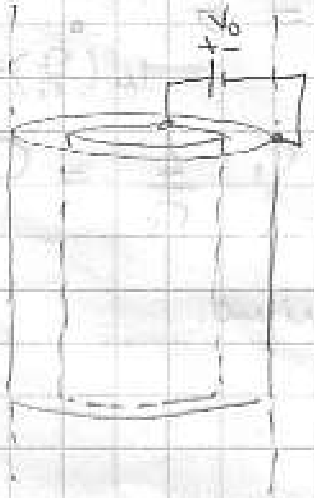
Pauta Ejercicio 4



a) Uso Laplace

$$\nabla^2 V = 0 \qquad \text{solo depende de } r$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \qquad \text{cte}$$

$$r\frac{\partial V}{\partial r} = A$$

$$\frac{\partial V}{\partial r} = \frac{A}{r}$$

$$V(r) = A\cdot\ln(r) + B.$$

$$V(r=r_1) = V_0 \qquad V(r=r_2) = 0 \Rightarrow \begin{cases} A\cdot\ln(r_1) + B = V_0 \\ A\ln(r_2) + B = 0 \end{cases}$$

$$\Rightarrow A = \frac{V_0}{\ln(r_1/r_2)} \qquad B = -\frac{V_0}{\ln(r_1/r_2)}\ln(r_2)$$

$$\Rightarrow \boxed{V(r) = \frac{V_0}{\ln(r_1/r_2)}\ln(r/r_2)}$$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} = \frac{V_0}{\ln(r_2/r_1)}\cdot\frac{1}{r}\hat{r}$$

$$\boxed{\vec{E} = \frac{V_0}{r\ln(r_2/r_1)}\hat{r}}$$

b) $$|\vec{E}| = \frac{V_0}{r\ln(r_2/r_1)} \Rightarrow |\vec{E}(r_1)| = \frac{V_0}{r_1\ln(r_2/r_1)}.$$

$$\frac{\partial|E(r_1)|}{\partial r_1} = V_0 \cdot \frac{-\frac{\partial}{\partial r_1}\left(r_1 \ln(r_2/r_1)\right)}{\left(r_1 \ln(r_2/r_1)\right)^2} = 0$$

$\ln\left(\frac{r_2}{r_1}\right) = \ln(r_2) - \ln(r_1)$

$$\frac{\partial}{\partial r_1}\left(r_1 \cdot \ln(r_2/r_1)\right) = \ln(r_2/r_1) - r_1 \cdot \frac{1}{r_1} = 0.$$

$$\Rightarrow \boxed{\ln(r_2/r_1) = 1} \Rightarrow \text{mínimo.}$$

c) $\sigma = \varepsilon_0 \cdot E\Big|_{r=r_1} = \varepsilon_0 \dfrac{V_0}{a \cdot \ln(r_2/r_1)}$

$$\rightarrow Q = \int_0^a\int_0^{2\pi} \sigma \, r_1 d\theta dz = \frac{\varepsilon_0 V_0}{\ln(r_2/r_1)} \cdot 2\pi.$$

$$\Rightarrow \boxed{C = \frac{Q}{V_0} = \frac{2\pi\varepsilon_0 \cdot 1}{\ln(r_2/r_1)}} \leftarrow \text{por unidad de longitud.}$$

d) $r_2 = r_1 + d \Rightarrow C = \dfrac{2\pi\varepsilon_0 \cdot L}{\ln\left(1 + \frac{d}{r_1}\right)}$

pero como $d \ll r_1 \Rightarrow \frac{d}{r_1} \ll 1$. Usando una aprox lineal de Taylor entorno a x_0

$$\ln(1+x)\Big|_{x_0} = \ln(1+x_0) + \frac{1}{1!}\frac{1}{1+x_0}(x-x_0) + O(x^2)$$

si $x_0 = 0$ y $x = d/r_1$

$$\rightarrow \ln(1 + d/r_1) \approx d/r_1$$

$$\rightarrow C \approx \frac{2\pi\varepsilon_0 L}{d/r_1} = \frac{2\pi\varepsilon_0 r_1 L}{d} = \varepsilon_0 \cdot \frac{2\pi r_1 L}{d}$$

$$\approx \varepsilon_0 \frac{A}{d}$$ cond. de placas planas.