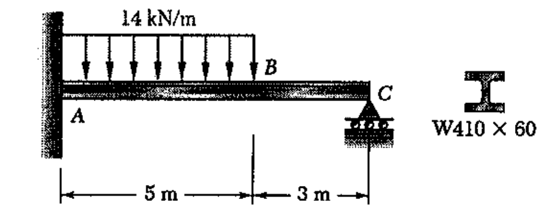
$EI\,y_C = \tfrac{1}{2}(-37.5)(1)^2 + \tfrac{1}{6}(78.75)(1)^3 - \tfrac{20}{3}(0.5)^3 - 0 - 0 + 0 + 0$

$= -6.4583 \text{ kN}\cdot\text{m}^3$

$y_C = -\dfrac{6.4583}{9100} = -0.710 \times 10^{-3}$ m $\qquad y_C = 0.710 \text{ mm}\downarrow$

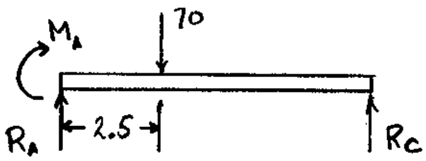
PROBLEM 9.58

9.58 For the beam and loading shown, determine (*a*) the reaction at *C*, (*b*) the deflection at *B*. Use $E = 200$ GPa.



[x=0, y=0] [x=8, y=0]

$[x=0, \frac{dy}{dx}=0]$



SOLUTION

Units: Forces in kN, lengths in m.

$+\uparrow \Sigma F_y = 0 \quad R_A - 70 + R_C = 0$

$R_A = 70 - R_C \quad \text{kN}$

$+\circlearrowleft M_A = 0 \quad -M_A - (70)(2.5) + 8R_C = 0$

$M_A = 8R_C - 175 \quad \text{kN·m}$

Reactions are statically indeterminate.

$$w(x) = 14 - 14\langle x-5\rangle^0 \quad \text{kN/m}$$

$$\frac{dV}{dx} = -w = -14 + 14\langle x-5\rangle^0 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = R_A - 14x + 14\langle x-5\rangle^1 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x-5\rangle^2 \quad \text{kN·m}$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - \tfrac{7}{3}x^3 + \tfrac{7}{3}\langle x-5\rangle^3 + C_1 \quad \text{kN·m}^2$$

$$EI\,y = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{7}{12}x^4 + \tfrac{7}{12}\langle x-5\rangle^4 + C_1 x + C_2 \quad \text{kN·m}^3$$

$[x=0, \frac{dy}{dx}=0] \quad 0+0+0+0+C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \quad 0+0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=8, y=0] \quad \tfrac{1}{2}M_A(8)^2 + \tfrac{1}{6}R_A(8)^3 - \tfrac{7}{12}(8)^4 + \tfrac{7}{12}(3)^4 + 0 + 0 = 0$

$$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28105}{12} = 0$$

$$170.667R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \qquad R_C = 11.536 \text{ kN}\uparrow$$

$$M_A = (8)(11.536) - 175 = -82.715 \text{ kN·m}$$

$$R_A = 70 - 11.536 = 58.464 \text{ kN}$$

Data: $E = 200\times 10^9$ Pa $\quad I = 216\times 10^6 \text{ mm}^4 = 216\times 10^{-6} \text{ m}^4$

$EI = (200\times 10^9)(216\times 10^{-6}) = 43.2\times 10^6 \text{ N·m}^2 = 43200 \text{ kN·m}^2$

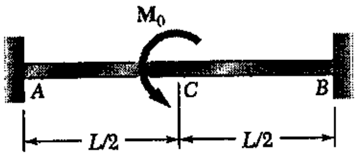
(b) Deflection at B $\quad$ (y at $x = 5$ m)

$$EI\,y_B = \tfrac{1}{2}(-82.715)(5)^2 + \tfrac{1}{6}(58.464)(5)^3 - \tfrac{7}{12}(5)^4 = -180.52 \text{ kN·m}^3$$

$$y_B = -\frac{180.52}{43200} = -4.18\times 10^{-3} \text{ m} \qquad y_B = 4.18 \text{ mm}\downarrow$$

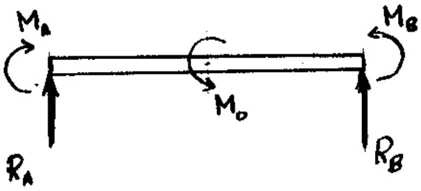
PROBLEM 9.59

9.59 For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the slope at *C*.



SOLUTION

Reactions are statically indeterminate



$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0\langle x - \tfrac{L}{2}\rangle^0$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - M_0\langle x - \tfrac{L}{2}\rangle^1 + C_1$$

$$EI\,y = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{2}M_0\langle x - \tfrac{L}{2}\rangle^2 + C_1 x + C_2$$

$[x = 0,\ \frac{dy}{dx} = 0]$ $\qquad C_1 = 0$

$[x = 0,\ y = 0]$ $\qquad C_2 = 0$

$[x = L,\ \frac{dy}{dx} = 0]$ $\qquad M_A L + \tfrac{1}{2}R_A L^2 - M_0\frac{L}{2} = 0$ $\qquad M_A = \tfrac{1}{2}M_0 - \tfrac{1}{2}R_A L$

$[x = L,\ y = 0]$ $\qquad \tfrac{1}{2}M_A L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{2}M_0\left(\tfrac{L}{2}\right)^2 = 0$

$$\tfrac{1}{2}\left(\tfrac{1}{2}M_0 - \tfrac{1}{2}R_A L\right)L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{8}M_0 L^2 = 0$$

$$\left(\tfrac{1}{4} - \tfrac{1}{6}\right)R_A L^3 = \left(\tfrac{1}{4} - \tfrac{1}{8}\right)M_0 L^2 \qquad R_A = \frac{3}{2}\frac{M_0}{L}\uparrow$$

$$M_A = \tfrac{1}{2}M_0 - \tfrac{1}{2}\cdot\tfrac{3}{2}\frac{M_0}{L}L = -\tfrac{1}{4}M_0$$

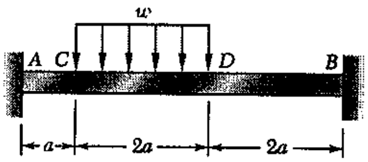
(b) Slope at C $\qquad \frac{dy}{dx}$ at $x = \frac{L}{2}$

$$\theta_C = \frac{1}{EI}\left\{\left(-\tfrac{1}{4}M_0\right)\tfrac{L}{2} + \tfrac{1}{2}\left(\tfrac{3}{2}\frac{M_0}{L}\right)\left(\tfrac{L}{2}\right)^2 + 0 + 0 = \right\} - \frac{1}{16}\frac{M_0 L}{EI}$$

$$\theta_C = \frac{1}{16}\frac{M_0 L}{EI}$$

PROBLEM 9.60

9.60 For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the deflection at *D*.



$[x=0, y=0]$
$[x=0, \frac{dy}{dx}=0]$

$[x=5a, y=0]$
$[x=5a, \frac{dy}{dx}=0]$

SOLUTION

$$w(x) = w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = R_A - w\langle x-a\rangle^1 + w\langle x-3a\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \tfrac{1}{2}w\langle x-a\rangle^2 + \tfrac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - \tfrac{1}{6}w\langle x-a\rangle^3 + \tfrac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{24}w\langle x-a\rangle^4 + \tfrac{1}{24}\langle x-3a\rangle^4 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0]$ $\quad 0+0+0+0+C_1=0 \qquad C_1=0$

$[x=0, y=0]$ $\quad 0+0+0+0+0+C_2=0 \qquad C_2=0$

$[x=5a, \frac{dy}{dx}=0]$ $\quad M_A(5a)+\tfrac{1}{2}R_A(5a)^2-\tfrac{1}{6}w(4a)^3+\tfrac{1}{6}w(2a)^3+0=0$

$$5M_A a + 12.5R_A a^2 = 9.3333\,wa^3 \qquad (1)$$

$[x=5a, y=0]$ $\quad \tfrac{1}{2}M_A(5a)^2+\tfrac{1}{6}R_A(5a)^3-\tfrac{1}{24}w(4a)^4+\tfrac{1}{24}(2a)^4+0+0=0$

$$12.5M_A a^2 + 20.8333R_A a^3 = 10\,wa^4 \qquad (2)$$

Solving (1) and (2) simultaneously

$$M_A = -1.3333\,wa^2$$

$$R_A = 1.280\,wa \uparrow$$

(b) Deflection at D $\quad$ (y at $x=3a$)

$$y_D = \frac{1}{EI}\left\{\tfrac{1}{2}M_A(3a)^2+\tfrac{1}{6}R_A(3a)^3-\tfrac{1}{24}w(2a)^4+0+0+0\right\}$$

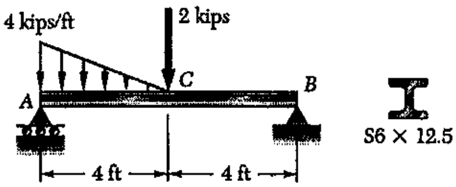
$$= \left[\tfrac{9}{2}(-1.3333)+\tfrac{1}{6}(1.28)(27)-\tfrac{1}{24}(16)\right]\frac{wa^4}{EI} = -0.907\frac{wa^4}{EI}$$

$$y_D = 0.907\frac{wa^4}{EI}\downarrow$$

PROBLEM 9.61

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.61 Beam and loading of Prob. 9.45.



SOLUTION

See solution to Prob. 9.45 for the derivation of the equations used in the following.

$EI = 4451 \text{ kip}\cdot\text{ft}^2$

$$EI\frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}\langle x-4\rangle^4 - \langle x-4\rangle^2 - 26.844 \qquad \text{kip}\cdot\text{ft}^2$$

$$EIy = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}\langle x-4\rangle^5 - \frac{1}{3}\langle x-4\rangle^3 - 26.844\,x \qquad \text{kip}\cdot\text{ft}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0 < x < 4$ ft.

$$EI\frac{dy}{dx} = \frac{23}{6}x_m^2 - \frac{2}{3}x_m^3 + \frac{1}{24}x_m^4 - 26.844 = 0 \qquad f,$$

Solve by iteration: $x_m =$ 4.0 3.73 3.735 $\qquad x_m = 3.735$ ft.

$df/dx =$ 9.33 9.42

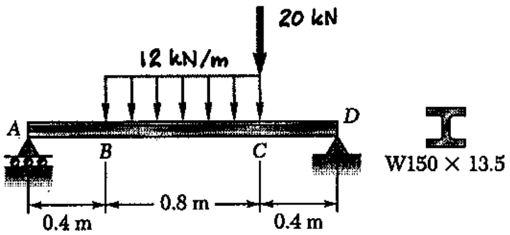
$$EIy_m = \frac{23}{18}(3.735)^3 - \frac{1}{6}(3.735)^4 + \frac{1}{120}(3.735)^5 - (26.844)(3.735) = -60.06 \text{ kip}\cdot\text{ft}^3$$

$$y_m = -\frac{60.06}{4451} = -13.49\times10^{-3} \text{ ft.} \qquad y_m = 0.1619 \text{ in.}$$

PROBLEM 9.62

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.62 Beam and loading of Prob. 9.46.



SOLUTION

See solution to Prob. 9.46 for the derivation of the equations used in the following.

$EI = EI = 1374 \text{ kN}\cdot\text{m}^2$

$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x-0.4\rangle^3 + 2\langle x-1.2\rangle^3 - 10\langle x-1.2\rangle^2 - 3.4080 \quad \text{kN}\cdot\text{m}^2$

$EIy = 1.63333x^3 - \frac{1}{2}\langle x-0.4\rangle^4 + \frac{1}{2}\langle x-1.2\rangle^4 - \frac{10}{8}\langle x-1.2\rangle^3 - 3.4080x \quad \text{kN}\cdot\text{m}^3$

To find the location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0.4 < x_m < 1.2$

$$4.9x_m^2 - 2(x_m - 0.4)^3 - 3.4080 = f(x_m) = 0$$

Solve by iteration

$x_m =$	0.8	0.858	0.857	0.8570
$df/dx =$	6.88	7.123	7.145	

$x_m = 0.8570 \text{ m}$ ◄

$EIy = (1.63333)(0.8570)^3 - \frac{1}{2}(0.8570 - 0.4)^4 - (3.4080)(0.8570)$

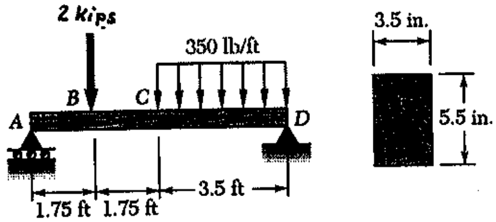
$= -1.9144 \quad \text{kN}\cdot\text{m}$

$y = -\frac{1.9144}{1374} = -1.393 \times 10^{-3} \text{ m} = 1.393 \text{ mm} \downarrow$ ◄

PROBLEM 9.63

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.63 Beam and loading of Prob. 9.47.



SOLUTION

See solution to Prob. 9.47 for the derivation of the equations used in the following.

$EI = 539.18 \text{ kip}\cdot\text{ft}^2$

$EI\dfrac{dy}{dx} = 0.903125x^2 - 1\langle x-1.75\rangle^2 - 0.05833\langle x-3.5\rangle^3 - 7.54779 \quad \text{kip}\cdot\text{ft}^2$

$EIy = 0.301042x^3 - \frac{1}{3}\langle x-1.75\rangle^3 - 0.014583\langle x-3.5\rangle^4 - 7.54779x \quad \text{kip}\cdot\text{ft}^3$

To find the location of maximum $|y|$, set $\dfrac{dy}{dx} = 0$. Assume $1.75 < x_m < 3.5$

$0.903125 x_m^2 - 1(x_m - 1.75)^2 - 7.547$

$0.096875 x_m^2 - 3.5 x_m + 10.61029 = 0$

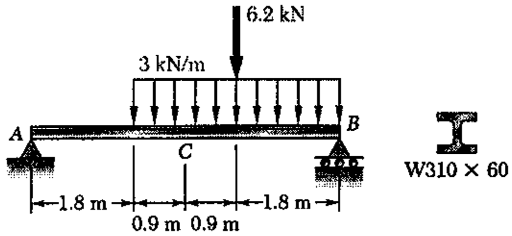
$x_m = \dfrac{3.5 - \sqrt{(3.5)^2 - (4)(0.096875)(10.61029)}}{(2)(0.096875)} = 3.340 \text{ ft}$ ◀

$EIy = (0.301042)(3.340)^3 - \frac{1}{3}(3.340 - 1.73)^3 - (7.54779)(3.340)$

$= -15.3328 \text{ kip ft}^3$

$y = -\dfrac{15.3328}{539.18} = -12.44 \times 10^{-3} \text{ ft} = 0.341 \text{ in.} \downarrow$ ◀

PROBLEM 9.64



9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.64 Beam and loading of Prob. 9.48.

SOLUTION

See solution to Prob. 9.48 for the derivation of the equations used in the following.

$EI = 25.8 \times 10^3$ kN·m

$EI \frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x-1.8\rangle^2 - 3.1\langle x-3.6\rangle^2 - 22.535$

$EI\,y = 0.9444x^3 - \frac{1}{8}\langle x-1.8\rangle^3 - 1.03333\langle x-3.6\rangle^3 - 22.535x$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $1.8 < x_m < 3.6$

$EI \frac{dy}{dx} = 2.8333\,x_m^2 - \frac{1}{2}(x_m - 1.8)^2 - 22.535 = 0$ $\quad F_1$

Solving by iteration: $x_m = 3, \quad 2.86, \quad 2.855$ $\qquad x_m = 2.855$ m

$df/dx = 15.8, \quad 15.15$

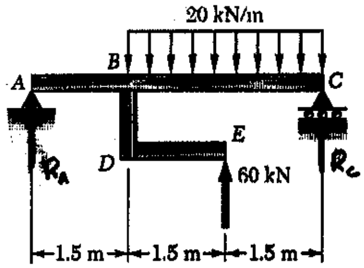
$EI\,y_m = 0.9444\,x_m^3 - \frac{1}{8}(x_m - 1.8)^3 - 22.535\,x_m$

$= (0.9444)(2.855)^3 - \frac{1}{8}(2.855-1.8)^3 - (22.535)(2.855) = -42.507$ kN·m³

$y_m = -\frac{42.507}{25.8\times10^3} = -1.648\times10^{-3}$ m $\qquad y_m = 1.648$ mm ↓

PROBLEM 9.65

9.65 The rigid bar *BDE* is welded at point *B* to the rolled steel beam *AC*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



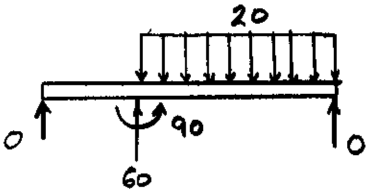
SOLUTION

$\circlearrowleft M_C = 0$

$-4.5 R_A + (20)(3)(1.5) - (60)(1.5) = 0$

$R_A = 0$

Units: Forces in kN, lengths in m



$$EI \frac{d^2y}{dx^2} = M = 60\langle x-1.5\rangle^1 - 90\langle x-1.5\rangle^0 - \tfrac{1}{2}(20)\langle x-1.5\rangle^2$$

$$EI \frac{dy}{dx} = 30\langle x-1.5\rangle^2 - 90\langle x-1.5\rangle^1 - (\tfrac{1}{6})(20)\langle x-1.5\rangle^3 + C_1$$

$$EI\,y = 10\langle x-1.5\rangle^3 - 45\langle x-1.5\rangle^2 - \tfrac{1}{24}(20)\langle x-1.5\rangle^4 + C_1 x + C_2$$

$[x=0, y=0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=4.5, y=0]$ $\quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5 C_1 + 0 = 0$

$C_1 = 45 \text{ kN}\cdot\text{m}^2$

Data: $E = 200\times10^9$ Pa, $I = 315\times10^6 \text{ mm}^4 = 315\times10^{-6} \text{ m}^4$

$EI = (200\times10^9)(315\times10^{-6}) = 63\times10^6 \text{ N}\cdot\text{m}^2 = 63000 \text{ kN}\cdot\text{m}^2$

(a) Slope at A $\quad$ ($\frac{dy}{dx}$ at $x = 0$)

$EI\,\theta_A = C_1 = 45 \text{ kN}\cdot\text{m}^2$

$\theta_A = \frac{45}{63000} = 0.714\times10^{-3}$ rad $\qquad \theta_A = 0.714\times10^{-3}$ rad $\measuredangle$ ◄

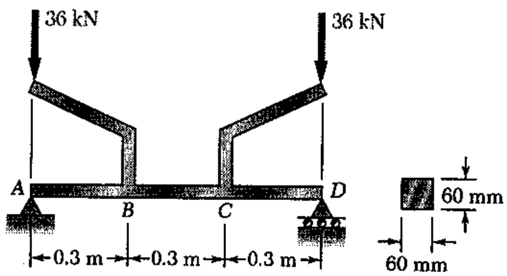
(b) Deflection at B $\quad$ (y at $x = 1.5$)

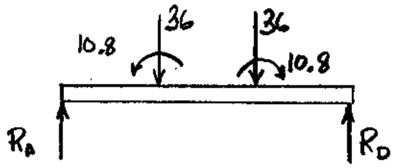
$EI\,y_B = (C_1)(1.5) = (45)(1.5) = 67.5 \text{ kN}\cdot\text{m}^3$

$y_B = \frac{67.5}{63000} = 1.071\times10^{-3}$ m $= 1.071$ mm $\uparrow$ ◄

PROBLEM 9.66

9.66 Rigid bars are welded to the steel rod *AD* as shown. For the loading shown, determine (*a*) the deflection at point *B*, (*b*) the slope at end *A*. Use $E = 200$ GPa.





SOLUTION

Units: Use kN for forces, m for lengths.

By symmetry $R_A = R_D$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_D - 36 - 36 = 0 \qquad R_A = 36 \text{ kN}$

$$EI\frac{d^2y}{dx^2} = M = 36x - 36\langle x-0.3\rangle^1 - 36\langle x-0.6\rangle^1 - 10.8\langle x-0.3\rangle^0 + 10.8\langle x-0.6\rangle^0$$

$$(\text{kN}\cdot\text{m}^2)\quad EI\frac{dy}{dx} = 18x^2 - 18\langle x-0.3\rangle^2 - 18\langle x-0.6\rangle^2 - 10.8\langle x-0.3\rangle^1 + 10.8\langle x-0.6\rangle^1 + C_1$$

$$(\text{kN}\cdot\text{m}^3)\quad EIy = 6x^3 - 6\langle x-0.3\rangle^3 - 6\langle x-0.6\rangle^3 - 5.4\langle x-0.3\rangle^2 + 5.4\langle x-0.6\rangle^2 + C_1x + C_2$$

$[x=0,\ y=0] \qquad 0+0+0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=0.9,\ y=0] \qquad (6)(0.9)^3 - (6)(0.6)^3 - (6)(0.3)^3 - (5.4)(0.6)^2 + (5.4)(0.3)^2 + 0.9C_1 + 0 = 0$

$C_1 = -1.62 \text{ kN}\cdot\text{m}^2$

Data: $E = 200\times10^9$ Pa, $I = bh^3 = \frac{1}{12}(60)(60)^3 = 1.08\times10^6 \text{ mm}^4 = 1.08\times10^{-6} \text{ m}^4$

$EI = (200\times10^9)(1.08\times10^{-6}) = 216\times10^3 \text{ N}\cdot\text{m}^2 = 216 \text{ kN}\cdot\text{m}^2$

(a) Deflection at B $\quad$ (y at $x = 0.3$)

$EIy_B = (6)(0.3)^3 - 0 - 0 - 0 + 0 - (1.62)(0.3) = -0.324 \text{ kN}\cdot\text{m}^3$

$y_B = -\frac{0.324}{216} = -1.500\times10^{-3} \text{ m} \qquad y_B = 1.500 \text{ mm}\downarrow$

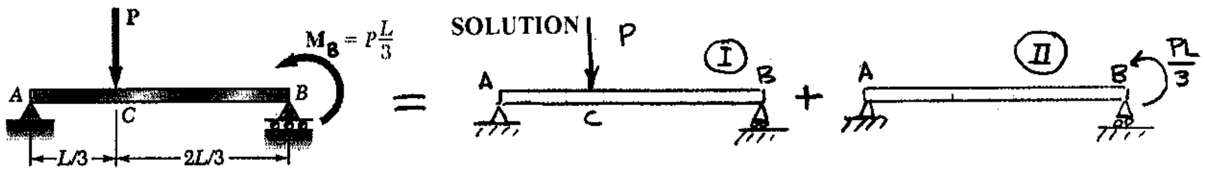
(b) Slope at A $\quad$ ($\frac{dy}{dx}$ at $x=0$)

$EI\theta_A = C_1 = -1.62 \text{ kN}\cdot\text{m}^2$

$\theta_A = -\frac{1.62}{216} = -7.50\times10^{-3} \text{ rad} \qquad \theta_A = 7.50\times10^{-3} \text{ rad}$

PROBLEM 9.67

9.67 and 9.68 For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.



Loading I: Case 5 $\quad a = \frac{L}{3}$, $b = \frac{2L}{3}$, $P = P$, $x = a$

$$y_C = -\frac{Pa^2b^2}{6EIL} = -\frac{P}{6EIL}\left(\frac{L}{3}\right)^2\left(\frac{2L}{3}\right)^2 = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2-b^2)}{6EIL} = -\frac{P}{6EIL}\left(\frac{2L}{3}\right)\left[L^2-\left(\frac{2L}{3}\right)^2\right] = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Case 7 $\quad M = -\frac{PL}{3} \quad x = \frac{L}{3}$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = +\frac{PL/3}{6EIL}\left\{\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right\} = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = +\frac{ML}{6EI} = \frac{-(PL/3)L}{6EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

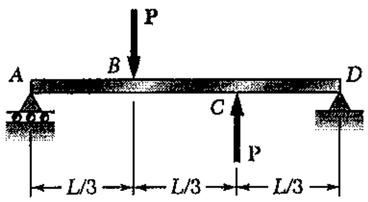
(a) Deflection at C: $\quad y_C = -\frac{4}{243}\frac{PL^3}{EI} - \frac{4}{243}\frac{PL^3}{EI} = -\frac{8}{243}\frac{PL^3}{EI}$

$$y_C = \frac{8}{243}\frac{PL^3}{EI} \downarrow$$

(b) Slope at A: $\quad \theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{1}{18}\frac{PL^2}{EI} = -\frac{19}{162}\frac{PL^2}{EI}$

$$\theta_A = \frac{19}{162}\frac{PL^2}{EI} \measuredangle$$

PROBLEM 9.68



9.67 and 9.68 For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.

SOLUTION

Loading I: Downward load P at B

Use Case 5 of Appendix D with

$P = P, \quad a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad L = L, \quad x = \frac{2L}{3}$

For $x < a$, given elastic curve is $y = \frac{Pb}{EIL}[x^3 - (L^2 - b^2)x]$

To obtain elastic curve for $x > a$ replace x by $L - x$ and interchange a and b to get

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)] \qquad \text{with } x = \frac{2L}{3} \text{ at point C}$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left(L^2 - \left(\frac{L}{3}\right)^2\right)\left(\frac{L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)[L^2 - (2L/3)^2]}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Upward load at C. Use Case 5 of Appendix D with

$P = -P, \quad a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad L = L, \quad x = a = \frac{2L}{3}$

$$y_C = -\frac{(-P)(2L/3)^2(L/3)^2}{3EIL} = \frac{4}{243}\frac{PL^3}{EI}$$

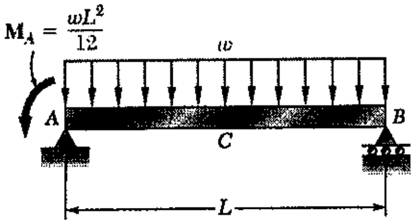
$$\theta_A = -\frac{(-P)(L/3)(L^2 - (L/3)^2)}{6EIL} = \frac{4}{81}\frac{PL^2}{EI}$$

(a) Deflection at C $\qquad y_C = -\frac{7}{486}\frac{PL^3}{EI} + \frac{4}{243}\frac{PL^3}{EI} = \frac{1}{486}\frac{PL^3}{EI} \uparrow$ ◄

(b) Slope at A $\qquad \theta_A = -\frac{5}{81}\frac{PL^2}{EI} + \frac{4}{81}\frac{PL^2}{EI} = \frac{1}{81}\frac{PL^2}{EI}$ ◄

PROBLEM 9.69

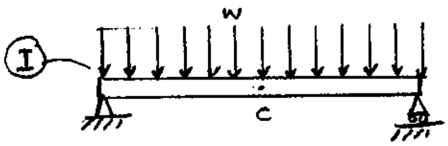
9.69 and 9.70 For the beam and loading shown, determine (*a*) the deflection at the midpoint *C*, (*b*) the slope at end *A*.



SOLUTION

Loading I: Case 6 in Appendix D

$$y_C = -\frac{5}{384}\frac{wL^4}{EI} \;; \qquad \theta_A = -\frac{1}{24}\frac{wL^3}{EI}$$

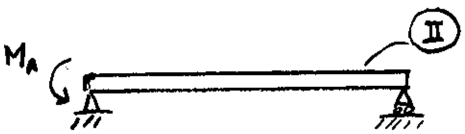


Loading II: Case 7 of Appendix D.

Note that center deflection is

$$y_C = -\frac{M_A}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right]$$

$$= \frac{1}{16}\frac{M_A L}{EI}$$



$$\theta_A = \frac{M_A L}{3EI}$$

with $M_A = \frac{wL^2}{12}$, $\qquad y_C = \frac{1}{192}\frac{wL^4}{EI}$, $\qquad \theta_A = \frac{1}{36}\frac{wL^3}{EI}$

(a) Deflection at C. $\quad y_C = -\frac{5}{384}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} = -\frac{1}{128}\frac{wL^4}{EI}$

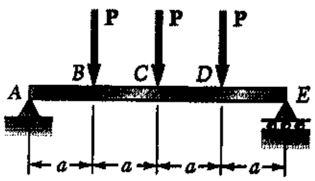
$$y_C = \frac{1}{128}\frac{wL^4}{EI} \downarrow$$

(b) Slope at A. $\quad \theta_A = -\frac{1}{24}\frac{wL^3}{EI} + \frac{1}{36}\frac{wL^3}{EI} = -\frac{1}{72}\frac{wL^3}{EI}$

$$\theta_A = \frac{1}{72}\frac{wL^3}{EI}$$

PROBLEM 9.70

9.69 and 9.70 For the beam and loading shown, determine (*a*) the deflection at the midpoint *C*, (*b*) the slope at end *A*.



SOLUTION

Loading I : Load at B

Case 5 in Appendix D.

$L = 4a, \quad a = a, \quad b = 3a, \quad x = 2a$

For $x > a$, replace x by $L - x$ and interchange a and b in expression for elastic curve given.

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_C = \frac{Pa}{6EI(4a)}\left[(2a)^3 - (16a^2 - a^2)(2a)\right] = -\frac{11}{12}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(3a)(16a^2 - 9a^2)}{6EI(4a)} = -\frac{7}{8}\frac{Pa^2}{EI}$$

Loading II Load at C Case 4 of Appendix D with $L = 4a$

$$y_C = -\frac{PL^3}{48EI} = -\frac{P(4a)^3}{48EI} = -\frac{4}{3}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{P(4a)^2}{16EI} = -\frac{Pa^2}{EI}$$

Loading III Load at D Case 5 of Appendix D

$L = 4a, \quad a = 3a, \quad b = a, \quad x = 2a$ at point C

$$y_C = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x] = \frac{Pa}{6EI(4a)}[(2a)^3 - (16a^2 - a^2)(2a)]$$

$$= -\frac{11}{12}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{Pa(16a^2 - a^2)}{6EI(4a)} = -\frac{5}{8}\frac{Pa^3}{EI}$$

(a) Deflection at C : $y_C = -\frac{11}{12}\frac{Pa^3}{EI} - \frac{4}{3}\frac{Pa^3}{EI} - \frac{11}{12}\frac{Pa^3}{EI} = -\frac{19}{6}\frac{Pa^3}{EI}$

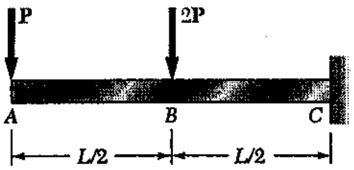
$$y_C = \frac{19}{6}\frac{Pa^3}{EI} \downarrow$$

(b) Slope at A : $\theta_A = -\frac{7}{8}\frac{Pa^2}{EI} - \frac{Pa^2}{EI} - \frac{5}{8}\frac{Pa^2}{EI} = -\frac{5}{2}\frac{Pa^2}{EI}$

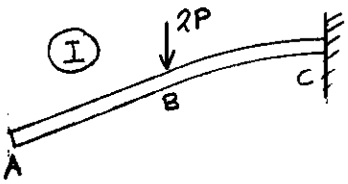
$$\theta_A = \frac{5}{2}\frac{Pa^2}{EI} \measuredangle$$

PROBLEM 9.71

9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



SOLUTION



Loading I: 2P downward at B.

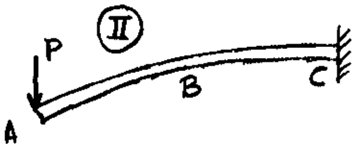
Case 1 of Appendix D applied to portion BC.

$$\theta_B' = \frac{(2P)(L/2)^2}{2EI} = \frac{1}{4}\frac{PL^2}{EI}$$

$$y_B' = \frac{(2P)(L/2)^3}{3EI} = \frac{1}{12}\frac{PL^3}{EI}$$

AB remains straight.

$$\theta_A' = \theta_B' = \frac{1}{4}\frac{PL^2}{EI}$$



$$y_A' = y_B' - \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{12}\frac{PL^3}{EI} - \frac{1}{8}\frac{PL^3}{EI} = -\frac{5}{24}\frac{PL^3}{EI}$$

Loading II P downward at A. Case 1 of Appendix D.

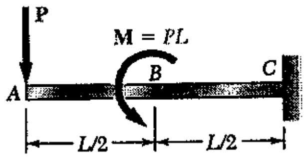
$$\theta_A'' = \frac{PL^2}{2EI} \quad , \quad y_A'' = -\frac{PL^3}{3EI}$$

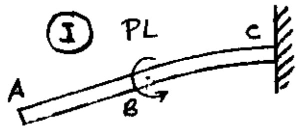
By superposition

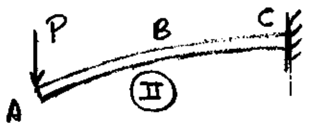
$$\theta_A = \theta_A' + \theta_A'' = \frac{1}{4}\frac{PL^2}{EI} + \frac{1}{2}\frac{PL^2}{EI} = \frac{3}{4}\frac{PL^2}{EI}$$

$$y_A = y_A' + y_A'' = -\frac{5}{24}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{13}{24}\frac{PL^3}{EI}$$

PROBLEM 9.72







9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION

Loading I: Counterclockwise couple PL at B.

Case 3 of Appendix D applied to portion BC.

$$\theta_B' = \frac{(PL)(L/2)}{EI} = \frac{1}{2}\frac{PL^2}{EI}$$

$$y_B' = \frac{(PL)(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^3}{EI}$$

AB remains straight.

$$\theta_A' = \theta_B' = \frac{1}{2}\frac{PL^2}{EI}$$

$$y_A' = y_B' - \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{8}\frac{PL^3}{EI} - \frac{1}{4}\frac{PL^3}{EI} = -\frac{3}{8}\frac{PL^3}{EI}$$

Loading II Case 1 of Appendix D.

$$\theta_A'' = \frac{PL^2}{2EI}, \qquad y_A'' = -\frac{PL^3}{3EI}$$

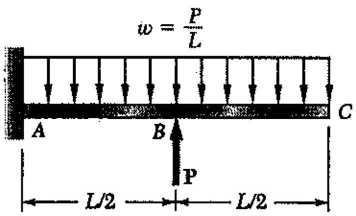
By superposition

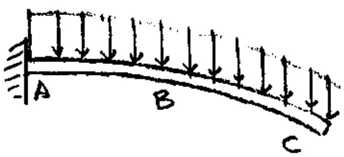
$$\theta_A = \theta_A' + \theta_A'' = \frac{1}{2}\frac{PL^2}{EI} + \frac{1}{2}\frac{PL^2}{EI} = \frac{PL^2}{EI} = \frac{PL^2}{EI}$$ ◄

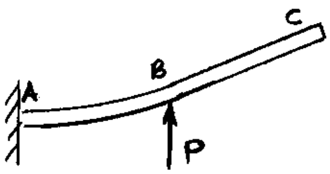
$$y_A = y_A' + y_A'' = -\frac{3}{8}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{17}{24}\frac{PL^3}{EI} = \frac{17}{24}\frac{PL^3}{3EI}\downarrow$$ ◄

PROBLEM 9.73

9.73 and 9.74 For the cantilever beam and loading shown, determine the slope and deflection at point C.







SOLUTION

Loading I Uniformly distributed downward loading with $w = P/L$.

Case 2 of Appendix D.

$$\theta_C' = -\frac{(P/L)L^3}{6EI} = -\frac{1}{6}\frac{PL^2}{EI}$$

$$y_C' = -\frac{(P/L)L^4}{8EI} = -\frac{1}{8}\frac{PL^3}{EI}$$

Loading II Upward concentrated load at P.

Case 1 of Appendix D applied to portion AB.

$$\theta_B'' = \frac{P(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^2}{EI}$$

$$y_B'' = \frac{P(L/2)^3}{3EI} = \frac{1}{24}\frac{PL^3}{EI}$$

Portion BC remains straight.

$$\theta_C'' = \theta_B'' = \frac{1}{8}\frac{PL^2}{EI}$$

$$y_C'' = y_B'' + \frac{L}{2}\theta_B'' = \frac{1}{24}\frac{PL^3}{EI} + \frac{1}{16}\frac{PL^3}{EI} = \frac{5}{48}\frac{PL^3}{EI}$$

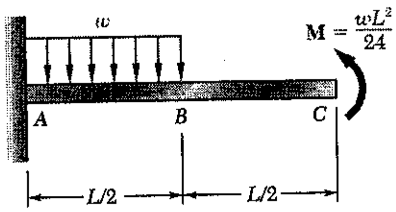
By superposition

$$\theta_C = \theta_C' + \theta_C'' = -\frac{1}{6}\frac{PL^2}{EI} + \frac{1}{8}\frac{PL^2}{EI} = -\frac{1}{24}\frac{PL^2}{EI} = \frac{PL^2}{24EI} \quad ◄$$

$$y_C = y_C' + y_C'' = -\frac{1}{8}\frac{PL^3}{EI} + \frac{5}{48}\frac{PL^3}{EI} = -\frac{1}{48}\frac{PL^3}{EI} = \frac{PL^3}{48EI} \downarrow \quad ◄$$

PROBLEM 9.74

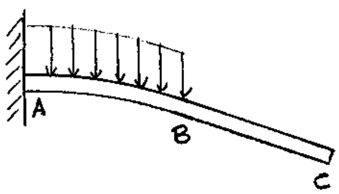
9.73 and 9.74 For the cantilever beam and loading shown, determine the slope and deflection at point C.



SOLUTION

Loading I: Downward distributed load w applied to portion AB.

Case 2 of Appendix D applied to portion AB.



$$\theta_B' = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$y_B' = -\frac{w(L/2)^4}{8EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

Portion BC remains straight.

$$\theta_C' = \theta_B' = -\frac{1}{48}\frac{wL^3}{EI}$$

$$y_C' = y_B' + \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{128}\frac{wL^4}{EI} - \frac{1}{96}\frac{wL^4}{EI} = -\frac{7}{384}\frac{wL^4}{EI}$$

Loading II: Counterclockwise couple $\frac{wL^2}{24}$ applied at C

Case 3 of Appendix D

$$\theta_C'' = \frac{(wL^2/24)L}{EI} = \frac{1}{24}\frac{wL^3}{EI}$$

$$y_C'' = \frac{(wL^2/24)L^2}{2EI} = \frac{1}{48}\frac{wL^4}{EI}$$

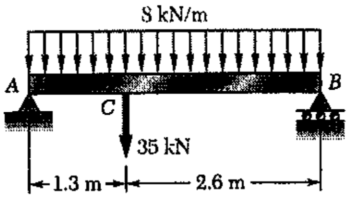
By superposition

$$\theta_C = \theta_C' + \theta_C'' = -\frac{1}{48}\frac{wL^3}{EI} + \frac{1}{24}\frac{wL^3}{EI} = \frac{1}{48}\frac{wL^3}{EI}$$

$$y_C = y_C' + y_C'' = -\frac{7}{384}\frac{wL^4}{EI} + \frac{1}{48}\frac{wL^4}{EI} = \frac{1}{384}\frac{wL^4}{EI} \uparrow$$

PROBLEM 9.75

9.75 For the W360 × 39 beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.



SOLUTION

Units: Forces in kN, lengths in m.

Loading I: 8 kN/m uniformly distributed.

Case 6: $w = 8$ kN/m, $L = 3.9$ m, $x = 1.3$ m

$$\theta_A = -\frac{wL^3}{24EI} = -\frac{(8)(3.9)^3}{24\,EI} = -\frac{19.773}{EI}$$

$$y_C = -\frac{w}{24EI}\left[x^4 - 2Lx^3 + L^3x\right] = -\frac{8}{24\,EI}\left[(1.3)^4 - (2)(3.9)(1.3)^3 + (3.9)^3(1.3)\right]$$

$$= -\frac{20.945}{EI}$$

Loading II 35 kN concentrated load at C. Case 5 of Appendix D

$P = 35$ kN, $L = 3.9$ m, $a = 1.3$ m, $b = 2.6$ m, $x = a = 1.3$ m

$$\theta_A = -\frac{Pb(L^2-b^2}{6EI\,L} = -\frac{(35)(2.6)(3.9^2-2.6)^2}{6\,EI\;(3.9)} = -\frac{32.861}{EI}$$

$$y_C = -\frac{Pa^2b^2}{3EIL} = -\frac{(35)(1.3)^2(2.6)^2}{3\,EI\;(3.9)} = -\frac{34.176}{EI}$$

Data: $E = 200 \times 10^9$, $I = 102.0 \times 10^6\ \text{mm}^4 = 102.0 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(102.0 \times 10^{-6}) = 20.4 \times 10^6\ \text{N}\cdot\text{m}^2 = 20400\ \text{kN}\cdot\text{m}^2$

(a) Slope at A $\quad \theta_A = -\dfrac{19.773 + 32.861}{20400} = -2.58 \times 10^{-3}$ rad

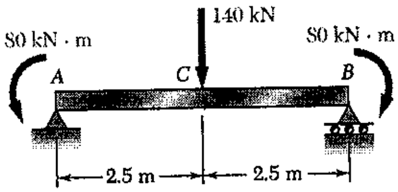
$$\theta_A = 2.58 \times 10^{-3}\ \text{rad}$$

(b) Deflection at C $\quad y_C = -\dfrac{20.934 + 34.176}{20400} = -2.70 \times 10^{-3}$ m

$$y_C = 2.70\ \text{mm} \downarrow$$

PROBLEM 9.76

9.76 For the W410 × 46.1 beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.



SOLUTION

Units: Forces in kN, lengths in m.

Loading I: Moment at B

Case 7 of Appendix D $\qquad M = 80$ kN·m, $L = 5.0$ m, $x = 2.5$ m

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = -\frac{80}{6EI(5.0)}\left[2.5^3 - (5.0)^2(2.5)\right] = \frac{125}{EI}$$

Loading II $\qquad$ Moment at A $\qquad$ Case 7 of Appendix D

$M = 80$ kN·m, $L = 5.0$ m, $x = 2.5$ m

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \qquad \text{(Same as loading I)}$$

Loading III $\qquad$ 140 kN concentrated load at C $\qquad P = 140$ kN

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N·m}^2 = 31200 \text{ kN·m}^2$$

(a) Slope at A $\qquad \theta_A = \dfrac{67.667 + 133.333 - 218.75}{31200} = -0.601 \times 10^{-3}$ rad

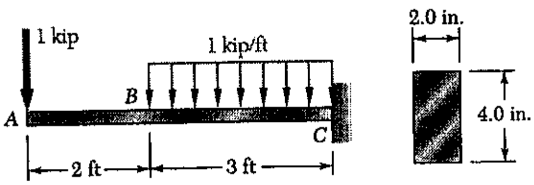
$$\theta_A = 0.601 \times 10^{-3} \quad ◄$$

(b) Deflection at C $\qquad y_C = \dfrac{125 + 125 - 364.583}{31200} = -3.67 \times 10^{-3}$ m

$$y_C = 3.67 \text{ mm} \downarrow \quad ◄$$

PROBLEM 9.77

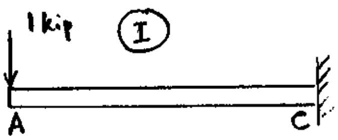
9.77 For the cantilever beam shown, determine the slope and deflection at end A. Use $E = 29 \times 10^6$ psi.



SOLUTION

Units: Forces in kips, lengths in ft.

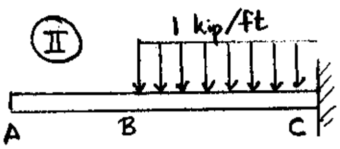
Loading I: Concentrated load at A



Case 1 of Appendix D.

$$\theta_A' = \frac{PL^2}{2EI} = \frac{(1)(5)^2}{2EI} = \frac{12.5}{EI}$$

$$y_A' = -\frac{PL^3}{3EI} = -\frac{(1)(5)^3}{3EI} = -\frac{41.667}{EI}$$



Loading II: Uniformly distributed load over portion BC.

Case 2 of Appendix D applied to portion BC

$$\theta_B'' = \frac{wL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

$$y_B'' = -\frac{wL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI}$$

Portion AB remains straight. $\theta_A'' = \theta_B'' = \frac{4.5}{EI}$

$$y_A'' = y_B'' - a\theta_B'' = -\frac{10.125}{EI} - (2)\left(\frac{4.5}{EI}\right) = -\frac{19.125}{EI}$$

By superposition

$$\theta_A = \theta_A' + \theta_A'' = \frac{12.5}{EI} + \frac{4.5}{EI} = \frac{17}{EI}$$

$$y_A = y_A' + y_A'' = -\frac{41.667}{EI} - \frac{19.125}{EI} = -\frac{60.792}{EI}$$

Data: $E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = \frac{1}{12}(2.0)(4.0)^3 = 10.667$ in^4

$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3$ kip-in^2 $= 2148$ kip$\cdot$ft^2

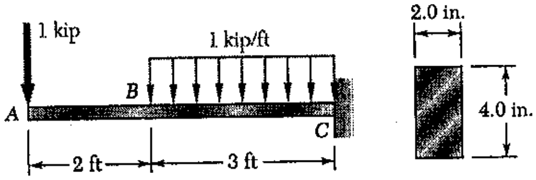
Slope at A $\quad \theta_A = \frac{17}{2148} = 7.91 \times 10^{-3}$ rad $\measuredangle$ ◄

Deflection at A $\quad y_A = -\frac{60.792}{2148} = -28.30 \times 10^{-3}$ ft

$= 0.340$ in. $\downarrow$ ◄

PROBLEM 9.78

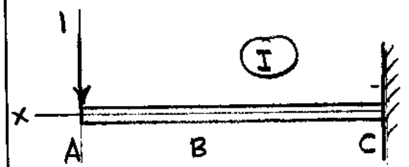
9.78 For the cantilever beam shown, determine the slope and deflection at point B. Use $E = 29 \times 10^6$ psi.



SOLUTION

Units: Forces in kips, lengths in ft.

Loading I: Concentrated load at A.



Case 1 of Appendix D.

$$y = \frac{P}{6EI}[x^3 - 3Lx^2]$$

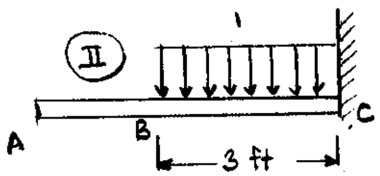
$$\frac{dy}{dx} = \frac{P}{6EI}[3x^2 - 6Lx]$$

with $P = 1$ kip, $L = 5$ ft, $x = 3$ ft.

$$y_B' = \frac{1}{6EI}[(3)^3 - (3)(5)(3)^2] = -\frac{18}{EI}$$

$$\left.\frac{dy}{dx}\right|_B' = \frac{1}{6EI}[(3)(3)^2 - (6)(5)(3)] = -\frac{10.5}{EI}$$

Adjusting the sign $\theta_B' = \dfrac{10.5}{EI}$



Loading II Uniformly distributed load over portion BC.

Case 2 of Appendix D applied to portion BC

$$y_B'' = -\frac{wL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI}$$

$$\theta_B'' = \frac{wL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

By superposition

$$\theta_B = \theta_B' + \theta_B'' = \frac{10.5}{EI} + \frac{4.5}{EI} = \frac{15}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{18}{EI} - \frac{10.125}{EI} = -\frac{28.125}{EI}$$

Data: $E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = \frac{1}{12}(2.0)(4.0)^3 = 10.667$ in^4

$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3$ kip·in^2 $= 2148$ kip·ft^2

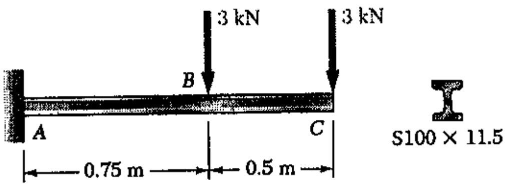
Slope at B $\quad \theta_B = \dfrac{15}{2148} = 6.98 \times 10^{-3}$ rad. ◄

Deflection at B $\quad y_B = -\dfrac{28.125}{2148} = -13.09 \times 10^{-3}$ ft

$= 0.1571$ in. ↓ ◄

PROBLEM 9.79

9.79 For the cantilever beam shown, determine the slope and deflection at end *C*. Use $E = 200$ GPa.

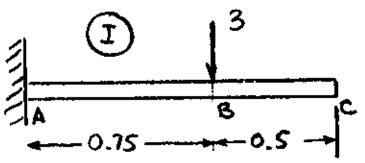


SOLUTION

Units: Forces in kN, lengths in m.

Loading I: Concentrated load at B

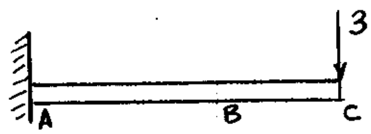
Case 1 of Appendix D applied to portion AB.



$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight



$$\theta_C' = \theta_B' = -\frac{0.84375}{EI}$$

$$y_C' = y_B' - (0.5)\theta_B' = -\frac{0.84375}{EI}$$

Loading II: Concentrated load at C. Case 1 of Appendix D.

$$\theta_A'' = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y_A'' = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition: $\theta_A = \theta_A' + \theta_A'' = -\frac{3.1875}{EI}$

$$y_A = y_A' + y_A'' = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $I = 2.53 \times 10^6\ \text{mm}^4 = 2.53 \times 10^{-6}\ \text{m}^4$

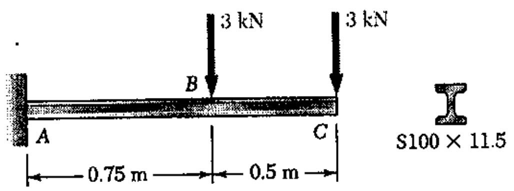
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3\ \text{N}\cdot\text{m}^2 = 506\ \text{kN}\cdot\text{m}^2$$

Slope at C $\quad \theta_C = -\frac{3.1875}{506} = -6.30 \times 10^{-3}$ rad $= 6.30 \times 10^{-3}$ rad

Deflection at C $\quad y_C = -\frac{2.796875}{506} = -5.53 \times 10^{-3}$ m $= 5.53$ mm ↓

PROBLEM 9.80

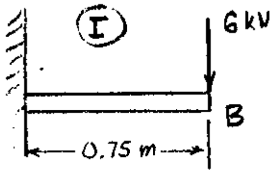
9.80 For the cantilever beam shown, determine the slope and deflection at point B. Use $E = 200$ GPa.



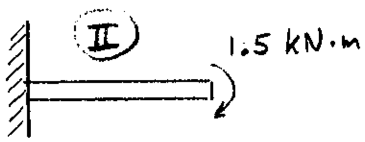
SOLUTION

Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.



Reducing the force at C to an equivalent force-couple system at B and adding the force already at B gives the loadings I and II shown.



Loading I: Case 1 of Appendix D

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition

$$\theta_B = \theta_B' + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $\quad I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

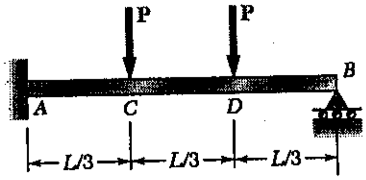
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at B $\qquad \theta_B = -\frac{2.8125}{506} = -5.56 \times 10^{-3}$ rad $= 5.56$ rad

Deflection at B $\qquad y_B = -\frac{1.265625}{506} = -2.50 \times 10^{-3}$ m $= 2.50$ mm ↓

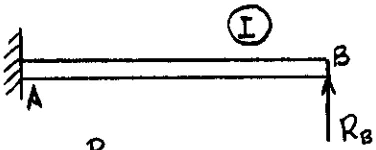
PROBLEM 9.81

9.81 and 9.82 For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.



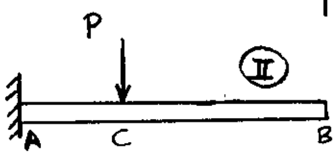
SOLUTION

Consider R_B as redundant and replace loading system by I, II, and III



Loading I   Case 1 of Appendix D applied to AB.

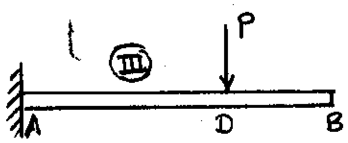
$$(y_B)_I = \frac{R_B L^3}{3EI}$$



Loading II   Case 1 applied to portion AC.

$$(\theta_C)_{II} = -\frac{P(L/3)^2}{2EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

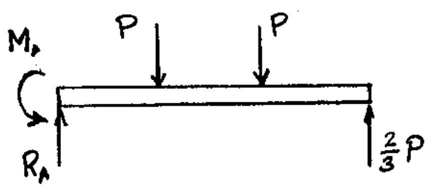
$$(y_C)_{II} = -\frac{P(L/3)^3}{3EI} = -\frac{1}{81}\frac{PL^3}{EI}$$



Portion CB remains straight

$$(y_B)_{II} = (y_C)_{II} + \frac{2L}{3}(\theta_C)_{II} = -\frac{4}{81}\frac{PL^3}{EI}$$

Loading III   Case 1 applied to portion AD



$$(\theta_D)_{III} = \frac{P(2L/3)^2}{2EI} = -\frac{2}{9}\frac{PL^2}{EI}$$

$$(y_D)_{III} = \frac{P(2L/3)^3}{3EI} = -\frac{8}{81}\frac{PL^3}{EI}$$

Portion DB remains straight

$$(y_C)_{III} = (y_D)_{III} + \frac{L}{3}(\theta_D)_{III} = -\frac{14}{81}\frac{PL^3}{EI}$$

Superposition and constraint. $y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$

$$\frac{1}{3}R_B L^3 - \frac{4}{81}\frac{PL^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = \frac{1}{3}\frac{R_B L^3}{EI} - \frac{2}{9}\frac{PL^3}{EI} = 0 \qquad R_B = \frac{2}{3}P\uparrow$$

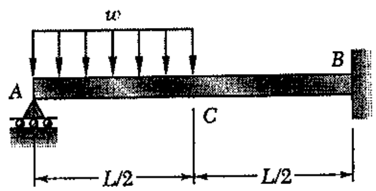
Statics

$+\uparrow \Sigma F_y = 0 \qquad R_A - P - P + \frac{2}{3}P = 0 \qquad R_A = \frac{4}{3}P\uparrow$

$\circlearrowleft \Sigma M_A = 0 \qquad M_A - P\left(\frac{L}{3}\right) - P\left(\frac{2L}{3}\right) + \left(\frac{2}{3}P\right)(L) = 0 \qquad M_A = \frac{1}{3}PL \circlearrowleft$

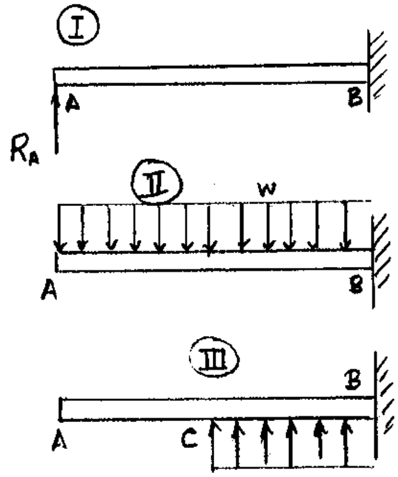
PROBLEM 9.82

9.81 and 9.82 For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.



SOLUTION

Beam is indeterminate to first degree. Consider R_A as redundant and replace the given loading by loadings I, II, and III.



Loading I: Case 1 of Appendix D

$$(y_A)_I = \frac{R_A L^3}{3EI}$$

Loading II: Case 2 of Appendix D

$$(y_A)_{II} = -\frac{wL^4}{8EI}$$

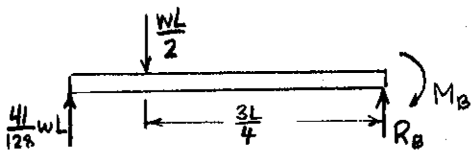
Loading III Case 2 of Appendix D (Portion CB)

$$(\theta_C)_{III} = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$(y_C)_{III} = \frac{w(L/2)^4}{8EI} = \frac{1}{128}\frac{wL^4}{EI}$$

Portion AC remains straight

$$(y_A)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{7}{384}\frac{wL^4}{EI}$$



Superposition and constraint $\quad y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III} = 0$

$$\frac{1}{3}\frac{R_A L^3}{EI} - \frac{1}{8}\frac{wL^4}{EI} + \frac{7}{384}\frac{wL^4}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{41}{384}\frac{wL^4}{EI} = 0 \qquad R_A = \frac{41}{128}wL \uparrow$$

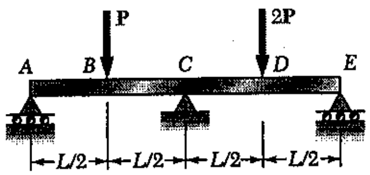
Statics

$$+\uparrow \sum F_y = 0 \quad \frac{41}{128}wL - \frac{1}{2}wL + R_B = 0 \qquad R_B = \frac{23}{128}wL \uparrow$$

$$+\circlearrowleft \sum M_B = 0 \quad -\left(\frac{41}{128}wL\right)L + \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_B = 0 \qquad M_B = \frac{7}{128}wL^2 \circlearrowright$$

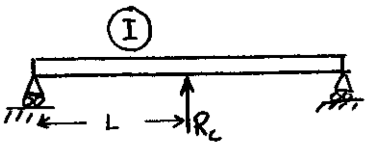
PROBLEM 9.83

9.83 and 9.84 For the uniform beam shown, determine the reaction at each of the three supports.



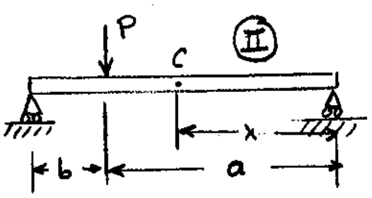
SOLUTION

Beam is indeterminate to first degree. Consider R_C to be the redundant reaction, and replace the loading by loadings I, II, and III.



Loading I Case 4 of Appendix D.

$$(y_C)_I = \frac{R_C(2L)^3}{48EI} = \frac{1}{6}\frac{R_C L^3}{EI}$$

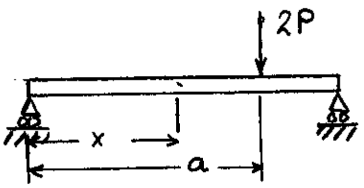


Loading II Case 5 of Appendix D.

$$(y_C)_{II} = \frac{Pb}{6EI(2L)}\left[x^3 - \{(2L)^2 - b^2\}x\right]$$

$$= \frac{P(L/2)}{12EIL}\left[L^3 - \{4L^2 - (\tfrac{L}{2})^2\}L\right]$$

$$= -\frac{11}{48}\frac{PL^3}{EI}$$



Loading III Case 5 of Appendix D.

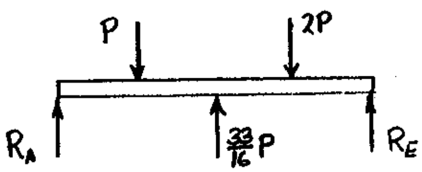
$(y_C)_{III}$ = twice that of loading II

$$(y_C)_{III} = -\frac{11}{24}\frac{PL^3}{EI}$$

Superposition and constraint $y_C = (y_C)_I + (y_C)_{II} + (y_C)_{III} = 0$

$$\frac{1}{6}\frac{R_C L^3}{EI} - \frac{11}{48}\frac{PL^3}{EI} - \frac{11}{24}\frac{PL^3}{EI} = \frac{1}{6}\frac{R_C L^3}{EI} - \frac{11}{16}\frac{PL^3}{EI} = 0 \qquad R_C = \frac{33}{16}P \uparrow$$

Statics



$+\circlearrowleft \Sigma M_E = 0$

$$-R_A(2L) + P(\tfrac{3L}{2}) - (\tfrac{33}{16}P)L + (2P)(\tfrac{L}{2}) = 0$$

$$R_A = \frac{7}{32}P \uparrow$$

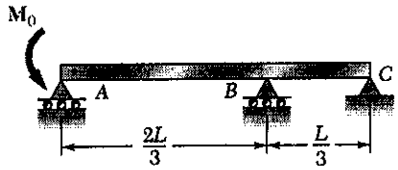
$+\uparrow \Sigma F_y = 0$

$$\frac{7}{32}P - P + \frac{33}{16}P - 2P + R_E = 0$$

$$R_E = \frac{23}{32}P \uparrow$$

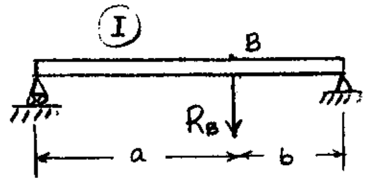
PROBLEM 9.84

9.83 and 9.84 For the uniform beam shown, determine the reaction at each of the three supports.



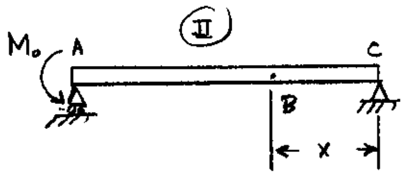
SOLUTION

Beam is statically indeterminate to first degree. Consider R_B to be the redundant reaction, and replace the loading by loadings I and II.



Loading I: Case 5 of Appendix D.

$$(y_B)_I = -\frac{R_B a^2 b^2}{3EIL} = -\frac{R_B (2L/3)^2 (L/3)^2}{3EIL} = -\frac{4}{243}\frac{R_B L^3}{EI}$$



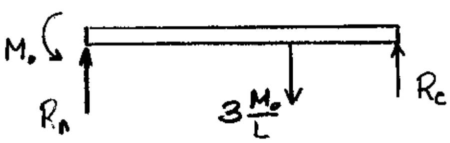
Loading II: Case 7 of Appendix D.

$$(y_B)_{II} = -\frac{M_0}{6EIL}(x^3 - L^2 x) = -\frac{M_0}{6EIL}\left[\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right]$$
$$= \frac{4}{81}\frac{M_0 L^2}{EI}$$

Superposition and constraint.

$$y_B = (y_B)_I + (y_B)_{II} = 0$$

$$-\frac{4}{243}\frac{R_B L^3}{EI} + \frac{4}{81}\frac{M_0 L^2}{EI} = 0 \qquad R_B = 3\frac{M_0}{L}\downarrow$$

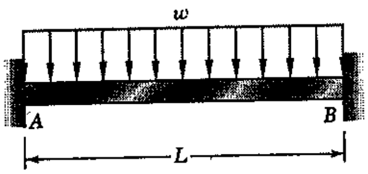


Statics

$$\circlearrowleft \Sigma M_C = 0$$
$$-R_A L + M_0 + 3\frac{M_0}{L}\cdot\frac{L}{3} = 0 \qquad R_A = 2\frac{M_0}{L}\uparrow$$
$$+\uparrow \Sigma F_y = 0$$
$$2\frac{M_0}{L} - 3\frac{M_0}{L} + R_C = 0 \qquad R_C = \frac{M_0}{L}\uparrow$$

PROBLEM 9.85

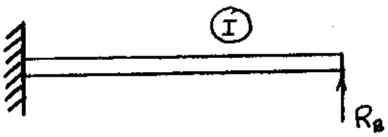
9.85* and *9.86 For the beam shown, determine the reaction at B.



SOLUTION

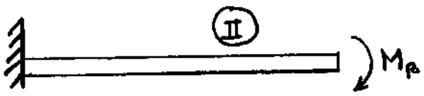
Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

Loading I: Case 1 of Appendix D.



$$(y_B)_I = \frac{R_B L^3}{3EI} \qquad (\theta_B)_I = \frac{R_B L^2}{2EI}$$

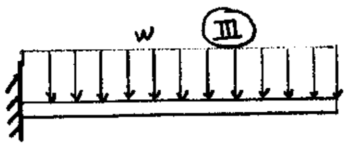
Loading II: Case 3 of Appendix D



$$(y_B)_{II} = -\frac{M_B L^2}{2EI} \qquad (\theta_B)_{II} = -\frac{M_B L}{EI}$$

Loading III: Case 2 of Appendix D

$$(y_B)_{III} = -\frac{wL^4}{8EI} \qquad (\theta_B)_{III} = -\frac{wL^3}{6EI}$$



Superposition and constraint

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$\frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B - \frac{wL^4}{8EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$\frac{L^2}{2EI} R_B - \frac{L}{EI} M_B - \frac{wL^3}{6EI} = 0 \qquad (2)$$

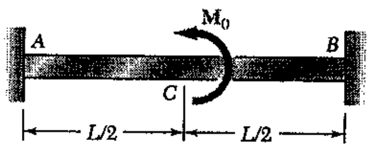
Solving (1) and (2) simultaneously

$$R_B = \tfrac{1}{2} wL \uparrow$$

$$M_B = \tfrac{1}{12} wL^2 \circlearrowright$$

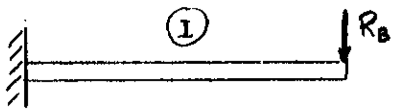
PROBLEM 9.86

9.85* and *9.86 For the beam shown, determine the reaction at *B*.



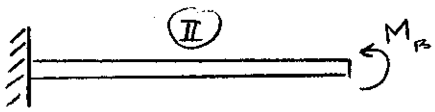
SOLUTION

Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



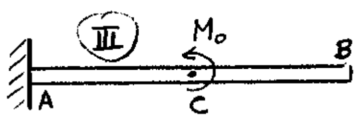
Loading I: Case 1 of Appendix D.

$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$



Loading II: Case 3 of Appendix D.

$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$



Loading III: Case 3 applied to portion AC.

$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8}\frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2}\frac{M_0 L^2}{EI}$$

Superposition and constraint

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI} R_B + \frac{L^2}{2EI} M_B + \frac{3}{8}\frac{M_0 L^2}{EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI} R_B + \frac{L}{EI} M_B + \frac{1}{2}\frac{M_0 L}{EI} = 0 \qquad (2)$$

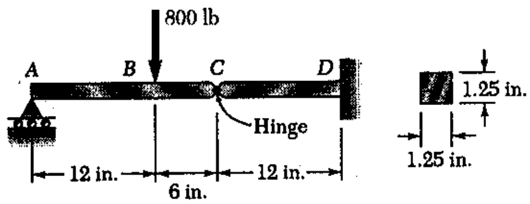
Solving (1) and (2) simultaneously

$$R_B = \frac{3}{2}\frac{M_0}{L} \downarrow \blacktriangleleft$$

$$M_B = \frac{1}{4} M_0 \circlearrowright \blacktriangleleft$$

PROBLEM 9.87

9.87 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point B. Use $E = 29 \times 10^6$ psi.

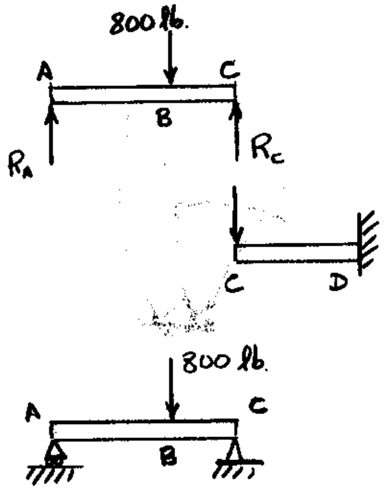


SOLUTION

Using free body ABC

$\circlearrowright \Sigma M_A = 0 \qquad 18 R_C - (12)(800) = 0$

$R_C = 533.33$ lb.



$E = 29 \times 10^6$ psi

$I = \frac{1}{12} bh^3 = \frac{1}{12}(1.25)(1.25)^3 = 0.20345 \text{ in}^4$

$EI = (29 \times 10^6)(0.20345) = 5.900 \times 10^6 \text{ lb} \cdot \text{in}^2$

Using cantilever beam CD with load R_C

Case 1 of Appendix D

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(533.33)(12)^3}{(3)(5.900 \times 10^6)} = -52.067 \times 10^{-3} \text{ in.}$$

Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D $\qquad P = 800$ lb. $L = 18$ in., $a = 12$ in., $b = 6$ in.

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(800)(6)(18^2 - 6^2)}{(6)(5.900 \times 10^6)(18)} = -2.1695 \times 10^{-3} \text{ rad.}$$

$$y_B' = -\frac{Pb^2a^2}{3EIL} = -\frac{(800)(6)^2(12)^2}{(3)(5.900 \times 10^6)(18)} = -13.017 \times 10^{-3} \text{ in}$$

Addition slope and deflection due to movement of point C

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{52.067 \times 10^{-3}}{18} = -2.8926 \times 10^{-3} \text{ rad.}$$

$$y_B'' = \frac{a}{L} y_C = -\frac{(12)(52.067 \times 10^{-3})}{18} = -34.711 \times 10^{-3} \text{ in.}$$

(a) Slope at A $\qquad \theta_A = \theta_A' + \theta_A'' = -2.1695 \times 10^{-3} - 2.8926 \times 10^{-3}$

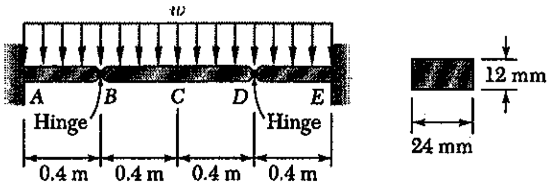
$= -5.06 \times 10^{-3}$ rad $= 5.06 \times 10^{-3}$ rad ◄

(b) Deflection at B $\qquad y_B = y_B' + y_B'' = -13.017 \times 10^{-3} - 34.711 \times 10^{-3}$

$= -47.7 \times 10^{-3}$ in. $= 47.7 \times 10^{-3}$ in. ↓ ◄

PROBLEM 9.88

9.88 A central beam *BD* is joined at hinges to two cantilever beams *AB* and *DE*. All beams have the cross section shown. For the loading shown, determine the largest allowable value of *w* if the deflection at *C* is not to exceed 3 mm. Use *E* = 200 GPa.



SOLUTION

Let $a = 0.4$ m

Cantilever beams AB and CD.

Cases 1 and 2 of Appendix D $\quad y_c = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = \frac{11}{24}\frac{wa^4}{EI}$

Beam BCD, with $L = 0.8$ m, assuming that points B and D do not move.

Case 6 of Appendi

$$y_c' = -\frac{5wL^4}{384\,EI}$$

Additional deflection due to movement of points B and D.

$$y_c'' = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

Total deflection at C $\quad y_c = y_c' + y_c''$

$$y_c = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

Data: $E = 200\times10^9$ Pa, $\quad I = \frac{1}{12}(24)(12)^3 = 3.456\times10^{-3}\text{ mm}^4 = 3.456\times10^{-9}\text{ m}^4$

$EI = (200\times10^9)(3.456\times10^{-9}) = 691.2\ \text{N}\cdot\text{m}^2$

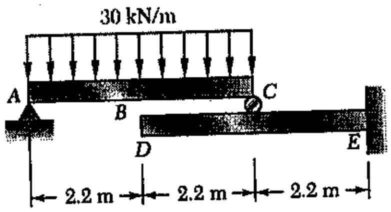
$y_c = -3\times10^{-3}$ m

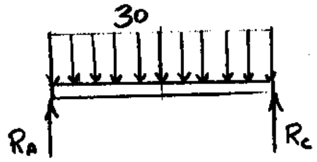
$$-3\times10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69\times10^{-6}\,w$$

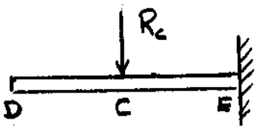
$w = 121.5$ N/m

PROBLEM 9.89

9.89 Beam *AC* rests on the cantilever beam *DE*, as shown. Knowing that a W410 × 38.8 rolled-steel shape is used for each beam, determine for the loading shown (*a*) the deflection at point *B*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.







SOLUTION

Units: Forces in kN, lengths in m.

Using free body ABC, $+\circlearrowleft \Sigma M_A = 0$

$$4.4 R_C - (4.4)(30)(2.2) = 0 \qquad R_C = 66.0 \text{ kN}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 127 \times 10^6 \text{ mm}^4 = 127 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(127 \times 10^{-6}) = 25.4 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$= 25400 \text{ kN}\cdot\text{m}^2$$

For slope and deflection at C, use Case 1, Appendix D applied to portion CE of beam DCE.

$$\theta_C = \frac{R_C L^2}{2EI} = \frac{(66.0)(2.2)^2}{(2)(25400)} = 6.2882 \times 10^{-3} \text{ rad}$$

$$y_C = -\frac{R_C L^3}{3EI} = \frac{(66.0)(2.2)^3}{(3)(25400)} = -9.2227 \times 10^{-3} \text{ m}$$

Deflection at B assuming that point C does not move.

Use Case 6 of Appendix D. $\quad (y_B)_1 = -\frac{5wL^4}{384EI} = -\frac{(5)(30)(4.4)^4}{(384)(25400)}$

$$= -5.7642 \times 10^{-3}$$

Additional deflection at B due to movement of point C

$$(y_B)_2 = \tfrac{1}{2} y_C = -4.6113 \times 10^{-3} \text{ m}$$

Total deflection at B $\quad y_B = (y_B)_1 + (y_B)_2 = -10.38 \times 10^{-3} \text{ m} = 10.38 \text{ mm} \downarrow$

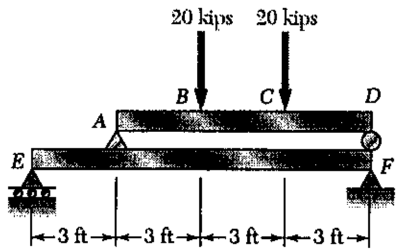
Portion DC of beam DCB remains straight.

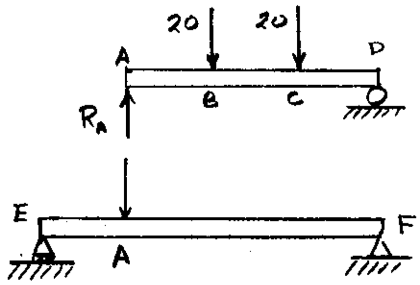
$$y_D = y_C - a\theta_C = -9.2227 \times 10^{-3} - (2.2)(6.2882 \times 10^{-3})$$

$$= -23.1 \times 10^{-3} \text{ m} = 23.1 \text{ mm} \downarrow$$

PROBLEM 9.90

9.90 Beam *AD* rests on beam *EF* as shown. Knowing that a W12 × 26 rolled-steel shape is used for each beam, determine for the loading shown the deflection at points *B* and *C*. Use $E = 29 \times 10^6$ psi.





SOLUTION

$E = 29 \times 10^3$ ksi $\qquad I = 204$ in^4

$EI = (29 \times 10^3)(204) = 5.916 \times 10^6$ kip·in^2 = 41083 kip·ft^2

For equilibrium of beam ABCD $\quad R_A = 20$ kips.

Deflection at point A is due to bending of beam EAF. Using Case 5 of

$$y_A = -\frac{Pa^2b^2}{3EIL} = -\frac{(20)(3)^2(9)^2}{(3)(EI)(12)} = -\frac{405}{EI} \text{ ft}$$

Assuming that beam ABCD is rigid

$$y_B' = \frac{6}{9}y_A = -\frac{270}{EI}\text{ft}, \quad y_C' = \frac{3}{9}y_A = -\frac{135}{EI}\text{ft}$$

Additional deflection at B due to bending of beam ABCD. Using Case 5

$$y_B'' = -\frac{P_B a^2 b^2}{3EIL} + \frac{P_C b}{6EIL}[x^3 - (L^2 - b^2)x]$$

$$= -\frac{(20)(3)^2(6)^2}{(3)(EI)(9)} + \frac{(20)(3)[(3)^3 - (9^2 - 3^2)(3)]}{(6)(EI)(9)} = -\frac{240}{EI} - \frac{210}{EI} = -\frac{450}{EI}\text{ft}$$

Additional deflection at C due to bending of beam ABCD.

By symmetry $\quad y_C'' = y_B'' = -\frac{450}{EI}$ ft

Total deflection at B

$$y_B = y_B' + y_B'' = -\frac{270}{EI} - \frac{450}{EI} = -\frac{720}{EI} = -\frac{720}{41083} = -17.525 \times 10^{-3}\text{ ft}$$

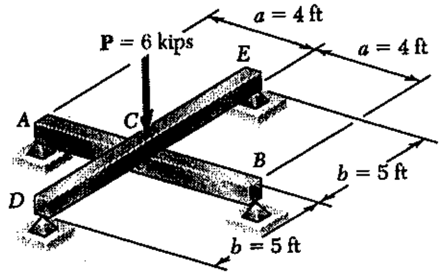
$$= 0.210 \text{ in.}\downarrow$$

$$y_C = y_C' + y_C'' = -\frac{135}{EI} - \frac{450}{EI} = -\frac{585}{EI} = -\frac{585}{41083} = -14.239 \times 10^{-3}\text{ ft}$$

$$= 0.171 \text{ in.}\downarrow$$

PROBLEM 9.91

9.91 For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *B*, (*b*) at *E*.



SOLUTION

Units: Forces in kips, lengths in ft.

For beam ACB, using Case 4 of Appendix D.

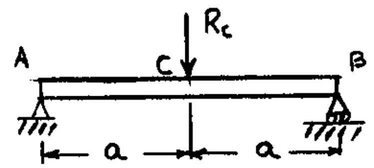
$$(y_C)_1 = -\frac{R_C(2a)^3}{48EI}$$

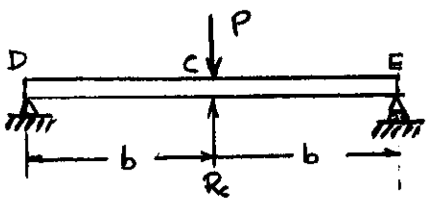
For beam DCE, using Case 4 of Appendix D.

$$(y_C)_2 = \frac{(R_C - P)(2b)^3}{48EI}$$

Matching deflections at C

$$-\frac{R_C(2a)^3}{48EI} = \frac{(R_C - P)(2b)^3}{48EI}$$





$$R_C = \frac{Pb^3}{a^3 + b^3} = \frac{(6)(5)^3}{4^3 + 5^3} = 3.968 \text{ kips}$$

$$P - R_C = 6 - 3.968 = 2.032 \text{ kips}$$

Using free body ACB $\circlearrowleft M_A = 0$ $\quad 2aR_B - aR_C = 0$

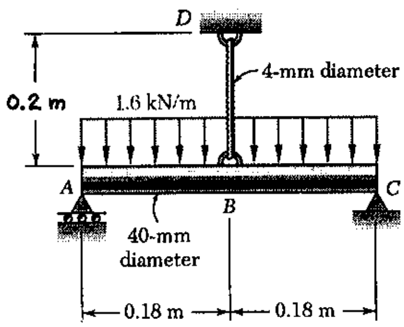
$$R_B = \tfrac{1}{2}R_C = 1.984 \text{ kips}$$

Using free body DCE $\circlearrowleft M_D = 0$ $\quad 2bR_E - b(P - R_C) = 0$

$$R_E = \tfrac{1}{2}(P - R_C) = 1.016 \text{ kips}$$

PROBLEM 9.92

9.92 Knowing that the rod *ABC* and the wire *BD* are both made of steel, determine (*a*) the deflection at *B*, (*b*) the reaction at *A*. Use $E = 200$ GPa.

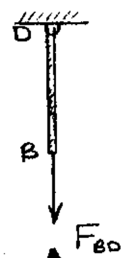


SOLUTION

Let F_{BD} be the tension in wire BD. The elongation of the wire is

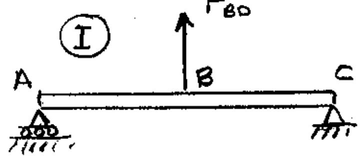
$$\delta_{BD} = \frac{F_{BD}\,l}{EA}$$

Beam ABC is subjected to loads F_{BD} (I) and w (II.)



Loading I: Case 4 of Appendix D.

$$(y_B)_I = \frac{F_{BD}L^3}{48\,EI}$$



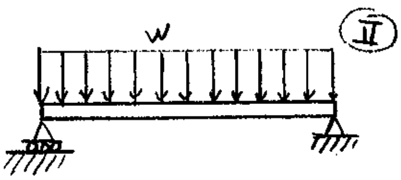
Loading II: Case 6 of Appendix D.

$$(y_B)_{II} = -\frac{5}{384}\frac{wL^4}{EI}$$

Deflection at B

$$-\delta_{BD} = y_B = (y_B)_I + (y_B)_{II}$$

$$-\frac{F_{BD}\,l}{EA} = \frac{F_{BD}L^3}{48\,EI} - \frac{5}{384}\frac{wL^4}{EI}$$

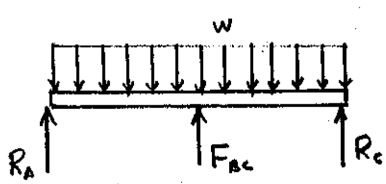


$$\left(\frac{l}{EA} + \frac{L^3}{48\,EI}\right)F_{BD} = \frac{5}{384}\frac{wL^4}{EI}$$

Data: $l = 0.2$ m

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(4)^2 = 12.566\ \text{mm}^2 = 12.566\times10^{-6}\ \text{m}^2$

$E = 200\times10^9$ Pa



$\frac{l}{EA} = 79.58\times10^{-9}$ m/N

$L = 0.36$ m $\qquad w = 1.6\times10^3$ N/m

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(\frac{40}{2}\right)^4 = 125.66\times10^3\ \text{mm}^4 = 125.66\times10^{-9}\ \text{m}^4$$

$$EI = (200\times10^9)(125.66\times10^{-9}) = 25.132\times10^3\ \text{N}\cdot\text{m}^2$$

$$\left[79.58\times10^{-9} + \frac{(0.36)^3}{(48)(25.132\times10^3)}\right]F_{BD} = \frac{(5)(1.6\times10^3)(0.36)^4}{(384)(25.132\times10^3)}$$

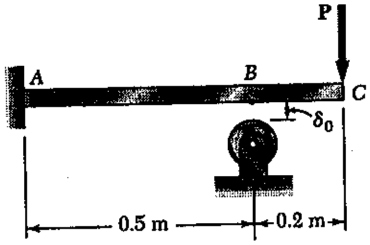
$$118.256\times10^{-9}\,F_{BD} = 13.923\times10^{-6} \qquad F_{BD} = 117.74\ \text{N}$$

(a) Deflection at B $\quad \delta_B = \frac{F_{BD}\,l}{EA} = (117.74)(79.58\times10^{-9}) = 9.37\times10^{-6}\ \text{m} = 0.00937\ \text{mm}\downarrow$

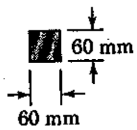
(b) $R_A = R_C = \frac{1}{2}\left[wL - F_{BD}\right] = \frac{1}{2}\left[(1600)(0.36) - 117.74\right] = 229\ \text{N}\uparrow$

PROBLEM 9.93

9.93 Before the load **P** was applied, a gap $\delta_0 = 0.5$ mm existed between the cantilever beam *AC* and the support at *B*. Knowing that $E = 200$ GPa, determine the magnitude of **P** for which the deflection at *C* is 1 mm.

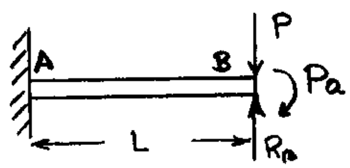


SOLUTION



Let length AB = L = 0.5 m
length BC = a = 0.2 m

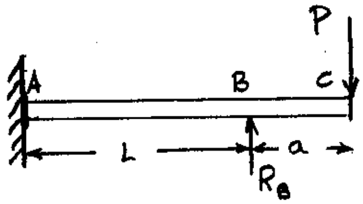
Consider portion AB of beam ABC.



The loading becomes forces P and R_B at B plus the couple Pa. The deflection at B is δ_0. Using Cases 1 and 3 of Appendix D.

$$\delta_0 = \frac{(P - R_B)L^3}{3EI} + \frac{PaL^2}{2EI}$$

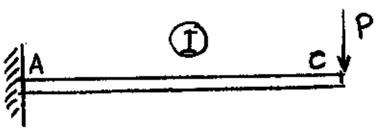
$$\left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)P - \frac{L^3}{3}R_B = EI\delta_0 \qquad (1)$$



The deflection at C depends on the deformation of beam ABC subjected to loads P and R_B.

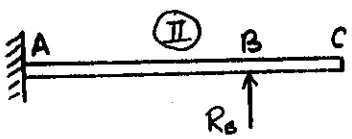
For loading I, using Case 1 of Appendix D

$$(\delta_C)_1 = \frac{P(L+a)^3}{3EI}$$



For loading II, using Case 1 of Appendix D

$$y_B = \frac{R_B L^3}{3EI} \qquad \theta_B = \frac{R_B L^2}{2EI}$$



Portion BC remains straight

$$y_C = y_B + a\theta_B = \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

By superposition the downward deflection at C is

$$\delta_C = \frac{P(L+a)^3}{3EI} - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

$$\frac{(L+a)^3}{3}P - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)R_B = EI\delta_C \qquad (2)$$

Data: $E = 200 \times 10^9$ Pa

$$I = \frac{1}{12}(60)(60)^3 = 1.08 \times 10^6 \text{ mm}^4 = 1.08 \times 10^{-6} \text{ m}^4$$

$$EI = 216 \times 10^3 \text{ N}\cdot\text{m}^2$$

$$\delta_0 = 0.5 \times 10^{-3} \text{ m}$$

$$\delta_C = 1.0 \times 10^{-3} \text{ m}$$

Using the data, eqs (1) and (2) become

$$0.06667\,P - 0.04167\,R_B = 108 \qquad (1)'$$

$$0.11433\,P - 0.06667\,R_B = 216 \qquad (2)'$$

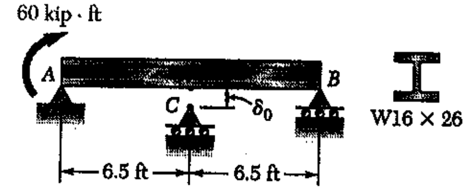
Solving simultaneously

$$P = 5.63 \times 10^3 \text{ N} = 5.63 \text{ kN} \downarrow$$

$$R_B = 6.42 \times 10^3 \text{ N}$$

PROBLEM 9.94

9.94 Before the 60-kip·ft couple was applied, a gap, $\delta_0 = 0.05$ in., existed between the W16 × 26 beam and the support at C. Knowing that $E = 29 \times 10^6$ psi, determine the reaction at each support after the couple is applied.



SOLUTION

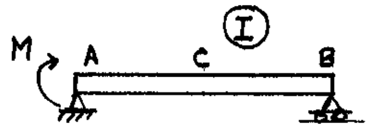
Units: Forces in kips, lengths in ft.

$\delta_0 = 0.05 \text{ in} = 4.1667 \times 10^{-3}$ ft

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = 301$ in^4

$EI = 8.729 \times 10^8$ kip·in^2 $= 60618$ kip·ft^2

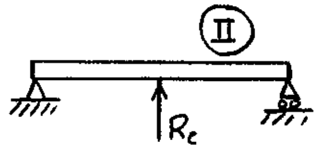


Loading I: Case 7 of Appendix D

$$y = -\frac{M}{6EIL}(x^3 - L^2x)$$

with $M = 60$ kip·ft, $L = 13$ ft, $x = 6.5$ ft

$$(y_c)_1 = -\frac{(60)[6.5^3 - (13)^2(6.5)]}{(6)(60618)(13)} = -10.454 \times 10^{-3} \text{ ft}$$



Loading II: Case 4 of Appendix D

$$(y_c)_2 = \frac{R_c L^3}{48\,EI} = \frac{(13)^3 R_c}{(48)(60618)}$$

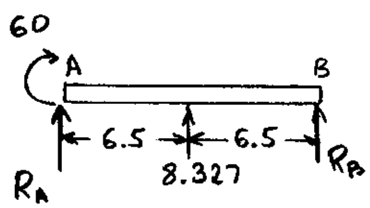
$$= 755.07 \times 10^{-6} R_B$$

Deflection at C

$$y_c = (y_c)_1 + (y_c)_2 = -\delta_0$$

$$-10.454 \times 10^{-3} + 755.07 \times 10^{-6} R_c = -4.1667 \times 10^{-3}$$

$R_c = 8.327$ kips ↑



Statics:

$\circlearrowleft\!+\ \Sigma M_B = 0$

$-13\,R_A - 60 - (6.5)(8.327) = 0$

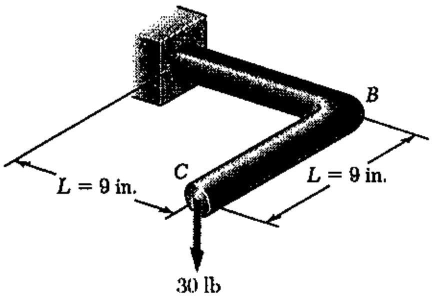
$R_A = -8.779$ kips $\qquad R_A = 8.779$ kips ↓

$\circlearrowleft\!+\ \Sigma M_A = 0$

$13\,R_A - 60 + (6.5)(8.327) = 0$

$R_B = 0.452$ kips ↑

PROBLEM 9.95



9.95 A 5/8-inch-diameter rod *ABC* was been bent into the shape shown. Determine the deflection of end *C* after the 30-lb force is applied. Use $E = 29 \times 10^6$ psi. and $G = 11.2 \times 10^6$ psi.

SOLUTION

Let 30 lb = P.

Consider torsion of rod AB.

$$\varphi_B = \frac{TL}{GJ} = \frac{(PL)L}{GJ} = \frac{PL^2}{GJ}$$

$$(y_C)_I = -L\varphi_B = -\frac{PL^3}{GJ}$$

Consider bending of AB (Case 1, App.D)

$$y_B = -\frac{PL^3}{3EI}$$

$$(y_C)_{II} = y_B = -\frac{PL^3}{3EI}$$

Consider bending of BC (Case 1, App.D)

$$(y_B)_{III} = -\frac{PL^3}{3EI}$$

Superposition

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III}$$

$$= -PL^3\left(\frac{1}{GJ} + \frac{1}{3EI} + \frac{1}{3EI}\right)$$

$$= -\frac{PL^3}{EI}\left(\frac{EI}{GJ} + \frac{2}{3}\right)$$

Data: $G = 11.2 \times 10^6$ psi, $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{5}{16}\right)^4 = 0.014980 \text{ in}^4$

$E = 29 \times 10^6$ psi, $I = \frac{1}{2}J = 0.007490 \text{ in}^4$

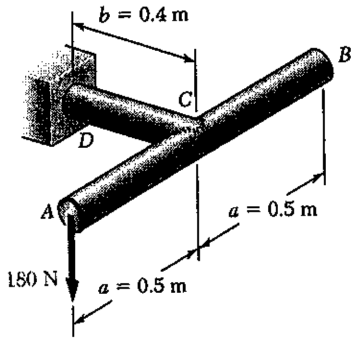
$EI = 217.21 \times 10^3 \text{ lb}\cdot\text{in}^2$ $\quad GJ = 167.78 \times 10^3 \text{ lb}\cdot\text{in}^2$

$$y_B = -\frac{(30)(9)^3}{217.21 \times 10^3}\left(\frac{217.21 \times 10^3}{167.78 \times 10^3} + \frac{2}{3}\right) = -0.1975 \text{ in.}$$

$$y_B = 0.1975 \text{ in.} \downarrow$$

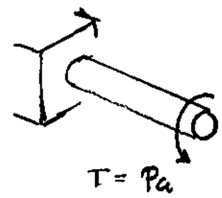
PROBLEM 9.96

9.96 Two 24-mm-diameter aluminum rods are welded together to form the T-shaped hanger shown. Knowing that $E = 70$ GPa and $G = 26$ GPa, determine the deflection at (*a*) end *A*, (*b*) end *B*.



SOLUTION

Consider torsion of rod CD $(180\ \mathrm{N} = P)$

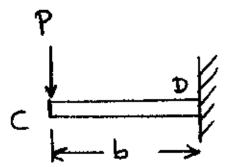


$$\varphi_c = \frac{TL}{GJ} = \frac{(Pa)b}{GJ}$$

$$(y_A)_I = -a\varphi_c = -\frac{Pa^2b}{GJ}$$

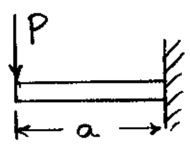
$$(y_B)_I = a\varphi_c = \frac{Pa^2b}{GJ}$$

Consider bending of rod CD



$$(y_A)_{II} = (y_B)_{II} = (y_C)_{II} = -\frac{Pb^3}{3EI} \quad \text{(Case 1, App D.)}$$

Consider bending of rod portion AC



$$(y_A)_{III} = -\frac{Pa^3}{3EI}$$

By superposition.

$$y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III}$$
$$= P\left\{-\frac{a^2b}{GJ} - \frac{b^3}{3EI} - \frac{a^3}{3EI}\right\}$$

$$y_B = (y_B)_I + (y_B)_{II}$$
$$= P\left\{\frac{a^2b}{GJ} - \frac{b^3}{3EI}\right\}$$

Data: $G = 26\times10^9\ \mathrm{Pa}$, $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(12)^4 = 32.572\times10^3\ \mathrm{mm}^4 = 32.572\times10^{-9}\ \mathrm{m}^4$

$E = 70\times10^9\ \mathrm{Pa}$, $I = \frac{1}{2}J = 16.286\times10^{-9}\ \mathrm{m}^4$

$GJ = 846.87\ \mathrm{N\cdot m^2}$ $\qquad EI = 1140.02\ \mathrm{N\cdot m^2}$

$a = 0.5\ \mathrm{m}$, $\quad b = 0.4\ \mathrm{m}$

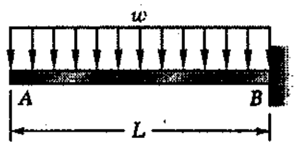
$$y_A = 180\left\{-\frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)} - \frac{(0.5)^3}{(3)(1140.02)}\right\} = -31.2\times10^{-3}\ \mathrm{m}$$
$$= 31.2\ \mathrm{mm}\downarrow$$

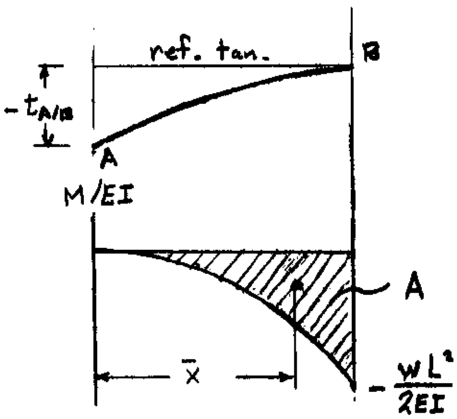
$$y_B = 180\left\{\frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)}\right\} = 17.89\times10^{-3}\ \mathrm{m}$$

$$17.89\ \mathrm{mm}\uparrow$$

PROBLEM 9.97

9.97 and *9.98* For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.





SOLUTION

Place reference tangent at B. $\theta_B = 0$

Draw $\frac{M}{EI}$ curve as parabola.

$$A = -\frac{1}{3}\left(\frac{wL^2}{2EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$

By first moment-area theorem

$$\theta_{B/A} = A = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6}\frac{wL^3}{EI} = \frac{1}{6}\frac{wL^3}{EI} \blacktriangleleft$$

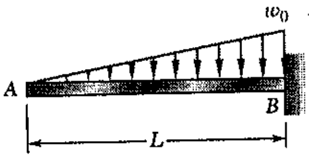
By second moment-area theorem

$$t_{A/B} = \bar{x}A = \left(\frac{3}{4}L\right)\left(-\frac{1}{6}\frac{wL^3}{EI}\right) = -\frac{1}{8}\frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8}\frac{wL^4}{EI} \blacktriangleleft$$

PROBLEM 9.98

9.97 and ***9.98*** For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

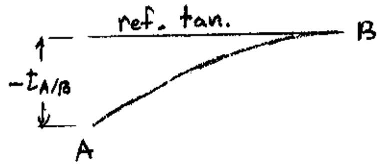


SOLUTION

Place reference tangent at B. $\theta_B = 0$

$$+\circlearrowleft \Sigma M_B = 0 \quad \left(\tfrac{1}{2} w_0 L\right)\tfrac{L}{3} + M_B = 0$$

$$M_B = -\tfrac{1}{6} wL^2$$

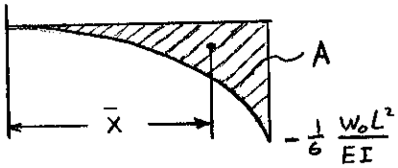


Draw $\dfrac{M}{EI}$ curve as cubic parabola.

$$A = -\tfrac{1}{4}\left(\tfrac{1}{6}\tfrac{w_0 L^2}{EI}\right)L = -\tfrac{1}{24}\tfrac{w_0 L^3}{EI}$$

$$\bar{x} = L - \tfrac{1}{5}L = \tfrac{4}{5}L$$

M/EI



By first moment-area theorem

$$\theta_{B/A} = A = -\tfrac{1}{24}\tfrac{w_0 L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \tfrac{1}{24}\tfrac{w_0 L^3}{EI} = \tfrac{1}{24}\tfrac{wL^3}{EI}$$ ◄

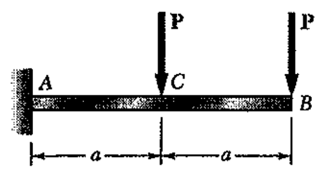
By second moment-area theorem

$$t_{A/B} = \bar{x}A = \left(\tfrac{4}{5}L\right)\left(-\tfrac{1}{24}\tfrac{w_0 L^3}{EI}\right) = -\tfrac{1}{30}\tfrac{w_0 L^4}{EI}$$

$$y_A = t_{A/B} = -\tfrac{1}{30}\tfrac{w_0 L^4}{EI}$$ ◄

PROBLEM 9.99

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.



SOLUTION

Place reference tangent at A. $\theta_A = 0$.

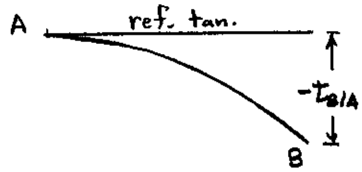
Draw $\frac{M}{EI}$ diagram by parts (two triangles)

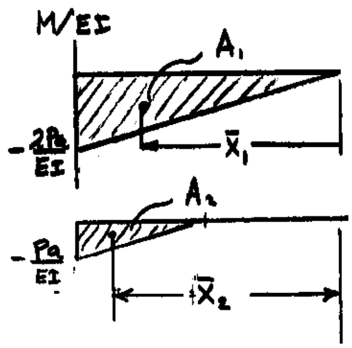
$$A_1 = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a) = -\frac{2Pa^2}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(2a) = \frac{4}{3}a$$

$$A_2 = \frac{1}{2}\left(-\frac{Pa}{EI}\right)a = -\frac{1}{2}\frac{Pa^2}{EI}$$

$$\bar{x}_2 = a + \frac{2}{3}a = \frac{5}{3}a$$





By first moment-area theorem

$$\theta_{B/A} = A_1 + A_2 = -\frac{2Pa^2}{EI} - \frac{1}{2}\frac{Pa^2}{EI} = -\frac{5}{2}\frac{Pa^2}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{5}{2}\frac{Pa^2}{EI}$$

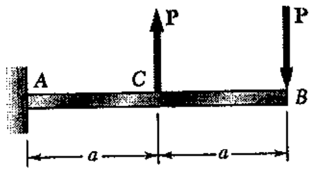
By second moment area theorem

$$t_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= \left(-\frac{2Pa^2}{EI}\right)\left(\frac{4}{3}a\right) + \left(-\frac{1}{2}\frac{Pa^2}{EI}\right)\left(\frac{5}{3}a\right) = -\frac{7}{2}\frac{Pa^3}{EI}$$

$$y_B = t_{B/A} = -\frac{7}{2}\frac{Pa^3}{EI}$$

PROBLEM 9.100

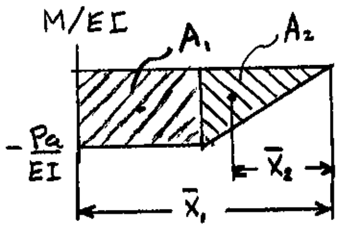


9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

Place reference tangent at A. $\theta_A = 0$

Draw V (shear) and $\frac{M}{EI}$ diagrams.



$$A_1 = -\left(\frac{Pa}{EI}\right)(a) = -\frac{Pa^2}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{Pa}{EI}\right)(a) = -\frac{1}{2}\frac{Pa^2}{EI}$$

$$\bar{x}_1 = a + \frac{1}{2}a = \frac{3}{2}a$$

$$\bar{x}_2 = \frac{2}{3}a$$

By first moment-area theorem

$$\theta_{B/A} = A_1 + A_2 = -\frac{Pa^2}{EI} - \frac{1}{2}\frac{Pa^2}{EI} = -\frac{3}{2}\frac{Pa^2}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{3}{2}\frac{Pa^2}{EI}$$

By second moment-area theorem

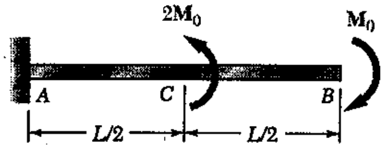
$$t_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2 = \left(-\frac{Pa^2}{EI}\right)\left(\frac{3}{2}a\right) + \left(-\frac{1}{2}\frac{Pa^2}{EI}\right)\left(\frac{2}{3}a\right)$$

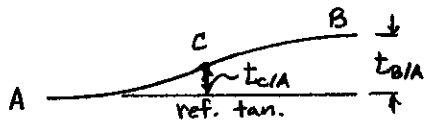
$$= -\frac{11}{6}\frac{Pa^3}{EI}$$

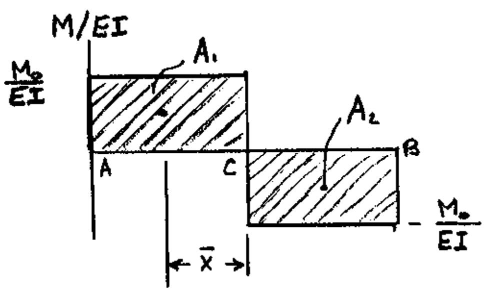
$$y_B = t_{B/A} = -\frac{11}{6}\frac{Pa^3}{EI}$$

PROBLEM 9.101

***9.101* and 102** For the uniform cantilever beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at *C*.







SOLUTION

Place reference tangent at A. $\theta_A = 0$

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{2}\frac{M_0 L}{EI}$$

$$A_2 = \left(-\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{2}\frac{M_0 L}{EI}$$

By first moment-area theorem

$$\theta_{B/A} = A_1 + A_2 = \frac{1}{2}\frac{M_0 L}{EI} - \frac{1}{2}\frac{M_0 L}{EI} = 0$$

$$\theta_B = \theta_A + \theta_{B/A} = 0$$ ◄

Deflection at C.

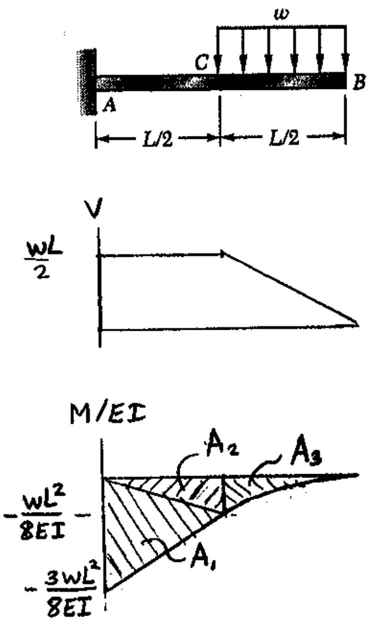
By second moment-area theorem

$$t_{C/A} = A_1 \bar{x} = \left(\frac{1}{2}\frac{M_0 L}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{8}\frac{M_0 L^2}{EI}$$

$$y_C = t_{C/A} = \frac{1}{8}\frac{M_0 L^2}{EI}$$ ◄

PROBLEM 9.102

***9.101* and 102** For the uniform cantilever beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at *C*.



SOLUTION

Place reference tangent at A. $\theta_A = 0$

Draw V (shear) and $\frac{M}{EI}$ diagrams.

(a) Slope at B

$$A_1 = -\frac{1}{2}\left(\frac{3wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{3}{32}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{32}\frac{wL^3}{EI}$$

$$A_3 = -\frac{1}{3}\left(\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\theta_{B/A} = A_1 + A_2 + A_3 = -\frac{7}{48}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{7}{48}\frac{wL^3}{EI}$$

(b) Deflection at C

$$\bar{x}_{1c} = \frac{2}{3}\cdot\frac{L}{2} = \frac{1}{3}L$$

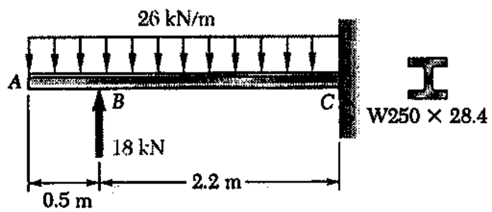
$$\bar{x}_{2c} = \frac{1}{3}\cdot\frac{L}{2} = \frac{1}{6}L$$

$$t_{C/A} = A_1\bar{x}_{1c} + A_2\bar{x}_{2c}$$
$$= \left(-\frac{3}{32}\frac{wL^3}{EI}\right)\left(\frac{1}{3}L\right) + \left(-\frac{1}{32}\frac{wL^3}{EI}\right)\left(\frac{1}{6}L\right) = -\frac{7}{192}\frac{wL^4}{EI}$$

$$y_C = t_{C/A} = -\frac{7}{192}\frac{wL^4}{EI}$$

PROBLEM 9.103

9.103 For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.



SOLUTION

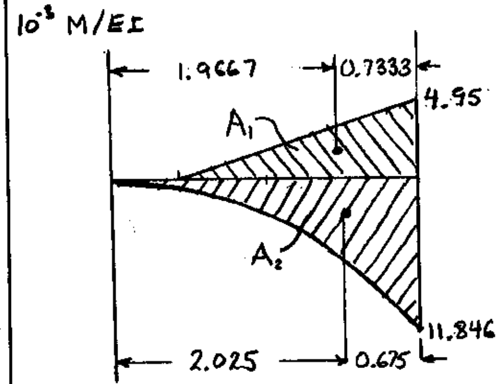
Units: Forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa

$I = 40.0 \times 10^6 \text{ mm}^4 = 40.0 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(40.0 \times 10^{-6}) = 8.00 \times 10^6 \text{ N}\cdot\text{m}^2$

$= 8000 \text{ kN}\cdot\text{m}^2$



Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8000} = 4.95 \times 10^{-3} \text{ m}^{-1}$$

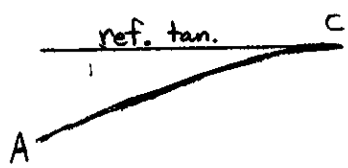
$$A_1 = \tfrac{1}{2}(4.95 \times 10^{-3})(2.2) = 5.445 \times 10^{-3}$$

$$\bar{x}_1 = \tfrac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8000)} = -11.846 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \tfrac{1}{3}(-11.846 \times 10^{-3})(2.7) = -10.662 \times 10^{-3}$$

$$\bar{x}_2 = \tfrac{1}{4}(2.7) = 0.675 \text{ m}$$



Draw reference tangent at C.

$$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$$

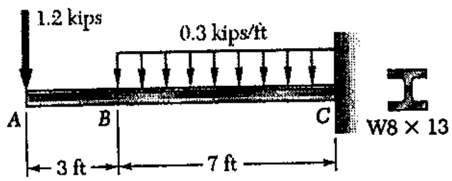
$$\theta_A = -A_1 - A_2 = -5.445 \times 10^{-3} + 10.662 \times 10^{-3}$$
$$= 5.22 \times 10^{-3} \text{ rad}$$

$$\theta_A = 5.22 \times 10^{-3} \measuredangle$$

$$y_A = y_C - \theta_C L + t_{A/C}$$
$$= 0 - 0 + A_1\bar{x}_1 + A_2\bar{x}_2$$
$$= 0 - 0 + (5.445 \times 10^{-3})(1.9667) - (10.662 \times 10^{-3})(2.025)$$
$$= -10.881 \times 10^{-3} \text{ m} = 10.88 \text{ mm} \downarrow$$

PROBLEM 9.104

9.104 For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 29 \times 10^6$ psi.



SOLUTION

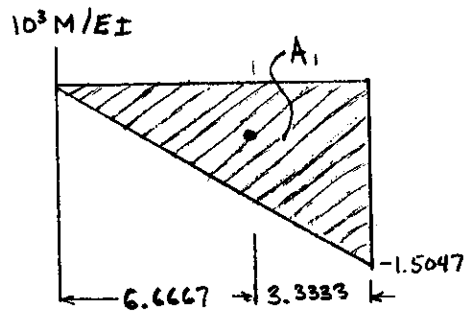
Units: Forces in kips, lengths in ft.

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = 39.6$ in^4

$EI = (29 \times 10^3)(39.6) = 1.1484 \times 10^6$ kip·in^2

$= 7975$ kip·ft^2

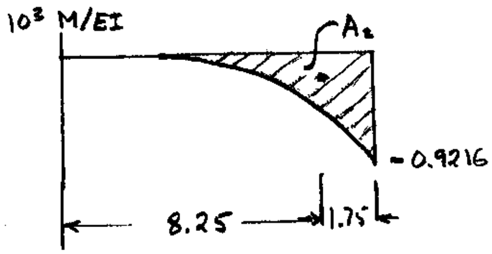


Draw $\frac{M}{EI}$ diagram by parts.

$\frac{M_1}{EI} = -\frac{(1.2)(10)}{7975} = -1.5047 \times 10^{-3}$ ft^{-1}

$A_1 = \frac{1}{2}(-1.5047 \times 10^{-3})(10) = -7.5235 \times 10^{-3}$

$\bar{x}_1 = \frac{1}{3}(10) = 3.3333$ ft

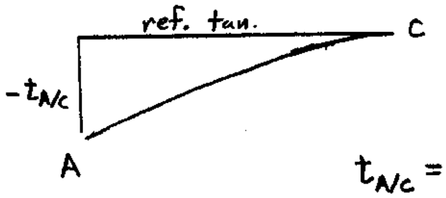


$\frac{M_2}{EI} = -\frac{(0.3)(7)^2}{(2)(7975)} = -0.9216 \times 10^{-3}$ ft^{-1}

$A_2 = \frac{1}{3}(-0.9216 \times 10^{-3})(7) = -2.1505 \times 10^{-3}$

$\bar{x}_2 = \frac{1}{4}(7) = 1.75$ ft.

Place reference tangent at C. $\theta_c = 0$



$\theta_{c/A} = A_1 + A_2 = -9.67 \times 10^{-3}$

$\theta_A = \theta_c - \theta_{c/A} = 9.67 \times 10^{-3}$ rad

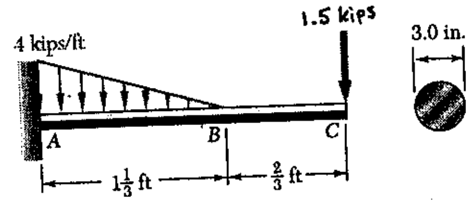
$t_{A/c} = (6.6667)(-7.5235 \times 10^{-3}) + (8.25)(-2.1505 \times 10^{-3})$

$= -67.90 \times 10^{-3}$ ft

$y_A = t_{A/c} = -67.90 \times 10^{-3}$ ft $= -0.814$ in.

PROBLEM 9.105

9.105 For the cantilever beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.



SOLUTION

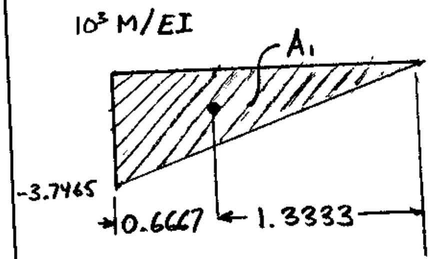
Units: Forces in kips, lengths in ft.

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}(1.5)^4 = 3.97608$ in^4

$EI = (29 \times 10^3)(3.97608) = 115.306$ kip·in^2

$= 800.74$ kip·ft^2

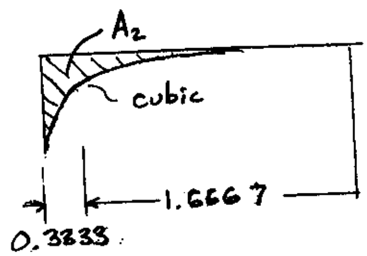


Draw $\frac{M}{EI}$ diagram by parts

$\frac{M_1}{EI} = -\frac{(1.5)(2)}{800.74} = -3.7465 \times 10^{-3}$ ft^{-1}

$A_1 = \frac{1}{2}(-3.7465 \times 10^{-3})(2) = -3.7465 \times 10^{-3}$

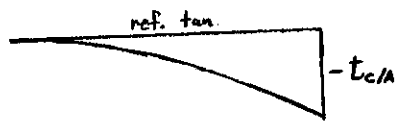
$\bar{x}_1 = \frac{1}{3}(2) = 0.66667$ ft



$\frac{M_2}{EI} = \frac{\frac{1}{2}(4)(\frac{4}{3})(\frac{1}{3} \cdot \frac{4}{3})}{800.74} = -1.4801 \times 10^{-3}$ ft^{-1}

$A_1 = \frac{1}{4}(-1.4801 \times 10^{-3})(\frac{4}{3}) = -0.49337 \times 10^{-3}$

$\bar{x} = \frac{1}{4} \cdot \frac{4}{3} = 0.33333$ ft



Place reference tangent at A. $\theta_A = 0$

$\theta_{C/A} = A_1 + A_2 = -4.24 \times 10^{-3}$ rad

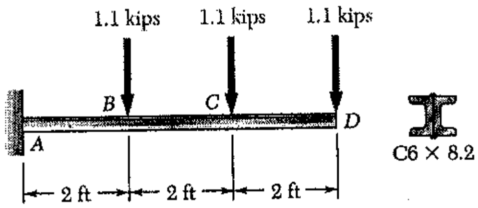
$\theta_C = \theta_A + \theta_{C/A} = -4.24 \times 10^{-3}$ rad ◄

$t_{C/A} = (1.3333)(-3.7465 \times 10^{-3})$
$+ (1.6667)(-0.49337 \times 10^{-3})$
$= -6.71 \times 10^{-3}$ ft

$y_C = y_A + (2)(\theta_A) + t_{C/A}$
$= 0 + 0 - 5.82 \times 10^{-3} = -5.82 \times 10^{-3}$ ft
$= 0.0698$ in. ↓ ◄

PROBLEM 9.106

9.106 Two C 6 × 8.2 channels are welded back to back and loaded as shown. Knowing that $E = 29 \times 10^6$ psi., determine (*a*) the slope at *D*, (*b*) the deflection at *D*.



SOLUTION

Units: Forces in kips, lengths in ft.

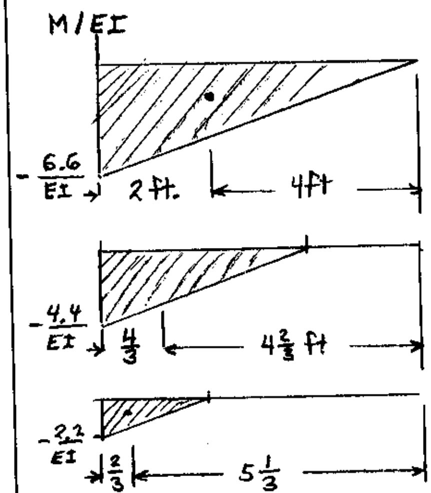
$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

$I = (2)(13.1) = 26.2 \text{ in}^4$

$EI = (29 \times 10^3)(26.2) = 759.8 \times 10^3 \text{ kip}\cdot\text{in}^2$

$= 5276 \text{ kip}\cdot\text{ft}^2$

Draw $\frac{M}{EI}$ diagram by parts.



$\frac{M_1}{EI} = -\frac{(1.1)(6)}{EI} = -\frac{6.6}{EI} \quad \text{ft}^{-1}$

$A_1 = \frac{1}{2}\left(\frac{6.6}{EI}\right)(6) = -\frac{19.8}{EI}$

$\bar{x}_1 = \frac{1}{3}(6) = 2 \text{ ft.}$

$\frac{M_2}{EI} = -\frac{(1.1)(4)}{EI} = -\frac{4.4}{EI} \quad \text{ft}^{-1}$

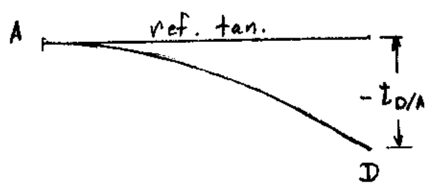
$A_2 = \frac{1}{2}\left(-\frac{4.4}{EI}\right)(4) = -\frac{8.8}{EI}$

$\bar{x}_2 = \frac{1}{3}(4) = \frac{4}{3} \text{ ft}$

$\frac{M_3}{EI} = -\frac{(1.1)(2)}{EI} = -\frac{2.2}{EI} \quad \text{ft}^{-1}$

$A_3 = \frac{1}{2}\left(-\frac{2.2}{EI}\right)(2) = -\frac{2.2}{EI}$

$\bar{x}_3 = \frac{1}{3}(2) = \frac{2}{3} \text{ ft.}$



Place reference tangent at A. $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{30.8}{EI} = -\frac{30.8}{5276} = -5.84 \times 10^{-3} \text{ rad.}$$

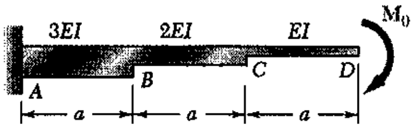
$$\theta_D = \theta_A + \theta_{D/A} = -5.84 \times 10^{-3} \text{ rad.}$$

$$t_{D/A} = \left(-\frac{19.8}{EI}\right)(4) + \left(-\frac{8.8}{EI}\right)\left(4\tfrac{2}{3}\right) + \left(-\frac{2.2}{EI}\right)\left(5\tfrac{1}{3}\right) = -\frac{132.0}{EI} = -\frac{132.0}{5276} = 25.02 \times 10^{-3} \text{ ft}$$

$$y_D = t_{D/A} = 25.02 \times 10^{-3} \text{ ft} = 0.300 \text{ in} \downarrow$$

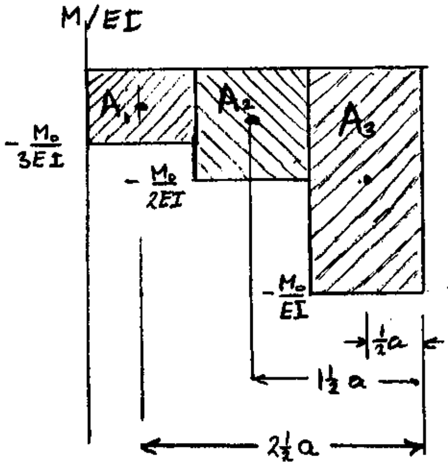
PROBLEM 9.107

9.107 For the cantilever beam and loading shown, determine the deflection and slope at end D caused by the couple $\mathbf{M}_0$.



SOLUTION

Draw $\frac{M}{EI}$ diagram.



$$A_1 = -\frac{M_0 a}{3EI}$$

$$A_2 = -\frac{M_0 a}{2EI}$$

$$A_3 = -\frac{M_0 a}{EI}$$

Place reference tangent at A. $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3$$

$$= -\frac{11}{6}\frac{M_0 a}{EI}$$

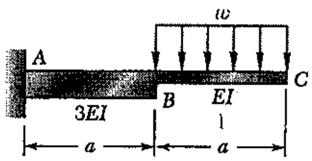
$$\theta_D = \theta_A + \theta_{D/A} = -\frac{11}{6}\frac{M_0 a}{EI}$$

$$t_{D/A} = -\left(\frac{M_0 a}{3EI}\right)\left(2\tfrac{1}{2}a\right) - \left(\frac{M_0 a}{2EI}\right)\left(1\tfrac{1}{2}a\right) - \left(\frac{M_0 a}{EI}\right)\left(\tfrac{1}{2}a\right) = -\frac{25}{12}\frac{M_0 a^2}{EI}$$

$$y_D = t_{D/A} = -\frac{25}{12}\frac{M_0 a^2}{EI}$$

PROBLEM 9.108

9.108 For the cantilever beam and loading shown, determine the deflection at (*a*) point *B*, (*b*) point *C*.



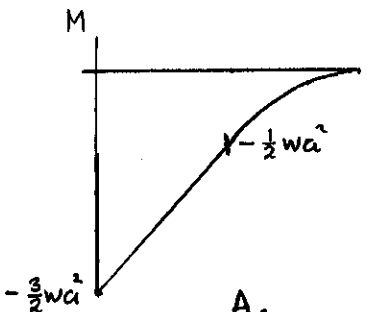
SOLUTION

Draw $\frac{M}{EI}$ diagram

$$A_1 = \frac{1}{2}\left(-\frac{1}{6}\frac{wa^2}{EI}\right)a = -\frac{1}{12}\frac{wa^3}{EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{1}{2}\frac{wa}{EI}\right)a = -\frac{1}{4}\frac{wa^3}{EI}$$

$$A_3 = \frac{1}{3}\left(-\frac{1}{2}\frac{wa^2}{EI}\right)a = -\frac{1}{6}\frac{wa^3}{EI}$$



Place reference tangent at A

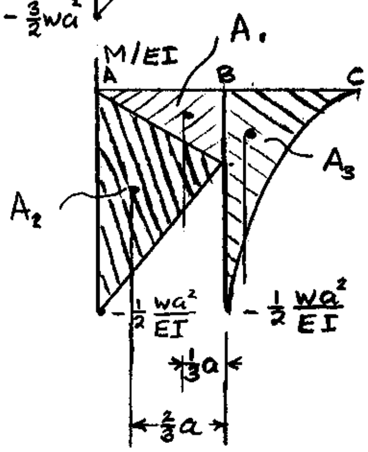
(a) Deflection at B.

$$t_{B/A} = A_1\left(\tfrac{1}{3}a\right) + A_2\left(\tfrac{2}{3}a\right)$$

$$= \left(-\frac{1}{12}\frac{wa^3}{EI}\right)\left(\tfrac{1}{3}a\right) + \left(-\frac{1}{4}\frac{wa^3}{EI}\right)\left(\tfrac{2}{3}a\right)$$

$$= -\frac{7}{36}\frac{wa^4}{EI}$$

$$y_B = t_{B/A} = -\frac{7}{36}\frac{wa^4}{EI}$$



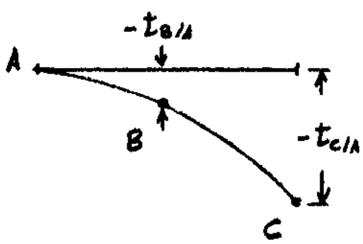
(b) Deflection at C.

$$t_{C/A} = A_1\left(a+\tfrac{1}{3}a\right) + A_2\left(a+\tfrac{2}{3}a\right) + A_3\left(a-\tfrac{1}{4}a\right)$$

$$= \left(-\frac{1}{12}\frac{wa^2}{EI}\right)\left(\tfrac{4}{3}a\right) + \left(-\frac{1}{4}\frac{wa^2}{EI}\right)\left(\tfrac{5}{3}a\right)$$

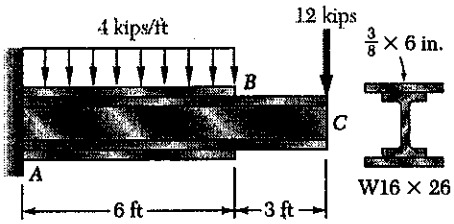
$$-\left(\frac{1}{6}\frac{wa^2}{EI}\right)\left(\tfrac{3}{4}a\right) = -\frac{47}{72}\frac{wa^4}{EI}$$

$$y_C = t_{C/A} = -\frac{47}{72}\frac{wa^4}{EI}$$



PROBLEM 9.109

9.109 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 29 \times 10^6$ psi., determine (*a*) the slope at end *C*, (*b*) the deflection at end *C*.



SOLUTION

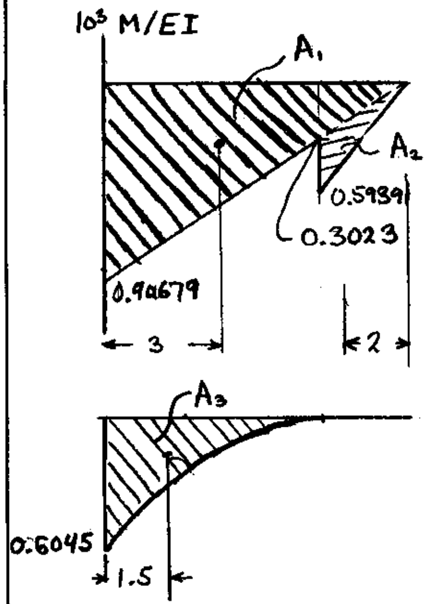
For W 16×26 rolled steel section

$d = 15.69 \text{ in}^2 \qquad I = 301 \text{ in}^4$

For the two cover plates

$$I = 2\left[\tfrac{1}{12}(6)\left(\tfrac{3}{8}\right)^3 + (6)\left(\tfrac{3}{8}\right)\left(\tfrac{15.69}{2} + \tfrac{3}{16}\right)^2\right]$$

$$= 290.4 \text{ in}^4$$



A to B $\quad EI_1 = (29\times10^3)(301 + 290.4) = 17.151 \times 10^6 \text{ kip}\cdot\text{in}^2$

$\qquad\qquad = 119101 \text{ kip}\cdot\text{ft}^2$

B to C $\quad EI_2 = (29\times10^3)(301) = 8.729\times10^6 \text{ kip}\cdot\text{in}^2$

$\qquad\qquad = 60618 \text{ kip}\cdot\text{ft}^2$

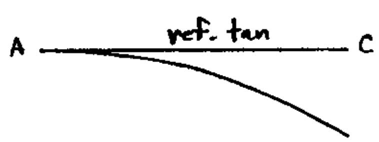
Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI_1} = -\frac{(12)(9)}{119101} = -0.90679\times10^{-3} \text{ ft}^{-1}$$

$$A_1 = -\tfrac{1}{2}(0.90679\times10^{-3})(9) = -4.081\times10^{-3}$$

$$\frac{M_2}{EI_1} = -\frac{(12)(3)}{119101} = -0.3023\times10^{-3} \text{ ft}^{-1}$$

$$\frac{M_2}{EI_1} = -\frac{(12)(3)}{60618} = -0.5939\times10^{-3} \text{ ft}^{-1}$$



$$A_2 = -\tfrac{1}{2}(0.5939 - 0.3023)(10^{-3})(3) = -0.437\times10^{-3}$$

$$\frac{M_3}{EI_1} = -\frac{(4)(6)(6)}{(2)(119101)} = -0.6045\times10^{-3} \text{ ft}^{-1}$$

$$A_3 = -\tfrac{1}{3}(0.6045\times10^{-3})(6) = -1.209\times10^{-3}$$

Place reference tangent at A where $y_A = 0, \; \theta_A = 0$

(a) $\theta_C = \theta_A + \theta_{C/A} = 0 + A_1 + A_2 + A_3 = -5.73\times10^{-3}$ rad ◄

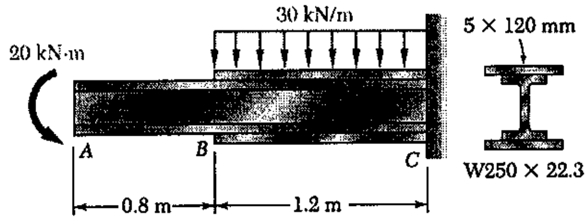
(b) $y_C = y_A + L\theta_A + t_{C/A}$

$= 0 + 0 - (4.081\times10^{-3})(6) - (0.437\times10^{-3})(2) - (1.209\times10^{-3})(7.5)$

$= -34.43\times10^{-3}$ ft $= 0.413$ in. ↓ ◄

PROBLEM 9.110

9.110 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*.



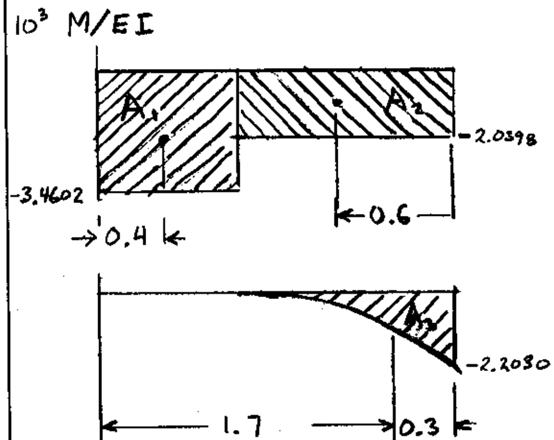
SOLUTION

Units: Forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa

From A to B $\quad I = 28.9 \times 10^6\ mm^4 = 28.9 \times 10^{-6}\ m^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6\ N \cdot m^2 = 5780\ kN \cdot m^2$



From B to C $\quad I = I_w + 2A_p d^2 + 2\bar{I}_p$

$A_p = 5 \times 120 = 600\ mm^2$

$d = \frac{254}{2} + \frac{5}{2} = 129.5\ mm$

$Ad^2 = 10.062 \times 10^6\ mm^4$

$\bar{I}_p = \frac{1}{12}(120)(5)^3 = 0.00125 \times 10^6\ mm^4$

$I = [28.9 + (2)(10.062) + (2)(0.00125)] \times 10^6\ mm^4 = 49.03 \times 10^6\ mm^4 = 49.03 \times 10^{-6}\ m^4$

$EI = (200 \times 10^9)(49.03 \times 10^{-6}) = 9.805 \times 10^6\ N \cdot m^2 = 9805\ kN \cdot m^2$

Draw M/EI diagram by parts

A to B $\quad \frac{M_1}{EI} = -\frac{20}{5780} = -3.4602 \times 10^{-3}\ m^{-1}$

B to C $\quad \frac{M_2}{EI} = -\frac{20}{9805} = -2.0398 \times 10^{-3}\ m^{-1}$

$\frac{M_3}{EI} = -\frac{(30)(1.2)^2}{(2)(9805)} = -2.2030 \times 10^{-3}\ m^{-1}$

$A_1 = (-3.4602 \times 10^{-3})(0.8) = -2.7682 \times 10^{-3}$

$A_2 = (-2.0398 \times 10^{-3})(1.2) = -2.4478 \times 10^{-3}$

$A_3 = \frac{1}{3}(-2.2030 \times 10^{-3})(1.2) = -0.8812 \times 10^{-3}$

Place reference tangent at C. $\quad \theta_C = 0$

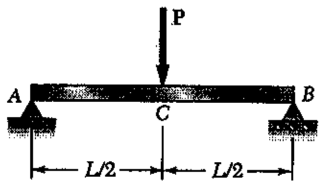
(a) Slope at A $\quad \theta_A = \theta_C - \theta_{A/C} = 0 - (A_1 + A_2 + A_3) = 6.10 \times 10^{-3}$ rad ◄

(b) Deflection at A

$y_A = t_{A/C} = (-2.7682 \times 10^{-3})(0.4) + (-2.4478 \times 10^{-3})(1.4) + (-0.8812 \times 10^{-3})(1.7)$

$= -6.03 \times 10^{-3}\ m = 6.03\ mm \downarrow$ ◄

PROBLEM 9.111



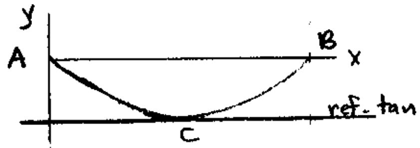
9.111 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetrical beam and loading.

Place reference tangent at C.

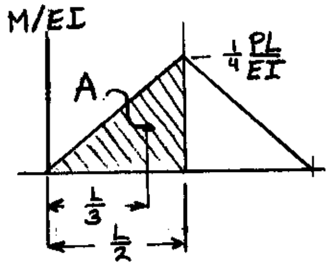
$\theta_C = 0 \quad , \quad y_C = -t_{A/C}$



Reactions $R_A = R_B = \frac{1}{2}P$

Bending moment at C $\quad M_C = \frac{1}{4}PL$

$A = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$



(a) Slope at A: $\quad \theta_A = \theta_C - \theta_{C/A}$

$\theta_A = 0 - \frac{1}{16}\frac{PL^2}{EI} = -\frac{1}{16}\frac{PL^2}{EI}$

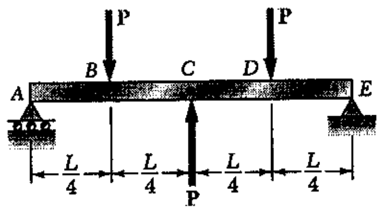
(b) Deflection at C

$y_C = -t_{A/C} = -A\left(\frac{L}{3}\right) = -\left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{L}{3}\right)$

$y_C = \frac{1}{48}\frac{PL^3}{EI}$

PROBLEM 9.112

9.111 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



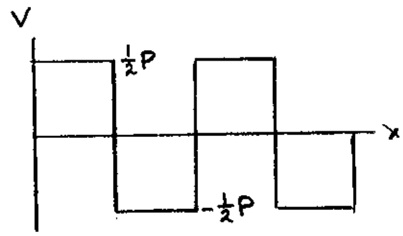
SOLUTION

Symmetrical beam and loading.

Place reference tangent at C. $\theta_C = 0$

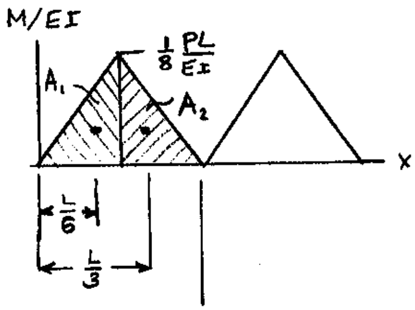
Reactions $R_A = R_E = \frac{1}{2}P$

Draw V (shear) and M/EI diagrams.



$$A_1 = A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\frac{L}{4} = \frac{1}{64}\frac{PL^2}{EI}$$

(a) Slope at A



$$\theta_A = \theta_C - \theta_{A/C} = 0 - A_1 - A_2$$

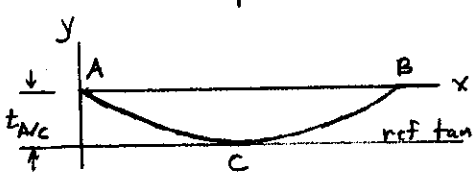
$$= -\frac{1}{32}\frac{PL^2}{EI}$$

(b) Deflection at C

$$y_C = -t_{A/C} = -\left(A_1 \frac{L}{6} + A_2 \frac{L}{3}\right)$$

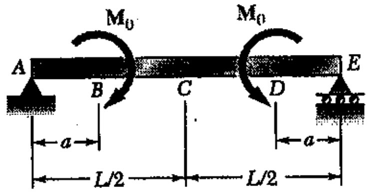
$$= -\left(\frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{6} + \frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{3}\right)$$

$$= -\frac{1}{128}\frac{PL^3}{EI}$$



PROBLEM 9.113

9.111 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

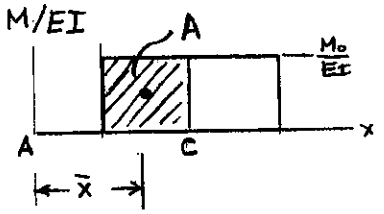


SOLUTION

Symmetrical beam and loading.

Place reference tangent at C. $\theta_C = 0$.

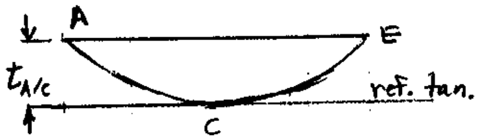
Draw $\frac{M}{EI}$ diagram.



(a) Slope at A $\qquad \theta_A$

$$A = \frac{M_0}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{2}\frac{M_0}{EI}(L - 2a)$$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - A = -\frac{1}{2}\frac{M_0}{EI}(L - 2a)$$



(b) Deflection at C

$$\bar{x} = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

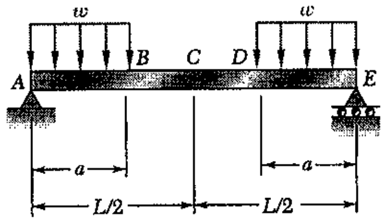
$$y_C = -t_{C/A} = A\bar{x}$$

$$= -\frac{1}{2}\frac{M_0}{EI}(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2)$$

PROBLEM 9.114

9.111 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



SOLUTION

Symmetric beam and loading.

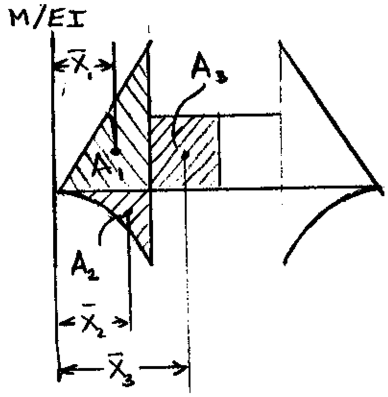
Place reference tangent at C. $\theta_C = 0$

Reactions $R_A = R_E = wa$

Bending moment

Over AB $\quad M = wax - \frac{1}{2}wa^2$

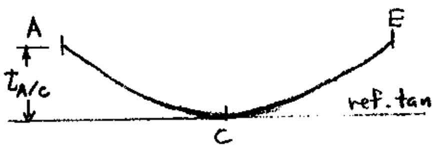
Over BD $\quad M = \frac{1}{2}wa^2$



Draw $\frac{M}{EI}$ diagram by parts

$$\frac{M_1}{EI} = \frac{wa^2}{EI} \qquad \frac{M_2}{EI} = -\frac{1}{2}\frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2}\frac{wa^2}{EI}$$



$$A_1 = \frac{1}{2}\frac{M_1}{EI}a = \frac{1}{2}\frac{wa^3}{EI}$$

$$A_2 = \frac{1}{3}\frac{M_2}{EI}a = -\frac{1}{6}\frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

(a) Slope at A. $\quad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$$= -\frac{1}{2}\frac{wa^3}{EI} + \frac{1}{6}\frac{wa^3}{EI} - \frac{1}{4}\frac{wa^2}{EI}(L - 2a) = -\frac{wa^2}{EI}\left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12}\frac{wa^2}{EI}(3L - 2a)$$

(b) Deflection at C $\qquad y_C = -t_{C/A}$

$$\bar{x}_1 = \frac{2}{3}a, \qquad \bar{x}_2 = \frac{3}{4}a, \qquad \bar{x}_3 = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_C = -t_{C/A} = -A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3$$

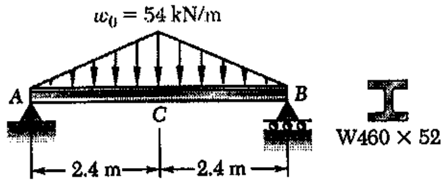
$$= -\left(\frac{1}{2}\frac{wa^3}{EI}\right)\left(\frac{2}{3}a\right) + \left(\frac{1}{6}\frac{wa^3}{EI}\right)\left(\frac{3}{4}a\right) - \frac{1}{4}\left(\frac{wa^2}{EI}\right)(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{3}\frac{wa^3}{EI} + \frac{1}{8}\frac{wa^3}{EI} - \frac{1}{16}\frac{wa^2}{EI}(L^2 - 4a^2)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48}\frac{wa^2}{EI}(3L^2 - 2a^2)$$

PROBLEM 9.115

9.115 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.



SOLUTION

Symmetric beam and loading. $R_A = R_B$

Place reference tangent at C. $\theta_C = 0$

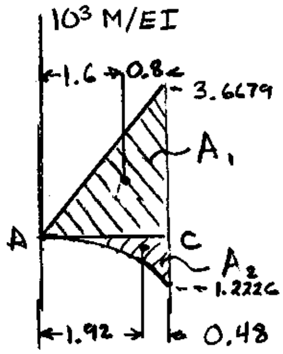
Units: Force in kN, lengths in m.

$E = 200 \times 10^9$ Pa

$I = 212 \times 10^6$ mm^4 $= 212 \times 10^{-6}$ m^4

$EI = (200\times10^9)(212\times10^{-6}) = 42.4 \times 10^6$ N·m^2

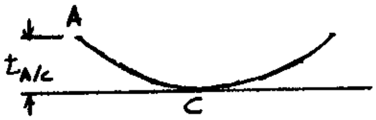
$= 42400$ kN·m^2



$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - \frac{1}{2}(54)(4.8) = 0$

$R_A = 64.8$ kN

$k = \frac{54}{2.4} = 22.5$ kN/m^2



For A to C $\qquad M = R_A x - \frac{1}{6} k x^3$

At C

$$\frac{M}{EI} = \frac{(64.8)(2.4)}{42400} - \frac{(22.5)(2.4)^3}{(6)(42400)}$$

$$= 3.6679\times10^{-3} - 1.2226\times10^{-3} \quad \text{m}^{-1}$$

Draw $\frac{M}{EI}$ diagram by parts.

$A_1 = \frac{1}{2}(3.6679\times10^{-3})(2.4) = 4.4015\times10^{-3}$

$A_2 = -\frac{1}{4}(1.2226\times10^{-3})(2.4) = -0.73356\times10^{-3}$

(a) Slope at A $\qquad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2)$

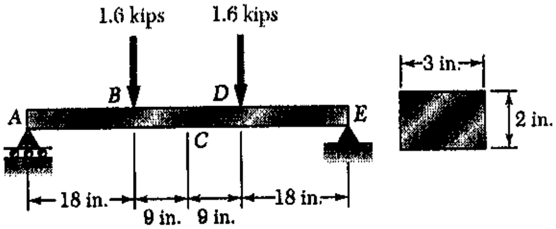
$= -4.4015\times10^{-3} + 0.73356\times10^{-3} = -3.67\times10^{-3}$ rad. ◄

(b) Deflection at C $\qquad y_C = -t_{A/C} = -[(4.4015\times10^{-3})(1.6) - (0.73356\times10^{-3})(1.92)]$

$= -5.63\times10^{-3}$ m $= 5.63$ mm ↓ ◄

PROBLEM 9.116

9.116 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C* of the beam. Use $E = 29 \times 10^6$ psi.



SOLUTION

$I = \frac{1}{12}(3)(2)^3 = 2.0 \text{ in}^4$

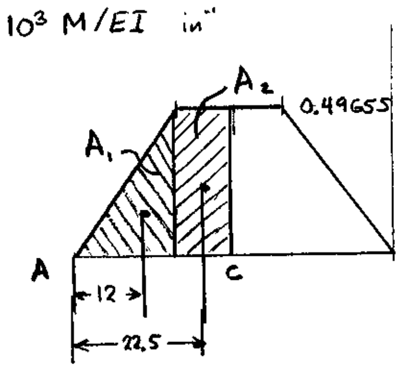
$E = 29 \times 10^3$ ksi

$EI = (29 \times 10^3)(2.0) = 58 \times 10^3 \text{ kip} \cdot \text{in}^2$

Symmetric beam and loading.

$R_A = R_E = 1.6$ kips

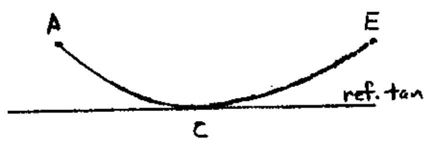
Draw $\frac{M}{EI}$ diagram.



$$\frac{M_{max}}{EI} = \frac{(1.6)(18)}{58 \times 10^3} = 0.49655 \times 10^{-3} \text{ in}^{-1}$$

$$A_1 = \tfrac{1}{2}(0.49655 \times 10^{-3})(18) = 4.469 \times 10^{-3}$$

$$A_2 = (0.49655 \times 10^{-3})(9) = 4.469 \times 10^{-3}$$



Place reference tangent at C. $\theta_C = 0$

(a) Slope at A $\quad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2) = -8.94 \times 10^{-3}$ rad ◄

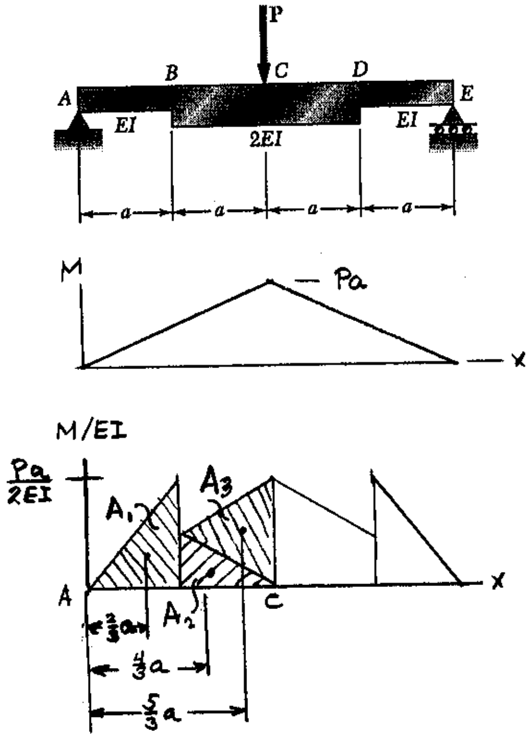
(b) Deflection at C $\quad |y_C| = t_{A/C} = (4.469 \times 10^{-3})(12) + (4.469 \times 10^{-3})(22.5)$

$= 0.1542$ in ↓ ◄

PROBLEM 9.117

9.117 and 9.118 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C* of the beam.

SOLUTION



Symmetric beam and loading. $R_A = R_E = \frac{1}{2}P$

$M_{max} = (\frac{1}{2}P)(2a) = Pa$

Draw M and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

Place reference tangent at C. $\theta_C = 0$

(a) Slope at A.

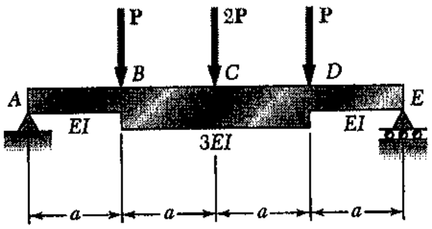
$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$
$$= -\frac{5}{8}\frac{Pa^2}{EI}$$

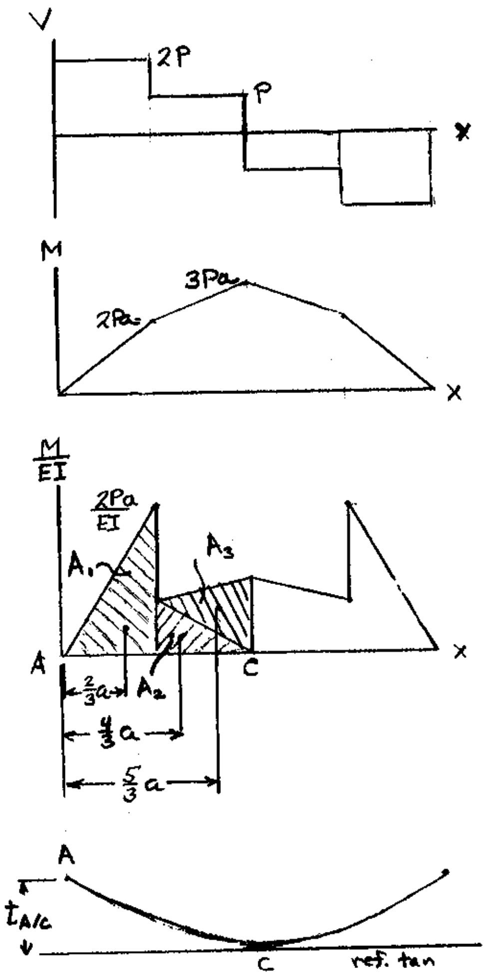
(b) Deflection at C

$$|y_C| = t_{A/C} = A_1\left(\tfrac{2}{3}a\right) + A_2\left(\tfrac{4}{3}a\right) + A_3\left(\tfrac{5}{3}a\right)$$
$$\frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI}$$
$$= \frac{3}{4}\frac{Pa^3}{EI} \downarrow$$

PROBLEM 9.118

9.117 and 9.118 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C* of the beam.





SOLUTION

Symmetric beam and loading. $R_A = R_E = 2P$.

Draw V, M, and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2}\left(\frac{2Pa}{EI}\right)a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{2}{3}\frac{Pa}{EI}\right)a = \frac{1}{3}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{EI}\right)a = \frac{1}{2}\frac{Pa^2}{EI}$$

Place reference tangent at C. $\theta_C = 0$

(a) Slope at A

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6}\frac{Pa^2}{EI}$$

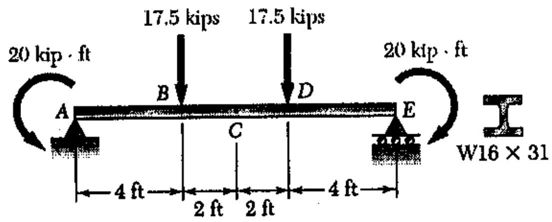
(b) Deflection at C.

$$|y_C| = t_{A/C} = A_1\left(\tfrac{2}{3}a\right) + A_2\left(\tfrac{4}{3}a\right) + A_3\left(\tfrac{5}{3}a\right)$$

$$= \frac{35}{18}\frac{Pa^3}{EI} \downarrow$$

PROBLEM 9.119

9.119 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C* of the beam. Use $E = 29 \times 10^6$ psi.



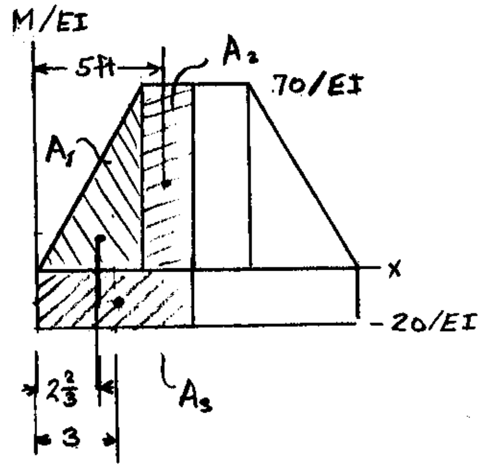
SOLUTION

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = 375$ in^4

$EI = (29\times10^3)(375) = 10.875 \times 10^6$ kip·in^2

$= 75521$ kip·ft^2



Symmetric beam and loading.

$R_A = R_E = 17.5$ kips

Bending moments:

$M_A = -20$ kip·ft

$M_B = -20 + (17.5)(4) = -20 + 70$ kip·ft

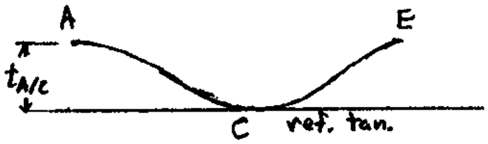
$M_C = -20 + 70$ kip·ft.

Draw M/EI diagram by parts.

$A_1 = \frac{1}{2}(70)(4) = 140/EI$

$A_2 = (70)(2) = 140/EI$

$A_3 = -(20)(6) = -120/EI$



Place reference tangent at C. $\theta_C = 0$

(a) Slope at A. $\theta_A = \theta_C - \theta_{C/A}$

$\theta_A = 0 - (A_1 + A_2 + A_3) = -160/EI$

$= -\frac{160}{75521} = -2.119 \times 10^{-3}$ rad.

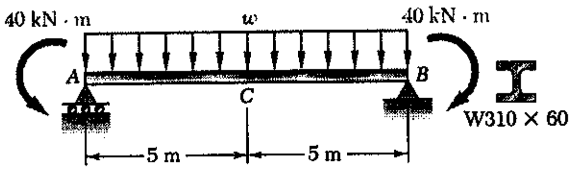
(b) Deflection at C $|y_C| = t_{A/C}$

$$|y_C| = \frac{1}{EI}\left\{(140)(2\tfrac{2}{3}) + (140)(5) - (120)(3)\right\} = \frac{713\frac{1}{3}}{EI}$$

$$= \frac{713\frac{1}{3}}{75521} = 9.445 \times 10^{-3} \text{ ft} = 0.1133 \text{ in.} \downarrow$$

PROBLEM 9.120

9.120 For the beam and loading shown and knowing that $w = 8$ kN/m, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C* of the beam. Use $E = 200$ GPa.



SOLUTION

$E = 200 \times 10^9$ Pa

$I = 129 \times 10^6$ mm^4 $= 129 \times 10^{-6}$ m^4

$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6$ N·m^2

$= 25800$ kN·m^2

Symmetrical beam and loading.

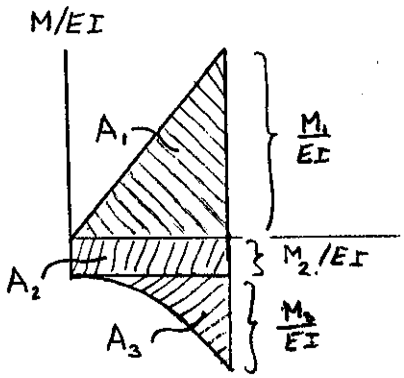
$R_A = R_B = \frac{1}{2}(8)(10) = 40$ kN

Bending moment

$M = 40x - 40 - \frac{1}{2}(8)x^2$

At $x = 5$

$M = 200 - 40 - 100$

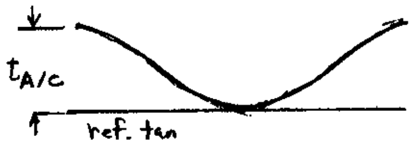


Draw $\frac{M}{EI}$ diagram by parts.

$\frac{M_1}{EI} = \frac{200}{25800} = 7.7519 \times 10^{-3}$ m^{-1}

$\frac{M_2}{EI} = \frac{40}{25800} = -1.5504 \times 10^{-3}$ m^{-1}

$\frac{M_3}{EI} = \frac{100}{25800} = -3.8760 \times 10^{-3}$ m^{-1}



$A_1 = \frac{1}{2}(7.7519 \times 10^{-3})(5) = 19.380 \times 10^{-3}$ $\qquad \bar{x}_1 = (\frac{2}{3})(5) = 3.3333$ m

$A_2 = -(1.5504)(5) = -7.7520 \times 10^{-3}$ $\qquad \bar{x}_2 = (\frac{1}{2})(5) = 2.5$ m

$A_3 = -\frac{1}{3}(3.8760)(5) = -6.4600 \times 10^{-3}$ $\qquad \bar{x}_3 = (\frac{3}{4})(5) = 3.75$ m

Place reference tangent at C. $\quad \theta_C = 0$

(a) Slope at A. $\quad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$\theta_A = -(19.380 \times 10^{-3} - 7.7520 \times 10^{-3} - 6.4600 \times 10^{-3}) = -5.17 \times 10^{-3}$ rad ◄

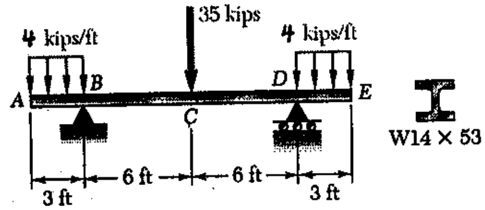
(b) Deflection at C $\quad |y_C| = t_{A/C}$

$= (19.380 \times 10^{-3})(3.3333) - (7.7520 \times 10^{-3})(2.5) - (6.4600 \times 10^{-3})(3.75)$

$= 21.0 \times 10^{-3}$ m $\quad = 21.0$ mm ↓ ◄

PROBLEM 9.121

9.121 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *A*. Use $E = 29 \times 10^6$ psi.

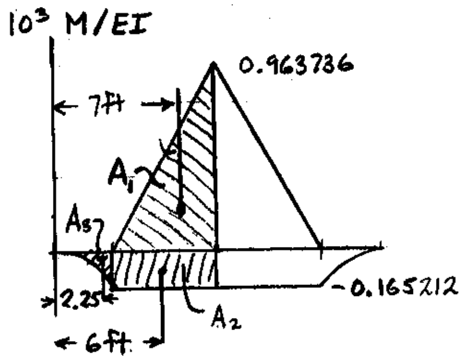


SOLUTION

$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

$I = 541 \text{ in}^4$

$EI = (29\times10^3)(541) = 15.689 \times 10^6 \text{ ksi}$
$= 108951 \text{ kip}\cdot\text{ft}^2$



Draw bending diagram by parts

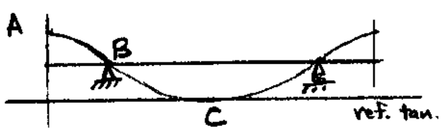
$$\frac{M_1}{EI} = \frac{(\frac{1}{2})(35)(6)}{108951} = 0.963736 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = (\tfrac{1}{2})(0.963736\times10^{-3})(6) = 2.8912\times10^{-3}$$

$$\frac{M_2}{EI} = -\frac{(4)(3)(1.5)}{108951} = -0.165212 \times 10^{-3} \text{ ft}^{-1}$$

$$A_2 = -(0.165212\times10^{-3})(6) = -0.99127\times10^{-3}$$

$$A_3 = -\tfrac{1}{3}(0.165212\times10^{-3})(3) = -0.165212\times10^{-3}$$



Place reference at symmetry point C.

(a) $\theta_C = \theta_A + \theta_{C/A} = 0$

$$\theta_A = -\theta_{C/A} = -A_1 - A_2 - A_3$$
$$= -2.8912\times10^{-3} + 0.99127\times10^{-3} + 0.165212\times10^{-3} = -1.735\times10^{-3} \text{ rad}$$

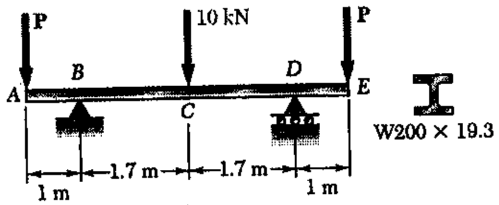
(b) $t_{A/C} = (7)(2.8912\times10^{-3}) + (6)(-0.99127\times10^{-3}) + (2.25)(-0.165212)$
$= 13.919 \times 10^{-3}$ ft

$t_{B/C} = (4)(2.8912\times10^{-3}) + (3)(-0.99127\times10^{-3})$
$= 8.591 \times 10^{-3}$ ft

$y_A = t_{A/C} - t_{B/C} = 5.328\times10^{-3} \text{ ft} = 0.0639 \text{ in.} \uparrow$

PROBLEM 9.122

9.122 Knowing that $P = 8$ kN, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.



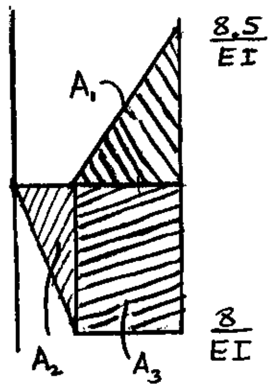
SOLUTION

$E = 200 \times 10^9$ Pa

$I = 16.6 \times 10^6 \text{ mm}^4 = 16.6 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(16.6 \times 10^{-6}) = 3.32 \times 10^6 \text{ N} \cdot \text{m}^2$

$= 3320 \text{ kN} \cdot \text{m}^2$



Symmetric beam and loading

$R_A = R_B = P + 5 = 8 + 5 = 13$ kN.

Bending moment

Over AB $\quad M = -Px = -8x$

Over BC $\quad M = -8x + 13(x - 1)$

$= 5(x-1) - 8$

Draw $\frac{M}{EI}$ diagram by parts

$A_1 = \frac{1}{2}\left(\frac{8.5}{EI}\right)(1.7) = \frac{7.225}{EI}$

$A_2 = -\frac{1}{2}\left(\frac{8}{EI}\right)(1) = -\frac{4}{EI}$

$A_3 = -\left(\frac{8}{EI}\right)(1.7) = -\frac{13.600}{EI}$

Place reference tangent at C $\qquad \theta_C = 0$

(a) Slope at A. $\quad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$\theta_A = -\left(\frac{7.225}{EI} - \frac{4}{EI} - \frac{13.600}{EI}\right) = \frac{10.375}{EI} = \frac{10.375}{3320} = 3.125 \times 10^{-3}$ rad ◄

(b) Deflection at C $\quad y_C = -t_{B/C}$

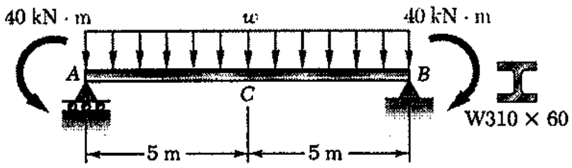
$= -(A_1\bar{x}_1 + A_3\bar{x}_3)$

$= -\left[\left(\frac{7.225}{EI}\right)\left(\frac{2}{3}(1.7)\right) - \left(\frac{13.600}{EI}\right)\left(\frac{1.7}{2}\right)\right] = \frac{3.3717}{EI} = \frac{3.3717}{3320}$

$= 1.016 \times 10^{-3}$ m $= 1.016$ mm ◄

PROBLEM 9.123

9.123 For the beam and loading of Prob. 9.120, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.



SOLUTION

Symmetric beam and loading.

$R_A = R_B = 5w \quad$ (w in kN/m)

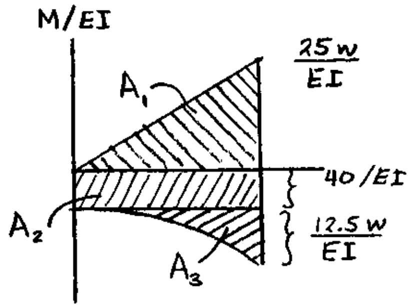
Bending moment in kN·m.

$M = 5wx - 40 - \frac{1}{2}wx^2$

At $x = 5$ m

$M = 25w - 40 - 12.5w$

Draw $\frac{M}{EI}$ diagram by parts.



$A_1 = \frac{1}{2}\left(\frac{25w}{EI}\right)(5) = \frac{62.5w}{EI}$

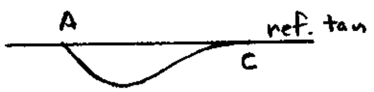
$A_2 = -\frac{(40)(5)}{EI} = -\frac{200}{EI}$

$A_3 = -\frac{1}{3}\left(\frac{12.5w}{EI}\right)(5) = -\frac{20.833w}{EI}$

$\bar{x}_1 = \frac{2}{3}(5) = 3.3333$ m

$\bar{x}_2 = \frac{1}{2}(5) = 2.5$ m

$\bar{x}_3 = \frac{3}{4}(5) = 3.75$ m



Place reference tangent at C.

Deflection at C is zero $\qquad t_{A/C} = y_A - y_C = 0$

$A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = 0$

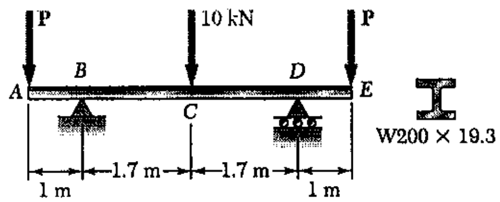
$\left(\frac{62.5w}{EI}\right)(3.3333) - \left(\frac{200}{EI}\right)(2.5) - \left(\frac{20.833w}{EI}\right)(3.75) = 0$

$\frac{130.21w}{EI} - \frac{500}{EI} = 0$

$w = \frac{500}{130.21} = 3.84$ kN/m

PROBLEM 9.124

9.124 **For the beam and loading of Prob. 9.122, determine the magnitude of the forces P for which the deflection is zero at end *A*. Use *E* = 200 GPa.**



SOLUTION

Symmetric beam and loading.

$$R_A = R_B = P + 5 \qquad (P \text{ in kN})$$

Bending moment

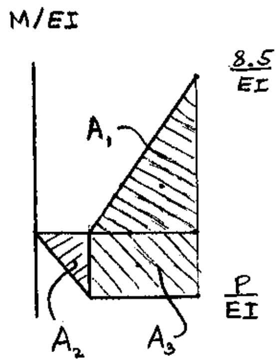
Over AB $\quad M = -Px \qquad$ kN·m

Over BC $\quad M = -Px + (P+5)(x-1)$

$$= 5(x-1) - P(1)$$

At $x = 2.7$ m

$$M = 8.5 - P(1)$$

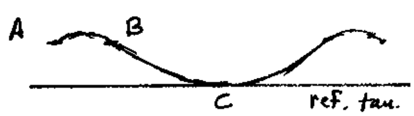


Draw $\frac{M}{EI}$ diagram by parts

$$A_1 = \frac{1}{2}\left(\frac{8.5}{EI}\right)(1.7) = \frac{7.225}{EI}$$

$$A_2 = -\frac{1}{2}\frac{P}{EI}(1) = -\frac{0.5P}{EI}$$

$$A_3 = -\left(\frac{P}{EI}\right)(1.7) = -\frac{1.7P}{EI}$$



Place reference tangent at C

$$y_A = y_B = 0$$

$$y_A - y_B = 0$$

$$t_{A/C} - t_{B/C} = 0$$

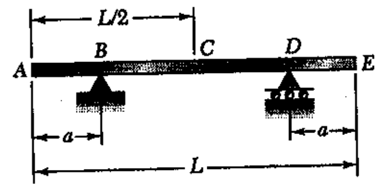
$$A_1\left(1 + \tfrac{2}{3}\cdot 1.7\right) + A_3\left(1 + \tfrac{1}{2}\cdot 1.7\right) + A_3\left(\tfrac{2}{3}\right) - A_1\left(\tfrac{2}{3}\cdot 1.7\right) - A_3\left(\tfrac{1}{2}\cdot 1.7\right) = 0$$

$$A_1(1) + A_3(1) + A_2\left(\tfrac{2}{3}\right) = 0$$

$$\frac{7.225}{EI} - \frac{1.7P}{EI} - \frac{0.33333P}{EI} = 0 \qquad P = \frac{7.225}{2.0333} = 3.55 \text{ kN}$$

PROBLEM 9.125

*9.125 A uniform rod AE is supported at two points B and D. Determine the distance a from the ends of the rod to the points of support if the downward deflections of points A, C, and E are to be equal.



SOLUTION

Let w = weight per unit length of rod.

Symmetrical beam and loading.

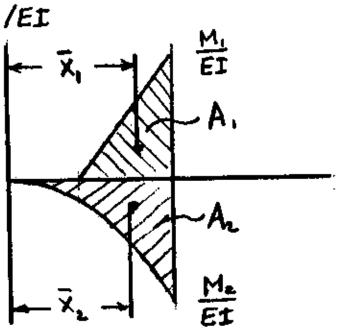
$$R_B = R_D = \tfrac{1}{2}wL$$

Bending moment:

Over AB $\quad M = -\frac{1}{2}wx^2$

Over BCD $\quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$

M/EI



Draw $\frac{M}{EI}$ diagram by parts

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL\left(\frac{L}{2}-a\right)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{w\left(\frac{L}{2}\right)^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3}\left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}\left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_c - y_c = t_{A/c} = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = 0$$

$$\frac{1}{16}\frac{wL(L-2a)^2}{EI}\,\frac{1}{3}(L+a) - \frac{1}{48}\frac{wL^3}{EI}\,\frac{3}{8}L = 0$$

Let $u = a/L$. Divide by $\frac{wL^4}{48\,EI}$

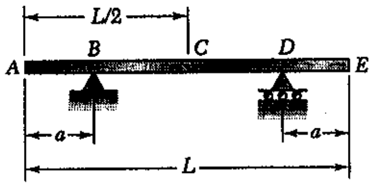
$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for $u = 0.22315$ $\qquad \frac{a}{L} = 0.223 \qquad a = 0.223\,L$

PROBLEM 9.126

*9.126 A uniform rod AE is supported at two points B and D. Determine the distance a for which the slope at ends A and E is to be zero.



SOLUTION

Let w = weight per unit length of rod.

Symmetrical beam and loading.

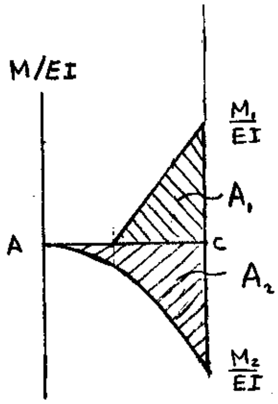
$$R_B = R_D = \frac{1}{2}wL$$

Bending moment

Over AB $\quad M = -\frac{1}{2}wx^2$

Over BCD $\quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$

Draw $\frac{M}{EI}$ diagram by parts.

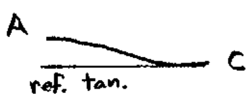


$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\frac{L}{2} = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at C. $\quad \theta_C = 0$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16}\frac{wL(L-2a)^2}{EI} + \frac{1}{48}\frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$

$$1 - 3(1-2u)^2 = 0$$

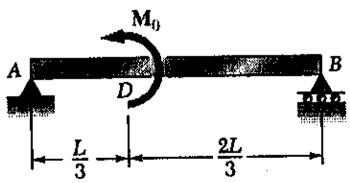
$$1 - 2u = \frac{\sqrt{3}}{3}$$

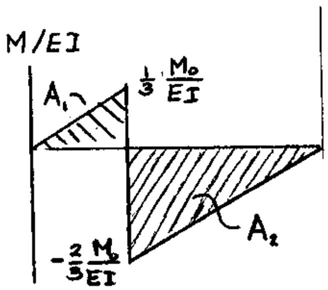
$$u = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right) = 0.21132$$

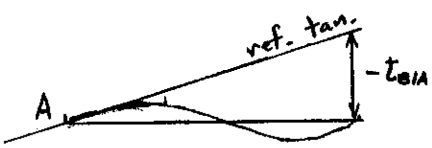
$$\frac{a}{L} = 0.211 \qquad\qquad a = 0.211\,L$$

PROBLEM 9.127

9.127 through 9.130 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.







SOLUTION

Reactions: $R_A = \frac{M_0}{L}\uparrow$, $R_B = \frac{M_0}{L}\downarrow$

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(\frac{1}{3}\frac{M_0}{EI}\right)\frac{L}{3} = \frac{1}{18}\frac{M_0L}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{2}{3}\frac{M_0}{EI}\right)\frac{2L}{3} = -\frac{2}{9}\frac{M_0L}{EI}$$

Place reference tangent at A

$$t_{B/A} = A_1\left(\frac{L}{9}+\frac{2L}{3}\right) + A_2\left(\frac{2}{3}\cdot\frac{2L}{3}\right)$$

$$= \frac{7}{162}\frac{M_0L^2}{EI} - \frac{8}{81}\frac{M_0L^2}{EI} = -\frac{1}{18}\frac{M_0L^2}{EI}$$

$$t_{D/A} = A_1\frac{L}{9} = \frac{1}{162}\frac{M_0L^2}{EI}$$

(a) Deflection at D

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

$$= \frac{1}{162}\frac{M_0L^2}{EI} - \frac{1}{3}\left(-\frac{1}{18}\frac{M_0L^2}{EI}\right)$$

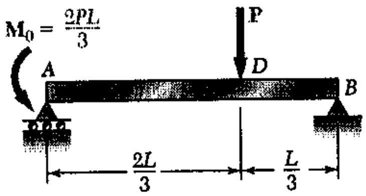
$$= \frac{2}{81}\frac{M_0L^2}{EI}\uparrow$$

(b) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{1}{18}\frac{M_0L}{EI}$$

PROBLEM 9.128

9.127 through 9.130 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

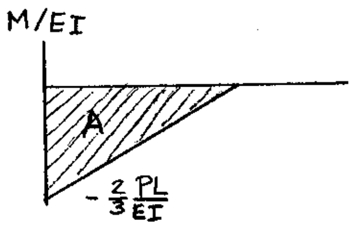


SOLUTION

$$\circlearrowleft \Sigma M_B = 0 \qquad \frac{2PL}{3} - R_A L + P\frac{L}{3} = 0 \qquad R_A = P$$

$$+\circlearrowleft \Sigma M_A = 0 \qquad \frac{2PL}{3} - P\frac{2L}{3} + R_B L = 0 \qquad R_B = 0$$

Draw $\frac{M}{EI}$ diagram. Reference tangent at A.



$$A = -\frac{1}{2}\cdot\left(\frac{2}{3}\frac{PL}{EI}\right)\left(\frac{2L}{3}\right) = -\frac{2}{9}\frac{PL^2}{EI}$$

$$t_{B/A} = \left(-\frac{2}{9}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\frac{2L}{3} + \frac{L}{3}\right) = -\frac{14}{81}\frac{PL^3}{EI}$$

$$t_{D/A} = \left(-\frac{2}{9}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{2L}{3}\right) = -\frac{8}{81}\frac{PL^3}{EI}$$

(a) Deflection at D

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

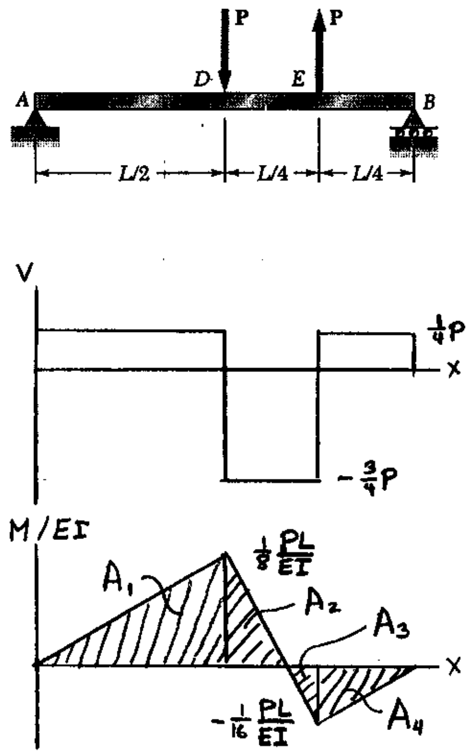
$$= -\frac{8}{81}\frac{PL^3}{EI} + \frac{2}{3}\cdot\frac{14}{81}\frac{PL^3}{EI} = \frac{4}{243}\frac{PL^3}{EI}\uparrow$$

(b) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{14}{81}\frac{PL^2}{EI}$$

PROBLEM 9.129

9.127 through 9.130 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.



SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \qquad R_A = \frac{1}{4}P \uparrow$

$\circlearrowleft \Sigma M_A = 0 \qquad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \qquad R_B = \frac{1}{4}P \downarrow$

Draw V (shear) diagram and $\frac{M}{EI}$ diagram.

$A_1 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$

$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{6}\right) = \frac{1}{96}\frac{PL^2}{EI}$

$A_3 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{12}\right) = -\frac{1}{384}\frac{PL^2}{EI}$

$A_4 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{4}\right) = -\frac{1}{128}\frac{PL^3}{EI}$

Place reference tangent at A.

$$t_{B/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{2L}{3}\right) + \left(\frac{1}{96}\frac{PL^2}{EI}\right)\left(\frac{L}{2} - \frac{1}{3}\cdot\frac{L}{6}\right) + \left(-\frac{1}{384}\frac{PL^2}{EI}\right)\left(\frac{L}{4} + \frac{1}{3}\frac{L}{12}\right) + \left(-\frac{1}{128}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{L}{4}\right)$$

$$= \frac{1}{48}\frac{PL^3}{EI} + \frac{1}{216}\frac{PL^3}{EI} - \frac{5}{6912}\frac{PL^3}{EI} - \frac{1}{768}\frac{PL^3}{EI} = \frac{3}{128}\frac{PL^3}{EI}$$

$$t_{D/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = \frac{1}{192}\frac{PL^3}{EI}$$

(a) Deflection at D.

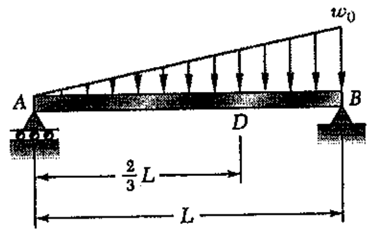
$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A} = \frac{1}{192}\frac{PL^3}{EI} - \frac{1}{2}\left(\frac{3}{128}\frac{PL^3}{EI}\right) = -\frac{5}{768}\frac{PL^3}{EI}$$

(b) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128}\frac{PL^2}{EI}$$

PROBLEM 9.130

9.127 through 9.130 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

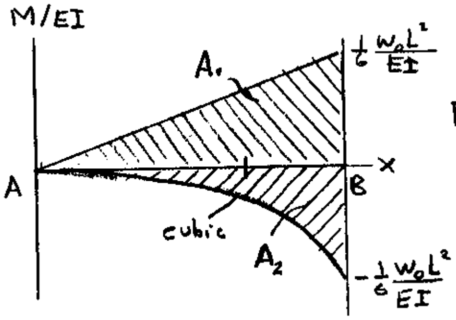


SOLUTION

$+\circlearrowleft M_B = 0 \qquad -R_A L + (\frac{1}{2} w_0 L)(\frac{L}{3}L) = 0 \qquad R_A = \frac{1}{6} w_0 L$

Bending moment $\qquad M = R_A x - \frac{1}{6}\frac{w_0}{L}x^3$

$= \frac{1}{6}\frac{w_0}{L}(L^2 x - x^3)$

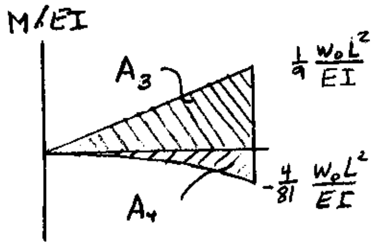


At $x = L \qquad M = \frac{1}{6} w_0 L^2 - \frac{1}{6} w_0 L^2$

Draw $\frac{M}{EI}$ diagram by parts.

$A_1 = \frac{1}{2}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{12}\frac{w_0 L^3}{EI} \qquad \bar{x}_1 = \frac{1}{3}L$

$A_2 = \frac{1}{4}\left(-\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0 L^3}{EI} \qquad \bar{x}_2 = \frac{1}{5}L$



Place reference tangent at A.

$t_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2$

$= \frac{1}{36}\frac{w_0 L^4}{EI} - \frac{1}{120}\frac{w_0 L^4}{EI} = \frac{7}{360}\frac{w_0 L^4}{EI}$

$y_B = L\theta_A + t_{B/A}$

$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7}{360}\frac{wL^3}{EI}$

$A_3 = \frac{1}{2}\left(\frac{1}{9}\frac{w_0 L^2}{EI}\right)\left(\frac{2}{3}L\right) = \frac{1}{27}\frac{w_0 L^3}{EI}$

$A_4 = \frac{1}{4}\left(-\frac{4}{81}\frac{w_0 L^2}{EI}\right)\left(\frac{2}{3}L\right) = -\frac{2}{243}\frac{w_0 L^3}{EI}$

$t_{D/A} = A_3\bar{x}_3 + A_4\bar{x}_4$

$= \left(\frac{1}{27}\frac{w_0 L^3}{EI}\right)\left(\frac{1}{3}\cdot\frac{2}{3}L\right) + \left(-\frac{2}{243}\frac{w_0 L^3}{EI}\right)\left(\frac{1}{5}\cdot\frac{2}{3}L\right) = \frac{2}{243}\frac{w_0 L^4}{EI} - \frac{4}{3645}\frac{w_0 L^4}{EI}$

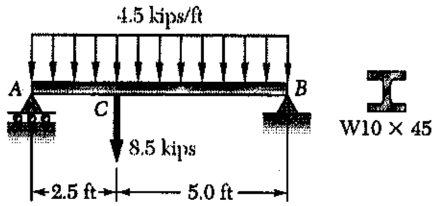
$= \frac{26}{3645}\frac{w_0 L^4}{EI}$

(a) $\quad y_D = t_{D/A} + \frac{2}{3}L\theta_A = \frac{26}{3645}\frac{w_0 L^4}{EI} + \left(\frac{2}{3}L\right)\left(-\frac{7}{360}\frac{w_0 L^3}{EI}\right) = -\frac{17}{2916}\frac{w_0 L^4}{EI}$

(b) $\qquad \theta_A = -\frac{7}{360}\frac{w_0 L^3}{EI}$

PROBLEM 9.131

9.131 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.



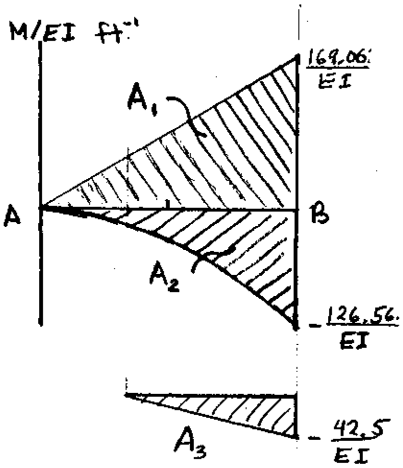
SOLUTION

$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

$I = 248 \text{ in}^4$

$EI = (29 \times 10^3)(248) = 7.192 \times 10^6 \text{ kip}\cdot\text{in}^2$

$= 49944 \text{ kip}\cdot\text{ft}^2$



$+\circlearrowleft \Sigma M_B = 0 \qquad -7.5 R_A + (4.5)(7.5)(\frac{7.5}{2}) + (8.5)(5.0) = 0$

$R_A = 22.542 \text{ kips} \uparrow$

Bending moment

Over AC $\quad M = 22.542x - 2.25x^2 \quad$ kip·ft

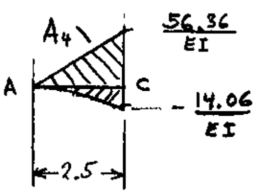
Over CB $\quad M = 22.542x - 2.25x^2 - 8.5(x - 2.5)$

Draw $\frac{M}{EI}$ diagram by parts.

$A_1 = \frac{1}{2}(7.5)(\frac{169.06}{EI}) = \frac{633.98}{EI}$

$A_2 = -\frac{1}{3}(7.5)(\frac{126.56}{EI}) = -\frac{316.40}{EI}$

$A_3 = -\frac{1}{2}(5)(\frac{42.5}{EI}) = -\frac{106.25}{EI}$



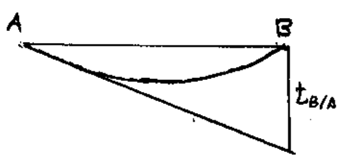
Place reference tangent at A.

$t_{B/A} = A_1(\frac{7.5}{3}) + (A_2)(\frac{7.5}{4}) + A_3(\frac{5}{3}) = \frac{814.62}{EI}$

$A_4 = \frac{1}{2}(\frac{56.36}{EI})(2.5) = \frac{70.44}{EI}$

$A_5 = -\frac{1}{3}(\frac{14.06}{EI})(2.5) = -\frac{11.72}{EI}$

$t_{C/A} = A_4(\frac{2.5}{3}) + A_5(\frac{2.5}{4}) = \frac{51.375}{EI}$



(a) Slope at A

$\theta_A = -\frac{t_{B/A}}{L} = -\frac{814.62}{7.5\,EI} = -\frac{108.62}{EI}$

$= -\frac{108.62}{49944} = -2.17 \times 10^{-3} \text{ rad}$ ◄

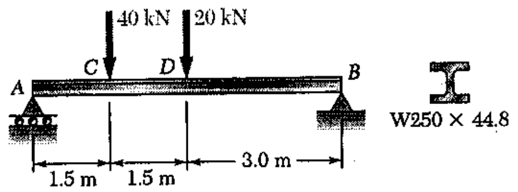
(b) Deflection at C $\qquad y_C = t_{C/A} - \frac{x_C}{L} t_{B/A}$

$y_C = \frac{51.375}{EI} - (\frac{2.5}{7.5})\frac{814.62}{EI} = -\frac{220.16}{EI} = -\frac{220.16}{49944} = -4.41 \times 10^{-3} \text{ ft}$

$= 0.0529 \text{ in.} \downarrow$ ◄

PROBLEM 9.132

9.132 and 9.133 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.



SOLUTION

$E = 200 \times 10^9$ Pa

$I = 71.1 \times 10^6 \ mm^4 = 71.1 \times 10^{-6} \ m^4$

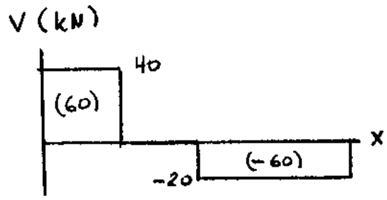
$EI = (200 \times 10^9)(71.1 \times 10^{-6}) = 14.22 \times 10^6 \ N \cdot m^2$

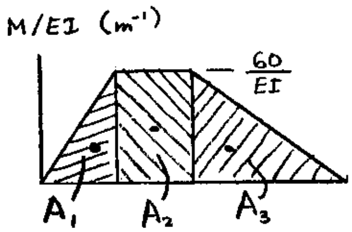
$= 14220 \ kN \cdot m^2$

$+\circlearrowleft \Sigma M_B = 0 \quad -6 R_A + (4.5)(40) + (3)(20) = 0$

$R_A = 40$ kN.

Draw shear and $\frac{M}{EI}$ diagrams.





$A_1 = \frac{1}{2}\left(\frac{60}{EI}\right)(1.5) = \frac{45}{EI}$

$A_2 = \left(\frac{60}{EI}\right)(1.5) = \frac{90}{EI}$

$A_3 = \frac{1}{2}\left(\frac{60}{EI}\right)(3) = \frac{90}{EI}$

Place reference tangent at A.

$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) + A_3(2.0) = \frac{742.5}{EI}$ m.

$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75) = \frac{157.5}{EI}$ m

(a) Slope at A $\quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14220}$

$= -8.70 \times 10^{-3}$ rad.

(b) Deflection at D.

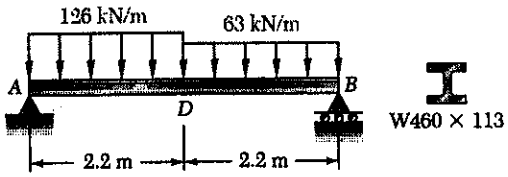
$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6}\right)\left(\frac{742.5}{EI}\right) = -\frac{213.75}{EI}$

$= -\frac{213.75}{14220} = -15.03 \times 10^{-3}$ m

$= 15.03$ mm $\downarrow$

PROBLEM 9.133

9.132 and 9.133 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*. Use *E* = 200 GPa.



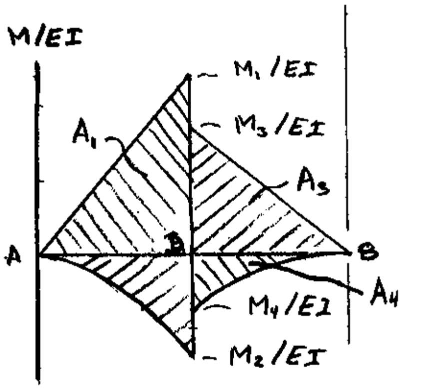
SOLUTION

$E = 200 \times 10^9$ Pa

$I = 556 \times 10^6\ \text{mm}^4 = 556 \times 10^{-6}\ \text{m}^4$

$EI = (200\times10^9)(556\times10^{-6}) = 111.2 \times 10^6\ \text{N}\cdot\text{m}^2$

$= 111200\ \text{kN}\cdot\text{m}^2$



$+\circlearrowleft \Sigma M_B = 0$

$-4.4 R_A + (126)(2.2)(3.3) + (63)(2.2)(1.1) = 0$

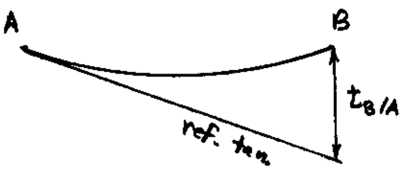
$R_A = 242.55\ \text{kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

$-(126)(2.2)(1.1) - (63)(2.2)(3.3) + 4.4 R_B = 0$

$R_B \quad 173.25\ \text{kN} \uparrow$

Draw $\frac{M}{EI}$ diagram by parts.



$\frac{M_1}{EI} = \frac{(242.55)(2.2)}{111200} = 4.7987 \times 10^{-3}\ \text{m}^{-1}$

$\frac{M_2}{EI} = -\frac{1}{2}\frac{(126)(2.2)^2}{111200} = -2.7421 \times 10^{-3}\ \text{m}^{-1}$

$\frac{M_3}{EI} = \frac{(173.25)(2.2)}{111200} = 3.4276 \times 10^{-3}\ \text{m}^{-1}$

$\frac{M_4}{EI} = -\frac{1}{2}\frac{(63)(2.2)^2}{111200} = -1.3710 \times 10^{-3}\ \text{m}^{-1}$

$A_1 = \frac{1}{2}\frac{M_1}{EI}(2.2) = 5.2785 \times 10^{-3}$

$A_2 = \frac{1}{3}\frac{M_2}{EI}(2.2) = -2.0109 \times 10^{-3}$

$A_3 = \frac{1}{2}\frac{M_3}{EI}(2.2) = 3.7704 \times 10^{-3}$

$A_4 = \frac{1}{3}\frac{M_4}{EI}(2.2) = -1.0054 \times 10^{-3}$

Place reference tangent at A

$t_{B/A} = A_1(2.9333) + A_2(2.75) + A_3(1.46667) + A_4(1.65) = 13.824 \times 10^{-3}\ \text{m}$

$t_{D/A} = A_1(0.7333) + A_2(0.55) = 2.7647 \times 10^{-3}\ \text{m}$

(a) Slope at A $\quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{13.824\times10^{-3}}{4.4} = -3.14 \times 10^{-3}\ \text{rad}$ ◄

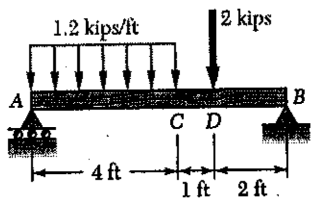
(b) Deflection at D $\quad y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$

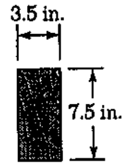
$y_D = 2.7647\times10^{-3} - \left(\frac{2.2}{4.4}\right)(13.824\times10^{-3}) = -4.15 \times 10^{-3}\ \text{m}$

$= 4.15\ \text{mm} \downarrow$ ◄

PROBLEM 9.134

9.134 For the timber beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point D. Use $E = 1.5 \times 10^6$ psi.



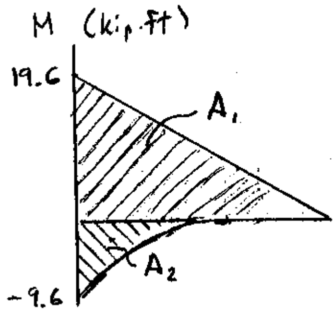


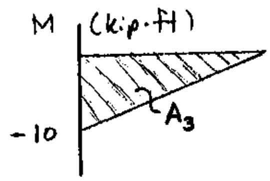
SOLUTION

$$I = \frac{1}{12}(3.5)(7.5)^3 = 123.047 \text{ in}^4$$

$$E = 1.5 \times 10^6 \text{ psi} = 1.5 \times 10^3 \text{ ksi}$$

$$EI = 184.57 \times 10 \text{ kip} \cdot \text{in}^2 = 1281.7 \text{ kip} \cdot \text{ft}^2$$





$$\circlearrowleft \Sigma M_A = 0 \qquad 7R_B - (2)(5) - (1.2)(4)(2) = 0$$

$$R_B = 2.8 \text{ kip}.$$

Draw bending moment diagram by parts.

$$M_1 = (2.8)(7) = 19.6 \text{ kip} \cdot \text{ft}$$

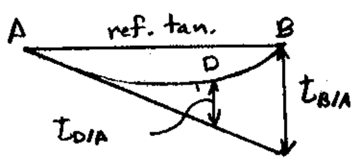
$$M_2 = -(1.2)(4)(2) = -9.6 \text{ kip} \cdot \text{ft}$$

$$M_3 = -(2)(5) = -10 \text{ kip} \cdot \text{ft}$$

$$A_1 = \frac{1}{2}(7)(19.6) = 68.6 \text{ kip} \cdot \text{ft}^2$$

$$A_2 = \frac{1}{3}(4)(-9.6) = -12.8 \text{ kip} \cdot \text{ft}^2$$

$$A_3 = \frac{1}{2}(5)(-10) = -25.0 \text{ kip} \cdot \text{ft}^2$$



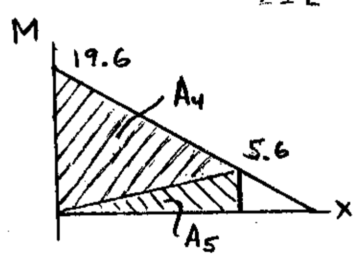
Draw reference tangent at A.

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$$

$$EI\, t_{B/A} = A_1\left(7 - \frac{7}{3}\right) + A_2(7-1) + A_3\left(7 - \frac{5}{3}\right) = 110.0 \text{ kip} \cdot \text{ft}^3$$

(a) $$\theta_A = -\frac{EI\, t_{B/A}}{EIL} = -\frac{110.0}{(1281.7)(7)} = -12.26 \times 10^{-3} \text{ rad}$$



$$A_4 = \frac{1}{2}(19.6)(5) = 49 \text{ kip} \cdot \text{ft}^2$$

$$A_5 = \frac{1}{2}(5.6)(5) = 14 \text{ kip} \cdot \text{ft}^2$$

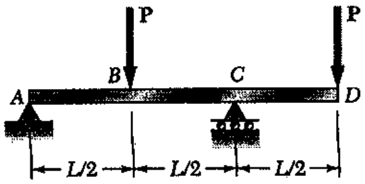
$$EI\, t_{D/A} = A_4\left(5 - \frac{5}{3}\right) + A_5\left(\frac{5}{3}\right) + A_2(5-1) + A_3\left(5 - \frac{5}{3}\right) = 52.133 \text{ kip} \cdot \text{ft}^3$$

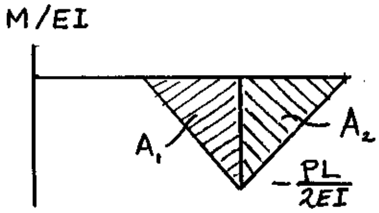
$$EI y_D = 52.133 - \frac{5}{7}(110.0) = -26.438 \text{ kip} \cdot \text{ft}^3$$

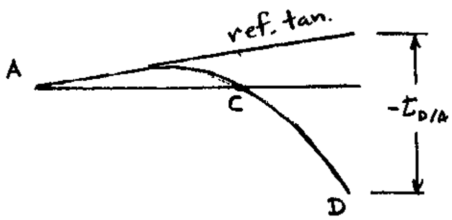
$$y_D = -\frac{26.438}{1281.7} = -20.63 \times 10^{-3} \text{ ft} = 0.248 \text{ in.} \downarrow$$

PROBLEM 9.135

9.135 **and** *9.136* For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.







SOLUTION

$$\circlearrowleft \Sigma M_C = 0 \qquad -R_A L + P\frac{L}{2} - P\frac{L}{2} = 0 \qquad R_A = 0.$$

Draw $\frac{M}{EI}$ diagram

$$A_1 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

Place reference tangent at A

$$t_{C/A} = A_1\cdot\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{1}{48}\frac{PL^3}{EI}$$

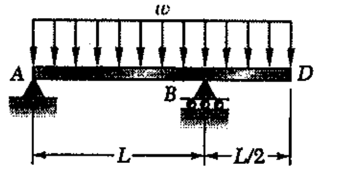
$$\text{(a)} \quad \theta_A = -\frac{t_{C/A}}{L} = \frac{1}{48}\frac{PL^2}{EI}$$

$$t_{D/A} = A_1\left(\frac{L}{2}+\frac{L}{6}\right) + A_2\left(\frac{2}{3}\cdot\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^3}{EI}$$

$$y_D = t_{D/A} - \frac{x_D}{L}t_{C/A} = -\frac{1}{8}\frac{PL^3}{EI} - \left(\frac{3}{2}\right)\left(-\frac{1}{48}\frac{PL^3}{EI}\right) = -\frac{3}{32}\frac{PL^3}{EI}$$

PROBLEM 9.136

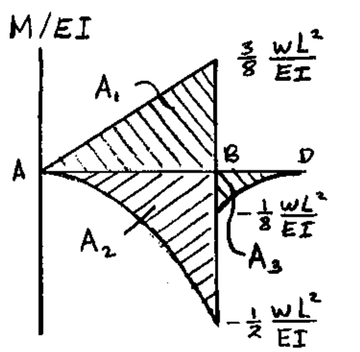
9.135* and *9.136 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.



SOLUTION

$$\circlearrowleft \Sigma M_B = 0 \qquad -R_A L + \left(\tfrac{3}{2}wL\right)\left(\tfrac{1}{4}L\right) = 0 \qquad R_A = \tfrac{3}{8}wL$$

Draw $\frac{M}{EI}$ diagram by parts.



$$A_1 = \tfrac{1}{2}\left(\tfrac{3}{8}\tfrac{wL^2}{EI}\right)L = \tfrac{3}{16}\tfrac{wL^3}{EI}$$

$$A_2 = -\tfrac{1}{3}\left(\tfrac{1}{2}\tfrac{wL^2}{EI}\right)L = -\tfrac{1}{6}\tfrac{wL^3}{EI}$$

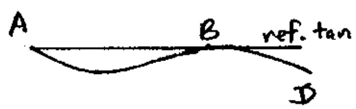
$$A_3 = -\tfrac{1}{3}\left(\tfrac{1}{8}\tfrac{wL^2}{EI}\right)\tfrac{L}{2} = -\tfrac{1}{48}\tfrac{wL^3}{EI}$$

Place reference tangent at A.

$$t_{B/A} = A_1\tfrac{L}{3} + A_2\tfrac{L}{4}$$

$$= \tfrac{1}{16}\tfrac{wL^4}{EI} - \tfrac{1}{24}\tfrac{wL^4}{EI} = \tfrac{1}{48}\tfrac{wL^4}{EI}$$

(a) Slope at A



$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{1}{48}\frac{wL^3}{EI}$$ ◄

$$t_{D/A} = A_1\left(\tfrac{L}{3}+\tfrac{L}{2}\right) + A_2\left(\tfrac{L}{4}+\tfrac{L}{2}\right) + A_3\left(\tfrac{3}{4}\cdot\tfrac{L}{2}\right)$$

$$= \tfrac{5}{32}\tfrac{wL^4}{EI} - \tfrac{1}{8}\tfrac{wL^4}{EI} - \tfrac{1}{128}\tfrac{wL^4}{EI} = \tfrac{3}{128}\tfrac{wL^4}{EI}$$

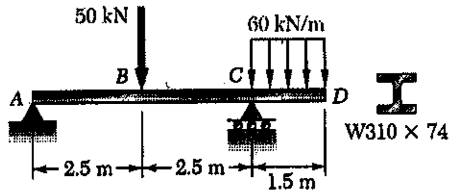
(b) Deflection at D.

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A} = \frac{3}{128}\frac{wL^4}{EI} - \frac{3}{2}\cdot\frac{1}{48}\frac{wL^4}{EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

$$y_D = \frac{1}{128}\frac{wL^4}{EI} \downarrow$$ ◄

PROBLEM 9.137

9.137 For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.



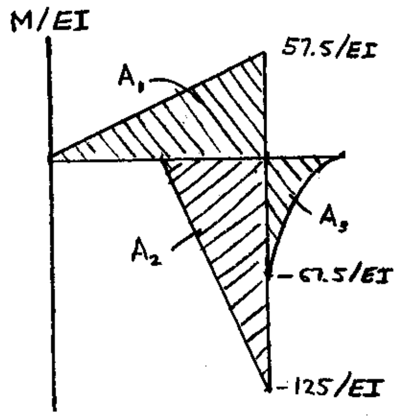
SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33.0 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$= 33000 \text{ kN}\cdot\text{m}^2$$



$$+\circlearrowleft \Sigma M_C = 0 \quad -5R_A + (50)(2.5) - (60)(1.5)(0.75) = 0$$

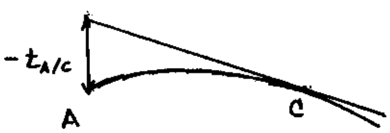
$$R_A = 11.5 \text{ kN}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{57.5}{EI}\right)(5) = \frac{143.75}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{125}{EI}\right)(2.5) = -\frac{156.25}{EI}$$

$$A_3 = -\frac{1}{3}\left(\frac{67.5}{EI}\right)(1.5) = -\frac{33.75}{EI}$$



Place reference tangent at C

$$t_{A/C} = A_1\left(\tfrac{2}{3}\cdot 5\right) + A_2\left(2.5 + \tfrac{2}{3}\cdot 2.5\right)$$

$$= -\frac{171.875}{EI} \text{ m}$$

(a) Slope at C

$$\theta_C = \frac{t_{A/C}}{L} = -\frac{171.875}{5EI} = -\frac{34.375}{EI}$$

$$= -\frac{34.375}{33000} = -1.042 \times 10^{-3} \text{ rad}$$

$$t_{D/C} = A_3\left(\tfrac{3}{4}\cdot 1.5\right) = -\frac{37.96875}{EI} \text{ m.}$$

(b) Deflection at D

$$y_D = \theta_C x_{D/C} + t_{D/C}$$

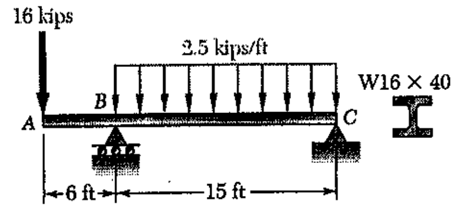
$$= -\left(\frac{34.375}{EI}\right)(1.5) - \frac{37.96875}{EI} = -\frac{89.53}{EI}$$

$$= -\frac{89.53}{33000} = -2.71 \times 10^{-3} \text{ m}$$

$$= 2.71 \text{ mm} \downarrow$$

PROBLEM 9.138

9.138 For the beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *A*. Use $E = 29 \times 10^6$ psi.



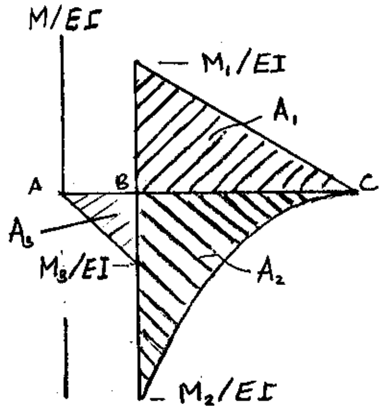
SOLUTION

$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ kips}$

$I = 518 \text{ in}^4$

$EI = (29 \times 10^3)(518) = 15.022 \times 10^6 \text{ kip} \cdot \text{in}^2$

$= 104319 \text{ kip} \cdot \text{ft}^2$



$+\circlearrowleft \Sigma M_B = 0 \qquad (16)(6) - (2.5)(15)(7.5) + 15 R_c = 0$

$R_c = 12.35 \text{ kN}$

Draw $\frac{M}{EI}$ diagram by parts

$M_1 = (12.35)(15) = 185.25 \text{ kip} \cdot \text{ft}$

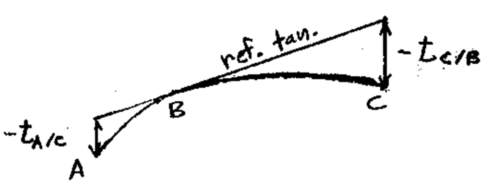
$M_2 = -\frac{1}{2}(2.5)(15)^2 = -281.25 \text{ kip} \cdot \text{ft}$

$M_3 = -(16)(6) = -96 \text{ kip} \cdot \text{ft}$

$A_1 = \frac{1}{2}(185.25)(15)/EI = 1389.375/EI$

$A_2 = -\frac{1}{3}(281.25)(15)/EI = -1406.25/EI$

$A_3 = -\frac{1}{2}(96)(6) = -288/EI$



Place reference tangent at B.

$t_{C/B} = A_1\left(\frac{2}{3} \cdot 15\right) + A_2\left(\frac{3}{4} \cdot 15\right)$

$= -1926.5625/EI$

$= -18.468 \times 10^{-3} \text{ ft.}$

(a) Slope at B $\qquad \theta_B = -\frac{t_{C/B}}{L} = \frac{18.468 \times 10^{-3}}{15} = 1.231 \times 10^{-3} \text{ rad}$ ◄

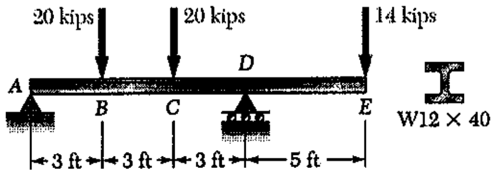
(b) $\quad t_{A/C} = A_3\left(\frac{2}{3} \cdot 6\right) = -1152/EI$

$y_A = t_{A/C} + \frac{x_{AB}}{L} t_{B/C} = -\frac{1152}{EI} - \frac{6}{15}\left(\frac{1926.5625}{EI}\right)$

$= -\frac{1922.6}{EI} = -18.43 \times 10^{-3} \text{ ft} = 0.221 \text{ in.} \downarrow$ ◄

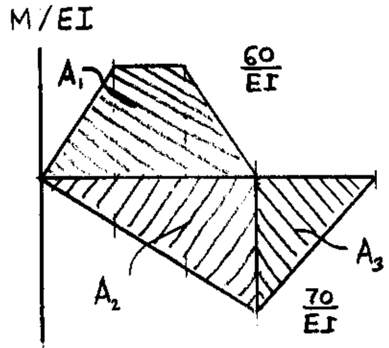
PROBLEM 9.139

9.139 For the beam and loading shown, determine (*a*) the slope at point *D*, (*b*) the deflection at point *E*. Use $E = 29 \times 10^6$ psi.



SOLUTION

Draw bending moment diagram as the sum of two diagrams: one for the pair of 20 kip loads and one for the 14 kip load.



$A_1 = [2 \cdot \frac{1}{2}(3)(60) + (3)(60)]/EI = 360/EI$

$A_2 = \frac{1}{2}(9)(70)/EI = -315/EI$

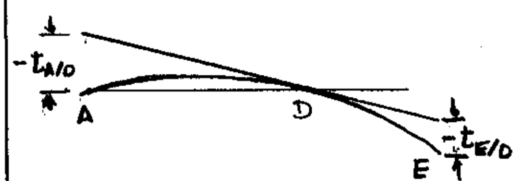
$A_3 = \frac{1}{2}(5)(70) = -175/EI$

$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

$I = 310 \text{ in}^4$

$EI = (29 \times 10^3)(310) = 8.99 \times 10^6 \text{ kip}\cdot\text{in}^2$

$= 62430 \text{ kip}\cdot\text{ft}^2$



Place reference tangent at D

$t_{A/D} = A_1(4.5) + A_2(6) = -270/EI$ ft.

(a) Slope at D $\qquad \theta_D = \frac{t_{A/D}}{L} = -\frac{270}{9EI} = -\frac{30}{EI}$

$= -0.48054 \times 10^{-3}$ rad

$t_{E/D} = A_3(\frac{2}{3} \cdot 5) = -583.333/EI = -9.3438 \times 10^{-3}$ ft

(b) Deflection at E

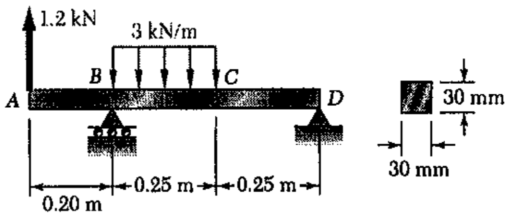
$y_E = L_{DE}\theta_D + t_{E/D}$

$= -(5)(0.48054 \times 10^{-3}) - 9.3438 \times 10^{-3} = -11.75 \times 10^{-3}$ ft

$= 0.1410$ in. ↓

PROBLEM 9.140

9.140 Knowing the beam *AD* is made of a solid steel bar, determine the (*a*) slope at point *B*, (*c*) the deflection at point *A*. Use $E = 200$ GPa.

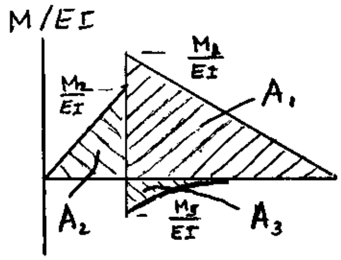


SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \tfrac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$
$$= 67.5 \times 10^{-9} \text{ m}^4$$

$$EI = (200 \times 10^9)(67.5 \times 10^{-9}) = 13500 \text{ N} \cdot \text{m}^2$$
$$= 13.5 \text{ kN} \cdot \text{m}^2$$



$$\circlearrowleft \Sigma M_B = 0 \qquad -(0.2)(1.2) - (3)(0.25)(0.125) + .5 R_D = 0$$

$$R_D = 0.6675 \text{ kN}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375 \text{ kN} \cdot \text{m}$$

$$M_2 = (1.2)(0.2) = 0.240 \text{ kN} \cdot \text{m}$$

$$M_3 = -\tfrac{1}{2}(3)(0.25)^2 = -0.09375 \text{ kN} \cdot \text{m}$$

$$A_1 = \tfrac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$

$$A_2 = \tfrac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$

$$A_3 = \tfrac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at B.

$$t_{D/B} = A_1\left(\tfrac{2}{3} \cdot 0.5\right) + A_3\left(\tfrac{3}{4} \cdot (0.25) + 0.25\right) = 0.024395/EI$$

(a) Slope at B

$$\theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5\, EI} = -\frac{0.048789}{EI}$$

$$= -3.6140 \times 10^{-3} \text{ rad.}$$

$$t_{A/B} = A_2\left(\tfrac{2}{3}(0.20)\right) = 0.0032/EI = 0.23704 \times 10^{-3} \text{ m}$$

(b) Deflection at A

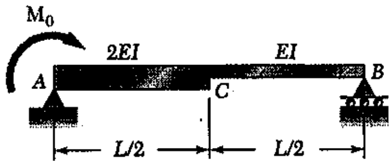
$$y_A = t_{A/B} - L_{AB}\,\theta_B$$

$$= 0.23704 \times 10^{-3} - (0.2)(-3.6140 \times 10^{-3}) = 0.960 \times 10^{-3} \text{ m}$$

$$= 0.960 \text{ mm} \uparrow$$

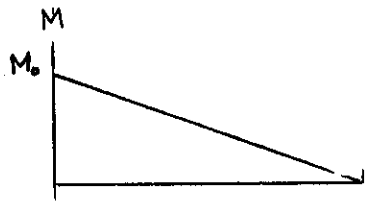
PROBLEM 9.141

9.141 and 9.142 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.



SOLUTION

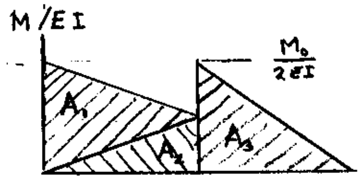
Draw bending moment and $\frac{M}{EI}$ diagrams.



$$A_1 = \tfrac{1}{2}\left(\tfrac{M_0}{2EI}\right)\left(\tfrac{L}{2}\right) = \tfrac{1}{8}\tfrac{M_0 L}{EI}$$

$$A_2 = \tfrac{1}{2}\left(\tfrac{M_0}{4EI}\right)\left(\tfrac{L}{2}\right) = \tfrac{1}{16}\tfrac{M_0 L}{EI}$$

$$A_3 = \tfrac{1}{2}\left(\tfrac{M_0}{2EI}\right)\left(\tfrac{L}{2}\right) = \tfrac{1}{8}\tfrac{M_0 L}{EI}$$



Place reference tangent at A.

$$t_{B/A} = A_1\left(\tfrac{L}{2} + \tfrac{2}{3}\tfrac{L}{2}\right) + A_2\left(\tfrac{L}{2} + \tfrac{1}{3}\tfrac{L}{2}\right) + A_3\left(\tfrac{2}{3}\tfrac{L}{2}\right)$$

$$= \left(\tfrac{1}{8}\tfrac{M_0 L}{EI}\right)\left(\tfrac{5}{6}L\right) + \left(\tfrac{1}{16}\tfrac{M_0 L}{EI}\right)\left(\tfrac{2}{3}L\right) + \left(\tfrac{1}{8}\tfrac{M_0 L}{EI}\right)\left(\tfrac{1}{3}L\right)$$

$$= \tfrac{3}{16}\tfrac{M_0 L^2}{EI}$$

(a) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{16}\frac{ML}{EI}$$

(b) Slope at B

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2 + A_3$$

$$= -\tfrac{3}{16}\tfrac{M_0 L}{EI} + \tfrac{1}{8}\tfrac{M_0 L}{EI} + \tfrac{1}{16}\tfrac{M_0 L}{EI} + \tfrac{1}{8}\tfrac{M_0 L}{EI}$$

$$= \tfrac{1}{8}\tfrac{M_0 L}{EI}$$

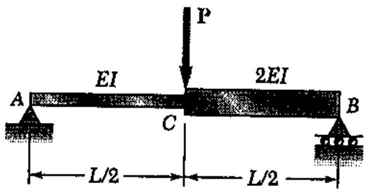
$$t_{C/A} = A_1\left(\tfrac{2}{3}\tfrac{L}{2}\right) + A_2\left(\tfrac{1}{3}\tfrac{L}{2}\right) = \left(\tfrac{1}{8}\tfrac{M_0 L}{EI}\right)\left(\tfrac{1}{3}L\right) + \left(\tfrac{1}{16}\tfrac{M_0 L}{EI}\right)\left(\tfrac{1}{6}L\right) = \tfrac{5}{96}\tfrac{M_0 L^2}{EI}$$

(c) Defection at C

$$y_C = t_{C/A} + \tfrac{L}{2}\theta_A = \tfrac{5}{96}\tfrac{M_0 L^2}{EI} - \tfrac{3}{32}\tfrac{M_0 L^2}{EI} = -\tfrac{1}{24}\tfrac{M_0 L^2}{EI} = \tfrac{1}{24}\tfrac{M_0 L^2}{EI}\downarrow$$

PROBLEM 9.142

9.141 and 9.142 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.

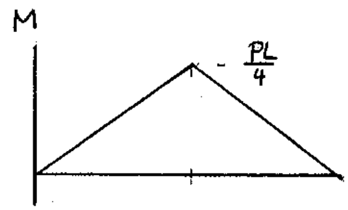


SOLUTION

Draw bending moment and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$$

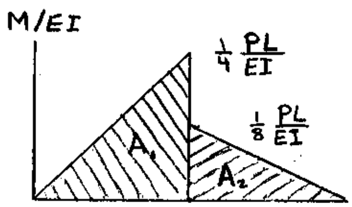
$$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$$



Place reference tangent at A

$$t_{B/A} = A_1\left(\frac{L}{2} + \frac{1}{3}\frac{L}{2}\right) + A_2\left(\frac{2}{3}\frac{L}{2}\right)$$

$$= \left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{2}{3}L\right) + \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{1}{3}L\right) = \frac{5}{96}\frac{PL^3}{EI}$$



(a) Slope at A

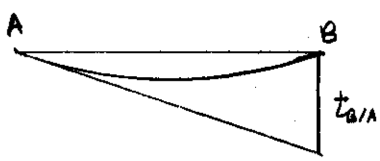
$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{96}\frac{PL^2}{EI}$$

(b) Slope at B

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2$$

$$= -\frac{5}{96}\frac{PL^2}{EI} + \frac{1}{16}\frac{PL^2}{EI} + \frac{1}{32}\frac{PL^2}{EI}$$

$$= \frac{1}{24}\frac{PL^2}{EI}$$



$$t_{C/A} = A_1\left(\frac{1}{3}\frac{L}{2}\right) = \left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{1}{6}L\right) = \frac{1}{96}\frac{PL^3}{EI}$$

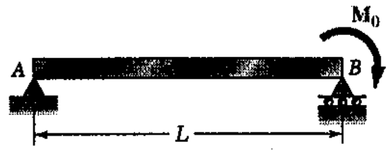
(c) Deflection at C

$$y_A = t_{C/A} - \frac{x_C}{L}t_{B/A} = \frac{1}{96}\frac{PL^3}{EI} - \frac{1}{2}\left(\frac{5}{96}\frac{PL^3}{EI}\right) = -\frac{1}{64}\frac{PL^3}{EI}$$

$$= \frac{1}{64}\frac{PL^3}{EI}\downarrow$$

PROBLEM 9.143

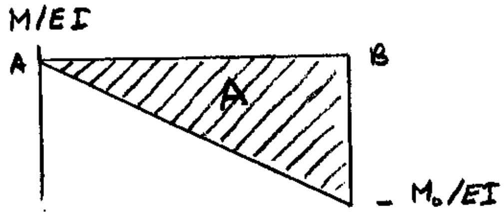
9.143 For the beam and loading shown, determine the magnitude and location of the maximum deflection.



SOLUTION

Draw $\frac{M}{EI}$ diagram

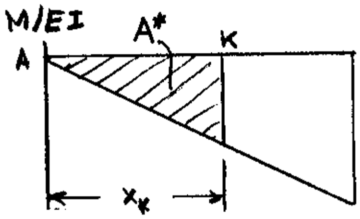
Place reference tangent at A.



$$A = \tfrac{1}{2}\left(-\frac{M_0}{EI}\right)L = -\frac{1}{2}\frac{M_0 L}{EI}$$

$$t_{B/A} = A\left(\frac{L}{3}\right) = -\frac{1}{6}\frac{M_0 L^2}{EI}$$

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{1}{6}\frac{M_0 L}{EI}$$



$$A^* = \tfrac{1}{2}\left(\frac{M_0}{EI}\frac{x_K}{L}\right)x_K = -\frac{1}{2}\frac{M_0 x_K^2}{EIL}$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A^* = 0$$

$$\frac{1}{6}\frac{M_0 L}{EI} - \frac{1}{2}\frac{M_0 x_K^2}{EIL} = 0$$

$$x_K = \frac{\sqrt{3}}{3}L$$

$$t_{K/A} = A^*\left(\tfrac{1}{3}x_K\right)$$

$$= -\frac{1}{2}\frac{M_0 x_K^2}{EIL}\left(\tfrac{1}{3}x_K\right)$$

$$= -\frac{1}{6}\frac{M_0 x_K^3}{EIL}$$

Maximum deflection

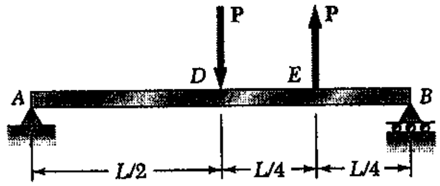
$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = -\frac{1}{6}\frac{M_0 x_K^3}{EIL} - \frac{x_K}{L}\left(\frac{M_0 L^2}{6EI}\right)$$

$$= \frac{M_0 x_K}{6EIL}\left(L^2 - x_K^2\right) = \frac{\sqrt{3}}{18}\frac{M_0}{EI}\left(L^2 - \tfrac{1}{3}L^2\right) = \frac{\sqrt{3}}{27}\frac{M_0 L^2}{EI}\uparrow$$

PROBLEM 9.144

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

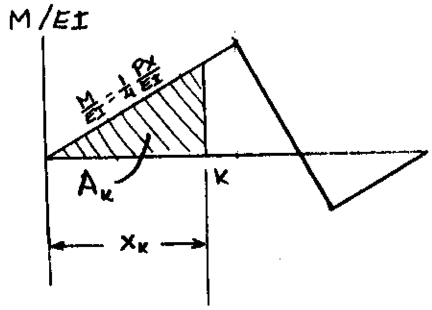
9.144 Beam and loading of Prob. 9.129



SOLUTION

Referring to the solution of Prob. 9.129

$$R_A = \frac{1}{4}P, \quad t_{B/A} = \frac{3}{128}\frac{PL^3}{EI}, \quad \theta_A = -\frac{3}{128}\frac{PL^2}{EI}$$



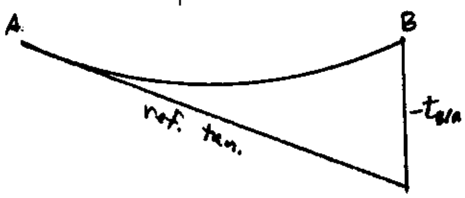
$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{3}{128}\frac{PL^2}{EI} + A_K$$

$$= -\frac{3}{128}\frac{PL^2}{EI} + \frac{1}{2}\left(\frac{1}{4}\frac{Px_K}{EI}\right)x_K$$

$$= \frac{P}{EI}\left(-\frac{3}{128}L^2 + \frac{1}{8}x_K^2\right) = 0$$

$$x_K = \sqrt{\frac{3}{16}}\,L = \frac{1}{4}\sqrt{3}\,L = 0.433\,L \quad ◄$$



$$t_{K/A} = A_K\left(\frac{1}{3}x_K\right) = \frac{1}{2}\left(\frac{1}{4}\frac{Px_K^2}{EI}\right)\frac{x_K}{3}$$

$$= \frac{1}{24}\frac{Px_K^3}{EI} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI}$$

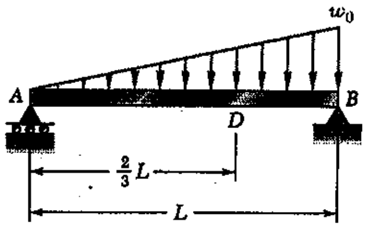
$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI} - \left(\frac{1}{4}\sqrt{3}\right)\frac{3}{128}\frac{PL^3}{EI} = -\frac{\sqrt{3}}{256}\frac{PL^3}{EI}$$

$$= 0.00677\frac{PL^3}{EI}\downarrow \quad ◄$$

PROBLEM 9.145

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.145 Beam and loading of Prob. 9.130

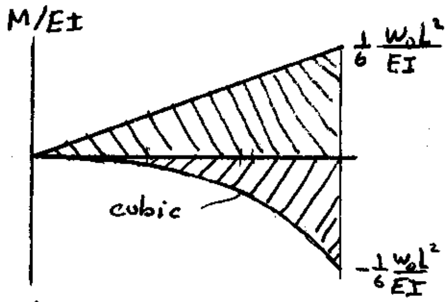


SOLUTION

$+\circlearrowleft M_B = 0 \qquad -R_A L + (\frac{1}{2} w_0 L)(\frac{1}{3} L) = 0 \qquad R_A = \frac{1}{6} w_0 L$

Bending moment $\quad M = R_A x - \frac{1}{6}\frac{w_0}{L} x^3$

$= \frac{1}{6}\frac{w_0}{L}(L^2 x - x^3)$

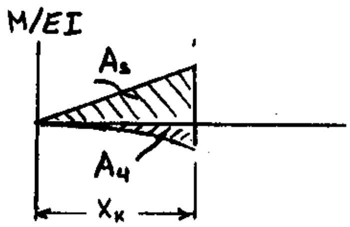


At $x = L \qquad M = \frac{1}{6} w_0 L^2 - \frac{1}{6} w_0 L^2$

Draw $\frac{M}{EI}$ diagram by parts

$A_1 = \frac{1}{2}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{12}\frac{w_0 L^3}{EI} \qquad \bar{x}_1 = \frac{1}{3} L$

$A_2 = \frac{1}{4}\left(-\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0 L^3}{EI} \qquad \bar{x}_2 = \frac{1}{5} L$



Place reference tangent at A.

$t_{B/A} = A_1 \bar{x}_1 + A_2 \bar{x}_2$

$= \frac{1}{36}\frac{w_0 L^4}{EI} - \frac{1}{120}\frac{w L^4}{EI} = \frac{7}{360}\frac{w_0 L^4}{EI}$

Slope at A $\qquad \theta_A = -\frac{t_{B/A}}{L} = -\frac{7}{360}\frac{w_0 L^3}{EI}$

$A_3 = A_1\left(\frac{x_K}{L}\right)^2 = \frac{1}{12}\frac{w_0 L^3}{EI} u^2$

$A_4 = A_2\left(\frac{x_K}{L}\right)^4 = -\frac{1}{24}\frac{w_0 L^3}{EI} u^4$

$\theta_{K/A} = A_3 + A_4 = \frac{w_0 L^3}{EI}\left(\frac{1}{12} u^2 - \frac{1}{24} u^4\right) = -\theta_A = \frac{7}{360}\frac{w_0 L^3}{EI}$

$u^4 - 2u^2 + \frac{7}{15} = 0 \qquad$ Solving for $u \qquad u = 0.51933$

$x_K = 0.51933 L$ ◄

$A_3 = \frac{1}{12}\frac{w_0 L^3}{EI}(0.51933)^2 = 0.0224753 \frac{w_0 L^3}{EI}, \qquad \bar{x}_3 = \frac{1}{3}(0.51933) L$

$A_4 = -\frac{1}{24}\frac{w_0 L^3}{EI}(0.51933)^4 = -0.0030308 \frac{w_0 L^3}{EI}, \qquad \bar{x}_4 = \frac{1}{5}(0.51933) L$

$t_{K/A} = A_3 \bar{x}_3 + A_4 \bar{x}_4 = 0.0035759 \frac{w_0 L^4}{EI}$

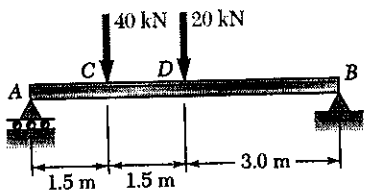
$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A} = 0.0035759 \frac{w_0 L^4}{EI} - (0.51933)\left(\frac{7}{360}\frac{w_0 L^4}{EI}\right)$

$= -0.00652 \frac{w_0 L^4}{EI} = 0.00652 \frac{w_0 L^4}{EI} \downarrow$ ◄

PROBLEM 9.146

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.146 Beam and loading of Prob. 9.132



SOLUTION

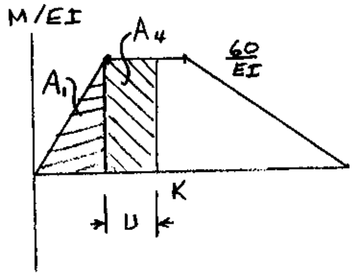
Referring to the solution to Prob. 9.129

$EI = 14220 \text{ kN}\cdot\text{m}^2$

$R_A = 40 \text{ kN}, \quad A_1 = \frac{45}{EI}$

$t_{B/A} = \frac{742.5}{EI} \text{ m}$

$\theta_A = -\frac{123.75}{EI}$



Let K be the location of the maximum deflection. Assume that K lies between C and D.

$$\theta_K = \theta_A + \theta_{K/A}$$
$$= -\frac{123.75}{EI} + A_1 + A_4$$
$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60\,u}{EI} = 0$$
$$u = \frac{123.75-45}{60} = 1.3125 \text{ m.}$$
$$x_K = 1.5 + u = 2.8125 \text{ m}$$

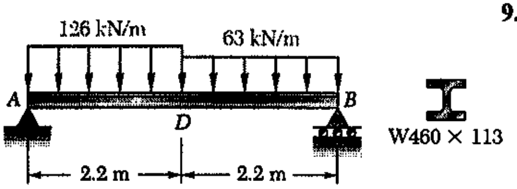
$$t_{K/A} = A_1(u + 0.5) + A_4\left(\tfrac{1}{2}u\right)$$
$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)(\frac{1}{2})(1.3125)}{EI} = \frac{133.242}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L}\,t_{B/A}$$
$$= \frac{133.242}{EI} - \frac{2.8125}{6}\left(\frac{742.5}{EI}\right) = -\frac{214.80}{EI} = -\frac{214.80}{14220}$$
$$= -15.11\times10^{-3} \text{ m} = 1.511 \text{ mm} \downarrow$$

PROBLEM 9.147

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.147 Beam and loading of Prob. 9.133



SOLUTION

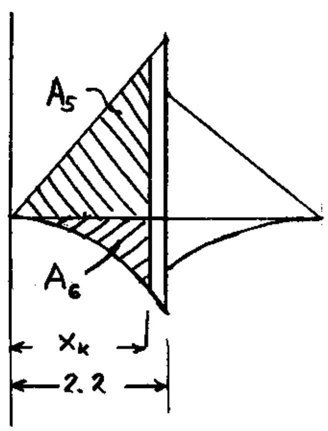
From the solution to Prob. 9.133

$EI = 111200 \text{ kN}\cdot\text{m}^2, \quad R_A = 242.55 \text{ kN}$

$t_{B/A} = 13.824 \times 10^{-3} \text{ m}$

$\theta_A = -\dfrac{t_{B/A}}{L} = -\dfrac{13.824 \times 10^{-3}}{4.4} = -3.1418 \times 10^{-3} \text{ rad.}$

M/EI



Over portion AD of the beam

$M = 242.55x - 63x^2 \text{ kN}\cdot\text{m}$

$\dfrac{M}{EI} = (2.1812x - 0.56655x^2) \times 10^{-3} \text{ m}^{-1}$

$\theta_{K/A} = \displaystyle\int_0^{x_K} \frac{M}{EI}\,dx$

$= (1.0906x_K^2 - 0.188849x_K^3) \times 10^{-3} \text{ rad}$

$\theta_K = \theta_A + \theta_{K/A} = -3.1418 \times 10^{-3} + (1.0906x_K^2 - 0.188849x_K^3) \times 10^{-3} = 0$

Solving for x_K $\quad x_K = 2.13907 \text{ m}$ $\quad$ $x_K = 2.14 \text{ m}$ ◄

$A_5 = 1.0906 \times 10^{-3}\, x_K^2 = 4.99017 \times 10^{-3}, \quad \bar{x}_5 = \frac{1}{3}x_K = 0.71302 \text{ m}$

$A_6 = -0.188849 \times 10^{-3}\, x_K^3 = -1.84837 \times 10^{-3}, \quad \bar{x}_6 = \frac{1}{4}x_K = 0.53477 \text{ m}$

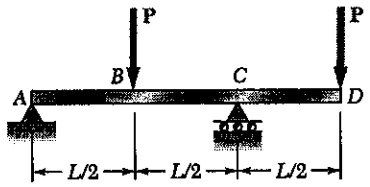
$t_{K/A} = A_5\bar{x}_5 + A_6\bar{x}_6 = 2.5696 \times 10^{-3} \text{ m}$

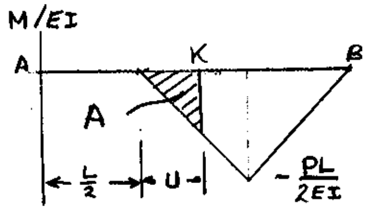
$y_K = t_{K/A} - \dfrac{x_K}{L}t_{B/A}$

$= 2.5696 \times 10^{-3} - \dfrac{2.13907}{4.4}(13.824 \times 10^{-3}) = -4.15 \times 10^{-3} \text{ m}$

$= 4.15 \text{ mm} \downarrow$ ◄

PROBLEM 9.148





9.148 **For the beam and loading of Prob. 9.135, determine the magnitude and location of the largest upward deflection in span** ***AC*****.**

SOLUTION

From solution of Problem 9.135

$$R_A = 0, \qquad \theta_A = \frac{1}{48}\frac{PL^2}{EI}$$

Draw M/EI diagram. Let K be location of maximum deflection.

$$\theta_K = \theta_A + \theta_{K/A} = \frac{1}{48}\frac{PL^2}{EI} + A = 0$$

where $\quad A = \frac{1}{2}\left(-\frac{PL}{2EI}\cdot\frac{2u}{L}\right)u = -\frac{1}{2}\frac{PLu^2}{EI}$

$$\frac{1}{2}\frac{PLu^2}{EI} = \frac{1}{48}\frac{PL^2}{EI}$$

$$u^2 = \frac{1}{24}L^2 \qquad u = 0.20412\,L$$

$$x_K = \frac{L}{2} + u = 0.704\,L$$

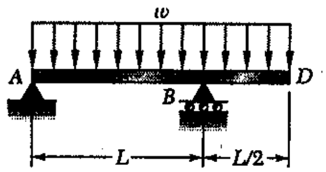
$$A = -\frac{1}{48}\frac{PL^2}{EI}$$

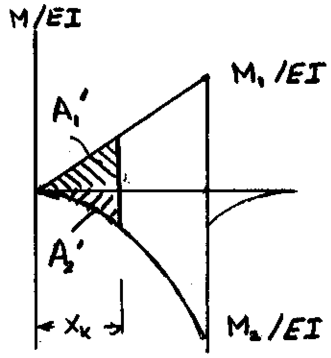
$$t_{A/K} = A\left(\frac{L}{2} + \frac{2}{3}u\right) = \left(-\frac{1}{48}\frac{PL^2}{EI}\right)(0.63608\,L) = -0.01325\frac{PL^3}{EI}$$

$$y_{max} = -t_{A/K} = 0.01325\frac{PL^3}{EI}$$

PROBLEM 9.149

9.149 For the beam and loading of Prob. 9.136, determine the magnitude and location of the largest downward deflection in span *AB*.





SOLUTION

From solution of Prob. 9.146

$$\frac{M_1}{EI} = \frac{3}{8}\frac{wL^2}{EI}, \quad \frac{M_2}{EI} = -\frac{1}{2}\frac{wL^2}{EI}, \quad \theta_A = -\frac{1}{48}\frac{wL^3}{EI}$$

Let K be the location of maximum deflection.

$$u = \frac{x_K}{L}$$

From $\frac{M}{EI}$ diagram

$$A_1' = \frac{1}{2}\left(\frac{M_1}{EI}u\right)(Lu) = \frac{3}{16}\frac{wL^3}{EI}u^2$$

$$A_2' = \frac{1}{3}\left(\frac{M_2}{EI}u^2\right)(Lu) = -\frac{1}{6}\frac{wL^3}{EI}u^3$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1' + A_2'$$

$$= -\frac{1}{48}\frac{wL^3}{EI} + \frac{3}{16}\frac{wL^3}{EI}u^2 - \frac{1}{6}\frac{wL^3}{EI}u^3$$

$$= -\left(\frac{1}{6}u^3 - \frac{3}{16}u^2 + \frac{1}{48}\right)\frac{wL^3}{EI} = 0$$

Multiplying by 48 $\qquad 8u^3 - 9u^2 + 1 = 0$

Solving for u $\qquad u = 0.421535 \qquad\qquad x_K = 0.4215\,L$ ◄

$$t_{A/K} = A_1'\left(\frac{2}{3}x_K\right) + A_2'\left(\frac{3}{4}x_K\right)$$

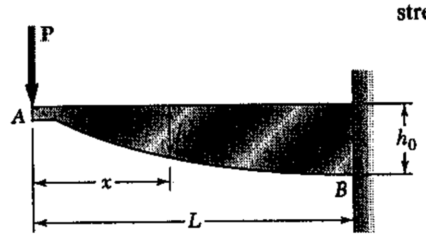
$$= \left(\frac{3}{16}\frac{wL^3}{EI}u^2\right)\left(\frac{2}{3}Lu\right) + \left(-\frac{1}{6}\frac{wL^3}{EI}u^3\right)\left(\frac{3}{4}Lu\right)$$

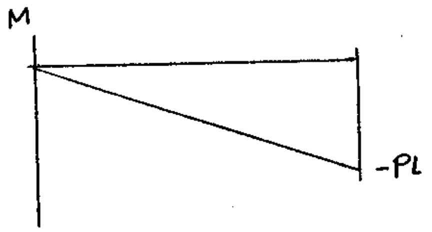
$$= \left(\frac{1}{8}u^3 - \frac{1}{8}u^4\right)\frac{wL^4}{EI} = 0.00542\frac{wL^4}{EI}$$

$$y_{max} = -t_{A/K} = -0.00542\frac{wL^4}{EI} = 0.00542\frac{wL^4}{EI}\downarrow$$ ◄

PROBLEM 9.150

***9.150** The cantilever AB is a beam of constant strength. It has a rectangular cross section of uniform width b and variable depth h. Express the deflection at end A in terms of P, L and the flexural rigidity EI_0 at B. (*Hint:* Since the beam is of constant strength, Mc/I has a constant value along AB.)





SOLUTION

Bending moment $\quad M = -Px$

$$M_o = -PL$$

$$M = M_o \frac{x}{L}$$

For a constant strength beam

$$\frac{Mc}{I} = \frac{(M_o x/L)(h/2)}{\frac{1}{12} b h^3} = \frac{6 M_o x/L}{h^2}$$

$$= \frac{6M_o}{h_o^2}$$

$$\left(\frac{h}{h_o}\right) = \left(\frac{x}{L}\right)^{\frac{1}{2}}$$

Moment of inertia $\quad I = \frac{1}{12} b h^3, \quad I_o = \frac{1}{12} b h_o^3$

$$\frac{I}{I_o} = \left(\frac{h}{h_o}\right)^3 = \left(\frac{x}{L}\right)^{3/2}$$

Curvature $\quad \dfrac{M}{EI} = \dfrac{M_o(x/L)}{EI_o (x/L)^{3/2}} = \dfrac{M_o}{EI_o}\left(\dfrac{L}{x}\right)^{1/2}$

$$= -\frac{PL}{EI_o}\left(\frac{L}{x}\right)^{1/2}$$

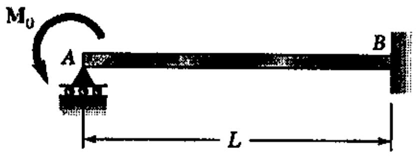
Deflection at A $\quad y_A = t_{A/B} = \int_0^L x \frac{M}{EI}\, dx$

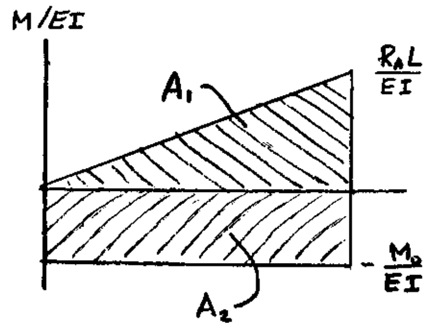
$$= -\frac{PL^{3/2}}{EI_o}\int_0^L x^{\frac{1}{2}}\, dx = -\frac{PL^{3/2}}{EI_o} \left.\frac{x^{3/2}}{3/2}\right|_0^L$$

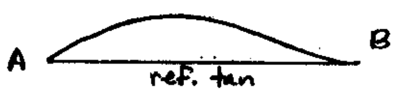
$$= -\frac{2}{3}\frac{PL^3}{EI_o} = \frac{2}{3}\frac{PL^3}{EI_o} \downarrow$$

PROBLEM 9.151

9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.







SOLUTION

Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads M_0 and R_A.

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right)L = \frac{1}{2}\frac{R_A L^2}{EI}$$

$$A_2 = -\left(\frac{M_0}{EI}\right)L = -\frac{M_0 L}{EI}$$

Place reference tangent at B

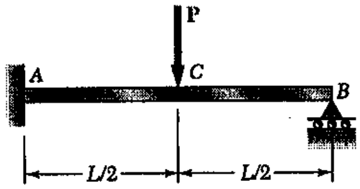
$$t_{B/A} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{1}{2}L\right)$$

$$= \frac{1}{3}\frac{R_A L^2}{EI} - \frac{1}{2}\frac{M_0 L^2}{EI} = 0$$

$$R_A = \frac{3}{2}\frac{M_0}{L} \uparrow$$

PROBLEM 9.152

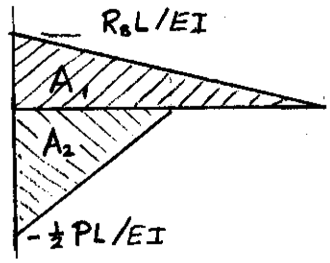
9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support B and treat R_B as redundant.

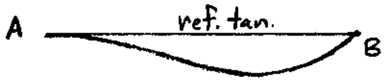
Draw M/EI diagram for loads P and R_B.



$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{1}{2}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

Place reference tangent at A.



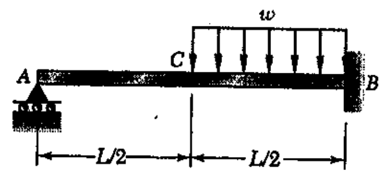
$$t_{B/A} = A_1\left(\tfrac{2}{3}L\right) + A_2\left(\tfrac{L}{2} + \tfrac{2}{3}\tfrac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI} = 0$$

$$R_B = \frac{5}{16}P \uparrow$$

PROBLEM 9.153

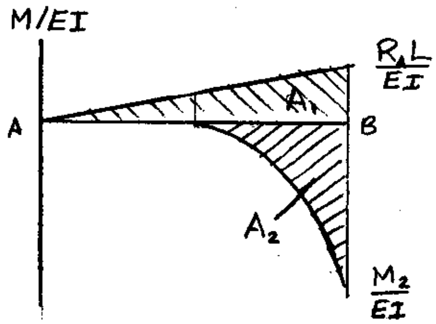
9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w.

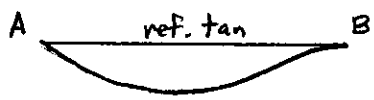


$$M_2 = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 = -\frac{1}{8}wL^2$$

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right)L = \frac{1}{2}\frac{R_A L^2}{EI}$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

Place reference tangent at B



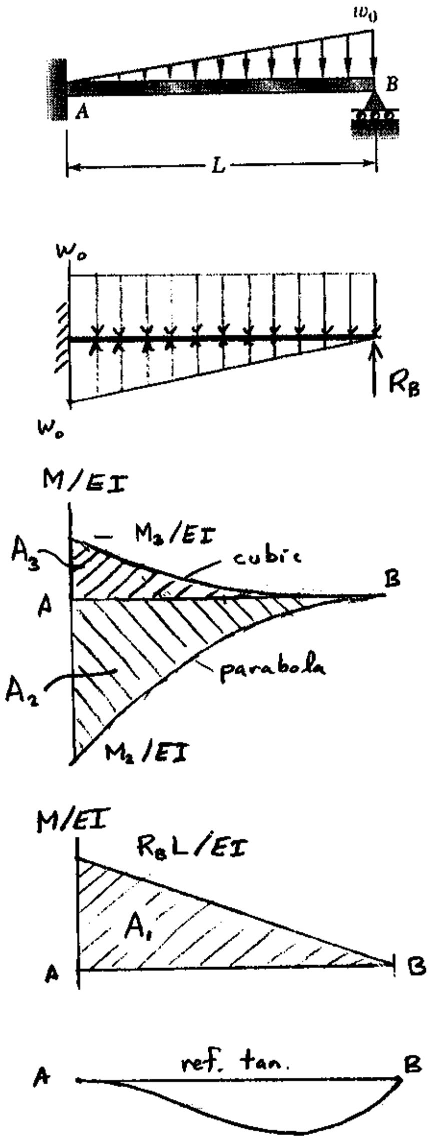
$$t_{A/B} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128}wL \uparrow$$

PROBLEM 9.154

9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left

Draw M/EI diagram for load w_o and R_B.

Use parts as shown.

$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2} w_o L^2$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{2}\frac{w_o L^2}{EI}\right)L = -\frac{1}{6}\frac{w_o L^3}{EI}$$

$$M_3 = \frac{1}{6}\frac{w_o}{L}L^3 = \frac{1}{6} w_o L^2$$

$$A_3 = \frac{1}{4}\left(\frac{1}{6}\frac{w_o L^2}{EI}\right)L = \frac{1}{24}\frac{w_o L^3}{EI}$$

Place reference tangent at A

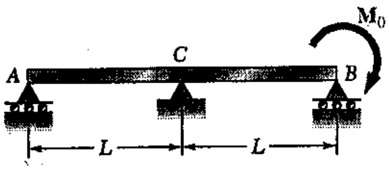
$$t_{B/A} = A_1\left(\tfrac{2}{3}L\right) + A_2\left(\tfrac{3}{4}L\right) + A_3\left(\tfrac{4}{5}L\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{1}{8}\frac{w_o L^4}{EI} + \frac{1}{30}\frac{w_o L^4}{EI} = 0$$

$$R_A = \frac{11}{40} w_o L = 0.275\, w_o L \uparrow$$

PROBLEM 9.155

9.155 and 9.156 For the beam and loading shown, determine the reaction at each support.

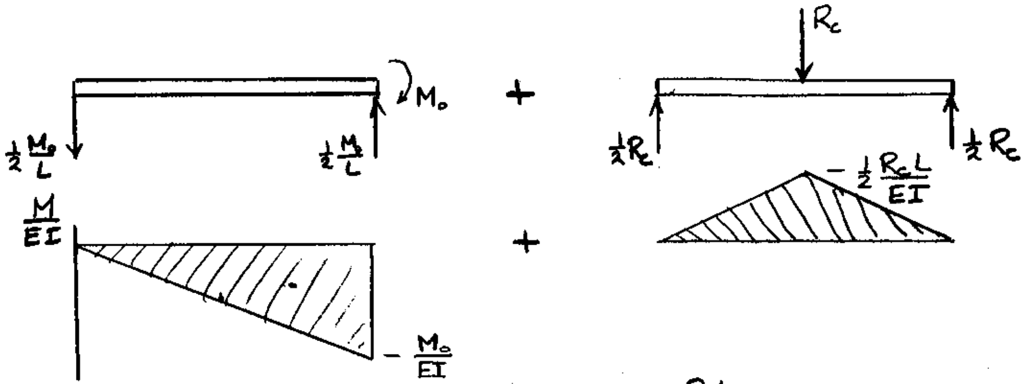


SOLUTION

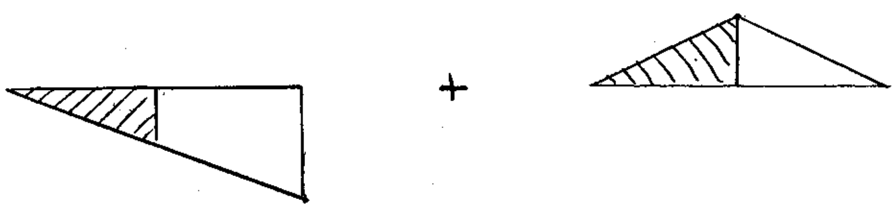
Remove support C and treat R_c as redundant.

Consider the loads M_0 and R_c separately

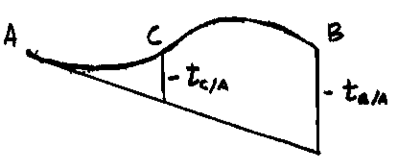
Place reference tangent at A.



$$t_{B/A} = -\frac{1}{2}\left(\frac{M_0}{EI}\right)(2L)\left(\frac{2L}{3}\right) + \frac{1}{2}\left(\frac{1}{2}\frac{R_cL}{EI}\right)(2L)(L) = -\frac{2}{3}\frac{M_0L^2}{EI} + \frac{1}{2}\frac{R_cL^3}{EI}$$



$$t_{C/A} = -\frac{1}{2}\left(\frac{1}{2}\frac{M_0}{EI}\right)(L)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{1}{2}\frac{R_cL}{EI}\right)(L)\left(\frac{L}{3}\right) = -\frac{1}{12}\frac{M_0L^2}{EI} + \frac{1}{12}\frac{R_cL^3}{EI}$$



$$y_C = t_{C/A} - \frac{x_{C/A}}{x_{B/A}}t_{B/A} = t_{C/A} - \frac{1}{2}t_{B/A} = 0$$

$$\left(-\frac{1}{12} + \frac{1}{3}\right)\frac{M_0L^2}{EI} + \left(\frac{1}{12} - \frac{1}{4}\right)\frac{R_cL^3}{EI} = 0$$

$$R_c = \frac{3}{2}\frac{M_0}{L} \downarrow$$

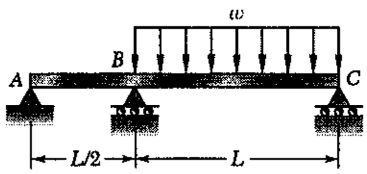
From Statics

$$R_A = \frac{1}{2}R_c - \frac{1}{2}\frac{M_0}{L} = \frac{1}{4}\frac{M_0}{L} \uparrow$$

$$R_B = \frac{1}{2}R_c + \frac{1}{2}\frac{M_0}{L} = \frac{5}{4}\frac{M_0}{L} \uparrow$$

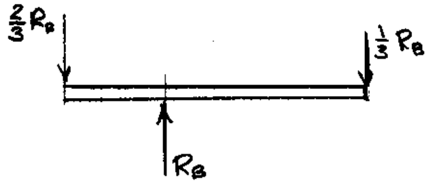
PROBLEM 9.156

9.155 and 9.156 For the beam and loading shown, determine the reaction at each support.



SOLUTION

Remove support B and consider R_B as redundant.
Consider loads R_B and w separately.
Place reference tangent at A.



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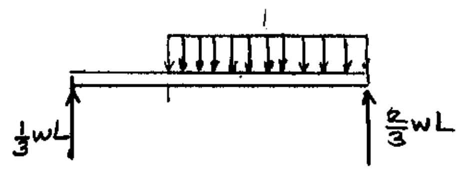

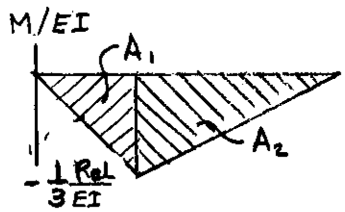

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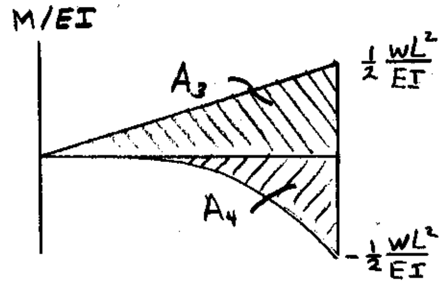


$$A_1 = \frac{1}{2}\cdot\left(-\frac{1}{3}\frac{R_BL}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{12}\frac{R_BL^2}{EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{1}{3}\frac{R_BL}{EI}\right)L = -\frac{1}{6}\frac{R_BL^2}{EI}$$

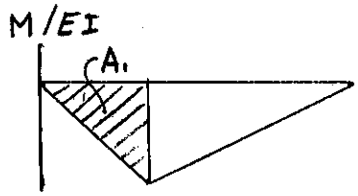
$$A_3 = \frac{1}{2}\left(\frac{1}{2}\frac{wL^2}{EI}\right)\frac{3L}{2} = \frac{3}{8}\frac{wL^3}{EI}$$

$$A_4 = \frac{1}{3}\left(-\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$t_{C/A} = A_1\left(L + \frac{1}{3}\frac{L}{2}\right) + A_2\left(\frac{2}{3}L\right) + A_3\left(\frac{1}{3}\cdot\frac{3L}{2}\right) + A_4\left(\frac{1}{4}L\right)$$

$$= -\frac{7}{72}\frac{R_BL^3}{EI} - \frac{1}{9}\frac{R_BL^3}{EI} + \frac{3}{16}\frac{wL^4}{EI} - \frac{1}{24}\frac{wL^4}{EI}$$

$$= -\frac{5}{24}\frac{R_BL^3}{EI} + \frac{7}{48}\frac{wL^3}{EI}$$



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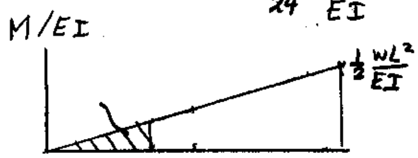


$$A_5 = \frac{1}{2}\left(\frac{1}{6}\frac{wL^2}{EI}\right)\frac{L}{2} = \frac{1}{24}\frac{wL^3}{EI}$$

$$t_{B/A} = A_1\left(\frac{1}{3}\frac{L}{2}\right) + A_5\left(\frac{1}{3}\frac{L}{2}\right) = -\frac{1}{72}\frac{R_BL^3}{EI} + \frac{1}{144}\frac{wL^4}{EI}$$

$$y_B = t_{B/A} - \frac{L/2}{3L/2}t_{C/A} = \left(-\frac{1}{72} + \frac{5}{72}\right)\frac{R_BL^3}{EI} + \left(\frac{1}{144} - \frac{7}{144}\frac{wL^4}{EI}\right)$$

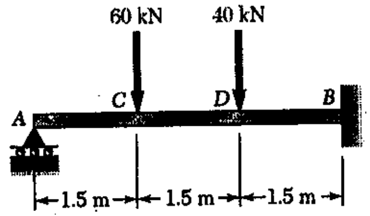
$$= \frac{1}{18}\frac{R_BL^3}{EI} - \frac{1}{24}\frac{wL^3}{EI} = 0 \qquad R_B = \frac{3}{4}wL \uparrow$$

$$R_A = \frac{1}{3}wL - \frac{2}{3}R_B = \frac{1}{3}wL - \frac{1}{2}wL = -\frac{1}{6}wL = \frac{1}{6}wL \downarrow$$

$$R_C = \frac{2}{3}wL - \frac{1}{3}R_B = \frac{2}{3}wL - \frac{1}{4}wL = \frac{5}{12}wL \uparrow$$

PROBLEM 9.157

9.157 and *9.158* Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

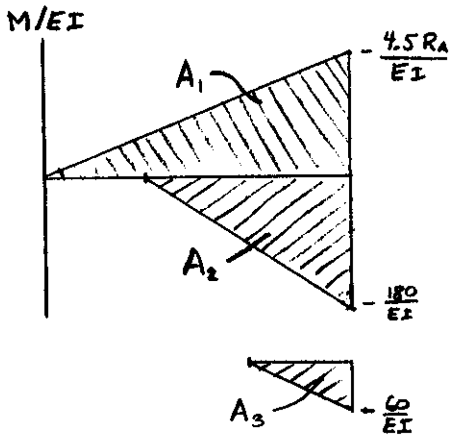
Remove support at A and treat R_A as redundant.

Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{4.5 R_A}{EI}\right)(4.5) = \frac{10.125 R_A}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{180}{EI}\right)(3.0) = -\frac{270}{EI}$$

$$A_3 = -\frac{1}{2}\left(\frac{60}{EI}\right)(1.5) = -\frac{45}{EI}$$

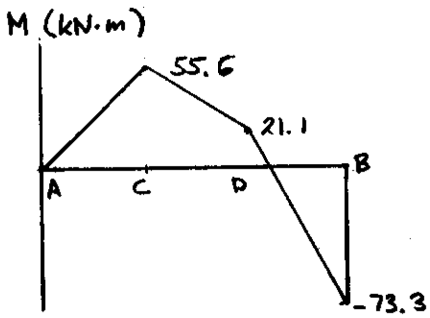


Place reference tangent at B. $\theta_B = 0$

$$t_{A/B} = A_1(3.0) + A_2(1.5 + 2.0) + A_3(3.0 + 1.0)$$

$$= \left\{30.375 R_A - 945 - 180\right\}\frac{1}{EI} = 0$$

$$R_A = 37.037 \text{ kN} \uparrow$$



$$M_A = 0$$

$$M_C = (1.5)(37.037) = 55.6 \text{ kN·m}$$

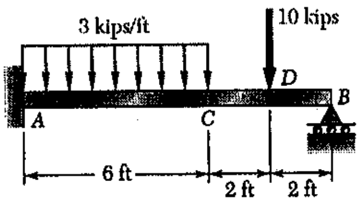
$$M_D = (3.0)(37.037) - (60)(1.5) = 21.1 \text{ kN·m}$$

$$M_B = (4.5)(37.037) - (60)(3) - (40)(1.5)$$

$$= -73.3 \text{ kN·m}$$

PROBLEM 9.158

9.157 and *9.158* Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support at B and treat R_B as redundant.

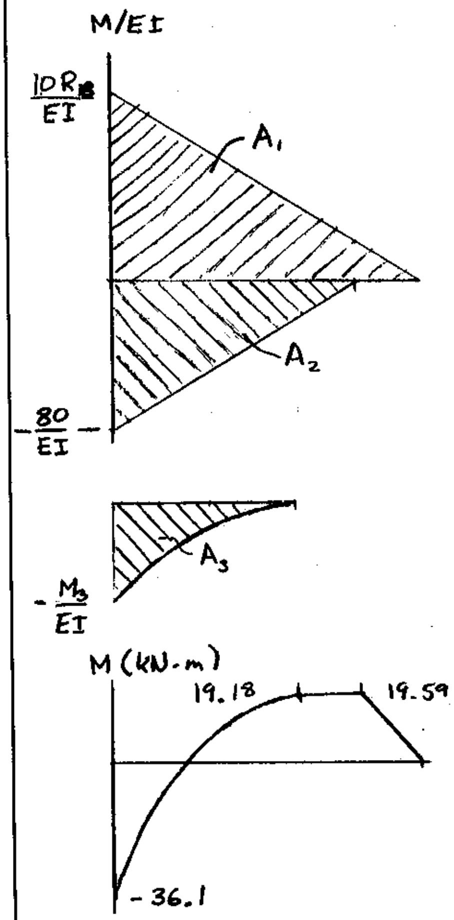
Draw bending moment diagram by parts

$$A_1 = \frac{1}{2}\left(\frac{10R_B}{EI}\right)(10) = \frac{50R_B}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{80}{EI}\right)(8) = -\frac{320}{EI}$$

$$M_3 = -\frac{1}{2}(3)(6)^2 = -54 \text{ kN}\cdot\text{m}$$

$$A_3 = \frac{1}{3}\left(-\frac{54}{EI}\right)(6) = -\frac{108}{EI}$$



Place reference tangent at A. $\theta_A = 0$

$$t_{B/A} = A_1\left(\tfrac{2}{3}\cdot 10\right) + A_2\left(\tfrac{2}{3}\cdot 8 + 2\right) + A_3\left(10 - \tfrac{1}{4}\cdot 6\right)$$

$$= \frac{333.33R_A}{EI} - \frac{2346.7}{EI} - \frac{918}{EI} = 0$$

$$R_A = 9.79 \text{ kN} \uparrow$$

$$M_B = 0$$

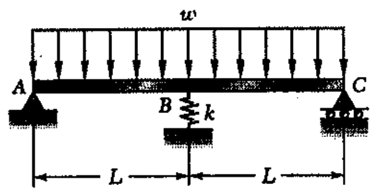
$$M_D = (9.79)(2) = 19.59 \text{ kN-m}$$

$$M_C = (9.79)(4) - (10)(2) = 19.18 \text{ kN}\cdot\text{m}$$

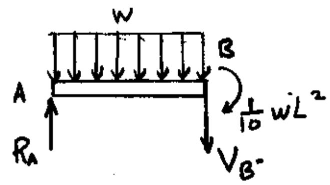
$$M_A = (9.79)(10) - (10)(8) - 54 = -36.1 \text{ kN}\cdot\text{m}$$

PROBLEM 9.159

9.159 For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.



SOLUTION



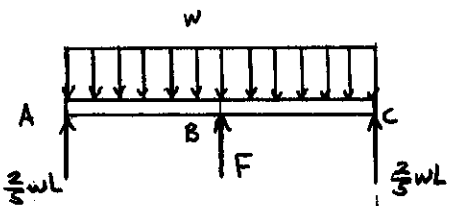
Using free body AB

$+\circlearrowleft M_B = 0$

$-R_A L + (wL)(\frac{L}{2}) - \frac{1}{10}wL^2 = 0$

$R_A = \frac{2}{5}wL \uparrow$

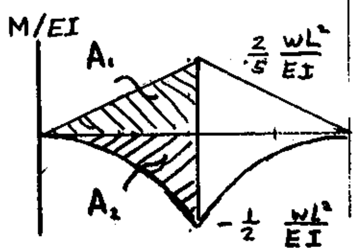
Symmetric beam and loading. $R_C = R_A$



Using free body ABC $\quad +\uparrow \Sigma F_y = 0$

$\frac{2}{5}wL + F + \frac{2}{5}wL - 2wL = 0$

$F = \frac{6}{5}wL$



Draw $\frac{M}{EI}$ diagram by parts

$A_1 = \frac{1}{2}\left(\frac{2}{5}\frac{wL^2}{EI}\right)L = \frac{1}{5}\frac{wL^3}{EI}$

$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$

Place reference tangent at B. $\quad \theta_B = 0$

$y_B = -t_{A/B}$

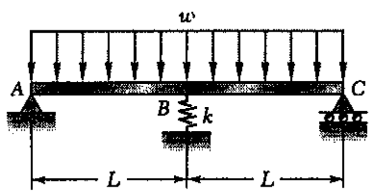
$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$

$= -\frac{1}{120}\frac{wL^4}{EI}$

$F = -k y_B$

$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120}\frac{wL^4}{EI}} = 144\frac{EI}{L^3}$

PROBLEM 9.160

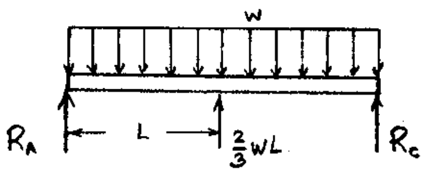


9.160 For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.

SOLUTION

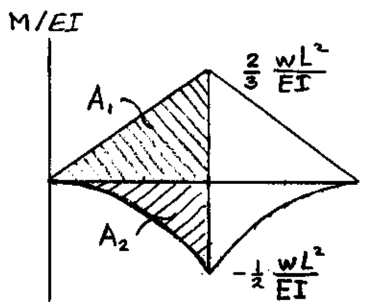
Symmetric beam and loading. $R_C = R_A$

Spring force $F = \frac{1}{3}(2wL) = \frac{2}{3}wL$



$+\uparrow \Sigma F_y = 0 \qquad R_A + F - 2wL + R_C = 0$

$$R_A = R_C = \frac{2}{3}wL$$



Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{3}\frac{wL^2}{EI}\right)L = \frac{1}{3}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

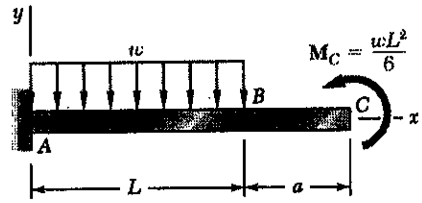
$$= -\frac{7}{72}\frac{wL^4}{EI}$$

$$F = -k\,y_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72}\frac{wL^4}{EI}} = \frac{48}{7}\frac{EI}{L^3}$$

PROBLEM 9.161

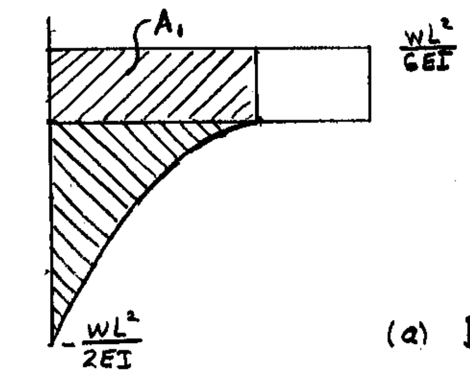
9.161 For the cantilever beam and loading shown, determine (*a*) the deflection at point *B*, (*b*) the slope at point *B*.



SOLUTION

Use moment area method.

Draw $\frac{M}{EI}$ diagram by parts.



Place reference tangent at A. $\theta_A = 0$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_{B/A}$$

$$y_B = y_A + L\theta_A + t_{B/A} = t_{B/A}$$

$$A_1 = \left(\frac{wL^2}{6EI}\right)(L) = \frac{1}{6}\frac{wL^3}{EI}$$

$$A_2 = \frac{1}{3}\left(-\frac{wL^2}{2EI}\right)(L) = -\frac{1}{6}\frac{wL^3}{EI}$$

(a) Deflection at B

$$y_B = t_{B/A} = A_1\left(\frac{L}{2}\right) + A_2\left(\frac{3}{4}L\right)$$

$$= \frac{1}{12}\frac{wL^4}{EI} - \frac{1}{8}\frac{wL^4}{EI} = -\frac{1}{24}\frac{wL^4}{EI}$$ ◄

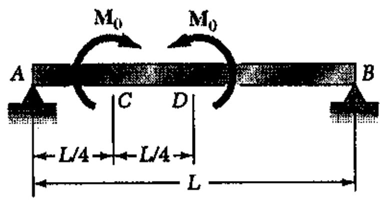
(b) Slope at B

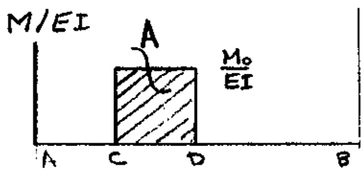
$$\theta_B = \theta_{B/A} = A_1 + A_2$$

$$= \frac{1}{6}\frac{wL^3}{EI} - \frac{1}{6}\frac{wL^3}{EI} = 0$$ ◄

PROBLEM 9.162

9.162 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.





SOLUTION

From Statics $R_A = R_B = 0.$

Draw $\frac{M}{EI}$ diagram

$$A = \left(\frac{M_0}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{4}\frac{M_0 L}{EI}$$

Place reference tangent at A.

$$t_{B/A} = A\left(\frac{L}{2}+\frac{L}{8}\right) = \frac{5}{32}\frac{M_0 L^2}{EI}$$

$$t_{D/A} = A\left(\frac{L}{8}\right) = \frac{1}{32}\frac{M_0 L^2}{EI}$$

(a) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32}\frac{M_0 L}{EI}$$

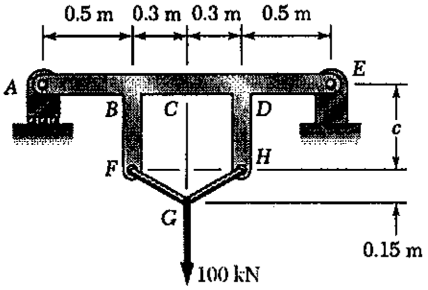
(b) Deflection at D

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A}$$

$$= \frac{1}{32}\frac{M_0 L^2}{EI} - \frac{5}{64}\frac{M_0 L^2}{EI} = -\frac{3}{64}\frac{M_0 L^2}{EI}$$

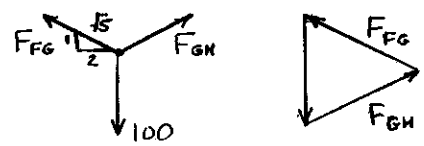
PROBLEM 9.163

9.163 The rigid bars *BF* and *DH* are welded to the rolled-steel beam *AE* as shown. Knowing that $c = 0.4$m, determine for the loading shown (*a*) the deflection at point *B*, (*b*) the deflection at the midpoint *C* of the beam. Use $E = 200$ GPa.

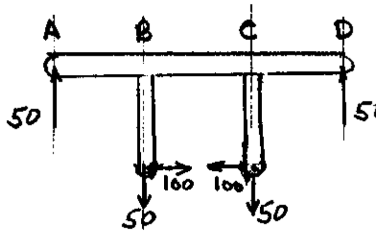


SOLUTION

Using joint G as a free body



By symmetry $F_{GH} = F_{FG}$



$$2F_{GHy} - 100 = 0 \qquad F_{GHy} = 50 \text{ kN}$$

$$F_{GHx} = 2F_{GHx} = 100 \text{ kN.}$$

Forces in kN. Lengths in m

$$V = 50 - 50\langle x-0.5\rangle^0 - 50\langle x-1.1\rangle^0 \qquad \text{kN}$$

$$M = 50x - 50\langle x-0.5\rangle^1 - 50\langle x-1.1\rangle^0 + 40\langle x-0.5\rangle^0 - 40\langle x-1.1\rangle \qquad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = 25x^2 - 25\langle x-0.5\rangle^2 - 25\langle x-1.1\rangle^2 - 40\langle x-0.5\rangle^1 + 40\langle x-1.1\rangle^1 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = \frac{25}{3}x^3 - \frac{25}{3}\langle x-0.5\rangle^3 - \frac{25}{3}\langle x-1.1\rangle^3 - 20\langle x-0.5\rangle^2 + 20\langle x-1.1\rangle^2 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$$

$[x=0, y=0]$ $\qquad C_2 = 0$

$[x=1.6, y=0]$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \quad \text{kN}\cdot\text{m}^2$$

For EIy_B, $x = 0.5$ m

$$EIy_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = \quad 0.1667 \text{ kN}\cdot\text{m}^3$$

For EIy_C, $x = 0.8$ m

$$EIy_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0$$
$$= -0.8417 \quad \text{kN}\cdot\text{m}^3$$

For W 100 × 19.3 rolled steel section $\quad I = 4.77\times10^6 \text{ mm}^4 = 4.77\times10^{-6} \text{ m}^4$

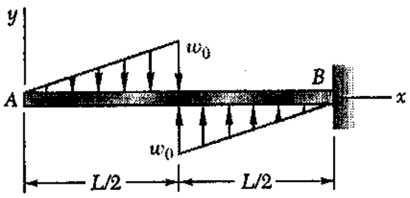
$$EI = (200\times10^9)(4.77\times10^{-6}) = 954\times10^3 \text{ N}\cdot\text{m}^2 = 954 \text{ kN}\cdot\text{m}^2$$

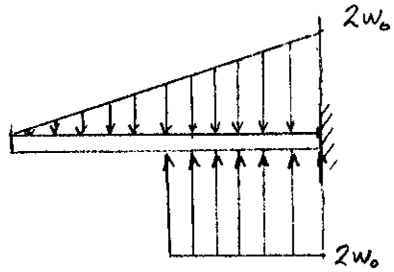
(a) $y_B = \dfrac{0.1667}{954} = 0.175\times10^{-3}$ m $= 0.175$ mm ↑

(b) $y_C = \dfrac{0.8417}{954} = 0.882\times10^{-3}$ m $= 0.882$ mm ↑

PROBLEM 9.164

9.164 For the beam and loading shown, determine the deflection at point *A*.





SOLUTION

Express loading in terms of singularity functions.

$$w = \frac{2w_0}{L}x - 2w_0\langle x - \tfrac{L}{2}\rangle^0$$

$$\frac{dV}{dx} = -w = -\frac{2w_0}{L}x + 2w_0\langle x - \tfrac{L}{2}\rangle^0$$

$$V = -\frac{w_0}{L}x^2 + 2w_0\langle x - \tfrac{L}{2}\rangle^1 + C_1$$

$$[x = 0,\ V = 0]$$

$$0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}x^2 + 2w_0\langle x - \tfrac{L}{2}\rangle^1$$

$$M = -\frac{1}{3}\frac{w_0}{L}x^3 + w_0\langle x - \tfrac{L}{2}\rangle^2 + C_2$$

$$[x = 0,\ M = 0]$$

$$0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{3}\frac{w_0}{L}x^3 + w_0\langle x - \tfrac{L}{2}\rangle^2$$

$$EI\frac{dy}{dx} = -\frac{1}{12}\frac{w_0}{L}x^4 + \frac{1}{3}w_0\langle x - \tfrac{L}{2}\rangle^3 + C_3$$

$$\left[x = L,\ EI\frac{dy}{dx} = 0\right] \quad -\frac{1}{12}w_0L^3 + \frac{1}{3}w_0\left(\frac{L}{2}\right)^3 + C_3 = 0 \qquad C_3 = \frac{1}{24}w_0L^3$$

$$EIy = -\frac{1}{60}\frac{w_0}{L}x^5 + \frac{1}{12}w_0\langle x - \tfrac{L}{2}\rangle^4 + C_3x + C_4$$

$$[x = L,\ EIy = 0] \quad -\frac{1}{60}w_0L^4 + \frac{1}{12}w_0\left(\frac{L}{2}\right)^4 + \frac{1}{24}w_0L^3\cdot L + C_4 = 0$$

$$C_4 = -\frac{29}{960}w_0L^4$$

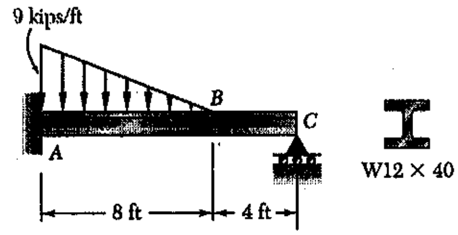
At point A, $x = 0$

$$EIy_A = 0 + 0 + 0 + C_4 = -\frac{29}{960}w_0L^4$$

$$y_A = -\frac{29}{960}\frac{w_0L^4}{EI}$$ ◄

PROBLEM 9.165

9.165 For the beam and loading shown, determine (*a*) the reaction at *C*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.



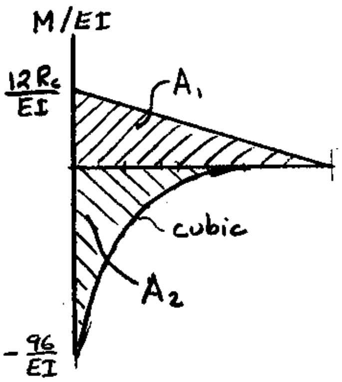
SOLUTION

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = 310$ in^4

$EI = (29 \times 10^3)(310) = 8.99 \times 10^6$ kip·in^2

$= 62430$ kip·ft^2



Statically indeterminate beam. Remove support at C and treat R_c as redundant.

Draw $\frac{M}{EI}$ diagram by parts.

For the uniformly varying load

$$k = \frac{9 \text{ kips/ft}}{8 \text{ ft}} = \frac{9}{8} \text{ kips/ft}^2$$

$$M_2 = -\frac{1}{6}ka^3 = -\frac{1}{6}\cdot\frac{9}{8}(8)^3 = -96 \text{ kip·ft}$$

$$A_1 = \frac{1}{2}\left(\frac{12R_c}{EI}\right)(12) = \frac{72R_c}{EI}$$

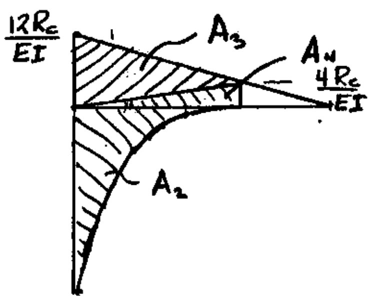
$$A_2 = \frac{1}{4}\left(-\frac{96}{EI}\right)(8) = -\frac{192}{EI}$$

Place reference tangent at A. $\theta_A = 0$

$y_A = 0$

$$y_C = y_A + \theta_A L + t_{C/A} = 0 + 0 + \left[A_1\left(\frac{2}{3}\cdot 12\right) + A_2\left(12 - \frac{1}{5}\cdot 8\right)\right]$$

$$= \frac{576R_c}{EI} - \frac{1996.8}{EI} = 0 \qquad R_c = 3.4667 \text{ kips} \uparrow$$



$$A_3 = \frac{1}{2}\left(\frac{12R_c}{EI}\right)(8) = \frac{48R_c}{EI}$$

$$A_4 = \frac{1}{2}\left(\frac{4R_c}{EI}\right)(8) = \frac{16R_c}{EI}$$

$$y_B = t_{B/A} = A_3\left(\frac{2}{3}\cdot 8\right) + A_4\left(\frac{1}{3}\cdot 8\right) + A_2\left(\frac{4}{5}\cdot 8\right)$$

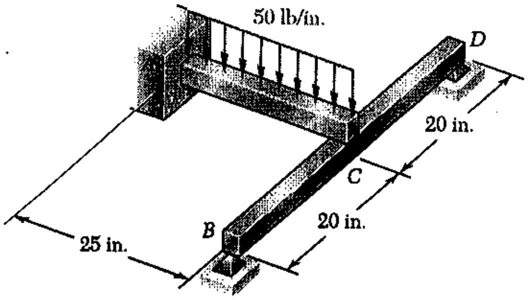
$$= 298\tfrac{2}{3}\frac{R_c}{EI} - \frac{6144}{5\,EI} = -\frac{193.42}{EI}$$

$$= -\frac{192.42}{62430} = -3.098 \times 10^{-3} \text{ ft}$$

$$= 0.0372 \text{ in.} \downarrow$$

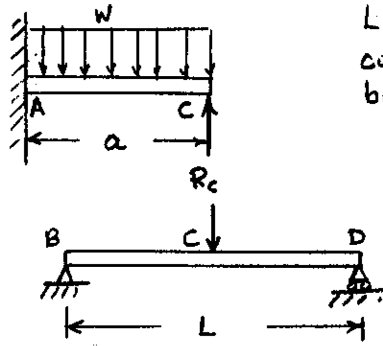
PROBLEM 9.166

9.166 For the loading shown, knowing that beams *AC* and *BD* have the same flexural rigidity, determine the reaction at *B*.



SOLUTION

Consider the two beams shown below.



Let R_c be the contact force between beams AC and BCD.

Applying Cases 1 and 2 of Appendix D to cantilever beam AC

$$y_c = \frac{R_c a^3}{3EI} - \frac{w a^4}{8EI}$$

Applying Case 4 of Appendix D to simply supported beam BCD.

$$y_c = -\frac{R_c L^3}{48\,EI}$$

Equating expressions for y_c

$$\frac{R_c a^3}{3EI} - \frac{w a^4}{8EI} = -\frac{R_c L^3}{48\,EI}$$

$$(16a^3 + L^3)R_c = 6wa^4$$

$$R_c = \frac{6\,wa}{16 + L^3/a^3}$$

Data: $w = 50$ lb/in, $a = 25$ in., $L = 20 + 20 = 40$ in.

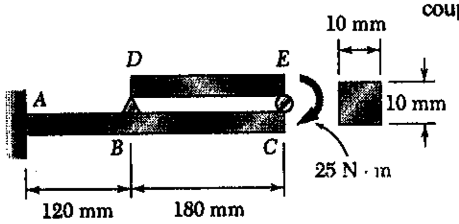
$$R_c = \frac{(6)(50)(25)}{16 + (40/25)^3} = 373.21 \text{ lb}$$

Using beam BCD as a free body

$\circlearrowleft \Sigma M_D = 0 \qquad -R_B L + R_c \frac{L}{2} = 0 \qquad R_B = \frac{1}{2} R_c = 186.6$ lb $\uparrow$

PROBLEM 9.167

9.167 Beam *DE* rests on the cantilever beam *AC* as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end *C* if the 25-N·m couple is applied (*a*) to end *E* of beam *DE*, (*b*) to end *C* of beam *AC*. Use $E = 200$ GPa.

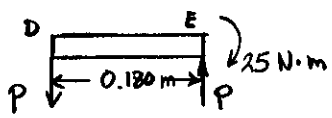


SOLUTION

$E = 200 \times 10^9$ Pa

$I = \frac{1}{12}(10)(10)^3 = 833.33\ \text{mm}^4 = 833.33 \times 10^{-12}\ \text{m}^4$

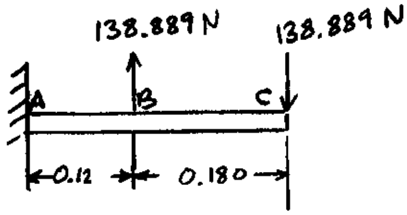
$EI = 166.667\ \text{N} \cdot \text{m}^2$



(a) Couple applied to beam DE

Free body DE $\circlearrowleft \Sigma M_D = 0$

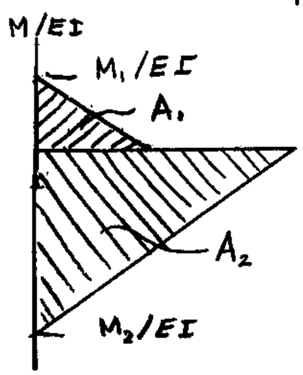
$.180\,P - 25 = 0 \qquad P = 138.889$ N



For beam ABC draw the $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(138.889)(0.12)}{166.667} = 100 \times 10^{-3}\ \text{m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(138.889)(0.30)}{166.667} = -250 \times 10^{-3}\ \text{m}^{-1}$$



$A_1 = \frac{1}{2}(100 \times 10^{-3})(0.12) = 6 \times 10^{-3}$

$A_2 = \frac{1}{2}(-250 \times 10^{-3})(0.30) = 37.5 \times 10^{-3}$

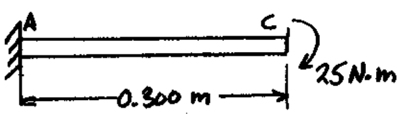
$y_A = 0 \qquad \theta_A = 0$

Place reference tangent at A

$y_C = y_A + L\theta_A + t_{C/A}$

$= 0 + 0 + A_1(0.180 + 0.080) + A_2(0.200)$

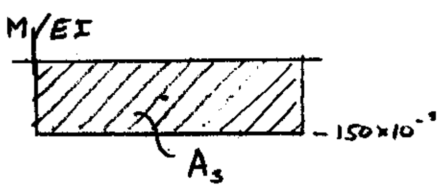
$= -5.94 \times 10^{-3}$ m $=$ 5.94 mm $\downarrow$ ◄



(b) Couple applied to beam AC

Draw $\frac{M}{EI}$ diagram

$$\frac{M}{EI} = \frac{25}{166.667} = 150 \times 10^{-3}\ \text{m}^{-1}$$



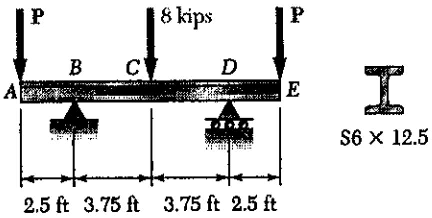
$A_3 = (-150 \times 10^{-3})(0.30) = -45 \times 10^{-3}$

$y_C = t_{C/A} = A_3(0.15) = -6.75 \times 10^{-3}$ m

$=$ 6.75 mm $\downarrow$ ◄

PROBLEM 9.168

9.168 For the beam and loading shown, determine the value of P for which the deflection is zero at end A of the beam. Use $E = 29 \times 10^6$ psi.



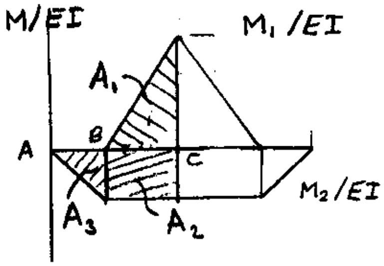
SOLUTION

Symmetric beam and loading. $\theta_c = 0$

Place reference tangent at C.

Draw $\frac{M}{EI}$ diagram by parts.

Assume EI in kip · ft²



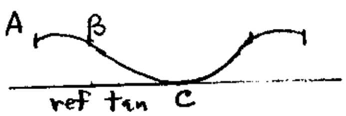
$M_1 = (4)(3.75) = 15$ kip · ft

$M_2 = -(P)(2.5) = -2.5P$ kip · ft

$A_1 = \frac{1}{2}\left(\frac{15}{EI}\right)(3.75) = \frac{28.125}{EI}$

$A_2 = -\frac{2.5P}{EI}(3.75) = -\frac{9.375P}{EI}$

$A_3 = \frac{1}{2}\left(-\frac{2.5P}{EI}\right)(2.5) = -\frac{3.125P}{EI}$



$y_A = y_c + t_{A/c}$

$y_B = y_c + t_{B/c}$

$y_A - y_B = t_{A/c} - t_{B/c} = 0$

$A_1(2.5 + 2.5) + A_2(2.5 + 1.875) + A_3\left(\frac{2}{3} \cdot 2.5\right)$
$- A_1(2.5) - A_2(1.875) = 0$

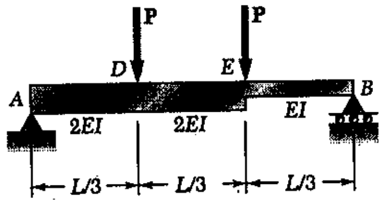
$A_1(2.5) + A_2(2.5) + A_3(1.66667) = 0$

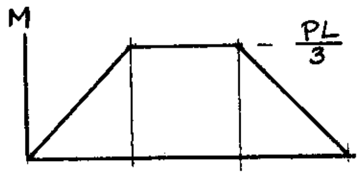
$\frac{70.3125}{EI} - \frac{23.4375P}{EI} - \frac{5.208333P}{EI} = 0$

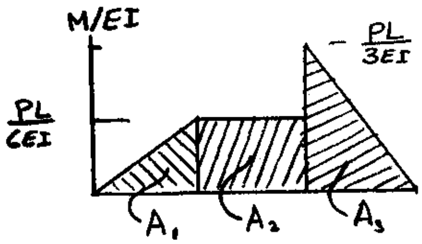
$P = 2.45$ kips

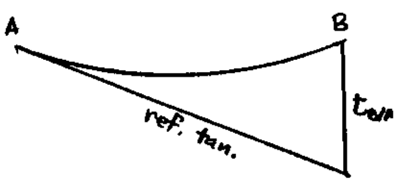
PROBLEM 9.169

9.169 For the beam and loading shown, determine the deflection (*a*) at point *D*, (*b*) at point *E*.









SOLUTION

Draw bending moment and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2}\left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{1}{36}\frac{PL^2}{EI}$$

$$A_2 = \left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{1}{18}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{PL}{3EI}\right)\left(\frac{L}{3}\right) = \frac{1}{18}\frac{PL^2}{EI}$$

Place reference tangent at A

$$t_{B/A} = A_1\left(\frac{2}{3}L + \frac{1}{3}\cdot\frac{1}{3}L\right) + A_2\left(\frac{L}{2}\right) + A_3\left(\frac{2}{3}\cdot\frac{L}{3}\right)$$

$$= \frac{7}{324}\frac{PL^3}{EI} + \frac{1}{36}\frac{PL^3}{EI} + \frac{1}{81}\frac{PL^3}{EI} = \frac{5}{81}\frac{PL^3}{EI}$$

$$t_{D/A} = A_1\left(\frac{1}{3}\cdot\frac{L}{3}\right)$$

$$= \frac{1}{324}\frac{PL^3}{EI}$$

$$t_{E/A} = A_1\left(\frac{1}{3}\cdot\frac{L}{3} + \frac{L}{3}\right) + A_2\left(\frac{1}{2}\cdot\frac{L}{3}\right)$$

$$= \frac{1}{81}\frac{PL^3}{EI} + \frac{1}{108}\frac{PL^3}{EI} = \frac{7}{324}\frac{PL^3}{EI}$$

(a) Deflection at D

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A} = \frac{1}{324}\frac{PL^3}{EI} - \frac{1}{3}\cdot\frac{5}{81}\frac{PL^3}{EI}$$

$$= -\frac{17}{972}\frac{PL^3}{EI} = -0.01749\frac{PL^3}{EI}$$

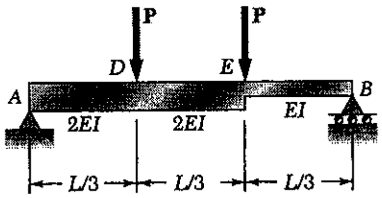
(b) Deflection at E

$$y_E = t_{E/A} - \frac{x_E}{L}t_{B/A} = \frac{7}{324}\frac{PL^3}{EI} - \frac{2}{3}\cdot\frac{5}{81}\frac{PL^3}{EI}$$

$$= -\frac{19}{972}\frac{PL^3}{EI} = -0.01955\frac{PL^3}{EI}$$

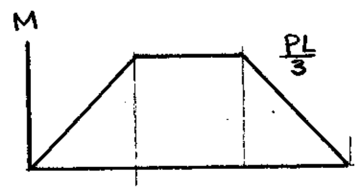
PROBLEM 9.170

9.170 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.



SOLUTION

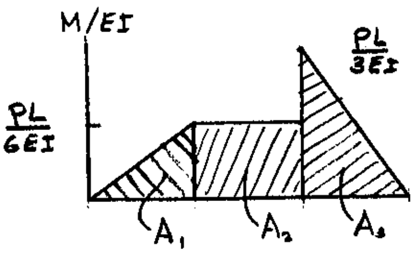
Draw bending moment and $\frac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2}\left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{1}{36}\frac{PL^2}{EI}$$

$$A_2 = \left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{1}{18}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{PL}{3EI}\right)\left(\frac{L}{3}\right) = \frac{1}{18}\frac{PL^3}{EI}$$



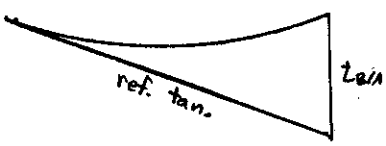
Place reference tangent at A

$$t_{B/A} = A_1\left(\frac{2}{3}L + \frac{1}{3}\cdot\frac{1}{3}L\right) + A_2\left(\frac{L}{2}\right) + A_3\left(\frac{2}{3}\cdot\frac{L}{3}\right)$$

$$= \frac{7}{324}\frac{PL^3}{EI} + \frac{1}{36}\frac{PL^3}{EI} + \frac{1}{81}\frac{PL^3}{EI} = \frac{5}{81}\frac{PL^3}{EI}$$

Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{81}\frac{PL^3}{EI}$$

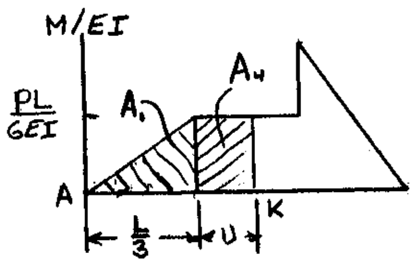


Deflection is maximum at point K.

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1 + A_4 = 0$$

assuming that point K lies between D and E.

$$A_4 = \frac{1}{6}\left(\frac{PL}{EI}\right)u = \frac{1}{6}\frac{PLu}{EI}$$



$$-\frac{5}{81}\frac{PL^2}{EI} + \frac{1}{36}\frac{PL^2}{EI} + \frac{1}{6}\frac{PLu}{EI} = 0$$

$$u = 6\left(\frac{5}{81} - \frac{1}{36}\right)L = \frac{11}{54}L$$

$$x_K = \frac{L}{3} + u = \left(\frac{1}{3} + \frac{11}{54}\right)L = \frac{29}{54}L$$

$$= 0.537\,L$$ ◄

$$A_4 = \frac{11}{324}\frac{PL}{EI}$$

$$t_{K/A} = A_1\left(\frac{1}{3}\cdot\frac{L}{3} + u\right) + A_4\left(\frac{u}{2}\right) = \frac{17}{1944}\frac{PL^3}{EI} + \frac{121}{34992}\frac{PL^3}{EI} = \frac{427}{34992}\frac{PL^3}{EI}$$

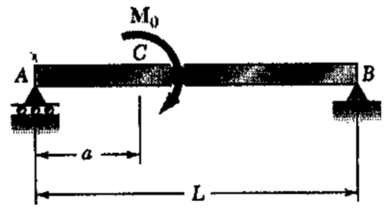
Maximum deflection

$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = \frac{427}{34992}\frac{PL^3}{EI} - \frac{29}{54}\cdot\frac{5}{81}\frac{PL^3}{EI} = -\frac{733}{34992}\frac{PL^3}{EI}$$

$$= -0.02095\frac{PL^3}{EI}$$ ◄

PROBLEM 9.171

9.171 For the beam and loading shown, determine (*a*) the value of *a* for which the slope at end *A* is zero, (*b*) the corresponding deflection at point *C*.



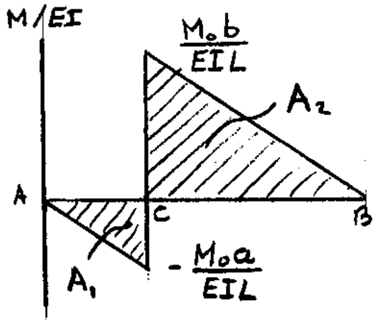
SOLUTION

Let $b = L - a$

Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0 + 0 + t_{B/A} = 0$$

Draw $\frac{M}{EI}$ diagram.



$$A_1 = -\frac{1}{2}\frac{M_0 a}{EIL}a = -\frac{1}{2}\frac{M_0 a^2}{EIL}$$

$$A_2 = \frac{1}{2}\frac{M_0 b}{EIL}b = \frac{1}{2}\frac{M_0 b^2}{EIL}$$

$$t_{B/A} = A_1\left(\frac{a}{3} + b\right) + A_2\left(\frac{2}{3}b\right)$$

$$= -\frac{1}{6}\frac{M_0 a^3}{EIL} - \frac{1}{2}\frac{M_0 a^2 b}{EIL} + \frac{1}{3}\frac{M_0 b^3}{EIL} = 0$$

Let $u = \frac{a}{b}$

$$u^3 + 3u^2 - 2 = 0$$

Solving for u: $\quad u = 0.73205$

$$\frac{a}{b} = \frac{a}{L-a} = 0.73205$$

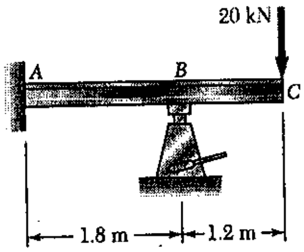
$$a = 0.73205(L-a)$$

$$a = \frac{0.73205}{1.73205}L = 0.42265\,L$$ ◀

$$A_1 = -\frac{1}{2}\frac{M_0 a^2}{EIL} = -0.089316\frac{M_0 L}{EI}$$

$$t_{C/A} = A_1\left(\frac{1}{3}a\right) = -0.01258\frac{M_0 L^2}{EI} = 0.01258\frac{M_0 L^2}{EI}\downarrow$$ ◀

PROBLEM 9.172



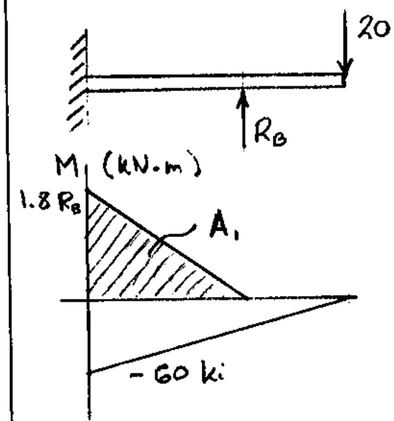
9.172 A hydraulic jack may be used to raise point *B* of the cantilever beam *ABC*. Knowing that after the 20-kN load is applied, point *C* is to have the same elevation as point *A*, determine (*a*) how much *B* should be raised, (*b*) the reaction at *B* after point *B* has been raised and the 20-kN load has been applied. Use $E = 200$ GPa.

SOLUTION

For W 130 × 23.8 $\quad I_x = 8.80 \times 10^6\ \text{mm}^4 = 8.80 \times 10^{-6}\ \text{m}^4$

$E = 200 \times 10^9$ Pa

$EI = (200 \times 10^9)(8.80 \times 10^{-6}) = 1.760 \times 10^6\ \text{N}\cdot\text{m}^2 = 1760\ \text{kN}\cdot\text{m}^2$



Let R_B be the jack force in kN.

$A_1 = \frac{1}{2}(1.8 R_B)(1.8) = 1.62\ R_B \quad \text{kN}\cdot\text{m}^2$

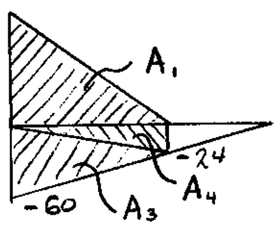
$A_2 = \frac{1}{2}(-60)(3) = -90\ \text{kN}\cdot\text{m}^2$

$EI\, t_{B/A} = (1.2 + 1.2)\, A_1 + (\frac{2}{3} \cdot 3)\, A_2$

$= 3.888\ R_B - 180 = 0 \quad \text{kN}\cdot\text{m}^3$

$R_B = \frac{180}{3.888} = 46.296\ \text{kN}$

$A_1 = 75\ \text{kN}\cdot\text{m}^2$



$A_3 = \frac{1}{2}(-60)(1.8) = -54\ \text{kN}\cdot\text{m}^2$

$A_4 = \frac{1}{2}(-24)(1.8) = -21.6\ \text{kN}\cdot\text{m}^2$

$EI\, t_{B/A} = 1.2\, A_1 + 1.2\, A_3 + 0.6\, A_4$

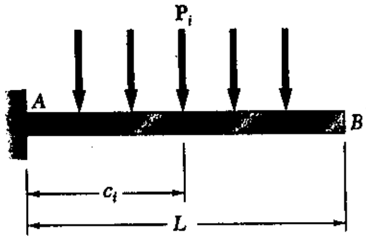
$= 12.24\ \text{kN}\cdot\text{m}^3$

(a) $y_B = t_{B/A} = \frac{EI\, t_{B/A}}{EI} = \frac{12.24}{1760} = 6.95 \times 10^{-3}\ \text{m}$

$= 6.95\ \text{mm}$

(b) $R_B = 46.3\ \text{kN}$

PROBLEM 9.C1



9.C1 Several concentrated loads can be applied to the cantilever beam *AB*. Write a computer program to calculate the slope and deflection of beam *AB* from $x = 0$ to $x = L$, using given increments Δx. Apply this program with increments $\Delta x = 50$ mm to the beam and loading of Probs. 9.79 and 9.80.

SOLUTION

FOR EACH LOAD, ENTER

P_i, c_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS

$R_A = R_A + P_i$

$M_A = M_A - P_i c_i$

COMPUTE SLOPE AND DEFLECTION

USE METHOD OF INTEGRATION:
STARTING WITH $x = 0$ AND UPDATING
THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A:

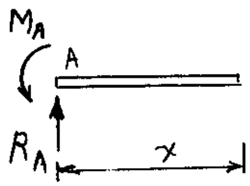
$$\theta = (1/EI)(R_A x^2/2.0 + M_A x)$$

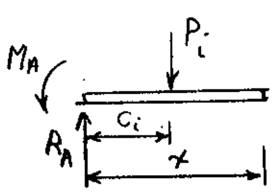
$$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$$

(2) DUE TO EACH LOAD WITH $c_i < x$:

$$\theta = -(1/EI)(P_i/2.0)(x - c_i)^2$$

$$y = -(1/EI)(P_i/6.0)(x - c_i)^3$$





AT $x = 0$, $y = \dfrac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION EQUAL ZERO

CONTINUED

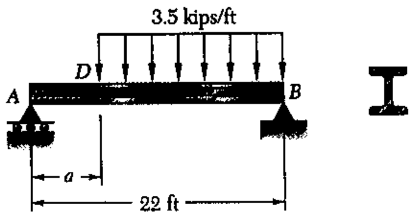
PROGRAM OUTPUT

Problem 9.79 and 9.80

At A: Force = 6.0 kN Couple = -6.0 kN·m

x m	Slope radians	Deflection m
.00	.000000	.000000
.05	-.000578	-.000015
.10	-.001126	-.000057
.15	-.001645	-.000127
.20	-.002134	-.000221
.25	-.002594	-.000340
.30	-.003024	-.000480
.35	-.003424	-.000642
.40	-.003794	-.000822
.45	-.004135	-.001021
.50	-.004447	-.001235
.55	-.004728	-.001465
.60	-.004980	-.001708
.65	-.005203	-.001962
.70	-.005395	-.002227
.75	-.005558	-.002501
.80	-.005699	-.002783
.85	-.005825	-.003071
.90	-.005936	-.003365
.95	-.006033	-.003664
1.00	-.006114	-.003968
1.05	-.006181	-.004275
1.10	-.006233	-.004586
1.15	-.006270	-.004898
1.20	-.006292	-.005213
1.25	-.006299	-.005527

PROBLEM 9.C2



9.C2 The 22-ft beam AB consists of a W21 × 62 rolled-steel shape and supports a 3.5 kips/ft distributed load as shown. Write a computer program and use it to calculate for values of a from 0 to 22 ft, using 1-ft increments, (*a*) the slope and deflection at D, (*b*) the location and magnitude of the maximum deflection. Use $E = 29 \times 10^6$ psi.

SOLUTION

ENTER LOAD w, LENGTH L, a

COMPUTE REACTION AT A

$$R_A = w\,(L-a)^2/(2.0\,L)$$

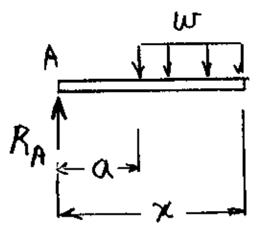
COMPUTE SLOPE AND DEFLECTION AT D

USING SINGULARITY FUNCTIONS:

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

$$\theta = (1/EI)(R_A a^2/2.0 + C_1)$$

$$y = (1/EI)(R_A a^3/6.0 + C_1 a)$$



$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w}{2}\langle x-a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{w}{24}\langle x-a\rangle^4 + C_1 x + C_2$$

FROM BOUNDARY CONDITIONS:

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

MAXIMUM y AT $\theta = 0$:

$$0 = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

IF $x_{max} \le a$

$$\frac{1}{2}R_A x^2 + C_1 = 0$$

$$x_{max} = \sqrt{\frac{-2.0\,C_1}{R_A}}$$

$$y_{max} = \frac{1}{6}R_A x_{max}^3 + C_1 x_{max}$$

COMPUTE LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

ASSUME $x < A$:

$$x_{max} = (-2.0\,C_1/R_A)^{\frac{1}{2}}$$

IF $x_{max} < a$, THEN

$$y_{max} = (1/EI)\left(\frac{1}{6}R_A x_{max}^3 + C_1 x_{max}\right)$$

IF $x_{max} > a$, THEN

BEGIN WITH $x = a$

$$\theta = (1/EI)\left(\frac{1}{2}R_A x - \frac{1}{6}(x-a)^3 + C_1\right)$$

INCREASE x BY SMALL AMOUNT UNTIL θ IS APPROXIMATELY 0

$$y_{max} = (1/EI)\left(\frac{1}{6}R_A x^3 - \frac{w}{24}(x-a)^4 + C_1 x\right)$$

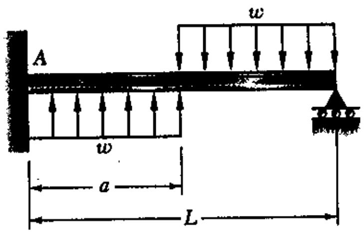
CONTINUED

PROBLEM 9.C2 CONTINUED

PROGRAM OUTPUT

a ft	theta D radians	yD in.	xm ft	ym in.
0.	-.00580	.000000	11.000	-.478290
1.	-.00569	-.068758	11.008	-.475922
2.	-.00539	-.133047	11.030	-.468860
3.	-.00494	-.189440	11.068	-.457231
4.	-.00439	-.235551	11.121	-.441245
5.	-.00378	-.269927	11.189	-.421192
6.	-.00314	-.291944	11.272	-.397443
7.	-.00250	-.301695	11.370	-.370441
8.	-.00188	-.299889	11.481	-.340699
9.	-.00131	-.287738	11.606	-.308795
10.	-.00080	-.266855	11.742	-.275364
11.	-.00036	-.239145	11.885	-.241090
12.	-.00001	-.206699	12.028	-.206700
13.	.00025	-.171684	12.159	-.172954
14.	.00043	-.136240	12.275	-.140603
15.	.00052	-.102374	12.376	-.110339
16.	.00054	-.071846	12.463	-.082792
17.	.00049	-.046069	12.537	-.058515
18.	.00039	-.026001	12.596	-.037987
19.	.00027	-.012036	12.643	-.021604
20.	.00014	-.003896	12.675	-.009677
21.	.00004	-.000530	12.695	-.002431
22.	.00000	.000000	12.702	.000000

PROBLEM 9.C3

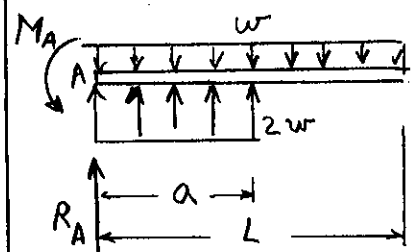


9.C3 The cantilever beam AB carries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam AB from $x = 0$ to $x = L$ using given increments Δx. Apply this program with increments $\Delta x = 100$ mm, assuming that $L = 2.4$ m, $w = 36$ kN/m, and (*a*) $a = 0.6$ m, (*b*) $a = 1.2$ m, (*c*) $a = 1.8$ m. Use $E = 200$ GPa.

SOLUTION

ENTER w, a, L

COMPUTE REACTION AT A

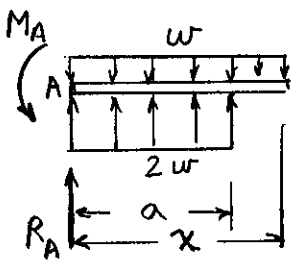


$$R_A = wL - 2.0wa$$
$$M_A = \frac{1}{2} wL^2 - \frac{1}{2} w a^2$$

COMPUTE SLOPE AND DEFLECTION

USE EQUATION OF ELASTIC CURVE

STARTING WITH $x=0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:



AT $x=0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION ARE ZERO

(1) DUE TO REACTIONS AT A

$$\theta = (1/EI)\left(\frac{1}{2} R_A x^2 + M_A x\right)$$
$$y = (1/EI)\left(\frac{1}{6} R_A x^3 + \frac{1}{2} M_A x\right)$$

(2) DUE TO LOAD w

$$\theta = -(1/EI)\left(\frac{1}{6} w x^3\right)$$
$$y = -(1/EI)\left(\frac{1}{24} w x^4\right)$$

(3) DUE TO LOAD $2w$

IF $x \leq a$

$$\theta = (1/EI)\left(\frac{1}{3} w x^3\right)$$
$$y = (1/EI)\left(\frac{1}{12} w x^4\right)$$

IF $x > a$

$$\theta = (1/EI)\left(\frac{1}{3} w x^3 - \frac{1}{3} w (x-a)^3\right)$$
$$y = (1/EI)\left(\frac{1}{12} w x^4 - \frac{1}{12} w (x-a)^4\right)$$

CONTINUED

PROBLEM 9.C3 CONTINUED

PROGRAM OUTPUT

Problem 9.C3 (a) a = 0.6 m

At A: Force = 43.2 kN Couple = -90.7 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000905	-.000046
.20	-.001762	-.000179
.30	-.002567	-.000396
.40	-.003318	-.000691
.50	-.004009	-.001058
.60	-.004638	-.001491
.70	-.005202	-.001983
.80	-.005703	-.002529
.90	-.006145	-.003122
1.00	-.006533	-.003756
1.10	-.006868	-.004427
1.20	-.007156	-.005128
1.30	-.007399	-.005856
1.40	-.007602	-.006607
1.50	-.007769	-.007376
1.60	-.007902	-.008160
1.70	-.008006	-.008955
1.80	-.008083	-.009760
1.90	-.008139	-.010571
2.00	-.008177	-.011387
2.10	-.008199	-.012206
2.20	-.008211	-.013027
2.30	-.008215	-.013848
2.40	-.008216	-.014669

Problem 9.C3 (b) a = 1.2 m

At A: Force = 0.0 kN Couple = -51.8 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000529	-.000026
.20	-.001055	-.000106
.30	-.001574	-.000237
.40	-.002081	-.000420
.50	-.002574	-.000653
.60	-.003048	-.000934
.70	-.003500	-.001262
.80	-.003926	-.001633
.90	-.004323	-.002046
1.00	-.004687	-.002497
1.10	-.005014	-.002982
1.20	-.005301	-.003498
1.30	-.005544	-.004041
1.40	-.005747	-.004606
1.50	-.005913	-.005189
1.60	-.006047	-.005787
1.70	-.006150	-.006398
1.80	-.006228	-.007017
1.90	-.006284	-.007642
2.00	-.006321	-.008273
2.10	-.006344	-.008906
2.20	-.006356	-.009541
2.30	-.006360	-.010177
2.40	-.006361	-.010813

CONTINUED

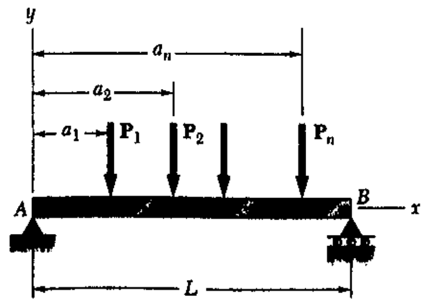
PROBLEM 9.C3 PROGRAM OUTPUTS CONTINUED

Problem 9.C3 (c) a = 1.8 m

At A: Force = -43.2 kN Couple = 13.0 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	.000111	.000006
.20	.000182	.000021
.30	.000215	.000041
.40	.000216	.000063
.50	.000187	.000083
.60	.000133	.000099
.70	.000056	.000109
.80	-.000039	.000110
.90	-.000149	.000101
1.00	-.000270	.000080
1.10	-.000398	.000046
1.20	-.000530	.000000
1.30	-.000662	-.000060
1.40	-.000790	-.000132
1.50	-.000911	-.000217
1.60	-.001021	-.000314
1.70	-.001116	-.000421
1.80	-.001193	-.000537
1.90	-.001248	-.000659
2.00	-.001286	-.000786
2.10	-.001309	-.000916
2.20	-.001320	-.001047
2.30	-.001325	-.001179
2.40	-.001325	-.001312

PROBLEM 9.C4



9.C4 The simply supported beam AB is of constant flexural rigidity EI and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to the beam and loading of (*a*) Prob. 9.14 with $\Delta x = 0.25$ m, (*b*) Prob. 9.15 with $\Delta x = 0.05$ m, (*c*) Prob. 9.132 with $\Delta x = 0.25$ m.

SOLUTION

FOR EACH LOAD, ENTER P_i, a_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS:

$M_A = M_A + P_i \, a_i$

$LOAD = LOAD + P_i$

THEN:

$R_B = M_A / L$

$R_A = LOAD - R_B$

COMPUTE SLOPE AND DEFLECTION

STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A

$\theta = (1/EI)(\frac{1}{2} R_A x^2)$

$y = (1/EI)(\frac{1}{6} R_A x^3)$

(2) DUE TO LOADS – CONSTANT PART

$CONST_1 = -\frac{1}{6} R_A L^2$

FOR 1 TO NUMBER LOADS

$CONST_2 = \frac{1}{6L} P_i (L - a_i)^3 + CONST_2$

THEN, TOTAL CONTRIBUTION FOR CONSTANT

$CONST = (1/EI)(CONST_1 + CONST_2)$

(3) DUE TO LOADS – REMAINING PART

IF $x \le a_i$

$\theta = (1/EI)(\frac{1}{2.0} R_A x^2)$

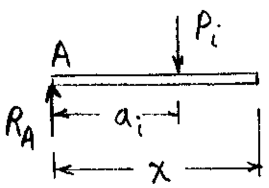
$y = (1/EI)(\frac{1}{6.0} R_A x^3)$

IF $x > a_i$

$\theta = (1/EI)(\frac{1}{2.0} R_A x^2 - \frac{1}{2.0} P_i (x - a_i)^2)$

$y = (1/EI)(\frac{1}{6.0} R_A x^3 - \frac{1}{6.0} P_i (x - a_i)^3)$

FOR LOAD P_i:



FOR $x < a_i$

$$EI \frac{d^2y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI\, y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

FOR $x > a_i$

$$EI \frac{d^2y}{dx^2} = R_A x - P_i (x - a_i)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P_i (x - a_i)^2 + C_3$$

$$EI\, y = \frac{1}{6} R_A x^3 - \frac{1}{6} (x - a_i)^3 + C_3 x + C_4$$

FROM BOUNDARY CONDITIONS

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L} (L - a_i)^3 - \frac{1}{6} R_A L^2$$

NOTE: R_A FOR LOAD P_i

CONTINUED

PROBLEM 9.C4 CONTINUED

PROGRAM OUTPUT

Problem 9.14

x m	theta rad*10**3	y mm
.000	-6.058	.000
.250	-5.831	-1.496
.500	-5.150	-2.878
.750	-4.014	-4.033
1.000	-2.423	-4.847
1.250	-.719	-5.235
1.500	.757	-5.225
1.750	2.007	-4.875
2.000	3.029	-4.241
2.250	3.824	-3.379
2.500	4.392	-2.348
2.750	4.733	-1.202
3.000	4.847	.000

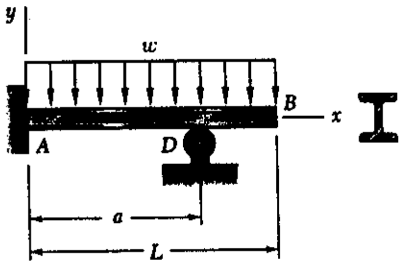
Problem 9.132

x m	theta rad*10**3	y mm
.000	-8.703	.000
.250	-8.615	-2.168
.500	-8.351	-4.293
.750	-7.911	-6.329
1.000	-7.296	-8.234
1.250	-6.505	-9.962
1.500	-5.538	-11.472
1.750	-4.483	-12.724
2.000	-3.428	-13.713
2.250	-2.373	-14.438
2.500	-1.319	-14.900
2.750	-.264	-15.098
3.000	.791	-15.032
3.250	1.802	-14.706
3.500	2.725	-14.138
3.750	3.560	-13.350
4.000	4.307	-12.365
4.250	4.967	-11.204
4.500	5.538	-9.889
4.750	6.021	-8.442
5.000	6.417	-6.886
5.250	6.725	-5.241
5.500	6.944	-3.531
5.750	7.076	-1.776
6.000	7.120	.000

Problem 9.15

x m	theta rad*10**3	y mm
.000	-2.490	.000
.050	-2.485	-.124
.100	-2.471	-.248
.150	-2.448	-.371
.200	-2.416	-.493
.250	-2.375	-.613
.300	-2.325	-.730
.350	-2.265	-.845
.400	-2.197	-.957
.450	-2.119	-1.065
.500	-2.032	-1.168
.550	-1.936	-1.268
.600	-1.831	-1.362
.650	-1.716	-1.451
.700	-1.593	-1.533
.750	-1.460	-1.610
.800	-1.318	-1.679
.850	-1.172	-1.741
.900	-1.025	-1.796
.950	-.879	-1.844
1.000	-.732	-1.884
1.050	-.586	-1.917
1.100	-.439	-1.943
1.150	-.293	-1.961
1.200	-.146	-1.972
1.250	.000	-1.976
1.300	.146	-1.972
1.350	.293	-1.961
1.400	.439	-1.943
1.450	.586	-1.917
1.500	.732	-1.884
1.550	.879	-1.844
1.600	1.025	-1.796
1.650	1.172	-1.741
1.700	1.318	-1.679
1.750	1.460	-1.610
1.800	1.593	-1.533
1.850	1.716	-1.451
1.900	1.831	-1.362
1.950	1.936	-1.268
2.000	2.032	-1.168
2.050	2.119	-1.065
2.100	2.197	-.957
2.150	2.265	-.845
2.200	2.325	-.730
2.250	2.375	-.613
2.300	2.416	-.493
2.350	2.448	-.371
2.400	2.471	-.248
2.450	2.485	-.124
2.500	2.490	.000

PROBLEM 9.C5



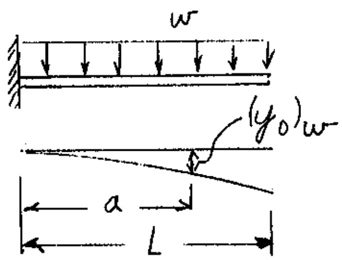
9.C5 The supports of beam AB consist of a fixed support at end A and a roller located at point D. Write a computer program to calculate the slope and deflection at the free end of the beam for values of a from 0 to L using given increments Δa. Apply this program to calculate the slope and deflection at point B for each of the following cases:

	L	Δa	w	E	Shape
(a)	12 ft	0.5 ft	1.6 kips/ft	29×10^6 psi	W16 × 57
(b)	3 m	0.2 m	18 kN/m	200 GPa	W460 × 113

SOLUTION

BEAM IS INDETERMINATE

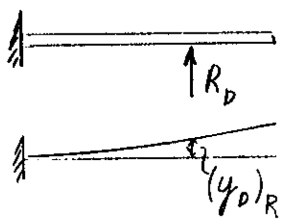
USE APPENDIX D AND SUPERPOSITION



DETERMINE REACTION AT D

DUE TO DISTRIBUTED LOAD

$$(y_D)_w = -\frac{w}{24EI}\left(a^4 - 4La^3 + 6L^2a^2\right)$$



DUE TO REDUNDANT LOAD:

$$(y_D)_R = \frac{R_D L^3}{3EI}$$

REDUNDANT REACTION:

SINCE $(y_D)_w + (y_D)_R = 0$:

$$R_D = \frac{3EI}{L^3}(y_D)_w$$

COMPUTE SLOPE AND DEFLECTION AT B

SUPERPOSE:

DUE TO DISTRIBUTED LOAD:

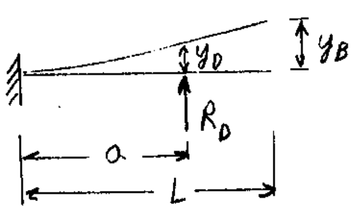
$$\theta_B = -\frac{wL^3}{6EI}$$

$$y_B = -\frac{wL^4}{8EI}$$

DUE TO R_D:

$$\theta_B = \frac{Pa^2}{2EI}$$

$$y_B = \frac{Pa^3}{3EI} + (L-a)\frac{Pa^2}{2EI}$$



$$\theta_B = \theta_D$$

$$y_B = y_D + (L-a)\theta_D$$

CONTINUED

PROBLEM 9.C5 CONTINUED

PROGRAM OUTPUT

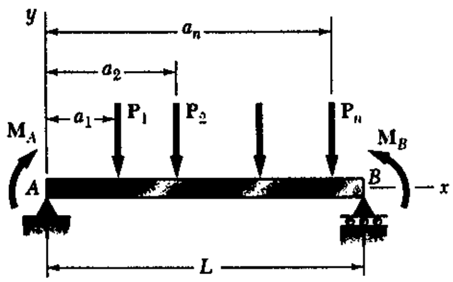
Problem 9.C5 (a)

a ft	theta B rad*10^-3	y at B in.
.0	-3.019	-.3260
.5	-2.743	-.2869
1.0	-2.483	-.2511
1.5	-2.238	-.2183
2.0	-2.007	-.1885
2.5	-1.790	-.1614
3.0	-1.586	-.1369
3.5	-1.395	-.1149
4.0	-1.216	-.0953
4.5	-1.049	-.0778
5.0	-.893	-.0624
5.5	-.748	-.0490
6.0	-.613	-.0374
6.5	-.488	-.0274
7.0	-.373	-.0191
7.5	-.266	-.0122
8.0	-.168	-.0067
8.5	-.077	-.0025
9.0	.006	.0006
9.5	.082	.0027
10.0	.152	.0037
10.5	.216	.0039
11.0	.274	.0033
11.5	.328	.0020
12.0	.377	.0000

Problem 9.C5 (b)

a m	theta B rad*10^-3	y at B mm
.0	-.728	-1.6389
.2	-.624	-1.3324
.4	-.529	-1.0663
.6	-.442	-.8374
.8	-.364	-.6426
1.0	-.293	-.4789
1.2	-.230	-.3435
1.4	-.174	-.2338
1.6	-.124	-.1472
1.8	-.079	-.0813
2.0	-.040	-.0337
2.2	-.006	-.0024
2.4	.023	.0149
2.6	.049	.0198
2.8	.072	.0143

PROBLEM 9.C6



9.C6 For the beam and loading shown, use the *Moment-Area Method* to write a computer program to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to calculate the slope and deflection at each concentrated load for the beam of (*a*) Prob. 9.76 with $\Delta x = 0.5$ m, (*b*) Prob. 9.116 with $\Delta x = 3$ in., (*c*) Prob. 9.119 with $\Delta x = 0.5$ ft.

SOLUTION

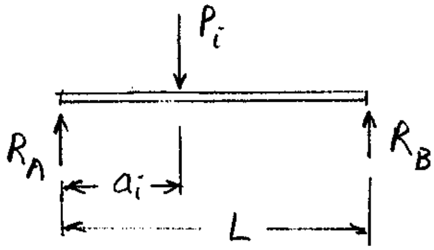
ENTER M_A AND M_B

FOR EACH LOAD ENTER P_i AND a_i

<u>DETERMINE REACTION AT A</u>

DUE TO MOMENTS AT ENDS:

$$(R_A)_1 = -(M_A - M_B)/L$$



DUE TO LOADS P_i:

FOR $i = 1$ TO NUMBER OF LOADS

$$R_B = R_B + P_i\, a_i/L$$

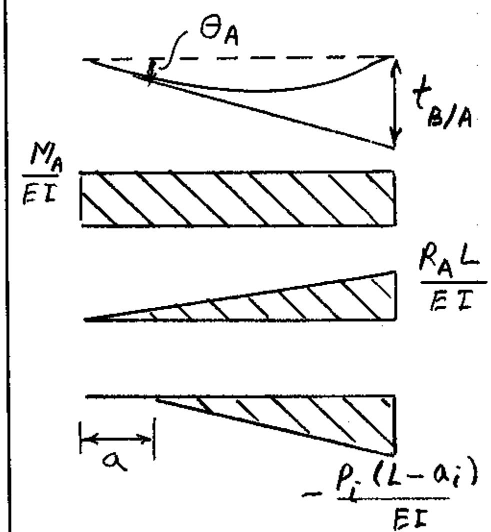
$$LOAD = LOAD + P_i$$

$$(R_A)_2 = LOAD - R_B$$

$$R_A = (R_A)_1 + (R_A)_2$$

<u>DETERMINE SLOPE AT A</u>

USE SECOND MOMENT-AREA THEOREM TO GET TANGENTIAL DEVIATION AT B



DUE TO M_A:

$$t_{B/A} = M_A L^2/(2.0\, EI)$$

DUE TO R_A:

$$t_{B/A} = R_A L^3/(6.0\, EI)$$

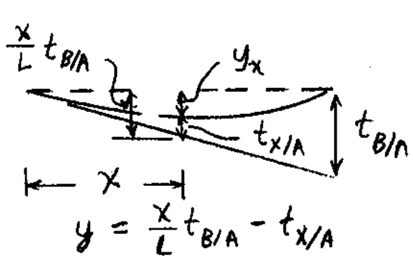
DUE TO LOADS P_i:

FOR $i = 1$ TO NUMBER OF LOADS

$$t_{B/A} = -P_i(L - a_i)^3/(6.0\, EI)$$

SUM $t_{B/A}$:

$$\theta_A = -t_{B/A}/L$$



<u>DETERMINE SLOPE AND DEFLECTIONS</u>

FOR $x = 0$ TO L, SUPERPOSE:

DUE TO M_A AND R_A:

$$\theta_x = \theta_A + (M_A x + R_A x^2/2.0)/EI$$

CONTINUED

PROBLEM 9.C6 CONTINUED

$$y_x = \frac{x}{L} t_{B/A} - M_A x^2/(2.0EI) - R_A x^3/(6.0EI)$$

DUE TO LOADS P_i :

DO FOR ALL LOADS WITH $a_i < X$

$$\theta_x = P_i (x - a_i)^2/(2.0\,EI)$$

$$y_x = P_i (x - a_i)^3/(6.0\,EI)$$

PROGRAM OUTPUT

Problem 9.76

x m	theta rad*1000	y at x mm
.000	-.600962	.000000
.500	-1.602564	.574252
1.000	-2.043269	1.509081
1.500	-1.923077	2.524039
2.000	-1.241987	3.338675
2.500	.000000	3.672543
3.000	1.241987	3.338676
3.500	1.923077	2.524039
4.000	2.043269	1.509082
4.500	1.602564	.574253
5.000	.600962	.000000

Problem 9.116

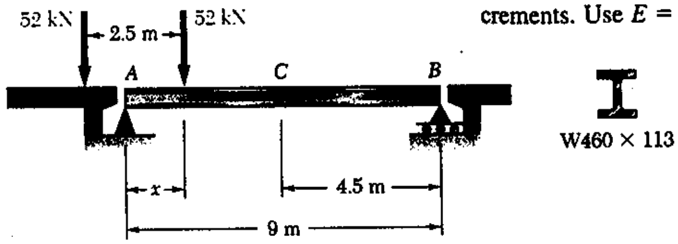
x ft	theta rad*1000	y at x in.
.000	-8.937931	.000000
.250	-8.813793	.026690
.500	-8.441380	.052634
.750	-7.820690	.077090
1.000	-6.951724	.099310
1.250	-5.834483	.118552
1.500	-4.468966	.134069
1.750	-2.979310	.145241
2.000	-1.489655	.151945
2.250	.000000	.154179
2.500	1.489655	.151945
2.750	2.979310	.145241
3.000	4.468966	.134069
3.250	5.834483	.118552
3.500	6.951724	.099310
3.750	7.820690	.077090
4.000	8.441380	.052634
4.250	8.813793	.026690
4.500	8.937931	.000000

Problem 9.119

x ft	theta rad*1000	y at x in.
.000	-2.118621	.000000
.500	-2.222069	.013051
1.000	-2.267586	.026549
1.500	-2.255172	.040146
2.000	-2.184828	.053495
2.500	-2.056552	.066248
3.000	-1.870345	.078058
3.500	-1.626207	.088577
4.000	-1.324138	.097457
4.500	-.993103	.104408
5.000	-.662069	.109374
5.500	-.331034	.112353
6.000	.000000	.113346
6.500	.331034	.112353
7.000	.662069	.109374
7.500	.993103	.104408
8.000	1.324138	.097457
8.500	1.626207	.088577
9.000	1.870345	.078058
9.500	2.056552	.066248
10.000	2.184828	.053495
10.500	2.255172	.040146
11.000	2.267586	.026549
11.500	2.222069	.013051
12.000	2.118621	.000000

PROBLEM 9.C7

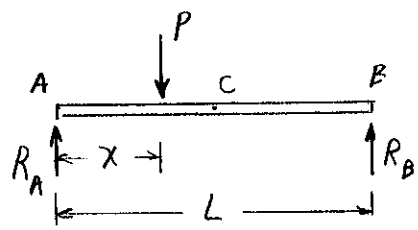
9.C7 Two 52-kN loads are maintained 2.5 m apart as they are moved slowly across beam *AB*. Write a computer program to calculate the deflection at the midpoint *C* of the beam for values of *x* from 0 to 9 m, using 0.5-m increments. Use E = 200 GPa.



SOLUTION

ENTER LOAD P, BEAM LENGTH L AND SPACE BETWEEN LOADS D

WILL SOLVE WITH MOMENT-AREA METHOD



DETERMINE DEFLECTION AT C

FOR $x = 0$ TO L

IF $0 \le x \le D$:

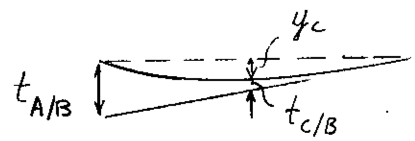
HAVE ONE LOAD TO LEFT OF C

$$R_B = Px/L$$

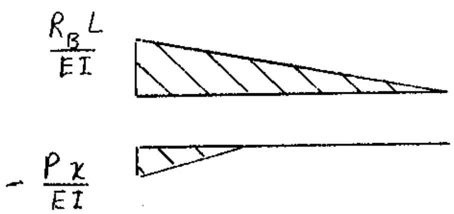
$$t_{A/B} = (R_B L^3 - Px^3)/(6.0EI)$$

$$t_{c/B} = R_B L^3/(48.0EI)$$

$$y_c = \frac{1}{2} t_{A/B} - t_{c/B}$$



$$y_c = \frac{1}{2} t_{A/B} - t_{c/B}$$



IF $D < x \le L/2$

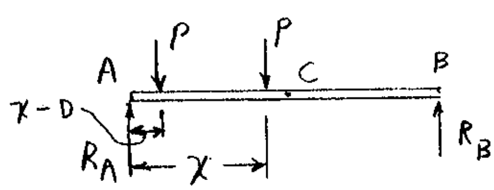
HAVE TWO LOADS TO LEFT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

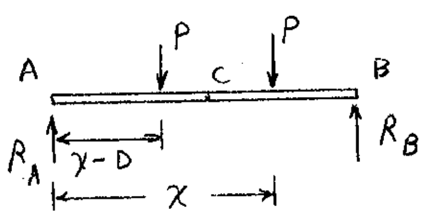
$$t_{c/B} = R_B L^3/(48.0EI)$$

$$y_c = \frac{1}{2} t_{A/B} - t_{c/B}$$



IF $L/2 < x \le (L/2 + D)$

HAVE ONE LOAD TO LEFT OF C AND ONE TO RIGHT OF C OR AT C



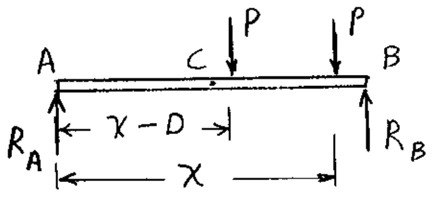
CONTINUED

$R_B = Px/L + P(x-D)/L$

$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$

$t_{C/B} = (R_B L^3/48.0 - P(x-\frac{L}{2})^3/6.0)/EI$

$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$



IF $(L/2 + D) < x < L$

HAVE BOTH LOADS TO RIGHT OF C

$R_B = Px/L + P(x-D)/L$

$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$

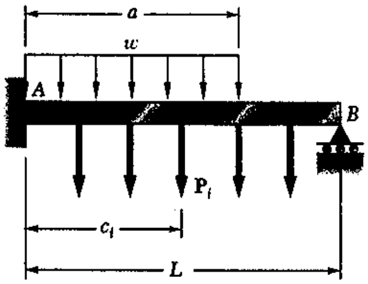
$t_{C/B} = (R_B L^3/48.0 - P(x-\frac{L}{2})^3/6.0 - P(x-D-\frac{L}{2})^3/6.0)/EI$

$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$

PROGRAM OUTPUT

x m	RB kN	ThetaB rad	YC mm
.0	.000	.00000	.00000
.5	2.889	.00315	1.17881
1.0	5.778	.00624	2.32839
1.5	8.667	.00921	3.41951
2.0	11.556	.01200	4.42296
2.5	14.444	.01456	5.30950
3.0	20.222	.01998	7.22872
3.5	26.000	.02499	8.94335
4.0	31.778	.02947	10.39493
4.5	37.556	.03331	11.52503
5.0	43.333	.03639	12.28492
5.5	49.111	.03859	12.66487
6.0	54.889	.03980	12.66487
6.5	60.667	.03989	12.28492
7.0	66.444	.03876	11.52503
7.5	72.222	.03629	10.39493
8.0	78.000	.03235	8.94335
8.5	83.778	.02684	7.22872
9.0	89.556	.01963	5.30950

PROBLEM 9.C8



9.C8 A uniformly distributed load w and several concentrated loads P_i may be applied to the cantilever beam AB. Write a computer program to determine the reaction at the roller support and apply this program to the beam and loading of (*a*) Prob. 9.57*a*, (*b*) Prob. 9.58*a*.

SOLUTION

THE BEAM IS INDETERMINATE

USE EQUATION OF ELASTIC CURVE

ENTER w AND FOR EACH LOAD P_i AND c_i

COMPUTE DISPLACEMENT AT B DUE TO LOADS

REACTION AT A:

DUE TO w

$$R_A = wa$$

$$M_A = \frac{1}{2} w a^2$$

FOR $i = 1$ TO NUMBER LOADS P_i

$$R_A = R_A - P_i$$

$$M_A = M_A - P_i c_i$$

FOR DISPLACEMENT AT B, SUPERPOSE:

DUE TO REACTION AT A

$$EI\, y_B = \frac{1}{6} R_A L^3 + \frac{1}{2} M_A L^2$$

DUE TO DISTRIBUTED LOADS

$$EI\, y_B = \frac{1}{24} \left(-wL^4 + w(L-a)^4\right)$$

DUE TO P_i

FOR $i = 1$ TO NUMBER LOADS

$$EI\, y_B = \frac{1}{6} P_i (L - c_i)^3$$

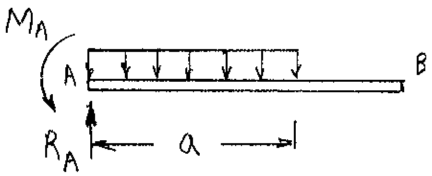
COMPUTE DISPLACEMENT AT B DUE TO UNIT R_B

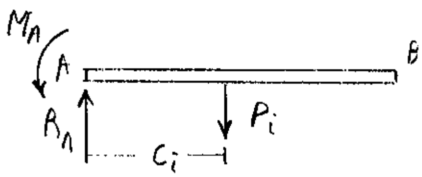
$$EI(y_B)_{UNIT} = \frac{1}{3} L^3$$

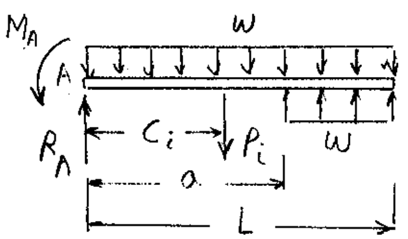
COMPUTE REACTION AT B

FROM $EI\, y_B + R_B EI\,(y_B)_{UNIT} = 0$

$$R_B = -y_B / (y_B)_{UNIT}$$

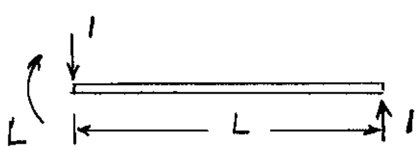






AT $x = 0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION ARE ZERO



$$EI \frac{d^2y}{dx^2} = -x + L$$

$$EI \frac{dy}{dx} = -\frac{1}{2}x^2 + Lx + C_1$$

$$EI\, y = -\frac{1}{6}x^3 + \frac{1}{2}Lx^2 + C_1 x + C_2$$

BOUNDARY CONDITIONS GIVE $C_1 = C_2 = 0$

CONTINUED

PROGRAM OUTPUT

```
Problem 9.57 (a)

    Reaction at Roller Support = 41.2500 kN

Problem 9.58 (a)

    Reaction at Roller Support = 11.5356 kN
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