P1/ forma 1:

a)

$$f(t) = \begin{cases} \cos(2t) & \text{si} \quad 0 \leq t < 2\pi \\ 0 & \text{si} \quad t \geq 2\pi \end{cases}$$

$$f(t) = \cos(2t) - \cos(2t)\, H(t-2\pi)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos(2t)\} - \mathcal{L}\{\cos(2t)\, H(t-2\pi)\}$$

$$= \frac{s}{s^2+4} - \mathcal{L}\{\cos(2t)\, H(t-2\pi)\}$$

Como $\cos(2t)$ es periódica con $T = 2\pi$

$$\Rightarrow \cos(2t) = \cos(2(t-2\pi))$$

por propiedad de desplazamiento temporal

$$\mathcal{L}\{f(t-a)\, H(t-a)\} = e^{-as} F(s)$$

$$\Rightarrow \mathcal{L}\{\cos(2(t-2\pi))\, H(t-2\pi)\} = e^{-2\pi s} \mathcal{L}\{\cos(2t)\} = e^{-2\pi s} \frac{s}{s^2+4}$$

$$\Rightarrow \mathcal{L}\{f(t)\} = \frac{s}{s^2+4}\left(1 - e^{-2\pi s}\right)$$

P1/ forma 2:

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st}\, dt = \int_0^{2\pi} \cos(2t)\, e^{-st}\, dt$$

Sea $I = \int_0^{2\pi} \cos(2t) e^{-st}\, dt$

$u = \cos(2t)$ $\quad dv = e^{-st}$

$du = -2\operatorname{sen}(2t)$ $\quad v = -\frac{1}{s} e^{-st}$

$$\Rightarrow I = \frac{-\cos(2t)}{s} e^{-st}\Big|_0^{2\pi} - \frac{2}{s}\int_0^{2\pi} e^{-st} \operatorname{sen}(2t)$$

$u = \operatorname{sen}(2t)$ $\quad dv = e^{-st}$

$du = 2\cos(2t)$ $\quad v = -\frac{1}{s} e^{-st}$

$$I = \frac{-\cos(2t)}{s} e^{-st}\Big|_0^{2\pi} + \frac{2}{s}\left[\frac{-\operatorname{sen}(2t)}{s} e^{-st}\Big|_0^{2\pi} + \frac{2}{s}\underbrace{\int_0^{2\pi} e^{-st}\cos(2t)\, dt}_{I}\right]$$

$$I = \frac{-1\, e^{-s\pi \cdot 2}}{s} + \cancel{\frac{e^{-s\cdot 0}}{s}}^{\,0} - \frac{2}{s}\left[\frac{2}{s} I\right]$$

$$I = \frac{1}{s} - \frac{e^{-2\pi s}}{s} - \frac{4}{s^2} I$$

$$I\left(1 + \frac{4}{s^2}\right) = \frac{1 - e^{-2\pi s}}{s}$$

$$I = \frac{\frac{1-e^{-2\pi s}}{s}}{\frac{s^2+4}{s^2}} = \frac{s\left(1 - e^{-2\pi s}\right)}{s^2+4}$$

b) $\mathcal{L}^{-1}\left(\frac{s}{(s^2+4)^2}\right) = \frac{1}{4} t \operatorname{sen}(2t) \Leftrightarrow \frac{s}{(s^2+4)^2} = \frac{1}{4}\mathcal{L}(t \operatorname{sen}(2t))$

$$\Rightarrow \frac{1}{4}\mathcal{L}(t \operatorname{sen}(2t)) = -\frac{1}{4}\frac{d}{ds}\mathcal{L}(\operatorname{sen}(2t))$$

$$= -\frac{1}{4}\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = -\frac{1}{2}\,\frac{-(s^2+4)'}{(s^2+4)^2}$$

$$= -\frac{1}{2}\,\frac{-2s}{(s^2+4)^2}$$

$$= \frac{s}{(s^2+4)^2}$$

c)

$$x'' + 4x = f(t)$$

$$x(0) = x'(0) = 0$$

$$\mathcal{L}(x'') + 4\mathcal{L}(x) = \mathcal{L}(f(t))$$

$$= s^2 \mathcal{L}(x) + 4\mathcal{L}(x) = \frac{s(1-e^{-2\pi s})}{s^2+4}$$

$$\mathcal{L}(x)(s^2+4) = \frac{s(1-e^{-2\pi s})}{s^2+4}$$

$$\mathcal{L}(x) = \frac{s(1-e^{-2\pi s})}{(s^2+4)^2}$$

$$\mathcal{L}(x) = \frac{s}{(s^2+4)^2} - \frac{se^{-2\pi s}}{(s^2+4)^2}$$

$$\Rightarrow x(t) = \frac{1}{4} t \operatorname{sen}(2t) - \mathcal{L}^{-1}\left\{ \frac{se^{-2\pi s}}{(s^2+4)^2} \right\}$$

usando $\mathcal{L}\{e^{-as} F(s)\} = f(t-a)\, u(t-a)$ tenemos:

$$\Rightarrow x(t) = \frac{1}{4} t \operatorname{sen}(2t) - \frac{1}{4}(t-2\pi) \operatorname{sen}(2(t-2\pi))\, u(t-2\pi)$$

d) $y(t) = \cos(t) + \int_0^t e^{-s} y(t-s)\, ds$

usando la convolución,

$$\mathcal{L}(y(t)) = \mathcal{L}(\cos(t)) + \mathcal{L}(e^{-t}) \cdot \mathcal{L}(y(t))$$

$$\mathcal{L}(y) = \frac{s}{s^2+1} + \frac{1}{s+1} \cdot \mathcal{L}(y)$$

$$\mathcal{L}(y(t))\left(1 - \frac{1}{s+1}\right) = \frac{s}{s^2+1}$$

$$\mathcal{L}(y(t))\left(\frac{s}{s+1}\right) = \frac{s}{s^2+1}$$

$$\mathcal{L}(y(t)) = \frac{s+1}{s^2+1}$$

$$\mathcal{L}(y(t)) = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$$y(t) = \cos(t) + \operatorname{sen}(t)$$