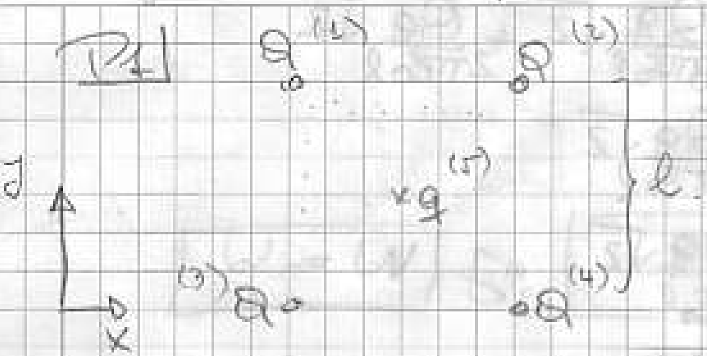
Pauta Ejercicio 3

Fecha / Date:

P1



$$\vec{F}_i = q_i \vec{E}_i$$

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0} \sum_{j\neq i}^{N} \frac{q_j (\vec{r}_i - \vec{r}_j)}{\|\vec{r}_i - \vec{r}_j\|^3}$$

a) revisemos una carga, digamos, la de arriba a la izq.

$$\vec{E}_{21} = \frac{Q}{4\pi\varepsilon_0 l^2}(-\hat{\imath})$$

$$\vec{E}_{31} = \frac{Q}{4\pi\varepsilon_0 l^2}\hat{\jmath}$$

$$\vec{E}_{41} = \frac{Q}{4\pi\varepsilon_0 (\sqrt{2}l)^2}\frac{(-\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$$

$$\vec{E}_{51} = \frac{q}{4\pi\varepsilon_0 \left(\frac{\sqrt{2}}{2}l\right)^2}\frac{(-\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$$

$$\Rightarrow \vec{F}_1 = Q\sum_{j=2}^{5}\vec{E}_{j1} = \frac{Q}{4\pi\varepsilon_0 l^2}\left[Q + \frac{Q}{2\sqrt{2}} + \frac{2q}{\sqrt{2}}\right](-\hat{\imath}+\hat{\jmath})$$

$$\vec{F}_1 = 0 \Rightarrow Q\left(1+\frac{\sqrt{2}}{4}\right) + \sqrt{2}q = 0 \Rightarrow \boxed{q = -\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)Q}$$

b) $U = \frac{1}{2}\sum_i^N q_i V_i$; $V_i = \frac{1}{4\pi\varepsilon_0}\sum_{j\neq i}^{N}\frac{q_j}{\|\vec{r}_i - \vec{r}_j\|}$

Notemos que para (1), (2), (3) y (4), el resultado es el mismo.

$$U_1 = U_2 = U_3 = U_4 = \frac{1}{2}Q\cdot\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{l} + \frac{Q}{l} + \frac{Q}{\sqrt{2}l} + \frac{q}{\frac{\sqrt{2}}{2}l}\right)$$

$$= Q\frac{1}{8\pi\varepsilon_0 l}\left[Q\left(2+\frac{\sqrt{2}}{2}\right) + q\sqrt{2}\right]$$

para (5), $U_5 = \frac{1}{2}q\cdot\frac{1}{4\pi\varepsilon_0}\cdot 4\frac{Q}{\frac{\sqrt{2}}{2}l} = \frac{\sqrt{2}qQ}{2\pi\varepsilon_0 l}$.

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$$\Rightarrow U = 4\cdot\frac{Q^2}{8\pi\varepsilon_0 l}\left(2+\frac{\sqrt{2}}{2}\right)+\frac{4Qq}{8\pi\varepsilon_0 l}\sqrt{2}+\frac{Qq}{2\pi\varepsilon_0 l}\sqrt{2}$$

$$= \frac{Q^2}{2\pi\varepsilon_0 l}\left(2+\frac{\sqrt{2}}{2}\right)+\frac{Qq\sqrt{2}}{\pi\varepsilon_0 l}$$

$$\boxed{U = \frac{Q^2}{4\pi\varepsilon_0 l}(4+\sqrt{2})+\frac{Qq\sqrt{2}}{\varepsilon_0 l}} \quad y \quad \boxed{W = U}$$

P2)
$$\vec{E} = \begin{cases} 0 & r<a \\ \dfrac{Q_a}{4\pi\varepsilon_0 r^2}\hat{r} & a<r<b \\ \dfrac{Q_a+Q_b}{4\pi\varepsilon_0 r^2}\hat{r} & r>b \end{cases}$$

$$\Rightarrow U = \frac{\varepsilon_0}{2}\iiint_{\text{todo el espacio}} |\vec{E}|^2 dV = \frac{\varepsilon_0}{2}\iiint |\vec{E}|^2 \cdot r^2 \operatorname{sen}\theta\, dr\, d\theta\, d\phi$$

$$U = \frac{\varepsilon_0}{2}\left[\int_0^{2\pi}\int_0^{\pi}\int_a^b \left(\frac{Q_a}{4\pi\varepsilon_0 r^2}\right)^2 r^2 \operatorname{sen}\theta\, dr\, d\theta\, d\phi + \int_0^{2\pi}\int_0^{\pi}\int_b^{\infty}\left(\frac{Q_a+Q_b}{4\pi\varepsilon_0 r^2}\right)^2 r^2\operatorname{sen}\theta\, dr\, d\theta\, d\phi\right]$$

$$U = \frac{\varepsilon_0}{2}\frac{1}{(4\pi\varepsilon_0)^2}\cdot 4\pi\cdot\left[Q_a^2\int_a^b\frac{dr}{r^2}+(Q_a+Q_b)^2\int_b^{\infty}\frac{dr}{r^2}\right]$$

$$U = \frac{1}{8\pi\varepsilon_0}\left[Q_a^2\left(-\frac{1}{r}\right)\Big|_a^b + (Q_a^2+2Q_aQ_b+Q_b^2)\left(-\frac{1}{r}\right)\Big|_b^{\infty}\right]$$

$$U = \frac{1}{8\pi\varepsilon_0}\left[Q_a^2\left(\frac{1}{a}-\cancel{\frac{1}{b}}\right)+(\cancel{Q_a^2}+2Q_aQ_b+Q_b^2)\cdot\frac{1}{b}\right]$$

$$U = \frac{1}{8\pi\varepsilon_0}\left[\frac{Q_a^2}{a}+\frac{Q_b^2}{b}+\frac{2Q_aQ_b}{b}\right]$$