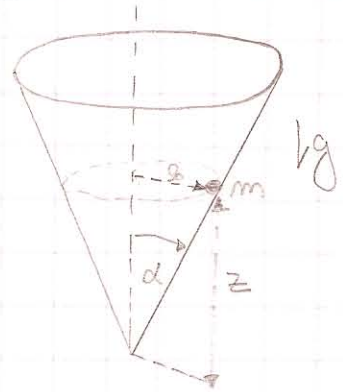


ELEGIMOS POLARES:

AUXILIAR FI21A

GABRIEL CUEVAS

$$\vec{r} = \rho\hat{\rho} + z\hat{z}$$
$$\dot{\vec{r}} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$$
$$\dot{\vec{r}}^2 = \dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2$$

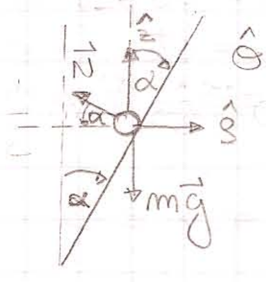
GEOMETRÍA: $\tan\alpha = \dfrac{\rho}{z}$

$$\Rightarrow z\tan\alpha = \rho \Rightarrow z = \rho\,\mathrm{cotan}\,\alpha$$
$$\Rightarrow \dot{z}\tan\alpha = \dot{\rho} \Rightarrow \dot{z} = \dot{\rho}\,\mathrm{cotan}\,\alpha$$
$$\Rightarrow \ddot{z}\tan\alpha = \ddot{\rho} \Rightarrow \ddot{z} = \ddot{\rho}\,\mathrm{cotan}\,\alpha$$

$$V = mgz$$

$$E = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + mgz$$
$$E = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{\rho}^2\,\mathrm{cotan}^2\alpha\right) + mg\,\mathrm{cotan}\,\alpha\,\rho$$
$$E = \frac{1}{2}m\left(\dot{\rho}^2\underbrace{\left(1 + \frac{\cos^2\alpha}{\sin^2\alpha}\right)}_{\mathrm{cosec}^2\alpha} + \rho^2\dot{\theta}^2\right) + mg\,\mathrm{cotan}\,\alpha\,\rho$$

DCL:



$\hat{\theta}\otimes$

$$(\hat{\rho})\quad -N\cos\alpha = m\left(\ddot{\rho} - \rho\dot{\theta}^2\right)$$
$$(\hat{\theta})\quad 0 = m\left(\rho\ddot{\theta} + 2\dot{\rho}\dot{\theta}\right)$$
$$(\hat{z})\quad N\sin\alpha - mg = m\ddot{z}$$

LA EC. EN $\theta \Rightarrow$

$$\frac{d}{dt}\left(\rho^2\dot{\theta}\right) = 0 \Rightarrow \rho^2\dot{\theta} = cte = h$$
$$\Rightarrow \boxed{\dot{\theta} = \frac{h}{\rho^2}}$$

$\dot{\theta}$ en E:

$$E = \frac{1}{2}m\left(\dot{\rho}^2\,\mathrm{cosec}^2\alpha + \frac{h^2}{\rho^2}\right) + mg\,\mathrm{cotan}\,\alpha\,\rho$$
$$E = \frac{1}{2}m\dot{\rho}^2\,\mathrm{cosec}^2\alpha + \frac{1}{2}m\frac{h^2}{\rho^2} + mg\,\mathrm{cotan}\,\alpha\,\rho$$

* MULTIPLICAMOS POR $\sin^2\alpha$ PARA DEJAR A $\dot{\rho}$ ACOMPAÑADO SÓLO DE $1/2m$

$$E\cdot\sin^2\alpha = \frac{1}{2}m\dot{\rho}^2 + \underbrace{\frac{1}{2}m\frac{h^2}{\rho^2}\sin^2\alpha + mg\,\mathrm{cotan}\,\alpha\,\rho\sin^2\alpha}_{V_{eff}}$$

LA EC. PARA ρ y z LAS PODEMOS ESCRIBIR COMO:

$$m\ddot{\rho} - m\frac{h^2}{\rho^3} = -N\cos\alpha \quad / \sin\alpha$$

$$m\ddot{\rho}\,\text{cotan}\,\alpha + mg = N\sin\alpha \quad / \cos\alpha$$

① $m\ddot{\rho}\sin\alpha - m\frac{h^2}{\rho^3}\sin\alpha = -N\sin\alpha\cos\alpha$

② $m\ddot{\rho}\frac{\cos^2\alpha}{\sin\alpha} + mg\cos\alpha = N\sin\alpha\cos\alpha$

① + ②

$$m\ddot{\rho}\,\text{cosec}\,\alpha - m\frac{h^2}{\rho^3}\sin\alpha + mg\cos\alpha = 0$$

$$(*) \Rightarrow \boxed{\ddot{\rho} - \frac{h^2}{\rho^3}\sin^2\alpha + g\sin\alpha\cos\alpha = 0}$$

EC. DE MOV. PARA ρ POR $\sum$ FUERZAS

$$\Rightarrow \boxed{\dot{\theta} = \frac{h}{\rho^2}}$$

EC. PARA $\dot{\theta}$ CON $h \equiv$ cte.

$$\Rightarrow \boxed{z = \rho\,\text{cotan}\,\alpha}$$

EC. PARA z

(b) MOV. CIRCULAR HORIZONTAL $\Rightarrow \rho = \rho_0 = $ cte (mov. circular)

$\Rightarrow z = z_0 = $ cte (mov. horizontal)

$\Rightarrow \dot{\rho} = \ddot{\rho} = \dot{z} = \ddot{z} = 0$

EN (*) $\quad -\frac{h^2}{\rho_0^3}\sin^2\alpha + g\sin\alpha\cos\alpha = 0$

MOV. CIRCULAR $\Rightarrow$

$$V_0 = \dot{\theta}\rho_0$$

$$\Rightarrow \dot{\theta} = \frac{V_0}{\rho_0} = \frac{h}{\rho_0^2}$$

$$\Rightarrow \boxed{h = \rho_0 V_0}$$

REEMPLAZANDO

$$-\frac{\rho_0^2 v_0^2}{\rho_0^3}\sin\alpha + g\cos\alpha = 0$$

$$\Rightarrow \boxed{\rho_0 = \frac{v_0^2}{g}\tan\alpha}$$

YA ENCONTRAMOS ρ_0, AHORA PERTURBAMOS ρ CERCA DE ρ_0

EC. MOV.:

$$\ddot{\rho} - \frac{h^2}{\rho^3}\sin^2\alpha + g\sin\alpha\cos\alpha = 0$$

NUEVAMENTE OCUPAREMOS "TAYLOR" (PASO IMPORTANTE)

$$\frac{1}{\rho^3} = \frac{1}{(\rho_0+(\rho-\rho_0))^3} = \frac{1}{\rho_0^3\left(1+\left(\frac{\rho}{\rho_0}-1\right)\right)^3}$$

$$\rho \approx \rho_0 \Rightarrow \frac{\rho}{\rho_0} \approx 1$$

$$\Rightarrow \underbrace{\left(\frac{\rho}{\rho_0}-1\right)}_{\text{(PEQUEÑO)}} << 1$$

$$\Rightarrow \frac{1}{\rho^3} = \frac{1}{\rho_0^3}\left(1+\left(\frac{\rho}{\rho_0}-1\right)\right)^{-3}$$

Y OCUPANDO $(1+\varepsilon)^n \approx 1+n\varepsilon$

$$\Rightarrow \frac{1}{\rho^3} \approx \frac{1}{\rho_0^3}\left(1-3\left(\frac{\rho}{\rho_0}-1\right)\right) = \frac{4}{\rho_0^3} - \frac{3\rho}{\rho_0^4}$$

EN LA EC. DE MOV.

$$\Rightarrow \ddot{\rho} + \frac{3h^2\sin^2\alpha}{\rho_0^4}\rho - \frac{4}{\rho_0^3}h^2\sin^2\alpha + g\sin\alpha\cos\alpha$$

$$\Rightarrow \omega_0 = \sqrt{3}\,\frac{h\sin\alpha}{\rho_0^2}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

$$\boxed{\ddot{q} + \omega_0^2 q = cte}$$

ESTO DEBE SUCEDER PARA TENER PEQUEÑAS OSCILACIONES

$$\Rightarrow \boxed{T_{peq} = \frac{2\pi\,\rho_0^2}{\sqrt{3}\,h\sin\alpha}}$$

$$h = \rho_0 v_0$$
$$\rho_0 = \frac{v_0^2}{g}\tan\alpha$$

$$\Rightarrow \boxed{T_{peq} = \frac{2\pi\,v_0}{\sqrt{3}\,g\cos\alpha}}$$

AHORA LO REALIZAREMOS POR EL MÉTODO DE U_{eff}

$$U_{eff} = \frac{1}{2} m \frac{h^2}{\rho^2} \sin^2\alpha + mg \cot\alpha \, \rho \sin^2\alpha$$

$$h = \rho_0 v_0$$

$$\frac{dU_{eff}}{d\rho} = -m\frac{h^2}{\rho^3}\sin^2\alpha + mg\cot\alpha \, \rho \sin^2\alpha$$

$$\frac{dU_{eff}}{d\rho} = 0 \Rightarrow \text{pto eq.}$$

$$-\frac{h^2}{\rho^3} + g\cot\alpha = 0$$

$$\Rightarrow \frac{h^2 \tan\alpha}{g} = \rho_0^3$$

Sabemos que cuando se alcanza ρ_0, la rapidez es v_0:

$$\Rightarrow h = \rho_0 v_0$$

$$\Rightarrow \frac{\rho_0^2 v_0^2 \tan\alpha}{g} = \rho_0^3$$

$$\Rightarrow \boxed{\rho_0 = \frac{v_0^2 \tan\alpha}{g}}$$

$$\frac{d^2U_{eff}}{d\rho^2} = 3m\frac{h^2}{\rho^4}\sin^2\alpha \Rightarrow \frac{d^2U_{eff}}{d\rho^2} = \frac{3m\rho_0^2 v_0^2 \sin^2\alpha}{\rho^4}$$

$$\left.\frac{d^2U_{eff}}{d\rho^2}\right|_{\rho_0} = \frac{3mv_0^2\sin^2\alpha}{\rho_0^2} = \frac{3m v_0^2 g^2 \sin^2\alpha}{v_0^4 \tan^2\alpha}$$

$$\left.\frac{d^2U_{eff}}{d\rho^2}\right|_{\rho_0} = \frac{3mg^2\cos^2\alpha}{v_0^2} > 0 \Rightarrow \text{eq. estable}$$

$$\omega_0 = \sqrt{\frac{1}{m}\left(\left.\frac{d^2U_{eff}}{d\rho^2}\right|_{\rho_0}\right)} = \frac{\sqrt{3}\, g\cos\alpha}{v_0}$$

$$\omega_0 = \frac{2\pi}{T} \Rightarrow \boxed{T_{peq} = \frac{2\pi v_0}{\sqrt{3}\, g\cos\alpha}}$$

QUE ES EL MISMO RESULTADO!!!