

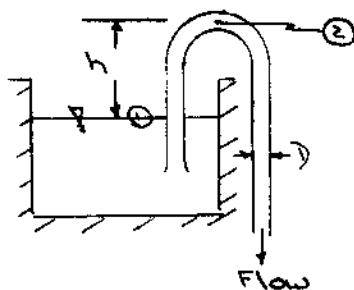
### Problem 6.40

6-3.4 [2]

Given: Water flow through siphon as shown

$$Q = 0.03 \text{ m}^3/\text{sec}, T = 20^\circ\text{C}, D = 75 \text{ mm}$$

Find: Maximum allowable height,  $h$ , such that  $P_2$  is above the vapor pressure of the water



Solution: Apply the Bernoulli equation along the streamline between locations 1 and 2 to determine  $h$  after employing the definition of volume flowrate to determine the flow speed in the tube

Basic equations:  $Q = \int v \cdot dA$  ( $Q$  is volume flow rate)

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + gZ_2$$

- Assumptions:
- (1) steady flow
  - (2) incompressible flow
  - (3) frictionless flow
  - (4) flow along a streamline
  - (5)  $Z_1 = 0$
  - (6)  $V_1 = 0$
  - (7) uniform flow in the tube

From the definition of  $Q$  and assumption 7,  $Q = V_2 A_2$ , and

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \cdot 0.03 \frac{\text{m}^3}{\text{s}} \cdot \frac{1}{(75 \cdot 10^{-3})^2} \text{ m/s} = 6.79 \text{ m/s}$$

From the Bernoulli equation,

$$h = Z_2 = \frac{1}{g} \left[ \frac{P_1 - P_2}{\rho} - \frac{V_2^2}{2g} \right]$$

For water at  $20^\circ\text{C}$ ,  $P_{\text{vapor}} = P_2 = 2.33 \text{ kPa}$  then

$$h = \frac{1}{g} \left[ \frac{P_1 - P_2 - V_2^2}{\rho} \right] = \frac{1}{9.81 \text{ m}} \left[ \frac{(101 - 2.33) \times 10^3 \text{ N/m}^2}{9.81 \text{ kg/m}^3 \cdot 999 \text{ kg/m}^3} - \frac{1}{2} \cdot \frac{(6.79)^2}{9.81 \text{ m}} \right]$$

$$h = 7.72 \text{ m}$$