Pauta P3 Examen

$$\rho_0 = \begin{cases} 0 & r<a \\ \frac{q}{\pi a^3} e^{-2r/a} & a<r<b \\ 0 & r>b \end{cases}$$

Calculemos la carga libre encerrada en función de la distancia radial r.

$r<a$

$$Q(r) = \iiint_0^r \rho_0 \cdot dV = 0.$$

$a<r<b$

$$Q(r) = \iiint_0^r \rho_0 dV = \int_0^{2\pi}\int_0^{\pi}\int_a^r \frac{q}{\pi a^3} e^{-2r/a} \cdot r^2 \operatorname{sen}\theta \, dr \, d\theta \, d\varphi$$

$$= \frac{q}{\pi a^3} \int_a^r r^2 e^{-2r/a} \cdot 4\pi \qquad \int_0^{2\pi}\int_0^{\pi} \operatorname{sen}\theta \, d\theta \, d\varphi$$

$$= \frac{4q}{a^3} \cdot \underbrace{\int_a^r r^2 \cdot e^{-2r/a}}_{\mathrm{I}}$$

I: por partes, sea $u = r^2$, $du = 2r$

$dv = e^{-2r/a}$; $v = -\frac{a}{2} e^{-2r/a}$

$$\mathrm{I} = -\frac{ar^2}{2} e^{-2r/a}\Big|_a^r + \underbrace{\int_a^r \cancel{2}r \cdot \frac{a}{\cancel{2}} e^{-2r/a} dr}_{\mathrm{II}}$$

II: por partes sea $u = r$, $du = 1$.

$dv = e^{-2r/a}$; $v = -\frac{a}{2} e^{-2r/a}$

$$\mathrm{II} = -\frac{a^2 r}{2} e^{-2r/a}\Big|_a^r + \int_a^r \frac{a^2}{2} \cdot e^{-2r/a} dr = -\frac{a^2 r}{2} e^{-2r/a}\Big|_a^r - \frac{a^3}{4} \cdot e^{-2r/a}\Big|_a^r$$

$$\Rightarrow \mathrm{I} = -\frac{ar^2}{2} e^{-2r/a} - \frac{a^2 r}{2} e^{-2r/a} - \frac{a^3}{4} \cdot e^{-2r/a}$$

$$\mathrm{I} = -\frac{a}{4} e^{-2r/a} (2r^2 + 2ar + a^2)\Big|_a^r$$

$$\mathrm{I} = -\frac{a}{4}\left[e^{-2r/a}(2r^2 + 2ar + a^2) - 5a^2 \cdot e^{-2}\right]$$

$$\Rightarrow \boxed{Q(r) = -\frac{q}{a^2}\left[e^{-2r/a}(2r^2 + 2ar + a^2) - 5a^2 e^{-2}\right]}$$

y como $\oint\vec{D}\cdot d\vec{S} = 4\pi r^2 \cdot D(r) = Q(r)$

$$\rightarrow \boxed{D(r) = \frac{Q(r)}{4\pi r^2}}$$ con $Q(r)$ calculado anterior%

$r>b$

$\rightarrow Q = Q(r=b) \Rightarrow \boxed{D(r) = \frac{Q}{4\pi r^2}}$

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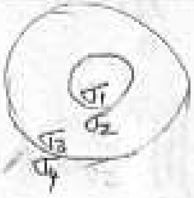
a) Densidad de carga libre en conductores. $\sigma_l = \vec{D}\cdot\hat{n}$



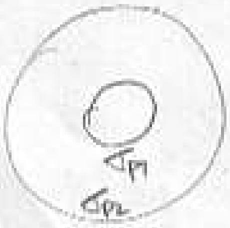
en $r=a$, $\sigma_l = \varepsilon_0\vec{E}\cdot\hat{n}|_{r=a} = {}^{+}E(r=a)$

en $r=b$, $\sigma_l = \varepsilon_0\vec{E}\cdot\hat{n}|_{r=b} = -E(r=b)$

Donde $\vec{E} = \dfrac{\vec{D}}{\varepsilon}$

b) $\vec{P} = \vec{D} - \varepsilon_0\vec{E} = \vec{D} - \varepsilon_0 \cdot \dfrac{\vec{D}}{\varepsilon} = \left(1 - \dfrac{\varepsilon_0}{\varepsilon}\right)\vec{D} = \dfrac{\varepsilon - \varepsilon_0}{\varepsilon}\vec{D}$

$$\rho_p = -\nabla\cdot\vec{P} = -\nabla\cdot\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\vec{D}\right) = -\frac{\varepsilon-\varepsilon_0}{\varepsilon}\underbrace{\nabla\cdot\vec{D}}_{\rho_0} = \frac{\varepsilon_0-\varepsilon}{\varepsilon}\cdot\frac{q}{\pi a^3}\cdot e^{-2r/a}$$



$$\sigma_{p1} = \vec{P}\cdot\underset{(-\hat{r})}{\hat{n}} = -P(r=a) = -\frac{\varepsilon-\varepsilon_0}{\varepsilon}D(r)\Big|_{r=a} = 0$$

$$\sigma_{p2} = \vec{P}\cdot\underset{(\hat{r})}{\hat{n}} = P(r=b) = -\frac{\varepsilon-\varepsilon_0}{\varepsilon}D(r)\Big|_{r=b}$$

c) $V = -\int_b^a \vec{E}\cdot d\vec{l} = \ldots$