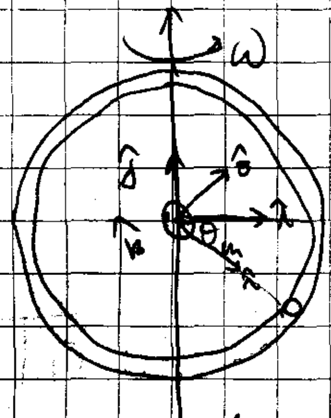
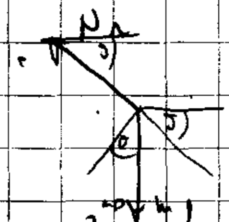
P5



$\downarrow g$ $\quad \vec{a}_0 =$ como el ejercicio 5



$$m\vec{a} = \left(-mR\dot{\theta}^2 - m\omega_0^2 R\cos^2\theta\right)\hat{r}$$
$$+\left(mR\ddot{\theta} + m\omega_0^2 R\cos\theta\,\mathrm{sen}\,\theta\right)\hat{\theta}$$
$$+\,m2R\omega_0\dot{\theta}\,\mathrm{sen}\,\theta\,\hat{k}$$

$$= \left(mg\,\mathrm{sen}\,\theta - N_r\right)\hat{r} - mg\cos\theta\,\hat{\theta} + N_k\hat{k}$$

$\hat{r})\quad mg\,\mathrm{sen}\,\theta - N_r = -mR\dot{\theta}^2 - m\omega^2 R\cos^2\theta$

$\hat{\theta})\quad -mg\cos\theta = mR\ddot{\theta} + m\omega^2 R\cos\theta\,\mathrm{sen}\,\theta$

$\hat{k})\quad N_k = m2R\omega_0\dot{\theta}\,\mathrm{sen}\,\theta$

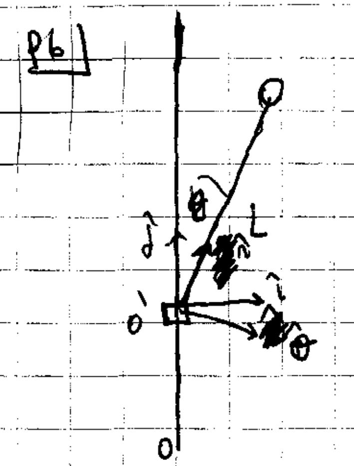
$$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = -\frac{g}{R}\int_0^{\theta_m}\cos\theta - \omega^2\int \mathrm{sen}\,\theta\cos\theta$$

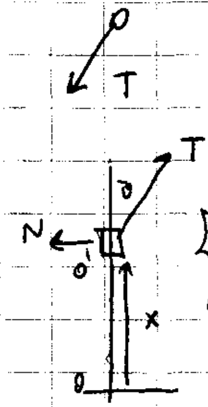
$$0 = -\frac{g}{R}\left(\mathrm{sen}\,\theta_m - \cancel{\mathrm{sen}\,0}\right) - \frac{\omega^2}{2}\left(\mathrm{sen}^2\theta_m - \cancel{\mathrm{sen}^2 0}\right)$$

$$\mathrm{sen}\,\theta_m = \frac{-2g}{\omega^2 R}$$

$$0 = \mathrm{sen}\,\theta_m\left(\frac{g}{R} + \frac{\omega^2}{2}\mathrm{sen}\,\theta_m\right) \qquad \frac{2g}{\omega^2 R} < 1$$

P6





$$\begin{cases} N = T\sin\theta \\ m\ddot{x} = T\cos\theta \end{cases}$$

$$\vec{a} = \frac{T\cos\theta}{m_{ANI}}\hat{j} + L\dot{\theta}^2(-\hat{r}) + L\ddot{\theta}\,\hat{\theta}$$

$$m\vec{a} = \frac{m}{m_{ANI}} T\cos\theta\,\hat{j} + mL\dot{\theta}^2(-\hat{r}) + mL\ddot{\theta}\,\hat{\theta}$$

$$= -T(\hat{r})$$

$$\hat{j} = \cos\theta(\hat{r}) - \sin\theta(\hat{\theta})$$

① $\hat{r}$) $-T = T\cos^2\theta - mL\dot{\theta}^2$

② $\hat{\theta}$) $0 = -T\sin\theta\cos\theta + mL\ddot{\theta}$

① $\Rightarrow$ $mL\dot{\theta}^2 = T(1+\cos^2\theta)$ es ts en ②

$$\cancel{mL}\ddot{\theta} = \cancel{mL}\dot{\theta}^2\frac{\sin\theta\cos\theta}{1+\cos^2\theta}$$

$\dot{\theta}\,d\dot{\theta}$

$\sin^2+\cos^2=1$

$$\int_{v_0/L}^{\dot{\theta}}\frac{d\dot{\theta}}{\dot{\theta}} = \int_0^{\theta}\frac{\sin\theta\cos\theta}{1+\cos^2\theta}\,d\theta \qquad -\frac{1}{2}\ln(1+\cos^2\theta)$$

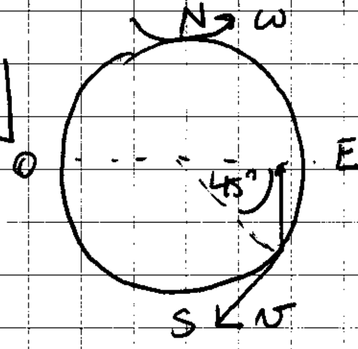
$\cos 2\theta = \cos^2 - \sin^2$
$= 2\cos^2 - 1$

$$\ln\left(\frac{\dot{\theta}}{v_0/L}\right) = -\frac{1}{2}\ln(1+\cos^2\theta)\Big|_0^{\theta}$$

$$= -\frac{1}{2}\left[\ln(1+\cos^2\theta) - \ln(1+\cos^2 0)\right] = -\frac{1}{2}\ln\left(\frac{1+\cos^2\theta}{2}\right)$$

$$\boxed{\dot{\theta} = \frac{v_0}{L}\sqrt{\frac{2}{1+\cos^2\theta}}} \qquad \boxed{T = \frac{mv_0^2}{L}\frac{2}{(1+\cos^2\theta)^2}} \qquad = \ln\left(\sqrt{\frac{2}{1+\cos^2\theta}}\right)$$

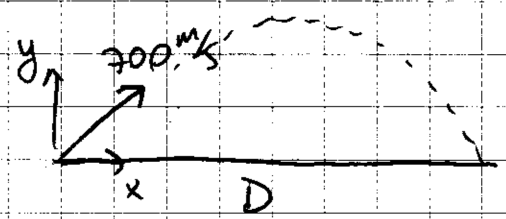
P7



CALCULE LA DESVIACION HORIZONTAL DE UN PROYECTIL DISPARADO DESDE 45° S HACIA EL SUR CON UN ∡ ELEV. 45°

Fza Coriolis vista desde la Tierra $= -2\vec{\omega}\times\vec{v}'$

En el H.S. apunta hacia el $\hat{E}$



tpo de vuelo = t_V

$$y = 700\,\frac{\sqrt{2}}{2}t_V - \frac{1}{2}g\,t_V^2 = 0$$

$$t_V = \frac{2\times 700\times\frac{\sqrt{2}}{2}}{9{,}8} \doteq 101\ s$$

alcance $= D = 700\times\cos 45°\times t_V = 700\times\frac{\sqrt{2}}{2}\times 101\,s =$

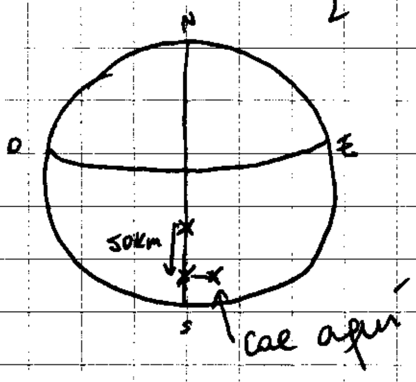
~ 50 Km.

desviación: acel. coriolis $= a_c = 2\omega v\,\text{sen}(\underset{45°}{\text{latitud}})$

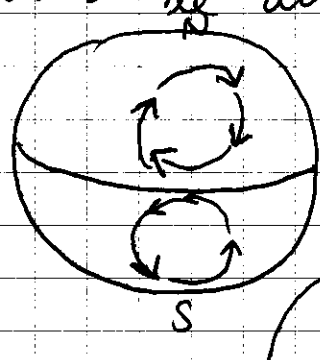
$$= 2\times\frac{2\pi}{86400}\times 700\times\frac{\sqrt{2}}{2} = 0{,}07199\ \frac{m}{s^2}$$

$$= \frac{1}{2}a_c t_V^2 = 0{,}5\times 0{,}07199\times 101^2 = 367\ m$$

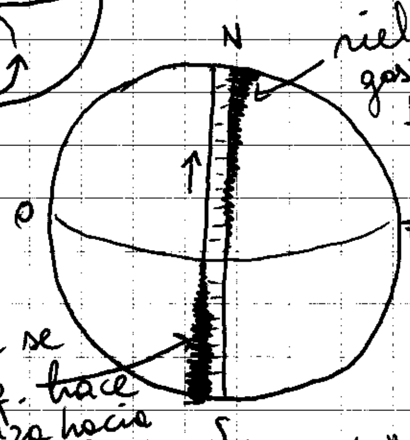
cae a 367 m. al este del objetivo



esto es igual que el movimiento de las masas de aire (debido a Δ presión)



tren que se mueve hacia el N en un meridiano:

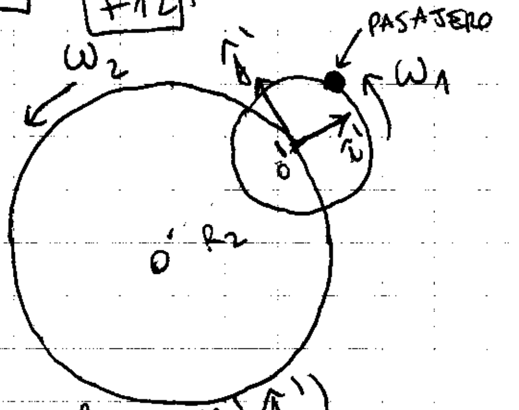


riel este se gasta p.q. hace fuerza hacia el oeste, "evitando" que tren se vaya al Este

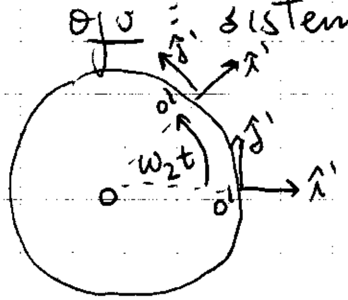
riel oeste se gasta p.q. hace fuerza hacia el este, "evitando" que el tren se vaya al oeste

P8 Parque de diversiones — carro gira con ω_1 c/r a la plataforma giratoria (ω_2)

$\boxed{F12}$

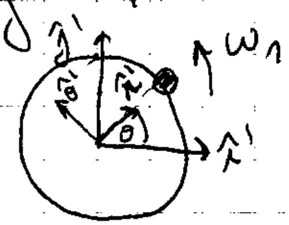


ojo: sistema $\hat{\imath}'\hat{\jmath}'$ gira con ω_2, el punto o' describe un círculo radio R_2 veloc. ang. de ω_2



$$\vec{a}_o = -\omega_2^2 R_2 (\hat{\imath}')$$

C/r al S.M. $\hat{\imath}'$ el pasajero se mueve en círculo radio R_1 veloc. ang. ω_1



$$\vec{r}\,' = R_1 \cos\theta\,\hat{\imath}' + R_1 \operatorname{sen}\theta\,\hat{\jmath}' = R_1\,\hat{r}'$$

más fácil expresarlo así

$$\vec{v}\,' = R_1\omega_1\,\hat{\theta}'$$

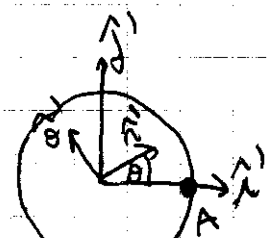
+ fácil

$$\vec{a}\,' = -R_1\omega_1^2\,\hat{r}'$$

coriolis $= 2\vec{\omega}\times\vec{v}\,' = 2\omega_2\hat{k}\times R_1\omega_1\hat{\theta}' = 2\omega_1\omega_2 R_1(-\hat{r}')$

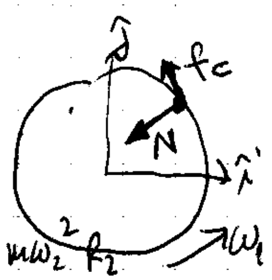
centrífuga: $\vec{\omega}\times\vec{r}\,' = \omega_2\hat{k}\times R_1\hat{r}' = \omega_2 R_1(\hat{\theta}')$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}\,') = \omega_2\hat{k}\times\omega_2 R_1(\hat{\theta}') = \omega_2^2 R_1(-\hat{r}')$$

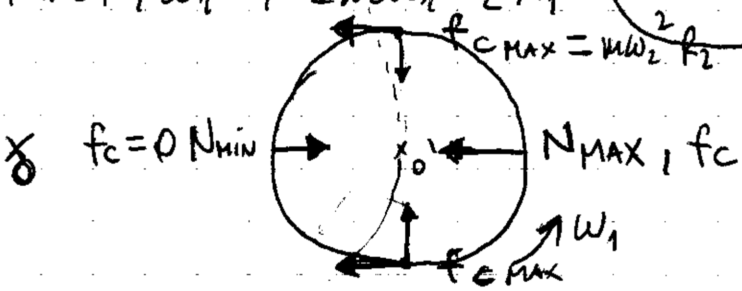


$\vec{a}_o$ conviene expresarlo en $\hat{r}'$ y $\hat{\theta}'$, para eso: $\hat{\imath}' = \cos\theta\,\hat{r}' - \operatorname{sen}\theta\,\hat{\theta}'$, suponiendo que el pasajero parte del punto A $\Rightarrow$ $\theta = \omega_1 t$

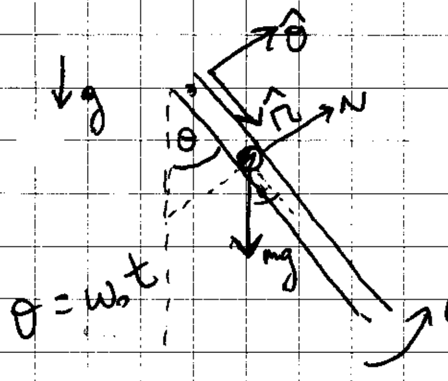
$$m\vec{a} = -\left(m\omega_2^2 R_2\cos(\omega_1 t) + mR_1\omega_1^2 + 2m\omega_1\omega_2 R_1 + m\omega_2^2 R_1\right)(\hat{r}') + \left(m\omega_2^2 R_2 \operatorname{sen}(\omega_1 t)\right)(\hat{\theta}')$$



$$N = m\omega_2^2(R_1 + R_2\cos\omega_1 t) + mR_1\omega_1^2 + 2m\omega_1\omega_2 R_1$$
$$f_c = m\omega_2^2 R_2 \operatorname{sen}(\omega_1 t)$$



pg | $\boxed{F33}$



$\vec{r} = r\hat{r}$

$\vec{v} = \dot r \hat r + r\omega_0\hat\theta$

$\vec a = (\ddot r - r\omega_0^2)\hat r + (2\dot r\omega_0)\hat\theta$

$\hat r)\ mg\cos(\omega_0 t) = m(\ddot r - r\omega_0^2)$ ①

$\hat\theta)\ N - mg\,sen(\omega_0 t) = m2\dot r\omega_0$ ②

ec ①: homogénea $\ddot r - r\omega_0^2 = 0$

$\Rightarrow r(t) = Ae^{\omega_0 t} + Be^{-\omega_0 t}$ (o equivalente: $A\cosh(\omega_0 t)+B\,senh(\omega_0 t)$)

particular $\ddot r - r\omega_0^2 = g\cos(\omega_0 t)$

probando: $r = C\cos\omega_0 t$

$\dot r = -C\omega_0\, sen(\omega_0 t)$

$\ddot r = -C\omega_0^2\cos(\omega_0 t)$

$\Rightarrow (-C\omega_0^2 - C\omega_0^2 - g)\cos(\omega_0 t)=0 \Rightarrow C = -\frac{g}{2\omega_0^2}$

solución $r(t) = Ae^{\omega_0 t}+Be^{-\omega_0 t} - \frac{g}{2\omega_0^2}\cos(\omega_0 t)$

c.i. $r(t=0)=0=A+B-\frac{g}{2\omega_0^2}$

$\dot r(t=0)=0 = A\omega_0 - B\omega_0 + \frac{g}{2\omega_0}\cancel{sen\,0}^{\,0} \Rightarrow A=B=\frac{g}{4\omega_0^2}$

$\Rightarrow r(t) = \frac{g}{2\omega_0^2}\left(\cosh(\omega_0t)-\cos(\omega_0 t)\right)$

$\dot r(t) = \frac{g}{2\omega_0}\left(senh(\omega_0 t)+sen(\omega_0 t)\right)$

ec ② $N(t) = mg\left(senh(\omega_0 t)+2\,sen(\omega_0 t)\right)$

la posición horizontal es alcanzada en $\omega_0 t = \frac{\pi}{2}$ $t=\frac{\pi}{2\omega_0}$

$L = \frac{g}{2\omega_0^2}\left(\cosh\frac\pi2 - \cancel{\cos\frac\pi2}^{\,0}\right) = 1{,}2546\,\frac{g}{\omega_0^2}$

la E. mecánica entregada a la partícula cuando alcanza pos. horizontal es $\frac12 mv^2$ con $v^2 = \dot r^2 + r^2\omega_0^2$ $(r=L)$

$\dot r = \frac{g}{2\omega_0}\left(senh(\frac\pi2)+sen\frac\pi2\right) = 1{,}6506\,\frac{g}{\omega_0}$ luego $v^2 = \frac{g^2}{\omega_0^2}\left(\underbrace{1{,}2546^2+1{,}6506^2}_{4{,}2986}\right)$

$\boxed{E = 2{,}1493\,\frac{mg^2}{\omega_0^2}}$