Pauta Ejercicio 1

P1 Ubicando el origen en el centro de la esfera

$\vec{r} = -a\hat{k}$

$\vec{r}' = a\hat{r} = a(\hat{\imath}\,\text{sen}\theta\cos\phi + \hat{\jmath}\,\text{sen}\theta\,\text{sen}\phi + \hat{k}\cos\theta)$

$dS = \sigma a^2 \text{sen}\theta\, d\theta\, d\phi$

$\vec{r} - \vec{r}' = a(-\hat{\imath}\,\text{sen}\theta\cos\phi - \hat{\jmath}\,\text{sen}\theta\,\text{sen}\phi - (1+\cos\theta)\hat{k})$

$\|\vec{r} - \vec{r}'\| = a\sqrt{\text{sen}^2\theta\cos^2\phi + \text{sen}^2\theta\,\text{sen}^2\phi + (1+\cos\theta)^2}$

$= a\sqrt{\text{sen}^2\theta + 1 + \cos^2\theta + 2\cos\theta}$

$= a\sqrt{2}\sqrt{1+\cos\theta}$

$= a\sqrt{2}\cdot\sqrt{2}\,\cos\frac{\theta}{2}$

$\Rightarrow d\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_0^{2\pi} \dfrac{dQ\,(\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$ las componentes en $\hat{\imath}$ y $\hat{\jmath}$ dan cero por simetría, luego

$d\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_0^{2\pi} \dfrac{\sigma a^2\,\text{sen}\theta\, d\theta\, d\phi\cdot a\overbrace{(1+\cos\theta)}^{2\cos^2\theta/2}(-\hat{k})}{a^3\cdot 2^3\cdot\cos^3\theta/2}$

$$\boxed{d\vec{E} = -\frac{1}{8\varepsilon_0}\frac{\sigma\,\text{sen}\theta\, d\theta\,\hat{k}}{\cos\theta/2}} = -\frac{1}{4\sqrt{2}\varepsilon_0}\frac{\sigma\,\text{sen}\theta\, d\theta}{\sqrt{1+\cos\theta}}\hat{k}$$

P2 en CGS, $K = 1\left[\dfrac{\text{dina}\cdot\text{cm}^2}{\text{u.e.c}^2}\right]$

en MKS, $K = 8{,}99\cdot 10^9\left[\dfrac{N\cdot m^2}{C^2}\right]$

$\text{dina} = \dfrac{g\cdot cm}{s^2} = \dfrac{10^{-3}\,kg\cdot 10^{-2}\,m}{s^2} = 10^{-5}\left[\dfrac{kg\,m}{s^2}\right] = 10^{-5}[N]$

$cm^2 = (10^{-2}[m])^2 = 10^{-4}[m^2]$

$\Rightarrow \dfrac{1\cdot 10^{-5}[N]\cdot 10^{-4}[m^2]}{1[uec]^2} = 9\,10^9\left[\dfrac{N\cdot m^2}{C^2}\right]$

$\Rightarrow \dfrac{10^{-9}}{9\cdot 10^9}[C^2] = 1[uec]^2 \Rightarrow \boxed{1[uec] = \frac{1}{3}10^{-9}[C]}$

$\Rightarrow \boxed{1[C] = 3\cdot 10^9[uec]}$