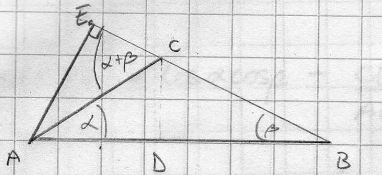
Pauta ejercicio 1

(a) pdq $\text{sen}(\alpha+\beta) = \text{sen}\,\alpha\cos\beta + \text{sen}\,\beta\cos\alpha$



$$\text{sen}(\alpha+\beta) = \frac{EA}{AC} \quad ①$$

pero $\dfrac{EA\cdot BC}{2} = \dfrac{AB\cdot CD}{2} \Rightarrow EA = \dfrac{AB\cdot CD}{BC.}$ ②

Por lo tanto $\text{sen}(\alpha+\beta) = \dfrac{AB\cdot CD}{BC\cdot AC}$ ③

Además se cumple que $AB = AD + DB.$ ④

Luego ④ en ③ $\Rightarrow \text{sen}(\alpha+\beta) = \dfrac{(AD+DB)\cdot CD}{BC\cdot AC}$

$$= \frac{AD}{AC}\cdot\frac{CD}{BC} + \frac{DB}{BC}\cdot\frac{CD}{AC}$$

pero sabemos que: $\dfrac{AD}{AC} = \cos\alpha$, $\dfrac{CD}{BC} = \text{sen}\,\beta$, $\dfrac{DB}{BC} = \cos\beta$, $\dfrac{CD}{AC} = \text{sen}\,\alpha$

$$\Rightarrow \text{sen}(\alpha+\beta) = \cos\alpha\,\text{sen}\,\beta + \cos\beta\cdot\text{sen}\,\alpha$$

$$= \text{sen}\,\alpha\cos\beta + \text{sen}\,\beta\cos\alpha$$

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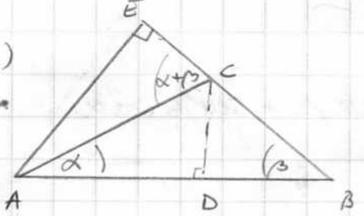
(b)



pdq: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$

Tenemos que

$$BE = EC + CB \quad / \cdot \frac{1}{AC} \quad (1)$$

$$\Rightarrow \frac{BE}{AC} = \frac{EC}{AC} + \frac{CB}{AC} \Rightarrow \frac{EC}{AC} = \frac{BE}{AC} - \frac{CB}{AC} \quad (2)$$

Además $\triangle ABE \sim \triangle CBD \Rightarrow \frac{BE}{AB} = \frac{BD}{CB} \Rightarrow \boxed{BE = \frac{AB \cdot BD}{CB}} \quad (3)$

Luego, (3) en (2)

$$\Rightarrow \frac{EC}{AC} = \frac{AB \cdot BD}{AC \cdot CB} - \frac{CB}{AC} \qquad \text{pero } AB = AD + DB$$

$$\Rightarrow \frac{EC}{AC} = \frac{AD \cdot BD}{AC \cdot CB} + \frac{DB^2}{AC \cdot CB} - \frac{CB}{AC}$$

Reconociendo algunos términos:

$$\frac{EC}{AC} = \cos(\alpha+\beta), \quad \frac{AD}{AC} = \cos\alpha, \quad \frac{BD}{CB} = \cos\beta$$

reemplazando:

$$\Rightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta + \cos\beta \cdot \frac{BD}{AC} - \frac{CB}{AC}$$

pero $BD = CB\cos\beta$

$$\Rightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta + \frac{CB}{AC}\cos^2\beta - \frac{CB}{AC}$$

$$= \cos\alpha\cos\beta + \frac{CB}{AC}\left(\cos^2\beta - 1\right)$$

pero $\cos^2\beta + \operatorname{sen}^2\beta = 1 \Rightarrow \cos^2\beta - 1 = -\operatorname{sen}^2\beta$

$$\Rightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \frac{CB}{AC}\operatorname{sen}\beta\cdot\operatorname{sen}\beta$$

Además $\operatorname{sen}\beta = \dfrac{CD}{CB}$

$$\Rightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \frac{\cancel{CB}}{AC}\frac{CD}{\cancel{CB}}\operatorname{sen}\beta$$

pero $\dfrac{CD}{AC} = \operatorname{sen}\alpha$, por lo que se concluye.

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

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