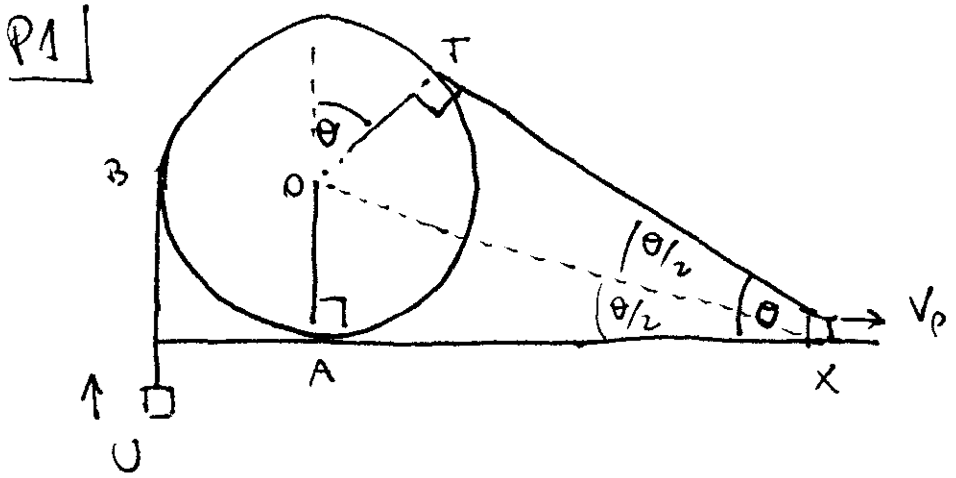
P1



a) $\overline{AX} = R + v_o t$ (por cond. inicial partícula der.)

por geom: $\tan\frac{\theta}{2} = \frac{\overline{OA}}{\overline{AX}} = \frac{R}{R+v_o t}$ (1)

en $\theta = \pi/3$ $\tan\pi/6 = 1/\sqrt{3} = \frac{R}{R+v_o\bar{t}}$

sale de Δ equilátero

en ese instante: $R + v_o\bar{t} = R\sqrt{3}$

derivando (1) c/r al tiempo:

$$\frac{\dot\theta}{2}\sec^2\frac{\theta}{2} = \frac{-R\,v_o}{(R+v_o t)^2}$$

$$\Rightarrow \quad \dot\theta = -\frac{2Rv_o}{(R+v_o t)^2}\cos^2\frac{\theta}{2}$$

evaluando en $t = \bar{t}$ $\theta = \pi/3$

$$\dot\theta\Big|_{\theta=\pi/3} = \frac{-2Rv_o}{(R\sqrt{3})^2}\cos^2\frac{\pi}{6} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$= -\frac{2Rv_o}{3R^2}\times\frac{3}{4} = \frac{-v_p}{2R}$$

(negativo, como debía ser)

P1 b) por geometría se tiene: $\overline{AX} = \overline{TX} = R + v_0 t$

$\widehat{BT} = R\left(\theta + \frac{\pi}{2}\right)$

$\overline{UB}$ = segmento de cuerda que cuelga

$$\overline{UB} + \widehat{BT} + \overline{TX} = \text{const.} = L \quad \text{(largo cuerda)}$$

$$\overline{UB} + R\left(\theta + \frac{\pi}{2}\right) + R + v_0 t = L \quad \Big/ \frac{d}{dt}$$

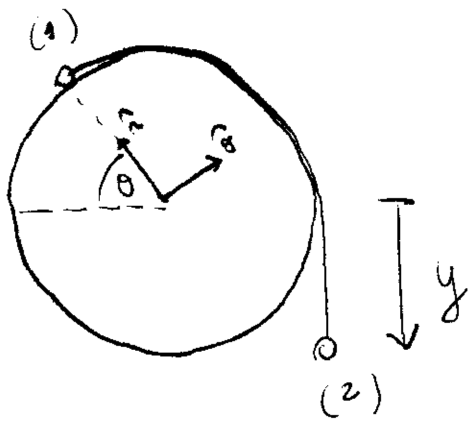
$$\Rightarrow \dot{\overline{UB}} + R\dot{\theta} + v_0 = 0$$

$$\Rightarrow \dot{\overline{UB}} = -R\dot{\theta} - v_0$$

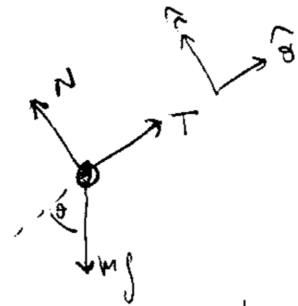
eval. en $\theta = \frac{\pi}{3}$ $\quad \dot{\theta} = -\frac{v_0}{2R}$

$$\dot{\overline{UB}} = -R\left(-\frac{v_0}{2R}\right) - v_0 = -\frac{v_0}{2}$$

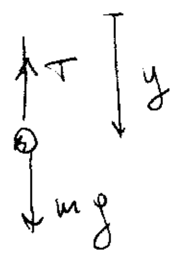
P2



DCL (1)



DCL (2)



$\vec F = m\vec a$ para (1): $\hat r)\quad N - mg\,\text{sen}\,\theta = -mR\dot\theta^2$ (1)

$\hat\theta)\quad T - mg\cos\theta = mR\ddot\theta$ (2)

para (2) $\quad mg - T = m\ddot y$

pero $\underline{y = R\theta} \Rightarrow mg - T = mR\ddot\theta$ (3)

(2) + (3) $\Rightarrow \quad mg(1-\cos\theta) = 2mR\ddot\theta$

pero $\ddot\theta = \dfrac{d\dot\theta}{d\theta}\dot\theta$

$$\Rightarrow \int_{\theta_{inic}=0}^{\dot\theta}\dot\theta\,d\dot\theta = \frac{g}{2R}\int_0^{\theta}(1-\cos\theta)\,d\theta$$

dado que $\cancel{\dot y_{inic}}^{\,0} = R\dot\theta_{inic} \Rightarrow \dot\theta_{inic} = 0$

$$\Rightarrow \frac{\dot\theta^2}{2} = \frac{g}{2R}(\theta - \text{sen}\,\theta)\Big|_0^{\theta}$$

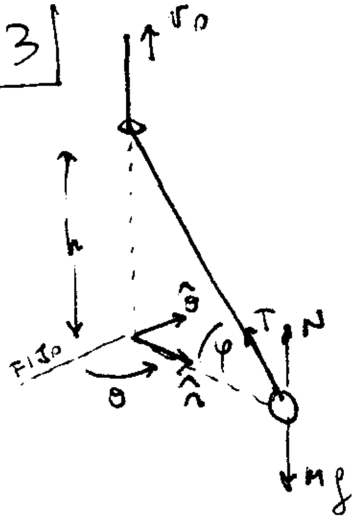
$$\boxed{\dot\theta^2 = \frac{g}{R}(\theta - \text{sen}\,\theta)}$$ esto en (1)

$N = mg\,\text{sen}\,\theta - mR\dot\theta^2 = mg\,\text{sen}\,\theta - mg(\theta - \text{sen}\,\theta)$

$\boxed{N = mg(2\,\text{sen}\,\theta - \theta)}$ con lo cual $N = 0$ para $\bar\theta$ t.q.: $\text{sen}\,\bar\theta = \bar\theta/2$ $<$ $\bar\theta = 0$, $\bar\theta = 1{,}897 \simeq 109°$

ahí se despega

P3



$\vec F = m\vec a \Rightarrow$

$\hat n)\quad -T\cos\varphi = m(\ddot\rho - \rho\dot\theta^2) \quad (1)$

$\hat\theta)\quad 0 = m(\rho\ddot\theta + 2\dot\rho\dot\theta) \quad (2)$

$\hat k)\quad N + T\,\text{sen}\,\varphi - mg = 0 \quad (3)$

de (2) : LADO DER. $= \dfrac{1}{\rho}\dfrac{d}{dt}(\rho^2\dot\theta) = 0$

$\Rightarrow \rho^2\dot\theta = cte = \rho_0^2\omega_0 = K$

$\Rightarrow \dot\theta = \dfrac{K}{\rho^2} \quad (4)$

veamos parte a)

$\sqrt{h^2+\rho^2} + v_0 t = \text{cnst} = \text{largo cuerda} \Big/ \dfrac{d}{dt}$

$\Rightarrow \dfrac{\rho\dot\rho}{\sqrt{h^2+\rho^2}} + v_0 = 0$

$\Rightarrow \dot\rho = -v_0\dfrac{\sqrt{h^2+\rho^2}}{\rho} \Big/ \dfrac{d}{dt}$

ocupando

$\ddot\rho = -v_0\left(\dfrac{1}{\rho}\dfrac{\rho\dot\rho}{\sqrt{h^2+\rho^2}} - \dfrac{\sqrt{h^2+\rho^2}}{\rho^2}\dot\rho\right)$

$\ddot\rho = -v_0\left(\dfrac{-v_0}{\rho} + v_0\dfrac{h^2+\rho^2}{\rho^3}\right)$

$\ddot\rho = -v_0^2\left(\dfrac{-\rho^2+h^2+\rho^2}{\rho^3}\right)$

$\ddot\rho = -v_0^2\dfrac{h^2}{\rho^3}$ (5) rpta parte a)

ocupando (4) y (5) en (1)

$-T\cos\varphi = m\left(-\dfrac{v_0^2h^2}{\rho^3} - \dfrac{K^2}{\rho^3}\right)$ pero por geom. $\cos\varphi = \dfrac{\rho}{\sqrt{h^2+\rho^2}}$

$\Rightarrow T = m(v_0^2h^2 + \rho_0^4\omega_0^2)\dfrac{\sqrt{h^2+\rho^2}}{\rho^4}$ rpta parte b)

P3 cont...

de (3) $N = mg - T \operatorname{sen}\varphi$ pero $\operatorname{sen}\varphi = \dfrac{h}{\sqrt{h^2+\rho^2}}$

usando esto y la ec. para T tenemos:

$$N = mg - m\left(v_0^2 h^2 + \rho_0^4 \omega_0^2\right)\frac{h}{\rho^4}$$

por lo tanto N se hace 0 para $\rho =$

$$\rho^* = \left[\frac{h}{g}\left(v_0^2 h^2 + \rho_0^4 \omega_0^2\right)\right]^{1/4}$$