$$xy'' - (2x+1)y' + (x+1)y = x^2 e^{2x} \quad / \cdot \frac{1}{x}$$

3. (a) $$y'' - \left(2+\frac{1}{x}\right)y' + \left(1+\frac{1}{x}\right)y = xe^{2x}$$

Parte 1 (a) conjunto fundamental

$$y_1(x) = e^x \quad , \quad y_2(x) = x^2 e^x$$

(1.0) $$W(e^x, x^2e^x) = \begin{vmatrix} e^x & x^2e^x \\ e^x & 2xe^x + x^2e^x \end{vmatrix} = e^{2x}\left[2x + x^2 - x^2\right]$$

$$= 2xe^{2x} \neq 0 \quad \text{si} \quad x \neq 0$$

(2.0) $$y_p(x) = -y_1(x)\int \frac{y_2(x)\, xe^{2x}}{W(e^x, x^2e^x)}\,dx + y_2(x)\int \frac{y_1(x)\, xe^{2x}}{W(e^x, x^2e^x)}\,dx$$

$$= -\frac{1}{2}e^x \int x^2 e^x\,dx + \frac{1}{2}x^2e^x \int e^x\,dx \quad , \quad y(x) = y_p + c_1e^x + c_2x^2e^x$$

3. (b) $y' = v$ (1.0)

$$v'' - 3v' + 2v = 1 + e^{3x}$$

$$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda_1 = \frac{3+\sqrt{9-8}}{2} = 2$$

$$\lambda_2 = \frac{3-1}{2} = 1$$

$$\Rightarrow \quad v_{p1} = \frac{1}{2}$$

$$v_{p2} = Ae^{3x}$$

$$9A - 9A + 2A = 1 \Rightarrow A = \frac{1}{2}$$

$$v_{p2} = \frac{1}{2}e^{3x}$$

$$v(x) = c_1e^{2x} + c_2e^x + \frac{1}{2} + \frac{1}{2}e^{3x}$$ (1.0)

$$y(x) = \int v(x) = c_1e^{2x} + c_2e^x + c_3 + \frac{x}{2} + \frac{1}{6}e^{3x}$$

(1.0)