Pauta Control 3
Electromagnetismo: FI2002-4
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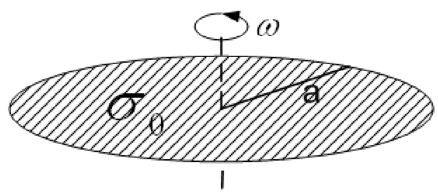
Pregunta 1

Parte a) y b) se resuelven con la misma forma integral del campo magnético:

$$\vec{B} = \frac{\mu_0}{4\pi} \int dq\vec{v} \times \frac{\vec{r} - \vec{r'}}{\left\| \vec{r} - \vec{r'} \right\|^3}$$

a)(2 puntos)
Campo magnético en el centro del disco:



$$dq = \sigma_0\, dS = \sigma_0\, r\, dr\, d\phi$$

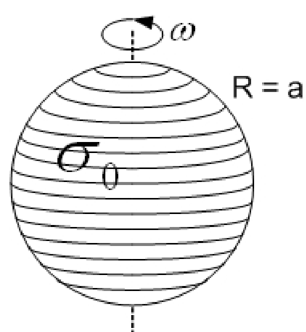
$$\vec{v} = r\,\omega\,\hat{\phi}$$

$$\vec{r} = 0$$

$$\vec{r'} = r\,\hat{r}$$

$$\vec{B}(0) = \frac{\mu_0}{4\pi} \int \sigma_0 r dr d\phi\, r\omega\hat{\phi} \times \frac{-r\hat{r}}{\|r\hat{r}\|^3} = \frac{\mu_0}{4\pi}\sigma_0\omega \int_0^a \int_0^{2\pi} \hat{k} dr d\phi = \frac{\mu_0\sigma_0}{2} a\omega\hat{k}$$

b)(2 puntos)
Campo magnético en el centro del casquete esférico:



$$dq = \sigma_0\, dS = \sigma_0\, a^2\, \sin\theta\, d\theta\, d\phi$$

$$\vec{v} = a\,\omega\, \sin\theta\,\hat{\phi}$$

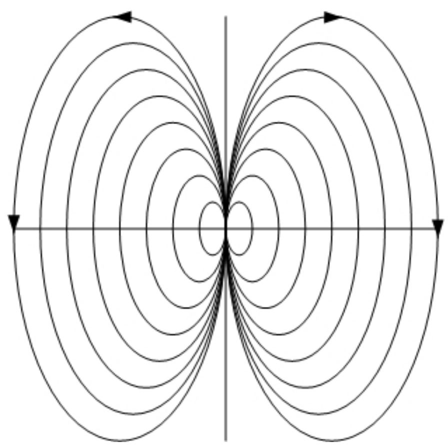
$$\vec{r} = 0$$

$$\vec{r'} = a\,\hat{r}$$

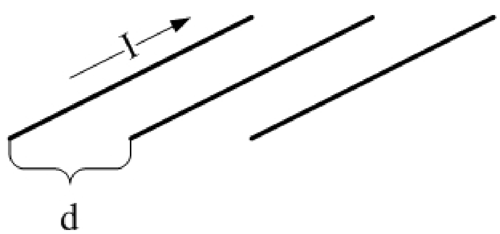
$$\vec{B}(0) = \frac{\mu_0}{4\pi}\int \sigma_0 a^2 \sin\theta d\theta d\phi \; a\omega \sin\theta \hat{\phi} \times \frac{-a\hat{r}}{\|a\hat{r}\|^3} = \frac{\mu_0}{4\pi}\sigma_0 \omega a \int_0^{\pi}\int_0^{2\pi} -\sin^2\theta \hat{\theta} d\phi d\theta$$

$$\vec{B}(0) = \frac{\mu_0}{4\pi}\sigma_0 \omega a \; 2\pi \hat{k}\int_0^{\pi} \sin^3\theta d\theta = \frac{2\mu_0\sigma_0}{3} a\omega\hat{k}$$

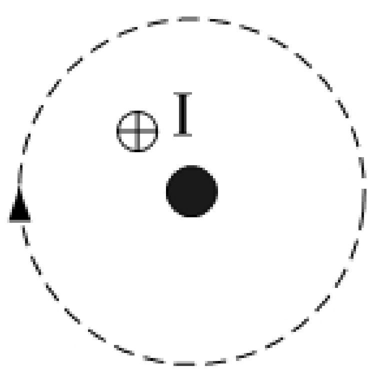
c)(2 puntos)
Lineas del campo:



Pregunta 2



Para encontra el campo hay que utilizar la ley de Ampere:



$$\vec{\nabla} \times \vec{} = \mu_0 \vec{J}$$

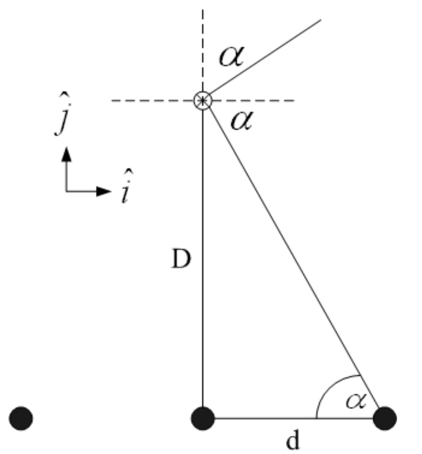
$$\Rightarrow \int_{\Gamma} \vec{B} = \mu_0 \int_S \vec{J} dS = \mu_0 I$$

$$\Rightarrow \vec{B}(r) = \frac{\mu_0 I}{2\pi \, r}\hat{\theta}$$

(1 puntos)

a)

Campo magnético generado por el alambre de la derecha en el punto D descompuesto en cada coordenada:



$$\vec{B} = \frac{\mu_0 I}{2\pi\, r}\hat{\theta}$$

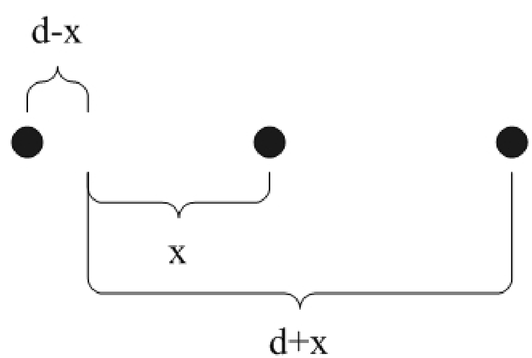
$$\frac{\mu_0 I}{2\pi\,\sqrt{d^2+D^2}}(\cos\alpha\,\hat{j} + \sin\alpha\,\hat{i})$$

$$\vec{B}_T = \frac{\mu_0 I}{2\pi}\left(\frac{\sin\alpha + \sin\alpha}{\sqrt{d^2+D^2}}\hat{i} + \frac{\hat{i}}{D} + \frac{\cos\alpha - \cos\alpha}{\sqrt{d^2+D^2}}\hat{j}\right)$$

$$\vec{B}_T = \frac{\mu_0 I}{2\pi}\left(\frac{1}{D} + 2\frac{D}{D^2+d^2}\right)\hat{i}$$

(1 puntos)

b)



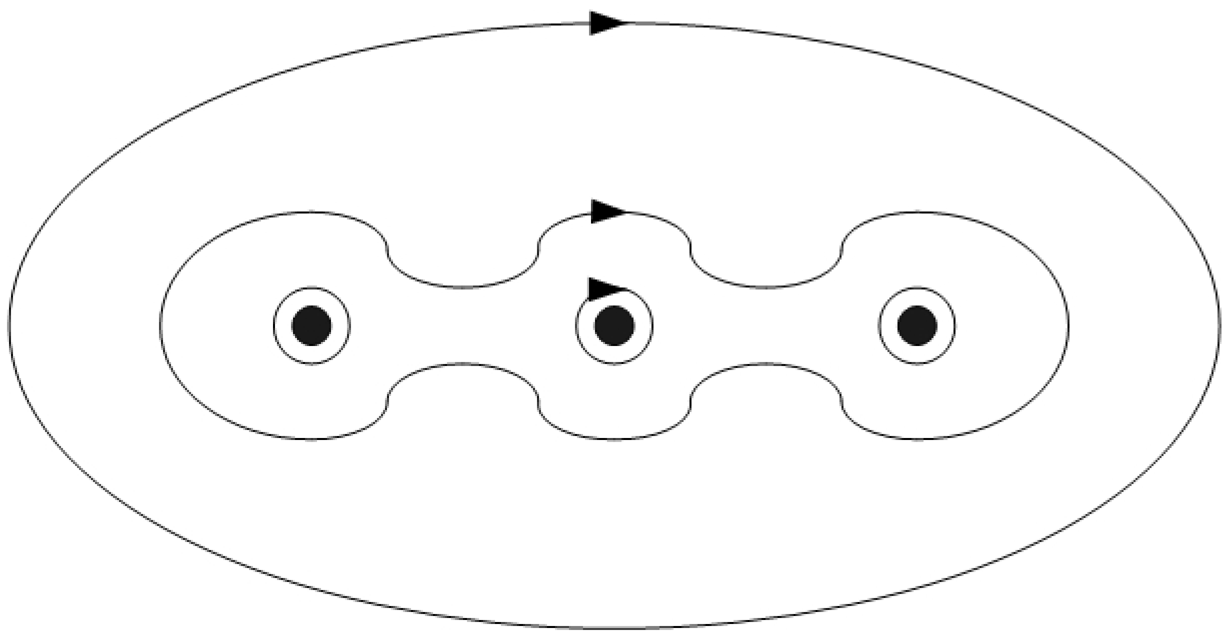
$$\vec{B}_T = \frac{\mu_0 I}{2\pi}\left(\frac{\hat{\theta}}{d-x} + \frac{\hat{\theta}'}{x} + \frac{\hat{\theta}''}{d+x}\right)$$

$$\vec{B}_T = \frac{\mu_0 I}{2\pi}\left(\frac{-1}{d-x} + \frac{1}{x} + \frac{1}{d+x}\right) = 0 \rightarrow \frac{-1}{d-x} + \frac{1}{x} + \frac{1}{d+x} = 0$$

$$x = \frac{\pm d}{\sqrt{3}}$$

(2 puntos)

c)



(2 puntos)