algunos problemas resueltos.

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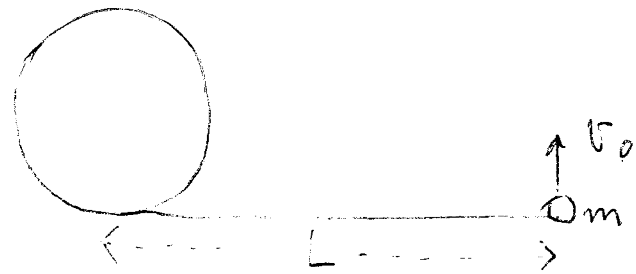

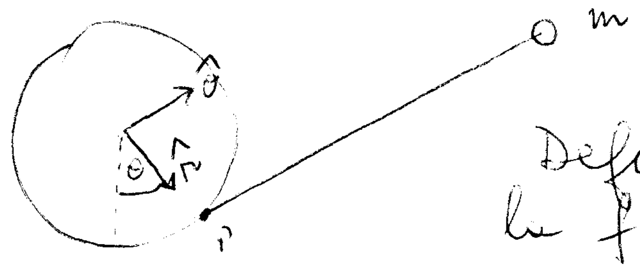

$\hat{r}$ apunta hacia punto P = punto donde cuerda deja de tocar el carrete

Definiendo θ como en la figura, la cuerda se ha enrollado $R\theta$

por lo tanto:

vector posición $m = \vec{r} = R\hat{r} + (L - R\theta)\hat{\theta}$

derivando c/r a t:

$$\vec{v} = R\dot{\theta}\,\hat{\theta} + -R\dot{\theta}\,\hat{\theta} + (L - R\theta)\dot{\theta}(-\hat{r})$$

$$\vec{v} = (L - R\theta)\dot{\theta}(-\hat{r})$$

$$\vec{a} = \left(-R\dot{\theta}^2 + (L - R\theta)\ddot{\theta}\right)(-\hat{r}) - (L - R\theta)\dot{\theta}^2(\hat{\theta})$$

$$\vec{F} = T(-\hat{\theta})$$ (la única fuerza es la tensión de la cuerda) según $-\hat{\theta}$

acordarse: $\boxed{\begin{array}{l} \dfrac{d\hat{r}}{dt} = \dot{\theta}\,\hat{\theta} \\ \dfrac{d\hat{\theta}}{dt} = \dot{\theta}(-\hat{r}) \end{array}}$

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$$\vec{F} = m\vec{a} \Rightarrow$$

$$\hat{r}) \quad -mR\dot{\theta}^2 + m(L-R\theta)\ddot{\theta} = 0 \qquad (1)$$

$$\hat{\theta}) \quad m(L-R\theta)\dot{\theta}^2 = T \qquad (2)$$

ocupando ec. (1):

$$(L-R\theta)\ddot{\theta} = R\dot{\theta}^2$$

usando que $\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\dot{\theta}$ entonces:

$$(L-R\theta)\frac{d\dot{\theta}}{d\theta}\dot{\theta} = R\dot{\theta}^2 \qquad \text{(ec. separable)}$$

$$\frac{d\dot{\theta}}{\dot{\theta}} = \frac{R}{L-R\theta}\,d\theta \qquad (*)$$

acordarse que $\vec{v}_{ini} = -v_0\,\hat{r}$ (ver fig.)

y: $\vec{v} = -(L - R\cancel{\theta_0})\dot{\theta}_{ini}\,(\hat{r})$

$$\Rightarrow v_0 = (L - \quad)\dot{\theta}_{ini}$$

$$\Rightarrow \dot{\theta}_{ini} = \frac{v_0}{L}$$

volviendo a (*) y integrando:

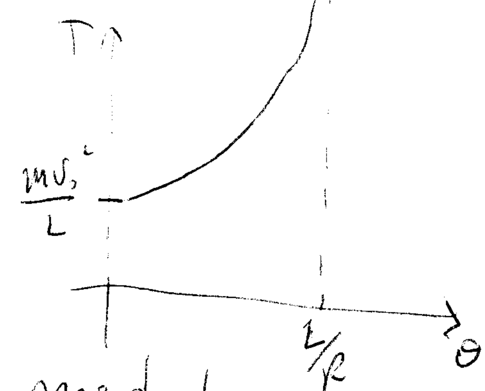
$$\ln \frac{\dot{\theta}}{v_0/L} = -\ln(L-R\theta)\Big|_0^{\theta}$$

$$\ln \frac{\dot{\theta}}{v_0/L} = \ln\left(\frac{L}{L-R\theta}\right) \qquad /\exp(x)$$

$$\Rightarrow \boxed{\dot{\theta} = \frac{v_0}{L-R\theta}}$$

esto en ec (2)

$$\Rightarrow \boxed{T(\theta) = \frac{m v_0^2}{L-R\theta}}$$



nótese que $T \to \infty$ a medida que $\theta \to L/R$ (en algún momento se corta la cuerda)

¡hagan el experimento: enrollen un hilo en su dedo con una masa en un extremo (una piedra p. ej.) y denle vuelta → les va a doler el dedo

¿T en función del tiempo?

tenemos: $$\frac{d\theta}{dt} = \frac{v_0}{L-R\theta}$$

$$\int_0^{\theta} d\theta \,(L - R\theta) = \int_0^t v_0 \, dt$$

$$\frac{1}{2}(L-R\theta)^2 \left(\frac{1}{-R}\right)\Big|_0^{\theta} = v_0 t$$

$$-\frac{1}{2R}\left[(L-R\theta)^2 - L^2\right] = v_0 t$$

$$\Rightarrow (L-R\theta)^2 = L^2 - 2Rv_0 t$$

de ahí uno despeja $\theta(t)$

$$L - R\theta = \sqrt{L^2 - 2Rv_0 t}$$

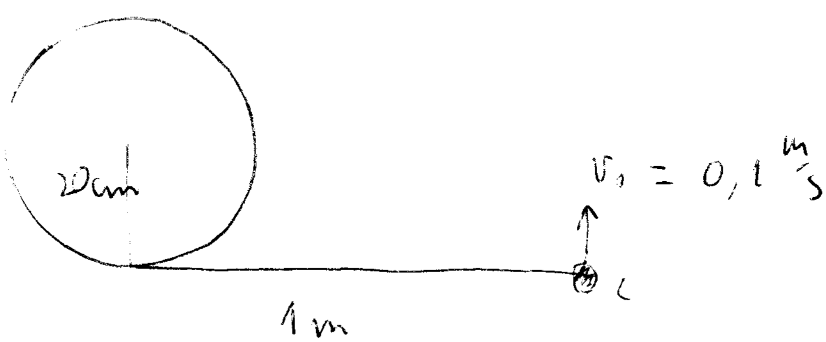
luego:

$$\boxed{T(t) = \frac{m v_0^2}{\sqrt{L^2 - 2Rv_0 t}}}$$

$$T = \infty \quad \text{en} \quad t = \frac{L^2}{2Rv_0}$$

eg.

$$t = \frac{1^2}{2 \times 0{,}2 \times 0{,}1} = 25\ s.$$



la cuerda se cortará antes de los 25 s. (hay que elegir una masa puntual chica)

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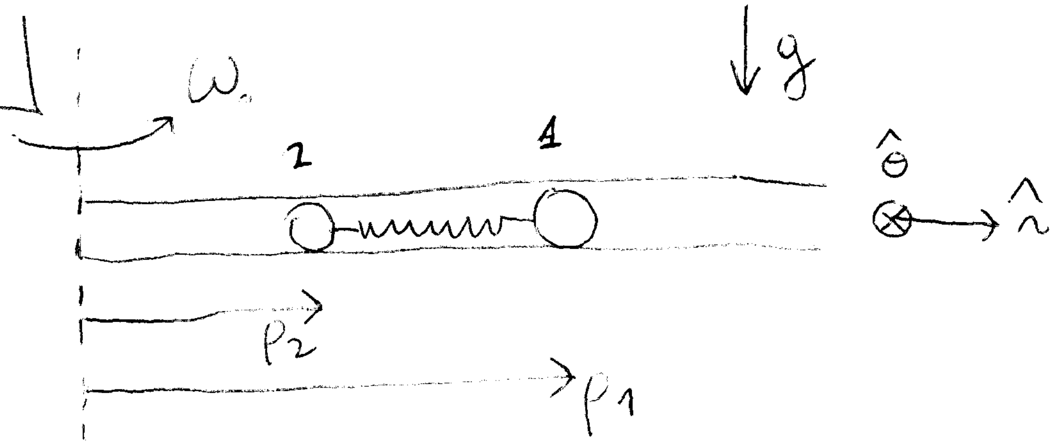


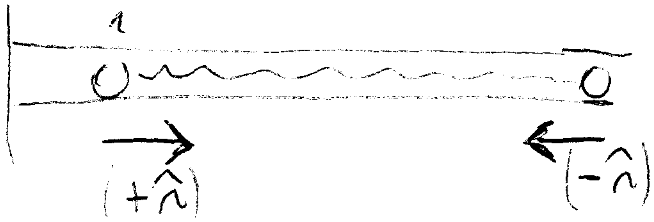
$$\vec{a}_1 = (\ddot{\rho}_1 - \rho_1 \omega_0^2)\,\hat{r} + 2\rho_1 \omega_0\,\hat{\theta}$$

fza. que siente $(1) = k(\rho_1 - \rho_2 - L_0)(-\hat{r})$

fza. que siente $(2) = k(\rho_1 - \rho_2 - L_0)(+\hat{r})$

esto funciona en los 2 casos:

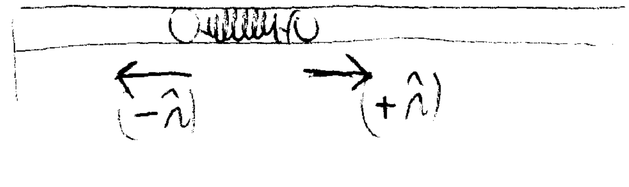
si $(\rho_1 - \rho_2) > L_0$ i.e. resorte estirado



concuerdan los signos

si $(\rho_1 - \rho_2) < L_0$ i.e. resorte comprimido

$\rho_1 - \rho_2 - L_0$ es negativo



signos O.K. ✓

$$\vec{F} = m\vec{a}$$

masa ① $\left\{ \begin{array}{lll} \hat{r}) & m(\ddot{\rho}_1 - \rho_1 \omega_0^2) & = -k(\rho_1 - \rho_2 - L_0) \\ \hat{\theta}) & m\, 2 \dot{\rho}_1 \omega_0 & = N^1_{\hat{\theta}} \\ \hat{k}) & m g & = N^1_z \end{array} \right.$ } normales estas ecs. no nos sirven

masa ② $\left\{ \begin{array}{lll} \hat{r}) & m(\ddot{\rho}_2 - \rho_2 \omega_0^2) & = +k(\rho_1 - \rho_2 - L_0) \\ \hat{\theta}) & m\, 2 \dot{\rho}_2 \omega_0 & = N^2_{\hat{\theta}} \\ \hat{k}) & m g & = N^2_z \end{array} \right.$ } normales → no nos sirven

$$\ddot{\rho}_1 - \rho_1 \omega_0^2 = -\frac{k}{m}(\rho_1 - \rho_2 - L_0)$$

$$\ddot{\rho}_2 - \rho_2 \omega_0^2 = +\frac{k}{m}(\rho_1 - \rho_2 - L_0)$$

si $m_1 \neq m_2$ se dificulta el álgebra

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si restamos.

$$(\ddot{\rho}_1 - \ddot{\rho}_2) - \omega_0^2(\rho_1 - \rho_2) = -\frac{2k}{m}(\rho_1 - \rho_2 - L_0)$$

si definimos $s = \rho_1 - \rho_2$ queda

$$\ddot{s} - \omega_0^2 s = -\frac{2k}{m}(s - L_0)$$

$$\Rightarrow \ddot{s} + \left(\frac{2k}{m} - \omega_0^2\right) s = \frac{2k}{m} L_0$$

Hay que identificar 2 casos:

(1) $\frac{2k}{m} - \omega_0^2 > 0$ ó $\omega_0 < \sqrt{\frac{2k}{m}}$

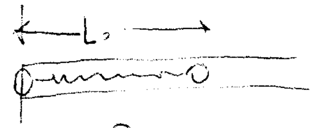
en ese caso la solución a la ec. dif. para "s" es del tipo $A\cos t + B\,\text{sen}\, t$

Sea $\Omega^2 = \frac{2k}{m} - \omega^2$

$$\ddot{s} + \Omega^2 s = \frac{2k}{m} L_0$$

$$\Rightarrow s(t) = A\cos\Omega t + B\,\text{sen}\,\Omega t + \overbrace{\frac{2kL_0}{m\Omega^2}}^{\text{particular}}$$

cond. inicial: $s(t=0) = L_0$



$$\dot{s}(t=0) = \dot{\rho}_1 - \dot{\rho}_0 = 0$$

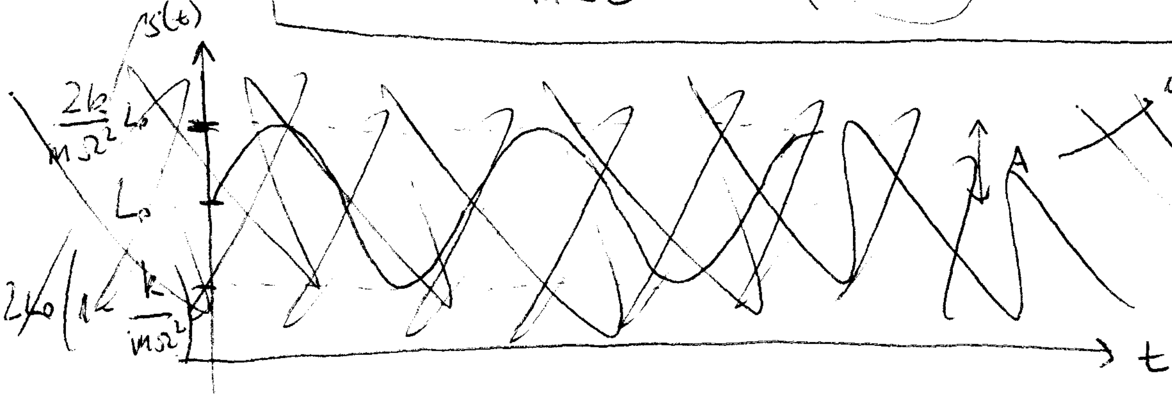
$$s(t=0) = A + \frac{2kL_0}{m\Omega^2} = L_0 \Rightarrow A = L_0\left(1 - \frac{2k}{m\Omega^2}\right)$$

$$\dot{s} = -A\Omega\,\text{sen}\,\Omega t + B\Omega\cos\Omega t$$

$$\dot{s}(t=0) = B\Omega = 0 \Rightarrow B = 0$$

$$\Omega = \sqrt{\frac{2k}{m} - \omega^2}$$

$$\therefore \boxed{s(t) = \frac{2kL_0}{m\Omega^2} - \left(\overbrace{\frac{2k}{m\Omega^2}}^{>1} L_0 - L_0\right)\cos\Omega t}$$



amplitud $= L_0\left(\frac{2k}{m\Omega^2} - 1\right)$

período $= \frac{2\pi}{\Omega}$

$$s(t=0) \underset{}{\overset{\cos\Omega t = 1}{=}} L_0$$

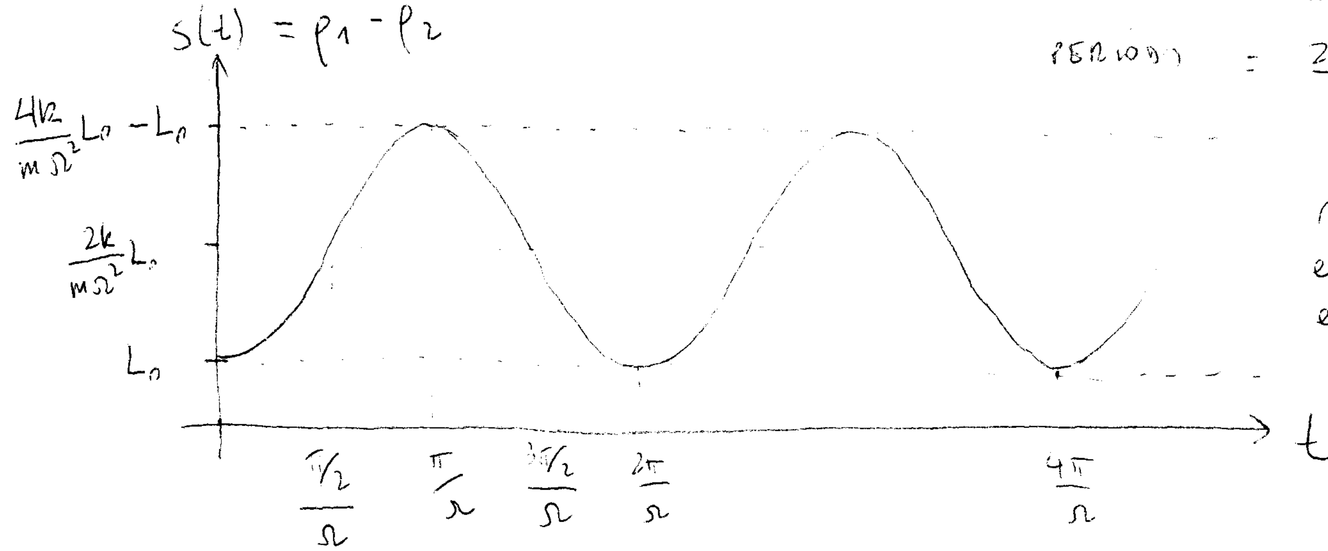
$$s\left(t=\frac{\pi/2}{\Omega}\right) \underset{\uparrow \cos\Omega t = 0}{=} \frac{2kL_0}{m\Omega^2}$$

$$s\left(t=\pi/\Omega\right) \underset{\uparrow \cos\Omega t = -1}{=} \frac{4kL_0}{m\Omega^2} - L_0$$

$$s\left(t=\frac{2\pi}{\Omega}\right) \underset{\uparrow \cos\Omega t = 1}{=} L_0$$

AMPLITUD $= L_0\left(\frac{2k}{m\Omega^2} - 1\right)$

PERIODO $= \frac{2\pi}{\Omega}$



resorte está estirado $\forall t$

② El 2º caso es: $\frac{2k}{m} - \omega_0^2 < 0$ ó $\omega_0 > \sqrt{\frac{2k}{m}}$

si definimos $\Omega^2 = \omega_0^2 - \frac{2k}{m}$, la ecuación queda

$$\ddot{s} - \Omega^2 s = \frac{2k}{m} L_0$$

cuya solución es del tipo:

particular

$$s(t) = \underbrace{A\cosh(\Omega t) + B\,\mathrm{senh}(\Omega t)}_{\text{o equivalentemente: } \tilde{A}e^{\Omega t} + \tilde{B}e^{-\Omega t}} - \frac{2kL_0}{m\Omega^2}$$

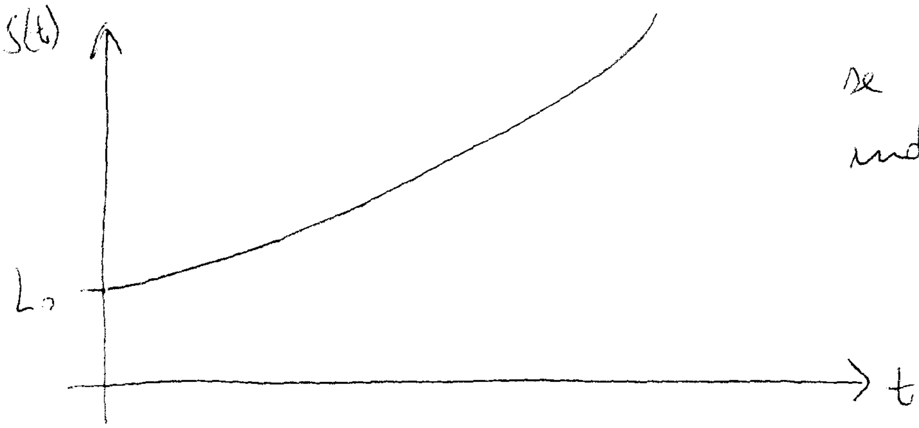
$$s(t=0) = A - \frac{2kL_0}{m\Omega^2} = L_0 \quad \Rightarrow \quad A = L_0\left(1 + \frac{2k}{m\Omega^2}\right)$$

$$\dot{s}(t) = A\Omega \operatorname{senh}(\Omega t) + B\Omega \cosh(\Omega t)$$

$$\dot{s}(t=0) = B\Omega = 0 \quad \Rightarrow \quad B = 0$$

$$\Omega = \sqrt{\omega_0^2 - \frac{2k}{m}}$$

$$\boxed{s(t) = \frac{-2kL_0}{m\Omega^2} + L_0\left(1 + \frac{2k}{m\Omega^2}\right)\cosh(\Omega t)}$$



se ~~s~~ estira indefinidamente

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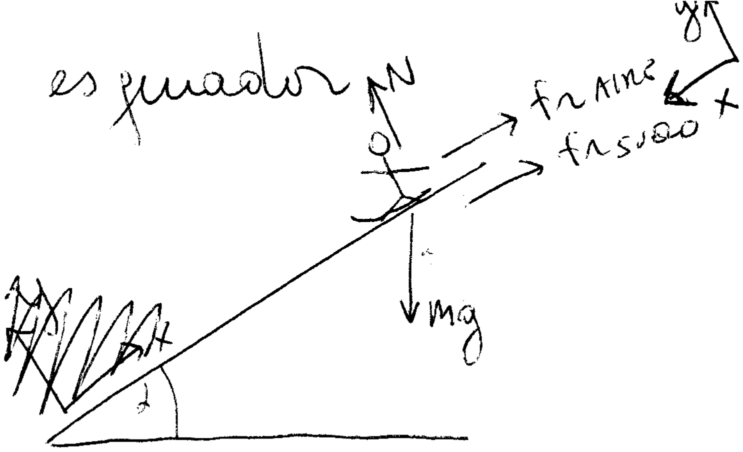


$$\hat{x})\quad m\ddot{x} = -fr_{AIRE} - fr_{SUELO} + mg\,sen\alpha$$

$$\hat{y})\quad 0 = N - mg\cos\alpha \Rightarrow N = mg\cos\alpha$$

$$fr_{SUELO} = \mu_c N = \mu_c mg\cos\alpha$$

$$fr_{AIRE} = cv = c\dot{x}$$

$$m\ddot{x} = mg\,sen\alpha - \mu_c mg\cos\alpha - c\dot{x}$$

$$\ddot{x} = \underbrace{g(sen\alpha - \mu_c\cos\alpha)}_{A} - \underbrace{\frac{c}{m}}_{B}\dot{x}$$

$$\ddot{x} = \frac{d\dot{x}}{dt}$$

$$\int \frac{d\dot{x}}{A - B\dot{x}} = \int^t dt$$

$$\left(-\frac{1}{B}\right)\ln(A - B\dot{x})\Big|_{\dot{x}_{ini} = v_0}^{\dot{x}} = t$$

$$\left(-\frac{1}{B}\right)\ln\left(\frac{A - B\dot{x}}{A - Bv_0}\right) = t$$

$$\frac{A - B\dot{x}}{A - Bv_0} = e^{-Bt}$$

$$A - B\dot{x} = (A - Bv_0)\, e^{-Bt}$$

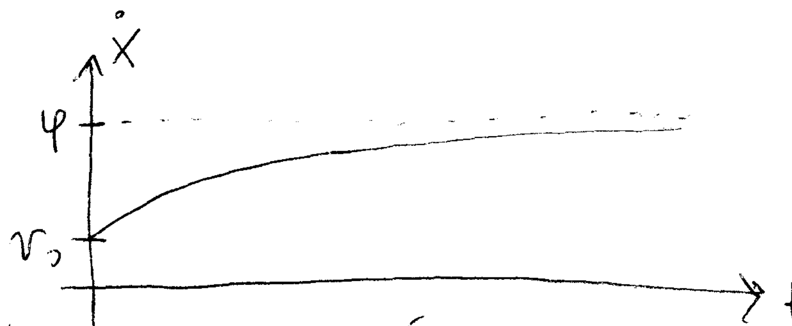
$$\boxed{\dot{x} = A/B - (A/B - v_0)\, e^{-B/t}}$$

donde $A/B = \frac{m}{c}(\operatorname{sen}\alpha - \mu_c \cos\alpha) \stackrel{def}{=} \varphi$

$$\boxed{\dot{x} = \varphi - (\varphi - v_0)\, e^{-\frac{c}{m}t}}$$

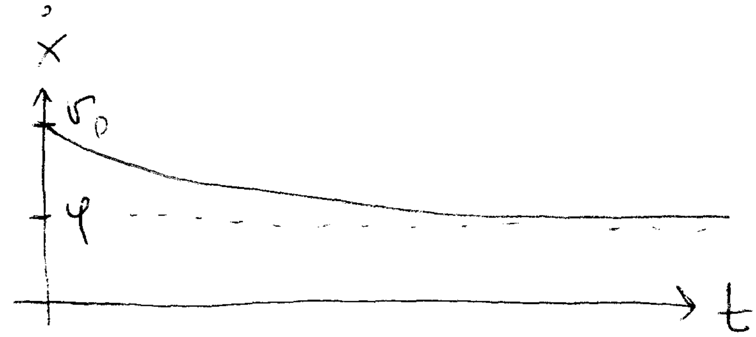
<u>Tres casos:</u>

1) $\varphi > v_0$



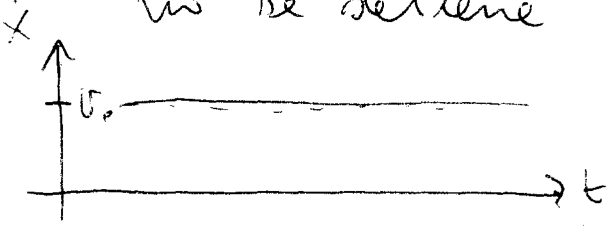
desciende cada vez más rápido hasta llegar a una veloc. terminal

2) $0 < \varphi < v_0$



desciende cada vez más lento pero no se detiene

$\varphi = v_0$



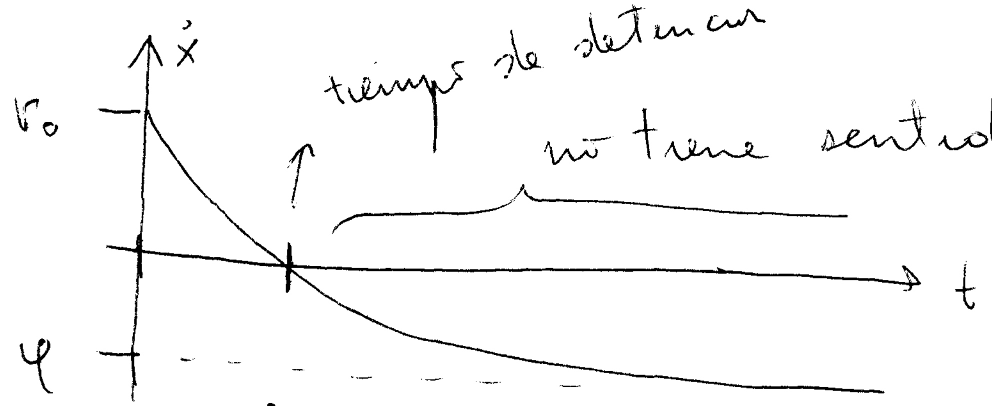
mantiene cte veloc inicial v_0

3) $\varphi < 0 \Rightarrow \operatorname{sen}\alpha - \mu_c \cos\alpha < 0$

$\Rightarrow \mu_c > \tan\alpha$

o sea cuando hay mucho <u>roce</u> con el suelo

en este caso:



$$\overset{neg}{\varphi} = \overbrace{(\varphi - v_0)}^{neg} e^{-c/m\, t}$$

$$\Rightarrow e^{-c/m\, t} = \frac{\varphi}{\varphi - v_0} \text{ (positivo) } = \frac{|\varphi|}{v_0 - \varphi}$$

$$-\frac{c}{m} t = \ln \frac{|\varphi|}{v_0 + |\varphi|}$$

$$\boxed{t_{\text{po detención}} = \frac{m}{c} \ln\left(\frac{v_0 + |\varphi|}{|\varphi|}\right)}$$