Pauta Ejercicio 2.

$$\rho = \begin{cases} 0 & r<a \\ \frac{\kappa}{r^2} & a<r<b \\ 0 & r>b. \end{cases}$$

a) $\oiint \vec{E}\cdot d\vec{s} = 4\pi r^2 E(r)$ para todas las zonas.

i) $r<a$: $Q_{enc}=0 \rightarrow \vec{E}=\vec{0}$.

ii) $a<r<b$: $Q_{enc} = \int_0^{2\pi}\int_0^{\pi}\int_a^r \frac{\kappa}{r^2} r^2 \operatorname{sen}\theta\, dr\, d\theta\, d\phi$

$= \kappa \cdot 4\pi \cdot (r-a)$

iii) $r>b$: $Q_{enc} = 4\pi\kappa(b-a)$.

$$\Rightarrow \boxed{\vec{E} = \begin{cases} \vec{0} & r<a \\ \frac{\kappa(r-a)}{\varepsilon_0 r^2}\hat{r} & a<r<b \\ \frac{\kappa(b-a)}{\varepsilon_0 r^2}\hat{r} & r>b \end{cases}}$$

b) $V = -\int_{\infty}^{r}\vec{E}\cdot d\vec{r} = -\int_{\infty}^{b}\vec{E}\cdot d\vec{r} - \int_{b}^{a}\vec{E}\cdot d\vec{r} - \int_{a}^{0}\vec{E}\cdot d\vec{r}$.

$$V = -\int_{\infty}^{r}\frac{\kappa(b-a)}{\varepsilon_0 r^2}dr - \int_{b}^{a}\frac{\kappa(r-a)}{\varepsilon_0 r^2}dr - \cancel{\int_{a}^{0} 0\, dr}.$$

$$= -\frac{\kappa(b-a)}{\varepsilon_0}\left(\frac{1}{r}\right)\Big|_{\infty}^{b} - \frac{\kappa}{\varepsilon_0}\left[\ln(r) + \frac{a}{r}\right]\Big|_{b}^{a}$$

$$= \frac{\kappa(b-a)}{\varepsilon_0 b} - \frac{\kappa}{\varepsilon_0}\left(\ln\left(\frac{a}{b}\right) + \frac{a}{a} - \frac{a}{b}\right)$$

$$\Rightarrow \boxed{V(r=0) = \frac{\kappa}{\varepsilon_0}\ln(b/a)}$$