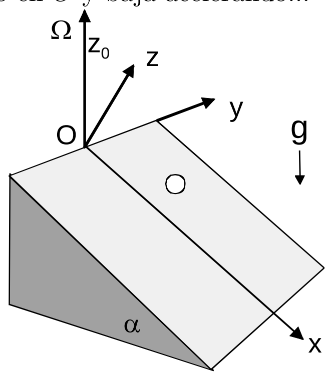
(agregar punto base)

Problema 1. Un plano inclinado en ángulo α respecto a la horizontal, rota en torno al eje z_0 vertical con velocidad angular constante de magnitud Ω. Una particula parte del reposo en O y baja acelerando...



Considerando el movimiento relativo al sistema rotante $Oxyz$ tenemos que

$$
\begin{aligned}
\vec{a}^{rel} &= \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} \\
\vec{v}^{rel} &= \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} \\
\vec{r} &= x\hat{\imath} + y\hat{\jmath} \\
\vec{\omega} &= \Omega\hat{k}_0 = \Omega(\cos\alpha\hat{k} - \sin\alpha\hat{\imath}) \\
\vec{F} &= N\hat{k} - mg(\cos\alpha\hat{k} - \sin\alpha\hat{\imath}) \\
\frac{d\vec{\omega}}{dt} &= 0
\end{aligned}
$$

luego usando el teorema de Coriolis despreciando los términos en Ω^2

$$\vec{a} = 2\vec{\omega} \times \vec{v}^{rel} + +\vec{a}^{rel}$$

y entonces

$$
\begin{aligned}
N\hat{k} - mg(\cos\alpha\hat{k} - \sin\alpha\hat{\imath}) &= m(2\Omega\hat{k}_0 \times (\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}) + \\
&\quad +\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}
\end{aligned}
$$

$2\vec{\omega}\times\vec{v}^{rel} = 2\Omega(-\sin\alpha, 0, \cos\alpha)\times(v_x, v_y, 0) = 2\Omega\,(-v_y\cos\alpha, v_x\cos\alpha, -v_y\sin\alpha)$
luego las componentes son

$$
\begin{aligned}
g\sin\alpha &= -2\Omega\dot{y}\cos\alpha + \ddot{x} \\
0 &= 2\Omega\dot{x}_x\cos\alpha + \ddot{y} \\
\frac{N}{m} - g\cos\alpha &= -2\Omega\dot{y}\sin\alpha
\end{aligned}
$$

$$
\begin{aligned}
g\sin\alpha &= -2\Omega\dot{y}\cos\alpha+\ddot{x} \\
0 &= 2\Omega\dot{x}\cos\alpha+\ddot{y}\Rightarrow\dot{y}=-2\Omega x\cos\alpha \\
\frac{N}{m}-g\cos\alpha &= -2\Omega\dot{y}\sin\alpha
\end{aligned}
$$

la primera da

$$
\begin{aligned}
\ddot{x} &= g\sin\alpha \\
\dot{x} &= gt\sin\alpha \\
\ddot{y} &= -2\Omega\dot{x}\cos\alpha=-2\Omega gt\sin\alpha\cos\alpha \\
\dot{y} &= -\Omega gt^2\sin\alpha\cos\alpha \\
y &= -\frac{1}{3}\Omega gt^3\sin\alpha\cos\alpha
\end{aligned}
$$

La normal será (despreciando Ω^2)

$$
\begin{aligned}
\frac{N}{m}-g\cos\alpha &= -2\Omega\dot{y}\sin\alpha \\
N &\simeq mg\cos\alpha
\end{aligned}
$$

Problema 2

Llamemos f_1 y f_2 las fuerzas de roce (hacia la izquierda). Las ecuaciones de movimiento son

$$
\begin{aligned}
3M\ddot{x}_1 &= -kx_1+k(x_2-x_1)-f_1 \\
f_1R &= \frac{1}{2}(3M)R^2\frac{\ddot{x}_1}{R} \\
2M\ddot{x}_2 &= -k(x_2-x_1)-f_2 \\
f_2R &= \frac{1}{2}(2M)R^2\frac{\ddot{x}_1}{R}
\end{aligned}
$$

al eliminar las fuerzas de roce

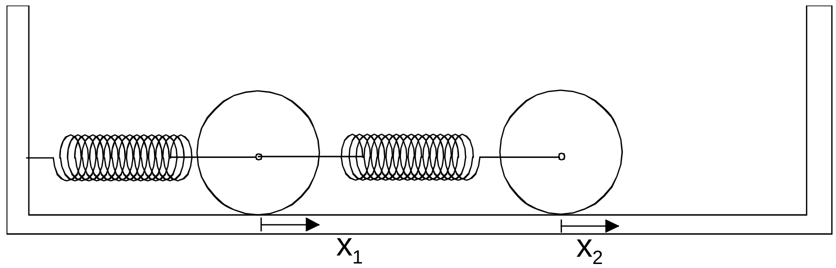
$$
\begin{aligned}
\frac{9}{2}\ddot{x}_1 &= \frac{k}{M}(x_2-2x_1) \\
3\ddot{x}_1 &= -\frac{k}{M}(x_2-x_1)
\end{aligned}
$$

al sustituir $x_1=Ae^{-i\omega t},\ x_2=Be^{-i\omega t}$ resulta

$$
\begin{aligned}
(-\frac{9}{2}\omega^2+\frac{2k}{M})A-\frac{k}{M}B &= 0 \\
\frac{k}{M}A+(3\omega^2-\frac{k}{M})B &= 0
\end{aligned}
$$

EL DETERMINANTE es

$$
(-\frac{9}{2}\omega^2+\frac{2k}{M})(3\omega^2-\frac{k}{M})+\frac{k^2}{M^2}=0
$$



cuyas soluciones son

$$\begin{aligned}
\omega_1^2 &= \frac{1}{9}\frac{k}{M} \\
\omega_2^2 &= \frac{2}{3}\frac{k}{M}
\end{aligned}$$

las razones resultan

$$\begin{aligned}
\frac{A}{B} &= \frac{-(3\omega^2 - \frac{k}{M})}{\frac{k}{M}} \\
\frac{A_1}{B_1} &= \frac{-(\frac{1}{3}\frac{k}{M} - \frac{k}{M})}{\frac{k}{M}} = \frac{2}{3} \\
\frac{A_2}{B_2} &= \frac{-(2\frac{k}{M} - \frac{k}{M})}{\frac{k}{M}} = -1
\end{aligned}$$

luego

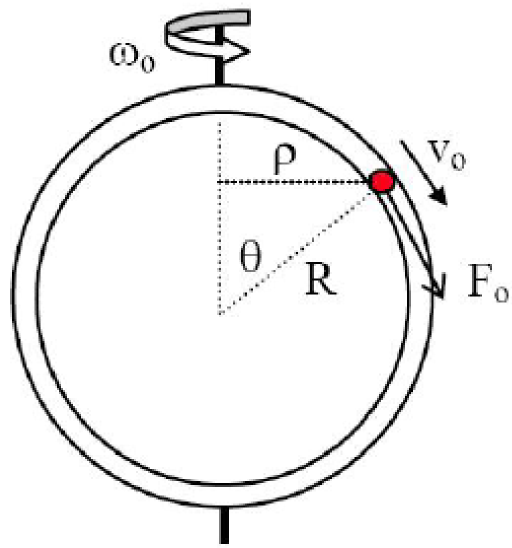
$$\begin{aligned}
x_1 &= A_1 e^{-i\omega_1 t} + A_2 e^{-i\omega_2 t} = \frac{2}{3} B_1 e^{-i\omega_1 t} - B_2 e^{-i\omega_2 t} \\
x_2 &= B_1 e^{-i\omega_1 t} + B_2 e^{-i\omega_2 t}
\end{aligned}$$

de donde se despejan las coordenadas normales

$$\begin{aligned}
x_1 &= \frac{2}{3}\xi_1 - \xi_2 \\
x_2 &= \xi_1 + \xi_2
\end{aligned}$$

$$\begin{aligned}
\xi_1 &= \frac{3}{5}x_2 + \frac{3}{5}x_1 \\
\xi_2 &= -\frac{3}{5}x_1 + \frac{2}{5}x_2
\end{aligned}$$

Problema 3



Para un sistema rotante con origen en el centro del aro

$$
\begin{aligned}
\vec{a} &= 2\vec{\omega} \times \vec{v}^{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{a}^{rel} \\
V &= \frac{1}{2} k \rho^2 \\
\vec{v}^{rel} &= R\dot{\theta}\hat{\theta} \\
\vec{a}^{rel} &= -R\dot{\theta}^2 \hat{r} + R\ddot{\theta}\hat{\theta} \\
\vec{\omega} \times (\vec{\omega} \times \vec{r}) &= -\omega_0^2 \vec{r}
\end{aligned}
$$

luego

$$m(2\vec{\omega} \times \vec{v}^{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{a}^{rel}) = \vec{N} - k\rho\hat{\rho}$$

$$m(2\omega_0 \hat{k}_0 \times R\dot{\theta}\hat{\theta} + \omega_0^2 R \hat{k}_0 \cdot \hat{r}\hat{k}_0 - \omega_0^2 \vec{r} - R\dot{\theta}^2 \hat{r} + R\ddot{\theta}\hat{\theta}) = \vec{N} - k\rho\hat{\rho}$$

para eliminar la normal multiplicamos $\cdot\hat{\theta}$ resultando

$$m(R\ddot{\theta} - \omega_0^2 R \cos\theta \sin\theta) = -k\rho\hat{\rho} \cdot \hat{\theta}$$

$$
\begin{aligned}
m(R\ddot{\theta} - \omega_0^2 R \cos\theta \sin\theta) &= -k\rho \cos\theta \\
m(R\ddot{\theta} - \omega_0^2 R \cos\theta \sin\theta) &= -kR \sin\theta \cos\theta \\
\ddot{\theta} + (\frac{k}{m} - \omega_0^2) \cos\theta \sin\theta &= 0
\end{aligned}
$$

Si además existe una fuerza $\vec{F}(\theta) = F_0(\theta)\hat{\theta}$

$$m(R\ddot{\theta} - \omega_0^2 R \cos\theta \sin\theta) = -kR \sin\theta \cos\theta + F_0(\theta)$$

la coondición $\ddot{\theta} = 0$ conduce a

$$(kR - m\omega_0^2 R) \cos\theta \sin\theta = F_0(\theta)$$