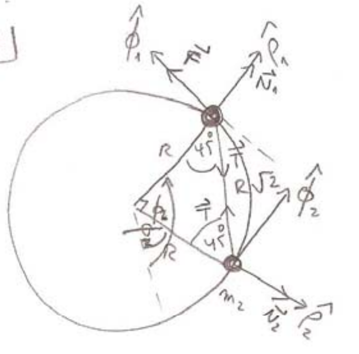
P1



$$\rho_1 = \rho_2 = R \Rightarrow \dot{\rho}_1 = \ddot{\rho}_1 = \dot{\rho}_2 = \ddot{\rho}_2 = 0$$

$$\phi_2 + \frac{\pi}{2} = \phi_1 = \phi$$

$$\Rightarrow \dot{\phi}_1 = \dot{\phi}_2 = \dot{\phi}$$

$$\Rightarrow \ddot{\phi}_1 = \ddot{\phi}_2 = \ddot{\phi}$$

ECS ①

① $\hat{\rho}_1)\ N_1 - T\cos\frac{\pi}{4} = -m_1 R\dot{\phi}^2$

② $\hat{\phi}_1)\ F - T\sin\frac{\pi}{4} = m_1 R\ddot{\phi}$

ECS ②

③ $\hat{\rho}_2)\ N_2 - T\cos\frac{\pi}{4} = -m_2 R\dot{\phi}^2$

④ $\hat{\phi}_2)\ T\sin\frac{\pi}{4} = m_2 R\ddot{\phi}$

②+④ $F = (m_1+m_2)R\ddot{\phi}$

$$\ddot{\phi} = \frac{F}{(m_1+m_2)R} \Rightarrow \boxed{\phi(t) = \frac{1}{2}\frac{F}{(m_1+m_2)R}t^2}$$

De ④ $\frac{T}{\sqrt{2}} = m_2 R\ddot{\phi} \Rightarrow \boxed{T = \frac{\sqrt{2}\,m_2 F}{(m_1+m_2)}}$

ECS ①

① $N_1 = -m_1 R\dot{\phi}_1^2$

② $F = m_1 R\ddot{\phi}_1 \Rightarrow \ddot{\phi}_1 = \frac{F}{m_1 R}$ $\qquad \dot{\phi}_1(t_0) = \frac{F\,t_0}{(m_1+m_2)R}$

$$\phi_1(t) = \frac{F t_0}{(m_1+m_2)R} t + \frac{1}{2} \frac{F}{m_1 R} t^2$$

$$\phi_2(t) = \frac{F t_0}{(m_1+m_2)R} t + \frac{3\pi}{2}$$

$$\phi_1(t^*) = \phi_2(t^*)$$

$$\Rightarrow \boxed{t^* = \sqrt{\frac{3 m_1 R \pi}{F}}}$$

P2)

$$\rho = z\tan\alpha \qquad \dot{z} = k\dot{\phi}$$

$$\dot{\rho} = \dot{z}\tan\alpha \qquad \dot{\phi} = \frac{\dot{z}}{k}$$

$$\vec{v} = \dot{z}\tan\alpha\,\hat{\rho} + \rho\frac{\dot{z}}{k}\hat{\phi} + \dot{z}\hat{k}$$

$$= \dot{z}\tan\alpha\,\hat{\rho} + z\tan\alpha\frac{\dot{z}}{k}\hat{\phi} + \dot{z}\hat{k}$$

$$= \dot{z}\left[\tan\alpha\,\hat{\rho} + \frac{z}{k}\tan\alpha\,\hat{\phi} + \hat{k}\right]$$

$$|\vec{v}| = v_0 = \dot{z}\sqrt{\tan^2\alpha + \left(\frac{z}{k}\tan\alpha\right)^2 + 1}$$

$$\dot{z} = \frac{v_0}{\sqrt{\tan^2\alpha\left(1+\frac{z^2}{k^2}\right)+1}}$$

$$\ddot{z} = -\frac{1}{2}\frac{\frac{v_0}{k^2}\,z\dot{z}z\tan^2\alpha}{\left[\tan^2\alpha\left(1+\frac{z^2}{k^2}\right)+1\right]^{3/2}}$$

$$\ddot{z} = -\frac{v_0^2}{k^2}\frac{z\tan^2\alpha}{\left[\tan^2\alpha\left(1+\frac{z^2}{k^2}\right)+1\right]^2}$$

P3

$$\theta = \alpha \Rightarrow \dot{\theta} = \ddot{\theta} = 0$$
$$\dot{\phi} = \omega_0 \Rightarrow \ddot{\phi} = 0$$

$\hat{r}$) $-mg\cos\alpha = m(\ddot{r} - r\omega_0^2 \sin^2\alpha)$

$\hat{\theta}$) $-N_\theta + mg\sin\alpha = -mr\omega_0^2 \sin\alpha\cos\alpha$

$\hat{\phi}$) $N_\phi = 2m\dot{r}\omega_0 \sin\alpha$

$$\ddot{r} = r\omega_0^2 \sin^2\alpha - g\cos\alpha$$

$$\int_0^{\dot{r}} \dot{r}\,d\dot{r} = \int_L^r (r\omega_0^2 \sin^2\alpha - g\cos\alpha)\,dr$$

$$\frac{\dot{r}^2}{2} = \frac{(r^2-L^2)}{2}\omega_0^2 \sin^2\alpha - g\cos\alpha\,(r-L)$$

$$\dot{r} = \left[(r-L)\left((r+L)\omega_0^2\sin^2\alpha - 2g\cos\alpha\right)\right]^{1/2}$$

$$N_\theta = mg\sin\alpha + mr\omega_0^2 \sin\alpha\cos\alpha$$

$$N_\phi = 2m\,\dot{r}(r)\,\omega_0 \sin\alpha$$

$$N_\phi = 2m\sqrt{(r-L)\left[(r+L)\omega_0^2\sin^2\alpha - 2g\cos\alpha\right]}\,\omega_0\sin\alpha$$

$$N = \sqrt{N_\theta^2 + N_\phi^2}$$

$$\Omega_0^2 = \omega_0^2 \sin^2\alpha$$

$$\ddot{r} - \Omega_0^2 r = -g\cos\alpha$$

$$r_h(t) = K_1 e^{(\omega_0 \sin\alpha)t} + K_2 e^{-(\omega_0 \sin\alpha)t}$$

$$r_p(t) = K$$

$$-\Omega_0^2 K = -g\cos\alpha$$

$$K = \frac{g\cos\alpha}{\omega_0^2 \sin^2\alpha}$$

$$r(t) = K_1 e^{(\omega_0 \sin\alpha)t} + K_2 e^{-(\omega_0 \sin\alpha)t} + \frac{g\cos\alpha}{\omega_0^2 \sin^2\alpha}$$

$$r(0) = r_0 = K_1 + K_2 + \frac{g\cos\alpha}{\omega_0^2 \sin^2\alpha}$$

$$\dot{r}(0) = 0 = \omega_0 \sin\alpha\,(K_1 - K_2) \Rightarrow K_1 = K_2$$

$$2K_1 = r_0 - \frac{g\cos\alpha}{\omega_0^2 \sin^2\alpha}$$

$$K_1 = \frac{r_0}{2} - \frac{g\cos\alpha}{2\omega_0^2 \sin^2\alpha}$$

$$r(t) = \left(\frac{r_0}{2} - \frac{g\cos\alpha}{2\omega_0^2 \sin^2\alpha}\right)\left[e^{(\omega_0 \sin\alpha)t} + e^{-(\omega_0 \sin\alpha)t}\right] + \frac{g\cos\alpha}{\omega_0^2 \sin^2\alpha}$$